On the Ramanujan's equations applied to various sectors of Particle Physics and Cosmology: new possible mathematical connections with the values of Pion mesons and other baryons and mesons.

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#### **Abstract**

In this research thesis, we have analyzed further Ramanujan formulas and described new possible mathematical connections with some sectors of Particle Physics (values of Pion mesons and other baryons and mesons) and Cosmology

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https://www.britannica.com/biography/Srinivasa-Ramanujan



https://biografieonline.it/foto-enrico-fermi

#### **Summary**

In this research thesis, we have analyzed further Ramanujan formulas and described new mathematical connections with some sectors of Particle Physics and Cosmology. We have described, as in previous papers, the possible and new connections between different formulas of Ramanujan's mathematics and some formulas concerning particle physics and cosmology. In the course of the discussion we describe and highlight the connections between some developments of Ramanujan equations and particles type solutions such as the mass of the Higgs boson and the masses of proton (or neutron), and other baryons and mesons. Principally solutions of Ramanujan equations, connected with the masses of the  $\pi$  mesons (139.57 and 134.9766 MeV) have been described and highlighted.

Further, we have described the connections between the mathematics of Ramanujan and different equations concerning some areas of theoretical physics and cosmology (see part "Replica Wormholes and the Entropy of Hawking Radiation"). Is our opinion, that the possible connections between the mathematical developments of some Rogers-Ramanujan continued fractions, various mathematical Ramanujan's expressions, the value of the dilaton and that of "the dilaton mass calculated as a type of Higgs boson that is equal about to 125 GeV", the Higgs boson mass itself and the masses of the  $\pi$  mesons (139.57 and 134.9766 MeV) are fundamental.

All the results of the most important connections are highlighted in blue throughout the drafting of the paper

#### From:

#### MANUSCRIPT BOOK 2 OF SRINIVASA RAMANUJAN

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$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{(1+\frac{\pi}{4})(1+\frac{\pi}{6})(1+\frac{\pi}{6})} \frac{dx}{dx} dx$$

$$= \frac{1}{1+x_{L}} + \frac{5}{3+x_{L}} - \frac{7}{6+x_{L}} + \frac{9}{10+x_{L}} - &C$$

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{(e^{i\pi x}\sqrt{i}-1)} - \frac{5}{\sqrt{i}} \frac{1}{(e^{i\pi x}\sqrt{i}-1)} - &C$$

$$+ \frac{1}{x} \left\{ \operatorname{Sech}(\frac{\pi}{x} \sqrt{1-\frac{x_{L}}{x}}) + \operatorname{Sech}(\frac{\pi}{x} \sqrt{1-\frac{x_{L}}{x}}) + \operatorname{Sech}(\frac{\pi}{x} \sqrt{9-\frac{x_{L}}{x}}) + &C \right\}$$

$$= \frac{1}{2\pi x} + \frac{\pi x}{6} - C, \quad \text{for all values of } x$$
where  $C = \frac{1}{2} + \frac{1}{3+\sqrt{8}} = \frac{1}{5+\sqrt{2}\lambda} + \frac{1}{7+\sqrt{48}} = \frac{8\lambda^{2}}{1+\sqrt{7+\sqrt{48}}} = \frac{1}{6(3+\sqrt{8})^{2}} - \frac{1}{16(5+\sqrt{24})^{2}} + \frac{1}{14(7+\sqrt{48})^{2}} - \frac{8\lambda^{2}}{16(5+\sqrt{24})^{2}} + \frac{1}{14(7+\sqrt{48})^{2}} = \frac{8\lambda^{2}}{16(5+\sqrt{24})^{2}}$ 

For x = 0.24, we obtain:

**Input:** 

$$\frac{1}{0.24^2} - \frac{3}{1 + 0.24^2} + \frac{5}{3 + 0.24^2} - \frac{7}{6 + 0.24^2} + \frac{9}{10 + 0.24^2}$$

#### **Result:**

15.89904193290744865691890961750664151215726023917181323420... 15.8990419329...

For x = 1/12 = 0.083, we obtain:

1/0.083^2-3/(1+0.083^2)+5/(3+0.083^2)-7/(6+0.083^2)+9/(10+0.083^2)

4

Input: 
$$\frac{1}{0.083^2} - \frac{3}{1 + 0.083^2} + \frac{5}{3 + 0.083^2} - \frac{7}{6 + 0.083^2} + \frac{9}{10 + 0.083^2}$$

#### **Result:**

143.5763746029481662180096635360300782826003184852433469694... 143.576374602948...

Where 4 is a Lucas number and the dimensions of a D4-brane

Input:

$$\frac{1}{0.083^2} - \frac{3}{1 + 0.083^2} + \frac{5}{3 + 0.083^2} - \frac{7}{6 + 0.083^2} + \frac{9}{10 + 0.083^2} - 4$$

#### **Result:**

139.5763746029481662180096635360300782826003184852433469694...

139.576374602948... result practically equal to the rest mass of Pion meson 139.57



$$1/2+1/(3+sqrt8)-1/(5+sqrt24)+1/(7+sqrt48) = C$$

Input: 
$$\frac{1}{2} + \frac{1}{3 + \sqrt{8}} - \frac{1}{5 + \sqrt{24}} + \frac{1}{7 + \sqrt{48}}$$

**Result:** 

$$\frac{1}{2} + \frac{1}{3+2\sqrt{2}} + \frac{1}{7+4\sqrt{3}} - \frac{1}{5+2\sqrt{6}}$$

## **Decimal approximation:**

0.642349130544656924681405334968897159021330195317921598288...

0.64234913054...

#### **Alternate forms:**

$$\frac{1}{2} \left( 11 - 4\sqrt{2} - 8\sqrt{3} + 4\sqrt{6} \right)$$

$$\frac{11}{2} - 2\sqrt{2} - 4\sqrt{3} + 2\sqrt{6}$$

$$\frac{1}{2} \left( 11 - 8\sqrt{3} + 8\sqrt{2 - \sqrt{3}} \right)$$

### **Minimal polynomial:**

$$16x^4 - 352x^3 + 344x^2 + 5224x - 3407$$

For x = 0.083, we obtain:

$$1/(2Pi*0.083) + (Pi*0.083)/6 - 0.64234913054$$

Input interpretation: 
$$\frac{1}{2\pi \times 0.083} + \frac{\pi \times 0.083}{6} - 0.64234913054$$

#### **Result:**

1.31864...

