Derivation of the correct dynamics of the Special Theory of Relativity and explanation of the right angle lever paradox

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Abstract:

This paper presents the derivation of correct dynamics of the Special Theory of Relativity for all spatial dimensions. The derived dynamics has no flaws, which have other dynamics, also Einstein’s dynamics, i.e. the lack of parallelism of the force vector with the acceleration vector, the right angle lever paradox is unsolvable and the incompatibility of energy conservation.

Key words: dynamics of bodies, equation of motion, momentum, kinetic energy, right angle lever paradox, Special Theory of Relativity

1. Introduction

The paper [6] presents an original method that enables to derive numerous dynamics for STR kinematics for one spatial dimension. Five examples of specific dynamics have been presented. This paper is a continuation of that research and presents a method of extending any dynamics for one spatial dimension to three spatial dimensions. For each of the five examples of one-dimensional STR dynamics derived in the paper [6], three-dimensional dynamics were derived (transformation of perpendicular force and equations of motion for perpendicular force).

2. Selected properties of STR kinematics

The determinations shown in Figure 1 have been adopted.

![Figure 1. Relative motion of inertial systems $U_1$ and $U_2$ ($v_{2/1} = -v_{1/2}$) as well as body acceleration $m_0$ seen from these systems.](image-url)
The inertial system $U_2$ moves in relation to the inertial system $U_1$ at the velocity of $v_{2/1}$. The inertial system $U_1$ moves in relation to the inertial system $U_2$ at the velocity of $v_{1/2}$. In STR, it is $v_{2/1} = -v_{1/2}$. The body with rest mass $m_0$ rests temporarily in $U_2$ system. This body performs an acceleration. In $U_2$ system, in which the body was resting temporarily, the acceleration has a value of $a_{2/2}$. The acceleration in relation to $U_1$ system, has a value of $a_{2/1}$. Index $i/j$ will mean that it is a body resting in $i$ system and is observed from $j$ system.

In the STR kinematics the following equations are derived from the Lorentz transformation [3]:

- transformation of dimensions parallel to velocity $v_{2/1}$ (Lorentz–FitzGerald contraction)
  \[ L_{2/2}^{\parallel} = \gamma L_{2/1}^{\parallel} \]  

- transformation of dimensions perpendicular to velocity $v_{2/1}$
  \[ L_{2/2}^{\perp} = L_{2/1}^{\perp} \]  

- transformation of acceleration parallel to velocity $v_{2/1}$
  \[ a_{2/2}^{\parallel} = \gamma^2 a_{2/1}^{\parallel} \]  

- transformation of acceleration perpendicular to velocity $v_{2/1}$
  \[ a_{2/2}^{\perp} = \gamma^2 a_{2/1}^{\perp} \]  

3. One-dimensional dynamics for STR

The STR dynamics for one spatial dimension were derived in the paper [6]. In accordance with the designations adopted there, the following equations apply to STR dynamics:

- definition of momentum
  \[ dp_{2/1} := F_{2/1} dt = m_{2/1} a_{2/1} dt = m_{2/1} \frac{dv_{2/1}}{dt} dt = m_{2/1} dv_{2/1} \]  

- definition of kinetic energy
  \[ dE_{2/1} := F_{2/1} dx = m_{2/1} a_{2/1} dx_{2/1} = m_{2/1} \frac{dv_{2/1}}{dt} dx_{2/1} = m_{2/1} \frac{dx_{2/1}}{dt} dv_{2/1} = m_{2/1} v_{2/1} dv_{2/1} \]  

- equation of motion in the own body system $U_2$ (Newton’s second law of motion)
  \[ F_{2/2}^{\parallel} = m_0 a_{2/2}^{\parallel} \quad \wedge \quad F_{2/2}^{\perp} = m_0 a_{2/2}^{\perp} \]  

- equation of motion of the body resting temporarily in $U_2$ system for the observer from $U_1$ system (these equations represent the generalized Newton’s second law of motion)
  \[ F_{2/1}^{\parallel} = m_0 f^{\parallel}(v_{2/1}) a_{2/1}^{\parallel} \quad \wedge \quad F_{2/1}^{\perp} = m_0 f^{\perp}(v_{2/1}) a_{2/1}^{\perp} \]  

The symbol $||$ indicates the force component parallel to velocity $v_{2/1}$, while the symbol $\perp$ indicates the component perpendicular to velocity $v_{2/1}$, where $v_{2/1}$ is the velocity of body in relation to the observer. In Newton’s dynamics $f^{\parallel}(v_{2/1}) = f^{\perp}(v_{2/1}) = 1$, while in Einstein’s STR dynamics $f^{\parallel}(v_{2/1}) = \gamma^3, f^{\perp}(v_{2/1}) = \gamma$.

