Abstract
With the vector form of $\zeta$, RH’s validity is direct.

1 Introduction

2 Proof

Let:

$$V(c) = (v_1(c), v_2(c), v_3(c), ...) \text{ where } v_n(c) = n^{-c} \text{ for } c \in \mathbb{C}$$

$\sigma = a + it$

$\circ$ be Hadamard product and $\bullet$ be dot product

$COS = (\cos(t \ln(1)), \cos(t \ln(2)), \cos(t \ln(3)), ...)$

$\delta$ be any real value

and take note of the identity:

$$n^\sigma = n^a \cos(t \ln(n)) + i \sin(t \ln(n))$$

If $\sigma + \delta^\sigma$ and $\sigma$ are both $\zeta$ roots, then $\zeta(\sigma) = \sum_{n=1}^\infty n^\sigma (\cos(t \ln(n)) + i \sin(t \ln(n))) = (V(a) \circ COS) \bullet V(0) + i((V(a) \circ ...) \bullet V(0)) = 0$. But for a complex number to be zero, both the real and imaginary components will have to be simultaneously zero. Therefore:

$$\sum_{n=1}^\infty n^\sigma (\cos(t \ln(n)) = 0 \quad \text{(and } \sum_{n=1}^\infty n^\sigma \sin(t \ln(n)) = 0)$$

$$\sum_{n=1}^\infty n^\sigma n^\delta (\cos(t \ln(n)) = 0 \quad \text{(and } \sum_{n=1}^\infty n^\sigma n^\delta \sin(t \ln(n))) = 0$$

It can be observed that the following vectors

$$V(a) \circ COS \quad \text{(1)}$$

$$V(a) \circ V(\delta) \circ COS \quad \text{(2)}$$

$$V(a) \circ COS \circ (V(\delta) - V(0))^{\dagger\dagger} \quad \text{(3)}$$

are linearly independent, so not coplanar unless $\delta = 0$. (Q. E. D.)

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$^\dagger$We ignore the symmetry and prove, equivalently, that there can not be more than one root among complex numbers with the same non-zero imaginary part $it$.

$^{\dagger\dagger}$Difference of (1) subtracted from (2).

$^1$If the assumption (of two distinct symmetric roots) holds, the dot products of (1), (2) and (3) with $V(0)$ will have to all result in 0, so (1), (2) and (3) will have to be all orthogonal to $V(0)$, implying they need be coplanar.