1.31864...

# Alternative representations:

$$\begin{split} &\frac{1}{2\,\pi\,0.083} + \frac{\pi\,0.083}{6} - 0.642349130540000 = \\ &-0.642349130540000 + \frac{14.94\,^{\circ}}{6} + \frac{1}{29.88\,^{\circ}} \\ &\frac{1}{2\,\pi\,0.083} + \frac{\pi\,0.083}{6} - 0.642349130540000 = \\ &-0.642349130540000 - \frac{1}{6}\times0.083\,i\log(-1) + -\frac{1}{0.166\,i\log(-1)} \\ &\frac{1}{2\,\pi\,0.083} + \frac{\pi\,0.083}{6} - 0.642349130540000 = \\ &-0.642349130540000 + \frac{1}{6}\times0.083\cos^{-1}(-1) + \frac{1}{0.166\cos^{-1}(-1)} \end{split}$$

$$\frac{1}{2\pi 0.083} + \frac{\pi 0.083}{6} - 0.642349130540000 =$$

$$0.0553333 \left(-8.34863 + \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}\right) \left(-3.26009 + \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}\right)$$

$$\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}$$

$$\frac{1}{2\pi 0.083} + \frac{\pi 0.083}{6} - 0.642349130540000 =$$

$$0.0276667 \left(-17.6973 + \sum_{k=1}^{\infty} \frac{2^k}{\binom{2k}{k}}\right) \left(-7.52019 + \sum_{k=1}^{\infty} \frac{2^k}{\binom{2k}{k}}\right)$$

$$-1 + \sum_{k=1}^{\infty} \frac{2^k}{\binom{2k}{k}}$$

$$\frac{1}{2\pi 0.083} + \frac{\pi 0.083}{6} - 0.642349130540000 =$$

$$0.05533333 \left(-8.34863 + \sum_{k=1}^{\infty} \tan^{-1} \left(\frac{1}{F_{1+2k}}\right)\right) \left(-3.26009 + \sum_{k=1}^{\infty} \tan^{-1} \left(\frac{1}{F_{1+2k}}\right)\right)$$

$$\sum_{k=1}^{\infty} \tan^{-1} \left(\frac{1}{F_{1+2k}}\right)$$

## **Integral representations:**

$$\begin{split} \frac{1}{2\,\pi\,0.083} + \frac{\pi\,0.083}{6} - 0.642349130540000 &= \\ 0.0276667 \left(-16.6973 + \int_0^\infty \frac{1}{1+t^2} \,dt\right) \left(-6.52019 + \int_0^\infty \frac{1}{1+t^2} \,dt\right) \\ -\frac{1}{2\,\pi\,0.083} + \frac{\pi\,0.083}{6} - 0.642349130540000 &= \\ 0.0276667 \left(-16.6973 + \int_0^\infty \frac{\sin(t)}{t} \,dt\right) \left(-6.52019 + \int_0^\infty \frac{\sin(t)}{t} \,dt\right) \\ -\frac{1}{2\,\pi\,0.083} + \frac{\pi\,0.083}{6} - 0.642349130540000 &= \\ 0.0553333 \left(-8.34863 + \int_0^1 \sqrt{1-t^2} \,dt\right) \left(-3.26009 + \int_0^1 \sqrt{1-t^2} \,dt\right) \\ -\frac{1}{2\,\pi\,0.083} + \frac{\pi\,0.083}{6} - 0.642349130540000 &= \\ 0.05533333 \left(-8.34863 + \int_0^1 \sqrt{1-t^2} \,dt\right) \left(-3.26009 + \int_0^1 \sqrt{1-t^2} \,dt\right) \end{split}$$

$$((((((1/(2Pi*0.083) + (Pi*0.083)/6 - 0.64234913054)))^2 + sqrt2)))*521/10^3)$$

Where 521 is a Lucas number. Note that 521 = 496 + 25, where 496 is the dimension of Lie's Group  $E_8$  X  $E_8$  and 25 corresponding to the dimensions of a D-25 brane

Input interpretation: 
$$\left( \left( \frac{1}{2\pi \times 0.083} + \frac{\pi \times 0.083}{6} - 0.64234913054 \right)^2 + \sqrt{2} \right) \times \frac{521}{10^3}$$

#### **Result:**

1.642724660893565725916220256844860859141606394336521851856...

$$1.64272466...\approx \zeta(2) = \frac{\pi^2}{6} = 1.644934...$$

$$\frac{\left(\left(\frac{1}{2\pi0.083} + \frac{\pi0.083}{6} - 0.642349130540000\right)^2 + \sqrt{2}\right)521}{0.301804 + \frac{18.907}{\pi^2} - \frac{4.0321}{\pi} - 0.00925903\pi + 0.0000996991\pi^2 + \frac{521\sqrt{z_0}}{2\pi0.083} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!} \quad \text{for not } \left(\left(z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0\right)\right)$$

$$\frac{\left(\left(\frac{1}{2\pi0.083} + \frac{\pi0.083}{6} - 0.642349130540000\right)^2 + \sqrt{2}\right)521}{0.301804 + \frac{18.907}{\pi^2} - \frac{4.0321}{\pi} - 0.00925903\pi + 0.0000996991\pi^2 + \frac{521\exp\left(i\pi\left[\frac{\arg(2-x)}{2\pi}\right]\right)\sqrt{x}\sum_{k=0}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!} \quad \text{for } (x \in \mathbb{R} \text{ and } x < 0)$$

$$\frac{\left(\left(\frac{1}{2\pi0.083} + \frac{\pi0.083}{6} - 0.642349130540000\right)^2 + \sqrt{2}\right)521}{1000} = 0.301804 + \frac{18.907}{\pi^2} - \frac{4.0321}{\pi} - 0.00925903\pi + 0.0000996991\pi^2 + \frac{10^3}{2\pi0} - \frac{10^3}{\pi^2} - \frac{10^3}{\pi^$$

