

**On some Ramanujan formulas concerning Highly composite numbers: new possible mathematical connections with various parameters of Particle Physics, Dark Matter, Dark Energy and Cosmology III.**

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**Abstract**

*In this research thesis, we have analyzed further Ramanujan formulas inherent Highly composite numbers and described new possible mathematical connections with various parameters of Particle Physics, Dark Matter, Dark Energy and Cosmology*

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[Replying to G. H. Hardy's suggestion that the number of a taxicab (1729) was a dull number:]  
No, it is a very interesting number, it is the smallest number expressible as a sum of two cubes in two different ways.

(Srinivasa Ramanujan)

izquotes.com

<https://sites.google.com/a/wusd.ws/mr-martinez-geometry-bls/mathemetician-of-the-month/srinivasaramanujan>

From:

**Highly composite numbers**

*Proceedings of the London Mathematical Society, 2, XIV, 1915, 347 – 409*

$$\begin{aligned}
\log dd(N) &= -\frac{\log p_1}{\log 2} \log \left\{ \left(1 - \frac{1}{2}\right) \left(1 - \frac{1}{3}\right) \dots \left(1 - \frac{1}{\lambda}\right) \right\} \\
&\quad + O \left\{ \sqrt{(\log p_1 \log \log p_1) \log \log \log p_1} \right\} \\
&\quad - \frac{\log p_1}{\log 2} \log \left\{ \left(1 - \frac{1}{\lambda'}\right) \left(1 - \frac{1}{\lambda''}\right) \dots \left(1 - \frac{1}{\varpi}\right) \right\} + O \left( \frac{\log p_1}{\log \log p_1} \right) \\
&= -\frac{\log p_1}{\log 2} \log \left\{ \left(1 - \frac{1}{2}\right) \left(1 - \frac{1}{3}\right) \dots \left(1 - \frac{1}{\varpi}\right) \right\} + O \left( \frac{\log p_1}{\log \log p_1} \right) \\
&= \frac{\log p_1}{\log 2} \left\{ \log \log \varpi + \gamma + O \left( \frac{1}{\log \varpi} \right) \right\} + O \left( \frac{\log p_1}{\log \log p_1} \right) \\
&= \frac{\log p_1}{\log 2} \left\{ \log \log \log p_1 + \gamma + O \left( \frac{1}{\log \log p_1} \right) \right\} + O \left( \frac{\log p_1}{\log \log p_1} \right) \\
&= \frac{\log \log N}{\log 2} \left\{ \log \log \log \log N + \gamma + O \left( \frac{1}{\log \log \log N} \right) \right\}, \\
&\hspace{15em} \dots\dots(180)
\end{aligned}$$

We have that:

$$\frac{\log \log N}{\log 2} \left\{ \log \log \log \log N + \gamma + O \left( \frac{1}{\log \log \log N} \right) \right\}$$

For  $N = 7.912454053394034144 \times 10^{21}$

$$(2^8 * 3^4 * 5^3 * 7^2 * 11 * 13 * 17 * 19 * 23 * 29 * 31 * 37 * 41 * 43)$$

**Input:**

$$2^8 \times 3^4 \times 5^3 \times 7^2 \times 11 \times 13 \times 17 \times 19 \times 23 \times 29 \times 31 \times 37 \times 41 \times 43$$

**Result:**

7912454053394034144000

**Scientific notation:**

$$7.912454053394034144 \times 10^{21}$$

$$7.912454053394034144 * 10^{21}$$

For  $O = 233$ , we obtain:

$\ln \ln (7.912454053394034144 \times 10^{21}) * 1/(\ln(2))^* (\ln \ln \ln \ln (7.912454053394034144 \times 10^{21}) + \text{Euler-Mascheroni constant} + 233(1/(\ln \ln \ln (7.912454053394034144 \times 10^{21}))))$

**Input interpretation:**

$$\log(\log(7.912454053394034144 \times 10^{21})) \times \frac{1}{\log(2)} \left( \log(\log(\log(\log(7.912454053394034144 \times 10^{21})))) + \gamma + 233 \times \frac{1}{\log(\log(\log(7.912454053394034144 \times 10^{21})))} \right)$$

$\log(x)$  is the natural logarithm  
 $\gamma$  is the Euler-Mascheroni constant

**Result:**

969.635284214442194090...  
 969.6352842...

$1/2((((\ln \ln (7.912454053394034144 \times 10^{21}) * 1/(\ln(2))^* (\ln \ln \ln \ln (7.912454053394034144 \times 10^{21}) + \text{Euler-Mascheroni constant} + 233(1/(\ln \ln \ln (7.912454053394034144 \times 10^{21}))))))))$

**Input interpretation:**

$$\frac{1}{2} \left( \log(\log(7.912454053394034144 \times 10^{21})) \times \frac{1}{\log(2)} \left( \log(\log(\log(\log(7.912454053394034144 \times 10^{21})))) + \gamma + 233 \times \frac{1}{\log(\log(\log(7.912454053394034144 \times 10^{21})))} \right) \right)$$

$\log(x)$  is the natural logarithm  
 $\gamma$  is the Euler-Mascheroni constant

**Result:**

484.817642107221097045...  
 484.8176421..... result very near to Holographic Ricci dark energy model, where

$\chi^2_{RDE} = 483.130.$

$\frac{1}{47} \left( \left( \left( \left( \ln \ln (7.912454053394034144 \times 10^{21}) \right) * \frac{1}{\ln(2)} \right) * (\ln \ln \ln \ln (7.912454053394034144 \times 10^{21}) + \text{Euler-Mascheroni constant} + 233 \left( \frac{1}{\ln \ln \ln (7.912454053394034144 \times 10^{21})} \right) \right) \right) \right) - 4$

**Input interpretation:**

$$\frac{1}{47} \left( \log(\log(7.912454053394034144 \times 10^{21})) \times \left( \frac{1}{\log(2)} \left( \log(\log(\log(\log(7.912454053394034144 \times 10^{21})))) + \gamma + 233 \times \frac{1}{\log(\log(\log(7.912454053394034144 \times 10^{21})))} \right) \right) \right) - 4$$

$\log(x)$  is the natural logarithm  
 $\gamma$  is the Euler-Mascheroni constant

**Result:**

16.6305379620094083849...

16.630537962.... result very near to the mass of the hypothetical light particle, the boson  $m_x = 16.84 \text{ MeV}$

$\left[ \frac{1}{729} \left( \left( \left( \left( \ln \ln (7.912454053394034144e+21) \right) * \frac{1}{\ln(2)} \right) * (\ln \ln \ln \ln (7.912454053394034144e+21) + \text{Euler-Mascheroni constant} + 233 \left( \frac{1}{\ln \ln \ln (7.912454053394034144e+21)} \right) \right) \right) \right) \right) - \frac{29-7}{10^2} - \frac{47-3}{10^4} \right]$

**Input interpretation:**

$$\frac{1}{729} \left( \log(\log(7.912454053394034144 \times 10^{21})) \times \left( \frac{1}{\log(2)} \left( \log(\log(\log(\log(7.912454053394034144 \times 10^{21})))) + \gamma + 233 \times \frac{1}{\log(\log(\log(7.912454053394034144 \times 10^{21})))} \right) \right) - \frac{29-7}{10^2} - \frac{47-3}{10^4} \right)$$

$\log(x)$  is the natural logarithm  
 $\gamma$  is the Euler-Mascheroni constant

**Result:**

1.10568955310623071892...

1.10568955...

$$\frac{1}{10^{52}} \left[ \left( \left( \left( \left( \left( \frac{1}{729} \left( \frac{1}{\ln(2)} \left( \ln \ln \ln \ln (7.912454e+21) + 0.5772156 + 233 \left( \frac{1}{\ln \ln \ln (7.912454e+21)} \right) \right) \right) \right) \right) \right) \right) - \frac{29-7}{10^2} - \frac{47-3}{10^4} \right]$$

**Input interpretation:**

$$\frac{1}{10^{52}} \left( \frac{1}{729} \left( \log(\log(7.912454 \times 10^{21})) \times \frac{1}{\log(2)} \left( \log(\log(\log(\log(7.912454 \times 10^{21})))) \right) + 0.5772156 + 233 \times \frac{1}{\log(\log(\log(7.912454 \times 10^{21})))} \right) \right) - \frac{29-7}{10^2} - \frac{47-3}{10^4}$$

log(x) is the natural logarithm

**Result:**

$$1.10568955... \times 10^{-52}$$

$$1.10568955... * 10^{-52}$$

result practically equal to the value of Cosmological Constant  $1.1056 * 10^{-52} \text{ m}^{-2}$

$$\left[ \frac{1}{7} \left( \left( \left( \left( \left( \ln \ln (7.912454053394034144e+21) * \frac{1}{\ln(2)} * \left( \ln \ln \ln \ln (7.912454053394034144e+21) + \text{Euler-Mascheroni constant} + 233 \left( \frac{1}{\ln \ln \ln (7.912454053394034144e+21)} \right) \right) \right) \right) \right) \right) + \frac{1}{\text{golden ratio}} \right]$$

**Input interpretation:**

$$\frac{1}{7} \left( \log(\log(7.912454053394034144 \times 10^{21})) \times \frac{1}{\log(2)} \left( \log(\log(\log(\log(7.912454053394034144 \times 10^{21})))) + \gamma + 233 \times \frac{1}{\log(\log(\log(7.912454053394034144 \times 10^{21})))} \right) \right) + \frac{1}{\phi}$$

log(x) is the natural logarithm  
 $\gamma$  is the Euler-Mascheroni constant  
 $\phi$  is the golden ratio

**Result:**

$$139.137360305098779718...$$

139.1373603.... result practically equal to the rest mass of Pion meson 139.57 MeV

Now, we have that:

Hence we have

$$d(N) < \frac{\{(1/n) \log(p_1 p_2 p_3 \cdots p_n N)\}^n}{\log p_1 \log p_2 \log p_3 \cdots \log p_n}, \quad (3)$$

for all values of  $N$ .

The following examples shew how close an approximation to  $d(N)$  may be given by the right-hand side of (3). If

$$N = 2^{72} \cdot 7^{25},$$

then, according to (3), we have

$$d(N) < 1898.00000685 \dots; \quad (11)$$

and as a matter of fact  $d(N) = 1898$ . Similarly, taking

$$N = 2^{568} \cdot 3^{358},$$

we have, by (3),

$$d(N) < 204271.000000372 \dots; \quad (12)$$

while the actual value of  $d(N)$  is 204271. In a similar manner, when

$$N = 2^{64} \cdot 3^{40} \cdot 5^{27},$$

we have, by (3),

$$d(N) < 74620.00412 \dots; \quad (13)$$

while actually

$$d(N) = 74620.$$

$$N = 2^5 \cdot 3^3 \cdot 5^2 \cdot 7 \cdot 11 \cdot 13.$$

$$(\log 4\pi - \gamma)\sqrt{x} \leq R(x)(\log x)^2 \leq (4 + \gamma - \log 4\pi)\sqrt{x}.$$

It can easily be verified that

$$\left. \begin{aligned} \log 4\pi - \gamma &= 1.954, \\ 4 + \gamma - \log 4\pi &= 2.046 \end{aligned} \right\}$$

approximately.

$$R(x) = \frac{2\sqrt{x} + \sum \frac{x^\rho}{\rho^2}}{(\log x)^2}$$

We have that:

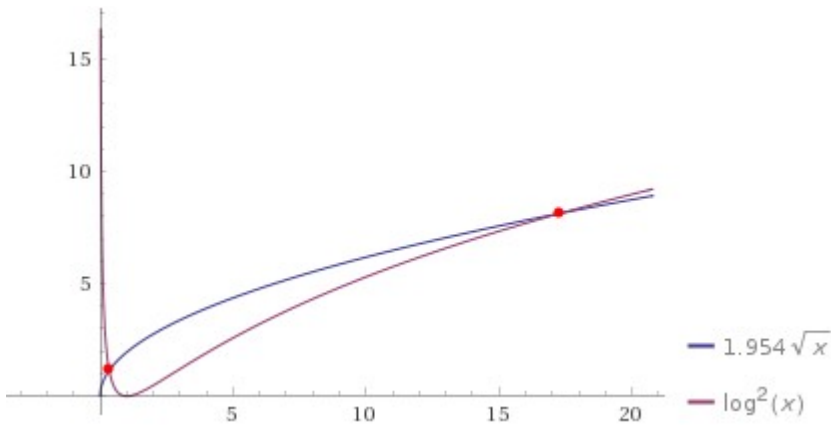
$$1.954 \cdot \sqrt{x} = (\ln x)^2$$

**Input:**

$$1.954 \sqrt{x} = \log^2(x)$$

$\log(x)$  is the natural logarithm

**Plot:**



**Solutions:**

$$x \approx 17.3084$$

$$x \approx 0.343072$$

$$x \approx 226.419$$



For  $x = 17.3084$  we obtain:

$$(\ln 17.3084)^2$$

**Input interpretation:**

$$\log^2(17.3084)$$

$\log(x)$  is the natural logarithm

**Result:**

8.129295437640103198741290135270881957463358718520398383624...

$$1.954 \cdot \sqrt{17.3084}$$

**Input interpretation:**

$$1.954 \sqrt{17.3084}$$

**Result:**

8.129297569556671112225226336330580536149371982971598491029...

8.1292975...

$$(((1.954 \cdot \sqrt{17.3084}))) / (((\ln 17.3084)^2)))$$

**Input interpretation:**

$$\frac{1.954 \sqrt{17.3084}}{\log^2(17.3084)}$$

$\log(x)$  is the natural logarithm

**Result:**

1.000000262251087350365684134344388654790853097333165364580...

1.00000026225108735..... value very near to the following Ramanujan continued fraction:

$$1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}} = \frac{e^{-2\pi\sqrt{5}}}{\frac{\sqrt{5}}{1 + [5^{3/4} (\phi - 1)^{5/2}]^{1/5}} - \phi} \tag{9}$$

$$= 1.000000791267 \dots \tag{10}$$

**Alternative representations:**

$$\frac{1.954 \sqrt{17.3084}}{\log^2(17.3084)} = \frac{1.954 \sqrt{17.3084}}{\log_e^2(17.3084)}$$

$$\frac{1.954 \sqrt{17.3084}}{\log^2(17.3084)} = \frac{1.954 \sqrt{17.3084}}{(\log(a) \log_a(17.3084))^2}$$

$$\frac{1.954 \sqrt{17.3084}}{\log^2(17.3084)} = \frac{1.954 \sqrt{17.3084}}{(-\text{Li}_1(-16.3084))^2}$$

**Series representations:**

$$\frac{1.954 \sqrt{17.3084}}{\log^2(17.3084)} = \frac{1.954 \sqrt{16.3084} \sum_{k=0}^{\infty} e^{-2.79168 k} \binom{\frac{1}{2}}{k}}{\log^2(17.3084)}$$

$$\frac{1.954 \sqrt{17.3084}}{\log^2(17.3084)} = \frac{1.954 \sqrt{16.3084} \sum_{k=0}^{\infty} e^{-2.79168 k} \binom{\frac{1}{2}}{k}}{\left(\log(16.3084) - \sum_{k=1}^{\infty} \frac{(-1)^k e^{-2.79168 k}}{k}\right)^2}$$

$$\frac{1.954 \sqrt{17.3084}}{\log^2(17.3084)} = \frac{1.954 \sqrt{16.3084} \sum_{k=0}^{\infty} \frac{(-0.0613181)^k \binom{-\frac{1}{2}}{k}}{k!}}{\left(\log(16.3084) - \sum_{k=1}^{\infty} \frac{(-1)^k e^{-2.79168 k}}{k}\right)^2}$$

**Integral representations:**

$$\frac{1.954 \sqrt{17.3084}}{\log^2(17.3084)} = \frac{1.954 \sqrt{17.3084}}{\left(\int_1^{17.3084} \frac{1}{t} dt\right)^2}$$

$$\frac{1.954 \sqrt{17.3084}}{\log^2(17.3084)} = \frac{7.816 i^2 \pi^2 \sqrt{17.3084}}{\left(\int_{-i \infty + \gamma}^{i \infty + \gamma} \frac{e^{-2.79168 s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds\right)^2} \text{ for } -1 < \gamma < 0$$

$R(x) = 1.00000026225108735.....$

And:

$$1/(((((((1.954*\text{sqrt}(17.3084)))) / ((((\ln 17.3084)^2))))))))$$

**Input interpretation:**

$$\frac{1}{\frac{1.954 \sqrt{17.3084}}{\log^2(17.3084)}}$$

log(x) is the natural logarithm

**Result:**

0.999999737748981425249095835028520713065345799056628356082...

0.999999737748981425249..... result very near to the value of the following Ramanujan continued fraction:

$$\frac{1}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}} = \left\{ \frac{\sqrt{5}}{1 + [5^{3/4} (\phi - 1)^{5/2} - 1]^{1/5}} - \phi \right\} e^{2\pi/\sqrt{5}} \tag{7}$$

$$= 0.99999920 \dots \tag{8}$$

**Alternative representations:**

$$\frac{1}{\frac{1.954 \sqrt{17.3084}}{\log^2(17.3084)}} = \frac{1}{\frac{1.954 \sqrt{17.3084}}{\log^2(17.3084)}}$$

$$\frac{1}{\frac{1.954 \sqrt{17.3084}}{\log^2(17.3084)}} = \frac{1}{\frac{1.954 \sqrt{17.3084}}{(\log(a) \log_a(17.3084))^2}}$$

$$\frac{1}{\frac{1.954 \sqrt{17.3084}}{\log^2(17.3084)}} = \frac{1}{\frac{1.954 \sqrt{17.3084}}{(-\text{Li}_1(-16.3084))^2}}$$

**Series representations:**

$$\frac{1}{\frac{1.954 \sqrt{17.3084}}{\log^2(17.3084)}} = \frac{0.511771 \log^2(17.3084)}{\sqrt{16.3084} \sum_{k=0}^{\infty} e^{-2.79168 k} \binom{\frac{1}{2}}{k}}$$

$$\frac{1}{\frac{1.954 \sqrt{17.3084}}{\log^2(17.3084)}} = \left( 0.511771 \left( \log^2(16.3084) - 2 \log(16.3084) \sum_{k=1}^{\infty} \frac{(-1)^k e^{-2.79168 k}}{k} + \left( \sum_{k=1}^{\infty} \frac{(-1)^k e^{-2.79168 k}}{k} \right)^2 \right) \right) / \left( \sqrt{16.3084} \sum_{k=0}^{\infty} e^{-2.79168 k} \binom{\frac{1}{2}}{k} \right)$$

$$\frac{1}{\frac{1.954 \sqrt{17.3084}}{\log^2(17.3084)}} = \left( 0.511771 \left( \log^2(16.3084) - 2 \log(16.3084) \sum_{k=1}^{\infty} \frac{(-1)^k e^{-2.79168 k}}{k} + \left( \sum_{k=1}^{\infty} \frac{(-1)^k e^{-2.79168 k}}{k} \right)^2 \right) \right) / \left( \sqrt{16.3084} \sum_{k=0}^{\infty} \frac{(-0.0613181)^k \binom{-\frac{1}{2}}{k}}{k!} \right)$$

**Integral representations:**

$$\frac{1}{\frac{1.954 \sqrt{17.3084}}{\log^2(17.3084)}} = \frac{0.511771 \left( \int_1^{17.3084} \frac{1}{t} dt \right)^2}{\sqrt{17.3084}}$$

$$\frac{1}{\frac{1.954 \sqrt{17.3084}}{\log^2(17.3084)}} = \frac{0.127943 \left( \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-2.79168 s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right)^2}{i^2 \pi^2 \sqrt{17.3084}} \text{ for } -1 < \gamma < 0$$

From which, we obtain:

$$\left( \frac{1}{\frac{1.954 \sqrt{17.3084}}{\log^2(17.3084)}} \right)^{4096}$$

**Input interpretation:**

$$\left( \frac{1}{\frac{1.954 \sqrt{17.3084}}{\log^2(17.3084)}} \right)^{4096}$$

$\log(x)$  is the natural logarithm

**Result:**

0.998926396412216970034319887577664418332086056026253895926...

0.99892639641221697..... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

$$\frac{1 + \sqrt[5]{\sqrt{\phi^5 4\sqrt{5^3} - 1}}}{\sqrt{5}} - \phi + 1$$

and to the dilaton value **0.989117352243 =  $\phi$**

And:

$$2 * \sqrt{\frac{1}{\log_{0.998926396412216} \left( \frac{1}{\frac{1.954 * \sqrt{17.3084}}{\log^2(17.3084)}} \right)}} - \pi + \frac{1}{\text{golden ratio}}$$

**Input interpretation:**

$$2 \sqrt{\frac{1}{\log_{0.998926396412216} \left( \frac{1}{\frac{1.954 \sqrt{17.3084}}{\log^2(17.3084)}} \right)}} - \pi + \frac{1}{\phi}$$

$\log(x)$  is the natural logarithm

$\log_b(x)$  is the base- $b$  logarithm

$\phi$  is the golden ratio

**Result:**

125.4764413352179586743591961101647426761086767734676081643...

125.4764413352.... result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for  $T = 0$  and to the Higgs boson mass 125.18 GeV

**Alternative representations:**

$$2 \sqrt{\frac{1}{\log_{0.9989263964122160000} \left( \frac{1}{\frac{1.954 \sqrt{17.3084}}{\log^2(17.3084)}} \right)}} - \pi + \frac{1}{\phi} =$$

$$\left( -\pi + \frac{1}{\phi} + 2 \sqrt{\frac{1}{\log \left( \frac{1}{\frac{1.954 \sqrt{17.3084}}{\log^2(17.3084)}} \right)}} \right) =$$

$$\frac{1}{\phi} - \pi + 2 \sqrt{\frac{\log(0.9989263964122160000)}{\log \left( \frac{0.511771 \log^2(17.3084)}{\sqrt{17.3084}} \right)}}$$

$$2 \sqrt{\frac{1}{\log_{0.9989263964122160000} \left( \frac{1}{\frac{1.954 \sqrt{17.3084}}{\log^2(17.3084)}} \right)}} - \pi + \frac{1}{\phi} =$$

$$\left( -\pi + \frac{1}{\phi} + 2 \sqrt{\frac{1}{\log_{0.9989263964122160000} \left( \frac{1}{\frac{1.954 \sqrt{17.3084}}{\log_e^2(17.3084)}} \right)}} \right) =$$

$$\frac{1}{\phi} - \pi + 2 \sqrt{\frac{1}{\log_{0.9989263964122160000} \left( \frac{0.511771 \log_e^2(17.3084)}{\sqrt{17.3084}} \right)}}$$

$$2 \sqrt{\frac{1}{\log_{0.9989263964122160000} \left( \frac{1}{\frac{1.954 \sqrt{17.3084}}{\log^2(17.3084)}} \right)}} - \pi + \frac{1}{\phi} =$$

$$\left( -\pi + \frac{1}{\phi} + 2 \sqrt{\frac{1}{\log_{0.9989263964122160000} \left( \frac{1}{\frac{1.954 \sqrt{17.3084}}{(\log(\alpha) \log_{\alpha}(17.3084))^2}} \right)}} = \right.$$

$$\left. \frac{1}{\phi} - \pi + 2 \sqrt{\frac{1}{\log_{0.9989263964122160000} \left( \frac{0.511771 \log^2(\alpha) \log_{\alpha}^2(17.3084)}{\sqrt{17.3084}} \right)}} \right)$$

### Series representations:

$$2 \sqrt{\frac{1}{\log_{0.9989263964122160000} \left( \frac{1}{\frac{1.954 \sqrt{17.3084}}{\log^2(17.3084)}} \right)}} - \pi + \frac{1}{\phi} =$$

$$\frac{1}{\phi} - \pi + 2 \exp \left( i \pi \left[ \frac{\arg \left( -x + \frac{1}{\log_{0.9989263964122160000} \left( \frac{0.511771 \log^2(17.3084)}{\sqrt{17.3084}} \right)} \right)}{2 \pi} \right] \right) \sqrt{x}$$

$$\sum_{k=0}^{\infty} \frac{(-1)^k x^{-k} \left( -x + \frac{1}{\log_{0.9989263964122160000} \left( \frac{0.511771 \log^2(17.3084)}{\sqrt{17.3084}} \right)} \right)^k}{k!} \left( -\frac{1}{2} \right)_k$$

for  $(x \in \mathbb{R} \text{ and } x < 0)$

$$2 \sqrt{\frac{1}{\log_{0.9989263964122160000} \left( \frac{1}{\frac{1.954 \sqrt{17.3084}}{\log^2(17.3084)}} \right)}} - \pi + \frac{1}{\phi} =$$

$$\frac{1}{\phi} - \pi + 2 \left( \frac{1}{z_0} \right)^{1/2} \left[ \arg \left( \frac{1}{\log_{0.9989263964122160000} \left( \frac{0.511771 \log^2(17.3084)}{\sqrt{17.3084}} \right)} - z_0 \right) / (2\pi) \right]$$

$$z_0^{1/2} \left[ 1 + \arg \left( \frac{1}{\log_{0.9989263964122160000} \left( \frac{0.511771 \log^2(17.3084)}{\sqrt{17.3084}} \right)} - z_0 \right) / (2\pi) \right]$$

$$\sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left( \frac{1}{\log_{0.9989263964122160000} \left( \frac{0.511771 \log^2(17.3084)}{\sqrt{17.3084}} \right)} - z_0 \right)^k}{k!} z_0^{-k}$$

### Integral representations:

$$2 \sqrt{\frac{1}{\log_{0.9989263964122160000} \left( \frac{1}{\frac{1.954 \sqrt{17.3084}}{\log^2(17.3084)}} \right)}} - \pi + \frac{1}{\phi} =$$

$$\frac{1}{\phi} - \pi + 2 \sqrt{\frac{1}{\log_{0.9989263964122160000} \left( \frac{0.511771 \left( \int_1^{17.3084} \frac{1}{t} dt \right)^2}{\sqrt{17.3084}} \right)}}$$

$$2 \sqrt{\frac{1}{\log_{0.9989263964122160000} \left( \frac{1}{\frac{1.954 \sqrt{17.3084}}{\log^2(17.3084)}} \right)}} - \pi + \frac{1}{\phi} =$$

$$\frac{1}{\phi} - \pi + 2 \sqrt{\frac{1}{\log_{0.9989263964122160000} \left( \frac{0.127943 \left( \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-2.79168 s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right)^2}{i^2 \pi^2 \sqrt{17.3084}} \right)}} \text{ for}$$

$$-1 < \gamma < 0$$



2\*sqrt[1/log base 0.998926396412216 (((1/((((((1.954\*sqrt(17.3084)))) / (((ln 17.3084)^2)))))))))]+11+1/golden ratio

**Input interpretation:**

$$2 \sqrt{\frac{1}{\log_{0.998926396412216} \left( \frac{1}{\frac{1.954 \sqrt{17.3084}}{\log^2(17.3084)}} \right)}} + 11 + \frac{1}{\phi}$$

log(x) is the natural logarithm

log<sub>b</sub>(x) is the base- b logarithm

φ is the golden ratio

**Result:**

139.6180339888077519128218394934442455603058461728427139852...

139.61803398..... result practically equal to the rest mass of Pion meson 139.57 MeV

**Alternative representations:**

$$2 \sqrt{\frac{1}{\log_{0.9989263964122160000} \left( \frac{1}{\frac{1.954 \sqrt{17.3084}}{\log^2(17.3084)}} \right)}} + 11 + \frac{1}{\phi} =$$

$$\left( \begin{array}{l} 11 + \frac{1}{\phi} + 2 \sqrt{\frac{1}{\log \left( \frac{1}{\frac{1.954 \sqrt{17.3084}}{\log^2(17.3084)}} \right)}} = \\ \log(0.9989263964122160000) \\ \log \left( \frac{0.511771 \log^2(17.3084)}{\sqrt{17.3084}} \right)} \end{array} \right)$$

$$2 \sqrt{\frac{1}{\log_{0.9989263964122160000} \left( \frac{1}{\frac{1.954 \sqrt{17.3084}}{\log^2(17.3084)}} \right)}} + 11 + \frac{1}{\phi} =$$

$$\left( \begin{array}{l} 11 + \frac{1}{\phi} + 2 \sqrt{\frac{1}{\log_{0.9989263964122160000} \left( \frac{1}{\frac{1.954 \sqrt{17.3084}}{\log_e^2(17.3084)}} \right)}} = \\ \log_{0.9989263964122160000} \left( \frac{0.511771 \log_e^2(17.3084)}{\sqrt{17.3084}} \right)} \end{array} \right)$$

$$2 \sqrt{\frac{1}{\log_{0.9989263964122160000} \left( \frac{1}{\frac{1.954 \sqrt{17.3084}}{\log^2(17.3084)}} \right)}} + 11 + \frac{1}{\phi} =$$

$$\left( 11 + \frac{1}{\phi} + 2 \sqrt{\frac{1}{\log_{0.9989263964122160000} \left( \frac{1}{\frac{1.954 \sqrt{17.3084}}{(\log(a) \log_{\alpha}(17.3084))^2}} \right)}} = \right.$$

$$\left. 11 + \frac{1}{\phi} + 2 \sqrt{\frac{1}{\log_{0.9989263964122160000} \left( \frac{0.511771 \log^2(a) \log_{\alpha}^2(17.3084)}{\sqrt{17.3084}} \right)}} \right)$$

### Series representations:

$$2 \sqrt{\frac{1}{\log_{0.9989263964122160000} \left( \frac{1}{\frac{1.954 \sqrt{17.3084}}{\log^2(17.3084)}} \right)}} + 11 + \frac{1}{\phi} =$$

$$11 + \frac{1}{\phi} + 2 \exp \left( i \pi \frac{\arg \left( -x + \frac{1}{\log_{0.9989263964122160000} \left( \frac{0.511771 \log^2(17.3084)}{\sqrt{17.3084}} \right)} \right)}{2 \pi} \right) \sqrt{x}$$

$$\sum_{k=0}^{\infty} \frac{(-1)^k x^{-k} \left( -x + \frac{1}{\log_{0.9989263964122160000} \left( \frac{0.511771 \log^2(17.3084)}{\sqrt{17.3084}} \right)} \right)^k}{k!} \left( -\frac{1}{2} \right)_k$$

for  $(x \in \mathbb{R} \text{ and } x < 0)$

$$\begin{aligned}
& 2 \sqrt{\frac{1}{\log_{0.9989263964122160000} \left( \frac{1}{\frac{1.954 \sqrt{17.3084}}{\log^2(17.3084)}} \right)}} + 11 + \frac{1}{\phi} = \\
& 11 + \frac{1}{\phi} + 2 \left( \frac{1}{z_0} \right)^{1/2} \left[ \arg \left( \frac{1}{\log_{0.9989263964122160000} \left( \frac{0.511771 \log^2(17.3084)}{\sqrt{17.3084}} \right)} - z_0 \right) / (2\pi) \right] \\
& \quad \left[ 1 + \arg \left( \frac{1}{\log_{0.9989263964122160000} \left( \frac{0.511771 \log^2(17.3084)}{\sqrt{17.3084}} \right)} - z_0 \right) / (2\pi) \right] \\
& \quad \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left( \frac{1}{\log_{0.9989263964122160000} \left( \frac{0.511771 \log^2(17.3084)}{\sqrt{17.3084}} \right)} - z_0 \right)^k}{k!} z_0^{-k}
\end{aligned}$$

### Integral representations:

$$\begin{aligned}
& 2 \sqrt{\frac{1}{\log_{0.9989263964122160000} \left( \frac{1}{\frac{1.954 \sqrt{17.3084}}{\log^2(17.3084)}} \right)}} + 11 + \frac{1}{\phi} = \\
& 11 + \frac{1}{\phi} + 2 \sqrt{\frac{1}{\log_{0.9989263964122160000} \left( \frac{0.511771 \left( \int_1^{17.3084} \frac{1}{t} dt \right)^2}{\sqrt{17.3084}} \right)}}
\end{aligned}$$

$$\begin{aligned}
& 2 \sqrt{\frac{1}{\log_{0.9989263964122160000} \left( \frac{1}{\frac{1.954 \sqrt{17.3084}}{\log^2(17.3084)}} \right)}} + 11 + \frac{1}{\phi} = \\
& 11 + \frac{1}{\phi} + 2 \sqrt{\frac{1}{\log_{0.9989263964122160000} \left( \frac{0.127943 \left( \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-2.79168s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right)^2}{i^2 \pi^2 \sqrt{17.3084}} \right)}} \quad \text{for} \\
& -1 < \gamma < 0
\end{aligned}$$

We have that:  
(after eq. 252)

$$\phi(N) = \left\{ \frac{\log(n+2)}{\log(n+1)} - 1 \right\} Li \left\{ \left( \frac{1}{n} \log N \right)^{n \frac{\log(n+2)}{\log(n+1)} - n} \right\} - \frac{\left( \frac{1}{n} \log N \right)^{n \frac{\log(n+2)}{\log(n+1)} - n}}{n \log \left( \frac{1}{n} \log N \right)} - R \left( \frac{1}{n} \log N \right) + \left\{ \frac{\sqrt{(\log N)}}{(\log \log N)^3} \right\}.$$

And:

$$N = 2^5 \cdot 3^3 \cdot 5^2 \cdot 7 \cdot 11 \cdot 13.$$

**Input:**

$$2^5 \times 3^3 \times 5^2 \times 7 \times 11 \times 13$$

**Result:**

$$21621600$$

**Scientific notation:**

$$2.16216 \times 10^7$$

For  $R = 1$ ,  $n = 3$  and  $N = 21621600$ , we obtain:

$$\left( \left( \left( \left( \left( \ln(5)/\ln(4) \right) - 1 \right) \right) \right) \right) Li \left( \left( \left( \left( \left( \frac{1}{3} \ln(21621600) \right)^{3 \cdot \left( \left( \left( \ln(5)/\ln(3) \right) \right) - 3 \right) \right) \right) \right) \right)$$