The dynamics derived in the paper [6] were parameterized by the parameter $x \in R$. In the five dynamics derived in that paper the following equations for momentum and kinetic energy apply:
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- Dynamics \( \{ x \} = \{ 0 \} \), in which for each observer \( F^\parallel/a_{2/1} = m_0 f^\parallel(v_{2/1}) = m_0 = \text{constans} \):

\[
P^{(0)}_{2/1} = p^{m}_{2/1} = m_0 v_{2/1}
\]

\[
E^{(0)}_{2/1} = E^{m}_{2/1} = \frac{m_0 v^2_{2/1}}{2}
\]

- Dynamics \( \{ x \} = \{ 1/2 \} \), in which for each observer \( F^\parallel/dv_{2/1} = m_0 f^\parallel(v_{2/1})/\Delta t = \text{constans} \):

\[
p^{(1/2)}_{2/1} = p^{m/\Delta t}_{2/1} = m_0 c \cdot \arcsin \left( \frac{v_{2/1}}{c} \right)
\]

\[
E^{(1/2)}_{2/1} = E^{m/\Delta t}_{2/1} = m_0 c^2 \left( 1 - \sqrt{1 - \left( \frac{v_{2/1}}{c} \right)^2} \right) = \frac{m_0 v^2_{2/1}}{2} \frac{2}{1 + \sqrt{1 - \left( \frac{v_{2/1}}{c} \right)^2}}
\]

- Dynamics \( \{ x \} = \{ 1 \} \), in which for each observer \( \Delta p = \text{constans} \):

\[
p^{(1)}_{2/1} = p^{sp}_{2/1} = \frac{m_0 c}{2} \ln \left( \frac{c + v_{2/1}}{c - v_{2/1}} \right) = m_0 v_{2/1} \ln \left( \frac{c + v_{2/1}}{c - v_{2/1}} \right)
\]

\[
E^{(1)}_{2/1} = E^{sp}_{2/1} = \frac{m_0 c^2}{2} \ln \left( \frac{1}{1 - \left( \frac{v_{2/1}}{c} \right)^2} \right) = \frac{m_0 v^2_{2/1}}{2} \ln \left( \frac{1}{1 - \left( \frac{v_{2/1}}{c} \right)^2} \right)^{\left( c/\gamma(2/1) \right)^2}
\]

- Dynamics \( \{ x \} = \{ 3/2 \} \), Einstein’s dynamics, in which for each observer \( F^\parallel = \text{constans} \):

\[
p^{(3/2)}_{2/1} = p^{p}_{2/1} = m_0 v_{2/1} \frac{1}{\sqrt{1 - \left( \frac{v_{2/1}}{c} \right)^2}}
\]

\[
E^{(3/2)}_{2/1} = E^{p}_{2/1} = m_0 c^2 \frac{1}{\sqrt{1 - \left( \frac{v_{2/1}}{c} \right)^2}} - m_0 c^2 = \frac{m_0 v^2_{2/1}}{2} \frac{2}{\sqrt{1 - \left( \frac{v_{2/1}}{c} \right)^2}} \left( 1 + \sqrt{1 - \left( \frac{v_{2/1}}{c} \right)^2} \right)
\]