Input interpretation: 
$$\frac{1}{10^{27}} \left( \left( \left( \frac{1}{2\pi \times 0.083} + \frac{\pi \times 0.083}{6} - 0.64234913054 \right)^2 + \sqrt{2} \right) \times \frac{521}{10^3} + \frac{29}{10^3} \right)$$

Where 521 and 29 are Lucas numbers. Note that 521 = 496 + 25, where 496 is the dimension of Lie's Group  $E_8$  X  $E_8$  and 25 corresponding to the dimensions of a D-25 brane

#### **Result:**

$$1.67172... \times 10^{-27}$$
  
 $1.67172... \times 10^{-27}$  kg

result practically equal to the value of the formula:

$$m_{p'} = 2 \times \frac{\eta}{R} m_P = 1.6714213 \times 10^{-24} \text{ gm}$$

that is the holographic proton mass (N. Haramein)

$$10^2 (((1/(2Pi*0.083) + (Pi*0.083)/6 - 0.64234913054))) + Pi$$

Where 10 is the number of dimensions in superstring theory

Input interpretation: 
$$10^{2} \left( \frac{1}{2 \pi \times 0.083} + \frac{\pi \times 0.083}{6} - 0.64234913054 \right) + \pi$$

#### **Result:**

135.005...

135.005.... result very near to the rest mass of Pion meson 139.57

### **Alternative representations:**

$$10^{2} \left( \frac{1}{2\pi 0.083} + \frac{\pi 0.083}{6} - 0.642349130540000 \right) + \pi =$$

$$180^{\circ} + 10^{2} \left( -0.642349130540000 + \frac{14.94^{\circ}}{6} + \frac{1}{29.88^{\circ}} \right)$$

$$10^{2} \left( \frac{1}{2\pi 0.083} + \frac{\pi 0.083}{6} - 0.642349130540000 \right) + \pi =$$

$$-i \log(-1) + 10^{2} \left( -0.642349130540000 - \frac{1}{6} \times 0.083 i \log(-1) + -\frac{1}{0.166 i \log(-1)} \right)$$

$$10^{2} \left( \frac{1}{2\pi 0.083} + \frac{\pi 0.083}{6} - 0.642349130540000 \right) + \pi =$$

$$\cos^{-1}(-1) + 10^{2} \left( -0.642349130540000 + \frac{1}{6} \times 0.083 \cos^{-1}(-1) + \frac{1}{0.166 \cos^{-1}(-1)} \right)$$

## **Integral representations:**

$$10^{2} \left( \frac{1}{2 \pi 0.083} + \frac{\pi 0.083}{6} - 0.642349130540000 \right) + \pi = \frac{4.76667 \left( 63.1898 - 13.4759 \int_{0}^{\infty} \frac{1}{1+t^{2}} dt + \left( \int_{0}^{\infty} \frac{1}{1+t^{2}} dt \right)^{2} \right)}{\int_{0}^{\infty} \frac{1}{1+t^{2}} dt}$$

$$10^{2} \left( \frac{1}{2 \pi 0.083} + \frac{\pi 0.083}{6} - 0.642349130540000 \right) + \pi = \frac{4.76667 \left( 63.1898 - 13.4759 \int_{0}^{\infty} \frac{\sin(t)}{t} dt + \left( \int_{0}^{\infty} \frac{\sin(t)}{t} dt \right)^{2} \right)}{\int_{0}^{\infty} \frac{\sin(t)}{t} dt}$$

$$10^{2} \left( \frac{1}{2\pi0.083} + \frac{\pi0.083}{6} - 0.642349130540000 \right) + \pi =$$

$$9.53333 \left( 15.7975 - 6.73793 \int_{0}^{1} \sqrt{1 - t^{2}} dt + \left( \int_{0}^{1} \sqrt{1 - t^{2}} dt \right)^{2} \right)$$

$$\int_{0}^{1} \sqrt{1 - t^{2}} dt$$

For x = 2, from the following expression, considering the symbol &, we obtain:

Input interpretation: 
$$\frac{1}{2^2} - \frac{3}{1+2^2} + \frac{5}{3+2^2} - \frac{7}{6+2^2} + \frac{9}{10+2^2} + \cdots$$

#### **Infinite sum:**

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1} (2 n - 1)}{\frac{1}{2} (n-1) n + 4} = 2 \pi \operatorname{sech} \left( \frac{\sqrt{31} \pi}{2} \right)$$

sech(x) is the hyperbolic secant function

## **Decimal approximation:**

0.001999617057621813260053366854580340306777114236479440058...

0.0019996170576...

### **Convergence tests:**

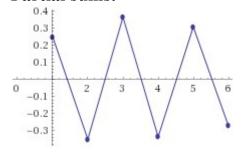
By the alternating series test, the series converges.

#### Partial sum formula:

$$\begin{split} \sum_{n=1}^{m} \frac{\left(-1\right)^{1+n} \left(-1+2 \, n\right)}{4+\frac{1}{2} \left(-1+n\right) n} &= 2 \left(\left(-1\right)^{m+1} \Phi \left(-1, \, 1, \, m+\frac{1}{2} \left(-1-i \, \sqrt{31} \, \right)+1\right)+\right. \\ &\left. \left(-1\right)^{m+1} \Phi \left(-1, \, 1, \, m+\frac{1}{2} \left(-1+i \, \sqrt{31} \, \right)+1\right)+\right. \\ &\left. \Phi \left(-1, \, 1, \, 1+\frac{1}{2} \left(-1+i \, \sqrt{31} \, \right)\right)+\Phi \left(-1, \, 1, \, 1+\frac{1}{2} \left(-1-i \, \sqrt{31} \, \right)\right) \right) \end{split}$$

 $\Phi(x, s, a)$  is the Lerch transcendent

#### Partial sums:



 $1/((((1+2^2/1)(1+2^2/3)(1+2^2/6)(1+2^2/10)*...)))$ 

**Input interpretation:** 

$$\frac{1}{(1+2^2)(1+\frac{2^2}{3})(1+\frac{2^2}{6})((1+\frac{2^2}{10})\times\cdots)}$$

**Result:** 

$$\frac{9}{175 \times \prod_{n=1}^{\infty} \frac{7}{5}}$$

 $1/((((1+2^2/1)(1+2^2/3)(1+2^2/6)(1+2^2/10)*...))) = 1/2^2-3/(1+2^2)+5/(3+2^2)-1/((((1+2^2/3)(1+2^2/3)(1+2^2/6)(1+2^2/10)*...))) = 1/2^2-3/(1+2^2)+5/(3+2^2)-1/(((1+2^2/3)(1+2^2/3)(1+2^2/6)(1+2^2/10)*...))) = 1/2^2-3/(1+2^2)+5/(3+2^2)-1/(((1+2^2/3)(1+2^2/$ 7/(6+2^2)+9/(10+2^2)-...

