Goldbach Conjecture

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Abstract

In this article, we use method of a modified sieve of Eratosthenes to prove the Goldbach conjecture.

We use $p_i$ for all the primes, $2, 3, 5, 7, 11, 13, \ldots$, $i=1, 2, 3, \ldots$.

Let $p_j < \sqrt{N}$, and $p_{j+1} > \sqrt{N}$,

$N$ is a large even integer.

Let set $M = \{ n \in (1, N); \text{and } n \text{ odd number} \}$

$P = \Pi_{2 \leq i \leq j} p_i$,

for $d \mid P$ set $M_d = \{ m ; md \in M \}$,

obviously, $|M_d| = \lfloor \frac{|M|}{d} \rfloor$,

if $d' \mid P$, set $M_{d,d'} = \{ m \in M_d , N-md = 0 \mod d' \}$,

By seiving of the Eratosthenes for all the primes ($p_i, i = 2, 3, \ldots j$),

The total of remaining numbers $n$ of $M$ which are those numbers in the following set,
\{n \in M; n \neq 0 \mod p \ \forall p \mid P, \text{ and } N - n \neq 0 \mod p \ \forall p \mid P\},

and it equals to,

\[ B(M) = \sum_{n \in M} (\sum_{d \mid (n, P)} \mu(d)) (\sum_{d' \mid (N - n, P)} \mu(d')) \]  

\hspace{1cm} (1)

we have,

\[ B(M) = \sum_{(d \mid P, d' \mid P)} \mu(d) \mu(d') (\sum_{(n \in M, d \mid (n, P), d' \mid (N - n, P))} 1), \]  

\hspace{1cm} (2)

The summation in the last blank is zero for those \(d \mid d'\) or \(d' \mid d\) when \(N \neq 0 \mod d'\), and \(d\) or \(d'\) is not equal 1.

For those are not zero, it is easy to prove that,

\[ |\left| M_{d, d'} \right| - \frac{|M|}{\text{LCM}(d, d')}| \leq 1, \]  

\hspace{1cm} (3)

we have,

\[ B(M) \approx \sum_{(d \mid P, d' \mid P)} \mu(d) \mu(d') \frac{|M|}{\text{LCM}(d, d')} = |M|\prod_{2 \leq i \leq j}(1 - \frac{2}{p_i}), \]  

\hspace{1cm} (4)

So there are approximately \((\frac{N - 1}{2})\prod_{2 \leq i \leq j}(1 - \frac{2}{p_i})\) such primes in the range \((1, N)\).

This number is larger than 3. There is at least one \(n\) which is not 1 or \(N-1\), and obviously it is a prime number.

Also \(N - n\) is a prime number too.

This proves the Goldbach conjecture.