Twin Prime Conjecture

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Abstract

In this article, we use method of a modified sieve of Eratosthenes to prove the twin prime conjecture.

We use $p_i$ for all the primes, 2,3,5,7,11,13,....., $i=1,2,3,.....$.

If a prime pair $(p_m, p_{m+1})$ is a twin prime, then it can be written as $(6k-1, 6k+1)$ for some k.

Theorem:

If $(p_j, p_{j+1})$ is a twin prime, then there are approximately $(\frac{p_j^2-1}{6})\prod_{3 \leq i \leq j}(1-\frac{2}{p_i})$ numbers of twin primes in the range of $(p_j+1, p_{j+1})$.

let set $M = \{m; m = 1, \ldots, \frac{p_j^2+1-1}{6}\}$,

set $N = \{n = 6m + 1; m \in M\}$,

$P = \prod_{3 \leq i \leq j} p_i$,

for $d \mid P$ set $M_d = \{ m \in M ; 6m+1 = 0, \mod d\}$,

obviously, $|M_d| = \lfloor \frac{|M|}{d} \rfloor$,

if $d' \mid P$, set $M_{d,d'} = \{m \in M_d , 6m+1 = 2, \mod d'\}$,
If \((p_j, p_{j+1})\) is a twin prime,

By sieving of the Eratosthenes for all the primes \((p_i, i = 3,4,\ldots,j)\),

The total of remaining numbers \(n\) of \(N\) which are those numbers in the following set,

\[
\{n \in N; n \not\equiv 0 \mod p \forall p \mid P, \text{ and } n \not\equiv 2 \mod p \forall p \mid P\},
\]

and it equals to,

\[
B(N) = \Sigma_{n \in N}(\Sigma_{d|(n,P)}\mu(d))(\Sigma_{d'|(n-2,P)}\mu(d'))
\]

we have,

\[
B(N) = \Sigma_{(d,P,d'P)}\mu(d)\mu(d')(\Sigma_{(n\in N,d|(n,P),d'|((n-2,P))}1),
\]

The summation in the last blank is zero for those \(d \mid d'\) or \(d' \mid d\) when \(d\) or \(d'\) is not equal 1.

It is easy to prove that,

\[
||M_{d,d'}| - \frac{|M|}{\text{LCM}(d,d')}| \leq 1,
\]

we have,

\[
B(N) \approx \Sigma_{(d,P,d'P)}\mu(d)\mu(d')\frac{|M|}{\text{LCM}(d,d')} = |M|\Pi_{(3 \leq i \leq j)}(1 - \frac{2}{p_i}),
\]

So there are approximately \((\frac{p_j^2 - 1}{6})\Pi_{(3 \leq i \leq j)}(1 - \frac{2}{p_i})\) twin primes in the range \((p_{j+1}, p_{j+1}^2)\).

This also proves the twin prime conjecture.