[... *Title and contents of Section 1 omitted for succinctness*]

Abstract

With the vector form of ζ, RH's validity is direct.

1 Introduction

...

2 Proof

Let:

$V(c) = (v_1(c), v_2(c), v_3(c), ...)$ where $v_n(c) = n^{-c}$ for $c \in \mathbb{C}$
$\sigma = a + it$
$\circ$ be Hadamard product and $\bullet$ be dot product
$COS = (\cos(t\ln(1)), \cos(t\ln(2)), \cos(t\ln(3)), ...)$
δ be any real value

and take note of the identity:

$$n^{\sigma} = n^{a}\cos(t\ln(n)) + i\sin(t\ln(n))$$

If $\sigma + \delta$[†] and σ are both ζ roots, then $\zeta(\sigma) = \Sigma_{n=1}^{\infty} n^{a}(\cos(t\ln(n)) + i\sin(t\ln(n))) = (V(a) \circ COS) \bullet V(0) + i((V(a) \circ ...) \bullet V(0)) = 0$. But for a complex number to be zero, both the real and imaginary components will have to be simultaneously zero. Therefore:

$$\Sigma_{n=1}^{\infty} n^{a}(\cos(t\ln(n)) = 0 \qquad (\text{and } \Sigma_{n=1}^{\infty} n^{a}\sin(t\ln(n)) = 0)$$

$$\Sigma_{n=1}^{\infty} n^{a}n^{\delta}(\cos(t\ln(n)) = 0 \qquad (\text{and } \Sigma_{n=1}^{\infty} n^{a}n^{\delta}\sin(t\ln(n))) = 0)$$

It can be observed that the following vectors

$$V(a) \circ COS \tag{1}$$
$$V(a) \circ V(\delta) \circ COS \tag{2}$$
$$V(a) \circ COS \circ (V(\delta) - V(0))^{\dagger\dagger} \tag{3}$$

are linearly independent, so not coplanar[‡] (unless $\delta = 0$). (*Q. E. D.*)

[†] We ignore the symmetry and prove, equivalently, that there can not be more than one root among complex numbers with the same non-zero imaginary part it.

[††] Difference of (1) subtracted from (2).

[‡] If the assumption (of two distinct symmetric roots) holds, the dot products of (1), (2) and (3) with $V(0)$ will have to all result in 0, so (1), (2) and (3) will have to be all orthogonal to $V(0)$, implying they need be coplanar.