ON THE SIGNIFICANCE OF PLANCK'S CONSTANT

Robert Spoljaric

Queensland, Australia

r.spoljaric08@gmail.com

Abstract

Planck's constant h has dimensions of energy (ergs) × time (seconds) called "actions." Action is a four-dimensional constant and is the same size for all observers in special relativity, even if those observers disagree as to the size of the energy and time components of that action. The various guessed at formalisms of non-relativistic quantum mechanics introduce h arbitrarily, empirically. But being inconsistent with special relativity would seem to render that *ad hoc* introduction out of context. In this brief article we shall generalise the Planck-Einstein relation $E = hf$ to reveal a new mass-free paradigm of Relativity, leaving little of the classical physics which preceded it. Furthermore, we historically avoid the misguided advent of non-relativistic quantum mechanics altogether, to finally reveal the true significance of h. The maths employed here has been used in [1] and [2].

Keywords: Planck's constant, Mass, The Light, Equivalence Identity, Bekenstein-Hawking formula, Paradigm, Relativity, Laws of physics

1. Introduction

In this article we use relativistic mass to generalise $E = hf$, before we reason our way to the ultimate conclusion that mass never *really* existed. What we will end up with are the foundations for a mass-free paradigm of Relativity. It will be shown that there is a natural evolution from classical physics to Relativity where the significance of Planck's four-dimensional constant will be apparent.

2. The Light

We begin with the fact that many practical applications of special relativity are found in the theory of relativistic mechanics. The basis of relativistic mechanics is the following four equations

$$M = m_0/\sqrt{1 - (v/c)^2} \tag{1}$$

which is a function of velocity v where M is the relativistic mass, m_0 the rest mass and c the speed of light in a vacuum. Multiplying both sides of Eq. (1) by the velocity vector $\boldsymbol{v}$ gives us the expression for relativistic momentum

$$\boldsymbol{p} = M\boldsymbol{v} = m_0\boldsymbol{v}/\sqrt{1 - (v/c)^2} \tag{2}$$

and multiplying both sides of Eq. (1) by c^2 gives the total energy of a particle

$$E = Mc^2 = m_0c^2/\sqrt{1 - (v/c)^2} \tag{3}$$

Ignoring M in Eqs. (2) and (3) and combining the two equations gives us

$$E = \sqrt{(m_0c^2)^2 + (pc)^2}$$

where we have ignored the negative root, which we shall justify below. *Of these four equations only Eqs. (2) and (3) are theoretically necessary where we arbitrarily ignore M, and as the basis of relativistic dynamics they are routinely confirmed in elementary-particle physics.*

We now introduce the energy of a photon $E = hf$ where f is frequency, and equating this with the expression above we get

$$hf = \sqrt{(m_0c^2)^2 + (pc)^2}$$

Now, if $m_0 = 0$ then using $f = c/\lambda$ we derive de Broglie's hypothesis

$$\lambda = h/p \tag{4}$$

where λ is wavelength, and if $p = 0$ we similarly derive the Compton wavelength

$$\lambda_c = h/m_0c \tag{5}$$

Historically this much was known before 1925, with the possible exception of the derivation of Eq. (5) as presented here. To generalise $E = hf$ we first rewrite Eq. (5) in terms of m_0 and then substituting for m_0 in Eq. (1) we obtain the expression

$$\lambda = h/M\sqrt{c^2 - v^2} \tag{6}$$

But as M is also a function of velocity v we have

$$\lambda = \frac{h}{\dfrac{m_0\sqrt{c^2 - v^2}}{\sqrt{1 - (v/c)^2}}} \tag{6a}$$

Rather than cancelling terms we *observe* that if $v = 0$ in Eq. (6a) we get the Compton wavelength. Thus when $v = 0$ the M used in Eq. (6) corresponds to m_0 in Eq. (6a), which leaves the case of $v > 0$ where the M used in Eq. (6) now corresponds to

