Revival of MOND or the Gravity Law without Universalism

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Abstract

In this note I argue that modified gravity can describe Dark Matter if one understands the modification of gravity as a tensor field $X^{\mu\nu} = X^{\mu\nu}(t, x, y, z)$ in the Einstein equations, i.e. as an additional mathematical parameter filling the Universe without correspondence to new particles. Notably, there are many different fields in nature, e.g. the Higgs field, the inflaton field, and the temperature distribution field $T(t, x, y, z)$.

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I. PREFACE

What is the nature of dark matter? Is it a particle, or do the phenomena attributed to dark matter actually require a modification of the laws of gravity? In this first publication in a series of papers I deal with this question without applying mathematical tools. Nevertheless, all my points are backed up by evidence. The next two publications entitled "Broken Geodesics and Dark Matter" and "Energy Localization Problem points out the vanishing of matter in the First Order Deviation Equation" are highly mathematical applications of the theory described in this short note. My approach goes beyond standard ΛCDM cosmology, trying to find a solution for problems indicated in Ref. [1]. However, ΛCDM is contained as a special case in Eq. (1).

Modified Newtonian dynamics (MOND) is a hypothesis that proposes a modification of Newton's laws to account for observed properties of galaxies. It is an alternative to the hypothesis of dark matter in terms of explaining why galaxies do not appear to obey the currently understood laws of physics. Created in 1982 and first published in 1983 by the Israel physicist Mordehai Milgrom [2], the hypothesis' original motivation was to explain why the velocities of stars in galaxies were observed to be larger than those expected by using Newtonian mechanics.

MOND is an example of a class of theories known as modified gravity, and it is an alternative to the hypothesis that the dynamics of galaxies are determined by massive, invisible dark matter halos. Since Milgrom's original proposal, MOND has successfully predicted a variety of galactic phenomena that are difficult to understand from a dark matter perspective [3]. However, MOND and its generalisations do not adequately account for observed properties of galaxy clusters, and no satisfactory cosmological model has been constructed from the hypothesis.

The accurate measurement of the speed of gravitational waves compared to the speed of light in 2017 ruled out many theories which used modified gravity to avoid dark matter [4]. However, according to the same study neither Milgrom's bi-metric formulation of MOND nor nonlocal MOND are ruled out.

II. COMMON FEATURE OF MOND PROPOSALS

Newton's law of universal gravitation usually states that every particle attracts every other particle in the universe with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centers. This is a general physical law derived from empirical observations by what Isaac Newton called inductive reasoning [5].

The common feature of all MOND proposals is this universalism. Given the energy-momentum tensor for "visible" (e.g., baryonic) matter, one perfectly determines Dark Matter. However, that seems to be not true because galaxies without Dark Matter are discovered [6]. In contrast to this, I introduce a non-universal law of gravitation in Eq. (2). According to this, there are places and times in the universe where the gravitational force cannot be calculated just from the properties of visible matter. To fix the problems of MOND, I suggest to include a tensor field of Dark Matter.

III. HOW TO MODIFY GRAVITY

A general expression for modified gravity can be written as

$$\mathrm{G}^{*\mu\nu} = 8\pi\, T^{\mu\nu}\,, \tag{1}$$

where the left hand side is the modified Einstein tensor. $T^{\mu\nu}$ is the energy–momentum tensor of visible matter. Without loss of generality one can rewrite Eq. (1) using the definition $8\pi\, X^{\mu\nu} = \mathrm{G}^{\mu\nu} - \mathrm{G}^{*\mu\nu}$,

$$\mathrm{G}^{\mu\nu} = 8\pi\left(T^{\mu\nu} + X^{\mu\nu}\right), \tag{2}$$

where the unmodified Einstein tensor is on the left hand side. In the following I call $X^{\mu\nu}$ a virtual term, in particular Virtual Matter. This term cannot be detected in particle detectors, as it is not visible matter but rather a pure mathematical modification of Einstein's equations. In case the covariant divergence $X^{\mu\nu}_{;\nu}$ vanishes, we will call it Dark Matter. In this sense, Dark Energy is a class of Dark Matter because $(\Lambda\, g^{\mu\nu})_{;\nu} = 0$.

My proposal is to allow the 10 independent functions $X^{\mu\nu} = X^{\mu\nu}(t, x, y, z)$ not to be universal, i.e. being not always the most popular expression of Dark Matter (which is dust-like tensor $X^{\mu}_{\nu} = \mathrm{diag}(-\rho, 0, 0, 0)$), but different in any given task and problem. What determines the shape of $X^{\mu\nu}$? Is it theoretical physics or the experiment or observation? My answer is, that it is both, as e.g. in Section IV the introduction of $X^{\mu\nu}$ turns out to be a solution to particular theoretical problems. Therefore, one can not blame my proposal for having no predictive power – despite the fact that the absolute generality of Eq. (2) fits any possible experiment or observation.

