Refutation of correspondence theory of van Benthem

Abstract: We evaluate 46 equations as not tautologous to refute the following conjectures: McKinsey's axiom; the completeness theorem in axiomatized Kripke frames; basic Kripke truth clause definitions translated from modal into classical logic; substitution yielding equivalent modal correspondence theory; negation of the modal failure description; Geach's axiom as a principle by familiar equivalence transformations; characteristic axiom, satisfiable condition; Löb's axiom; transitivity as $K_2$-derivable from Löb's axiom; definition of a class of Kripke frames using $K_2$-deduction; Hamblin's axiom and definition of discreteness of time; Stalanerker's axiom of excluded middle and definition of linearity of alternative worlds; weak excluded middle defining local convergence as directedness; the method of substitutions from modal to classical logic; directedness (confluence) and connectedness; the stability principle and Kreisel-Putnam axiom; some counterparts of de re/de dicto interchange principles; orthogonally closed quantum logic sets and the ortho-modularity axiom. This litany of denied conjectures refutes the correspondence theory of van Benthem to form a non tautologous fragment of the universal logic $VŁ4$.

We assume the method and apparatus of Meth8/$VŁ4$ with Tautology as the designated proof value, $F$ as contradiction, $N$ as truthity (non-contingency), and $C$ as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

From: van Benthem, J. (1999). Correspondence theory. staff.science.uva.nl/j.vanbenthem/docs/CT.pdf

Comment: Equations are not uniquely named, so we refer by page number in snipped excerpts from the text which unfortunately was published as an image.

I. Introduction to the subject
Correspondences

Indeed, $S4$ may also be shown to be the modal logic of the partial orders; which matches the most famous modal logic with perhaps the most basic type of classical relational structure. Such matchings extend to logics higher up in the $S4$-spectrum. For instance, $S4.2$ with its additional axiom

$$\Box\Box p \rightarrow \Box p$$

is complete with respect to those frames which are reflexive, transitive and directed, or confluent.

$$\forall xyz ((Rxz \land Rxz) \rightarrow \exists u (Ryu \land Rzu))$$

Again, the latter condition is a 'diamond property' of classical fame.

Let $p, q, r, s, t, u, v, w, x, y, z$: $P, Q, R, p, q, u, v, C, x, y, z$
((r&(x&y))&(r&(x&z)))>((r&(y&u))&(r&(z&u))) ;
TTTT TTTT TTTT TTTT (112)
TTTT CCCC TTTT CCCC ( 2 ) \times 4
TTTT TTTT TTTT TTTT ( 2 )

(325.3)

Remark 325: Eq. 325.3 is not tautologous. This refutes S4.2 with its additional axiom as complete for those designated frames.

Not all correspondences are equally simple. For instance, S4.2 has a companion logic S4.1 obtained by enriching S4 with the 'McKinsey Axiom'
\( \Box \ Diamond p \rightarrow \Diamond \Box p \). This converse of the S4.2 axiom turns out to be much more complex. A well-known completeness theorem says that S4.1 axiomatises the modal theory of those Kripke frames which are reflexive, transitive as well as atomic:
\[
\forall x \exists y (Rxy \land \forall z (Ryz \rightarrow z = y)).
\]

(326.1)

Remark 326: Eqs 326.1 and 326.2 are not tautologous. These refute the McKinsey axiom and the completeness theorem that S4.1 axiomatizes specified Kripke frames.

Modal formulas as conditions on the alternative relation

some valuation, the clauses of the basic Kripke truth definition amount to a translation from modal formulas into classical ones involving R. Thus e.g.,

\( \Box p \rightarrow p \) becomes \( \forall y (Rxy \rightarrow Py) \rightarrow Px \)
\( \Diamond p \rightarrow \Box \Diamond p \) becomes \( \forall y (Rxy \rightarrow Py) \rightarrow \forall y |Rxy \rightarrow \forall z (Ryz \rightarrow Pz)) \)

while the McKinsey Axiom \( \Box \Diamond p \rightarrow \Diamond \Box p \) becomes
\( \forall y (Rxy \rightarrow \exists z (Ryz \land Pz)) \rightarrow \exists y (Rxy \land \forall z (Ryz \rightarrow Pz)) \)

(327.2)