Input interpretation: 
$$\frac{1}{\left(1+2^2\right)\left(1+\frac{2^2}{3}\right)\left(1+\frac{2^2}{6}\right)\left(\left(1+\frac{2^2}{10}\right)\times\cdots\right)} = \frac{1}{2^2} - \frac{3}{1+2^2} + \frac{5}{3+2^2} - \frac{7}{6+2^2} + \frac{9}{10+2^2} - \cdots$$

**Result:** 

$$\frac{9}{175 \times \prod_{n=1}^{\infty} \frac{7}{5}} = 2 \pi \operatorname{sech} \left( \frac{\sqrt{31} \pi}{2} \right)$$

**Input:** 

$$2\pi\operatorname{sech}\left(\frac{1}{2}\left(\sqrt{31}\ \pi\right)\right)$$

 $\operatorname{sech}(x)$  is the hyperbolic secant function

**Exact result:** 

$$2 \pi \operatorname{sech} \left( \frac{\sqrt{31} \pi}{2} \right)$$

## **Decimal approximation:**

0.001999617057621813260053366854580340306777114236479440058...

0.0019996170576...

#### **Alternate forms:**

$$\frac{2\pi}{\cosh\left(\frac{\sqrt{31}\pi}{2}\right)}$$

$$\frac{4\pi\cosh\left(\frac{\sqrt{31}\pi}{2}\right)}{1+\cosh(\sqrt{31}\pi)}$$

$$\frac{4 \pi}{e^{-(\sqrt{31} \pi)/2} + e^{(\sqrt{31} \pi)/2}}$$

 $\cosh(x)$  is the hyperbolic cosine function

## **Alternative representations:**

$$2\pi\operatorname{sech}\left(\frac{\sqrt{31} \pi}{2}\right) = \frac{2\pi}{\cosh\left(\frac{\pi\sqrt{31}}{2}\right)}$$

$$2\pi\operatorname{sech}\left(\frac{\sqrt{31}\ \pi}{2}\right) = 2\pi\operatorname{csc}\left(\frac{\pi}{2} + \frac{1}{2}\ i\pi\sqrt{31}\right)$$

$$2\pi\operatorname{sech}\left(\frac{\sqrt{31}\pi}{2}\right) = \frac{2\pi}{\cos\left(\frac{1}{2}i\pi\sqrt{31}\right)}$$

# **Series representations:**

$$2\pi \operatorname{sech}\left(\frac{\sqrt{31} \pi}{2}\right) = 2\sum_{k=0}^{\infty} \frac{(-1)^k (1+2k)}{8+k+k^2}$$

$$2\pi \operatorname{sech}\left(\frac{\sqrt{31} \pi}{2}\right) = -4\pi \sum_{k=1}^{\infty} (-1)^k q^{-1+2k} \text{ for } q = e^{\left(\sqrt{31} \pi\right)/2}$$

$$2 \pi \operatorname{sech}\left(\frac{\sqrt{31} \pi}{2}\right) = 4 e^{-\left(\sqrt{31} \pi\right)/2} \pi \sum_{k=0}^{\infty} (-1)^k e^{-\sqrt{31} k \pi}$$

# **Integral representation:**

$$2\pi \operatorname{sech}\left(\frac{\sqrt{31} \pi}{2}\right) = 4 \int_0^\infty \frac{t^{i\sqrt{31}}}{1+t^2} dt$$

 $(1/4)*1/(((2 \pi \text{ sech}((\text{sqrt}(31) \pi)/2))))+11+3+1/\text{golden ratio})$ 

Where 11 and 3 are Lucas numbers

Input: 
$$\frac{1}{4} \times \frac{1}{2 \pi \operatorname{sech} \left(\frac{1}{2} \left(\sqrt{31} \pi\right)\right)} + 11 + 3 + \frac{1}{\phi}$$

 $\operatorname{sech}(x)$  is the hyperbolic secant function

ø is the golden ratio

#### **Exact result:**

$$\frac{1}{\phi} + 14 + \frac{\cosh\left(\frac{\sqrt{31} \pi}{2}\right)}{8 \pi}$$

cosh(x) is the hyperbolic cosine function

## **Decimal approximation:**

139.6419724709162115699630652093636492431933614860570506324...

139.64197247.... result practically equal to the rest mass of Pion meson 139.57

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#### **Alternate forms:**

$$\frac{1}{2} \left(27 + \sqrt{5}\right) + \frac{\cosh\left(\frac{\sqrt{31} \pi}{2}\right)}{8 \pi}$$

$$14 + \frac{2}{1+\sqrt{5}} + \frac{\cosh\left(\frac{\sqrt{31} \pi}{2}\right)}{8 \pi}$$

$$\frac{1}{\phi} + 14 + \frac{e^{-\left(\sqrt{31} \pi\right)/2}}{16 \pi} + \frac{e^{\left(\sqrt{31} \pi\right)/2}}{16 \pi}$$

# **Alternative representations:**

$$\frac{1}{\left(2\,\pi\,\mathrm{sech}\!\left(\frac{\sqrt{31}\,\pi}{2}\right)\!\right)4} + 11 + 3 + \frac{1}{\phi} = 14 + \frac{1}{\phi} + \frac{1}{\frac{4\,(2\,\pi)}{\cosh\!\left(\frac{\pi\,\sqrt{31}}{2}\right)}}$$

$$\frac{1}{\left(2\,\pi\,\text{sech}\!\left(\frac{\sqrt{31}\,\pi}{2}\right)\!\right)4} + 11 + 3 + \frac{1}{\phi} = 14 + \frac{1}{\phi} + \frac{1}{\frac{4\,(2\,\pi)}{\cos\!\left(\frac{1}{2}\,i\,\pi\,\sqrt{31}\right)}}$$

$$\frac{1}{\left(2\,\pi\,\mathrm{sech}\!\left(\frac{\sqrt{31}\,\pi}{2}\right)\!\right)4} + 11 + 3 + \frac{1}{\phi} = 14 + \frac{1}{\phi} + \frac{1}{4\left(2\,\pi\,\mathrm{csc}\!\left(\frac{\pi}{2} + \frac{1}{2}\,i\,\pi\,\sqrt{31}\,\right)\!\right)}$$