IV. EVIDENCES OF THE NECESSITY OF $X^{\mu\nu}$ FOR FIXING PROBLEMS

A. Fixing singularities

Using known facts from General Relativity, it is indeed possible and easy to solve the mystery. Any singularity is simply a mathematical blow up of the theory of Relativity. To fix this and to make the theory physical rather than mathematical, I am using a virtual term $\psi(r)$ in the Schwarzschild Black Hole metrics,

$$ds^2 = -\left(1 - \frac{2M}{r+\psi(r)}\right) dt^2 + \frac{dr^2}{1 - \frac{2M}{r+\psi(r)}} + r^2\, d\Omega^2\,, \tag{3}$$

where $\psi(r > 2M) = 0$, $\psi(r \leq 2M) = \epsilon\,(2M - r)$ for $0 \leq r < \infty$ and small $\epsilon > 0$.

The tensor $X^{\mu\nu}$ can be calculated from Eqs. (2) and (3) for $T^{\mu\nu} = 0$. The demand to fulfil the "energy conditions" (weak, strong, and others) is not applicable to the virtual matter $X^{\mu\nu}$, as it is not subject to measurements. So one would not measure a negative energy. Notably, the known concepts of "phantom fields" [7] and "exotic matter" [8] have problems with energy conditions, but seeing as the examples of Virtual Matter, they have no such problems. Any possible instability of my MOND proposal is simply removed by properly chosen variations of the arbitrary virtual term $X^{\mu\nu}$.

B. Fixing abrupt geodesics

If one releases a particle in Kerr, Kerr–Newman, or Reissner–Nordström spacetime with zero initial velocity $u^r = u^\theta = u^\phi = 0$ (in case of photon $u^\theta = u^\phi = 0$, $u^r < 0$), it will reach an abrupt end of the trajectory at the radius $r = r_m > 0$, because there is $(u^r)^2 < 0$ for $r < r_m$. The curvature singularity is at $r = 0$. The details are found in Ref. [9] which uses velocity expressions from Ref. [10]. Note that in case of a motion inside the equatorial plane

$\theta = \pi/2$ the abrupt end geodesics are present for Kerr-Newman and Reissner-Nordström spacetimes. The abrupt geodesics mean the vanishing of matter.

Notably, the Einstein's vision of a steady-state universe theory contains the "formation of matter from empty space", so that the density of matter in the expanding universe is kept constant. But it turned out that Einstein's equations are violated by such an assumption [11]. Indeed, the appearance of matter in "forward time" is equivalent to the vanishing of matter in "backwards time". If so, one needs to use a properly chosen virtual term $X^{\mu\nu}$ to revisit the Einstein's proposal.

C. Fixing the test-particle formalism

The known geodesic equation $u^{\mu}_{;\nu}\, u^{\nu} = 0$ of a test-particle motion, silently assumes, that the background spacetime is fixed, that there is no backreaction from high-speed (e.g. near Black Holes) geodesic motion on the spacetime. The absence of backreaction is only possible if there is $X^{\mu\nu} \neq 0$ in the Eq.(2).

D. Fixing the static universe

I am talking not about the real Universe (the one we are living in), but about an imaginable one, which is constructed from the known laws of nature.

It is known that the pressure in the perfect fluid model allows us to have a static drop of fluid in empty spacetime. Is expected that pressure as the resistance of matter counterparts gravity, and so a static universe filled with a perfect fluid should be allowed. Hereby, in case of a flat Friedmann universe metric the $X^{\mu\nu} = -T^{\mu\nu} \neq 0$ can be necessary.

It is interesting that while trying to construct the steady-state universe, Albert Einstein found an example for a non-zero $8\pi\, X^{\mu\nu} = -\Lambda\, g^{\mu\nu} \neq 0$, naming it later the "biggest blunder" of his life without even realizing the entire potential and usefulness of this discovery (e.g. the possibility of interstellar travel) [12].

V. CONCLUSIONS

One should include such a concept as virtual terms, i.e. mathematical insertions into the equations and laws of nature which are made not from fundamental premises but "by hand" in order to fit the theory under observation. An example for such insertions are Dark Matter and Dark Energy. Therefore, these cannot be directly detected, but it is possible to measure their effect on nature. As a prime example, the Dark Matter anomaly has acted on the space-time grid in such an amount that it created an additional force of attraction of stars to the center of their galaxy. By the way, the proton radius measured by many experimenters was different in different years. This riddle did not find yet a solution [13]. I, personally, would solve this problem with a virtual insertion Ψ into the radius value, $r = R + \Psi$.

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