Remark 327: Eq. 327.2 is not tautologous. This refutes that "the clauses of the basic
Kripke truth definition amount to a translation from modal formulas into classical one involving \( R^* \). Eq. 327.3 is not tautologous. The refutes the McKinsey axiom (already refuted in Eq. 326.1) from becoming the consequent as rendered.

\[
\forall x Rxx \text{ with } \forall x \forall y (Rxy \rightarrow Fy) \rightarrow P_x).
\]

\[
\begin{array}{l}
(r & (\#x & \#x)) ; \\
\text{FFFF FFFF FFFF (16)}
\end{array}
\]

\[
\begin{array}{l}
(r & (\#x & \#y)) > (p & \#y)) > (p & \#x)) ; \\
\text{FFFF FFFF FFFF FFFF (16)}
\end{array}
\]

Remark 328: Eqs. 328.2 and 328.3 are not tautologous. This refutes the cumbersome mapping of second-order transcriptions as equivalent from axioms of modal logic. In fact the rendering of \( r & (\#x & \#x) \) can be further shortened to the equivalent of \( r & \#x \), that is \( \forall x Rxx \) to \( \forall x Rx \).

Modal correspondence theory

At first sight, proving first-order definability seems a simple matter: just find an equivalent, and show that it works. Still, there is the question how much system there is to this activity. For instance, Examples 1–3 exhibited regularities in their proofs. And indeed, closer inspection reveals that reflexivity, transitivity and directedness may be obtained from the second-order transcriptions of the S4.2-axioms through certain substitutions of ‘minimal’ definable assignments.

The heuristics behind this method is simply this. If, e.g. \( \Box p \rightarrow p \) is true at \( x \), then the most ‘parsimonious’ way of verifying the antecedent (i.e. by having \( V(p) = \{ y | Rxy \} \) carries maximal information about the whole implication. This essentially, is why the substitution of \( Rxu \) for \( Pu \) in

\[
\forall x \forall y (\forall y (Rxy \rightarrow Fy) \rightarrow P_x)
\]

yields the equivalent formula

\[
\forall x (\forall y (Rxy \rightarrow Rx) \rightarrow Rx).
\]

\[
\begin{array}{l}
((r & (\#x & \#y)) > (p & \#y)) > (p & \#x)) ; \\
\text{FFFF FFFF FFFF FFFF (16)}
\end{array}
\]

Remark 329.3: Eq. 329.3 is not tautologous, hence refuting that the specified substitution yields the equivalent formula as claimed.
This time, the heuristics consists in imagining a situation where the property fails, together with a way of ‘maximally exploiting’ this failure through modal formulas. In the above particular case, supposing that $Rxy, Rzx, \neg Ryz, \neg Rzy$, one sets $\square p$ true at $y$ (with $p$ false at $z$) and $\square q$ true at $z$ (with $q$ false at $y$). This has the effect of verifying the following formula at $x$:

$$\Diamond (\square p \land \neg q) \land \Diamond (q \land \neg p).$$

Now, the original property itself will correspond to the negation of this modal ‘failure description’, i.e.

$$\neg (\Diamond (\square p \land \neg q) \land \Diamond (q \land \neg p)).$$

By some familiar equivalence transformations, this becomes

$$\square (p \rightarrow q) \lor \square (q \rightarrow p),$$

a principle known from the literature as Geach’s Axiom.

$$\% (#p \land \neg q) \land \% (#q \land \neg p); \quad CCCC CCCC CCCC CCCC \quad (330.2)$$

Remark 330.2: Eq. 330.2 is not tautologous, hence refuting the it as a formula.

$$\neg (\% (#p \land \neg q) \land \% (#q \land \neg p)) = (v = v); \quad NNNN NNNN NNNN NNNN \quad (330.3)$$

Remark 330.3: Eqs. 330.3 is not tautologous, hence refuting the original property itself as corresponding to the negation of the modal failure description.

$$\# (#p < q) \land \# (#q > p); \quad NFNF NFNF NFNF NFNF \quad (330.4)$$

Greath’s axiom becomes that below via machinations ... and not so!

Remark 330.4: Eq. 330.4 is not tautologous, hence refuting that by some familiar equivalence transformations Geach’s axiom is a principle.