$$m_0/\sqrt{1 - (v/c)^2} \tag{\#}$$

in Eq. (6a). Therefore, if Eq. (6a) is the Compton wavelength when $v = 0$, then holding m_0 fixed and *discarding* (#) when $v > 0$ gives us the generalized Compton wavelength

$$\lambda_{GC} = h/m_0\sqrt{c^2 - v^2} \tag{7}$$

Thus all that remains of Eq. (1) is (#) whose absence is accounted for below. But first rewriting Eq. (7) in terms of m_0 and substituting into Eq. (2) we find a *qualitatively* different expression where frequency has replaced the concept of rest mass

$$\boldsymbol{p} = hf\boldsymbol{v}/(c^2 - v^2)$$

Excluding Eq. (3), then, leaves (#) as the only expression using rest mass, and since (#) in itself is meaningless it is incumbent upon us to account for its absence. Therefore, starting with Eq. (4)

$$\lambda = h/p$$

and substituting Newton's definition of momentum $p = mv$ for p gives us the basis of wave mechanics or de Broglie equation

$$\lambda = h/mv$$

And substituting (#) for m puts (#) in context giving the relativistic expression

$$\lambda = \frac{h\sqrt{1-(v/c)^2}}{m_0 v} \qquad v > 0$$

Finally rewriting this expression in terms of m_0 and substituting into the magnitude of Eq. (2) we find

$$p = \frac{hv\sqrt{1-(v/c)^2}}{\lambda v\sqrt{1-(v/c)^2}} = \frac{h}{\lambda}$$

Therefore, the absence of (#) entails the absence of: 1) de Broglie's equation, 2) $p = mv$ and 3) Eq. (2). Consistency now dictates we substitute Eq. (7) into Eq. (3) and then to use the wave vector $\boldsymbol{k}$ where $k = 2\pi/\lambda$, the Dirac constant $\hbar = h/2\pi$, and the angular frequency $\omega = 2\pi f = kc$, to give us our sought for generalisation of $E = hf$

$$\begin{aligned} \boldsymbol{p} &= \hbar\boldsymbol{k} \\ E &= \hbar\omega c^2/(c^2 - v^2) \\ \boldsymbol{p} &= \hbar\omega\boldsymbol{v}/(c^2 - v^2) \qquad v > 0 \end{aligned}$$

The absence of Eq. (2) now necessitates the use of these irreducible relations, and hence the need to translate rest mass to angular frequency ω where

$$\omega = 2\pi c/\lambda_{GC} = m_0 c\sqrt{c^2 - v^2}/\hbar \qquad\qquad v \geq 0$$

Additionally that *physical* absence means we are not free to mathematically retrieve Eq. (2) and thereby undo the relations

$$\boldsymbol{p} = \frac{\hbar\omega\boldsymbol{v}}{(c^2 - v^2)} = \frac{m_0\boldsymbol{v}c\sqrt{c^2 - v^2}}{(c^2 - v^2)} = \frac{m_0\boldsymbol{v}c}{\sqrt{c^2 - v^2}} = \frac{m_0\boldsymbol{v}}{\sqrt{1 - (v/c)^2}}$$

Now rather than assigning a zero-rest mass to the photon we use the rest mass of a particle to find the relativistic angular frequency of that particle. Furthermore as $v \to c$ we *see* that $\omega \to 0$, and this "relativistic Doppler shift" is consistent with time-dilation.

In contrast, ignoring Eq. (1) and arbitrarily substituting the Compton wavelength directly into Eq. (2) gives us

$$\boldsymbol{p} = \frac{hf\boldsymbol{v}}{\sqrt{c^2 - v^2}}$$

However, this expression is erroneous as f is not velocity-dependent and remains unchanged as $v \to c$.