- Dynamics \( \{ x \} = \{ 2 \} \), in which for each observer \( F^\parallel/\Delta t = \text{constans} \):

\[
p^{(2)}_{2/1} = p^{F/\Delta t}_{2/1} = m_0 v_{2/1} \left[ \frac{1}{2} - \ln \left( \frac{c + v_{2/1}}{c - v_{2/1}} \right) \right]
\]

\[
E^{(2)}_{2/1} = E^{F/\Delta t}_{2/1} = \frac{m_0 c^2}{2} \frac{1}{1 - \left( \frac{v_{2/1}}{c} \right)^2} - \frac{m_0 c^2}{2} = \frac{m_0 v^2_{2/1}}{2} \frac{1}{2 - \left( \frac{v_{2/1}}{c} \right)^2}
\]

The force transformation for the component parallel to velocity \( v_{2/1} \) in dynamics \( \{ x \} \) has a form of [6]:

\[
F^{(x)}_{2/1} = \gamma^{2x-3} F^{(1)}_{2/1}
\]

The equation of motion (8) for the component parallel to velocity \( v_{2/1} \) in dynamics \( \{ x \} \) has a form of [6]:

\[
F^{(x)}_{2/1} = m_0 \gamma^{2x} a^{(1)}_{2/1}
\]
4. Derivation of three-dimensional dynamics for STR

For each dynamic, the momentum equation, e.g. (9), (11), (13), (15) or (17), has a form of

\[
p_{2/1}^{(x)} = m_0 v_{2/1} g^{(x)}(v_{2/1})
\]

where \( g^{(x)}(v_{2/1}) \) is a dimensionless function.

If the velocity vector of body changes, then the momentum vector of this body changes. This is shown in Figure 2.

![Fig. 2. Change of body momentum resting temporarily in \( U_2 \) system seen by an observer from \( U_1 \) system.]

Based on the definition of momentum resting temporarily in \( U_2 \) system seen by an observer from \( U_1 \) system.

\[
\begin{align*}
F^{(x)}_{2/1} &= \frac{dp_{2/1}^{(x)}}{dt} = \frac{d(m_0 v_{2/1} g^{(x)}(v_{2/1}))}{dt} = m_0 \left[ \frac{dv_{2/1}^{(x)} g^{(x)}(v_{2/1})}{dt} + v_{2/1} \frac{d(g^{(x)}(v_{2/1}))}{dv_{2/1}} \right] \\
F^{(x)}_{2/1} &= m_0 \left[ (a_{2/1}^\parallel + a_{2/1}^\perp) g^{(x)}(v_{2/1}) + v_{2/1} \frac{d(g^{(x)}(v_{2/1}))}{dv_{2/1}} \right] \\
F^{(x)}_{2/1} &= m_0 g^{(x)}(v_{2/1}) a_{2/1}^\parallel + m_0 \left[ g^{(x)}(v_{2/1}) a_{2/1}^\parallel + \frac{d(g^{(x)}(v_{2/1}))}{dv_{2/1}} \right] \frac{dv_{2/1}}{dt} v_{2/1} 
\end{align*}
\]

Because

\[
v_{2/1} \parallel a_{2/1}^\parallel
\]

thus, it follows from (24) that

\[
F^{(x)}_{2/1} = m_0 g^{(x)}(v_{2/1}) a_{2/1}^\parallel
\]

The equation (26) can be derived directly from (22), if it is observed that a force acting on the body perpendicular to its velocity \( v_{2/1} \) does not change velocity value \( v_{2/1} \), but only its direction. In this case \( g^{(x)}(v_{2/1}) = \text{constans} \). On this basis, the following is immediately obtained (26).

The equation (26) is an equation of motion for the force perpendicular to the velocity of body \( v_{2/1} \). For the five dynamics derived in the paper [6], the explicit forms of equations of motion are as follows:

- Dynamics \( \{x\} = \{0\} \)

\[
F^{(x)}_{2/1} = F^{(0)}_{2/1} = m_0 a_{2/1}^\parallel
\]

- Dynamics \( \{x\} = \{1/2\} \)

\[
F^{(x)}_{2/1} = F^{(1/2)}_{2/1} = m_0 \frac{\arcsin(v_{2/1}/c)}{v_{2/1}/c} a_{2/1}^\parallel
\]

- Dynamics \( \{x\} = \{1\} \)
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\[ F_{2/1}^{\perp} = F_{2/1}^{\perp F} = m_0 \ln \left( \frac{c + v_{2/1}}{c - v_{2/1}} \right)^{2v_{2/1}} a_{2/1}^{\perp} \]  

