Goldbach Conjecture

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Abstract

In this article, we use method of a modified sieve of Eratosthenes to prove the Goldbach conjecture.

We use $p_i$ for all the primes, 2,3,5,7,11,13,......, $i=1,2,3,.....$

Let $p_j < \sqrt{N}$, and $p_{j+1} > \sqrt{N}$,

$N$ is a large even integer.

Let set $M = \{n \in (1, N); \text{ and } n \text{ odd number}\}$

$P = \prod_{2 \leq i \leq j} p_i,$

for $d \mid P$ set $M_d = \{m ; md \in M\}$,

obviously, $|M_d| = \left\lfloor \frac{|M|}{d} \right\rfloor$,

if $d' \mid P$, set $M_{d,d'} = \{m \in M_d , N-md = 0, \text{ mod } d'\}$,

By seiving of the Eratosthenes for all the primes ($p_i, i = 2,3,...,j$),

The total of remaining numbers $n$ of $M$ which are those numbers in the following set,
\( \{n \in M; n \neq 0 \mod p \forall p \mid P, \text{ and } N - n \neq 0 \mod p \forall p \mid P\}, \)

and it equals to,

\[
B(M) = \sum_{n \in M}(\sum_{d \mid (n,P)}\mu(d))(\sum_{d' \mid (N-n,P)}\mu(d'))
\]

we have,

\[
B(M) = \sum_{(d,P,d'|P)\mu(d)\mu(d')}(\sum_{(n \in M,d \mid (n,P),d' \mid (N-n,P))}\cdot 1)
\]

The summation in the last blank is zero for those \( d \mid d' \) or \( d' \mid d \) when \( d \) or \( d' \) is not equal 1.

It is easy to prove that,

\[
||M_{d,d'}| - \frac{|M|}{\text{LCM}(d,d')}| \leq 1,
\]

we have,

\[
B(M) \approx \sum_{(d,P,d'|P)\mu(d)\mu(d')\frac{|M|}{\text{LCM}(d,d')}} = |M|R(2 \leq i \leq j)(1 - \frac{2}{p_i})
\]

So there are approximately \((\frac{N-1}{2})R(2 \leq i \leq j)(1 - \frac{2}{p_i})\) such primes in the range \((1, N)\).

This number is larger than 3. There is at least one \( n \) which is not 1 or \( N-1 \), and obviously it is a prime number.

Also \( N - n \) is a prime number too.

This proves the Goldbach conjecture.