Correspondence and completeness

FACT 5a The modal logic $L$ with characteristic axioms

$$\square p \rightarrow p$$
$$\square \Diamond p \rightarrow \Diamond \square p$$
$$(\Diamond p \land \square (p \rightarrow \square p)) \rightarrow p$$

is first-order definable: its frames are just those satisfying the condition

$$\forall x y (Rxy \iff x = y).$$

$$\# % p > \% # p; \quad NNNN NNNN NNNN NNNN \quad (333.2)$$

Remark 333.2: Eq. 333.2 is not tautologous and hence not a characteristic axiom.

$$(r \land \# (x \land y)) = \# (x = y); \quad FFFF FFFF FFFF FFFF (16)$$
$$NNNN NNNN NNNN NNNN (32)$$

Remark 333.4: Eq. 333.4 is not tautologous and hence not a satisfiable condition.

$$\# (\# p > p) > \# p; \quad CTCT CTCT CTCT CTCT \quad (333.6)$$
Remark 333.6: Eq. 333.6 is not tautologous and hence refutes Löb’s axiom.

The semantic import of the latter will be established in Section 2.2: it holds in those Kripke frames whose alternative relation is transitive, while possessing a well-founded cone. Moreover, transitivity is $K_2$-derivable from Löb’s Axiom, by the substitution of

\[ Rru \land \forall y(Rxy \rightarrow Rxy) \text{ for } Pu. \]

\[ (r\land(x\land u))\land((r\land(u\land y))\rightarrow(r\land(x\land y))) ; \]

\[ (r\land(x\land u))\land((r\land(u\land y))\rightarrow(r\land(x\land y))) ; \]

\[ FFFF FFFF FFFF FFFF (16) \]

\[ FFFF FFFF FFFF FFFF (2) \times 4 \]

\[ FFFF TTTT FFFF TTTT (2) \]  \hspace{1cm} (334.2)

Remark 334.2: Eq. 334.2 is not tautologous and hence refutes the conjecture that transitivity is $K_2$-derivable from Löb’s axiom.

FACT 6. The modal axiom

\[ \Box \Box \bot \land \Box \Box (\Box \Box p \rightarrow p) \rightarrow p, \]

with $\bot$ the falsum, defines the same class of Kripke frames as $\Box \Box \bot \land \Box \bot$. But, the latter formula is not $K$-derivable from the former — even though it is $K_2$-derivable.

Again, there is a correspondence involved here. But the idea is illustrated by a simple $K_2$-deduction at the back of this result:

1. $\forall P(\forall y(Rxy \rightarrow (\forall z(Ryz \rightarrow Pz) \rightarrow Py)) \rightarrow Px)$  \hspace{1cm} (‘$\forall \Box (p \rightarrow p) \rightarrow p$’).
2. $\forall y(Rxy \rightarrow (\forall z(Ryz \rightarrow z \neq x) \rightarrow y \neq x)) \rightarrow x \neq y$ \hspace{1cm} (x $\neq$ u for Pu).
3. $\exists y(Rxy \land \forall z(Ryz \rightarrow z \neq x) \land y = x)$.
4. $\exists Rxy \land \forall z(Ryz \rightarrow z \neq x)$
5. $Rxx \land \forall z(Rxz \rightarrow x \neq x)$
6. $x \neq x$: a contradiction ($\bot$).

\%#(v@v)&#(v@v) ; \hspace{1cm} FFFF FFFF FFFF FFFF \hspace{1cm} (334.3)

\%#(v@v)&#((#p>p)>p) ; \hspace{1cm} FFFF FFFF FFFF FFFF \hspace{1cm} (334.4)

\[ (((r\land(x\land y))\rightarrow(r\land(#y\land z))\rightarrow(#p\land #y))\rightarrow(#p\land #x)) ; \]

\[ TTTT TTTT TTTT TTTT (32) \]

\[ TTTT TTTT TTTT (16) \]

\[ TTTT TTTT (48) \]

\[ TTTT TTTT (16) \]

\[ TTTT CTCT TTTT CTCT (16) \]  \hspace{1cm} (334.5.1)

\[ ((r\land(x\land y))\rightarrow(((r\land(#y\land #z))\rightarrow(#z@x))\rightarrow(#y@x))\rightarrow(x@x)) ; \]

\[ TTTT TTTT TTTT TTTT (112) \]

\[ TTTT CCCC TTTT CCCC (16) \]  \hspace{1cm} (334.5.2)

\[ \neg (((r\land(x\land y))\rightarrow((r\land(#y\land #z))\rightarrow(#z@x))\rightarrow(#y@x))\rightarrow(v@v) ; \]

\[ TTTT TTTT TTTT (16) \]

\[ FFFF FFFF FFFF FFFF (16) \]

\[ CCCC CCCC CCCC CCCC (16) \]

\[ NNNN NNNN NNNN NNNN (16) \]

\[ TTTT TTTT TTTT (16) \]
(r&(x&%y))&(((r&(%y&#z))>(#z@x))&(%y@x)) ;