(29)

- Dynamics \( \{x\} = \{3/2\} \), Einstein’s dynamics

\[ F_{2/1}^{\perp(3/2)} = F_{2/1}^{\perp F} = m_0 \frac{1}{\sqrt[4]{\left( \frac{c + v_{2/1}}{c - v_{2/1}} \right)^2 - 1}} a_{2/1}^{\perp} \]  

(30)

- Dynamics \( \{x\} = \{2\} \)

\[ F_{2/1}^{\perp(2)} = F_{2/1}^{\perp F/m} = m_0 \frac{1}{2} \left[ \frac{1}{1 - \left( \frac{c + v_{2/1}}{c - v_{2/1}} \right)^2} + \ln \left( \frac{c + v_{2/1}}{c - v_{2/1}} \right)^{2v_{2/1}} \right] a_{2/1}^{\perp} \]  

(31)

Now the transformation of force perpendicular to velocity \( v_{2/1} \) will be determined. The equations of motion (7) are replaced by (3) and (4). Then the following is obtained

\[ F_{2/2}^{\parallel} = m_0 \gamma^2 a_{2/1}^{\parallel} \quad \land \quad F_{2/2}^{\perp} = m_0 \gamma^2 a_{2/1}^{\perp} \]  

(32)

If to divide the motion equation (20) by the first equation (32), then for dynamics \( \{x\} \) the already known transformation of force for the parallel component (19) is obtained. If the equation of motion (26) is divided by the first equation (32), then for dynamics \( \{x\} \) the force transformation for the perpendicular component is obtained in a form of

\[ F_{2/1}^{\perp(x)} = \frac{g^{(x)}(v_{2/1})}{\gamma^2} F_{2/2}^{\perp} \]  

(33)

The calculations presented in this chapter show that the dynamics for three spatial dimensions result unambiguously from the dynamics for one spatial dimension.

5. STR dynamics paradoxes

Papers [1], [2], [4] and [5] present the right angle lever paradox, Figure 3. The lever is fixed to the ground at R point by means of a rotary support.

![Diagram](image)

Fig. 3. Right angle lever paradox.

a) lever seen from the own system \( U_2 \), b) lever seen from the moving inertial system \( U_1 \).

For the observer from \( U_2 \) system (resting system), both lever arms have the same length, i.e.

\[ L_{2/2}^{\perp} = L_{2/2}^{\perp} \]  

(34)
The lever is subject to two forces applied to the ends of arms in directions perpendicular to these arms and two reaction forces applied at the support point R. In $U_2$ system, the lever is in equilibrium, i.e. the sum of torques is 0. That is

$$L_{2/2}^\perp F_{2/2}^\perp = L_{2/2}^\parallel F_{2/2}^\parallel \implies F_{2/2}^\parallel = F_{2/2}^\perp$$ (35)

For an observer from the inertial system $U_1$, the lever moves at velocity of $v_{2/1}$ parallel to one arm. According to the transformation of dimensions (1), (2) and force transformations (19), (33), for the observer from $U_2$ system, two moments of force act on the lever:

$$L_{2/1}^\parallel F_{2/1}^\parallel(x) = \frac{g^\parallel(x)(v_{2/1})}{\gamma} L_{2/2}^\parallel F_{2/2}^\parallel$$  
$$L_{2/1}^\perp F_{2/1}^\perp(x) = \gamma^{2x-3} L_{2/2}^\perp F_{2/2}^\perp$$ (37)

The moments of force (36) and (37) are equal only in such dynamics $\{x\}$, in which due to (34) and (35) there is an equality of

$$\frac{g^\parallel(x)(v_{2/1})}{\gamma^3} = \gamma^{2x-3} \iff g^\parallel(x)(v_{2/1}) = \gamma^{2x}$$ (38)

From the paper [6] (equation (126)) and from (21) it follows that

$$g^\parallel(x)(v_{2/1}) = \frac{1}{v_{2/1}} \int_0^{v_{2/1}} \gamma^{2x} dv_{2/1}$$ (39)