## **Series representations:**

$$\frac{1}{\left(2\pi\operatorname{sech}\left(\frac{\sqrt{31}\,\pi}{2}\right)\right)4} + 11 + 3 + \frac{1}{\phi} = 14 + \frac{1}{\phi} + \frac{\sum_{k=0}^{\infty} \frac{\left(\frac{31}{4}\right)^k \pi^{2k}}{(2k)!}}{8\,\pi}$$

$$\frac{1}{\left(2\pi\operatorname{sech}\left(\frac{\sqrt{31}\,\pi}{2}\right)\right)4} + 11 + 3 + \frac{1}{\phi} = 14 + \frac{1}{\phi} + \frac{i\sum_{k=0}^{\infty} \frac{\left(\frac{1}{2}\left(-i+\sqrt{31}\right)\pi\right)^{1+2\,k}}{(1+2\,k)!}}{8\,\pi}$$

$$\frac{1}{\left(2\,\pi\,\mathrm{sech}\left(\frac{\sqrt{31}\,\pi}{2}\right)\right)4} + 11 + 3 + \frac{1}{\phi} = 14 + \frac{1}{\phi} + \frac{\sum_{k=0}^{\infty}I_{2\,k}\left(\frac{1}{2}\right)T_{2\,k}\left(\sqrt{31}\,\pi\right)(2 - \delta_k)}{8\,\pi}$$

## **Integral representations:**

$$\frac{1}{\left(2\pi\operatorname{sech}\left(\frac{\sqrt{31}\,\pi}{2}\right)\right)4} + 11 + 3 + \frac{1}{\phi} = 14 + \frac{1}{\phi} + \frac{1}{8\,\pi}\int_{\frac{i\pi}{2}}^{\frac{\sqrt{31}\,\pi}{2}} \sinh(t)\,dt$$

$$\frac{1}{\left(2\pi\operatorname{sech}\left(\frac{\sqrt{31}\,\pi}{2}\right)\right)4} + 11 + 3 + \frac{1}{\phi} = 14 + \frac{2}{1+\sqrt{5}} + \frac{1}{8\,\pi} + \frac{\sqrt{31}}{16} \int_{0}^{1} \sinh\left(\frac{1}{2}\,\sqrt{31}\,\pi\,t\right)dt$$

$$\frac{1}{\left(2\pi\operatorname{sech}\left(\frac{\sqrt{31}\pi}{2}\right)\right)4} + 11 + 3 + \frac{1}{\phi} = 14 + \frac{2}{1+\sqrt{5}} - \frac{i}{16\pi^{3/2}} \int_{-i\,\omega+\gamma}^{i\,\omega+\gamma} \frac{e^{\left(31\pi^2\right)/(16\,s)+s}}{\sqrt{s}} \,ds \text{ for } \gamma > 0$$

 $(((1/2^2-3/(1+2^2)+5/(3+2^2)-7/(6+2^2)+9/(10+2^2)+...)))/(((2 \pi \operatorname{sech}((\operatorname{sqrt}(31)$  $\pi)/2))))$ 

Input interpretation:  

$$\frac{\frac{1}{2^2} - \frac{3}{1+2^2} + \frac{5}{3+2^2} - \frac{7}{6+2^2} + \frac{9}{10+2^2} + \cdots}{2 \pi \operatorname{sech} \left(\frac{1}{2} \left(\sqrt{31} \pi\right)\right)}$$

sech(x) is the hyperbolic secant function

#### **Result:**

1

1 result that can be interpreted as the photon spin

We have that:

For x = 1/12 = 0.083, we obtain:

$$1-Pi/2+1/(6(3+sqrt8)^2)-1/(10(5+sqrt24)^2)+1/(14(7+sqrt48)^2)$$

**Input:** 

$$1 - \frac{\pi}{2} + \frac{1}{6(3+\sqrt{8})^2} - \frac{1}{10(5+\sqrt{24})^2} + \frac{1}{14(7+\sqrt{48})^2}$$

$$1 + \frac{1}{6(3+2\sqrt{2})^2} + \frac{1}{14(7+4\sqrt{3})^2} - \frac{1}{10(5+2\sqrt{6})^2} - \frac{\pi}{2}$$

## **Decimal approximation:**

-0.56654243434547778978801159476609237831534974246486940743...

-0.566542434.....

Property: 
$$1 + \frac{1}{6(3+2\sqrt{2})^2} + \frac{1}{14(7+4\sqrt{3})^2} - \frac{1}{10(5+2\sqrt{6})^2} - \frac{\pi}{2}$$
 is a transcendental number

#### **Alternate forms:**

$$\frac{1}{210} \left( 1231 - 420\sqrt{2} - 840\sqrt{3} + 420\sqrt{6} - 105\pi \right)$$

$$\frac{1231}{210} - 2\sqrt{2} - 4\sqrt{3} + 2\sqrt{6} - \frac{\pi}{2}$$

$$\frac{1}{210} \left( 1231 - 840\sqrt{3} + 840\sqrt{2} - \sqrt{3} \right) - \frac{\pi}{2}$$

$$1 - \frac{\pi}{2} + \frac{1}{6(3 + \sqrt{8})^2} - \frac{1}{10(5 + \sqrt{24})^2} + \frac{1}{14(7 + \sqrt{48})^2} = 1$$

$$1 - \frac{\pi}{2} + \frac{1}{6(3 + \sqrt{7} \sum_{k=0}^{\infty} 7^{-k} \left(\frac{1}{2}\right)^2} - \frac{1}{10(5 + \sqrt{23} \sum_{k=0}^{\infty} 23^{-k} \left(\frac{1}{2}\right)^2} + \frac{1}{14(7 + \sqrt{47} \sum_{k=0}^{\infty} 47^{-k} \left(\frac{1}{2}\right)^2}$$