**Input:**

$$\left( \frac{\log(5)}{\log(4)} - 1 \right) li \left( \frac{1}{3} \log^3 (\log(5)/\log(3) - 3) (21621600) \right)$$

$\log(x)$  is the natural logarithm

$li(x)$  is the logarithmic integral

**Decimal approximation:**

$$-7.918550312975138281085838579694890949730781209501119... \times 10^{-9}$$

$$-7.918550312975138281085838579694890949730781209501119 \times 10^{-9}$$

**Alternate forms:**

$$\frac{\log\left(\frac{5}{4}\right) \operatorname{li}\left(\frac{1}{3} \log^{-9+(3 \log(5))/\log(3)}(21\,621\,600)\right)}{\log(4)}$$

$$-\frac{(\log(4) - \log(5)) \operatorname{li}\left(\frac{1}{3} \log^{3(-3+\log(5)/\log(3))}(21\,621\,600)\right)}{\log(4)}$$

$$\left(\frac{\log(5)}{2 \log(2)} - 1\right) \operatorname{li}\left(\frac{1}{3} (5 \log(2) + 3 \log(3) + 2 \log(5) + \log(1001))^{-9+(3 \log(5))/\log(3)}\right)$$

**Alternative representations:**

$$\begin{aligned} &\left(\frac{\log(5)}{\log(4)} - 1\right) \operatorname{li}\left(\frac{1}{3} \log^{3(\log(5)/\log(3)-3)}(21\,621\,600)\right) = \\ &\operatorname{Ei}\left(\log\left(\frac{1}{3} \log^{3(-3+\log(5)/\log(3))}(21\,621\,600)\right)\right) \left(-1 + \frac{\log_e(5)}{\log_e(4)}\right) \end{aligned}$$

$$\begin{aligned} &\left(\frac{\log(5)}{\log(4)} - 1\right) \operatorname{li}\left(\frac{1}{3} \log^{3(\log(5)/\log(3)-3)}(21\,621\,600)\right) = \\ &\operatorname{Ei}\left(\log\left(\frac{1}{3} \log^{3(-3+\log(5)/\log(3))}(21\,621\,600)\right)\right) \left(-1 + \frac{\log(a) \log_a(5)}{\log(a) \log_a(4)}\right) \end{aligned}$$

$$\begin{aligned} &\left(\frac{\log(5)}{\log(4)} - 1\right) \operatorname{li}\left(\frac{1}{3} \log^{3(\log(5)/\log(3)-3)}(21\,621\,600)\right) = \\ &\operatorname{li}\left(\frac{1}{3} \log_e^{3(-3+\log_e(5)/\log_e(3))}(21\,621\,600)\right) \left(-1 + \frac{\log_e(5)}{\log_e(4)}\right) \end{aligned}$$

**Series representations:**

$$\begin{aligned} &\left(\frac{\log(5)}{\log(4)} - 1\right) \operatorname{li}\left(\frac{1}{3} \log^{3(\log(5)/\log(3)-3)}(21\,621\,600)\right) = \\ &\frac{2 \gamma \log\left(\frac{5}{4}\right) + i \pi \log\left(\frac{5}{4}\right)}{\log(16)} - \frac{\log\left(\frac{5}{4}\right) \log\left(\frac{1}{\log\left(\frac{1}{3} \log^{-9+(3 \log(5))/\log(3)}(21\,621\,600)\right)}\right)}{\log(16)} + \\ &\frac{\log\left(\frac{5}{4}\right) \log\left(-\log\left(\frac{1}{3} \log^{-9+(3 \log(5))/\log(3)}(21\,621\,600)\right)\right)}{\log(16)} + \\ &\frac{2 \log\left(\frac{5}{4}\right) \sum_{k=1}^{\infty} \frac{\log^k\left(\frac{1}{3} \log^{-9+(3 \log(5))/\log(3)}(21\,621\,600)\right)}{k k!}}{\log(16)} \end{aligned}$$

$$\left(\frac{\log(5)}{\log(4)} - 1\right) \operatorname{li}\left(\frac{1}{3} \log^3(\log(5)/\log(3)-3)(21\,621\,600)\right) =$$

$$\frac{2 \log\left(\frac{5}{4}\right) \left( \operatorname{Res}_{s=0} \frac{\Gamma(s) \left( \frac{\log(3)}{\log^2(3) + \log\left(\frac{19683}{125}\right) \log(\log(21\,621\,600))} \right)^s}{s} \right)}{\log(16)}$$

$$\frac{2 \log\left(\frac{5}{4}\right) \sum_{j=1}^{\infty} \operatorname{Res}_{s=-j} \frac{\Gamma(s) \left( \frac{\log(3)}{\log^2(3) + \log\left(\frac{19683}{125}\right) \log(\log(21\,621\,600))} \right)^s}{s}}{\log(16)}$$

$$\left(\frac{\log(5)}{\log(4)} - 1\right) \operatorname{li}\left(\frac{1}{3} \log^3(\log(5)/\log(3)-3)(21\,621\,600)\right) =$$

$$-\left( \left( \log(3) - \log(4) - \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{3}\right)^k}{k} + \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{4}\right)^k}{k} \right) \right.$$

$$\left. \left( 2\gamma + i\pi - \log\left( \frac{1}{\log\left(\frac{1}{3} \log^3(-3 + \log(5)/\log(3))(21\,621\,600)\right)} \right) + \right.$$

$$\left. \log\left(-\log\left(\frac{1}{3} \log^3(-3 + \log(5)/\log(3))(21\,621\,600)\right)\right) + \right.$$

$$\left. \left. 2 \sum_{k=1}^{\infty} \frac{\log^k\left(\frac{1}{3} \log^{-9+(3 \log(5))/\log(3)}(21\,621\,600)\right)}{k k!} \right) \right) / 2$$

$$\left( \log(3) - \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{3}\right)^k}{k} \right)$$

### Integral representations:

$$\left(\frac{\log(5)}{\log(4)} - 1\right) \operatorname{li}\left(\frac{1}{3} \log^3(\log(5)/\log(3)-3)(21\,621\,600)\right) =$$

$$\frac{\log\left(\frac{5}{4}\right)}{\log(4)} \int_0^1 \log^{-9+(3 \log(5))/\log(3)}(21\,621\,600) \frac{1}{\log(t)} dt$$

$$\left(\frac{\log(5)}{\log(4)} - 1\right) \operatorname{li}\left(\frac{1}{3} \log^3(\log(5)/\log(3)-3)(21\,621\,600)\right) =$$

$$\frac{\left( \int_1^4 \frac{1}{t} dt - \int_1^5 \frac{1}{t} dt \right) \int_0^1 \log^{-9+(3 \log(5))/\log(3)}(21\,621\,600) \frac{1}{\log(t)} dt}{\int_1^4 \frac{1}{t} dt}$$

$$\left(\frac{\log(5)}{\log(4)} - 1\right) \operatorname{li}\left(\frac{1}{3} \log^{3(\log(5)/\log(3)-3)}(21\,621\,600)\right) = \frac{i \log\left(\frac{5}{4}\right)}{\pi \log(16)} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(s)^2 \left(-\log\left(\frac{1}{3} \log^{-9+(3\log(5)/\log(3))}(21\,621\,600)\right)\right)^{-s}}{\Gamma(1+s)} ds \text{ for } 0 < \gamma$$

$$-\left(\frac{1}{3} \ln(21621600)^{(3 * ((\ln(5)/\ln(3)) - 3))}\right) / \left(\frac{3 \ln(1/3 * \ln(21621600))}{\ln(21621600) + \left(\frac{\sqrt{\ln(21621600)}}{\ln(\ln(21621600))}\right)}\right)^3 - (1/3 \ln(21621600)) + \left(\frac{\sqrt{\ln(21621600)}}{\ln(\ln(21621600))}\right)^3$$

**Input:**

$$\left[ \frac{\frac{1}{3} \log^{3(\log(5)/\log(3)-3)}(21\,621\,600)}{3 \log\left(\frac{1}{3} \log(21\,621\,600)\right)} - \frac{1}{3} \log(21\,621\,600) + \frac{\sqrt{\log(21\,621\,600)}}{\log^3(\log(21\,621\,600))} \right]$$

$\log(x)$  is the natural logarithm

**Exact result:**

$$-\frac{\log^{3(\log(5)/\log(3)-3)}(21\,621\,600)}{9 \log\left(\frac{\log(21\,621\,600)}{3}\right)} - \frac{\sqrt{\log(21\,621\,600)}}{\log^3(\log(21\,621\,600))} + \frac{\log(21\,621\,600)}{3}$$

**Decimal approximation:**

5.447773802891567009002511540670266419804633783231593000653...

5.447773802891567009002511540670266419804633783231593000653

**Alternate forms:**

$$\frac{\sqrt{\log(21\,621\,600)} \left( \sqrt{\log(21\,621\,600)} \log^3(\log(21\,621\,600)) - 3 \right)}{3 \log^3(\log(21\,621\,600))} - \frac{\log^{3(\log(5)/\log(3)-9)}(21\,621\,600)}{9 \log\left(\frac{\log(21\,621\,600)}{3}\right)}$$

$$\left( -9 \log^{19/2}(21\,621\,600) \log\left(\frac{\log(21\,621\,600)}{3}\right) - \log^{3(\log(5)/\log(3))}(21\,621\,600) \log^3(\log(21\,621\,600)) + 3 \log^{10}(21\,621\,600) \log\left(\frac{\log(21\,621\,600)}{3}\right) \log^3(\log(21\,621\,600)) \right) / \left( 9 \log^9(21\,621\,600) \log\left(\frac{\log(21\,621\,600)}{3}\right) \log^3(\log(21\,621\,600)) \right)$$



$$-\frac{\sqrt{5 \log(2) + 3 \log(3) + 2 \log(5) + \log(1001)}}{\log^3(5 \log(2) + 3 \log(3) + 2 \log(5) + \log(1001))} + \frac{5 \log(2)}{3} + \log(3) + \frac{2 \log(5)}{3} + \frac{\log(1001)}{3} - \frac{(5 \log(2) + 3 \log(3) + 2 \log(5) + \log(1001))^{(3 \log(5))/\log(3)-9}}{9 \log\left(\frac{1}{3} (5 \log(2) + 3 \log(3) + 2 \log(5) + \log(1001))\right)}$$

**Alternative representations:**

$$-\left( \frac{\log^3(\log(5)/\log(3)-3)(21\,621\,600)}{\left(3 \log\left(\frac{\log(21\,621\,600)}{3}\right)\right)^3} - \frac{\log(21\,621\,600)}{3} + \frac{\sqrt{\log(21\,621\,600)}}{\log^3(\log(21\,621\,600))} \right) = \frac{\log_e(21\,621\,600)}{3} - \frac{\log_e^{3(-3+\log_e(5)/\log_e(3))}(21\,621\,600)}{3 \left(3 \log_e\left(\frac{\log(21\,621\,600)}{3}\right)\right)} - \frac{\sqrt{\log_e(21\,621\,600)}}{\log_e^3(\log(21\,621\,600))}$$

$$-\left( \frac{\log^3(\log(5)/\log(3)-3)(21\,621\,600)}{\left(3 \log\left(\frac{\log(21\,621\,600)}{3}\right)\right)^3} - \frac{\log(21\,621\,600)}{3} + \frac{\sqrt{\log(21\,621\,600)}}{\log^3(\log(21\,621\,600))} \right) = \frac{1}{3} \log(a) \log_a(21\,621\,600) - \frac{(\log(a) \log_a(21\,621\,600))^{3(-3+(\log(a) \log_a(5))/(\log(a) \log_a(3)))}}{3 \left(3 \log(a) \log_a\left(\frac{\log(21\,621\,600)}{3}\right)\right)} - \frac{\sqrt{\log(a) \log_a(21\,621\,600)}}{(\log(a) \log_a(\log(21\,621\,600)))^3}$$

$$-\left( \frac{\log^3(\log(5)/\log(3)-3)(21\,621\,600)}{\left(3 \log\left(\frac{\log(21\,621\,600)}{3}\right)\right)^3} - \frac{\log(21\,621\,600)}{3} + \frac{\sqrt{\log(21\,621\,600)}}{\log^3(\log(21\,621\,600))} \right) = \frac{\text{Li}_1(-21\,621\,599)}{3} - \frac{(-\text{Li}_1(-21\,621\,599))^{3(-3--\text{Li}_1(-4)/\text{Li}_1(-2))}}{3 \left(-3 \text{Li}_1\left(1 - \frac{\log(21\,621\,600)}{3}\right)\right)} - \frac{\sqrt{-\text{Li}_1(-21\,621\,599)}}{(-\text{Li}_1(1 - \log(21\,621\,600)))^3}$$

**Input interpretation:**

$$-7.918550312975138281085838579694890949730781209501119 \times 10^{-9} + 5.447773802891567009002511540670266419804633783231593000653$$

**Result:**

5.447773794973016696027373259584427840109742833500811791151...  
5.4477737949730166.....

$$(-7.918550312975138281 \times 10^{-9} + 5.447773802891567009)^{3-18-4}$$

**Input interpretation:**

$$(-7.918550312975138281 \times 10^{-9} + 5.447773802891567009)^3 - 18 - 4$$

**Result:**

139.6803344549422804211944404273052921112358488089286944978...

139.6803344549..... result practically equal to the rest mass of Pion meson 139.57 MeV

$$(-7.918550312975138281 \times 10^{-9} + 5.447773802891567009)^3 - 47 + 11$$

**Input interpretation:**

$$(-7.918550312975138281 \times 10^{-9} + 5.447773802891567009)^3 - 47 + 11$$

**Result:**

125.6803344549422804211944404273052921112358488089286944978...

125.68033445.... result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T = 0 and to the Higgs boson mass 125.18 GeV

Now, from:

$$ddd(n) > (\log n)^{\log \log \log \log n}$$

We obtain:

$$((\ln(3))^{\ln \ln \ln \ln(3)})$$

**Input:**

$$\log^{\log(\log(\log(\log(3))))}(3)$$

log(x) is the natural logarithm

**Exact result:**

$$\log^{\log(-\log(\log(\log(3)))) + i\pi}(3)$$

**Decimal approximation:**

1.0372926227460690906405231048871661077563928270497470233... +  
0.31571943313737053101208072851421568350398832859656639367... i

## Polar coordinates:

$r \approx 1.08428$  (radius),  $\theta \approx 16.9286^\circ$  (angle)

1.08428

## Alternate form:

$$\log^{\log(-\log(\log(\log(3))))}(3) \cos(\pi \log(\log(3))) + i \log^{\log(-\log(\log(\log(3))))}(3) \sin(\pi \log(\log(3)))$$

## General form

$$\log^{\log(\log(\log(\log(3))))}(3) = e^{-2\pi^2 n + 2i\pi n \log(-\log(\log(\log(3)))) + \log(\log(3)) \log(-\log(\log(\log(3)))) + i\pi \log(\log(3))} \text{ for } n \in \mathbf{Z}$$

(the choice of  $n$  is determined by the branch of the logarithm used for complex exponentiation)

## Alternative representations:

$$\log^{\log(\log(\log(\log(3))))}(3) = \log^{\log(\log(3))}(\log(\log(3)))$$

$$\log^{\log(\log(\log(\log(3))))}(3) = \log_e^{\log_e(\log(\log(\log(3))))}(3)$$

$$\log^{\log(\log(\log(\log(3))))}(3) = (\log(a) \log_a(3))^{\log(a) \log_a(\log(\log(\log(3))))}$$

## Series representations:

$$\log^{\log(\log(\log(\log(3))))}(3) = \left( \log(2) - \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{2}\right)^k}{k} \right)^{i\pi + \log(-1 - \log(\log(\log(3)))) - \sum_{k=1}^{\infty} \left( \frac{1}{1 + \log(\log(3))} \right)^k / k}$$

$$\log^{\log(\log(\log(\log(3))))}(3) = \left( 2i\pi \left\lfloor \frac{\arg(3-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (3-x)^k x^{-k}}{k} \right)^{i\pi + 2i\pi \left\lfloor \frac{|\arg(-x - \log(\log(\log(3))))|}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \left( (-1)^k x^{-k} (-x - \log(\log(\log(3))))^k \right) / k}$$

for  
 $x < 0$

$$\log^{\log(\log(\log(\log(3))))}(3) = \left( 2i\pi \left\lfloor \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right\rfloor + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (3-z_0)^k z_0^{-k}}{k} \right)^{i\pi + 2i\pi \left\lfloor \frac{\left(\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)\right)}{2\pi} \right\rfloor + \log(z_0) - \sum_{k=1}^{\infty} \left( (-1)^k (-\log(\log(\log(3))) - z_0)^k z_0^{-k} \right) / k}$$

**Integral representations:**

$$\log^{\log(\log(\log(\log(\log(3)))))(3)} = \left( \int_1^3 \frac{1}{t} dt \right)^{i\pi + \int_1^{-\log(\log(\log(3)))} 1/t dt}$$

$$\log^{\log(\log(\log(\log(\log(3)))))(3)} = (2\pi)^{-i\pi + i/(2\pi)} \int_{-i\infty + \gamma}^{i\infty + \gamma} (\Gamma(-s)^2 \Gamma(1+s) (-1 - \log(\log(\log(3))))^{-s}) / \Gamma(1-s) ds$$

$$\left( -i \int_{-i\infty + \gamma}^{i\infty + \gamma} \frac{2^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right)^{i\pi - i/(2\pi)} \int_{-i\infty + \gamma}^{i\infty + \gamma} (\Gamma(-s)^2 \Gamma(1+s) (-1 - \log(\log(\log(3))))^{-s}) / \Gamma(1-s) ds$$

for  $-1 < \gamma < 0$

$$[((\ln(3))^{\ln \ln \ln \ln(3)})^6 + (47/10^3)i]$$

**Input:**

$$\log^{\log(\log(\log(\log(\log(3)))))(3)}^6 + \frac{47}{10^3} i$$

$\log(x)$  is the natural logarithm

$i$  is the imaginary unit

**Exact result:**

$$\frac{47i}{1000} + \log^{6 \log(-\log(\log(\log(3)))) + 6i\pi}(3)$$

**Decimal approximation:**

$$-0.3259535806127655849521310685046535199855794916084654645... + 1.638920063836049724274182295374681755596340931790306841... i$$

**Alternate forms:**

$$\frac{47i}{1000} + \log^{\log(\log^6(\log(\log(3)))) + 6i\pi}(3)$$

$$\frac{47i}{1000} + \log^{6(\log(-\log(\log(\log(3)))) + i\pi)}(3)$$

$$\frac{47i + 1000 \log^{6 \log(-\log(\log(\log(3)))) + 6i\pi}(3)}{1000}$$

**Alternative representations:**

$$\log^{\log(\log(\log(\log(\log(3)))))(3)}^6 + \frac{i47}{10^3} = \frac{47i}{10^3} + \log^{\log(\log(3))}(\log(\log(3)))^6$$

$$\log^{\log(\log(\log(\log(3))))}(3)^6 + \frac{i 47}{10^3} = \frac{47 i}{10^3} + \log_e^{\log(\log(\log(\log(3))))}(3)^6$$

$$\log^{\log(\log(\log(\log(3))))}(3)^6 + \frac{i 47}{10^3} = \frac{47 i}{10^3} + \left( \log(a) \log_a(3) \right)^{\log(a) \log_a(\log(\log(\log(3))))} 6$$

### Series representations:

$$\begin{aligned} \log^{\log(\log(\log(\log(3))))}(3)^6 + \frac{i 47}{10^3} &= \frac{1}{1000} \\ &\left( 1000 \left( \log(2) - \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{2}\right)^k}{k} \right)^{6 i \pi + 6 \log(-1 - \log(\log(\log(3))))} \right. \\ &\quad \left. + 47 i \left( \log(2) - \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{2}\right)^k}{k} \right)^{6 \sum_{k=1}^{\infty} \left( \frac{1}{1 + \log(\log(\log(3)))} \right)^k / k} \right) \\ &\left( \log(2) - \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{2}\right)^k}{k} \right)^{-6 \sum_{k=1}^{\infty} \left( \frac{1}{1 + \log(\log(\log(3)))} \right)^k / k} \end{aligned}$$

$$\begin{aligned} \log^{\log(\log(\log(\log(3))))}(3)^6 + \frac{i 47}{10^3} &= \\ \frac{1}{1000} \left( 47 i + 1000 \left( \operatorname{Res}_{s=0} \frac{2^{-s} \Gamma(-s) \Gamma(1+s)}{s} + \sum_{j=1}^{\infty} \operatorname{Res}_{s=j} \frac{2^{-s} \Gamma(-s) \Gamma(1+s)}{s} \right)^{6 i \pi + 6 (\operatorname{Res}_{s=0} (\Gamma(-s) \Gamma(1+s) (-1 - \log(\log(\log(3))))^{-s}) / s) + 6 \sum_{j=1}^{\infty} \operatorname{Res}_{s=j} (\Gamma(-s) \Gamma(1+s) (-1 - \log(\log(\log(3))))^{-s}) / s} \right) \end{aligned}$$

$$\begin{aligned} \log^{\log(\log(\log(\log(3))))}(3)^6 + \frac{i 47}{10^3} &= \frac{1}{1000} \left( 1000 \left( 2 i \pi \left\lfloor \frac{\arg(3-x)}{2 \pi} \right\rfloor + \log(x) - \right. \right. \\ &\quad \left. \left. \sum_{k=1}^{\infty} \frac{(-1)^k (3-x)^k x^{-k}}{k} \right)^{6 i \pi + 12 i \pi \left\lfloor \frac{\arg(-x - \log(\log(\log(3))))}{2 \pi} \right\rfloor + 6 \log(x)} \right. \\ &\quad \left. + 47 i \left( 2 i \pi \left\lfloor \frac{\arg(3-x)}{2 \pi} \right\rfloor + \log(x) - \right. \right. \\ &\quad \left. \left. \sum_{k=1}^{\infty} \frac{(-1)^k (3-x)^k x^{-k}}{k} \right)^{6 \sum_{k=1}^{\infty} \left( \frac{(-1)^k x^{-k} (-x - \log(\log(\log(3))))^k}{k} \right)} \right) \left( 2 \right. \\ &\quad \left. i \pi \left\lfloor \frac{\arg(3-x)}{2 \pi} \right\rfloor + \log(x) - \right. \\ &\quad \left. \sum_{k=1}^{\infty} \frac{(-1)^k (3-x)^k x^{-k}}{k} \right)^{-6 \sum_{k=1}^{\infty} \left( \frac{(-1)^k x^{-k} (-x - \log(\log(\log(3))))^k}{k} \right)} \end{aligned} \quad \text{for } x < 0$$

**Integral representations:**

$$\log^{\log(\log(\log(\log(\log(3)))))}(3)^6 + \frac{i 47}{10^3} = \frac{47 i + 1000 \left( \int_1^3 \frac{1}{t} dt \right)^{6 i \pi + 6} \int_1^{-\log(\log(\log(3)))} 1/t dt}{1000}$$

$$\begin{aligned} \log^{\log(\log(\log(\log(\log(3)))))}(3)^6 + \frac{i 47}{10^3} = & \\ \frac{1}{125} \times 2^{-3-6 i \pi} \pi^{-6 i \pi} \left( 125 \times 2^{3+(3 i)/\pi} \int_{-i \infty + \gamma}^{i \infty + \gamma} (\Gamma(-s)^2 \Gamma(1+s) (-1-\log(\log(\log(3))))^{-s}) / \Gamma(1-s) ds \right. & \\ \pi^{(3 i)/\pi} \int_{-i \infty + \gamma}^{i \infty + \gamma} (\Gamma(-s)^2 \Gamma(1+s) (-1-\log(\log(\log(3))))^{-s}) / \Gamma(1-s) ds & \\ \left. \left( -i \int_{-i \infty + \gamma}^{i \infty + \gamma} \frac{2^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right)^{6 i \pi} + \right. & \\ 47 i (2 \pi)^{6 i \pi} \left( -i \int_{-i \infty + \gamma}^{i \infty + \gamma} \frac{2^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} \right. & \\ \left. ds \right)^{(3 i)/\pi} \int_{-i \infty + \gamma}^{i \infty + \gamma} (\Gamma(-s)^2 \Gamma(1+s) (-1-\log(\log(\log(3))))^{-s}) / \Gamma(1-s) ds & \\ \left. \left( -i \int_{-i \infty + \gamma}^{i \infty + \gamma} \frac{2^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right)^{-(3 i)/\pi} \int_{-i \infty + \gamma}^{i \infty + \gamma} (\Gamma(-s)^2 \Gamma(1+s) (-1-\log(\log(\log(3))))^{-s}) / \Gamma(1-s) ds \right. & \\ \left. \text{for } -1 < \gamma < 0 \right. & \end{aligned}$$

The result is:

$$1/10^{27}(-0.325953580612765584952131 + 1.638920063836049724274i)$$

**Input interpretation:**

$$\frac{1}{10^{27}} \times (-0.325953580612765584952131 + 1.638920063836049724274 i)$$

*i* is the imaginary unit

**Result:**

$$- 3.259535806127655849521... \times 10^{-28} + 1.638920063836049724274... \times 10^{-27} i$$

**Polar coordinates:**

$$r = 1.671019064032078592690 \times 10^{-27}$$

(radius),  $\theta = 101.24838952855295406620^\circ$  (angle)

1.67101906...\*10<sup>-27</sup> result practically equal to the value of the formula:

$$m_{p'} = 2 \times \frac{\eta}{R} m_p = 1.6714213 \times 10^{-27} \text{ kg}$$

that is the holographic proton mass (N. Hamein)

Now, we have that:

$$\sqrt{(1 + a_{\lambda'})} - \sqrt{(1 + a_{\lambda})} > \frac{\sqrt{(\log p_1)}}{2\lambda(\log \lambda)^{\frac{3}{2}}} - \sqrt{2}. \quad (121)$$

for  $\lambda = 5$ ,  $\mu = 8$ ,  $a_{\lambda} \leq (\ln \mu / \ln \lambda)$  and  $p_1 = 7$ , we obtain:

$$(\ln(8))/(\ln(5))$$

**Input:**

$$\frac{\log(8)}{\log(5)}$$

$\log(x)$  is the natural logarithm

**Decimal approximation:**

1.292029674220179152010319706291896896209375796239281347965...

1.29202967...

**Alternate form:**

$$\frac{3 \log(2)}{\log(5)}$$

**Alternative representations:**

$$\frac{\log(8)}{\log(5)} = \frac{\log_e(8)}{\log_e(5)}$$

$$\frac{\log(8)}{\log(5)} = \frac{\log(a) \log_a(8)}{\log(a) \log_a(5)}$$

$$\frac{\log(8)}{\log(5)} = \frac{-\text{Li}_1(-7)}{-\text{Li}_1(-4)}$$

### Series representations:

$$\frac{\log(8)}{\log(5)} = \frac{\log(7) - \sum_{k=1}^{\infty} \frac{(-\frac{1}{7})^k}{k}}{\log(4) - \sum_{k=1}^{\infty} \frac{(-\frac{1}{4})^k}{k}}$$

$$\frac{\log(8)}{\log(5)} = \frac{2\pi \left[ \frac{\text{arg}(8-x)}{2\pi} \right] - i \log(x) + i \sum_{k=1}^{\infty} \frac{(-1)^k (8-x)^k x^{-k}}{k}}{2\pi \left[ \frac{\text{arg}(5-x)}{2\pi} \right] - i \log(x) + i \sum_{k=1}^{\infty} \frac{(-1)^k (5-x)^k x^{-k}}{k}} \quad \text{for } x < 0$$

$$\frac{\log(8)}{\log(5)} = \frac{2\pi \left[ \frac{\pi - \text{arg}\left(\frac{1}{z_0}\right) - \text{arg}(z_0)}{2\pi} \right] - i \log(z_0) + i \sum_{k=1}^{\infty} \frac{(-1)^k (8-z_0)^k z_0^{-k}}{k}}{2\pi \left[ \frac{\pi - \text{arg}\left(\frac{1}{z_0}\right) - \text{arg}(z_0)}{2\pi} \right] - i \log(z_0) + i \sum_{k=1}^{\infty} \frac{(-1)^k (5-z_0)^k z_0^{-k}}{k}}$$

### Integral representations:

$$\frac{\log(8)}{\log(5)} = \frac{\int_1^8 \frac{1}{t} dt}{\int_1^5 \frac{1}{t} dt}$$

$$\frac{\log(8)}{\log(5)} = \frac{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{7^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds}{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{4^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds} \quad \text{for } -1 < \gamma < 0$$

$$(((\text{sqrt}(\ln(7)))) / (((2*5(\ln(5))^{1.5}))) - \text{sqrt}2$$

### Input:

$$\frac{\sqrt{\log(7)}}{2 \times 5 \log^{1.5}(5)} - \sqrt{2}$$



**Result:**

-1.345893...

-1.345893...

**Alternative representations:**

$$\frac{\sqrt{\log(7)}}{2 \times 5 \log^{1.5}(5)} - \sqrt{2} = -\sqrt{2} + \frac{\sqrt{\log_e(7)}}{10 \log_e^{1.5}(5)}$$

$$\frac{\sqrt{\log(7)}}{2 \times 5 \log^{1.5}(5)} - \sqrt{2} = -\sqrt{2} + \frac{\sqrt{\log(a) \log_a(7)}}{10 (\log(a) \log_a(5))^{1.5}}$$

$$\frac{\sqrt{\log(7)}}{2 \times 5 \log^{1.5}(5)} - \sqrt{2} = -\sqrt{2} + \frac{\sqrt{-\text{Li}_1(-6)}}{10 (-\text{Li}_1(-4))^{1.5}}$$

**Series representations:**

$$\begin{aligned} \frac{\sqrt{\log(7)}}{2 \times 5 \log^{1.5}(5)} - \sqrt{2} = \\ \sum_{k=0}^{\infty} -\frac{1}{10 k! \log^{1.5}(5)} (-1)^k x^{-k} \left( 10 (2-x)^k \exp\left(i\pi \left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor\right) \log^{1.5}(5) - \right. \\ \left. \exp\left(i\pi \left\lfloor \frac{\arg(-x + \log(7))}{2\pi} \right\rfloor\right) (-x + \log(7))^k \right) \left(-\frac{1}{2}\right)_k \sqrt{x} \quad \text{for } (x \in \mathbb{R} \text{ and } x < 0) \end{aligned}$$

$$\begin{aligned} \frac{\sqrt{\log(7)}}{2 \times 5 \log^{1.5}(5)} - \sqrt{2} = \sum_{k=0}^{\infty} -\frac{1}{10 k! \log^{1.5}(5)} (-1)^k \left(-\frac{1}{2}\right)_k z_0^{1/2-k} \\ \left( 10 \log^{1.5}(5) (2-z_0)^k \left(\frac{1}{z_0}\right)^{1/2 \lfloor \arg(2-z_0)/(2\pi) \rfloor} z_0^{1/2 \lfloor \arg(2-z_0)/(2\pi) \rfloor} - \right. \\ \left. (\log(7) - z_0)^k \left(\frac{1}{z_0}\right)^{1/2 \lfloor \arg(\log(7)-z_0)/(2\pi) \rfloor} z_0^{1/2 \lfloor \arg(\log(7)-z_0)/(2\pi) \rfloor} \right) \end{aligned}$$

$$\frac{\sqrt{\log(7)}}{2 \times 5 \log^{1.5}(5)} - \sqrt{2} = - \left( \left( \sqrt{x} \left( 10 \exp\left(i\pi \left[ \frac{\arg(2-x)}{2\pi} \right] \right) \left( \log(4) - \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{4}\right)^k}{k} \right)^{1.5} \sum_{k=0}^{\infty} \frac{(-1)^k (2-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} - \exp\left(i\pi \left[ \frac{\arg(-x + \log(7))}{2\pi} \right] \right) \sum_{k=0}^{\infty} \frac{(-1)^k x^{-k} (-x + \log(7))^k \left(-\frac{1}{2}\right)_k}{k!} \right) \right) / \left( 10 \left( \log(4) - \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{4}\right)^k}{k} \right)^{1.5} \right) \text{ for } (x \in \mathbb{R} \text{ and } x < 0)$$

**Integral representations:**

$$\frac{\sqrt{\log(7)}}{2 \times 5 \log^{1.5}(5)} - \sqrt{2} = - \frac{10 \left( \int_1^5 \frac{1}{t} dt \right)^{1.5} \sqrt{2} - \sqrt{\int_1^7 \frac{1}{t} dt}}{10 \left( \int_1^5 \frac{1}{t} dt \right)^{1.5}}$$

$$\frac{\sqrt{\log(7)}}{2 \times 5 \log^{1.5}(5)} - \sqrt{2} = \frac{\left( \frac{1}{i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{4^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right)^{1.5} \sqrt{2} - 0.282843 \sqrt{\frac{1}{2i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{6^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds}}{\left( \frac{1}{i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{4^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right)^{1.5}}$$

for  $-1 < \gamma < 0$

$$\text{sqrt}(1+x) - \text{sqrt}(1+1.29202967422) > (((\text{sqrt}(\ln(7)))) / (((2*5(\ln(5))^{1.5}))) - \text{sqrt}2$$

**Input interpretation:**

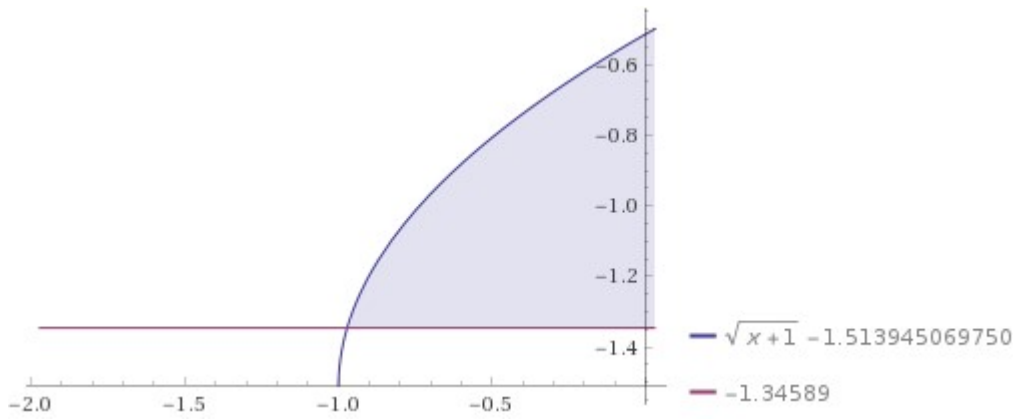
$$\sqrt{1+x} - \sqrt{1+1.29202967422} > \frac{\sqrt{\log(7)}}{2 \times 5 \log^{1.5}(5)} - \sqrt{2}$$

$\log(x)$  is the natural logarithm

**Result:**

$$\sqrt{x+1} - 1.513945069750 > -1.34589$$

**Inequality plot:**



**Alternate forms:**

$$x > -0.971759$$

$$\sqrt{x+1} > 0.168052$$

$$1.000000000000 \left( 1.000000000000 \sqrt{x+1} - 1.51394506975 \right) > -1.34589$$

**Alternate form assuming x>0:**

$$\sqrt{x+1} > 0.168052$$

**Solution:**

$$x > -0.971759$$

$$\text{sqrt}(1+0.971759) - \text{sqrt}(1+1.29202967422)$$

**Input interpretation:**

$$\sqrt{1+0.971759} - \sqrt{1+1.29202967422}$$

**Result:**

$$-0.109752\dots$$

$$-0.109752\dots$$

We obtain:

$$\left( \left( \left( \left( \sqrt{\ln(7)} \right) \right) / \left( \left( 2 * 5 (\ln(5))^{1.5} \right) \right) - \sqrt{2} \right) \right) / \left( \left( \sqrt{1+0.971759} \right) - \sqrt{1+1.29202967422} \right) \right)$$

**Input interpretation:**

$$\frac{\frac{\sqrt{\log(7)}}{2 \times 5 \log^{1.5}(5)} - \sqrt{2}}{\sqrt{1 + 0.971759} - \sqrt{1 + 1.29202967422}}$$

$\log(x)$  is the natural logarithm

**Result:**

12.2631...