We now conclude that the basis of relativistic dynamics ($v > 0$) is consistent with the de Broglie relations ($v = 0$), and if Eq. (2) does not exist, then it is these relations that are really *being confirmed in elementary-particle physics!*

Clearly the coining of the term "photon" by Lewis in 1926 is inappropriate as the term refers to the smallest unit of *radiant* energy, but the relations just derived imply the term should be extended to include the energy of *matter* as well. To avoid confusion we shall simply define the Light as the electromagnetic energy and momentum of a particle of *radiation*, or a particle of *matter*. Wave-particle ambiguity is avoided.

We mentioned above that we would justify our ignoring the negative root in the derivation of

$$E^* = \pm\sqrt{(m_0c^2)^2 + (pc)^2}$$

The first thing to note is that no such expression exists now that we have the Light. In hindsight, then, Dirac's equation from 1928 assumes that E^* does exist, and it associates the negative root with *antimatter*, which leaves the question of why we don't observe an equal amount of *antimatter* in the universe? However, looking at

$$E = \hbar\omega c^2/(c^2 - v^2) \qquad\qquad v \geq 0$$

we observe that E is positive in the interval $0 < v < c$ corresponding to *matter*, and E is negative in the interval $c < v < c\sqrt{2}$, which must correspond to *antimatter*. Thus E shows the whereabouts of *antimatter* to be in a mirror-universe reflected by the speed of c. Furthermore, when $v = 0$ and $v = c\sqrt{2}$ we find $E = \pm\hbar\omega$, which is a difference of $E = 2\hbar\omega$. Special relativity implies that since *matter* is 'massless' it must be moving at speed *c* in its own rest frame, which seems to be a contradiction, for how can *matter* be both at rest - relative to an observer - and moving at speed *c*? As *matter* is the origin in its own rest frame we can extend x-y axes from that 'origin' to have a plane. We now represent *matter* moving at speed *c* up a vertical time axis t but with no unit lengths marked, which begins at *O*. Rotating the axis around *O* perpendicularly gives us the time axis $t\,i$ where $i = \sqrt{-1}$ and by symmetry *matter*' (with x'-y' axes perpendicular to x-y giving a perpendicular plane) moving along this axis at speed *c*. The relative velocity between these planes is thus $\sqrt{c^2 + c^2} = c\sqrt{2}$. However since the energy of *matter*' is in the interval $c < v \leq c\sqrt{2}$ it must be negative, and thus *matter*' corresponds to *antimatter*. To show this using special relativity we choose seconds as a unit of time and note that the number of second's t' passing for *antimatter* relative to the viewpoint of *matter* as we count $t = 1$ second passing for *matter* is given by the reciprocal of the time-dilation formula with $v = c\sqrt{2}$

$$t' = \sqrt{1 - (2c^2/c^2)} = i$$

Therefore, generalising the energy of a photon by completely exhausting the equations of relativistic mechanics gives us the Light

Matter

$$E_+ = \hbar\omega c^2/(c^2 - v^2)$$
$$\boldsymbol{p}_+ = \hbar\omega\boldsymbol{v}/(c^2 - v^2) \qquad\qquad v > 0$$

Radiation

$$E_0 = \hbar\omega \qquad\qquad \text{(de Broglie relations)}$$
$$\boldsymbol{p}_0 = \hbar\boldsymbol{k}$$

Antimatter

$$E_- = i\hbar\omega c^2/(c^2 - v^2)$$
$$\boldsymbol{p}_- = i\hbar\omega\boldsymbol{v}/(c^2 - v^2) \qquad\qquad v > 0$$

A new mass-free paradigm of Relativity begins with the Light. Does the Light imply PCT asymmetry? We leave this unanswered, and focus upon the fact that the Light entails a revision of the mass-based physics used today.

3. Equivalence Identity

As a particle theory the Standard Model 'unifies' electromagnetism, weak, and strong nuclear interactions. Thus the Light entails a fundamental revision of the Standard Model as the Standard Model must be derived anew beginning with the Light in this new paradigm of Relativity. And that brings us to the 'force' of gravity.