$$1 - \frac{\pi}{2} + \frac{1}{6(3 + \sqrt{8})^2} - \frac{1}{10(5 + \sqrt{24})^2} + \frac{1}{14(7 + \sqrt{48})^2} = 1$$

$$1 - \frac{\pi}{2} + \frac{1}{6(3 + \sqrt{7} \sum_{k=0}^{\infty} \frac{(-\frac{1}{2})^k (-\frac{1}{2})_k}{k!})^2} - \frac{1}{10(5 + \sqrt{23} \sum_{k=0}^{\infty} \frac{(-\frac{1}{23})^k (-\frac{1}{2})_k}{k!})^2} + \frac{1}{14(7 + \sqrt{47} \sum_{k=0}^{\infty} \frac{(-\frac{1}{47})^k (-\frac{1}{2})_k}{k!})^2}$$

$$\begin{split} 1 - \frac{\pi}{2} + \frac{1}{6\left(3 + \sqrt{8}\right)^2} - \frac{1}{10\left(5 + \sqrt{24}\right)^2} + \frac{1}{14\left(7 + \sqrt{48}\right)^2} &= \\ 1 - \frac{\pi}{2} + \frac{1}{6\left(3 + \sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(8 - z_0)^kz_0^{-k}}{k!}\right)^2} - \\ &\frac{1}{10\left(5 + \sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(24 - z_0)^kz_0^{-k}}{k!}\right)^2} + \\ &\frac{1}{14\left(7 + \sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(48 - z_0)^kz_0^{-k}}{k!}\right)^2} \text{ for not } \left(\left(z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0\right)\right) \end{split}$$

$$-1/(((1-Pi/2+1/(6(3+sqrt8)^2)-1/(10(5+sqrt24)^2)+1/(14(7+sqrt48)^2))))*76+1/golden ratio$$

Where 76 is a Lucas number

$$-\frac{76}{1-\frac{\pi}{2}+\frac{1}{6(3+\sqrt{8})^2}-\frac{1}{10(5+\sqrt{24})^2}+\frac{1}{14(7+\sqrt{48})^2}+\frac{1}{\phi}}$$

φ is the golden ratio

Result: 
$$\frac{1}{\phi} - \frac{76}{1 + \frac{1}{6(3+2\sqrt{2})^2} + \frac{1}{14(7+4\sqrt{3})^2} - \frac{1}{10(5+2\sqrt{6})^2} - \frac{\pi}{2}}$$

## **Decimal approximation:**

134.7650905773745897871094135928998181319929018953260271255...

134.765090577.... result practically equal to the rest mass of Pion meson 134.976

## **Property:**

$$\frac{1}{\phi} - \frac{76}{1 + \frac{1}{6(3+2\sqrt{2})^2} + \frac{1}{14(7+4\sqrt{3})^2} - \frac{1}{10(5+2\sqrt{6})^2} - \frac{\pi}{2}}$$
 is a transcendental number

#### **Alternate forms:**

$$\left(33\,151 - 420\,\sqrt{2} - 840\,\sqrt{3} - 1231\,\sqrt{5} + 420\,\sqrt{6} + 420\,\sqrt{10} + 840\,\sqrt{15} - 420\,\sqrt{30} - 105\,\pi + 105\,\sqrt{5}\,\pi\right) \right)$$

$$\left(2\left(-1231 + 420\,\sqrt{2} + 840\,\sqrt{3} - 420\,\sqrt{6} + 105\,\pi\right)\right)$$

$$\frac{1}{\phi} - \frac{76}{\frac{1231}{210} - 2\,\sqrt{2} - 4\,\sqrt{3} + 2\,\sqrt{6} - \frac{\pi}{2}}$$

$$\frac{1}{\phi} + \frac{76}{-\frac{1231}{210} + 2\,\sqrt{2} + 4\,\sqrt{3} - 2\,\sqrt{6} + \frac{\pi}{2}}$$

$$\frac{76 (-1)}{1 - \frac{\pi}{2} + \frac{1}{6(3 + \sqrt{8})^2} - \frac{1}{10(5 + \sqrt{24})^2} + \frac{1}{14(7 + \sqrt{48})^2}} + \frac{1}{\phi} = \frac{1}{\phi} - \frac{1}{1 - \frac{\pi}{2} + \frac{1}{6(3 + \sqrt{7})} + \frac{1}{2(5 + \sqrt{24})^2} - \frac{76}{10(5 + \sqrt{23})} + \frac{76}{2(5 + \sqrt{23})} + \frac{1}{14(7 + \sqrt{47})} + \frac{1}{14(7 + \sqrt{47})} + \frac{1}{14(7 + \sqrt{47})} + \frac{1}{14(7 + \sqrt{47})^2} + \frac{76(-1)}{1 - \frac{\pi}{2} + \frac{1}{6(3 + \sqrt{8})^2} - \frac{1}{10(5 + \sqrt{24})^2} + \frac{1}{14(7 + \sqrt{48})^2}} + \frac{1}{\phi} = \frac{1}{\phi} - \frac{1}{\phi} - \frac{\pi}{2} + \frac{1}{\phi} + \frac{1}{\phi$$

$$\frac{76 (-1)}{1 - \frac{\pi}{2} + \frac{1}{6(3 + \sqrt{8})^2} - \frac{1}{10(5 + \sqrt{24})^2} + \frac{1}{14(7 + \sqrt{48})^2} + \frac{1}{\phi} = \frac{1}{4(7 + \sqrt{48})^2} - \frac{1}{6(3 + \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (8 - z_0)^k z_0^{-k}}{k!}\right)^2 - \frac{1}{10(5 + \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (24 - z_0)^k z_0^{-k}}{k!}\right)^2} + \frac{1}{14(7 + \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (48 - z_0)^k z_0^{-k}}{k!}\right)^2}$$
for not  $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$ 

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4. 
$$T^{2} + x^{2} + \frac{x^{4}}{12} + \frac{x^{4}}{2^{2} + x^{2} + \frac{x^{4}}{2^{2}}} + \frac{x^{4}}{3^{2} + x^{2} + \frac{x^{4}}{3^{2}}} + \frac{x^{4}}{3^{2}} + \frac{x^{4}}{3^{2}$$

For x = 2, we obtain:

### **Input:**

$$\frac{\pi}{4\sqrt{3}} \times \frac{\sinh(2\pi\sqrt{3}) - \sqrt{3}\sin(2\pi)}{\cosh(2\pi\sqrt{3}) - \cos(2\pi)}$$

 $\sinh(x)$  is the hyperbolic sine function

 $\cosh(x)$  is the hyperbolic cosine function

#### **Exact result:**

$$\frac{\pi \sinh(2\sqrt{3} \pi)}{4\sqrt{3} (\cosh(2\sqrt{3} \pi) - 1)}$$

## **Decimal approximation:**

0.453466871624258724623634815745739322304887984526058956146...