12.2631... result very near to the black hole entropy 12.1904

**Alternative representations:**

$$\frac{\frac{\sqrt{\log(7)}}{2 \times 5 \log^{1.5}(5)} - \sqrt{2}}{\sqrt{1 + 0.971759} - \sqrt{1 + 1.292029674220000}} = \frac{-\sqrt{2} + \frac{\sqrt{\log_e(7)}}{10 \log_e^{1.5}(5)}}{\sqrt{1.97176} - \sqrt{2.292029674220000}}$$

$$\frac{\frac{\sqrt{\log(7)}}{2 \times 5 \log^{1.5}(5)} - \sqrt{2}}{\sqrt{1 + 0.971759} - \sqrt{1 + 1.292029674220000}} = \frac{-\sqrt{2} + \frac{\sqrt{\log(a) \log_a(7)}}{10 (\log(a) \log_a(5))^{1.5}}}{\sqrt{1.97176} - \sqrt{2.292029674220000}}$$

$$\frac{\frac{\sqrt{\log(7)}}{2 \times 5 \log^{1.5}(5)} - \sqrt{2}}{\sqrt{1 + 0.971759} - \sqrt{1 + 1.292029674220000}} = \frac{-\sqrt{2} + \frac{\sqrt{-\text{Li}_1(-6)}}{10 (-\text{Li}_1(-4))^{1.5}}}{\sqrt{1.97176} - \sqrt{2.292029674220000}}$$

### Series representations:

$$\frac{\frac{\sqrt{\log(7)}}{2 \times 5 \log^{1.5}(5)} - \sqrt{2}}{\sqrt{1 + 0.971759} - \sqrt{1 + 1.292029674220000}} =$$

$$-\left( \exp\left(i\pi \left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor\right) \log^{1.5}(5) \sum_{k=0}^{\infty} \frac{(-1)^k (2-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} - \right.$$

$$\left. 0.1 \exp\left(i\pi \left\lfloor \frac{\arg(-x + \log(7))}{2\pi} \right\rfloor\right) \sum_{k=0}^{\infty} \frac{(-1)^k x^{-k} (-x + \log(7))^k \left(-\frac{1}{2}\right)_k}{k!} \right) /$$

$$\left( \log^{1.5}(5) \sum_{k=0}^{\infty} \frac{1}{k!} (-1)^k x^{-k} \left( (1.97176 - x)^k \exp\left(i\pi \left\lfloor \frac{\arg(1.97176 - x)}{2\pi} \right\rfloor\right) - \right.$$

$$\left. \left. (2.292029674220000 - x)^k \exp\left(i\pi \left\lfloor \frac{\arg(2.292029674220000 - x)}{2\pi} \right\rfloor\right) \right) \right)$$

$$\left. \left(-\frac{1}{2}\right)_k \right) \text{ for } (x \in \mathbb{R} \text{ and } x < 0)$$

$$\frac{\frac{\sqrt{\log(7)}}{2 \times 5 \log^{1.5}(5)} - \sqrt{2}}{\sqrt{1 + 0.971759} - \sqrt{1 + 1.292029674220000}} =$$

$$-\left( \sqrt{x} \left( 10 \exp\left(i\pi \left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor\right) \left( \log(4) - \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{4}\right)^k}{k} \right)^{1.5} \sum_{k=0}^{\infty} \frac{(-1)^k (2-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} - \right.$$

$$\left. \exp\left(i\pi \left\lfloor \frac{\arg(-x + \log(7))}{2\pi} \right\rfloor\right) \sum_{k=0}^{\infty} \frac{(-1)^k x^{-k} (-x + \log(7))^k \left(-\frac{1}{2}\right)_k}{k!} \right) /$$

$$\left( 10 \left( \log(4) - \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{4}\right)^k}{k} \right)^{1.5} \sum_{k=0}^{\infty} \frac{1}{k!} (-1)^k x^{-k} \left( (1.97176 - x)^k \right.$$

$$\left. \exp\left(i\pi \left\lfloor \frac{\arg(1.97176 - x)}{2\pi} \right\rfloor\right) - (2.292029674220000 - x)^k \right.$$

$$\left. \left. \exp\left(i\pi \left\lfloor \frac{\arg(2.292029674220000 - x)}{2\pi} \right\rfloor\right) \right) \right)$$

$$\left. \left(-\frac{1}{2}\right)_k \sqrt{x} \right) \text{ for } (x \in \mathbb{R} \text{ and } x < 0)$$

$$\begin{aligned}
& \frac{\frac{\sqrt{\log(7)}}{2 \times 5 \log^{1.5}(5)} - \sqrt{2}}{\sqrt{1 + 0.971759} - \sqrt{1 + 1.292029674220000}} = \\
& - \left( \left( 10 \exp\left(i\pi \left[ \frac{\arg(2-x)}{2\pi} \right] \right) \left( 2i\pi \left[ \frac{\arg(5-x)}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (5-x)^k x^{-k}}{k} \right)^{1.5} \right. \right. \\
& \quad \left. \left. \sum_{k=0}^{\infty} \frac{(-1)^k (2-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} - \right. \right. \\
& \quad \left. \left. \exp\left(i\pi \left[ \frac{\arg(-x + \log(7))}{2\pi} \right] \right) \sum_{k=0}^{\infty} \frac{(-1)^k x^{-k} (-x + \log(7))^k \left(-\frac{1}{2}\right)_k}{k!} \right) / \right. \\
& \quad \left( 10 \left( 2i\pi \left[ \frac{\arg(5-x)}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (5-x)^k x^{-k}}{k} \right)^{1.5} \right. \\
& \quad \left. \sum_{k=0}^{\infty} \frac{1}{k!} (-1)^k x^{-k} \left( (1.97176-x)^k \exp\left(i\pi \left[ \frac{\arg(1.97176-x)}{2\pi} \right] \right) - \right. \right. \\
& \quad \left. \left. (2.292029674220000-x)^k \right. \right. \\
& \quad \left. \left. \exp\left(i\pi \left[ \frac{\arg(2.292029674220000-x)}{2\pi} \right] \right) \right) \right) \\
& \quad \left. \left. \left(-\frac{1}{2}\right)_k \right) \right) \text{ for } (x \in \mathbb{R} \text{ and } x < 0)
\end{aligned}$$

### Integral representations:

$$\begin{aligned}
& \frac{\frac{\sqrt{\log(7)}}{2 \times 5 \log^{1.5}(5)} - \sqrt{2}}{\sqrt{1 + 0.971759} - \sqrt{1 + 1.292029674220000}} = \\
& \frac{10 \left( \int_1^{5\frac{1}{t}} \frac{1}{t} dt \right)^{1.5} \sqrt{2} - \sqrt{\int_1^{7\frac{1}{t}} \frac{1}{t} dt}}{10 \left( \int_1^{5\frac{1}{t}} \frac{1}{t} dt \right)^{1.5} \left( \sqrt{1.97176} - \sqrt{2.292029674220000} \right)}
\end{aligned}$$

$$\begin{aligned}
& \frac{\frac{\sqrt{\log(7)}}{2 \times 5 \log^{1.5}(5)} - \sqrt{2}}{\sqrt{1 + 0.971759} - \sqrt{1 + 1.292029674220000}} = \\
& \frac{\left( \frac{1}{i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{4^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right)^{1.5} \sqrt{2} - 0.282843 \sqrt{\frac{1}{2i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{6^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds}}{\left( \frac{1}{i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{4^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right)^{1.5} \left( \sqrt{1.97176} - \sqrt{2.292029674220000} \right)} \\
& \text{for } -1 < \gamma < 0
\end{aligned}$$

And:

**Input interpretation:**

$$\frac{\sqrt{1 + 0.971759} - \sqrt{1 + 1.29202967422}}{\frac{\sqrt{\log(7)}}{2 \times 5 \log^{1.5}(5)} - \sqrt{2}}$$

$\log(x)$  is the natural logarithm

**Result:**

0.0815456...

0.0815456...

**Alternative representations:**

$$\frac{\sqrt{1 + 0.971759} - \sqrt{1 + 1.292029674220000}}{\frac{\sqrt{\log(7)}}{2 \times 5 \log^{1.5}(5)} - \sqrt{2}} = \frac{\sqrt{1.97176} - \sqrt{2.292029674220000}}{-\sqrt{2} + \frac{\sqrt{\log_e(7)}}{10 \log_e^{1.5}(5)}}$$

$$\frac{\sqrt{1 + 0.971759} - \sqrt{1 + 1.292029674220000}}{\frac{\sqrt{\log(7)}}{2 \times 5 \log^{1.5}(5)} - \sqrt{2}} = \frac{\sqrt{1.97176} - \sqrt{2.292029674220000}}{-\sqrt{2} + \frac{\sqrt{\log(a) \log_a(7)}}{10 (\log(a) \log_a(5))^{1.5}}}$$

$$\frac{\sqrt{1 + 0.971759} - \sqrt{1 + 1.292029674220000}}{\frac{\sqrt{\log(7)}}{2 \times 5 \log^{1.5}(5)} - \sqrt{2}} = \frac{\sqrt{1.97176} - \sqrt{2.292029674220000}}{-\sqrt{2} + \frac{\sqrt{-\text{Li}_1(-6)}}{10 (-\text{Li}_1(-4))^{1.5}}}$$

### Series representations:

$$\begin{aligned}
 & \frac{\sqrt{1+0.971759} - \sqrt{1+1.292029674220000}}{2 \times 5 \log^{1.5}(5)} - \sqrt{2} \\
 & - \left( \left( 10 \log^{1.5}(5) \sum_{k=0}^{\infty} \frac{1}{k!} (-1)^k x^{-k} \left( (1.97176 - x)^k \exp\left(i\pi \left[ \frac{\arg(1.97176 - x)}{2\pi} \right]\right) \right) \right. \right. \\
 & \quad \left. \left. (2.292029674220000 - x)^k \exp\left(i\pi \left[ \frac{\arg(2.292029674220000 - x)}{2\pi} \right]\right) \right) \right) \left(-\frac{1}{2}\right)_k / \\
 & \left( \sum_{k=0}^{\infty} \frac{1}{k!} (-1)^k x^{-k} \left( 10 (2 - x)^k \exp\left(i\pi \left[ \frac{\arg(2 - x)}{2\pi} \right]\right) \log^{1.5}(5) - \right. \right. \\
 & \quad \left. \left. \exp\left(i\pi \left[ \frac{\arg(-x + \log(7))}{2\pi} \right]\right) (-x + \log(7))^k \right) \right) \\
 & \left. \left(-\frac{1}{2}\right)_k \right) \text{ for } (x \in \mathbb{R} \text{ and } x < 0)
 \end{aligned}$$

$$\begin{aligned}
 & \frac{\sqrt{1+0.971759} - \sqrt{1+1.292029674220000}}{2 \times 5 \log^{1.5}(5)} - \sqrt{2} \\
 & - \left( \left( 10 \left( \log(4) - \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{4}\right)^k}{k} \right)^{1.5} \sum_{k=0}^{\infty} \frac{1}{k!} (-1)^k x^{-k} \left( (1.97176 - x)^k \right. \right. \right. \\
 & \quad \left. \left. \exp\left(i\pi \left[ \frac{\arg(1.97176 - x)}{2\pi} \right]\right) - (2.292029674220000 - x)^k \right. \right. \\
 & \quad \left. \left. \exp\left(i\pi \left[ \frac{\arg(2.292029674220000 - x)}{2\pi} \right]\right) \right) \right) \left(-\frac{1}{2}\right)_k \sqrt{x} \Big/ \\
 & \left( \sqrt{x} \left( 10 \exp\left(i\pi \left[ \frac{\arg(2 - x)}{2\pi} \right]\right) \left( \log(4) - \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{4}\right)^k}{k} \right)^{1.5} \right. \right. \\
 & \quad \left. \left. \sum_{k=0}^{\infty} \frac{(-1)^k (2 - x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} - \exp\left(i\pi \left[ \frac{\arg(-x + \log(7))}{2\pi} \right]\right) \right) \right. \\
 & \quad \left. \left. \sum_{k=0}^{\infty} \frac{(-1)^k x^{-k} (-x + \log(7))^k \left(-\frac{1}{2}\right)_k}{k!} \right) \right) \text{ for } (x \in \mathbb{R} \text{ and } x < 0)
 \end{aligned}$$



$$\begin{aligned}
& \frac{\sqrt{1+0.971759} - \sqrt{1+1.292029674220000}}{2 \times 5 \log^{1.5}(5)} = \\
& \frac{\sqrt{\log(7)} - \sqrt{2}}{2 \times 5 \log^{1.5}(5)} - \left( \left( 10 \log^{1.5}(5) \sum_{k=0}^{\infty} \frac{1}{k!} (-1)^k \left(-\frac{1}{2}\right)_k z_0^{-k} \left( (1.97176 - z_0)^k \right. \right. \right. \\
& \quad \left. \left. \left. \left(\frac{1}{z_0}\right)^{1/2 [\arg(1.97176 - z_0)/(2\pi)]} z_0^{1/2 [\arg(1.97176 - z_0)/(2\pi)]} - \right. \right. \\
& \quad \left. \left. \left. (2.292029674220000 - z_0)^k \left(\frac{1}{z_0}\right)^{1/2 [\arg(2.292029674220000 - z_0)/(2\pi)]} z_0^{1/2 [\arg(2.292029674220000 - z_0)/(2\pi)]} \right) \right) / \left( \sum_{k=0}^{\infty} \frac{1}{k!} (-1)^k \left(-\frac{1}{2}\right)_k z_0^{-k} \right. \right. \\
& \quad \left. \left. \left. \left( 10 \log^{1.5}(5) (2 - z_0)^k \left(\frac{1}{z_0}\right)^{1/2 [\arg(2 - z_0)/(2\pi)]} z_0^{1/2 [\arg(2 - z_0)/(2\pi)]} - \right. \right. \right. \\
& \quad \left. \left. \left. \left. (\log(7) - z_0)^k \left(\frac{1}{z_0}\right)^{1/2 [\arg(\log(7) - z_0)/(2\pi)]} z_0^{1/2 [\arg(\log(7) - z_0)/(2\pi)]} \right) \right) \right) \right) \right)
\end{aligned}$$

### Integral representations:

$$\begin{aligned}
& \frac{\sqrt{1+0.971759} - \sqrt{1+1.292029674220000}}{2 \times 5 \log^{1.5}(5)} = \\
& \frac{\sqrt{\log(7)} - \sqrt{2}}{2 \times 5 \log^{1.5}(5)} - \frac{10 \left( \int_1^5 \frac{1}{t} dt \right)^{1.5} \left( \sqrt{1.97176} - \sqrt{2.292029674220000} \right)}{10 \left( \int_1^5 \frac{1}{t} dt \right)^{1.5} \sqrt{2} - \sqrt{\int_1^7 \frac{1}{t} dt}}
\end{aligned}$$

$$\begin{aligned}
& \frac{\sqrt{1+0.971759} - \sqrt{1+1.292029674220000}}{2 \times 5 \log^{1.5}(5)} = \\
& \frac{\sqrt{\log(7)} - \sqrt{2}}{2 \times 5 \log^{1.5}(5)} - \frac{\left( \frac{1}{i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{4^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right)^{1.5} \left( \sqrt{1.97176} - \sqrt{2.292029674220000} \right)}{\left( \frac{1}{i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{4^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right)^{1.5} \sqrt{2} - 0.282843 \sqrt{\frac{1}{2i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{6^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds}} \\
& \text{for } -1 < \gamma < 0
\end{aligned}$$

Now, we have that:

$$-\log \left\{ \left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{5}\right) \cdots \left(1 - \frac{1}{p}\right) \right\} = \log \log p + \gamma + O\left(\frac{1}{\log p}\right), \quad (177)$$

For  $p = 11$ , we obtain:

$$-\ln\left(\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{5}\right)\left(1 - \frac{1}{7}\right)\left(1 - \frac{1}{11}\right)\right) = \ln \ln (11) + 0.5772156649 + O(1/(\ln(11)))$$

Now:

$$\left(-\ln\left(\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{5}\right)\left(1 - \frac{1}{7}\right)\left(1 - \frac{1}{11}\right)\right)\right)$$

**Input:**

$$-\log\left(\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{5}\right)\left(1 - \frac{1}{7}\right)\left(1 - \frac{1}{11}\right)\right)$$

$\log(x)$  is the natural logarithm

**Exact result:**

$$\log\left(\frac{77}{16}\right)$$

**Decimal approximation:**

1.571216699613902611498367835575602757156791046078257347195...

1.5712166996...

**Property:**

$\log\left(\frac{77}{16}\right)$  is a transcendental number

**Alternate forms:**

$$\log(77) - 4 \log(2)$$

$$-4 \log(2) + \log(7) + \log(11)$$

**Alternative representations:**

$$-\log\left(\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{5}\right)\left(1 - \frac{1}{7}\right)\left(1 - \frac{1}{11}\right)\right) = -\log_e\left(\frac{1}{3}\left(1 - \frac{1}{5}\right)\left(1 - \frac{1}{7}\right)\left(1 - \frac{1}{11}\right)\right)$$

$$-\log\left(\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{5}\right)\left(1 - \frac{1}{7}\right)\left(1 - \frac{1}{11}\right)\right) = -\log(a) \log_a\left(\frac{1}{3}\left(1 - \frac{1}{5}\right)\left(1 - \frac{1}{7}\right)\left(1 - \frac{1}{11}\right)\right)$$

$$-\log\left(\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{5}\right)\left(1 - \frac{1}{7}\right)\left(1 - \frac{1}{11}\right)\right) = \text{Li}_1\left(1 - \frac{1}{3}\left(1 - \frac{1}{5}\right)\left(1 - \frac{1}{7}\right)\left(1 - \frac{1}{11}\right)\right)$$

### Series representations:

$$-\log\left(\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{5}\right)\left(1 - \frac{1}{7}\right)\left(1 - \frac{1}{11}\right)\right) = \log\left(\frac{61}{16}\right) - \sum_{k=1}^{\infty} \frac{\left(-\frac{16}{61}\right)^k}{k}$$

$$-\log\left(\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{5}\right)\left(1 - \frac{1}{7}\right)\left(1 - \frac{1}{11}\right)\right) = 2i\pi \left\lfloor \frac{\arg\left(\frac{77}{16} - x\right)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{77}{16} - x\right)^k x^{-k}}{k} \quad \text{for } x < 0$$

$$-\log\left(\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{5}\right)\left(1 - \frac{1}{7}\right)\left(1 - \frac{1}{11}\right)\right) = \left\lfloor \frac{\arg\left(\frac{77}{16} - z_0\right)}{2\pi} \right\rfloor \log\left(\frac{1}{z_0}\right) + \log(z_0) + \left\lfloor \frac{\arg\left(\frac{77}{16} - z_0\right)}{2\pi} \right\rfloor \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{77}{16} - z_0\right)^k z_0^{-k}}{k}$$

### Integral representations:

$$-\log\left(\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{5}\right)\left(1 - \frac{1}{7}\right)\left(1 - \frac{1}{11}\right)\right) = \int_1^{\frac{77}{16}} \frac{1}{t} dt$$

$$-\log\left(\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{5}\right)\left(1 - \frac{1}{7}\right)\left(1 - \frac{1}{11}\right)\right) = -\frac{i}{2\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\left(\frac{16}{61}\right)^s \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds$$

for  $-1 < \gamma < 0$

For  $O = x$ , we obtain the following equation:

$$\left(-\ln\left(\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{5}\right)\left(1 - \frac{1}{7}\right)\left(1 - \frac{1}{11}\right)\right)\right) = \ln \ln(11) + 0.5772156649 + x \left(\frac{1}{\ln(11)}\right)$$

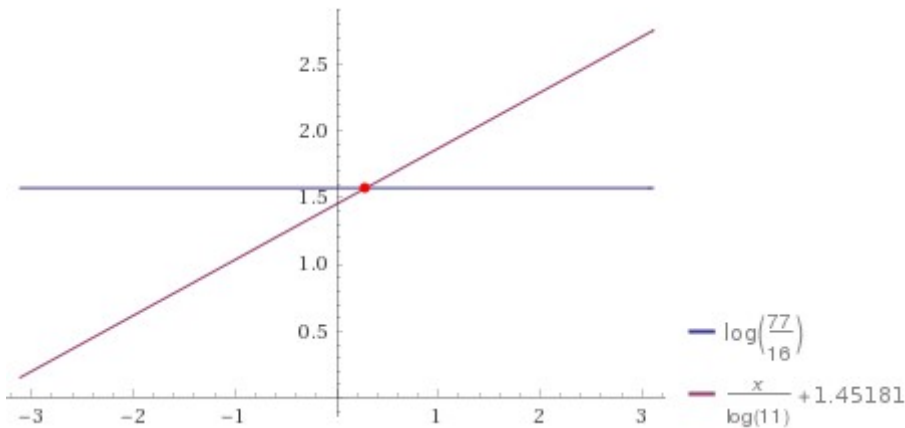
### Input interpretation:

$$-\log\left(\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{5}\right)\left(1 - \frac{1}{7}\right)\left(1 - \frac{1}{11}\right)\right) = \log(\log(11)) + 0.5772156649 + x \times \frac{1}{\log(11)}$$

$\log(x)$  is the natural logarithm

**Result:**

$$\log\left(\frac{77}{16}\right) = \frac{x}{\log(11)} + 1.45181$$

**Plot:****Alternate forms:**

$$\log\left(\frac{77}{16}\right) = 0.417032x + 1.45181$$

$$\log\left(\frac{77}{16}\right) = 0.417032(x + 3.48128)$$

$$0.11941 - \frac{x}{\log(11)} = 0$$

**Alternate form assuming  $x > 0$ :**

$$-4 \log(2) + \log(7) + \log(11) = \frac{x}{\log(11)} + 1.45181$$

**Solution:**

$$x \approx 0.286332$$

$$0.286332$$

thence:

$$\ln \ln(11) + 0.5772156649 + 0.286332 \left( \frac{1}{\ln(11)} \right)$$

**Input interpretation:**

$$\log(\log(11)) + 0.5772156649 + 0.286332 \times \frac{1}{\log(11)}$$

$\log(x)$  is the natural logarithm

**Result:**

1.571216766524976218615076662164065464382893231096006263247...

1.571216766...

**Alternative representations:**

$$\log(\log(11)) + 0.577216 + \frac{0.286332}{\log(11)} = 0.577216 + \log_e(\log(11)) + \frac{0.286332}{\log_e(11)}$$

$$\log(\log(11)) + 0.577216 + \frac{0.286332}{\log(11)} = 0.577216 + \log(a) \log_a(\log(11)) + \frac{0.286332}{\log(a) \log_a(11)}$$

$$\log(\log(11)) + 0.577216 + \frac{0.286332}{\log(11)} = 0.577216 - \text{Li}_1(1 - \log(11)) + \frac{0.286332}{\text{Li}_1(-10)}$$

**Series representations:**

$$\log(\log(11)) + 0.577216 + \frac{0.286332}{\log(11)} = 0.577216 + 2i\pi \left[ \frac{\arg(-x + \log(11))}{2\pi} \right] + \frac{0.286332}{\log(x) + \frac{2i\pi \left[ \frac{\arg(11-x)}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (11-x)^k x^{-k}}{k}}{\sum_{k=1}^{\infty} \frac{(-1)^k x^{-k} (-x + \log(11))^k}{k}} \text{ for } x < 0$$

$$\log(\log(11)) + 0.577216 + \frac{0.286332}{\log(11)} = 0.577216 + \left[ \frac{\arg(\log(11) - z_0)}{2\pi} \right] \log\left(\frac{1}{z_0}\right) + \log(z_0) + \left[ \frac{\arg(\log(11) - z_0)}{2\pi} \right] \log(z_0) + \frac{0.286332}{\log(z_0) + \left[ \frac{\arg(11-z_0)}{2\pi} \right] \left( \log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (11-z_0)^k z_0^{-k}}{k}}{\sum_{k=1}^{\infty} \frac{(-1)^k (\log(11) - z_0)^k z_0^{-k}}{k}}$$

$$\log(\log(11)) + 0.577216 + \frac{0.286332}{\log(11)} =$$

$$\left( 0.286332 + 0.577216 \log(10) + \log(10) \log(-1 + \log(11)) - 0.577216 \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{10}\right)^k}{k} - \right.$$

$$\left. \log(-1 + \log(11)) \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{10}\right)^k}{k} - \log(10) \sum_{k=1}^{\infty} \frac{(-1)^k (-1 + \log(11))^{-k}}{k} + \right.$$

$$\left. \sum_{k_1=1}^{\infty} \sum_{k_2=1}^{\infty} \frac{(-1)^{k_1+k_2} 10^{-k_1} (-1 + \log(11))^{-k_2}}{k_1 k_2} \right) / \left( \log(10) - \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{10}\right)^k}{k} \right)$$

### Integral representations:

$$\log(\log(11)) + 0.577216 + \frac{0.286332}{\log(11)} =$$

$$\frac{0.286332 + 0.577216 \int_1^{11} \frac{1}{t} dt + \int_0^1 \int_0^1 \frac{1}{(1+10t_1)(1+(-1+\log(11))t_2)} dt_2 dt_1}{\int_1^{11} \frac{1}{t} dt}$$

$$\log(\log(11)) + 0.577216 + \frac{0.286332}{\log(11)} =$$

$$\left( 0.572664 \left( i^2 \pi^2 + 1.00795 i \pi \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{10^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds + \right. \right.$$

$$\left. 0.873112 \left( \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{10^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right) \right.$$

$$\left. \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2 \Gamma(1+s) (-1 + \log(11))^{-s}}{\Gamma(1-s)} ds \right) /$$

$$\left( i \pi \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{10^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right) \text{ for } -1 < \gamma < 0$$

We note that:

$$1 + 1 / (((((\ln \ln(11) + 0.5772156649 + 0.286332(1/(\ln(11))))))))) - 18/10^3$$

### Input interpretation:

$$1 + \frac{1}{\log(\log(11)) + 0.5772156649 + 0.286332 \times \frac{1}{\log(11)}} - \frac{18}{10^3}$$

$\log(x)$  is the natural logarithm

**Result:**

1.618449420159687368092156155396148950634541840570133136469...

1.61844942... result that is a very good approximation to the value of the golden ratio 1,618033988749...