(r&(x&x))&((r&(x&#z))>(#z@x)) ;

x@x ;

Remark 334.1-6: The entire conjecture is the implication chain of 334.5.1 imply .5.2 imply .5.3 imply .5.4 imply .5.5. imply .5.6. (334.5.7.1)

((((((r&(x&#y))>((r&(#y&#z))>(#p&#z)))>(#p&x))>(r&(x&#y))>
(((r&(x&#z))>(#z@x))>(#y@x))>(x@x)))>((r&(x&%y))&(((r&(%y&#z))>(#z@x))&(%y@x))))>
((r&(x&x))&((r&(x&#z))>(#z@x))))>(x@x) ;

Remark 334.3-5.7.2: Eqs. 334.3-5.7.2 are not tautologous, with 334.3, .5.4, and .5.6 as contradictory. This refutes the conjecture of the specified modal axiom as definition of a class of Kripke frames using K₂-deduction.

Variations and generalisations

EXAMPLE D (‘Hamblin’s Axiom’). (p \land H p) \rightarrow F H p defines discreteness of Time:

∀x∃y > x∀z < y (z = x \lor z < x).

In the logic of counterfactual conditionals, conditional inferences are related to the behaviour of the comparative similar ordering C among alternative worlds.

((#x&%y)>(x&#z))<(y&((z=x)+(z<x))) ;

Remark 337: Eq. 337 is not tautologous. This refutes Hamblin’s axiom and definition of discreteness of time.

EXAMPLE Il (Stalnaker’s Axiom of ‘Conditional Excluded Middle’).

(p \rightarrow q) \lor (p \rightarrow \neg q) defines linearity of alternative worlds:

∀xyz[y = z \lor Cxyz \lor Cxz\neg].
\[(p \lor q) \land (p \supset q) = ((\#y = \#z) \lor (w \land (\#x \land (\#y \land (\#z \land \#y)))) \lor (w \land (\#x \land (\#z \land \#y))))\];
\[
\text{TTTT TTTT TTTT TTTT (32)}
\]
\[
\text{CCCC CCCC CCCC CCCC (64)}
\]
\[
\text{TTTT TTTT TTTT TTTT (32)}
\]

**Remark 337:** Eq. 337.3 is not tautologous. This refutes Stalaner’s axiom of excluded middle and definition of linearity of alternative worlds.

EXAMPLE 12 (‘Weak Excluded Middle’). \(\lnot p \lor \lnot \lnot p\) defines ‘local convergence’ of growing stages, i.e. directedness:

\[
\forall xyz ((x \subseteq y \land x \subseteq z) \rightarrow \exists u (y \subseteq u \land z \subseteq u)).
\]
\[
\lnot (p \lor \lnot \lnot p) = \lnot ((\#y < \#x) \lor (\#z < \#x)) \lor (\lnot (\%u < \#y) \lor (\lnot \lnot (\%u < \#y)));
\]

\[
\text{NNNN NNNN NNNN NNNN (2)} \times 8
\]
\[
\text{FFFF FFFF FFFF FFFF (2)}
\]
\[
\text{NNNN NNNN NNNN NNNN (16)}
\]
\[
\text{NNNN NNNN NNNN NNNN (2)} \times 4
\]
\[
\text{FFFF FFFF FFFF FFFF (2)}
\]
\[
\text{NNNN NNNN NNNN NNNN (16)}
\]
\[
\text{NNNN NNNN NNNN NNNN (2)} \times 4
\]
\[
\text{FFFF FFFF FFFF FFFF (2)}
\]
\[
\text{NNNN NNNN NNNN NNNN (32)}
\]

**Remark 338.2:** Eq. 338.2 is not tautologous, refuting the conjecture of weak excluded middle to define local convergence of growing stages as directedness.