0.4534668716242587....

#### **Alternate forms:**

$$\frac{\pi \coth(\sqrt{3} \pi)}{4\sqrt{3}}$$

$$\frac{\left(e^{2\sqrt{3}\pi} - e^{-2\sqrt{3}\pi}\right)\pi}{8\sqrt{3}\left(\frac{1}{2}\left(e^{-2\sqrt{3}\pi} + e^{2\sqrt{3}\pi}\right) - 1\right)}$$

coth(x) is the hyperbolic cotangent function

## **Alternative representations:**

$$\frac{\left(\sinh(2\pi\sqrt{3}\,) - \sqrt{3}\,\sin(2\pi)\right)\pi}{\left(\cosh(2\pi\sqrt{3}\,) - \cos(2\pi)\right)\left(4\sqrt{3}\,\right)} = \frac{\pi\left(\frac{1}{2}\left(-e^{-2\pi\sqrt{3}}\, + e^{2\pi\sqrt{3}}\,\right) - \frac{\left(-e^{-2\,i\,\pi} + e^{2\,i\,\pi}\right)\sqrt{3}}{2\,i}\right)}{\left(-\cosh(-2\,i\,\pi) + \frac{1}{2}\left(e^{-2\pi\sqrt{3}}\, + e^{2\pi\sqrt{3}}\,\right)\right)\left(4\sqrt{3}\,\right)}$$

$$\frac{\left(\sinh(2\pi\sqrt{3}) - \sqrt{3}\,\sin(2\pi)\right)\pi}{\left(\cosh(2\pi\sqrt{3}) - \cos(2\pi)\right)\left(4\sqrt{3}\right)} = \frac{\pi\left(\frac{1}{2}\left(-e^{-2\pi\sqrt{3}} + e^{2\pi\sqrt{3}}\right) + \cos\left(\frac{5\pi}{2}\right)\sqrt{3}\right)}{\left(\frac{1}{2}\left(-e^{-2i\pi} - e^{2i\pi}\right) + \frac{1}{2}\left(e^{-2\pi\sqrt{3}} + e^{2\pi\sqrt{3}}\right)\right)\left(4\sqrt{3}\right)}$$

$$\frac{\left(\sinh(2\pi\sqrt{3}) - \sqrt{3}\,\sin(2\pi)\right)\pi}{\left(\cosh(2\pi\sqrt{3}) - \cos(2\pi)\right)\left(4\sqrt{3}\right)} = \frac{\pi\left(\frac{1}{2}\left(-e^{-2\pi\sqrt{3}} + e^{2\pi\sqrt{3}}\right) - \frac{\left(-e^{-2i\pi} + e^{2i\pi}\right)\sqrt{3}}{2i}\right)}{\left(\cos(-2i\pi\sqrt{3}) + \frac{1}{2}\left(-e^{-2i\pi} - e^{2i\pi}\right)\right)\left(4\sqrt{3}\right)}$$

$$\frac{\left(\sinh(2\pi\sqrt{3}) - \sqrt{3}\sin(2\pi)\right)\pi}{\left(\cosh(2\pi\sqrt{3}) - \cos(2\pi)\right)\left(4\sqrt{3}\right)} = \frac{i\pi\sum_{k=0}^{\infty} \frac{\left(\frac{1}{2}\left(-i+4\sqrt{3}\right)\pi\right)^{2k}}{(2k)!}}{4\sqrt{3}\left(-1+\sum_{k=0}^{\infty} \frac{12^{k}\pi^{2k}}{(2k)!}\right)}$$

$$\frac{\left(\sinh(2\pi\sqrt{3}) - \sqrt{3}\sin(2\pi)\right)\pi}{\left(\cosh(2\pi\sqrt{3}) - \cos(2\pi)\right)\left(4\sqrt{3}\right)} = \frac{\pi\sum_{k=0}^{\infty} \frac{3^{1/2+k}(2\pi)^{1+2k}}{(1+2k)!}}{4\sqrt{3}\left(-1 + \sum_{k=0}^{\infty} \frac{12^{k}\pi^{2k}}{(2k)!}\right)}$$

$$\frac{\left(\sinh\left(2\,\pi\,\sqrt{3}\,\right) - \sqrt{3}\,\sin\left(2\,\pi\right)\right)\pi}{\left(\cosh\left(2\,\pi\,\sqrt{3}\,\right) - \cos(2\,\pi)\right)\left(4\,\sqrt{3}\,\right)} = \frac{\pi\,\sum_{k=0}^{\infty}\,\frac{3^{1/2\,(1+2\,k)}\,(2\,\pi)^{1+2\,k}}{(1+2\,k)!}}{4\,\sqrt{3}\left(-1 + i\,\sum_{k=0}^{\infty}\,\frac{\left(-\frac{i\,\pi}{2} + 2\,\sqrt{3}\,\pi\right)^{1+2\,k}}{(1+2\,k)!}\right)}$$