**Alternative representations:**

$$1 + \frac{1}{\log(\log(11)) + 0.577216 + \frac{0.286332}{\log(11)}} - \frac{18}{10^3} =$$

$$1 - \frac{18}{10^3} + \frac{1}{0.577216 + \log_e(\log(11)) + \frac{0.286332}{\log_e(11)}}$$

$$1 + \frac{1}{\log(\log(11)) + 0.577216 + \frac{0.286332}{\log(11)}} - \frac{18}{10^3} =$$

$$1 - \frac{18}{10^3} + \frac{1}{0.577216 + \log(a) \log_a(\log(11)) + \frac{0.286332}{\log(a) \log_a(11)}}$$

$$1 + \frac{1}{\log(\log(11)) + 0.577216 + \frac{0.286332}{\log(11)}} - \frac{18}{10^3} =$$

$$1 - \frac{18}{10^3} + \frac{1}{0.577216 - \text{Li}_1(1 - \log(11)) + -\frac{0.286332}{\text{Li}_1(-10)}}$$

**Series representations:**

$$1 + \frac{1}{\log(\log(11)) + 0.577216 + \frac{0.286332}{\log(11)}} - \frac{18}{10^3} =$$

$$\frac{491}{500} + \frac{1}{0.577216 + \log(-1 + \log(11)) + \frac{0.286332}{\log(10) - \sum_{k=1}^{\infty} \frac{(-1)^k}{k}} - \sum_{k=1}^{\infty} \frac{(-1)^k (-1 + \log(11))^{-k}}{k}}$$

$$1 + \frac{1}{\log(\log(11)) + 0.577216 + \frac{0.286332}{\log(11)}} - \frac{18}{10^3} =$$

$$\frac{491}{500} + 1 \left/ \left( 0.577216 + 2 i \pi \left\lfloor \frac{\arg(-x + \log(11))}{2 \pi} \right\rfloor + \frac{0.286332}{2 i \pi \left\lfloor \frac{\arg(11-x)}{2 \pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (11-x)^k x^{-k}}{k}} \right) \right.$$

$$\left. \sum_{k=1}^{\infty} \frac{(-1)^k x^{-k} (-x + \log(11))^k}{k} \right) \text{ for } x < 0$$

$$1 + \frac{1}{\log(\log(11)) + 0.577216 + \frac{0.286332}{\log(11)}} - \frac{18}{10^3} = \frac{491}{500} + 1 / \left( \frac{0.577216 + \log(z_0) + \left\lfloor \frac{\arg(\log(11) - z_0)}{2\pi} \right\rfloor \left( \log\left(\frac{1}{z_0}\right) + \log(z_0) \right) + \frac{0.286332}{\log(z_0) + \left\lfloor \frac{\arg(11-z_0)}{2\pi} \right\rfloor \left( \log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (11-z_0)^k z_0^{-k}}{k}}{\sum_{k=1}^{\infty} \frac{(-1)^k (\log(11) - z_0)^k z_0^{-k}}{k}} \right)$$

**Integral representation:**

$$1 + \frac{1}{\log(\log(11)) + 0.577216 + \frac{0.286332}{\log(11)}} - \frac{18}{10^3} = \left( 0.982 \left( i^2 \pi^2 + 2.78618 i \pi \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{10^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds + 0.873112 \left( \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{10^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right) \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2 \Gamma(1+s) (-1 + \log(11))^{-s}}{\Gamma(1-s)} ds \right) \right) / \left( i^2 \pi^2 + 1.00795 i \pi \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{10^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds + 0.873112 \left( \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{10^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right) \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2 \Gamma(1+s) (-1 + \log(11))^{-s}}{\Gamma(1-s)} ds \right) \text{ for } -1 < \gamma < 0$$

And:

$$1/10^{27} \left( \left( \left( 1 + 1 / \left( \left( \ln \ln(11) + 0.5772156649 + 0.286332 / \ln(11) \right) \right) \right) \right) + (29+7)/10^3 \right)$$

**Input interpretation:**

$$\frac{1}{10^{27}} \left( 1 + \frac{1}{\log(\log(11)) + 0.5772156649 + 0.286332 \times \frac{1}{\log(11)}} + \frac{29+7}{10^3} \right)$$

log(x) is the natural logarithm



**Result:**

$$1.6724494... \times 10^{-27}$$

1.6724494... \* 10<sup>-27</sup> result practically equal to the proton mass in kg

**Alternative representations:**

$$\frac{1 + \frac{1}{\log(\log(11)) + 0.577216 + \frac{0.286332}{\log(11)}} + \frac{29+7}{10^3}}{10^{27}} = \frac{1 + \frac{36}{10^3} + \frac{1}{0.577216 + \log_e(\log(11)) + \frac{0.286332}{\log_e(11)}}}{10^{27}}$$

$$\frac{1 + \frac{1}{\log(\log(11)) + 0.577216 + \frac{0.286332}{\log(11)}} + \frac{29+7}{10^3}}{10^{27}} = \frac{1 + \frac{36}{10^3} + \frac{1}{0.577216 + \log(a) \log_a(\log(11)) + \frac{0.286332}{\log(a) \log_a(11)}}}{10^{27}}$$

$$\frac{1 + \frac{1}{\log(\log(11)) + 0.577216 + \frac{0.286332}{\log(11)}} + \frac{29+7}{10^3}}{10^{27}} = \frac{1 + \frac{36}{10^3} + \frac{1}{0.577216 - \text{Li}_1(1 - \log(11)) + \frac{0.286332}{\text{Li}_1(-10)}}}{10^{27}}$$

**Series representations:**

$$\frac{1 + \frac{1}{\log(\log(11)) + 0.577216 + \frac{0.286332}{\log(11)}} + \frac{29+7}{10^3}}{10^{27}} = \frac{259}{250\,000\,000\,000\,000\,000\,000\,000\,000} +$$

$$1 / \left( 1000\,000\,000\,000\,000\,000\,000\,000\,000 \left( 0.577216 + \log(-1 + \log(11)) + \right.$$

$$\left. \frac{0.286332}{\log(10) - \sum_{k=1}^{\infty} \frac{(-\frac{1}{10})^k}{k}} - \sum_{k=1}^{\infty} \frac{(-1)^k (-1 + \log(11))^{-k}}{k} \right)$$

$$\begin{aligned}
& \frac{1 + \frac{1}{\log(\log(11)) + 0.577216 + \frac{0.286332}{\log(11)}} + \frac{29+7}{10^3}}{10^{27}} = \\
& \frac{259}{250\,000\,000\,000\,000\,000\,000\,000\,000} + 1 / \\
& \left( 1\,000\,000\,000\,000\,000\,000\,000\,000\,000 \left( 0.577216 + 2i\pi \left[ \frac{\arg(-x + \log(11))}{2\pi} \right] + \right. \right. \\
& \quad \left. \frac{\log(x) + \frac{0.286332}{2i\pi \left[ \frac{\arg(11-x)}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (11-x)^k x^{-k}}{k}}{\sum_{k=1}^{\infty} \frac{(-1)^k x^{-k} (-x + \log(11))^k}{k}} \right) \right) \text{ for } x < 0
\end{aligned}$$

$$\begin{aligned}
& \frac{1 + \frac{1}{\log(\log(11)) + 0.577216 + \frac{0.286332}{\log(11)}} + \frac{29+7}{10^3}}{10^{27}} = \frac{259}{250\,000\,000\,000\,000\,000\,000\,000\,000} + \\
& 1 / \left( 1\,000\,000\,000\,000\,000\,000\,000\,000\,000 \right. \\
& \quad \left( 0.577216 + \log(z_0) + \left[ \frac{\arg(\log(11) - z_0)}{2\pi} \right] \left( \log\left(\frac{1}{z_0}\right) + \log(z_0) \right) + \right. \\
& \quad \left. \frac{0.286332}{\log(z_0) + \left[ \frac{\arg(11-z_0)}{2\pi} \right] \left( \log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (11-z_0)^k z_0^{-k}}{k}}{\sum_{k=1}^{\infty} \frac{(-1)^k (\log(11) - z_0)^k z_0^{-k}}{k}} \right) \right)
\end{aligned}$$

**Integral representation:**

$$\begin{aligned}
 & \frac{1 + \frac{1}{\log(\log(11)) + 0.577216 + \frac{0.286332}{\log(11)}} + \frac{29+7}{10^3}}{10^{27}} = \\
 & \left( 1.036 \times 10^{-27} i^2 \pi^2 + 2.79046 \times 10^{-27} i \pi \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{10^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds + \right. \\
 & \quad \left. 9.04544 \times 10^{-28} \left( \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{10^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right) \right. \\
 & \quad \left. \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2 \Gamma(1+s) (-1 + \log(11))^{-s}}{\Gamma(1-s)} ds \right) / \\
 & \left( i^2 \pi^2 + 1.00795 i \pi \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{10^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds + \right. \\
 & \quad \left. 0.873112 \left( \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{10^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right) \right. \\
 & \quad \left. \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2 \Gamma(1+s) (-1 + \log(11))^{-s}}{\Gamma(1-s)} ds \right) \text{ for } -1 < \gamma < 0
 \end{aligned}$$

1/10^52 (((((((ln ln (11) + 0.5772156649 + 0.286332(1/(ln(11)))))))- (4Pi/27) - (golden ratio)/10^4)))

**Input interpretation:**

$$\frac{1}{10^{52}} \left( \left( \log(\log(11)) + 0.5772156649 + 0.286332 \times \frac{1}{\log(11)} \right) - 4 \times \frac{\pi}{27} - \frac{\phi}{10^4} \right)$$

log(x) is the natural logarithm

φ is the golden ratio

**Result:**

$$1.105634... \times 10^{-52}$$

1.105634... \* 10<sup>-52</sup> result practically equal to the value of Cosmological Constant

$$1.1056 * 10^{-52} \text{ m}^{-2}$$

**Alternative representations:**

$$\begin{aligned}
 & \frac{\left( \log(\log(11)) + 0.577216 + \frac{0.286332}{\log(11)} \right) - \frac{4\pi}{27} - \frac{\phi}{10^4}}{10^{52}} = \\
 & \frac{0.577216 + \log_e(\log(11)) - \frac{4\pi}{27} + \frac{0.286332}{\log_e(11)} - \frac{\phi}{10^4}}{10^{52}}
 \end{aligned}$$

$$\frac{(\log(\log(11)) + 0.577216 + \frac{0.286332}{\log(11)}) - \frac{4\pi}{27} - \frac{\phi}{10^4}}{10^{52}} =$$

$$\frac{0.577216 + \log(a) \log_a(\log(11)) - \frac{4\pi}{27} + \frac{0.286332}{\log(a) \log_a(11)} - \frac{\phi}{10^4}}{10^{52}}$$

$$\frac{(\log(\log(11)) + 0.577216 + \frac{0.286332}{\log(11)}) - \frac{4\pi}{27} - \frac{\phi}{10^4}}{10^{52}} =$$

$$\frac{0.577216 - \text{Li}_1(1 - \log(11)) - \frac{4\pi}{27} + \frac{0.286332}{\text{Li}_1(-10)} - \frac{\phi}{10^4}}{10^{52}}$$

### Series representations:

$$\frac{(\log(\log(11)) + 0.577216 + \frac{0.286332}{\log(11)}) - \frac{4\pi}{27} - \frac{\phi}{10^4}}{10^{52}} =$$

$$-\left(1. \times 10^{-56} \left( -2863.32 - 5772.16 \log(10) + \phi \log(10) + 1481.48 \pi \log(10) - \right.$$

$$10\,000 \log(10) \log(-1 + \log(11)) + 5772.16 \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{10}\right)^k}{k} - \phi \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{10}\right)^k}{k} -$$

$$1481.48 \pi \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{10}\right)^k}{k} + 10\,000 \log(-1 + \log(11)) \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{10}\right)^k}{k} +$$

$$10\,000 \log(10) \sum_{k=1}^{\infty} \frac{(-1)^k (-1 + \log(11))^{-k}}{k} - 10\,000$$

$$\left. \sum_{k_1=1}^{\infty} \sum_{k_2=1}^{\infty} \frac{(-1)^{k_1+k_2} 10^{-k_1} (-1 + \log(11))^{-k_2}}{k_1 k_2} \right) \Bigg/ \left( \log(10) - \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{10}\right)^k}{k} \right)$$



$$\frac{(\log(\log(11)) + 0.577216 + \frac{0.286332}{\log(11)}) - \frac{4\pi}{27} - \frac{\phi}{10^4}}{10^{52}} =$$

$$\left( 5.72664 \times 10^{-53} i^2 \pi^2 + 5.77216 \times 10^{-53} i \pi \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{10^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds - \right.$$

$$1 \times 10^{-56} \phi i \pi \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{10^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds - 1.48148 \times 10^{-53} i \pi^2$$

$$\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{10^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds + 5 \times 10^{-53} \left( \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{10^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right)$$

$$\left. \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2 \Gamma(1+s) (-1 + \log(11))^{-s}}{\Gamma(1-s)} ds \right) /$$

$$\left( i \pi \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{10^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right) \text{ for } -1 < \gamma < 0$$

We have that:

$$= - \frac{\log p_1}{\log 2} \log \left\{ \left(1 - \frac{1}{2}\right) \left(1 - \frac{1}{3}\right) \cdots \left(1 - \frac{1}{\lambda}\right) \right\}$$

$$+ O\{\sqrt{(\log p_1 \log \log p_1)} \log \log \log p_1\}.$$

$$-((\ln(7)/\ln(2))) \ln((1-1/2)(1-1/3)(1-1/5))+((((\sqrt{\ln(7)\ln \ln(7))})\ln \ln \ln(7))))$$

**Input:**

$$-\frac{\log(7)}{\log(2)} \log\left(\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{5}\right)\right) + \sqrt{\log(7) \log(\log(7))} \log(\log(\log(7)))$$

$\log(x)$  is the natural logarithm

**Exact result:**

$$\frac{\log\left(\frac{15}{4}\right) \log(7)}{\log(2)} + \sqrt{\log(7) \log(\log(7))} \log(\log(\log(7)))$$

**Decimal approximation:**

3.247545488142725567782775943872167292974537996998315914452...

3.247545488...

**Alternate forms:**

$$-2 \log(7) + \frac{\log(7) \log(15)}{\log(2)} + \sqrt{\log(7) \log(\log(7))} \log(\log(\log(7)))$$

$$\frac{\log\left(\frac{15}{4}\right) \log(7) + \log(2) \sqrt{\log(7) \log(\log(7))} \log(\log(\log(7)))}{\log(2)}$$

$$\frac{\sqrt{\log(7)} \left( (-2 \log(2) + \log(3) + \log(5)) \sqrt{\log(7)} + \log(2) \sqrt{\log(\log(7))} \log(\log(\log(7))) \right)}{\log(2)}$$

### Alternative representations:

$$\begin{aligned} & - \frac{\log\left(\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{5}\right)\right) \log(7)}{\log(2)} + \sqrt{\log(7) \log(\log(7))} \log(\log(\log(7))) = \\ & - \frac{\log^2(a) \log_a(7) \log_a\left(\frac{1}{3}\left(1 - \frac{1}{5}\right)\right)}{\log(a) \log_a(2)} + \log(a) \log_a(\log(\log(7))) \sqrt{\log^2(a) \log_a(7) \log_a(\log(7))} \end{aligned}$$

$$\begin{aligned} & - \frac{\log\left(\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{5}\right)\right) \log(7)}{\log(2)} + \sqrt{\log(7) \log(\log(7))} \log(\log(\log(7))) = \\ & - \frac{\log_e(7) \log_e\left(\frac{1}{3}\left(1 - \frac{1}{5}\right)\right)}{\log_e(2)} + \log_e(\log(\log(7))) \sqrt{\log_e(7) \log_e(\log(7))} \end{aligned}$$

$$\begin{aligned} & - \frac{\log\left(\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{5}\right)\right) \log(7)}{\log(2)} + \sqrt{\log(7) \log(\log(7))} \log(\log(\log(7))) = \\ & \frac{-\text{Li}_1(-6) \text{Li}_1\left(1 + \frac{1}{3}\left(-1 + \frac{1}{5}\right)\right)}{-\text{Li}_1(-1)} - \text{Li}_1(1 - \log(\log(7))) \sqrt{\text{Li}_1(-6) \text{Li}_1(1 - \log(7))} \end{aligned}$$

### Series representations:

$$\begin{aligned} & - \frac{\log\left(\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{5}\right)\right) \log(7)}{\log(2)} + \sqrt{\log(7) \log(\log(7))} \log(\log(\log(7))) = \\ & \frac{1}{\log(4)} \left( 2 \log\left(\frac{15}{4}\right) \log(7) + \right. \\ & \left. \log(2) \sqrt{6 \log(\log(7))} \log(\log(\log(7))) \sum_{k=0}^{\infty} 6^k \binom{-\frac{1}{2} + k}{k} \sum_{j=0}^k \frac{2(-1)^j \binom{k}{j} p_{j,k}}{-1 + 2j} \right) \end{aligned}$$

$$\text{for } \left( c_k = \frac{(-1)^k \log(\log(7))}{1 + k} \text{ and } p_{j,0} = 1 \text{ and} \right.$$

$$\left. \log(\log(7)) p_{j,k} = \frac{\sum_{m=1}^k (-k + m + j m) c_m p_{j,k-m}}{k} \text{ and } k \in \mathbb{Z} \text{ and } k > 0 \right)$$

$$\begin{aligned}
& - \frac{\log\left(\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{5}\right)\right) \log(7)}{\log(2)} + \sqrt{\log(7) \log(\log(7)) \log(\log(\log(7)))} = \\
& \frac{1}{2 \log(2)} \left( 2 \log\left(\frac{15}{4}\right) \log(7) + e^{i\pi \left[1/2 - \arg\left(\frac{1}{e}\right)/(2\pi)\right]} \log(2) \sqrt{6 \log(\log(7))} \right. \\
& \left. \log(\log(\log(7))) \sum_{k=0}^{\infty} 6^k \binom{-\frac{1}{2} + k}{k} \sum_{j=0}^k \frac{2(-1)^j \binom{k}{j} p_{j,k}}{-1 + 2j} \right) \\
& \text{for } \left( c_k = \frac{(-1)^k \log(\log(7))}{1+k} \text{ and } p_{j,0} = 1 \text{ and} \right. \\
& \left. \log(\log(7)) p_{j,k} = \frac{\sum_{m=1}^k (-k+m+j)m c_m p_{j,k-m}}{k} \text{ and } k \in \mathbb{Z} \text{ and } k > 0 \right)
\end{aligned}$$

$$\begin{aligned}
& - \frac{\log\left(\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{5}\right)\right) \log(7)}{\log(2)} + \sqrt{\log(7) \log(\log(7)) \log(\log(\log(7)))} = \\
& \left( \left( 2i\pi \left[ \frac{\arg\left(\frac{15-x}{4}\right)}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{15-x}{4}\right)^k x^{-k}}{k} \right) \right. \\
& \left. \left( 2i\pi \left[ \frac{\arg(7-x)}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (7-x)^k x^{-k}}{k} \right) \right) / \\
& \left( 2i\pi \left[ \frac{\arg(2-x)}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k} \right) + \\
& \sqrt{\left( \left( 2i\pi \left[ \frac{\arg(7-x)}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (7-x)^k x^{-k}}{k} \right) \right. \\
& \left. \left( 2i\pi \left[ \frac{\arg(-x + \log(7))}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k x^{-k} (-x + \log(7))^k}{k} \right) \right) \\
& \left( 2i\pi \left[ \frac{\arg(-x + \log(\log(7)))}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k x^{-k} (-x + \log(\log(7)))^k}{k} \right) \\
& \text{for } x < 0
\end{aligned}$$

### Integral representation:

$$\begin{aligned}
& - \frac{\log\left(\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{5}\right)\right) \log(7)}{\log(2)} + \sqrt{\log(7) \log(\log(7)) \log(\log(\log(7)))} = \\
& \frac{4 \int_0^1 \int_0^1 \frac{1}{(4+11t_1)(1+6t_2)} dt_2 dt_1 + \int_0^1 \int_0^1 \frac{1}{(1+t_1)(1+(-1+\log(\log(7)))t_2)} dt_2 dt_1}{\int_1^2 \frac{1}{t} dt}
\end{aligned}$$



$$1/10^{27}((((1/2[-((\ln(7)/\ln(2))) \ln((1-1/2)(1-1/3)(1-1/5))+((((\sqrt{\ln(7)\ln \ln(7))\ln \ln \ln(7)))]+(47+2)/10^3))))))$$

**Input:**

$$\frac{1}{10^{27}} \left( \frac{1}{2} \left( -\frac{\log(7)}{\log(2)} \log\left(\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{5}\right)\right) + \sqrt{\log(7) \log(\log(7)) \log(\log(\log(7)))} \right) + \frac{47+2}{10^3} \right)$$

log(x) is the natural logarithm

**Exact result:**

$$\frac{\frac{49}{1000} + \frac{1}{2} \left( \frac{\log\left(\frac{15}{4}\right)\log(7)}{\log(2)} + \sqrt{\log(7) \log(\log(7)) \log(\log(\log(7)))} \right)}{1\,000\,000\,000\,000\,000\,000\,000\,000\,000}$$

**Decimal approximation:**

$$1.6727727440713627838913879719360836464872689984991579... \times 10^{-27}$$

1.672772744... \* 10<sup>-27</sup> result practically equal to the proton mass in kg

**Alternate forms:**

$$\frac{\frac{49}{1000} + \frac{\log\left(\frac{15}{4}\right)\log(7)}{\log(4)} + \frac{1}{2} \sqrt{\log(7) \log(\log(7)) \log(\log(\log(7)))}}{1\,000\,000\,000\,000\,000\,000\,000\,000\,000}$$

$$\frac{\frac{49}{1\,000\,000\,000\,000\,000\,000\,000\,000\,000} + \frac{\log\left(\frac{15}{4}\right)\log(7)}{2\,000\,000\,000\,000\,000\,000\,000\,000\,000 \log(2)} + \frac{\sqrt{\log(7) \log(\log(7)) \log(\log(\log(7)))}}{2\,000\,000\,000\,000\,000\,000\,000\,000\,000}}$$

$$\frac{\frac{49}{1000} + \frac{\log\left(\frac{15}{4}\right)\log(7)+\log(2) \sqrt{\log(7) \log(\log(7)) \log(\log(\log(7)))}}{\log(4)}}{1\,000\,000\,000\,000\,000\,000\,000\,000\,000}$$

**Alternative representations:**

$$\begin{aligned}
 & \frac{\frac{1}{2} \left( -\frac{\log(7) \log\left(\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{5}\right)\right)}{\log(2)} + \sqrt{\log(7) \log(\log(7)) \log(\log(\log(7)))} \right) + \frac{47+2}{10^3}}{10^{27}} = \\
 & \frac{1}{10^{27}} \left( \frac{49}{10^3} + \frac{1}{2} \left( -\frac{\log^2(a) \log_a(7) \log_a\left(\frac{1}{3}\left(1-\frac{1}{5}\right)\right)}{\log(a) \log_a(2)} + \right. \right. \\
 & \quad \left. \left. \log(a) \log_a(\log(\log(7))) \sqrt{\log^2(a) \log_a(7) \log_a(\log(7))} \right) \right) \\
 & \frac{\frac{1}{2} \left( -\frac{\log(7) \log\left(\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{5}\right)\right)}{\log(2)} + \sqrt{\log(7) \log(\log(7)) \log(\log(\log(7)))} \right) + \frac{47+2}{10^3}}{10^{27}} = \\
 & \frac{\frac{49}{10^3} + \frac{1}{2} \left( -\frac{\log_e(7) \log_e\left(\frac{1}{3}\left(1-\frac{1}{5}\right)\right)}{\log_e(2)} + \log_e(\log(\log(7))) \sqrt{\log_e(7) \log_e(\log(7))} \right)}{10^{27}} \\
 & \frac{\frac{1}{2} \left( -\frac{\log(7) \log\left(\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{5}\right)\right)}{\log(2)} + \sqrt{\log(7) \log(\log(7)) \log(\log(\log(7)))} \right) + \frac{47+2}{10^3}}{10^{27}} = \\
 & \frac{\frac{49}{10^3} + \frac{1}{2} \left( \frac{-\text{Li}_1(-6) \text{Li}_1\left(1+\frac{1}{3}\left(-1+\frac{1}{5}\right)\right)}{-\text{Li}_1(-1)} - \text{Li}_1(1-\log(\log(7))) \sqrt{\text{Li}_1(-6) \text{Li}_1(1-\log(7))} \right)}{10^{27}}
 \end{aligned}$$

**Series representations:**

$$\begin{aligned}
 & \frac{\frac{1}{2} \left( -\frac{\log(7) \log\left(\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{5}\right)\right)}{\log(2)} + \sqrt{\log(7) \log(\log(7)) \log(\log(\log(7)))} \right) + \frac{47+2}{10^3}}{10^{27}} = \\
 & \left( 49 \log(2) + 500 \log\left(\frac{15}{4}\right) \log(7) + 250 \log(2) \sqrt{6 \log(\log(7))} \right. \\
 & \quad \left. \log(\log(\log(7))) \sum_{k=0}^{\infty} 6^k \binom{-\frac{1}{2}+k}{k} \sum_{j=0}^k -\frac{2(-1)^j \binom{k}{j} p_{j,k}}{-1+2j} \right) / \\
 & (100 \log(2)) \\
 & \text{for } \left( c_k = \frac{(-1)^k \log(\log(7))}{1+k} \text{ and } p_{j,0} = 1 \text{ and} \right. \\
 & \quad \left. \log(\log(7)) p_{j,k} = \frac{\sum_{m=1}^k (-k+m+jm) c_m p_{j,k-m}}{k} \text{ and } k \in \mathbb{Z} \text{ and } k > 0 \right)
 \end{aligned}$$



**Integral representation:**

$$\frac{\frac{1}{2} \left( -\frac{\log(7)\log\left(\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{5}\right)\right)}{\log(2)} + \sqrt{\log(7)\log(\log(7))\log(\log(\log(7)))} \right) + \frac{47+2}{10^3}}{49 \int_1^2 \frac{1}{t} dt + 4 \int_0^1 \int_0^1 \frac{10^{27}}{(4+11t_1)(1+6t_2)} dt_2 dt_1 + \int_0^1 \int_0^1 \frac{1}{(1+t_1)(1+(-1+\log(\log(7)))t_2)} dt_2 dt_1} \int_1^2 \frac{1}{t} dt =$$

$[-((\ln(7)/\ln(2))) \ln((1-1/2)(1-1/3)(1-1/5))+((((\text{sqrt}(\ln(7)\ln \ln(7)))\ln \ln \ln(7))))]^6+18+\text{golden ratio}$

**Input:**

$$\left( -\frac{\log(7)}{\log(2)} \log\left(\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{5}\right)\right) + \sqrt{\log(7)\log(\log(7))\log(\log(\log(7)))} \right)^6 + 18 + \phi$$

$\log(x)$  is the natural logarithm

$\phi$  is the golden ratio

**Exact result:**

$$\phi + 18 + \left( \frac{\log\left(\frac{15}{4}\right)\log(7)}{\log(2)} + \sqrt{\log(7)\log(\log(7))\log(\log(\log(7)))} \right)^6$$

**Decimal approximation:**

1192.708371242980323083269117163676486992065586995446696719...

1192.70837124... result practically equal to the rest mass of Sigma baryon 1192.642

**Alternate forms:**

$$18 + \frac{1}{2} (1 + \sqrt{5}) + \left( \frac{\log\left(\frac{15}{4}\right)\log(7)}{\log(2)} + \sqrt{\log(7)\log(\log(7))\log(\log(\log(7)))} \right)^6$$

$$\begin{aligned}
& \frac{37}{2} + \frac{\sqrt{5}}{2} + \frac{\log^6\left(\frac{15}{4}\right) \log^6(7)}{\log^6(2)} + \frac{6 \log^5\left(\frac{15}{4}\right) \log^{11/2}(7) \sqrt{\log(\log(7))} \log(\log(\log(7)))}{\log^5(2)} + \\
& \frac{\log^3(7) \log^3(\log(7)) \log^6(\log(\log(7))) + 15 \log^2\left(\frac{15}{4}\right) \log^4(7) \log^2(\log(7)) \log^4(\log(\log(7)))}{\log^2(2)} + \\
& \frac{20 \log^3\left(\frac{15}{4}\right) \log^{9/2}(7) \log^{3/2}(\log(7)) \log^3(\log(\log(7)))}{\log^3(2)} + \\
& \frac{6 \log\left(\frac{15}{4}\right) \log^{7/2}(7) \log^{5/2}(\log(7)) \log^5(\log(\log(7)))}{\log(2)} + \\
& \frac{15 \log^4\left(\frac{15}{4}\right) \log^5(7) \log(\log(7)) \log^2(\log(\log(7)))}{\log^4(2)}
\end{aligned}$$

$$\begin{aligned}
\phi + \frac{1}{\log^6(2)} & \left( 6 \log(2) \log^{11/2}(7) (2 \log(2) - \log(15))^4 (\log(15) - 2 \log(2)) \right. \\
& \sqrt{\log(\log(7))} \log(\log(\log(7))) + 2 \log^6(2) (9 + 32 \log^6(7)) + \\
& \log^6(7) \log^6(15) - 192 \log^5(2) \log^6(7) \log(15) - 12 \log(2) \log^6(7) \log^5(15) - \\
& 160 \log^3(2) \log^6(7) \log^3(15) + \log^6(2) \log^3(7) \log^3(\log(7)) \log^6(\log(\log(7))) + \\
& 15 \log^2(2) \log^5(7) (2 \log(2) - \log(15))^4 \log(\log(7)) \log^2(\log(\log(7))) + \\
& 15 \log^4(2) \log^4(7) (2 \log(2) - \log(15))^2 \log^2(\log(7)) \log^4(\log(\log(7))) + 20 \log^3(2) \\
& \left. \log^{9/2}(7) (2 \log(2) - \log(15))^2 (\log(15) - 2 \log(2)) \log^{3/2}(\log(7)) \log^3(\log(\log(7))) + \right. \\
& \left. 6 \log^5(2) \log^{7/2}(7) (\log(15) - 2 \log(2)) \log^{5/2}(\log(7)) \log^5(\log(\log(7))) + \right. \\
& \left. 240 \log^4(2) \log^6(7) \log^2(15) + 60 \log^2(2) \log^6(7) \log^4(15) \right)
\end{aligned}$$

**Alternative representations:**

$$\begin{aligned}
& \left( -\frac{\log\left(\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{5}\right)\right) \log(7)}{\log(2)} + \sqrt{\log(7) \log(\log(7))} \log(\log(\log(7))) \right)^6 + 18 + \phi = \\
& 18 + \phi + \left( -\frac{\log_e(7) \log_e\left(\frac{1}{3}\left(1 - \frac{1}{5}\right)\right)}{\log_e(2)} + \log_e(\log(\log(7))) \sqrt{\log_e(7) \log_e(\log(7))} \right)^6
\end{aligned}$$

$$\begin{aligned}
& \left( -\frac{\log\left(\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{5}\right)\right) \log(7)}{\log(2)} + \sqrt{\log(7) \log(\log(7))} \log(\log(\log(7))) \right)^6 + 18 + \phi = \\
& 18 + \phi + \left( -\frac{\log^2(a) \log_a(7) \log_a\left(\frac{1}{3}\left(1 - \frac{1}{5}\right)\right)}{\log(a) \log_a(2)} + \right. \\
& \left. \log(a) \log_a(\log(\log(7))) \sqrt{\log^2(a) \log_a(7) \log_a(\log(7))} \right)^6
\end{aligned}$$

$$\left( -\frac{\log\left(\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{5}\right)\right)\log(7)}{\log(2)} + \sqrt{\log(7)\log(\log(7))\log(\log(\log(7)))} \right)^6 + 18 + \phi =$$

$$18 + \phi + \left( \frac{-\text{Li}_1(-6)\text{Li}_1\left(1+\frac{1}{3}\left(-1+\frac{1}{5}\right)\right)}{-\text{Li}_1(-1)} - \text{Li}_1(1-\log(\log(7)))\sqrt{\text{Li}_1(-6)\text{Li}_1(1-\log(7))} \right)^6$$

$[-((\ln(7)/\ln(2))) \ln((1-1/2)(1-1/3)(1-1/5)) + (((\sqrt{\ln(7)\ln \ln(7)})\ln \ln \ln(7)))]^6 - 199 + 47 - \text{golden ratio}$

**Input:**

$$\left( -\frac{\log(7)}{\log(2)} \log\left(\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{5}\right)\right) + \sqrt{\log(7)\log(\log(7))\log(\log(\log(7)))} \right)^6 - 199 + 47 - \phi$$

$\log(x)$  is the natural logarithm

$\phi$  is the golden ratio

**Exact result:**

$$-\phi - 152 + \left( \frac{\log\left(\frac{15}{4}\right)\log(7)}{\log(2)} + \sqrt{\log(7)\log(\log(7))\log(\log(\log(7)))} \right)^6$$

**Decimal approximation:**

1019.472303265480533386859943494945210756624968635835170994...

1019.472303... result practically equal to the rest mass of Phi meson 1019.445

**Alternate forms:**

$$\frac{1}{2}(-305 - \sqrt{5}) + \left( \frac{\log\left(\frac{15}{4}\right)\log(7)}{\log(2)} + \sqrt{\log(7)\log(\log(7))\log(\log(\log(7)))} \right)^6$$

$$\begin{aligned}
& -\frac{305}{2} - \frac{\sqrt{5}}{2} + \frac{\log^6\left(\frac{15}{4}\right)\log^6(7)}{\log^6(2)} + \frac{6\log^5\left(\frac{15}{4}\right)\log^{11/2}(7)\sqrt{\log(\log(7))\log(\log(\log(7)))}}{\log^5(2)} + \\
& \frac{\log^3(7)\log^3(\log(7))\log^6(\log(\log(7))) + 15\log^2\left(\frac{15}{4}\right)\log^4(7)\log^2(\log(7))\log^4(\log(\log(7)))}{\log^2(2)} + \\
& \frac{20\log^3\left(\frac{15}{4}\right)\log^{9/2}(7)\log^{3/2}(\log(7))\log^3(\log(\log(7)))}{\log^3(2)} + \\
& \frac{6\log\left(\frac{15}{4}\right)\log^{7/2}(7)\log^{5/2}(\log(7))\log^5(\log(\log(7)))}{\log(2)} + \\
& \frac{15\log^4\left(\frac{15}{4}\right)\log^5(7)\log(\log(7))\log^2(\log(\log(7)))}{\log^4(2)}
\end{aligned}$$

$$\begin{aligned}
& -\frac{1}{2\log^6(2)} \\
& \left(305\log^6(2) + \sqrt{5}\log^6(2) - 2\log^6\left(\frac{15}{4}\right)\log^6(7) - 12\log(2)\log^5\left(\frac{15}{4}\right)\log^{11/2}(7) \right. \\
& \quad \left. \sqrt{\log(\log(7))\log(\log(\log(7)))} - 2\log^6(2)\log^3(7)\log^3(\log(7))\log^6(\log(\log(7))) - \right. \\
& \quad 30\log^4(2)\log^2\left(\frac{15}{4}\right)\log^4(7)\log^2(\log(7))\log^4(\log(\log(7))) - \\
& \quad 40\log^3(2)\log^3\left(\frac{15}{4}\right)\log^{9/2}(7)\log^{3/2}(\log(7))\log^3(\log(\log(7))) - \\
& \quad 12\log^5(2)\log\left(\frac{15}{4}\right)\log^{7/2}(7)\log^{5/2}(\log(7))\log^5(\log(\log(7))) - \\
& \quad \left. 30\log^2(2)\log^4\left(\frac{15}{4}\right)\log^5(7)\log(\log(7))\log^2(\log(\log(7))) \right)
\end{aligned}$$

### Alternative representations:

$$\begin{aligned}
& \left(-\frac{\log\left(\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{5}\right)\right)\log(7)}{\log(2)} + \sqrt{\log(7)\log(\log(7))\log(\log(\log(7)))}\right)^6 - 199 + 47 - \\
& \phi = -152 - \phi + \left(-\frac{\log_e(7)\log_e\left(\frac{1}{3}\left(1-\frac{1}{5}\right)\right)}{\log_e(2)} + \log_e(\log(\log(7)))\sqrt{\log_e(7)\log_e(\log(7))}\right)^6
\end{aligned}$$

$$\begin{aligned}
& \left(-\frac{\log\left(\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{5}\right)\right)\log(7)}{\log(2)} + \sqrt{\log(7)\log(\log(7))\log(\log(\log(7)))}\right)^6 - \\
& 199 + 47 - \phi = -152 - \phi + \left(-\frac{\log^2(a)\log_a(7)\log_a\left(\frac{1}{3}\left(1-\frac{1}{5}\right)\right)}{\log(a)\log_a(2)} + \right. \\
& \quad \left. \log(a)\log_a(\log(\log(7)))\sqrt{\log^2(a)\log_a(7)\log_a(\log(7))}\right)^6
\end{aligned}$$

$$\left( -\frac{\log\left(\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{5}\right)\right)\log(7)}{\log(2)} + \sqrt{\log(7)\log(\log(7))\log(\log(\log(7)))} \right)^6 - \left( \frac{199+47-\phi = -152-\phi + (-\text{Li}_1(-6)\text{Li}_1\left(1+\frac{1}{3}\left(-1+\frac{1}{5}\right)\right))}{-\text{Li}_1(-1)} - \text{Li}_1(1-\log(\log(7)))\sqrt{\text{Li}_1(-6)\text{Li}_1(1-\log(7))} \right)^6$$

Now, we have that:

$$\begin{aligned} & Li\{\log N - (\log N)^{\log(\frac{3}{2})/\log 2}\} + O(\log N)^{\frac{5}{12}} \\ &= Li(\log N) - \frac{(\log N)^{\log(\frac{3}{2})/\log 2}}{\log \log N} + O\left\{\frac{(\log N)^{\frac{5}{12}}}{\log \log N}\right\} + O\left\{\frac{(\log N)^{\{2\log(\frac{3}{2})/\log 2\}-1}}{(\log \log N)^2}\right\} \\ &= Li(\log N) - \frac{(\log N)^{\log(\frac{3}{2})/\log 2}}{\log \log N} + O(\log N)^{\frac{5}{12}}; \end{aligned}$$

For N = 21621600, we obtain:

$$Li(\ln(21621600)) - \frac{(\ln(21621600))^{\ln(1.5)/\ln(2)}}{\ln(\ln(21621600))} + (\ln(21621600))^{5/12}$$

**Input:**

$$li(\log(21621600)) - \frac{\log^{\log(1.5)/\log(2)}(21621600)}{\log(\log(21621600))} + \log^{5/12}(21621600)$$

$\log(x)$  is the natural logarithm

$li(x)$  is the logarithmic integral

**Result:**

10.2359...