2. Modality

2.2 Correspondence I: from modal to classical logic

*The method of substitutions*

**THEOREM 48.** Modal formulas \(\varphi \rightarrow \psi\) are in \(M1\), provided that

1. \(\varphi\) is constructed from the forms \(p, \Box p, \Box \Box p, \ldots, \bot, T\), using only \(\land\) and \(\lor\) and \(\Box\), while

2. \(\varphi\) is constructed from proposition letters, \(\bot, T\), using \(\land\), \(\lor\), \(\Box\) and \(\Diamond\).

This theorem accounts for cases such as

\(\Diamond(p \land \Box q) \rightarrow \Box(p \lor \Diamond p \lor q)\)

which defines

\[
\forall xyz (Ryz \rightarrow Rzx \rightarrow (z = y \lor Rzy \lor Ryz));
\]
\[
\% (p \land \#p) \lor (p + (\%p + q));
\]
\[
\text{NNNN NNNN NNNN NNNN (354.3)}
\]
\[
\text{NNNN NNNN NNNN NNNN} (354.4)
\]

**Remark 354:** Eqs 354.3 and 354.4 are not tautologous, although both truthty. This refutes that the selected conjecture is not a theorem as claimed.
EXAMPLE 30. Write $\Diamond p \rightarrow \Box q$ as
\[
\forall x \forall y (Rxy \land \forall z (Ryz \rightarrow Pz)) \rightarrow \forall u (Rxu \rightarrow \exists v (Ruv \land Pv)).
\]
Rewrite this to the equivalent
\[
\forall xy (Rxy \rightarrow \forall (\forall z (Ryz \rightarrow Pz)) \rightarrow \forall u (Rxu \rightarrow \exists (Ruv \land Pv))).
\]
Substitute for $P : \lambda z. Ryz$, to obtain
\[
\forall xy (Rxy \rightarrow (\forall z (Ryz \rightarrow Ryz) \rightarrow \forall u (Rxu \rightarrow \exists (Ruv \land Ryv))).
\]
This is equivalent to
\[
\forall xy (Rxy \rightarrow \forall u (Rxu \rightarrow \exists (Ruv \land Ryv))),
\]
i.e. directedness (confluence).
Write $\Diamond (p \land \Box q) \rightarrow \Box (p \lor \Diamond p \lor q)$ as
\[
\forall xy (Rxy \rightarrow \forall (P(y \land \forall z (Ryz \rightarrow Qz)) \rightarrow \forall u (Rxu \rightarrow (Pu \lor 
\forall \exists v (Ruv \land Pv) \lor Qu))).
\]
Substitute for $P : \lambda z. y = z$, and for $Q : \lambda z. Ryz$, to obtain (an equivalent of)
the earlier connectedness.
Write $\Diamond (p \land \Box q) \rightarrow p$ as
\[
\forall xy (Rxy \rightarrow \forall (Py \land \forall z (Ryz \rightarrow Pz)) \rightarrow Px)).
\]
Substitute for $P : \lambda z. y = z \lor Ryz$, to obtain (an equivalent of)
\[
\forall xy (Rxy \rightarrow (Ryx \lor y = x)).
\]
Write $\Box p \rightarrow \Box p$ as
\[
\forall xy (\forall (Rxy \rightarrow \forall z (Ryz \rightarrow Pz)) \rightarrow \forall u (Rxu \rightarrow Pu).)
\]
\[
\begin{align*}
\%#p >& \%#p ; \\
(\%#p > #%p) &= (((r&(x Enrique)) & (\%y & (\%y & (\%z))) > (p & #z)) > ((r&(x & #u)) > ((r&(u & v)) & (p & %v)))) ;
\end{align*}
\]
\[
\begin{array}{cccccccc}
\text{TTTT} & \text{TTTT} & \text{TTTT} & \text{TTTT} & \text{TTTT} & \text{TTTT} (48) \\
\text{TTTT} & \text{TTTT} & \text{TTTT} & \text{TTTT} & \text{TTTT} (2) \times 2 \\
\text{TTTT} & \text{CCCC} & \text{TTTT} & \text{CCCC} (2) \\
\text{TTTT} & \text{TTTT} & \text{TTTT} & \text{TTTT} (2) \\
\text{TTTT} & \text{CTCT} & \text{TTTT} & \text{CTCT} (2) \\
\text{TTTT} & \text{TTTT} & \text{TTTT} & \text{TTTT} (48) \\
\text{TTTT} & \text{TTTT} & \text{TTTT} & \text{TTTT} (2) \times 2 \\
\text{TTTT} & \text{TCTC} & \text{TTTT} & \text{TCTC} (2) \\
\text{TTTT} & \text{TTTT} & \text{TTTT} & \text{TTTT} (4) \\
\end{array}
\]
\[
(\%#p > #%p) = (((r&(x Enrique)) & (\%y & (\%y & (\%z))) > (p & #z)) > ((r&(x & #u)) > ((r&(u & v)) & (p & %v)))) ;
\]
\[
\begin{array}{cccccccc}
\text{TTTT} & \text{TTTT} & \text{TTTT} & \text{TTTT} (16) \times 3 \\
\text{TTTT} & \text{TTTT} & \text{TTTT} & \text{TTTT} (2) \\
\text{TTTT} & \text{CCCC} & \text{TTTT} & \text{CCCC} (2) \\
\text{TTTT} & \text{TTTT} & \text{TTTT} & \text{TTTT} (2) \\
\text{TTTT} & \text{CTCT} & \text{TTTT} & \text{CTCT} (2) \\
\text{TTTT} & \text{TTTT} & \text{TTTT} & \text{TTTT} (16) \\
\text{TTTT} & \text{TTTT} & \text{TTTT} & \text{TTTT} (2) \times 2 \\
\text{TTTT} & \text{TCTC} & \text{TTTT} & \text{TCTC} (2) \\
\text{TTTT} & \text{TTTT} & \text{TTTT} & \text{TTTT} (4) \\
\end{array}
\]
\[
(\%#p > #%p) = (((r&(x Enrique)) & (\%y & (\%y & (\%z))) > (p & #z)) > ((r&(x & #u)) > ((r&(u & v)) & (p & %v)))) ;
\]
\[
\begin{array}{cccccccc}
\text{TTTT} & \text{TTTT} & \text{TTTT} & \text{TTTT} (2) \times 2 \\
\text{TTTT} & \text{TCTC} & \text{TTTT} & \text{TCTC} (2) \\
\text{TTTT} & \text{TTTT} & \text{TTTT} & \text{TTTT} (4) \\
\end{array}
\]
\[
(\%#p > #%p) = (((r&(x Enrique)) & (\%y & (\%y & (\%z))) > (p & #z)) > ((r&(x & #u)) > ((r&(u & v)) & (p & %v)))) ;
\]
Remark 355.1-.5: Eqs. 355.1 is supposed to imply 355.2 to imply 355.3 to imply 355.4 to imply 355.5.