On this basis, condition (38) takes the form of

$$\frac{1}{v_{2/1}} \int_0^{v_{2/1}} \gamma^{2x} dv_{2/1} = \gamma^{2x}$$ (40)

$$\int_0^{v_{2/1}} \gamma^{2x} dv_{2/1} = v_{2/1} \gamma^{2x}$$ (41)

After differentiating the sides by the velocity $v_{2/1}$ the following is obtained

$$\gamma^{2x} = \frac{dv_{2/1}}{dv_{2/1}} \gamma^{2x} + v_{2/1} \frac{d\gamma^{2x}}{dv_{2/1}}$$ (42)

$$0 = v_{2/1} \frac{d\gamma^{2x}}{dv_{2/1}}$$ (43)

Equality must be true for every velocity $v_{2/1}$. This is only possible if

$$\gamma^{2x} = \text{const}(v_{2/1}) \implies x = 0$$ (44)

It follows that only for one dynamics $\{x\} = \{0\}$ for the observer from the moving inertial system $U_1$, the moments of force applied to the lever are balanced. For all other dynamics, including Einstein’s, moments of force in the system of moving observer are not balanced. Therefore, according to the moving observer, the lever should rotate. The right angle lever paradox is that if the lever does not rotate in the resting system, it does not rotate for an observer from any other inertial reference system. That is, either in dynamics $\{x\} \neq \{0\}$ should be understood differently in terms of force moment or dynamics $\{x\} \neq \{0\}$ are contradictory.
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It can be shown that for the STR dynamics under consideration the following applies (numerically verified)

\[
\left( \forall \{x\} > \{0\} \quad g^{(x)}(v_{2/1}) \leq \gamma^{2x} \right) \quad \Rightarrow \quad \left( \forall \{x\} > \{0\} \quad L^{\mathcal{L}}_{2/1} F^{\mathcal{L}\{x\}}_{2/1} \leq L^{\mathcal{L}}_{2/1} F^{\mathcal{L}\{x\}}_{2/1} \right) \quad (45)
\]

The right angle lever paradox shows that the dynamics \{x\} \neq \{0\} do not meet the energy conservation principle. This problem was probably not noticed in the previous literature. It can be shown by the thought experiment shown in Figure 4. In this experiment the lever has the form of a circle which moves in relation to the observer with the velocity of \(v_{2/1}\). From (45) it appears that if to act on the circle with a force perpendicular to the velocity of \(v_{2/1}\) (doing some work), it will be possible to receive a larger work with the force that the circle acts parallel to the velocity \(v_{2/1}\). These forces act on a path equal to the circumference of a circle with the appropriate radius.

Fig. 4. A thought experiment showing that dynamics \{x\} \neq \{0\} do not meet the principle of energy conservation.

If the work performed has a value of

\[
W_p = s_p F^{\mathcal{L}\{x\}}_{2/1} = \frac{2\pi}{T/dt_1} L^{\mathcal{L}}_{2/1} F^{\mathcal{L}\{x\}}_{2/1} \quad (46)
\]

then the work that can be receive has according to (45) the following values

\[
W_r = s_r F^{\mathcal{L}\{x\}}_{2/1} = \frac{2\pi}{T/dt_1} L^{\mathcal{L}}_{2/1} F^{\mathcal{L}\{x\}}_{2/1} > W_i \quad (47)
\]

In addition, a force perpendicular to velocity \(v_{2/1}\) will change the direction of this velocity, i.e. the circle shown in Figure 4 will move around the circle. In turn the reaction force, parallel to velocity \(v_{2/1}\), will increase the value of this velocity, i.e. in spite of energy surplus, the velocity \(v_{2/1}\) will increase.

Fig. 5. In dynamics \{x\} \neq \{0\} the acceleration vector may have a different direction than force. This illustration refers to the dynamics \{x\} > \{0\}. 
The right angle lever paradox results from the fact that in dynamics \( \{ x \} \neq \{ 0 \} \) the vector of acceleration may not be parallel to the force vector causing the acceleration. Then the body accelerates in a slightly different direction than the direction of force. This is shown in Figure 5. In the inertial system \( U_2 \), in which the body temporarily rests, the force \( F_{2/2} \) and acceleration \( a_{2/2} \) are parallel to each other. This must be the case in own body system, because the STR should meet the correspondence principle in relation to Newton’s mechanics. But for the observer from inertial system \( U_1 \) the force \( F_{2/1} \) and acceleration \( a_{2/1} \) are not parallel to each other.