10.2359...



### Alternative representations:

$$\begin{aligned} \operatorname{li}(\log(21\,621\,600)) - \frac{\log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} + \log^{5/12}(21\,621\,600) = \\ -\Gamma(0, -\log(\log(21\,621\,600))) - \log(-\log(\log(21\,621\,600))) + \\ \frac{1}{2} \left( \log(\log(\log(21\,621\,600))) - \log\left(\frac{1}{\log(\log(21\,621\,600))}\right) \right) + \\ \log^{5/12}(21\,621\,600) - \frac{\log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} \end{aligned}$$

$$\begin{aligned} \operatorname{li}(\log(21\,621\,600)) - \frac{\log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} + \log^{5/12}(21\,621\,600) = \\ \operatorname{li}(\log_e(21\,621\,600)) + \log_e^{5/12}(21\,621\,600) - \frac{\log_e^{\log_e(1.5)/\log_e(2)}(21\,621\,600)}{\log_e(\log(21\,621\,600))} \end{aligned}$$

$$\begin{aligned} \operatorname{li}(\log(21\,621\,600)) - \frac{\log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} + \log^{5/12}(21\,621\,600) = \\ \operatorname{Ei}(\log(\log(21\,621\,600))) + \log_e^{5/12}(21\,621\,600) - \frac{\log_e^{\log_e(1.5)/\log_e(2)}(21\,621\,600)}{\log_e(\log(21\,621\,600))} \end{aligned}$$

### Series representations:

$$\begin{aligned} \operatorname{li}(\log(21\,621\,600)) - \frac{\log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} + \log^{5/12}(21\,621\,600) = \\ \gamma + \log^{5/12}(21\,621\,600) - \frac{\log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} - \frac{1}{2} \log\left(\frac{1}{\log(\log(21\,621\,600))}\right) + \\ \frac{1}{2} \log(\log(\log(21\,621\,600))) + \sum_{k=1}^{\infty} \frac{\log^k(\log(21\,621\,600))}{k k!} \end{aligned}$$

$$\begin{aligned} \operatorname{li}(\log(21\,621\,600)) - \frac{\log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} + \log^{5/12}(21\,621\,600) = \\ \left( -2 \log^{\log(1.5)/\log(2)}(21\,621\,600) + 2 \gamma \log(\log(21\,621\,600)) + 2 \log^{5/12}(21\,621\,600) \right. \\ \left. \log(\log(21\,621\,600)) - \log\left(\frac{1}{-1 + \log(21\,621\,600)}\right) \log(\log(21\,621\,600)) + \right. \\ \left. \log(-1 + \log(21\,621\,600)) \log(\log(21\,621\,600)) + 2 \log(\log(21\,621\,600)) \right. \\ \left. \sum_{k=0}^{\infty} \frac{(-1)^k (1 - \log(21\,621\,600))^{1+k} \sum_{j=1}^{1+k} \frac{B_j S_k^{(-1+j)}}{j}}{(1+k)!} \right) / (2 \log(\log(21\,621\,600))) \end{aligned}$$

$$\begin{aligned}
& \text{li}(\log(21\,621\,600)) - \frac{\log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} + \log^{5/12}(21\,621\,600) = \\
& \log^{5/12}(21\,621\,600) - \frac{\log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} - \frac{1}{2} \log\left(\frac{1}{\log(\log(21\,621\,600))}\right) - \\
& \log(-\log(\log(21\,621\,600))) + \frac{1}{2} \log(\log(\log(21\,621\,600))) - \\
& \text{Res}_{s=0} \frac{\Gamma(s) (-\log(\log(21\,621\,600)))^{-s}}{s} - \sum_{j=1}^{\infty} \text{Res}_{s=-j} \frac{\Gamma(s) (-\log(\log(21\,621\,600)))^{-s}}{s}
\end{aligned}$$

### Integral representations:

$$\begin{aligned}
& \text{li}(\log(21\,621\,600)) - \frac{\log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} + \log^{5/12}(21\,621\,600) = \\
& \int_0^{\log(21\,621\,600)} \frac{1}{\log(t)} dt + \log^{5/12}(21\,621\,600) - \frac{\log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))}
\end{aligned}$$

$$\begin{aligned}
& \text{li}(\log(21\,621\,600)) - \frac{\log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} + \log^{5/12}(21\,621\,600) = \\
& \frac{1}{\int_1^{\log(21\,621\,600)} \frac{1}{t} dt} \\
& \left( -\left(\int_1^{21\,621\,600} \frac{1}{t} dt\right) \left(\int_1^{1.5} \frac{1}{t} dt\right) / \left(\int_1^2 \frac{1}{t} dt\right) + \left(\int_1^{21\,621\,600} \frac{1}{t} dt\right)^{5/12} \int_1^{\log(21\,621\,600)} \frac{1}{t} dt + \right. \\
& \left. \int_0^1 \int_0^1 \frac{1}{\log(\log(21\,621\,600) t_2) (1 + (-1 + \log(21\,621\,600)) t_1)} dt_2 dt_1 \right)
\end{aligned}$$

$$\begin{aligned}
& \text{li}(\log(21\,621\,600)) - \frac{\log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} + \log^{5/12}(21\,621\,600) = \\
& \left( 2^{-1} \left( \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{0.693147s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right) / \left( \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right) \right. \\
& \quad \left( -4i\pi \left( \frac{1}{i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{21\,621\,599^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} \right. \right. \\
& \quad \quad \left. \left. ds \right) \left( \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{0.693147s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right) / \left( \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right) \right) + \\
& \quad 2^{7/12} \left( \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{0.693147s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right) / \left( \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right) \\
& \quad \left( \frac{1}{i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{21\,621\,599^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right)^{5/12} \\
& \quad \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2 \Gamma(1+s) (-1 + \log(21\,621\,600))^{-s}}{\Gamma(1-s)} ds + \\
& \quad 2^{1+} \left( \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{0.693147s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right) / \left( \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right) \\
& \quad \left( \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2 \Gamma(1+s) (-1 + \log(21\,621\,600))^{-s}}{\Gamma(1-s)} ds \right) \\
& \quad \left. \int_0^{\log(21\,621\,600)} \frac{1}{\log(t)} dt \right) / \\
& \quad \left( \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2 \Gamma(1+s) (-1 + \log(21\,621\,600))^{-s}}{\Gamma(1-s)} ds \right) \text{ for} \\
& \quad -1 < \\
& \quad \gamma < \\
& \quad 0
\end{aligned}$$

$$2\text{Pi} + (((\text{Li}(\ln(21621600))) - ((\ln(21621600))^{\ln(1.5)/\ln(2)})) / ((\ln(\ln(21621600)))) + (\ln(21621600))^{5/12}))$$

**Input:**

$$2\pi + \left( \text{li}(\log(21\,621\,600)) - \frac{\log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} + \log^{5/12}(21\,621\,600) \right)$$

$\log(x)$  is the natural logarithm

$\text{li}(x)$  is the logarithmic integral

**Result:**

16.5191...

16.5191... result very near to the black hole entropy 16.8741 and to the mass of the hypothetical light particle, the boson  $m_x = 16.84$  MeV

**Alternative representations:**

$$2\pi + \left( \text{li}(\log(21\,621\,600)) - \frac{\log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} + \log^{5/12}(21\,621\,600) \right) =$$

$$2\pi - \Gamma(0, -\log(\log(21\,621\,600))) - \log(-\log(\log(21\,621\,600))) +$$

$$\frac{1}{2} \left( \log(\log(\log(21\,621\,600))) - \log\left(\frac{1}{\log(\log(21\,621\,600))}\right) \right) +$$

$$\log^{5/12}(21\,621\,600) - \frac{\log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))}$$

$$2\pi + \left( \text{li}(\log(21\,621\,600)) - \frac{\log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} + \log^{5/12}(21\,621\,600) \right) =$$

$$2\pi + \text{li}(\log_e(21\,621\,600)) + \log_e^{5/12}(21\,621\,600) - \frac{\log_e^{\log_e(1.5)/\log_e(2)}(21\,621\,600)}{\log_e(\log(21\,621\,600))}$$

$$2\pi + \left( \text{li}(\log(21\,621\,600)) - \frac{\log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} + \log^{5/12}(21\,621\,600) \right) =$$

$$2\pi + \text{Ei}(\log(\log(21\,621\,600))) + \log_e^{5/12}(21\,621\,600) - \frac{\log_e^{\log_e(1.5)/\log_e(2)}(21\,621\,600)}{\log_e(\log(21\,621\,600))}$$

**Series representations:**

$$2\pi + \left( \text{li}(\log(21\,621\,600)) - \frac{\log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} + \log^{5/12}(21\,621\,600) \right) = \gamma + 2\pi +$$

$$\log^{5/12}(21\,621\,600) - \frac{\log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} - \frac{1}{2} \log\left(\frac{1}{\log(\log(21\,621\,600))}\right) +$$

$$\frac{1}{2} \log(\log(\log(21\,621\,600))) + \sum_{k=1}^{\infty} \frac{\log^k(\log(21\,621\,600))}{k k!}$$

$$2\pi + \left( \text{li}(\log(21\,621\,600)) - \frac{\log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} + \log^{5/12}(21\,621\,600) \right) =$$

$$\left( -2 \log^{\log(1.5)/\log(2)}(21\,621\,600) + 2\gamma \log(\log(21\,621\,600)) + \right.$$

$$4\pi \log(\log(21\,621\,600)) + 2 \log^{5/12}(21\,621\,600) \log(\log(21\,621\,600)) -$$

$$\log\left(\frac{1}{-1 + \log(21\,621\,600)}\right) \log(\log(21\,621\,600)) +$$

$$\log(-1 + \log(21\,621\,600)) \log(\log(21\,621\,600)) + 2 \log(\log(21\,621\,600))$$

$$\left. \sum_{k=0}^{\infty} \frac{(-1)^k (1 - \log(21\,621\,600))^{1+k} \sum_{j=1}^{1+k} \frac{B_j S_k^{(-1+j)}}{j}}{(1+k)!} \right) / (2 \log(\log(21\,621\,600)))$$

$$2\pi + \left( \text{li}(\log(21\,621\,600)) - \frac{\log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} + \log^{5/12}(21\,621\,600) \right) =$$

$$2\pi + \log^{5/12}(21\,621\,600) - \frac{\log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} -$$

$$\frac{1}{2} \log\left(\frac{1}{\log(\log(21\,621\,600))}\right) - \log(-\log(\log(21\,621\,600))) +$$

$$\frac{1}{2} \log(\log(\log(21\,621\,600))) - \text{Res}_{s=0} \frac{\Gamma(s) (-\log(\log(21\,621\,600)))^{-s}}{s} -$$

$$\sum_{j=1}^{\infty} \text{Res}_{s=-j} \frac{\Gamma(s) (-\log(\log(21\,621\,600)))^{-s}}{s}$$

### Integral representations:

$$2\pi + \left( \text{li}(\log(21\,621\,600)) - \frac{\log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} + \log^{5/12}(21\,621\,600) \right) =$$

$$2\pi + \int_0^{\log(21\,621\,600)} \frac{1}{\log(t)} dt + \log^{5/12}(21\,621\,600) - \frac{\log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))}$$

$$2\pi + \left( \text{li}(\log(21\,621\,600)) - \frac{\log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} + \log^{5/12}(21\,621\,600) \right) =$$

$$\frac{1}{\int_1^{\log(21\,621\,600)} \frac{1}{t} dt} \left( - \left( \int_1^{21\,621\,600} \frac{1}{t} dt \right) \left( \int_1^{1.5} \frac{1}{t} dt \right) / \left( \int_1^2 \frac{1}{t} dt \right) + \right.$$

$$2\pi \int_1^{\log(21\,621\,600)} \frac{1}{t} dt + \left( \int_1^{21\,621\,600} \frac{1}{t} dt \right)^{5/12} \int_1^{\log(21\,621\,600)} \frac{1}{t} dt +$$

$$\left. \int_0^1 \int_0^1 \frac{1}{\log(\log(21\,621\,600) t_2) (1 + (-1 + \log(21\,621\,600)) t_1)} dt_2 dt_1 \right)$$

$$\begin{aligned}
& 2\pi + \left( \text{li}(\log(21\,621\,600)) - \frac{\log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} + \log^{5/12}(21\,621\,600) \right) = \\
& \left( 2^{-1} \left( \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{0.693147s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right) \middle/ \left( \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right) \right. \\
& \left( -4i\pi \left( \frac{1}{i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{21\,621\,599^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} \right. \right. \\
& \left. \left. ds \right) \left( \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{0.693147s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right) \middle/ \left( \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right) \right) + \\
& 2^{2+} \left( \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{0.693147s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right) \middle/ \left( \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right) \pi \\
& \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2 \Gamma(1+s) (-1 + \log(21\,621\,600))^{-s}}{\Gamma(1-s)} ds + \\
& 2^{7/12+} \left( \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{0.693147s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right) \middle/ \left( \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right) \\
& \left( \frac{1}{i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{21\,621\,599^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right)^{5/12} \\
& \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2 \Gamma(1+s) (-1 + \log(21\,621\,600))^{-s}}{\Gamma(1-s)} ds + \\
& 2^{1+} \left( \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{0.693147s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right) \middle/ \left( \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right) \\
& \left( \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2 \Gamma(1+s) (-1 + \log(21\,621\,600))^{-s}}{\Gamma(1-s)} ds \right) \\
& \left. \int_0^{\log(21\,621\,600)} \frac{1}{\log(t)} dt \right) \middle/ \\
& \left( \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2 \Gamma(1+s) (-1 + \log(21\,621\,600))^{-s}}{\Gamma(1-s)} ds \right) \text{ for} \\
& -1 < \\
& \gamma < \\
& 0
\end{aligned}$$

$$\begin{aligned}
& 7 * (((2\pi + ((\text{Li}(\ln(21\,621\,600)) - \\
& ((\ln(21\,621\,600))^{\ln(1.5)/\ln(2)}) / ((\ln(\ln(21\,621\,600)))) + (\ln(21\,621\,600))^{5/12})))))) \\
& + 11 - \text{golden ratio}
\end{aligned}$$

**Input:**

$$7 \left( 2\pi + \left( \text{li}(\log(21\,621\,600)) - \frac{\log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} + \log^{5/12}(21\,621\,600) \right) \right) + 11 - \phi$$

$\log(x)$  is the natural logarithm

$\text{li}(x)$  is the logarithmic integral

$\phi$  is the golden ratio

**Result:**

125.016...

125.016... result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for  $T = 0$  and to the Higgs boson mass 125.18 GeV

**Alternative representations:**

$$7 \left( 2\pi + \left( \text{li}(\log(21\,621\,600)) - \frac{\log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} + \log^{5/12}(21\,621\,600) \right) \right) + 11 - \phi = 11 - \phi + 7 \left( 2\pi - \Gamma(0, -\log(\log(21\,621\,600))) - \log(-\log(\log(21\,621\,600))) + \frac{1}{2} \left( \log(\log(\log(21\,621\,600))) - \log\left(\frac{1}{\log(\log(21\,621\,600))}\right) \right) + \log^{5/12}(21\,621\,600) - \frac{\log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} \right)$$

$$7 \left( 2\pi + \left( \text{li}(\log(21\,621\,600)) - \frac{\log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} + \log^{5/12}(21\,621\,600) \right) \right) + 11 - \phi = 11 - \phi + 7 \left( 2\pi + \text{li}(\log_e(21\,621\,600)) + \log_e^{5/12}(21\,621\,600) - \frac{\log_e^{\log_e(1.5)/\log_e(2)}(21\,621\,600)}{\log_e(\log(21\,621\,600))} \right)$$

$$7 \left( 2\pi + \left( \text{li}(\log(21\,621\,600)) - \frac{\log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} + \log^{5/12}(21\,621\,600) \right) \right) + 11 - \phi = 11 - \phi + 7 \left( 2\pi + \text{Ei}(\log(\log(21\,621\,600))) + \log_e^{5/12}(21\,621\,600) - \frac{\log_e^{\log_e(1.5)/\log_e(2)}(21\,621\,600)}{\log_e(\log(21\,621\,600))} \right)$$

**Series representations:**

$$7 \left( 2\pi + \left( \operatorname{li}(\log(21\,621\,600)) - \frac{\log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} + \log^{5/12}(21\,621\,600) \right) \right) +$$

$$\frac{11 - \phi}{7} \log^{\log(1.5)/\log(2)}(21\,621\,600) - \frac{11 + 7\gamma - \phi + 14\pi + 7 \log^{5/12}(21\,621\,600) - 1}{\log(\log(21\,621\,600))} - \frac{7}{2} \log\left(\frac{1}{\log(\log(21\,621\,600))}\right) +$$

$$\frac{7}{2} \log(\log(\log(21\,621\,600))) + 7 \sum_{k=1}^{\infty} \frac{\log^k(\log(21\,621\,600))}{k k!}$$

$$7 \left( 2\pi + \left( \operatorname{li}(\log(21\,621\,600)) - \frac{\log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} + \log^{5/12}(21\,621\,600) \right) \right) +$$

$$11 - \phi = \left( -14 \log^{\log(1.5)/\log(2)}(21\,621\,600) + 22 \log(\log(21\,621\,600)) + \right.$$

$$14\gamma \log(\log(21\,621\,600)) - 2\phi \log(\log(21\,621\,600)) +$$

$$28\pi \log(\log(21\,621\,600)) + 14 \log^{5/12}(21\,621\,600) \log(\log(21\,621\,600)) -$$

$$7 \log\left(\frac{1}{-1 + \log(21\,621\,600)}\right) \log(\log(21\,621\,600)) +$$

$$7 \log(-1 + \log(21\,621\,600)) \log(\log(21\,621\,600)) + 14 \log(\log(21\,621\,600))$$

$$\left. \sum_{k=0}^{\infty} \frac{(-1)^k (1 - \log(21\,621\,600))^{1+k} \sum_{j=1}^{1+k} \frac{B_j S_k^{(-1+j)}}{j}}{(1+k)!} \right) / (2 \log(\log(21\,621\,600)))$$

$$7 \left( 2\pi + \left( \operatorname{li}(\log(21\,621\,600)) - \frac{\log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} + \log^{5/12}(21\,621\,600) \right) \right) + 11 -$$

$$\phi = 11 - \phi + 14\pi + 7 \log^{5/12}(21\,621\,600) - \frac{7 \log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} -$$

$$\frac{7}{2} \log\left(\frac{1}{\log(\log(21\,621\,600))}\right) - 7 \log(-\log(\log(21\,621\,600))) +$$

$$\frac{7}{2} \log(\log(\log(21\,621\,600))) - 7 \left( \operatorname{Res}_{s=0} \frac{\Gamma(s) (-\log(\log(21\,621\,600)))^{-s}}{s} \right) -$$

$$7 \sum_{j=1}^{\infty} \operatorname{Res}_{s=-j} \frac{\Gamma(s) (-\log(\log(21\,621\,600)))^{-s}}{s}$$

7\*(((2Pi+(((Li(ln(21621600))-  
((ln(21621600))^(ln(1.5)/ln(2)))/((ln(ln(21621600))))+(ln(21621600))^(5/12))))))))  
+29-Pi-golden ratio



**Input:**

$$7 \left( 2\pi + \left( \text{li}(\log(21\,621\,600)) - \frac{\log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} + \log^{5/12}(21\,621\,600) \right) \right) + 29 - \pi - \phi$$

$\log(x)$  is the natural logarithm

$\text{li}(x)$  is the logarithmic integral

$\phi$  is the golden ratio

**Result:**

139.874...

139.874... result practically equal to the rest mass of Pion meson 139.57 MeV

**Alternative representations:**

$$7 \left( 2\pi + \left( \text{li}(\log(21\,621\,600)) - \frac{\log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} + \log^{5/12}(21\,621\,600) \right) \right) + 29 - \pi - \phi = 29 - \phi - \pi + 7 \left( 2\pi - \Gamma(0, -\log(\log(21\,621\,600))) - \log(-\log(\log(21\,621\,600))) + \frac{1}{2} \left( \log(\log(\log(21\,621\,600))) - \log\left(\frac{1}{\log(\log(21\,621\,600))}\right) \right) + \log^{5/12}(21\,621\,600) - \frac{\log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} \right)$$

$$7 \left( 2\pi + \left( \text{li}(\log(21\,621\,600)) - \frac{\log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} + \log^{5/12}(21\,621\,600) \right) \right) + 29 - \pi - \phi = 29 - \phi - \pi + 7 \left( 2\pi + \text{li}(\log_e(21\,621\,600)) + \log_e^{5/12}(21\,621\,600) - \frac{\log_e^{\log_e(1.5)/\log_e(2)}(21\,621\,600)}{\log_e(\log(21\,621\,600))} \right)$$

$$7 \left( 2\pi + \left( \text{li}(\log(21\,621\,600)) - \frac{\log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} + \log^{5/12}(21\,621\,600) \right) \right) + 29 - \pi - \phi = 29 - \phi - \pi + 7 \left( 2\pi + \text{Ei}(\log(\log(21\,621\,600))) + \log_e^{5/12}(21\,621\,600) - \frac{\log_e^{\log_e(1.5)/\log_e(2)}(21\,621\,600)}{\log_e(\log(21\,621\,600))} \right)$$

**Series representations:**

$$7 \left( 2\pi + \left( \text{li}(\log(21\,621\,600)) - \frac{\log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} + \log^{5/12}(21\,621\,600) \right) \right) +$$

$$\frac{29 - \pi - \phi}{7} = \frac{29 + 7\gamma - \phi + 13\pi + 7 \log^{5/12}(21\,621\,600) - \log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} - \frac{7}{2} \log \left( \frac{1}{\log(\log(21\,621\,600))} \right) +$$

$$\frac{7}{2} \log(\log(\log(21\,621\,600))) + 7 \sum_{k=1}^{\infty} \frac{\log^k(\log(21\,621\,600))}{k k!}$$

$$7 \left( 2\pi + \left( \text{li}(\log(21\,621\,600)) - \frac{\log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} + \log^{5/12}(21\,621\,600) \right) \right) +$$

$$29 - \pi - \phi = \left( -14 \log^{\log(1.5)/\log(2)}(21\,621\,600) + 58 \log(\log(21\,621\,600)) + \right.$$

$$14\gamma \log(\log(21\,621\,600)) - 2\phi \log(\log(21\,621\,600)) +$$

$$26\pi \log(\log(21\,621\,600)) + 14 \log^{5/12}(21\,621\,600) \log(\log(21\,621\,600)) -$$

$$7 \log \left( \frac{1}{-1 + \log(21\,621\,600)} \right) \log(\log(21\,621\,600)) +$$

$$7 \log(-1 + \log(21\,621\,600)) \log(\log(21\,621\,600)) + 14 \log(\log(21\,621\,600))$$

$$\left. \sum_{k=0}^{\infty} \frac{(-1)^k (1 - \log(21\,621\,600))^{1+k} \sum_{j=1}^{1+k} \frac{B_j S_k^{(-1+j)}}{j}}{(1+k)!} \right) / (2 \log(\log(21\,621\,600)))$$

$$7 \left( 2\pi + \left( \text{li}(\log(21\,621\,600)) - \frac{\log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} + \log^{5/12}(21\,621\,600) \right) \right) + 29 -$$

$$\pi - \phi = 29 - \phi + 13\pi + 7 \log^{5/12}(21\,621\,600) - \frac{7 \log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} -$$

$$\frac{7}{2} \log \left( \frac{1}{\log(\log(21\,621\,600))} \right) - 7 \log(-\log(\log(21\,621\,600))) +$$

$$\frac{7}{2} \log(\log(\log(21\,621\,600))) - 7 \left( \text{Res}_{s=0} \frac{\Gamma(s) (-\log(\log(21\,621\,600)))^{-s}}{s} \right) -$$

$$7 \sum_{j=1}^{\infty} \text{Res}_{s=-j} \frac{\Gamma(s) (-\log(\log(21\,621\,600)))^{-s}}{s}$$

From:

$$\log D(N) = \log 2 \cdot \text{Li}(\log N) + \log\left(\frac{3}{2}\right) \text{Li}\{(\log n)^{\log(\frac{3}{2}) \log 2}\}$$

$$- \log 2 \frac{(\log N)^{\log(\frac{3}{2})/\log 2}}{\log \log N} - \log 2 \cdot R(\log N) + O \left\{ \frac{\sqrt{(\log N)}}{(\log \log N)^3} \right\}. \quad (235)$$

For  $R = 1$ ,  $n = 3$  and  $N = 21621600$ , we obtain:

$$\ln(2) * \text{Li}(\ln(21621600)) + \ln(1.5) \text{Li}((\ln(3))^{\ln(1.5)\ln(2)}) - \\ \ln(2) * ((\ln(21621600)^{(\ln(1.5)/\ln(2))}) / (\ln(\ln(21621600)))) - \\ \ln(2) * (\ln(21621600)) + (((\sqrt{\ln(21621600)})) / (((\ln(\ln(21621600)))^3))))$$

$$\ln(2) * \text{Li}(\ln(21621600)) + \ln(1.5) \text{Li}((\ln(3))^{\ln(1.5)\ln(2)})$$

**Input:**

$$\log(2) \text{li}(\log(21621600)) + \log(1.5) \text{li}(\log^{\log(1.5)\log(2)}(3))$$

$\log(x)$  is the natural logarithm

$\text{li}(x)$  is the logarithmic integral

**Result:**

4.897258078969881864501895621153137289856846183658961212908...

4.8972580789698818645...

4.8972580789698818645 –

$$\ln(2) * ((\ln(21621600)^{(\ln(1.5)/\ln(2))}) / (\ln(\ln(21621600)))) - \\ \ln(2) * (\ln(21621600)) + (((\sqrt{\ln(21621600)})) / (((\ln(\ln(21621600)))^3))))$$

**Input interpretation:**

$$4.8972580789698818645 - \log(2) \times \frac{\log^{\log(1.5)/\log(2)}(21621600)}{\log(\log(21621600))} -$$

$$\log(2) \log(21621600) + \frac{\sqrt{\log(21621600)}}{\log^3(\log(21621600))}$$

$\log(x)$  is the natural logarithm

**Result:**

-7.90880...

-7.90880...

### Alternative representations:

$$4.89725807896988186450000 - \frac{\log(2) \log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} -$$

$$\frac{\log(2) \log(21\,621\,600) + \sqrt{\log(21\,621\,600)}}{\log^3(\log(21\,621\,600))} =$$

$$4.89725807896988186450000 - \log_e(2) \log_e(21\,621\,600) - \frac{\log_e(2) \log_e^{\log_e(1.5)/\log_e(2)}(21\,621\,600)}{\log_e(\log(21\,621\,600))} + \frac{\sqrt{\log_e(21\,621\,600)}}{\log_e^3(\log(21\,621\,600))}$$

$$4.89725807896988186450000 - \frac{\log(2) \log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} -$$

$$\frac{\log(2) \log(21\,621\,600) + \sqrt{\log(21\,621\,600)}}{\log^3(\log(21\,621\,600))} =$$

$$4.89725807896988186450000 - \log^2(a) \log_a(2) \log_a(21\,621\,600) - \frac{\log(a) \log_a(2) (\log(a) \log_a(21\,621\,600))^{\log(a) \log_a(1.5)/(\log(a) \log_a(2))}}{\log(a) \log_a(\log(21\,621\,600))} + \frac{\sqrt{\log(a) \log_a(21\,621\,600)}}{(\log(a) \log_a(\log(21\,621\,600)))^3}$$

$$4.89725807896988186450000 - \frac{\log(2) \log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} -$$

$$\frac{\log(2) \log(21\,621\,600) + \sqrt{\log(21\,621\,600)}}{\log^3(\log(21\,621\,600))} =$$

$$4.89725807896988186450000 - \text{Li}_1(-21\,621\,599) \text{Li}_1(-1) + \frac{\text{Li}_1(-1) (-\text{Li}_1(-21\,621\,599))^{(-\text{Li}_1(-0.5))/(-\text{Li}_1(-1))}}{\text{Li}_1(1 - \log(21\,621\,600))} + \frac{\sqrt{-\text{Li}_1(-21\,621\,599)}}{(-\text{Li}_1(1 - \log(21\,621\,600)))^3}$$

### Series representations:

$$4.89725807896988186450000 - \frac{\log(2) \log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} -$$

$$\frac{\log(2) \log(21\,621\,600) + \sqrt{\log(21\,621\,600)}}{\log^3(\log(21\,621\,600))} = 4.89725807896988186450000 -$$

$$\frac{\log(2) \log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} +$$

$$\frac{\sqrt{-1 + \log(21\,621\,600)} \sum_{k=0}^{\infty} \binom{\frac{1}{2}}{k} (-1 + \log(21\,621\,600))^{-k}}{\log^3(\log(21\,621\,600))}$$

$$4.89725807896988186450000 - \frac{\log(2) \log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} -$$

$$\frac{\log(2) \log(21\,621\,600) + \frac{\sqrt{\log(21\,621\,600)}}{\log^3(\log(21\,621\,600))}}{\log(2) \log^{\log(1.5)/\log(2)}(21\,621\,600)} -$$

$$\frac{\log(2) \log(21\,621\,600) - \frac{\log(2) \log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} +$$

$$\frac{\exp\left(i\pi \left\lfloor \frac{\arg(-x + \log(21\,621\,600))}{2\pi} \right\rfloor\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k x^{-k} (-x + \log(21\,621\,600))^k \left(-\frac{1}{2}\right)_k}{k!}}{\log^3(\log(21\,621\,600))}$$

for  $(x \in \mathbb{R} \text{ and } x < 0)$

$$4.89725807896988186450000 - \frac{\log(2) \log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} -$$

$$\frac{\log(2) \log(21\,621\,600) + \frac{\sqrt{\log(21\,621\,600)}}{\log^3(\log(21\,621\,600))}}{\log(2) \log^{\log(1.5)/\log(2)}(21\,621\,600)} = 4.89725807896988186450000 -$$

$$\frac{\log(2) \log(21\,621\,600) - \frac{\log(2) \log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} +$$

$$\left( \left( \frac{1}{z_0} \right)^{1/2 \lfloor \arg(\log(21\,621\,600) - z_0) / (2\pi) \rfloor} z_0^{1/2 (1 + \lfloor \arg(\log(21\,621\,600) - z_0) / (2\pi) \rfloor)} \right.$$

$$\left. \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (\log(21\,621\,600) - z_0)^k z_0^{-k}}{k!} \right) / \log^3(\log(21\,621\,600))$$

2\*(((4.8972580789698818645 -  
ln(2)\*((ln(21621600)^((ln(1.5)/(ln(2)))))))/((ln(ln(21621600)))))-  
ln(2)\*(ln(21621600))+((((sqrt(ln(21621600)))/(((ln(ln(21621600))^3)))))))))^2+1/  
golden ratio

**Input interpretation:**

$$2 \left( 4.8972580789698818645 - \log(2) \times \frac{\log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} - \right.$$

$$\left. \frac{\log(2) \log(21\,621\,600) + \frac{\sqrt{\log(21\,621\,600)}}{\log^3(\log(21\,621\,600))}}{\log(2) \log^{\log(1.5)/\log(2)}(21\,621\,600)} \right)^2 + \frac{1}{\phi}$$

$\log(x)$  is the natural logarithm

$\phi$  is the golden ratio

**Result:**

125.716...