Remark 355.6.2: The respective equations 355.1-.5 are not tautologous, not equivalent, and do not form an implicative chain to 355.6.2 which is refuted as directedness (confluence) and connectedness.

Much more forbidding principles than these have been proposed as intermediate axioms. But surprisingly, these usually turned out to be first-order definable:

EXAMPLE 85. (i) The Stability Principle \((\neg \neg p \rightarrow p) \rightarrow (p \vee \neg p)\) defines

\[
\forall x \exists y z (x \subseteq y \land x \subseteq z \land \neg \exists u (y \subseteq u \land z \subseteq u) \land \\
\land \forall u (\forall s (s \subseteq t \land z \subseteq t) \rightarrow \neg \exists v (x \subseteq v \land y \subseteq v)).
\]

(ii) The Kreisel-Putnam Axiom \((\neg p \rightarrow (q \vee r)) \rightarrow ((\neg p \rightarrow q) \vee (\neg p \rightarrow r))\) defines

\[
\forall x \exists y z (x \subseteq y \land x \subseteq z \land \neg y \subseteq z \land \neg z \subseteq y \land \\
\land \forall u (x \subseteq u \land u \subseteq y \land u \subseteq z) \rightarrow \exists v (u \subseteq v \land \neg y \subseteq v \land \neg z \subseteq v)).
\]

LET \(p, q, r, s, t, u, v, w, x, y, z: P, Q, R, p, q, u, v, C, x, y, z.\) 

