Refutation of categoricity by Feferman-Vaught theorem via Morley homogeneous systems

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Abstract: We evaluate the first definition of the Feferman-Vaught theorem and the Morely homogeneous system, both as not tautologous. This refutes categoricity. These results form a non tautologous fragment of the universal logic $VŁ4$.

We assume the method and apparatus of Meth8/$VŁ4$ with $Tautology$ as the designated proof value, $F$ as contradiction, $N$ as truthity (non-contingency), and $C$ as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET $\sim$ Not, $\neg$; $+$ Or, $\lor$, $\cup$; $-$ Not Or; $&$ And, $\land$, $\cap$, $\cdot$, $\otimes$; $\setminus$ Not $\land$; $>$ Imply, greater than, $\rightarrow$, $\Rightarrow$, $\succ$, $\supset$; $<$ Not Imply, less than, $\in$, $\subset$, $\subseteq$, $\preceq$; $\approx$, $\equiv$, $\equiv$, $\approx$; $\not\equiv$ Not Equivalent, $\neq$, $\oplus$;

$\%$ possibility, for one or some, $\exists$, $\exists!$, $\Diamond$, $M$; $\#$ necessity, for every or all, $\forall$, $\Box$, $L$;

$(z=z)$ $T$ as tautology, $T$, ordinal 3; $(z@z)$ $F$ as contradiction, $\emptyset$, Null, $\bot$, zero;

$(\%z>\#z)$ $N$ as non-contingency, $\Delta$, ordinal 1; $(\%z<\#z)$ $C$ as contingency, $\nabla$, ordinal 2;

$\sim(y<x)$ $(x\leq y)$, $(x\leq y)$, $(x\subseteq y)$; $(A=B)$ $(A\sim B)$.

Note for clarity, we usually distribute quantifiers onto each designated variable.


Abstract. This paper aims to provide an exposition of the Feferman-Vaught theorem, closely following the presentation in Hodges .. and Chang-Keisler.. The Feferman-Vaught theorem is a method for analysing the first-order theory of a complicated structure by analysing its components. ...

Definition 1. A filter on a set $X$ is a collection $F \subseteq P(X)$ such that: i) $X \in F$ and $\emptyset \in F$. ii) if $A,B \in F$, then $A \cap B \in F$. iii) if $A \in F$ and $A \subseteq B \in P(X)$, then $B \in F$.

(1.1.1)

LET $p, q, r, s, x$: $P, F, A, B, X$.

$(q<(p\&x))>$

$(((x/q)&\neg((s@s)<q))\&( ((r\land s)<q))\&( ((r<s)\&\neg(s<r)<(p\&x)))>(s<p)))$ ;

$TTFF$ $TTFF$ $TTFF$ $TTFF$ $\text{(16)}$

$TTFT$ $TTFT$ $TTFT$ $TTFT$ $\text{(16)}$ (1.1.2)

Remark 1.1.2: Eq. 1.1.2 is not tautologous, refuting the definition of the filter.

We say that a filter $F$ is an ultrafilter if either $A \in F$ or $X\setminus A \in F$ for every $A \in P(X)$.

(1.2.1)

$(q<(p\&x))>((#q<(p\&x))>((r<q)+((x'r)<q)s)$ ;

$TTCC$ $TTCC$ $TTCC$ $TTCC$ $\text{(16)}$

$TTCT$ $TTCT$ $TTCT$ $TTCT$ $\text{(16)}$ (1.2.2)

Remark 1.2.2: Eq. 1.2.2 is not tautologous, refuting the definition of the ultrafilter.

Introduction. Is a theory categorical in one uncountable power necessarily categorical in every uncountable power? The principal result of this paper is an affirmative answer to that question. We actually prove a stronger result, namely: If a theory is categorical in some uncountable power then every uncountable model of that theory is saturated. (Terminology used in the Introduction will be defined in the body of the paper; roughly speaking, a model is saturated, or universal homogeneous, if it contains an element of every possible elementary type relative to its subsystems of strictly smaller power.)

5. Saturated models and categoricity in power. … ([Fn] 22)

In [M. Morley, M; Vaught, R. (1962). Homogeneous universal models, Math. Scand. 11), 37-57.] universal homogeneous systems are considered. … . If $K$ is a class of similar relational systems and $A \in K$ then:

1. $A$ is universal for $K$ if $A$ contains an isomorphic image of every $B \in K$ with $\kappa(B) \leq \kappa(A)$, (5.1.1)
2. $A$ is homogeneous in $K$ if whenever $B_1, B_2 \in K$, $B_1, B_2 \subseteq A$. (5.2.1)

Let $p, q, r, s$: $A, K, B_1, B_2$.

$(p \land q) \land ((r \land s) \land \neg (p \land (r \land s)))$; TFFT TFTT TFTT TTTT (5.2.2)

Remark 5.2.2: Eq. 5.2.2 is not tautologous. It refutes the homogeneous part of universal homogeneous systems.