125.716... result very near to the dilaton mass calculated as a type of Higgs boson:  
125 GeV for  $T = 0$  and to the Higgs boson mass 125.18 GeV

**Alternative representations:**

$$\begin{aligned}
& 2 \left( 4.89725807896988186450000 - \frac{\log(2) \log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} - \right. \\
& \quad \left. \log(2) \log(21\,621\,600) + \frac{\sqrt{\log(21\,621\,600)}}{\log^3(\log(21\,621\,600))} \right)^2 + \frac{1}{\phi} = \\
& \frac{1}{\phi} + 2 \left( 4.89725807896988186450000 - \log_e(2) \log_e(21\,621\,600) - \right. \\
& \quad \left. \frac{\log_e(2) \log_e^{\log_e(1.5)/\log_e(2)}(21\,621\,600)}{\log_e(\log(21\,621\,600))} + \frac{\sqrt{\log_e(21\,621\,600)}}{\log_e^3(\log(21\,621\,600))} \right)^2 \\
& 2 \left( 4.89725807896988186450000 - \frac{\log(2) \log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} - \right. \\
& \quad \left. \log(2) \log(21\,621\,600) + \frac{\sqrt{\log(21\,621\,600)}}{\log^3(\log(21\,621\,600))} \right)^2 + \frac{1}{\phi} = \\
& \frac{1}{\phi} + 2 \left( 4.89725807896988186450000 - \log^2(a) \log_a(2) \log_a(21\,621\,600) - \right. \\
& \quad \frac{\log(a) \log_a(2) (\log(a) \log_a(21\,621\,600))^{\log(a) \log_a(1.5)/(\log(a) \log_a(2))}}{\log(a) \log_a(\log(21\,621\,600))} + \\
& \quad \left. \frac{\sqrt{\log(a) \log_a(21\,621\,600)}}{(\log(a) \log_a(\log(21\,621\,600)))^3} \right)^2 \\
& 2 \left( 4.89725807896988186450000 - \frac{\log(2) \log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} - \right. \\
& \quad \left. \log(2) \log(21\,621\,600) + \frac{\sqrt{\log(21\,621\,600)}}{\log^3(\log(21\,621\,600))} \right)^2 + \frac{1}{\phi} = \\
& \frac{1}{\phi} + 2 \left( 4.89725807896988186450000 - \text{Li}_1(-21\,621\,599) \text{Li}_1(-1) + \right. \\
& \quad \left. - \frac{\text{Li}_1(-1) (-\text{Li}_1(-21\,621\,599))^{(-\text{Li}_1(-0.5))/(-\text{Li}_1(-1))}}{\text{Li}_1(1 - \log(21\,621\,600))} + \right. \\
& \quad \left. \frac{\sqrt{-\text{Li}_1(-21\,621\,599)}}{(-\text{Li}_1(1 - \log(21\,621\,600)))^3} \right)^2
\end{aligned}$$

**Series representations:**

$$2 \left( 4.89725807896988186450000 - \frac{\log(2) \log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} - \log(2) \log(21\,621\,600) + \frac{\sqrt{\log(21\,621\,600)}}{\log^3(\log(21\,621\,600))} \right)^2 + \frac{1}{\phi} =$$

$$\frac{1}{\phi} + 2 \left( -4.89725807896988186450000 + \log(2) \log(21\,621\,600) + \frac{\log(2) \log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} - \frac{\sqrt{-1 + \log(21\,621\,600)} \sum_{k=0}^{\infty} \binom{\frac{1}{2}}{k} (-1 + \log(21\,621\,600))^{-k}}{\log^3(\log(21\,621\,600))} \right)^2$$

$$2 \left( 4.89725807896988186450000 - \frac{\log(2) \log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} - \log(2) \log(21\,621\,600) + \frac{\sqrt{\log(21\,621\,600)}}{\log^3(\log(21\,621\,600))} \right)^2 + \frac{1}{\phi} =$$

$$\frac{1}{\phi} + 2 \left( 4.89725807896988186450000 - \log(2) \log(21\,621\,600) - \frac{\log(2) \log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} + \frac{\exp\left(i\pi \left\lfloor \frac{\arg(-x + \log(21\,621\,600))}{2\pi} \right\rfloor\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k x^{-k} (-x + \log(21\,621\,600))^k \left(-\frac{1}{2}\right)_k}{k!}}{\log^3(\log(21\,621\,600))} \right)^2$$

for  $(x \in \mathbb{R}$  and  $x < 0)$

$$2 \left( 4.89725807896988186450000 - \frac{\log(2) \log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} - \left( \log(2) \log(21\,621\,600) + \frac{\sqrt{\log(21\,621\,600)}}{\log^3(\log(21\,621\,600))} \right)^2 + \frac{1}{\phi} = \right.$$

$$\frac{1}{\phi} + 2 \left( 4.89725807896988186450000 - \log(2) \log(21\,621\,600) - \frac{\log(2) \log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} + \left( \left( \frac{1}{z_0} \right)^{1/2 [\arg(\log(21\,621\,600) - z_0)/(2\pi)]} z_0^{1/2 (1 + [\arg(\log(21\,621\,600) - z_0)/(2\pi)])} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (\log(21\,621\,600) - z_0)^k z_0^{-k}}{k!} \right) / \log^3(\log(21\,621\,600)) \right)^2$$

2\*(((4.8972580789698818645 - ln(2)\*(((ln(21621600)^((ln(1.5)/(ln(2)))))))/((ln(ln(21621600)))))-ln(2)\*(ln(21621600))+((((sqrt(ln(21621600)))/(((ln(ln(21621600))^3)))))))))^2+11+Pi+1/golden ratio

**Input interpretation:**

$$2 \left( 4.8972580789698818645 - \log(2) \times \frac{\log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} - \left( \log(2) \log(21\,621\,600) + \frac{\sqrt{\log(21\,621\,600)}}{\log^3(\log(21\,621\,600))} \right)^2 + 11 + \pi + \frac{1}{\phi} \right.$$

log(x) is the natural logarithm

φ is the golden ratio

**Result:**

139.858...

139.858... result practically equal to the rest mass of Pion meson 139.57 MeV



**Alternative representations:**

$$2 \left( 4.89725807896988186450000 - \frac{\log(2) \log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} - \log(2) \log(21\,621\,600) + \frac{\sqrt{\log(21\,621\,600)}}{\log^3(\log(21\,621\,600))} \right)^2 + 11 + \pi + \frac{1}{\phi} =$$

$$11 + \pi + \frac{1}{\phi} + 2 \left( 4.89725807896988186450000 - \log_e(2) \log_e(21\,621\,600) - \frac{\log_e(2) \log_e^{\log_e(1.5)/\log_e(2)}(21\,621\,600)}{\log_e(\log(21\,621\,600))} + \frac{\sqrt{\log_e(21\,621\,600)}}{\log_e^3(\log(21\,621\,600))} \right)^2$$

$$2 \left( 4.89725807896988186450000 - \frac{\log(2) \log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} - \log(2) \log(21\,621\,600) + \frac{\sqrt{\log(21\,621\,600)}}{\log^3(\log(21\,621\,600))} \right)^2 + 11 + \pi + \frac{1}{\phi} =$$

$$11 + \pi + \frac{1}{\phi} + 2 \left( 4.89725807896988186450000 - \log^2(a) \log_a(2) \log_a(21\,621\,600) - \frac{\log(a) \log_a(2) (\log(a) \log_a(21\,621\,600))^{\log(a) \log_a(1.5)/(\log(a) \log_a(2))}}{\log(a) \log_a(\log(21\,621\,600))} + \frac{\sqrt{\log(a) \log_a(21\,621\,600)}}{(\log(a) \log_a(\log(21\,621\,600)))^3} \right)^2$$

$$2 \left( 4.89725807896988186450000 - \frac{\log(2) \log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} - \log(2) \log(21\,621\,600) + \frac{\sqrt{\log(21\,621\,600)}}{\log^3(\log(21\,621\,600))} \right)^2 + 11 + \pi + \frac{1}{\phi} =$$

$$11 + \pi + \frac{1}{\phi} + 2 \left( 4.89725807896988186450000 - \text{Li}_1(-21\,621\,599) \text{Li}_1(-1) + \frac{\text{Li}_1(-1) (-\text{Li}_1(-21\,621\,599))^{(-\text{Li}_1(-0.5))/(-\text{Li}_1(-1))}}{\text{Li}_1(1 - \log(21\,621\,600))} + \frac{\sqrt{-\text{Li}_1(-21\,621\,599)}}{(-\text{Li}_1(1 - \log(21\,621\,600)))^3} \right)^2$$

**Series representations:**

$$2 \left( 4.89725807896988186450000 - \frac{\log(2) \log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} - \frac{\log(2) \log(21\,621\,600) + \frac{\sqrt{\log(21\,621\,600)}}{\log^3(\log(21\,621\,600))}}{\log(\log(21\,621\,600))} \right)^2 + 11 + \pi + \frac{1}{\phi} =$$

$$11 + \frac{1}{\phi} + \pi + 2 \left( 4.89725807896988186450000 - \log(2) \log(21\,621\,600) - \frac{\log(2) \log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} + \frac{\sqrt{-1 + \log(21\,621\,600)} \sum_{k=0}^{\infty} \binom{\frac{1}{2}}{k} (-1 + \log(21\,621\,600))^{-k}}{\log^3(\log(21\,621\,600))} \right)^2$$

$$2 \left( 4.89725807896988186450000 - \frac{\log(2) \log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} - \frac{\log(2) \log(21\,621\,600) + \frac{\sqrt{\log(21\,621\,600)}}{\log^3(\log(21\,621\,600))}}{\log(\log(21\,621\,600))} \right)^2 + 11 + \pi + \frac{1}{\phi} =$$

$$11 + \frac{1}{\phi} + \pi + 2 \left( 4.89725807896988186450000 - \log(2) \log(21\,621\,600) - \frac{\log(2) \log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} + \frac{\exp\left(i\pi \left\lfloor \frac{\arg(-x + \log(21\,621\,600))}{2\pi} \right\rfloor\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k x^{-k} (-x + \log(21\,621\,600))^k \left(-\frac{1}{2}\right)_k}{k!}}{\log^3(\log(21\,621\,600))} \right)^2$$

for  $(x \in \mathbb{R}$  and  $x < 0)$

$$\begin{aligned}
& 2 \left( 4.89725807896988186450000 - \frac{\log(2) \log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} - \right. \\
& \quad \left. \log(2) \log(21\,621\,600) + \frac{\sqrt{\log(21\,621\,600)}}{\log^3(\log(21\,621\,600))} \right)^2 + 11 + \pi + \frac{1}{\phi} = \\
& 11 + \frac{1}{\phi} + \pi + 2 \left( 4.89725807896988186450000 - \log(2) \log(21\,621\,600) - \right. \\
& \quad \frac{\log(2) \log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} + \\
& \quad \left( \frac{1}{z_0} \right)^{1/2} \frac{e^{i \arg(\log(21\,621\,600) - z_0)/(2\pi)}}{z_0^{1/2 (1 + \arg(\log(21\,621\,600) - z_0)/(2\pi))}} \\
& \quad \left. \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (\log(21\,621\,600) - z_0)^k z_0^{-k}}{k!} \right) / \\
& \quad \left. \log^3(\log(21\,621\,600)) \right)^2
\end{aligned}$$

Now, we have that:

$$\phi(N) = \frac{\log\left(\frac{9}{8}\right) \sqrt{(\log N)}}{\log 2 \log \log N} + \frac{4 \log\left(\frac{3}{2}\right)}{(\log \log N)^2} - R(\log N) + O\left\{ \frac{\sqrt{(\log N)}}{(\log \log N)^3} \right\}$$

For  $R = 1$ ,  $O = 1$  and  $N = 21621600$ , we obtain:

$$\begin{aligned}
& \ln(9/8) / \ln(2) * (((\sqrt{\ln(21621600)})/((\ln(\ln(21621600)))))) + ((4\ln(3/2))) / \\
& (\ln(\ln(21621600)))^2 - \\
& (\ln(21621600)) + (((\sqrt{\ln(21621600)})/((\ln(\ln(21621600))^3))))
\end{aligned}$$

**Input:**

$$\begin{aligned}
& \frac{\log\left(\frac{9}{8}\right) \sqrt{\log(21\,621\,600)}}{\log(2) \log(\log(21\,621\,600))} + \\
& \frac{4 \log\left(\frac{3}{2}\right)}{\log^2(\log(21\,621\,600))} - \log(21\,621\,600) + \frac{\sqrt{\log(21\,621\,600)}}{\log^3(\log(21\,621\,600))}
\end{aligned}$$

$\log(x)$  is the natural logarithm

**Exact result:**

$$\frac{\sqrt{\log(21\,621\,600)}}{\log^3(\log(21\,621\,600))} + \frac{4 \log\left(\frac{3}{2}\right)}{\log^2(\log(21\,621\,600))} - \frac{\log\left(\frac{9}{8}\right) \sqrt{\log(21\,621\,600)}}{\log(2) \log(\log(21\,621\,600))} - \log(21\,621\,600) + \frac{\log\left(\frac{9}{8}\right) \sqrt{\log(21\,621\,600)}}{\log(2) \log(\log(21\,621\,600))}$$

**Decimal approximation:**

-16.2572081572853343602690100150900036039645625339046371117...

-16.2572081572....

**Alternate forms:**

$$-\left( \left( \log(2) \log(21\,621\,600) \log^3(\log(21\,621\,600)) - \log\left(\frac{9}{8}\right) \sqrt{\log(21\,621\,600)} \log^2(\log(21\,621\,600)) - \log(2) \sqrt{\log(21\,621\,600)} - 4 \log\left(\frac{3}{2}\right) \log(2) \log(\log(21\,621\,600)) \right) / (\log(2) \log^3(\log(21\,621\,600))) \right)$$

$$\frac{\sqrt{5 \log(2) + 3 \log(3) + 2 \log(5) + \log(1001)}}{\log^3(5 \log(2) + 3 \log(3) + 2 \log(5) + \log(1001))} + \frac{4 \log(3) - 4 \log(2)}{\log^2(5 \log(2) + 3 \log(3) + 2 \log(5) + \log(1001))} - 5 \log(2) - 3 \log(3) - 2 \log(5) - \log(1001) + \frac{(2 \log(3) - 3 \log(2)) \sqrt{5 \log(2) + 3 \log(3) + 2 \log(5) + \log(1001)}}{\log(2) \log(5 \log(2) + 3 \log(3) + 2 \log(5) + \log(1001))}$$

$$\left( -\log(2) (5 \log(2) + 3 \log(3) + 2 \log(5) + \log(7) + \log(11) + \log(13)) \right. \\ \left. \frac{\log^3(5 \log(2) + 3 \log(3) + 2 \log(5) + \log(7) + \log(11) + \log(13)) + (2 \log(3) - 3 \log(2)) \sqrt{5 \log(2) + 3 \log(3) + 2 \log(5) + \log(7) + \log(11) + \log(13)}{\log^2(5 \log(2) + 3 \log(3) + 2 \log(5) + \log(7) + \log(11) + \log(13))} + \log(2) \sqrt{5 \log(2) + 3 \log(3) + 2 \log(5) + \log(7) + \log(11) + \log(13)} - 4 \log(2) \right. \\ \left. (\log(2) - \log(3)) \log(5 \log(2) + 3 \log(3) + 2 \log(5) + \log(7) + \log(11) + \log(13)) \right) / (\log(2) \log^3(5 \log(2) + 3 \log(3) + 2 \log(5) + \log(7) + \log(11) + \log(13)))$$

**Alternative representations:**

$$\begin{aligned} & \frac{\sqrt{\log(21\,621\,600)} \log\left(\frac{9}{8}\right)}{\log(\log(21\,621\,600)) \log(2)} + \frac{4 \log\left(\frac{3}{2}\right)}{\log^2(\log(21\,621\,600))} - \log(21\,621\,600) + \\ & \frac{\sqrt{\log(21\,621\,600)}}{\log^3(\log(21\,621\,600))} = -\log(a) \log_a(21\,621\,600) + \frac{4 \log(a) \log_a\left(\frac{3}{2}\right)}{(\log(a) \log_a(\log(21\,621\,600)))^2} + \\ & \frac{\log(a) \log_a\left(\frac{9}{8}\right) \sqrt{\log(a) \log_a(21\,621\,600)}}{(\log(a) \log_a(2)) (\log(a) \log_a(\log(21\,621\,600)))} + \frac{\sqrt{\log(a) \log_a(21\,621\,600)}}{(\log(a) \log_a(\log(21\,621\,600)))^3} \end{aligned}$$

$$\begin{aligned} & \frac{\sqrt{\log(21\,621\,600)} \log\left(\frac{9}{8}\right)}{\log(\log(21\,621\,600)) \log(2)} + \frac{4 \log\left(\frac{3}{2}\right)}{\log^2(\log(21\,621\,600))} - \\ & \log(21\,621\,600) + \frac{\sqrt{\log(21\,621\,600)}}{\log^3(\log(21\,621\,600))} = -\log_e(21\,621\,600) + \\ & \frac{4 \log_e\left(\frac{3}{2}\right)}{\log_e^2(\log(21\,621\,600))} + \frac{\log_e\left(\frac{9}{8}\right) \sqrt{\log_e(21\,621\,600)}}{\log_e(2) \log_e(\log(21\,621\,600))} + \frac{\sqrt{\log_e(21\,621\,600)}}{\log_e^3(\log(21\,621\,600))} \end{aligned}$$

$$\begin{aligned} & \frac{\sqrt{\log(21\,621\,600)} \log\left(\frac{9}{8}\right)}{\log(\log(21\,621\,600)) \log(2)} + \frac{4 \log\left(\frac{3}{2}\right)}{\log^2(\log(21\,621\,600))} - \log(21\,621\,600) + \\ & \frac{\sqrt{\log(21\,621\,600)}}{\log^3(\log(21\,621\,600))} = \text{Li}_1(-21\,621\,599) - \frac{4 \text{Li}_1\left(-\frac{1}{2}\right)}{(-\text{Li}_1(1 - \log(21\,621\,600)))^2} - \\ & - \frac{\text{Li}_1\left(1 - \frac{9}{8}\right) \sqrt{-\text{Li}_1(-21\,621\,599)}}{\text{Li}_1(-1) (-\text{Li}_1(1 - \log(21\,621\,600)))} + \frac{\sqrt{-\text{Li}_1(-21\,621\,599)}}{(-\text{Li}_1(1 - \log(21\,621\,600)))^3} \end{aligned}$$

## Series representations:

$$\begin{aligned}
& \frac{\sqrt{\log(21\,621\,600)} \log\left(\frac{9}{8}\right)}{\log(\log(21\,621\,600)) \log(2)} + \\
& \frac{4 \log\left(\frac{3}{2}\right) - \log(21\,621\,600) + \frac{\sqrt{\log(21\,621\,600)}}{\log^3(\log(21\,621\,600))}}{\log^2(\log(21\,621\,600))} = \\
& -2i\pi \left[ \frac{\arg(21\,621\,600 - x)}{2\pi} \right] - \log(x) + \sum_{k=1}^{\infty} \frac{(-1)^k (21\,621\,600 - x)^k x^{-k}}{k} + \\
& \frac{\sqrt{2i\pi \left[ \frac{\arg(21\,621\,600 - x)}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (21\,621\,600 - x)^k x^{-k}}{k}}}{\left(2i\pi \left[ \frac{\arg(-x + \log(21\,621\,600))}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k x^{-k} (-x + \log(21\,621\,600))^k}{k} \right)^3} + \\
& \frac{8i\pi \left[ \frac{\arg\left(\frac{3}{2} - x\right)}{2\pi} \right]}{4 \log(x)} + \\
& \frac{\left(2i\pi \left[ \frac{\arg(-x + \log(21\,621\,600))}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k x^{-k} (-x + \log(21\,621\,600))^k}{k} \right)^2}{4 \log(x)} + \\
& \frac{\left(2i\pi \left[ \frac{\arg(-x + \log(21\,621\,600))}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k x^{-k} (-x + \log(21\,621\,600))^k}{k} \right)^2}{4 \sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{3}{2} - x\right)^k x^{-k}}{k}} - \\
& \frac{\left(2i\pi \left[ \frac{\arg(-x + \log(21\,621\,600))}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k x^{-k} (-x + \log(21\,621\,600))^k}{k} \right)^2}{\left(2i\pi \left[ \frac{\arg\left(\frac{9}{8} - x\right)}{2\pi} \right] \right)} + \\
& \sqrt{2i\pi \left[ \frac{\arg(21\,621\,600 - x)}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (21\,621\,600 - x)^k x^{-k}}{k}} \Bigg/ \\
& \left( \left(2i\pi \left[ \frac{\arg(2 - x)}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (2 - x)^k x^{-k}}{k} \right) \right. \\
& \left. \left(2i\pi \left[ \frac{\arg(-x + \log(21\,621\,600))}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k x^{-k} (-x + \log(21\,621\,600))^k}{k} \right) \right) + \\
& \left( \log(x) \sqrt{2i\pi \left[ \frac{\arg(21\,621\,600 - x)}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (21\,621\,600 - x)^k x^{-k}}{k}} \right) \Bigg/ \\
& \left( \left(2i\pi \left[ \frac{\arg(2 - x)}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (2 - x)^k x^{-k}}{k} \right) \right. \\
& \left. \left(2i\pi \left[ \frac{\arg(-x + \log(21\,621\,600))}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k x^{-k} (-x + \log(21\,621\,600))^k}{k} \right) \right) - \left( \sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{9}{8} - x\right)^k x^{-k}}{k} \right) \\
& \sqrt{2i\pi \left[ \frac{\arg(21\,621\,600 - x)}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (21\,621\,600 - x)^k x^{-k}}{k}} \Bigg/ \\
& \left( \left(2i\pi \left[ \frac{\arg(2 - x)}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (2 - x)^k x^{-k}}{k} \right) \right. \\
& \left. \left(2i\pi \left[ \frac{\arg(-x + \log(21\,621\,600))}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k x^{-k} (-x + \log(21\,621\,600))^k}{k} \right) \right) \Bigg) \text{ for } x < 0
\end{aligned}$$

$$\begin{aligned}
& \frac{\sqrt{\log(21621600) \log\left(\frac{9}{8}\right)}}{\log(\log(21621600)) \log(2)} + \\
& \frac{4 \log\left(\frac{3}{2}\right)}{\log^2(\log(21621600))} - \log(21621600) + \frac{\sqrt{\log(21621600)}}{\log^3(\log(21621600))} = \\
& -2i\pi \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] - \log(z_0) + \sum_{k=1}^{\infty} \frac{(-1)^k (21621600 - z_0)^k z_0^{-k}}{k} + \\
& \sqrt{2i\pi \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (21621600 - z_0)^k z_0^{-k}}{k}} \\
& \left( 2i\pi \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (\log(21621600) - z_0)^k z_0^{-k}}{k} \right)^3 + \\
& \frac{8i\pi \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right]}{4 \log(z_0)} + \\
& \left( 2i\pi \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (\log(21621600) - z_0)^k z_0^{-k}}{k} \right)^2 + \\
& \frac{4 \log(z_0)}{4 \sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{3}{2} - z_0\right)^k z_0^{-k}}{k}} - \\
& \left( 2i\pi \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (\log(21621600) - z_0)^k z_0^{-k}}{k} \right)^2 - \\
& \frac{2i\pi \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right]}{\left( 2i\pi \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (\log(21621600) - z_0)^k z_0^{-k}}{k} \right)^2} + \\
& \sqrt{2i\pi \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (21621600 - z_0)^k z_0^{-k}}{k}} \Bigg/ \\
& \left( \left( 2i\pi \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (2 - z_0)^k z_0^{-k}}{k} \right) \right. \\
& \left. \left( 2i\pi \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (\log(21621600) - z_0)^k z_0^{-k}}{k} \right) \right) + \\
& \left( \log(z_0) \sqrt{2i\pi \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (21621600 - z_0)^k z_0^{-k}}{k}} \right) \Bigg/ \\
& \left( \left( 2i\pi \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (2 - z_0)^k z_0^{-k}}{k} \right) \right. \\
& \left. \left( 2i\pi \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (\log(21621600) - z_0)^k z_0^{-k}}{k} \right) \right) - \left( \sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{9}{8} - z_0\right)^k z_0^{-k}}{k} \right) \\
& \sqrt{2i\pi \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (21621600 - z_0)^k z_0^{-k}}{k}} \Bigg/ \\
& \left( \left( 2i\pi \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (2 - z_0)^k z_0^{-k}}{k} \right) \right. \\
& \left. \left( 2i\pi \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (\log(21621600) - z_0)^k z_0^{-k}}{k} \right) \right) \Bigg)
\end{aligned}$$





$$-\left(\frac{\ln(9/8)}{\ln(2)} * \left(\frac{\sqrt{\ln(21621600)}}{\ln(\ln(21621600))}\right)\right) + \left(\frac{4\ln(3/2)}{\ln(\ln(21621600))}\right)^2 - \left(\frac{\ln(21621600) + \left(\frac{\sqrt{\ln(21621600)}}{\ln(\ln(21621600))}\right)^3}{\ln(21621600)} - \frac{1}{\text{golden ratio}}\right)$$

**Input:**

$$-\left(\frac{\log\left(\frac{\phi}{8}\right)}{\log(2)} \times \frac{\sqrt{\log(21\,621\,600)}}{\log(\log(21\,621\,600))} + \frac{4 \log\left(\frac{3}{2}\right)}{\log^2(\log(21\,621\,600))} - \log(21\,621\,600) + \frac{\sqrt{\log(21\,621\,600)}}{\log^3(\log(21\,621\,600))} - \frac{1}{\phi}\right)$$

$\log(x)$  is the natural logarithm

$\phi$  is the golden ratio

**Exact result:**

$$\frac{1}{\phi} - \frac{\sqrt{\log(21\,621\,600)}}{\log^3(\log(21\,621\,600))} - \frac{4 \log\left(\frac{3}{2}\right)}{\log^2(\log(21\,621\,600))} + \frac{\log\left(\frac{\phi}{8}\right) \sqrt{\log(21\,621\,600)}}{\log(2) \log(\log(21\,621\,600))} - \log(21\,621\,600) + \frac{\sqrt{\log(21\,621\,600)}}{\log^3(\log(21\,621\,600))} - \frac{1}{\phi}$$

**Decimal approximation:**

16.87524214603522920847359684945564172168487171371039997386...

16.875242146.... result practically equal to the black hole entropy 16.8741 and to the mass of the hypothetical light particle, the boson  $m_x = 16.84$  MeV

**Alternate forms:**

$$\frac{1}{2} \left( -1 + \sqrt{5} - \frac{8 \log\left(\frac{3}{2}\right)}{\log^2(\log(21\,621\,600))} \right) - \frac{\sqrt{\log(21\,621\,600)} \left( \log\left(\frac{\phi}{8}\right) \log^2(\log(21\,621\,600)) + \log(2) \right)}{\log(2) \log^3(\log(21\,621\,600))} + \log(21\,621\,600) - \frac{2 \left( -\log^2(\log(21\,621\,600)) + 2 \log\left(\frac{3}{2}\right) + 2 \sqrt{5} \log\left(\frac{3}{2}\right) \right)}{(1 + \sqrt{5}) \log^2(\log(21\,621\,600))} - \frac{\sqrt{\log(21\,621\,600)} \left( \log\left(\frac{\phi}{8}\right) \log^2(\log(21\,621\,600)) + \log(2) \right)}{\log(2) \log^3(\log(21\,621\,600))} + \log(21\,621\,600)$$

$$\begin{aligned}
& \left( 2 \log(2) \log^3(\log(21\,621\,600)) + \log(2) \log(21\,621\,600) \log^3(\log(21\,621\,600)) + \right. \\
& \quad \sqrt{5} \log(2) \log(21\,621\,600) \log^3(\log(21\,621\,600)) - \\
& \quad \log\left(\frac{9}{8}\right) \sqrt{\log(21\,621\,600)} \log^2(\log(21\,621\,600)) - \\
& \quad \log\left(\frac{9}{8}\right) \sqrt{5 \log(21\,621\,600)} \log^2(\log(21\,621\,600)) - \\
& \quad \log(2) \sqrt{\log(21\,621\,600)} - \log(2) \sqrt{5 \log(21\,621\,600)} - \\
& \quad \left. 4 \log\left(\frac{3}{2}\right) \log(2) \log(\log(21\,621\,600)) - 4 \sqrt{5} \log\left(\frac{3}{2}\right) \log(2) \log(\log(21\,621\,600)) \right) / \\
& \left( (1 + \sqrt{5}) \log(2) \log^3(\log(21\,621\,600)) \right)
\end{aligned}$$

### Alternative representations:

$$\begin{aligned}
& - \left( \frac{\sqrt{\log(21\,621\,600)} \log\left(\frac{9}{8}\right)}{\log(\log(21\,621\,600)) \log(2)} + \frac{4 \log\left(\frac{3}{2}\right)}{\log^2(\log(21\,621\,600))} - \right. \\
& \quad \left. \log(21\,621\,600) + \frac{\sqrt{\log(21\,621\,600)}}{\log^3(\log(21\,621\,600))} - \frac{1}{\phi} \right) = \\
& \log(a) \log_a(21\,621\,600) + \frac{1}{\phi} - \frac{4 \log(a) \log_a\left(\frac{3}{2}\right)}{(\log(a) \log_a(\log(21\,621\,600)))^2} - \\
& \frac{\log(a) \log_a\left(\frac{9}{8}\right) \sqrt{\log(a) \log_a(21\,621\,600)}}{(\log(a) \log_a(2)) (\log(a) \log_a(\log(21\,621\,600)))} - \frac{\sqrt{\log(a) \log_a(21\,621\,600)}}{(\log(a) \log_a(\log(21\,621\,600)))^3} \\
& - \left( \frac{\sqrt{\log(21\,621\,600)} \log\left(\frac{9}{8}\right)}{\log(\log(21\,621\,600)) \log(2)} + \frac{4 \log\left(\frac{3}{2}\right)}{\log^2(\log(21\,621\,600))} - \right. \\
& \quad \left. \log(21\,621\,600) + \frac{\sqrt{\log(21\,621\,600)}}{\log^3(\log(21\,621\,600))} - \frac{1}{\phi} \right) = \\
& \log_e(21\,621\,600) + \frac{1}{\phi} - \frac{4 \log_e\left(\frac{3}{2}\right)}{\log_e^2(\log(21\,621\,600))} - \\
& \frac{\log_e\left(\frac{9}{8}\right) \sqrt{\log_e(21\,621\,600)}}{\log_e(2) \log_e(\log(21\,621\,600))} - \frac{\sqrt{\log_e(21\,621\,600)}}{\log_e^3(\log(21\,621\,600))}
\end{aligned}$$

$$\begin{aligned}
& - \left( \frac{\sqrt{\log(21\,621\,600)} \log\left(\frac{9}{8}\right)}{\log(\log(21\,621\,600)) \log(2)} + \frac{4 \log\left(\frac{3}{2}\right)}{\log^2(\log(21\,621\,600))} - \right. \\
& \quad \left. \log(21\,621\,600) + \frac{\sqrt{\log(21\,621\,600)}}{\log^3(\log(21\,621\,600))} - \frac{1}{\phi} \right) = \\
& -\text{Li}_1(-21\,621\,599) + \frac{1}{\phi} + \frac{4 \text{Li}_1\left(-\frac{1}{2}\right)}{(-\text{Li}_1(1 - \log(21\,621\,600)))^2} + \\
& - \frac{\text{Li}_1\left(1 - \frac{9}{8}\right) \sqrt{-\text{Li}_1(-21\,621\,599)}}{\text{Li}_1(-1) (-\text{Li}_1(1 - \log(21\,621\,600)))} - \frac{\sqrt{-\text{Li}_1(-21\,621\,599)}}{(-\text{Li}_1(1 - \log(21\,621\,600)))^3}
\end{aligned}$$

**Series representations:**

$$\begin{aligned}
& \left( \frac{\sqrt{\log(21621600)} \log\left(\frac{\rho}{8}\right)}{\log(\log(21621600)) \log(2)} + \frac{4 \log\left(\frac{3}{2}\right)}{\log^2(\log(21621600))} - \right. \\
& \quad \left. \log(21621600) + \frac{\sqrt{\log(21621600)}}{\log^3(\log(21621600))} - \frac{1}{\phi} \right) = \\
& \frac{2}{1 + \sqrt{5}} + 2i\pi \left[ \frac{\arg(21621600 - x)}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (21621600 - x)^k x^{-k}}{k} - \\
& \frac{\sqrt{2i\pi \left[ \frac{\arg(21621600 - x)}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (21621600 - x)^k x^{-k}}{k}}}{\left( 2i\pi \left[ \frac{\arg(-x + \log(21621600))}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k x^{-k} (-x + \log(21621600))^k}{k} \right)^3} - \\
& \frac{8i\pi \left[ \frac{\arg\left(\frac{3}{2} - x\right)}{2\pi} \right]}{\left( 2i\pi \left[ \frac{\arg(-x + \log(21621600))}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k x^{-k} (-x + \log(21621600))^k}{k} \right)^2} - \\
& \frac{4 \log(x)}{\left( 2i\pi \left[ \frac{\arg(-x + \log(21621600))}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k x^{-k} (-x + \log(21621600))^k}{k} \right)^2} + \\
& \frac{4 \sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{3}{2} - x\right)^k x^{-k}}{k}}{\left( 2i\pi \left[ \frac{\arg(-x + \log(21621600))}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k x^{-k} (-x + \log(21621600))^k}{k} \right)^2} - \\
& \left( 2i\pi \left[ \frac{\arg\left(\frac{\rho}{8} - x\right)}{2\pi} \right] \right. \\
& \quad \left. \sqrt{2i\pi \left[ \frac{\arg(21621600 - x)}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (21621600 - x)^k x^{-k}}{k}} \right) / \\
& \left( \left( 2i\pi \left[ \frac{\arg(2 - x)}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (2 - x)^k x^{-k}}{k} \right) \right. \\
& \quad \left. \left( 2i\pi \left[ \frac{\arg(-x + \log(21621600))}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k x^{-k} (-x + \log(21621600))^k}{k} \right) \right) - \\
& \left( \log(x) \sqrt{2i\pi \left[ \frac{\arg(21621600 - x)}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (21621600 - x)^k x^{-k}}{k}} \right) / \\
& \left( \left( 2i\pi \left[ \frac{\arg(2 - x)}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (2 - x)^k x^{-k}}{k} \right) \right. \\
& \quad \left. \left( 2i\pi \left[ \frac{\arg(-x + \log(21621600))}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k x^{-k} (-x + \log(21621600))^k}{k} \right) \right) + \left( \left( \sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{\rho}{8} - x\right)^k x^{-k}}{k} \right) \right. \\
& \quad \left. \sqrt{2i\pi \left[ \frac{\arg(21621600 - x)}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (21621600 - x)^k x^{-k}}{k}} \right) / \\
& \left( \left( 2i\pi \left[ \frac{\arg(2 - x)}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (2 - x)^k x^{-k}}{k} \right) \right. \\
& \quad \left. \left( 2i\pi \left[ \frac{\arg(-x + \log(21621600))}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k x^{-k} (-x + \log(21621600))^k}{k} \right) \right) \Bigg) \text{ for } x < 0
\end{aligned}$$

$$\begin{aligned}
& \left( \frac{\sqrt{\log(21\,621\,600)} \log\left(\frac{9}{8}\right)}{\log(\log(21\,621\,600)) \log(2)} + \frac{4 \log\left(\frac{3}{2}\right)}{\log^2(\log(21\,621\,600))} - \right. \\
& \quad \left. \frac{\log(21\,621\,600) + \sqrt{\log(21\,621\,600)} - \frac{1}{\phi}}{\log^3(\log(21\,621\,600))} \right) = \\
& \frac{2}{1 + \sqrt{5}} + 2i\pi \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (21\,621\,600 - z_0)^k z_0^{-k}}{k} \\
& \sqrt{2i\pi \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (21\,621\,600 - z_0)^k z_0^{-k}}{k}} \\
& \frac{\left( 2i\pi \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (\log(21\,621\,600) - z_0)^k z_0^{-k}}{k} \right)^3}{8i\pi \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right]} \\
& \frac{\left( 2i\pi \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (\log(21\,621\,600) - z_0)^k z_0^{-k}}{k} \right)^2}{4 \log(z_0)} \\
& \frac{\left( 2i\pi \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (\log(21\,621\,600) - z_0)^k z_0^{-k}}{k} \right)^2}{4 \sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{3}{2} - z_0\right)^k z_0^{-k}}{k}} \\
& \frac{\left( 2i\pi \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (\log(21\,621\,600) - z_0)^k z_0^{-k}}{k} \right)^2}{\left( 2i\pi \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] \right)} \\
& \sqrt{2i\pi \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (21\,621\,600 - z_0)^k z_0^{-k}}{k}} \\
& \left( \left( 2i\pi \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (2 - z_0)^k z_0^{-k}}{k} \right) \right. \\
& \quad \left. \left( 2i\pi \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (\log(21\,621\,600) - z_0)^k z_0^{-k}}{k} \right) \right) \\
& \left( \log(z_0) \sqrt{2i\pi \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (21\,621\,600 - z_0)^k z_0^{-k}}{k}} \right) \\
& \left( \left( 2i\pi \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (2 - z_0)^k z_0^{-k}}{k} \right) \right. \\
& \quad \left. \left( 2i\pi \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (\log(21\,621\,600) - z_0)^k z_0^{-k}}{k} \right) \right) + \left( \sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{9}{8} - z_0\right)^k z_0^{-k}}{k} \right) \\
& \sqrt{2i\pi \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (21\,621\,600 - z_0)^k z_0^{-k}}{k}} \\
& \left( \left( 2i\pi \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (2 - z_0)^k z_0^{-k}}{k} \right) \right. \\
& \quad \left. \left( 2i\pi \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (\log(21\,621\,600) - z_0)^k z_0^{-k}}{k} \right) \right)
\end{aligned}$$



$$7 * - \left( \frac{\ln(9/8)}{\ln(2)} * \left( \frac{\sqrt{\ln(21621600)}}{\ln(\ln(21621600))} \right) + \frac{4 \ln(3/2)}{\ln(\ln(21621600))} \right) - \frac{4 \log(\frac{3}{2})}{\log^2(\log(21621600))} - \log(21621600) + \frac{\sqrt{\log(21621600)}}{\log^3(\log(21621600))} - \frac{1}{\phi} + 7$$

**Input:**

$$7 \times (-1) \left( \frac{\log(\frac{9}{8})}{\log(2)} \times \frac{\sqrt{\log(21621600)}}{\log(\log(21621600))} + \frac{4 \log(\frac{3}{2})}{\log^2(\log(21621600))} - \log(21621600) + \frac{\sqrt{\log(21621600)}}{\log^3(\log(21621600))} - \frac{1}{\phi} \right) + 7$$

log(x) is the natural logarithm

φ is the golden ratio

**Exact result:**

$$7 - 7 \left( -\frac{1}{\phi} + \frac{\sqrt{\log(21621600)}}{\log^3(\log(21621600))} + \frac{4 \log(\frac{3}{2})}{\log^2(\log(21621600))} - \log(21621600) + \frac{\log(\frac{9}{8}) \sqrt{\log(21621600)}}{\log(2) \log(\log(21621600))} \right)$$

**Decimal approximation:**

125.1266950222466044593151779461894920517941019959727998170...