\((\neg p \rightarrow p) \rightarrow (p \rightarrow \neg p) = ((\neg (\neg y < #x) \land (\neg z < #x)) \land (\neg (\neg u < y) \land (\neg u < z))) \land ((\neg x < #u) \land (\neg t < #z)) \land (\neg (\neg v < #u) \land (\neg v < #y))).
Let $p, q, s$: $A, x, s$.

\begin{align*}
\neg(p\&q) & > \neg(p\&q); 2. \quad \text{TNTN TNTN TNTN TNTN} \quad (392.2) \\
\neg(p\&q) & > \neg(p\&q); 4. \text{same as 2.} \quad \text{TNTN TNTN TNTN TNTN} \quad (393.1)
\end{align*}

\begin{align*}
#(\neg(p\&q)=(s=s)) & > #(\neg(p\&q)=(s=s)) \quad \text{NNNF NNFN NNFN NNFN} \quad (392.2) \\
#(\neg(p\&q)=(s=s)) & > #(\neg(p\&q)=(s=s)) \quad \text{NNNF NNFN NNFN NNFN} \quad (393.2)
\end{align*}

Remark 393: Eqs. 392.2, 393.1-3 are not tautologous. This means the selected de re/de dicto interchange principle counterparts are identical and refuted, as is the Gödel translation which is not valid in the first place.
**Post-Script:** quantum logic.

Correspondences have not proved uniformly successful in intensional contexts. It seems only fair to finish with a more problematic example.

A possible worlds semantics for quantum logic was proposed in [Goldblatt, 1974]. Kripke frames are now regarded as sets of 'states' of some physical system, provided with a relation of 'orthogonality' ($\perp$). From its physical motivation, two pre-conditions follow for $\perp$, viz. *irreflexivity* and *symmetry*. But in addition, there is also a restriction to 'admissible ranges' for propositions, in the sense that these sets $X \subseteq W$ are to be orthogonally closed:

$$\forall x \in (W - X) \exists y \in (W - X) (\neg x \perp y \wedge \forall z \in X \, \perp z).$$

The key truth clauses are those for conjunction (interpreted as usual), and negation, interpreted as follows:

$$\neg \varphi \text{ is true at } x \text{ if } x \text{ is orthogonal to all } \varphi\text{-worlds.}$$

This semantics validates the usual principles for quantum logic, when $\vee$ is defined in terms of $\neg$, $\wedge$ by the De Morgan law. But, one key principle remains invalid, viz. the ortho-modularity axiom

$$p \leftrightarrow (p \wedge q) \vee (p \wedge \neg(p \wedge q)).$$

**Let** $p, q, s, x, y, z$: $W, X, s, x, y, z$.

$$((\#x<(p-q)) \& (%y<(p-q))) \& (((x\&(s@s)\&y)) \& ((\#z<p) \& ((s@s)\&z))) ;$$

**FFFF FFFF FFFF FFFF**

(395.2)

**Let** $p, q$: $p, q$.

$$p=((p\&q)+(p\&\neg(p\&q))) ;$$

**TTTT TTTT TTTT TTTT**

(395.3)

**Remark 395.2-.3:** Eqs. 395.2 and 395.3 appear as the opposite of the conjectures van Benthem is trying to make, hence refuting the conjectures of orthogonally closed quantum logic sets and the ortho-modularity axiom.

The 46 equations evaluated refute the following conjectures:

McKinsey's axiom; the completeness theorem in axiomatized Kripke frames; basic Kripke truth clause definitions translated from modal into classical logic; substitution yielding equivalent modal correspondence theory; negation of the modal failure description; Geach’s axiom as a principle by familiar equivalence transformations; characteristic axiom, satisfiable condition; Löb’s axiom; transivity as $K_2$-derivable from Löb’s axiom; definition of a class of Kripke frames using $K_2$-deduction; Hamblin’s axiom and definition of discreteness of time; Stalanerk’s axiom of excluded middle and definition of linearity of alternative worlds; weak excluded middle defining local convergence as directedness; the method of substitutions from modal to classical logic; directedness (confluence) and connectedness; the stability principle and Kreisel-Putnam axiom; some counterparts of de re/de dicto interchange principles; orthogonally closed quantum logic sets and the ortho-modularity axiom.

This refutes the correspondence theory of van Benthem.