This can be shown in the following way. As in our own body system the force \( F_{2/2} \) and the acceleration \( a_{2/2} \) are parallel to each other, thus the following occurs

\[
\frac{F_{2/2}}{F_{2/2}^r} = \frac{a_{2/2}}{a_{2/2}^r}
\]

(48)

From the transformation of forces (19), (33) and the transformation of accelerations (3)-(4) the following is obtained (assuming that the vector \( F_{2/2} \) is not perpendicular to velocity \( v_{2/1} \))

\[
\tan \alpha_F = \frac{F_{2/1}}{F_{2/2}^r} = \frac{g^{[x]}(v_{2/1})^F}{\gamma^{2x-1}} \quad \wedge \quad \tan \alpha_a = \frac{a_{2/1}}{a_{2/2}^r} = \frac{a_{2/2}^r}{a_{2/2}^r} = \frac{\gamma^{2x}a_{2/2}}{a_{2/2}^r} = \gamma^{2x}a_{2/2}
\]

(49)

The angles of force slope and acceleration will be the same in the moving inertial system \( U_1 \), only in dynamics \( \{ x \} \), which meet the condition

\[
\frac{g^{[x]}(v_{2/1})}{\gamma^{2x-1}} = \gamma \Rightarrow g^{[x]}(v_{2/1}) = \gamma^{2x}
\]

(50)

Calculations (38)-(44) show that such a dynamic is only \( \{ x \} = \{ 0 \} \).

It is important to note that the force vectors \( F_{2/1} \) and \( F_{2/2} \) represent the same force, but measured from different reference systems. The acceleration vectors \( a_{2/1} \) and \( a_{2/2} \) represent the same acceleration, but measured from different reference systems. For the observer from \( U_2 \) system, the acceleration vectors and forces are parallel. Nevertheless, in dynamics \( \{ x \} \neq \{ 0 \} \) for a moving observer, these vectors are not parallel. For the moving observer, one line (direction in space) is divided into two different lines (two directions in space). This seems to be impossible. Moreover, the Lorentz transformation maintains parallelism. If two vectors have the same direction in space, then this is the case for every observer. If in dynamics \( \{ x \} \neq \{ 0 \} \) is different, then it is contrary to the STR kinematics and seems impossible to explain.

It can also be shown that for dynamics \( \{ x \} > \{ 0 \} \) (numerically verified)

\[
v_{2/1} \rightarrow c \Rightarrow (\tan \alpha_F \rightarrow 0 \wedge \tan \alpha_a \rightarrow \pm \infty)
\]

(51)

From (51) it appears that if the accelerated body moves at a high velocity \( v_{2/1} \), then in order to change the direction of its movement, for example, in such a way that the body moves along a trajectory close to a circle, it is necessary to act on it with a force almost parallel to velocity \( v_{2/1} \). But such a force will change the value of velocity \( v_{2/1} \), not its direction.

6. Conclusions

The paper shows that from each STR dynamic for one spatial dimension, there is a clear dynamic for three spatial dimensions. The equations for the perpendicular force transformation and the equation of motion for the perpendicular force to body speed have been derived.

It was shown that only in one STR dynamics \( \{ x \} = \{ 0 \} \) there is no right angle lever paradox. In all other STR dynamics this paradox occurs, and therefore they are incompatible with energy conservation laws. In all dynamics \( \{ x \} \neq \{ 0 \} \) for a moving observer, the force vector does not need to be parallel to the acceleration vector caused by this force. This is contrary to the Lorentz
transformation, which maintains the parallelism of vectors. It follows that all dynamics \( x \neq \{0\} \), including Einstein’s dynamics, are inconsistent with STR kinematics. Only one dynamics \( x = \{0\} \) is devoid of these defects.

Bibliography


* * *

Other important publications


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