125.1266950222466.... result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T = 0 and to the Higgs boson mass 125.18 GeV

**Alternate forms:**

$$7 \left( 1 + \frac{2}{1 + \sqrt{5}} - \frac{\sqrt{\log(21621600)}}{\log^3(\log(21621600))} \left( \frac{1}{\log^3(\log(21621600))} + \frac{\log(\frac{9}{8})}{\log(2) \log(\log(21621600))} \right) - \frac{4 \log(\frac{3}{2})}{\log^2(\log(21621600))} + \log(21621600) \right)$$

$$7 + \frac{14}{1 + \sqrt{5}} - \frac{7 \sqrt{\log(21\,621\,600)}}{\log^3(\log(21\,621\,600))} - \frac{28 \log\left(\frac{3}{2}\right)}{\log^2(\log(21\,621\,600))} +$$

$$\frac{7 \log\left(\frac{9}{8}\right) \sqrt{\log(21\,621\,600)}}{\log(2) \log(\log(21\,621\,600))}$$

$$\frac{7(3 \log^2(\log(21\,621\,600)) + \sqrt{5} \log^2(\log(21\,621\,600)) - 4 \log\left(\frac{3}{2}\right) - 4 \sqrt{5} \log\left(\frac{3}{2}\right))}{(1 + \sqrt{5}) \log^2(\log(21\,621\,600))} -$$

$$\frac{7 \sqrt{\log(21\,621\,600)} \left(\log\left(\frac{9}{8}\right) \log^2(\log(21\,621\,600)) + \log(2)\right)}{\log(2) \log^3(\log(21\,621\,600))} + 7 \log(21\,621\,600)$$

### Alternative representations:

$$7(-1) \left( \frac{\sqrt{\log(21\,621\,600)} \log\left(\frac{9}{8}\right)}{\log(\log(21\,621\,600)) \log(2)} + \frac{4 \log\left(\frac{3}{2}\right)}{\log^2(\log(21\,621\,600))} - \right.$$

$$\left. \log(21\,621\,600) + \frac{\sqrt{\log(21\,621\,600)}}{\log^3(\log(21\,621\,600))} - \frac{1}{\phi} \right) + 7 =$$

$$7 - 7 \left( -\log(a) \log_a(21\,621\,600) - \frac{1}{\phi} + \frac{4 \log(a) \log_a\left(\frac{3}{2}\right)}{(\log(a) \log_a(\log(21\,621\,600)))^2} + \right.$$

$$\frac{\log(a) \log_a\left(\frac{9}{8}\right) \sqrt{\log(a) \log_a(21\,621\,600)}}{(\log(a) \log_a(2)) (\log(a) \log_a(\log(21\,621\,600)))} +$$

$$\left. \frac{\sqrt{\log(a) \log_a(21\,621\,600)}}{(\log(a) \log_a(\log(21\,621\,600)))^3} \right)$$

$$7(-1) \left( \frac{\sqrt{\log(21\,621\,600)} \log\left(\frac{9}{8}\right)}{\log(\log(21\,621\,600)) \log(2)} + \frac{4 \log\left(\frac{3}{2}\right)}{\log^2(\log(21\,621\,600))} - \right.$$

$$\left. \log(21\,621\,600) + \frac{\sqrt{\log(21\,621\,600)}}{\log^3(\log(21\,621\,600))} - \frac{1}{\phi} \right) + 7 =$$

$$7 - 7 \left( -\log_e(21\,621\,600) - \frac{1}{\phi} + \frac{4 \log_e\left(\frac{3}{2}\right)}{\log_e^2(\log(21\,621\,600))} + \right.$$

$$\frac{\log_e\left(\frac{9}{8}\right) \sqrt{\log_e(21\,621\,600)}}{\log_e(2) \log_e(\log(21\,621\,600))} + \frac{\sqrt{\log_e(21\,621\,600)}}{\log_e^3(\log(21\,621\,600))} \left. \right)$$



$$7(-1) \left( \frac{\sqrt{\log(21\,621\,600)} \log\left(\frac{9}{8}\right)}{\log(\log(21\,621\,600)) \log(2)} + \frac{4 \log\left(\frac{3}{2}\right)}{\log^2(\log(21\,621\,600))} - \right. \\ \left. \log(21\,621\,600) + \frac{\sqrt{\log(21\,621\,600)}}{\log^3(\log(21\,621\,600))} - \frac{1}{\phi} \right) + 7 = \\ 7 - 7 \left( \operatorname{Li}_1(-21\,621\,599) - \frac{1}{\phi} - \frac{4 \operatorname{Li}_1\left(-\frac{1}{2}\right)}{(-\operatorname{Li}_1(1 - \log(21\,621\,600)))^2} - \right. \\ \left. - \frac{\operatorname{Li}_1\left(1 - \frac{9}{8}\right) \sqrt{-\operatorname{Li}_1(-21\,621\,599)}}{\operatorname{Li}_1(-1) (-\operatorname{Li}_1(1 - \log(21\,621\,600)))} + \frac{\sqrt{-\operatorname{Li}_1(-21\,621\,599)}}{(-\operatorname{Li}_1(1 - \log(21\,621\,600)))^3} \right)$$

$$7^* - \left( \frac{\ln(9/8)}{\ln(2)} * \left( \frac{\sqrt{\ln(21621600)}}{(\ln(\ln(21621600)))} \right) \right) + \left( \frac{4 \ln(3/2)}{(\ln(\ln(21621600)))^2} - \right. \\ \left. (\ln(21621600)) + \left( \frac{\sqrt{\ln(21621600)}}{(\ln(\ln(21621600))^3)} \right) - 1 / \text{golden ratio} \right) + 18 + 3$$

**Input:**

$$7 \times (-1) \left( \frac{\log\left(\frac{9}{8}\right)}{\log(2)} \times \frac{\sqrt{\log(21\,621\,600)}}{\log(\log(21\,621\,600))} + \frac{4 \log\left(\frac{3}{2}\right)}{\log^2(\log(21\,621\,600))} - \right. \\ \left. \log(21\,621\,600) + \frac{\sqrt{\log(21\,621\,600)}}{\log^3(\log(21\,621\,600))} - \frac{1}{\phi} \right) + 18 + 3$$

$\log(x)$  is the natural logarithm

$\phi$  is the golden ratio

**Exact result:**

$$21 - 7 \left( -\frac{1}{\phi} + \frac{\sqrt{\log(21\,621\,600)}}{\log^3(\log(21\,621\,600))} + \right. \\ \left. \frac{4 \log\left(\frac{3}{2}\right)}{\log^2(\log(21\,621\,600))} - \log(21\,621\,600) + \frac{\log\left(\frac{9}{8}\right) \sqrt{\log(21\,621\,600)}}{\log(2) \log(\log(21\,621\,600))} \right)$$

**Decimal approximation:**

139.1266950222466044593151779461894920517941019959727998170...

139.1266950222466.... result practically equal to the rest mass of Pion meson  
139.57 MeV

**Alternate forms:**

$$7 \left( 3 + \frac{2}{1 + \sqrt{5}} - \frac{\sqrt{\log(21\,621\,600)}}{\log^3(\log(21\,621\,600))} \left( \frac{1}{\log^3(\log(21\,621\,600))} + \frac{\log\left(\frac{9}{8}\right)}{\log(2) \log(\log(21\,621\,600))} \right) - \frac{4 \log\left(\frac{3}{2}\right)}{\log^2(\log(21\,621\,600))} + \log(21\,621\,600) \right)$$

$$21 + \frac{14}{1 + \sqrt{5}} - \frac{7 \sqrt{\log(21\,621\,600)}}{\log^3(\log(21\,621\,600))} - \frac{28 \log\left(\frac{3}{2}\right)}{\log^2(\log(21\,621\,600))} + 7 \log(21\,621\,600) - \frac{7 \log\left(\frac{9}{8}\right) \sqrt{\log(21\,621\,600)}}{\log(2) \log(\log(21\,621\,600))}$$

$$\frac{7(5 \log^2(\log(21\,621\,600)) + 3 \sqrt{5} \log^2(\log(21\,621\,600)) - 4 \log\left(\frac{3}{2}\right) - 4 \sqrt{5} \log\left(\frac{3}{2}\right))}{(1 + \sqrt{5}) \log^2(\log(21\,621\,600))} - \frac{7 \sqrt{\log(21\,621\,600)} \left( \log\left(\frac{9}{8}\right) \log^2(\log(21\,621\,600)) + \log(2) \right)}{\log(2) \log^3(\log(21\,621\,600))} + 7 \log(21\,621\,600)$$

**Alternative representations:**

$$7(-1) \left( \frac{\sqrt{\log(21\,621\,600)} \log\left(\frac{9}{8}\right)}{\log(\log(21\,621\,600)) \log(2)} + \frac{4 \log\left(\frac{3}{2}\right)}{\log^2(\log(21\,621\,600))} - \log(21\,621\,600) + \frac{\sqrt{\log(21\,621\,600)}}{\log^3(\log(21\,621\,600))} - \frac{1}{\phi} \right) + 18 + 3 =$$

$$21 - 7 \left( -\log(a) \log_a(21\,621\,600) - \frac{1}{\phi} + \frac{4 \log(a) \log_a\left(\frac{3}{2}\right)}{(\log(a) \log_a(\log(21\,621\,600)))^2} + \frac{\log(a) \log_a\left(\frac{9}{8}\right) \sqrt{\log(a) \log_a(21\,621\,600)}}{(\log(a) \log_a(2)) (\log(a) \log_a(\log(21\,621\,600)))} + \frac{\sqrt{\log(a) \log_a(21\,621\,600)}}{(\log(a) \log_a(\log(21\,621\,600)))^3} \right)$$

$$\begin{aligned}
& 7(-1) \left( \frac{\sqrt{\log(21\,621\,600)} \log\left(\frac{9}{8}\right)}{\log(\log(21\,621\,600)) \log(2)} + \frac{4 \log\left(\frac{3}{2}\right)}{\log^2(\log(21\,621\,600))} - \right. \\
& \quad \left. \log(21\,621\,600) + \frac{\sqrt{\log(21\,621\,600)}}{\log^3(\log(21\,621\,600))} - \frac{1}{\phi} \right) + 18 + 3 = \\
& 21 - 7 \left( -\log_e(21\,621\,600) - \frac{1}{\phi} + \frac{4 \log_e\left(\frac{3}{2}\right)}{\log_e^2(\log(21\,621\,600))} + \right. \\
& \quad \left. \frac{\log_e\left(\frac{9}{8}\right) \sqrt{\log_e(21\,621\,600)}}{\log_e(2) \log_e(\log(21\,621\,600))} + \frac{\sqrt{\log_e(21\,621\,600)}}{\log_e^3(\log(21\,621\,600))} \right) \\
& 7(-1) \left( \frac{\sqrt{\log(21\,621\,600)} \log\left(\frac{9}{8}\right)}{\log(\log(21\,621\,600)) \log(2)} + \frac{4 \log\left(\frac{3}{2}\right)}{\log^2(\log(21\,621\,600))} - \right. \\
& \quad \left. \log(21\,621\,600) + \frac{\sqrt{\log(21\,621\,600)}}{\log^3(\log(21\,621\,600))} - \frac{1}{\phi} \right) + 18 + 3 = \\
& 21 - 7 \left( \text{Li}_1(-21\,621\,599) - \frac{1}{\phi} - \frac{4 \text{Li}_1\left(-\frac{1}{2}\right)}{(-\text{Li}_1(1 - \log(21\,621\,600)))^2} - \right. \\
& \quad \left. - \frac{\text{Li}_1\left(1 - \frac{9}{8}\right) \sqrt{-\text{Li}_1(-21\,621\,599)}}{\text{Li}_1(-1) (-\text{Li}_1(1 - \log(21\,621\,600)))} + \frac{\sqrt{-\text{Li}_1(-21\,621\,599)}}{(-\text{Li}_1(1 - \log(21\,621\,600)))^3} \right)
\end{aligned}$$

Now, we have that:

$$\begin{aligned}
\phi(N) &= \frac{\log\left(\frac{3}{2}\right)}{\log 2} \text{Li}\{(\log N)^{\log\left(\frac{3}{2}\right)/\log 2}\} - \frac{(\log N)^{\log\left(\frac{3}{2}\right)/\log 2}}{\log \log N} - R(\log N) \\
&\quad + O\left\{\frac{\sqrt{(\log N)}}{(\log \log N)^3}\right\}.
\end{aligned}$$

$$\begin{aligned}
& \ln(3/2) / \ln(2) \text{Li}(\ln(21621600))^{\ln(1.5)/\ln(2)} - \\
& ((\ln(21621600))^{\ln(1.5)/\ln(2)}) / ((\ln(\ln(21621600)))) - \\
& (\ln(21621600)) + (((\sqrt{\ln(21621600)})) / (((\ln(\ln(21621600))))))^3
\end{aligned}$$

**Input:**

$$\frac{\log\left(\frac{3}{2}\right)}{\log(2)} \operatorname{li}(\log(21\,621\,600))^{\log(1.5)/\log(2)} - \frac{\log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} - \log(21\,621\,600) + \frac{\sqrt{\log(21\,621\,600)}}{\log^3(\log(21\,621\,600))}$$

$\log(x)$  is the natural logarithm

$\operatorname{li}(x)$  is the logarithmic integral

**Result:**

-16.4632...

-16.4632.... result very near to the mass of the hypothetical light particle, the boson  $m_x = 16.84$  MeV with minus sign

**Alternative representations:**

$$\frac{\operatorname{li}(\log(21\,621\,600))^{\log(1.5)/\log(2)} \log\left(\frac{3}{2}\right)}{\log(2)} - \frac{\log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} - \log(21\,621\,600) + \frac{\sqrt{\log(21\,621\,600)}}{\log^3(\log(21\,621\,600))} = -\log(21\,621\,600) + \frac{\log\left(\frac{3}{2}\right) \operatorname{Ei}(\log(\log(21\,621\,600))^{\log(1.5)/\log(2)})}{\log(2)} - \frac{\log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} + \frac{\sqrt{\log(21\,621\,600)}}{\log^3(\log(21\,621\,600))}$$

$$\frac{\operatorname{li}(\log(21\,621\,600))^{\log(1.5)/\log(2)} \log\left(\frac{3}{2}\right)}{\log(2)} - \frac{\log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} - \log(21\,621\,600) + \frac{\sqrt{\log(21\,621\,600)}}{\log^3(\log(21\,621\,600))} = -\log(a) \log_a(21\,621\,600) + \frac{\log(a) \log_a\left(\frac{3}{2}\right) \operatorname{li}(\log(a) \log_a(21\,621\,600))^{\log(a) \log_a(1.5)/(\log(a) \log_a(2))}}{\log(a) \log_a(2)} - \frac{(\log(a) \log_a(21\,621\,600))^{\log(a) \log_a(1.5)/(\log(a) \log_a(2))}}{\log(a) \log_a(\log(21\,621\,600))} + \frac{\sqrt{\log(a) \log_a(21\,621\,600)}}{(\log(a) \log_a(\log(21\,621\,600)))^3}$$

$$\frac{\text{li}(\log(21\,621\,600))^{\log(1.5)/\log(2)} \log\left(\frac{3}{2}\right)}{\log(2)} - \frac{\log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} - \log(21\,621\,600) + \frac{\sqrt{\log(21\,621\,600)}}{\log^3(\log(21\,621\,600))} = -\log(21\,621\,600) - \frac{\log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} + \frac{1}{\log(2)} \log\left(\frac{3}{2}\right) \left( -\Gamma(0, -\log(\log(21\,621\,600))) - \log(-\log(\log(21\,621\,600))) + \frac{1}{2} \left( \log(\log(\log(21\,621\,600))) - \log\left(\frac{1}{\log(\log(21\,621\,600))}\right) \right) \right)^{\log(1.5)/\log(2)} + \frac{\sqrt{\log(21\,621\,600)}}{\log^3(\log(21\,621\,600))}$$

$$-8 * \left( \frac{\ln(3/2)}{\ln(2)} \text{Li}(\ln(21621600))^{\ln(1.5)/\ln(2)} - \frac{(\ln(21621600))^{\ln(1.5)/\ln(2)}}{(\ln(\ln(21621600)))} - (\ln(21621600)) + \frac{(\sqrt{\ln(21621600)})}{(\ln(\ln(21621600)))^3} \right) + 8$$

**Input:**

$$-8 \left( \frac{\log\left(\frac{3}{2}\right)}{\log(2)} \text{li}(\log(21\,621\,600))^{\log(1.5)/\log(2)} - \frac{\log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} - \log(21\,621\,600) + \frac{\sqrt{\log(21\,621\,600)}}{\log^3(\log(21\,621\,600))} \right) + 8$$

$\log(x)$  is the natural logarithm

$\text{li}(x)$  is the logarithmic integral

**Result:**

139.705...

139.705... result practically equal to the rest mass of Pion meson 139.57 MeV

**Alternative representations:**

$$\begin{aligned}
 & -8 \left( \frac{\operatorname{li}(\log(21\,621\,600))^{\log(1.5)/\log(2)} \log\left(\frac{3}{2}\right)}{\log(2)} - \frac{\log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} - \right. \\
 & \quad \left. \log(21\,621\,600) + \frac{\sqrt{\log(21\,621\,600)}}{\log^3(\log(21\,621\,600))} \right) + 8 = \\
 & 8 - 8 \left( -\log(21\,621\,600) + \frac{\log\left(\frac{3}{2}\right) \operatorname{Ei}(\log(\log(21\,621\,600)))^{\log(1.5)/\log(2)}}{\log(2)} - \right. \\
 & \quad \left. \frac{\log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} + \frac{\sqrt{\log(21\,621\,600)}}{\log^3(\log(21\,621\,600))} \right) \\
 \\
 & -8 \left( \frac{\operatorname{li}(\log(21\,621\,600))^{\log(1.5)/\log(2)} \log\left(\frac{3}{2}\right)}{\log(2)} - \right. \\
 & \quad \left. \frac{\log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} - \log(21\,621\,600) + \frac{\sqrt{\log(21\,621\,600)}}{\log^3(\log(21\,621\,600))} \right) + \\
 & 8 = 8 - 8 \left( -\log(a) \log_a(21\,621\,600) + \right. \\
 & \quad \frac{\log(a) \log_a\left(\frac{3}{2}\right) \operatorname{li}(\log(a) \log_a(21\,621\,600))^{\log(a) \log_a(1.5)/(\log(a) \log_a(2))}}{\log(a) \log_a(2)} - \\
 & \quad \frac{(\log(a) \log_a(21\,621\,600))^{\log(a) \log_a(1.5)/(\log(a) \log_a(2))}}{\log(a) \log_a(\log(21\,621\,600))} + \\
 & \quad \left. \frac{\sqrt{\log(a) \log_a(21\,621\,600)}}{(\log(a) \log_a(\log(21\,621\,600)))^3} \right)
 \end{aligned}$$

$$\begin{aligned}
& -8 \left( \frac{\text{li}(\log(21\,621\,600))^{\log(1.5)/\log(2)} \log\left(\frac{3}{2}\right)}{\log(2)} - \frac{\log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} - \right. \\
& \quad \left. \log(21\,621\,600) + \frac{\sqrt{\log(21\,621\,600)}}{\log^3(\log(21\,621\,600))} \right) + 8 = \\
& 8 - 8 \left( -\log(21\,621\,600) - \frac{\log^{\log(1.5)/\log(2)}(21\,621\,600)}{\log(\log(21\,621\,600))} + \right. \\
& \quad \frac{1}{\log(2)} \log\left(\frac{3}{2}\right) \left( -\Gamma(0, -\log(\log(21\,621\,600))) - \right. \\
& \quad \quad \log(-\log(\log(21\,621\,600))) + \frac{1}{2} \left( \log(\log(\log(21\,621\,600))) - \right. \\
& \quad \quad \quad \left. \left. \left. \log\left(\frac{1}{\log(\log(21\,621\,600))}\right) \right)^{\log(1.5)/\log(2)} \right) \right) + \right. \\
& \quad \left. \frac{\sqrt{\log(21\,621\,600)}}{\log^3(\log(21\,621\,600))} \right)
\end{aligned}$$

From:

<https://blog.wolfram.com/2013/05/01/after-100-years-ramanujan-gap-filled/>

$S\left(e^{-\frac{\pi}{\sqrt{5}}}\right)$	$\sqrt[5]{\sqrt{1 + \frac{(\sqrt{5}-1)^{10}}{1024}} - \frac{1}{32}(\sqrt{5}-1)^5}$	Ramanujan
$S\left(e^{-\frac{\pi}{\sqrt{15}}}\right)$	$\frac{\sqrt[5]{-3+5\sqrt{5} + \sqrt{30(5-\sqrt{5})}}}{2^{2/5}}$	Ramanathan
$S\left(e^{-\sqrt{\frac{3}{5}}\pi}\right)$	$\frac{\sqrt[5]{-3-5\sqrt{5} + \sqrt{30(5+\sqrt{5})}}}{2^{2/5}}$	Ramanathan
$S(e^{-\pi/5})$	$\frac{\sqrt[5]{9-\sqrt{5}+3\sqrt{2(5-\sqrt{5})}}}{\sqrt[5]{2}}$	Ramanathan
$S\left(e^{-\frac{\pi}{\sqrt{35}}}\right)$	$\sqrt[5]{-7+5\sqrt{5} + \sqrt{35(5-2\sqrt{5})}}$	Ramanujan
$S\left(e^{-\sqrt{\frac{7}{5}}\pi}\right)$	$\sqrt[5]{-7-5\sqrt{5} + \sqrt{35(5+2\sqrt{5})}}$	Ramanujan
$S\left(e^{-\frac{\pi}{3\sqrt{5}}}\right)$	$(568 - 325\sqrt{3} - 260\sqrt{5} + 150\sqrt{15} + 2\sqrt{(5(65750 - 37960\sqrt{3} - 29393\sqrt{5} + 16970\sqrt{15})}))^{(1/5)}$	Marichev
$S\left(e^{-\frac{3\pi}{\sqrt{5}}}\right)$	$\sqrt[5]{\sqrt{a^2+1} - a}$ where $a = -568 - 325\sqrt{3} + 260\sqrt{5} + 150\sqrt{15}$	Berndt
$S\left(e^{-\frac{\pi}{\sqrt{55}}}\right)$	$\left(-\frac{81}{8} - \frac{45\sqrt{5}}{8} + \frac{25}{4}\sqrt{\frac{1}{2}(19+9\sqrt{5})} + \frac{3}{4}\sqrt{\left(5\left(225+103\sqrt{5} - 5\sqrt{22(181+81\sqrt{5})}\right)\right)}\right)^{(1/5)}$	Marichev
$S\left(e^{-\sqrt{\frac{11}{5}}\pi}\right)$	$\sqrt[5]{\sqrt{a^2+1} - a}$ where $a = \frac{1}{8}\left(81+45\sqrt{5} + 25\sqrt{38+18\sqrt{5}}\right)$	Berndt
$S\left(e^{-\sqrt{\frac{13}{5}}\pi}\right)$	$\sqrt[5]{\sqrt{a^2+1} - a}$ where $a = \frac{1}{2}\left(\left(\frac{(\sqrt{\sqrt{65}+7}-\sqrt{\sqrt{65}-1})\sqrt{5}}{(\sqrt{\sqrt{65}+9}-\sqrt{\sqrt{65}+7})^2}\right)^3 - 11\right)$	Berndt
$S\left(e^{-\frac{\pi}{5\sqrt{3}}}\right)$	$\sqrt[5]{3+\sqrt{5} + \sqrt{15+6\sqrt{5}}}$	Marichev
$S\left(e^{-\frac{\sqrt{3}\pi}{5}}\right)$	$\sqrt[5]{3-\sqrt{5} + \sqrt{3(5-2\sqrt{5})}}$	Marichev



We have that:

$$\left(\left(\left(\left(\left(1+\frac{(\sqrt{5}-1)^{10}}{1024}\right)^{1/2} - \frac{1}{32}(\sqrt{5}-1)^5\right)\right)\right)\right)^{1/5}$$

**Input:**

$$\sqrt[5]{\sqrt{1 + \frac{(\sqrt{5}-1)^{10}}{1024} - \frac{1}{32}(\sqrt{5}-1)^5}}$$

**Decimal approximation:**

0.982151564550401278618628523902506593809168530490215164666...

0.98215156455.... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}}}{1 + \sqrt[5]{\sqrt{\varphi^5 \sqrt[4]{5^3} - 1}}} - \varphi + 1 = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

and to the dilaton value **0.989117352243 =  $\phi$**

**Alternate forms:**

$$\frac{1}{2} \sqrt[5]{\sqrt{10(25 - 11\sqrt{5}) - 5\sqrt{5} + 11} 2^{4/5}}$$

$$\sqrt[5]{\frac{1}{2} \left( 11 - 5\sqrt{5} + \sqrt{250 - 110\sqrt{5}} \right)}$$

$$\sqrt[5]{\frac{1}{2} \left( 11 - 5\sqrt{5} + \sqrt{10(25 - 11\sqrt{5})} \right)}$$

**Minimal polynomial:**

$$x^{20} - 22x^{15} - 6x^{10} + 22x^5 + 1$$

**Continued fraction:**

$$\begin{array}{r}
 1 \\
 \hline
 1 + \frac{1}{55 + \frac{1}{36 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{10 + \frac{1}{1 + \frac{1}{1 + \frac{1}{8 + \frac{1}{1 + \frac{1}{78 + \frac{1}{1 + \frac{1}{1 + \frac{1}{2 + \frac{1}{3 + \frac{1}{7 + \frac{1}{21 + \frac{1}{1 + \frac{1}{\dots}}}}}}}}}}}}}}}}}}}}}}}}
 \end{array}$$

$$\left( \left( (-7 + 5\sqrt{5} + (35(5 - 2\sqrt{5}))^{1/2} \right) \right)^{1/5}$$

**Input:**

$$\sqrt[5]{-7 + 5\sqrt{5} + \sqrt{35(5 - 2\sqrt{5})}}$$

**Decimal approximation:**

1.533433933531672725045223292893931793005891204687420376743...  
 1.53343393353167.....

**Alternate form:**

$$\sqrt[5]{\sqrt{175 - 70\sqrt{5}} + 5\sqrt{5} - 7}$$

**Minimal polynomial:**

$$x^{20} + 28x^{15} - 306x^{10} - 28x^5 + 1$$

**Continued fraction:**

$$\begin{array}{c}
 1 \\
 \hline
 1 + \frac{1}{\hline} \\
 \hline
 1 + \frac{1}{\hline} \\
 \hline
 6 + \frac{1}{\hline} \\
 \hline
 1 + \frac{1}{\hline} \\
 \hline
 43 + \frac{1}{\hline} \\
 \hline
 3 + \frac{1}{\hline} \\
 \hline
 5 + \frac{1}{\hline} \\
 \hline
 33 + \frac{1}{\hline} \\
 \hline
 1 + \frac{1}{\hline} \\
 \hline
 15 + \frac{1}{\hline} \\
 \hline
 4 + \frac{1}{\hline} \\
 \hline
 1 + \frac{1}{\hline} \\
 \hline
 1 + \frac{1}{\hline} \\
 \hline
 1 + \frac{1}{\hline} \\
 \hline
 1 + \frac{1}{\hline} \\
 \hline
 1 + \frac{1}{\hline} \\
 \hline
 1 + \frac{1}{\hline} \\
 \hline
 1 + \frac{1}{\hline} \\
 \hline
 6 + \frac{1}{\hline} \\
 \hline
 1 + \frac{1}{\hline} \\
 \hline
 6 + \frac{1}{\hline} \\
 \hline
 \dots
 \end{array}$$

$$(((-7 - 5\sqrt{5} + (35(5 + 2\sqrt{5}))^{1/2}))^{1/2})^{1/5}$$

**Input:**

$$\sqrt[5]{-7 - 5\sqrt{5} + \sqrt{35(5 + 2\sqrt{5})}}$$

**Decimal approximation:**

0.487312978391510968364287898438916092421960166897653822602...

0.48731297839151.....

**Alternate form:**

$$\sqrt[5]{\sqrt{175 + 70\sqrt{5}} - 5\sqrt{5} - 7}$$

**Minimal polynomial:**

$$x^{20} + 28x^{15} - 306x^{10} - 28x^5 + 1$$

**Continued fraction:**

$$\begin{array}{c}
1 \\
\hline
2 + \frac{1}{19 + \frac{1}{4 + \frac{1}{1 + \frac{1}{6 + \frac{1}{1 + \frac{1}{12 + \frac{1}{1 + \frac{1}{2 + \frac{1}{1 + \frac{1}{8 + \frac{1}{13 + \frac{1}{2 + \frac{1}{5 + \frac{1}{1 + \frac{1}{3 + \frac{1}{3 + \frac{1}{6 + \frac{1}{1 + \frac{1}{14 + \frac{1}{5 + \frac{1}{\dots}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}} \\
\end{array}$$

$$\left(\left(\left(-\frac{81}{8} - \frac{45\sqrt{5}}{8} + \frac{25}{4}\sqrt{\frac{1}{2}(19+9\sqrt{5})}\right)^{1/2} + \frac{3}{4}\left(\left(5(225+103\sqrt{5}-5\sqrt{22(181+81\sqrt{5})}\right)\right)^{1/2}\right)\right)^{1/5}$$

**Input:**

$$\left(-\frac{81}{8} - \frac{45\sqrt{5}}{8} + \frac{25}{4}\sqrt{\frac{1}{2}(19+9\sqrt{5})} + \frac{3}{4}\sqrt{5\left(225+103\sqrt{5}-5\sqrt{22(181+81\sqrt{5})}\right)}\right)^{1/5}$$

**Result:**

$$\left(-\frac{81}{8} - \frac{45\sqrt{5}}{8} + \frac{25}{4}\sqrt{\frac{1}{2}(19+9\sqrt{5})} + \frac{3}{4}\sqrt{5\left(225+103\sqrt{5}-5\sqrt{22(181+81\sqrt{5})}\right)}\right)^{1/5}$$

**Decimal approximation:**

1.584291893060286610795091161449694716780476770797317977602...  
1.584291893060286.....

