We take
\[ \mathbb{N} = \{0, 1, 2, 3, \ldots\} \]
the set of natural numbers obtained by Peano’s axioms.

\[ \mathbb{Z} = \{p - q | \forall p, q \in \mathbb{N}\} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\} \]
the set of integer numbers

\[ \mathbb{Q} = \{\frac{r}{s} | r \in \mathbb{Z}, s \in \mathbb{Z} - \{0\}\} \]
the set of rational numbers

\[ \mathbb{R} = \text{the set of real numbers constructed by rational numbers through Dedekind’s sections.} \]

A function, \( f \), is a relation between two sets where to each member of first set is linked at most one member of second set.

A real succession \((a_n)_{n \in \mathbb{N}}\) is a function from \( \mathbb{N} \) to \( \mathbb{R} \) defined by \( n \mapsto a_n \). The most simply succession is \((n)_{n \in \mathbb{N}}\) this succession is divergent.

Let \((a_n)_{n \in \mathbb{N}}\) a real succession is say a serie the sum \( \sum_{n \in \mathbb{N}} a_n = a_0 + a_1 + a_2 + \ldots \)

The natural set can be sayed like the successions of partial sum of the sum \( 0+1+1+1+1+\ldots \) in fact \( s_0 = 0, s_1 = 0+1 = 1, s_2 = 0+1+1 = 2, \ldots \) so a serie \( \sum_{n \in \mathbb{N}} a_n \geq 0+1+1+1+\ldots \) is a divergent serie because not exist a real number grater than a real succession \((n)_{n \in \mathbb{N}}\) so not exist a real grater than the succession of the partial sum of the serie. Conversely, if \( \sum_{n \in \mathbb{N}} a_n \leq 0+1+1+1+1+\ldots \) the real exists for the construction of real numbers, in general this real depends by infinity.

For example the armonic serie \( 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots \) have for domain the set of natural numbers for codomain the real set that contained the natural set obtained from the sum \( 0+1+1+1+\ldots \) so the harmonic serie is minor than \( 0+1+1+1+\ldots \) so a real number in codomain grater than succession of the partial sum of harmonic serie exists. In general the serie \( \sum_{n \in \mathbb{N}} \frac{1}{n} \) converges for \( s > 0 \).

The same discuss is true for the functions in fact if get a function from reals to reals, for example if we get the esponential function we note that it diverges more fast of the function that a real \( x \) maps the real \( x \) so exist a real a such that the limit of \( \exp(x) \) for \( x \) tends to a is infinity and for the inverse of \( \exp(x) \) is true the opposite that is it changes the domain with codomain.