Newton's second law of motion ($F = ma$) contains all three of Newton's laws of motion, i.e., the third law ($F = -F$); and first law ($F = 0$), or law of inertia. Inertial frames are those in which the law of inertia holds.

In special relativity inertial frames extend throughout all space. In general relativity inertial frames are considered to be freely falling frames moving with neither acceleration nor gravity where the observer experiences weightlessness and tidal forces are considered 'non-local'. But suppose it could be shown *a priori* that $F = ma$ does not exist, then the absence of an inertial frame means our weightless 'at rest' observer could not deny that tidal effects *are* local. This would have no effect upon the Light as it is defined in the subatomic realm where the nonuniformity of the gravitational field is negligible and space-time considered flat. *However*, the absence of inertial frames entails the absence of special relativity, and that just leaves the constant *c* as encoded in the Light from Maxwell's wave theory.

Consider, then, that as the derivation of the Light also consistently accounts for the absence of $p = mv$ from physics, this in turn implies the absence of Newton's second law, for mathematically we have

$$F = \frac{d(mv)}{dt} = m_I a$$

where F is force, m_I is inertial mass and a is acceleration. Following on from Newton's second law we also have

$$W = m_G g$$

where W is the weight of a terrestrial body, m_G is its gravitational mass, and g is the local acceleration of free fall. If we ignore air resistance, then by Galileo's empirical law of falling bodies we put $F = W$ and obtain

$$a = \frac{m_G}{m_I} g$$

General relativity necessitates that a body's acceleration under gravity be independent of its mass. This is finally realised, for $m_G = m_I$ *a priori* as the Light entails locally there *is* only

$$a = g \qquad \text{(Equivalence Identity)}$$

Therefore, if we assume the gravitational constant, G, is used in this mass-free paradigm of Relativity, then all that remains of Newtonian mechanics is G. In contrast to general relativity, then, Relativity transcends any notion of mass and necessitates that tidal effects are local. And all that now remains to remind us of the preceding classical physics of Newton and Maxwell are the constants G and c respectively. To this extent it is clear that general relativity did not go far enough.

If this is correct, then there never was anything else but this to understand, and what does not follow on from this is erroneous. Therefore, since this much could have been discovered before 1925 the misguided advent of non-relativistic quantum mechanics - and thus everything that was built upon that - could have been avoided!

Finally, from the totality of those physics theories used today, that just leaves the classical physics of statistical mechanics and thermodynamics unmentioned. But the nonexistence of Newton's three 'laws' of motion entails a revision of these two remaining theories as well.

4. Bekenstein-Hawking formula

Does Relativity subsume these last two remaining vestiges of classical physics? The following suggests that it does. Consider the Bekenstein-Hawking formula [3] for a black hole

$$S_{BH} = (A/4) \times (kc^3/G\hbar)$$

where k is Boltzman's constant from statistical mechanics, and A is the surface area of the event horizon. If we assume this formula holds true in Relativity, then we have a consistent unification of G, c, k, $\hbar$ and *entropy* to remind us of the mass-based paradigm of classical physics that was.

5. Conclusion

In hindsight, the evolution from the three-dimensional idealisations of classical physics to Relativity was inevitable as tidal effects were *always* local, and we reasoned our way to that conclusion by showing 'mass' was illusory! The true significance of Planck's four-dimensional constant is that it consistently leaves four-dimensional Relativity as our only description of the *matter/antimatter* universe(s).

References

[1] Spoljaric, R., Hadronic Journal 34, Number 2, 125 (2011)

[2] Spoljaric, R., Global Journal of Science Frontier Research (A), Volume 4, Issue 7, Version 1, 13 (2014)

[3] J. D. Bekenstein (2008), *Bekenstein-Hawking entropy*, Scholarpedia, 3 (10): 7375