**Alternate forms:**

$$\frac{1}{2} \left( 25 \sqrt{2(19 + 9\sqrt{5})} - 45\sqrt{5} + \right. \\ \left. 6 \sqrt{5 \left( -5 \sqrt{22(181 + 81\sqrt{5})} + 103\sqrt{5} + 225 \right) - 81} \right)^{(1/5) 2^{2/5}}$$

$$\sqrt[5]{\boxed{\begin{matrix} \text{root of } x^8 + 81x^7 - 1778x^6 + 19683x^5 - \\ 107805x^4 - 19683x^3 - 1778x^2 - 81x + 1 \text{ near } x = 9.98104 \end{matrix}}}$$

$$\frac{1}{2^{3/5}} \left( \left( -81 - 45\sqrt{5} + 25\sqrt{2(19 + 9\sqrt{5})} + \right. \right. \\ \left. \left. 6 \sqrt{5 \left( 225 + 103\sqrt{5} - 5\sqrt{22(181 + 81\sqrt{5})} \right)} \right) \right)^{(1/5)}$$

**Minimal polynomial:**

$$x^{40} + 81x^{35} - 1778x^{30} + 19683x^{25} - 107805x^{20} - 19683x^{15} - 1778x^{10} - 81x^5 + 1$$

**Continued fraction:**

$$1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{6 + \cfrac{1}{1 + \cfrac{1}{4 + \cfrac{1}{1 + \cfrac{1}{4 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{11 + \cfrac{1}{1 + \cfrac{1}{4 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{4 + \cfrac{1}{6 + \cfrac{1}{\dots}}$$

$$\left( \left( (568 - 325\sqrt{3} - 260\sqrt{5} + 150\sqrt{15} + 2 * ((5(65750 - 37960\sqrt{3} - 29393\sqrt{5} + 16970\sqrt{15})))^{1/2}) \right)^{(1/2) \right)^{(1/5)}$$



For  $a = -568 - 325\sqrt{3} + 260\sqrt{5} + 150\sqrt{15}$

$\left(\left(\left(\sqrt{\left(-568 - 325\sqrt{3} + 260\sqrt{5} + 150\sqrt{15}\right)^2 + 1}\right) - (-568 - 325\sqrt{3} + 260\sqrt{5} + 150\sqrt{15})\right)\right)^{1/5}$

**Input:**

$$\left(\sqrt{\left(-568 - 325\sqrt{3} + 260\sqrt{5} + 150\sqrt{15}\right)^2 + 1} - (-568 - 325\sqrt{3} + 260\sqrt{5} + 150\sqrt{15})\right)^{(1/5)}$$

**Result:**

$$\left(568 + 325\sqrt{3} - 260\sqrt{5} - 150\sqrt{15} + \sqrt{1 + \left(-568 - 325\sqrt{3} + 260\sqrt{5} + 150\sqrt{15}\right)^2}\right)^{(1/5)}$$

**Decimal approximation:**

0.436879941888840499316219672919154029445259002822252793774...

0.4368799418888404993.....

**Alternate forms:**

$$\left(\sqrt{1315000 + 759200\sqrt{3} - 587860\sqrt{5} - 339400\sqrt{15} + 325\sqrt{3} - 260\sqrt{5} - 150\sqrt{15} + 568}\right)^{(1/5)}$$

$$\left(568 + 325\sqrt{3} - 260\sqrt{5} - 150\sqrt{15} + 2\sqrt{5\left(65750 + 37960\sqrt{3} - 29393\sqrt{5} - 16970\sqrt{15}\right)}\right)^{(1/5)}$$

$$\left(568 + 325\sqrt{3} - 260\sqrt{5} - 150\sqrt{15} + \sqrt{1 + \left(568 + 325\sqrt{3} - 260\sqrt{5} - 150\sqrt{15}\right)^2}\right)^{(1/5)}$$

**Minimal polynomial:**

$$x^{40} - 4544x^{35} - 196028x^{30} + 5407808x^{25} - 29175930x^{20} - 5407808x^{15} - 196028x^{10} + 4544x^5 + 1$$

**Continued fraction:**

$$\begin{array}{c}
 1 \\
 \hline
 2 + \frac{1}{\hline} \\
 3 + \frac{1}{\hline} \\
 2 + \frac{1}{\hline} \\
 5 + \frac{1}{\hline} \\
 1 + \frac{1}{\hline} \\
 6 + \frac{1}{\hline} \\
 3 + \frac{1}{\hline} \\
 1 + \frac{1}{\hline} \\
 4 + \frac{1}{\hline} \\
 8 + \frac{1}{\hline} \\
 2 + \frac{1}{\hline} \\
 2 + \frac{1}{\hline} \\
 1 + \frac{1}{\hline} \\
 3 + \frac{1}{\hline} \\
 1 + \frac{1}{\hline} \\
 6 + \frac{1}{\hline} \\
 4 + \frac{1}{\hline} \\
 6 + \frac{1}{\hline} \\
 7 + \frac{1}{\hline} \\
 1 + \frac{1}{\hline} \\
 1 + \frac{1}{\hline} \\
 27 + \frac{1}{\hline} \\
 \dots
 \end{array}$$

$$((3+\sqrt{5}+(\sqrt{15+6\sqrt{5}})))^{1/5}$$

**Input:**

$$\sqrt[5]{3 + \sqrt{5} + \sqrt{15 + 6\sqrt{5}}}$$

**Decimal approximation:**

1.602464692075199162701844965538622490476260432697997939681...

1.6024646920751991627... result practically equal to the elementary charge without exponent

**Alternate form:**

$$\sqrt[5]{3 + \sqrt{5} + \sqrt{3(5 + 2\sqrt{5})}}$$

**Minimal polynomial:**

$$x^{20} - 12x^{15} + 14x^{10} + 12x^5 + 1$$





**Continued fraction:**

$$\begin{array}{l}
 1 + \cfrac{1}{6 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{8 + \cfrac{1}{1 + \cfrac{1}{46 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{1 + \cfrac{1}{14 + \cfrac{1}{17 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{\dots}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}} \\
 \end{array}$$

$$\left( \left( \left( \left( \left( -3 + 5\sqrt{5} + (30(5 - \sqrt{5}))^{1/2} \right)^{1/5} \right) \right) \right) \right) / (2)^{(2/5)}$$

**Input:**

$$\frac{\sqrt[5]{-3 + 5\sqrt{5} + \sqrt{30(5 - \sqrt{5})}}}{2^{2/5}}$$

**Decimal approximation:**

1.340072390306138126503073359233198144708174724298375381579...

1.3400723903061381265...

**Alternate forms:**

$$\frac{1}{2} \sqrt[5]{\sqrt{30(5 - \sqrt{5})} + 5\sqrt{5} - 3} 2^{3/5}$$

root of  $x^{20} + 3x^{15} - 31x^{10} - 3x^5 + 1$  near  $x = 1.34007$

**Minimal polynomial:**

$$x^{20} + 3x^{15} - 31x^{10} - 3x^5 + 1$$

### Continued fraction:

$$\begin{array}{c}
 \phantom{1+} \phantom{\frac{1}{2+\frac{1}{1+\frac{1}{15+\frac{1}{1+\frac{1}{4+\frac{1}{1+\frac{1}{1+\frac{1}{2+\frac{1}{2+\frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{4+\frac{1}{2+\frac{1}{2+\frac{1}{185+\frac{1}{42+\frac{1}{3+\frac{1}{7+\frac{1}{\dots}} \\
 1+ \frac{1}{2+\frac{1}{1+\frac{1}{15+\frac{1}{1+\frac{1}{4+\frac{1}{1+\frac{1}{1+\frac{1}{2+\frac{1}{2+\frac{1}{1+\frac{1}{1+\frac{1}{4+\frac{1}{2+\frac{1}{2+\frac{1}{185+\frac{1}{42+\frac{1}{3+\frac{1}{7+\frac{1}{\dots}}
 \end{array}$$

$$\left(\left(\left(\left(\left(-3-5\sqrt{5}+(30(5+\sqrt{5}))^{1/2}\right)^{1/5}\right)\right)^{1/5}\right)\right)^{1/5} / (2)^{(2/5)}$$

### Input:

$$\frac{\sqrt[5]{-3-5\sqrt{5} + \sqrt{30(5+\sqrt{5})}}}{2^{2/5}}$$

### Decimal approximation:

0.673271499377838882300517039899565654880002651514020129820...  
 0.673271499377838....

### Alternate forms:

$$\frac{1}{2} \sqrt[5]{\sqrt{30(5+\sqrt{5})} - 5\sqrt{5} - 3} 2^{3/5}$$

$$\text{root of } x^{20} + 3x^{15} - 31x^{10} - 3x^5 + 1 \text{ near } x = 0.673271$$

### Minimal polynomial:

$$x^{20} + 3x^{15} - 31x^{10} - 3x^5 + 1$$



$$\sqrt[5]{\text{root of } x^4 - 18x^3 + 74x^2 + 18x + 1 \text{ near } x = 6.90868}$$

**Minimal polynomial:**

$$x^{20} - 18x^{15} + 74x^{10} + 18x^5 + 1$$

**Continued fraction:**

$$1 + \frac{1}{2 + \frac{1}{8 + \frac{1}{2 + \frac{1}{1 + \frac{1}{1 + \frac{1}{15 + \frac{1}{1 + \frac{1}{1 + \frac{1}{3 + \frac{1}{4 + \frac{1}{1 + \frac{1}{1 + \frac{1}{3 + \frac{1}{1 + \frac{1}{29 + \frac{1}{17 + \frac{1}{2 + \frac{1}{3 + \frac{1}{7 + \frac{1}{5 + \frac{1}{\dots}}$$

$$\left[ \left( \left( \left( \left( \left( \left( \frac{1}{8} \left( 81 + 45\sqrt{5} + 25(\sqrt{38 + 18\sqrt{5}}) \right) \right)^2 + 1 \right) \right)^2 \right) \right)^2 \right)^2 \right)^{1/2} - \left( \frac{1}{8} \left( 81 + 45\sqrt{5} + 25(\sqrt{38 + 18\sqrt{5}}) \right) \right) \right]^{(1/5)}$$

**Input:**

$$\sqrt[5]{\sqrt{\left( \frac{1}{8} \left( 81 + 45\sqrt{5} + 25\sqrt{38 + 18\sqrt{5}} \right) \right)^2 + 1} - \frac{1}{8} \left( 81 + 45\sqrt{5} + 25\sqrt{38 + 18\sqrt{5}} \right)}$$

**Exact result:**

$$\left( \frac{1}{8} \left( -81 - 45\sqrt{5} - 25\sqrt{38 + 18\sqrt{5}} \right) + \sqrt{1 + \frac{1}{64} \left( 81 + 45\sqrt{5} + 25\sqrt{38 + 18\sqrt{5}} \right)^2} \right)^{(1/5)}$$

**Decimal approximation:**

0.397550321973103166162432323106038598430300874787887343768...

0.397550321973103166.....

**Alternate forms:**

$$\sqrt[5]{\text{root of } x^8 + 81x^7 - 1778x^6 + 19683x^5 - 107805x^4 - 19683x^3 - 1778x^2 - 81x + 1 \text{ near } x = 0.00993026}$$

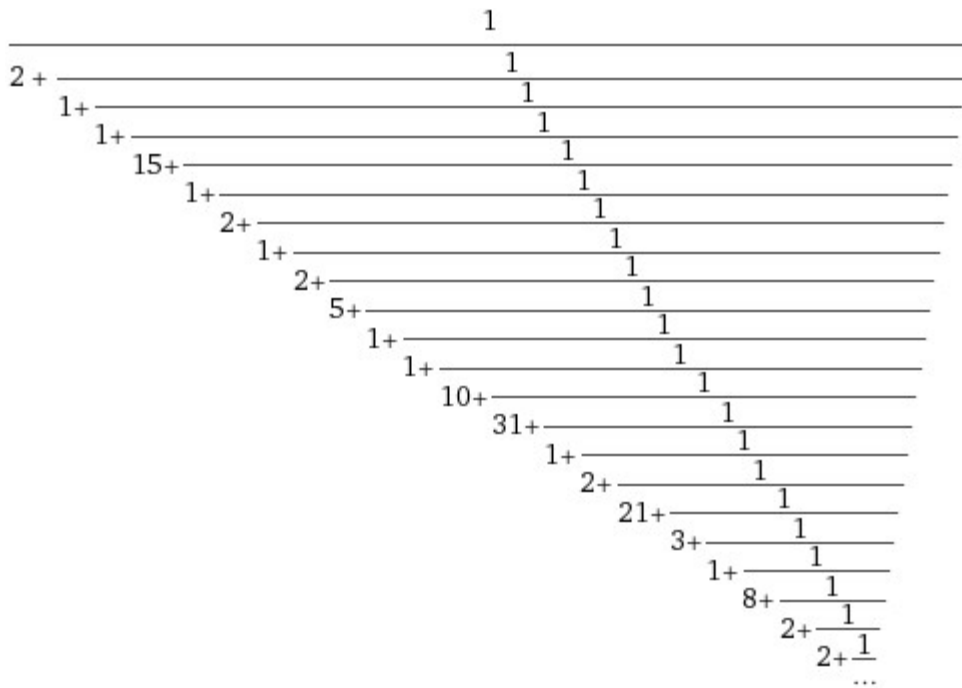
$$\frac{1}{2^{3/5}} \left( \left( -81 - 45\sqrt{5} - 25\sqrt{2(19+9\sqrt{5})} + 3\sqrt{10(450+206\sqrt{5}+45\sqrt{2(19+9\sqrt{5})}+25\sqrt{10(19+9\sqrt{5})})} \right)^{1/5} \right)$$

$$\frac{1}{2^{7/10}} \left( \left( -25\sqrt{38-4i\sqrt{11}} - \sqrt{2} \left( 81+45\sqrt{5}+25\sqrt{i(2\sqrt{11}+-19i)} \right) + \sqrt{5 \left( -500i\sqrt{11}+10\sqrt{2(38-4i\sqrt{11})} \left( 81+45\sqrt{5}+25\sqrt{i(2\sqrt{11}+-19i)} \right) + 4 \left( 4050+729\sqrt{5}+125i\sqrt{11}+45(9+5\sqrt{5})\sqrt{i(2\sqrt{11}+-19i)} \right) \right) \right) \right)^{1/5}$$

**Minimal polynomial:**

$$x^{40} + 81x^{35} - 1778x^{30} + 19683x^{25} - 107805x^{20} - 19683x^{15} - 1778x^{10} - 81x^5 + 1$$

**Continued fraction:**



$$\frac{1}{2}[\frac{(((((((\sqrt{65+7})^{1/2}) - (\sqrt{65-1})^{1/2}) \cdot \sqrt{5}) / ((((((\sqrt{65+9})^{1/2}) - (\sqrt{65+7})^{1/2}) \cdot 2))))))^{3-11}}{11}]$$

**Input:**

$$\frac{1}{2} \left( \frac{\left( \left( \sqrt{\sqrt{65+7}} - \sqrt{\sqrt{65-1}} \right) \sqrt{5} \right)^3}{\left( \left( \sqrt{\sqrt{65+9}} - \sqrt{\sqrt{65+7}} \right) \times 2 \right)^3} - 11 \right)$$

**Exact result:**

$$\frac{1}{2} \left( \frac{5 \sqrt{5} \left( \sqrt{7+\sqrt{65}} - \sqrt{\sqrt{65}-1} \right)^3}{8 \left( \sqrt{9+\sqrt{65}} - \sqrt{7+\sqrt{65}} \right)^3} - 11 \right)$$

**Decimal approximation:**

76.77116559762173762472407948396825354078251084656824071086...

**Alternate forms:**

root of $x^8 + 12494x^7 - 847203x^6 - 7374258x^5 - 96224680x^4 - 2214196942x^3 - 19686288803x^2 - 73531567694x - 101008925599$ near $x = 76.7712$
--

$$\frac{1}{2} \left( \frac{5\sqrt{5} \left( \sqrt{\sqrt{65}-1} - \sqrt{7+\sqrt{65}} \right)^3}{8 \left( \sqrt{7+\sqrt{65}} - \sqrt{9+\sqrt{65}} \right)^3} - 11 \right)$$

$$\left( -25\sqrt{5(\sqrt{65}-1)} - 25\sqrt{13(\sqrt{65}-1)} + \right.$$

$$\left. 88\sqrt{7+\sqrt{65}} + 5\sqrt{5(7+\sqrt{65})} + 25\sqrt{13(7+\sqrt{65})} \right) /$$

$$\left( 4 \left( \sqrt{9+\sqrt{65}} - \sqrt{7+\sqrt{65}} \right)^3 \right) - \frac{11(15+2\sqrt{65})}{\left( \sqrt{9+\sqrt{65}} - \sqrt{7+\sqrt{65}} \right)^2}$$

**Minimal polynomial:**

$$x^8 + 12494x^7 - 847203x^6 - 7374258x^5 - 96224680x^4 - 2214196942x^3 - 19686288803x^2 - 73531567694x - 101008925599$$

$$\left( \left( \left( \left( \left( 76.771165597621737624724^2 + 1 \right) \right)^{1/2} - 76.771165597621737624724 \right) \right) \right)^{1/5}$$

**Input interpretation:**

$$\sqrt[5]{\sqrt{76.771165597621737624724^2 + 1} - 76.771165597621737624724}$$

**Result:**

0.3653849793177824269...

0.3653849793177824269...

**Continued fraction:**

$$\cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{1 + \cfrac{1}{1015 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{6 + \cfrac{1}{58 + \cfrac{1}{\dots}}}}}}}}}}}}}}}}$$



From this sum, we obtain:

$$0.98215156455 + 1.53343393353167 + 0.48731297839151 + 1.584291893060286 + 1.5658292280247776 + 0.4368799418888404993 + 1.6024646920751991627$$

**Input interpretation:**

$$0.98215156455 + 1.53343393353167 + 0.48731297839151 + 1.584291893060286 + 1.5658292280247776 + 0.4368799418888404993 + 1.6024646920751991627$$

**Result:**

$$8.192364231522283262$$

$$8.192364231522283262$$

$$(8.192364231522283262 + 1.1512532253508328497 + 1.3400723903061381265 + 0.673271499377838 + 1.47190230137890442 + 0.397550321973103166 + 0.3653849793177824269)$$

**Input interpretation:**

$$8.192364231522283262 + 1.1512532253508328497 + 1.3400723903061381265 + 0.673271499377838 + 1.47190230137890442 + 0.397550321973103166 + 0.3653849793177824269$$

**Result:**

$$13.5917989492268822511$$

$$13.5917989492268822511$$

In [atomic physics](#), **Rydberg unit of energy**, symbol Ry, corresponds to the energy of the photon whose wavenumber is the Rydberg constant, i.e. the ionization energy of the hydrogen atom.

$$1 \text{ Ry} \equiv hcR_{\infty} = \frac{m_e e^4}{8\epsilon_0^2 h^2} = 13.605\,693\,009(84) \text{ eV} \approx 2.179 \times 10^{-18} \text{ J.}$$

We have also:

**Input interpretation:**

$$1.1512532253508328497 \times 1.3400723903061381265 \times 0.673271499377838 \times 1.47190230137890442 \times 0.397550321973103166 \times 0.3653849793177824269$$

**Result:**

$$0.222080861566500571484756810056558028957477116776177917321...$$

0.222080861566500571...

**Input interpretation:**

$0.98215156455 \times 1.53343393353167 \times 0.48731297839151 \times 1.584291893060286 \times$   
 $1.5658292280247776 \times 0.4368799418888404993 \times 1.6024646920751991627$

**Result:**

1.274622940613291493036434736655864996568907435192470923854...  
1.27462294061329....

**Input interpretation:**

$0.222080861566500571484756810056558028957477116776177917321 \times$   
 $1.274622940613291493036434736655864996568907435192470923854$

**Result:**

0.283069360823826267102517802398034350207117652397036200187...  
0.283069360823826.....

And:

$1/0.98215156455 + 1/1.53343393353167 + 1/0.48731297839151 +$   
 $1/1.584291893060286 + 1/1.5658292280247776 + 1/0.4368799418888404993 +$   
 $1/1.6024646920751991627$

**Input interpretation:**

$\frac{1}{0.98215156455} + \frac{1}{1.53343393353167} + \frac{1}{0.48731297839151} +$   
 $\frac{1}{1.584291893060286} + \frac{1}{1.5658292280247776} +$   
 $\frac{1}{0.4368799418888404993} + \frac{1}{1.6024646920751991627}$

**Result:**

7.905206359518329467967784264198793045502344235017542302462...  
7.90520635951832946.....

$(1/1.1512532253508328497 + 1/1.3400723903061381265 + 1/0.673271499377838 +$   
 $1/1.47190230137890442 + 1/0.397550321973103166 + 1/0.3653849793177824269)$

**Input interpretation:**

$$\frac{1}{1.1512532253508328497} + \frac{1}{1.3400723903061381265} + \frac{1}{0.673271499377838} + \frac{1}{1.47190230137890442} + \frac{1}{0.397550321973103166} + \frac{1}{0.3653849793177824269}$$

**Result:**

9.031769013934306462087446035243170079654329298316796820387...  
 9.0317690139343064....

(7.905206359518329467967784264198793045502344235017542302462 + 9.031769013934306462087446035243170079654329298316796820387)

**Input interpretation:**

$$7.905206359518329467967784264198793045502344235017542302462 + 9.031769013934306462087446035243170079654329298316796820387$$

**Result:**

16.93697537345263593005523029944196312515667353333433912284...

**Repeating decimal:**

16.936975373452635930055230299441963125156673533334339122849

16.9369753734526.... result very near to the mass of the hypothetical light particle, the boson  $m_x = 16.84 \text{ MeV}$

We have that:

$$(8.192364231522283262 + 1.1512532253508328497 + 1.3400723903061381265 + 0.673271499377838 + 1.47190230137890442 + 0.397550321973103166 + 0.3653849793177824269)^3 + 7$$

Where 7 is a Lucas number

**Input interpretation:**

$$(8.192364231522283262 + 1.1512532253508328497 + 1.3400723903061381265 + 0.673271499377838 + 1.47190230137890442 + 0.397550321973103166 + 0.3653849793177824269)^3 + 7$$

**Result:**

2517.908144490570791187700846063198954587925846619788913818...

2517.90814449..... result practically equal to the rest mass of charmed Sigma baryon  
2517.9

$$(8.192364231522283262 + 1.1512532253508328497 + 1.3400723903061381265 + 0.673271499377838 + 1.47190230137890442 + 0.397550321973103166 + 0.3653849793177824269)^3 - 728$$

Where  $728 = 9^3 - 1$  (Ramanujan cube)

**Input interpretation:**

$$(8.192364231522283262 + 1.1512532253508328497 + 1.3400723903061381265 + 0.673271499377838 + 1.47190230137890442 + 0.397550321973103166 + 0.3653849793177824269)^3 - 728$$

**Result:**

1782.908144490570791187700846063198954587925846619788913818...

**Repeating decimal:**

1782.908144490570791187700846063198954587925846619788913818831

1782.90814..... result in the range of the hypothetical mass of Gluino (gluino = 1785.16 GeV).

$$(8.192364231 + 1.151253225 + 1.340072390 + 0.673271499 + 1.471902301 + 0.397550321 + 0.365384979)^3 - 744 - 34 - \pi - \frac{1}{\text{golden ratio}}$$

**Input interpretation:**

$$(8.192364231 + 1.151253225 + 1.340072390 + 0.673271499 + 1.471902301 + 0.397550321 + 0.365384979)^3 - 744 - 34 - \pi - \frac{1}{\phi}$$

$\phi$  is the golden ratio

**Result:**

1729.14852...

1729.14852...

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–

Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

**Alternative representations:**

$$(8.19236 + 1.15125 + 1.34007 + 0.673271 + 1.4719 + 0.39755 + 0.365385)^3 - 744 - 34 - \pi - \frac{1}{\phi} = -778 - \pi + 13.5918^3 - \frac{1}{2 \cos(216^\circ)}$$

$$(8.19236 + 1.15125 + 1.34007 + 0.673271 + 1.4719 + 0.39755 + 0.365385)^3 - 744 - 34 - \pi - \frac{1}{\phi} = -778 - 180^\circ + 13.5918^3 - \frac{1}{2 \cos(216^\circ)}$$

$$(8.19236 + 1.15125 + 1.34007 + 0.673271 + 1.4719 + 0.39755 + 0.365385)^3 - 744 - 34 - \pi - \frac{1}{\phi} = -778 - \pi + 13.5918^3 - \frac{1}{2 \cos(\frac{\pi}{5})}$$

**Series representations:**

$$(8.19236 + 1.15125 + 1.34007 + 0.673271 + 1.4719 + 0.39755 + 0.365385)^3 - 744 - 34 - \pi - \frac{1}{\phi} = 1732.91 - \frac{1}{\phi} - 4 \sum_{k=0}^{\infty} \frac{(-1)^k}{1 + 2k}$$

$$(8.19236 + 1.15125 + 1.34007 + 0.673271 + 1.4719 + 0.39755 + 0.365385)^3 - 744 - 34 - \pi - \frac{1}{\phi} = 1734.91 - \frac{1}{\phi} - 2 \sum_{k=1}^{\infty} \frac{2^k}{\binom{2k}{k}}$$

$$(8.19236 + 1.15125 + 1.34007 + 0.673271 + 1.4719 + 0.39755 + 0.365385)^3 - 744 - 34 - \pi - \frac{1}{\phi} = 1732.91 - \frac{1}{\phi} - \sum_{k=0}^{\infty} \frac{2^{-k} (-6 + 50k)}{\binom{3k}{k}}$$

**Integral representations:**

$$(8.19236 + 1.15125 + 1.34007 + 0.673271 + 1.4719 + 0.39755 + 0.365385)^3 - 744 - 34 - \pi - \frac{1}{\phi} = 1732.91 - \frac{1}{\phi} - 2 \int_0^{\infty} \frac{1}{1 + t^2} dt$$

$$(8.19236 + 1.15125 + 1.34007 + 0.673271 + 1.4719 + 0.39755 + 0.365385)^3 - 744 - 34 - \pi - \frac{1}{\phi} = 1732.91 - \frac{1}{\phi} - 4 \int_0^1 \sqrt{1 - t^2} dt$$

$$(8.19236 + 1.15125 + 1.34007 + 0.673271 + 1.4719 + 0.39755 + 0.365385)^3 - 744 - 34 - \pi - \frac{1}{\phi} = 1732.91 - \frac{1}{\phi} - 2 \int_0^{\infty} \frac{\sin(t)}{t} dt$$

$$(8.192364231 + 1.151253225 + 1.340072390 + 0.673271499 + 1.471902301 + 0.397550321 + 0.365384979)^3 - 1729 + 1/\text{golden ratio}$$

**Input interpretation:**

$$(8.192364231 + 1.151253225 + 1.340072390 + 0.673271499 + 1.471902301 + 0.397550321 + 0.365384979)^3 - 1729 + \frac{1}{\phi}$$

$\phi$  is the golden ratio

**Result:**

782.526177...

782.526177... result practically equal to the rest mass of Omega meson 782.65

**Alternative representations:**

$$(8.19236 + 1.15125 + 1.34007 + 0.673271 + 1.4719 + 0.39755 + 0.365385)^3 - 1729 + \frac{1}{\phi} = -1729 + 13.5918^3 + \frac{1}{2 \sin(54^\circ)}$$

$$(8.19236 + 1.15125 + 1.34007 + 0.673271 + 1.4719 + 0.39755 + 0.365385)^3 - 1729 + \frac{1}{\phi} = -1729 + 13.5918^3 + -\frac{1}{2 \cos(216^\circ)}$$

$$(8.19236 + 1.15125 + 1.34007 + 0.673271 + 1.4719 + 0.39755 + 0.365385)^3 - 1729 + \frac{1}{\phi} = -1729 + 13.5918^3 + -\frac{1}{2 \sin(666^\circ)}$$

$$(8.192364231 + 1.151253225 + 1.340072390 + 0.673271499 + 1.471902301 + 0.397550321 + 0.365384979)^2 * \text{golden ratio}^2$$

**Input interpretation:**

$$(8.192364231 + 1.151253225 + 1.340072390 + 0.673271499 + 1.471902301 + 0.397550321 + 0.365384979)^2 \phi^2$$

$\phi$  is the golden ratio

**Result:**

483.647741...

483.647741... result practically equal to Holographic Ricci dark energy model, where

$$\chi_{\text{RDE}}^2 = 483.130.$$

**Alternative representations:**

$$(8.19236 + 1.15125 + 1.34007 + 0.673271 + 1.4719 + 0.39755 + 0.365385)^2 \phi^2 = 13.5918^2 (2 \sin(54^\circ))^2$$

$$(8.19236 + 1.15125 + 1.34007 + 0.673271 + 1.4719 + 0.39755 + 0.365385)^2 \phi^2 = 13.5918^2 (-2 \cos(216^\circ))^2$$

$$(8.19236 + 1.15125 + 1.34007 + 0.673271 + 1.4719 + 0.39755 + 0.365385)^2 \phi^2 = 13.5918^2 (-2 \sin(666^\circ))^2$$

$$(8.192364231 + 1.151253225 + 1.340072390 + 0.673271499 + 1.471902301 + 0.397550321 + 0.365384979) \times 11^{-7} - e$$

**Input interpretation:**

$$(8.192364231 + 1.151253225 + 1.340072390 + 0.673271499 + 1.471902301 + 0.397550321 + 0.365384979) \times 11^{-7} - e$$

**Result:**

139.7915066...

139.7915066... result practically equal to the rest mass of Pion meson 139.57 MeV

$$(8.192364231 + 1.151253225 + 1.340072390 + 0.673271499 + 1.471902301 + 0.397550321 + 0.365384979) \times 11^{-18} - 2\pi$$

**Input interpretation:**

$$(8.192364231 + 1.151253225 + 1.340072390 + 0.673271499 + 1.471902301 + 0.397550321 + 0.365384979) \times 11^{-18} - 2\pi$$

**Result:**

125.2266031...

125.2266031... result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T = 0 and to the Higgs boson mass 125.18 GeV

$$1/10^{52} \left( \left( \left( \left( 8.192364231 + 1.151253225 + 1.340072390 + 0.673271499 + 1.471902301 + 0.397550321 + 0.365384979 \right) / 11 - 13/10^2 \right) \right) \right)$$

**Input interpretation:**

$$\frac{1}{10^{52}} \left( \frac{1}{11} (8.192364231 + 1.151253225 + 1.340072390 + 0.673271499 + 1.471902301 + 0.397550321 + 0.365384979) - \frac{13}{10^2} \right)$$

**Result:**

$$1.105618086 \times 10^{-52}$$

1.105618086 \* 10<sup>-52</sup> result practically equal to the value of Cosmological Constant

$$1.1056 \cdot 10^{-52} \text{ m}^{-2}$$

With regard 744, we have that:

Several remarkable properties of  $j$  have to do with its  $q$ -expansion (Fourier series expansion), written as a Laurent series in terms of  $q = e^{2\pi i\tau}$  (the square of the nome), which begins:

$$j(\tau) = q^{-1} + 744 + 196884q + 21493760q^2 + 864299970q^3 + 20245856256q^4 + \dots$$

Note that  $j$  has a simple pole at the cusp, so its  $q$ -expansion has no terms below  $q^{-1}$ .

All the Fourier coefficients are integers, which results in several almost integers, notably Ramanujan's constant:

$$e^{\pi\sqrt{163}} \approx 640320^3 + 744.$$

Thence:

$$744 = e^{(\pi \cdot \sqrt{163})} - 640320^3$$

Indeed:

$$e^{(\pi \cdot \sqrt{163})} - 640320^3$$

**Input:**

$$e^{\pi \sqrt{163}} - 640320^3$$

**Exact result:**

$$e^{\sqrt{163} \pi} - 262537412640768000$$



### Decimal approximation:

743.99999999999992500725971981856888793538563373369908627075...

$$743.999... \approx 744$$

### Property:

$-262537412640768000 + e^{\sqrt{163}\pi}$  is a transcendental number

### Series representations:

$$e^{\pi\sqrt{163}} - 640320^3 = -262537412640768000 + e^{\pi\sqrt{162} \sum_{k=0}^{\infty} 162^{-k} \binom{1/2}{k}}$$

$$e^{\pi\sqrt{163}} - 640320^3 = -262537412640768000 + e^{\pi\sqrt{162} \sum_{k=0}^{\infty} \frac{(-1/162)^k (-1/2)_k}{k!}}$$

$$e^{\pi\sqrt{163}} - 640320^3 = -262537412640768000 + \exp\left(\frac{\pi \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 162^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{2\sqrt{\pi}}\right)$$

## References

### **Highly composite numbers**

*Proceedings of the London Mathematical Society*, 2, XIV, 1915, 347 – 409