### **Integral representations:**

$$\frac{\left(\sinh(2\pi\sqrt{3}) - \sqrt{3}\sin(2\pi)\right)\pi}{\left(\cosh(2\pi\sqrt{3}) - \cos(2\pi)\right)\left(4\sqrt{3}\right)} = \frac{\pi\int_{0}^{1}\cosh(2\sqrt{3}\pi t) dt}{4\sqrt{3}\int_{0}^{1}\sinh(2\sqrt{3}\pi t) dt}$$

$$\frac{\left(\sinh(2\pi\sqrt{3}) - \sqrt{3}\,\sin(2\pi)\right)\pi}{\left(\cosh(2\pi\sqrt{3}) - \cos(2\pi)\right)\left(4\sqrt{3}\right)} = \frac{\pi^2 \int_0^1 \cosh(2\sqrt{3}\pi\,t)\,dt}{2\left(-1 + \int_{\frac{i\pi}{2}}^{2\sqrt{3}\pi} \sinh(t)\,dt\right)}$$

$$\frac{\left(\sinh(2\pi\sqrt{3}) - \sqrt{3}\sin(2\pi)\right)\pi}{\left(\cosh(2\pi\sqrt{3}) - \cos(2\pi)\right)\left(4\sqrt{3}\right)} = -\frac{i\sqrt{\frac{\pi}{3}}\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{e^{\left(3\pi^2\right)/s+s}}{s^{3/2}}\,ds}{16\int_{0}^{1}\sinh(2\sqrt{3}\pi\,t)\,dt} \quad \text{for } \gamma > 0$$

# Multiple-argument formulas:

$$\frac{\left(\sinh\left(2\pi\sqrt{3}\right) - \sqrt{3}\sin\left(2\pi\right)\right)\pi}{\left(\cosh\left(2\pi\sqrt{3}\right) - \cos\left(2\pi\right)\right)\left(4\sqrt{3}\right)} = \frac{\pi\coth\left(\sqrt{3}\pi\right)}{4\sqrt{3}}$$

$$\frac{\left(\sinh\left(2\,\pi\,\sqrt{3}\,\right)-\sqrt{3}\,\sin\left(2\,\pi\right)\right)\pi}{\left(\cosh\left(2\,\pi\,\sqrt{3}\,\right)-\cos\left(2\,\pi\right)\right)\left(4\,\sqrt{3}\,\right)} = \frac{\pi\,\cosh\left(\sqrt{3}\,\pi\right)\sinh\left(\sqrt{3}\,\pi\right)}{2\,\sqrt{3}\,\left(-2+2\,\cosh^2\left(\sqrt{3}\,\pi\right)\right)}$$

$$\frac{\left(\sinh(2\pi\sqrt{3}) - \sqrt{3}\sin(2\pi)\right)\pi}{\left(\cosh(2\pi\sqrt{3}) - \cos(2\pi)\right)(4\sqrt{3})} = \frac{\pi \operatorname{csch}^{2}(\sqrt{3}\pi)\left(3\sinh\left(\frac{2\pi}{\sqrt{3}}\right) + 4\sinh^{3}\left(\frac{2\pi}{\sqrt{3}}\right)\right)}{8\sqrt{3}}$$

Where 29 is a Lucas number and 16 is the difference between 26 and 10, where in bosonic string theory, spacetime is 26-dimensional, while in superstring theory it is 10-dimensional

#### Input:

$$\exp^{16} \left( \frac{\pi}{4\sqrt{3}} \times \frac{\sinh(2\pi\sqrt{3}) - \sqrt{3}\sin(2\pi)}{\cosh(2\pi\sqrt{3}) - \cos(2\pi)} \right) - 29 + \frac{1}{\phi}$$

sinh(x) is the hyperbolic sine function

 $\cosh(x)$  is the hyperbolic cosine function

φ is the golden ratio

#### **Exact result:**

$$\frac{1}{\phi} - 29 + e^{\frac{4\pi \sinh\left(2\sqrt{3}\pi\right)}{\sqrt{3}\left(\cosh\left(2\sqrt{3}\pi\right)-1\right)}}$$

#### **Decimal approximation:**

1387.446243586492327751485699773040770815595398403027547115...

1387.4462435.... result practically equal to the rest mass of Sigma baryon 1387.2

## **Alternate forms:**

$$\frac{1}{\phi} - 29 + e^{\left(4\pi \coth\left(\sqrt{3}\pi\right)\right)/\sqrt{3}}$$

$$\frac{1}{2}\left(\sqrt{5} - 59\right) + e^{\frac{4\pi \sinh\left(2\sqrt{3}\pi\right)}{\sqrt{3}\left(\cosh\left(2\sqrt{3}\pi\right) - 1\right)}}$$

$$\frac{4\left(1+e^{2\sqrt{3}\pi}\right)\pi}{\sqrt{3}\left(e^{2\sqrt{3}\pi}-1\right)}\phi+1$$

coth(x) is the hyperbolic cotangent function

## **Alternative representations:**

$$\begin{split} \exp^{16} & \left( \frac{\left( \sinh\left( 2\,\pi\,\sqrt{3}\,\right) - \sqrt{3}\,\sin(2\,\pi) \right)\pi}{\left( \cosh\left( 2\,\pi\,\sqrt{3}\,\right) - \cos(2\,\pi) \right) \left( 4\,\sqrt{3}\,\right)} \right) - 29 + \frac{1}{\phi} = \\ & - 29 + \frac{1}{\phi} + \exp^{16} \left( \frac{\pi \left( \frac{1}{2} \left( -e^{-2\,\pi\,\sqrt{3}}\, + e^{2\,\pi\,\sqrt{3}}\,\right) - \frac{\left( -e^{-2\,i\,\pi} + e^{2\,i\,\pi} \right)\sqrt{3}}{2\,i} \right)}{\left( -\cosh(-2\,i\,\pi) + \frac{1}{2} \left( e^{-2\,\pi\,\sqrt{3}}\, + e^{2\,\pi\,\sqrt{3}}\,\right) \right) \left( 4\,\sqrt{3}\,\right)} \right) \end{split}$$

$$\begin{split} &\exp^{16}\!\left(\frac{\left(\sinh\!\left(2\,\pi\,\sqrt{3}\,\right)-\sqrt{3}\,\sin\!\left(2\,\pi\right)\right)\pi}{\left(\cosh\!\left(2\,\pi\,\sqrt{3}\,\right)-\cos\!\left(2\,\pi\right)\right)\left(4\,\sqrt{3}\,\right)}\right) - 29 + \frac{1}{\phi} = \\ &-29 + \frac{1}{\phi} + \exp^{16}\!\left(\frac{\pi\left(\frac{1}{2}\left(-e^{-2\,\pi\,\sqrt{3}}\,+e^{2\,\pi\,\sqrt{3}}\,\right) + \cos\!\left(\frac{5\,\pi}{2}\right)\sqrt{3}\,\right)}{\left(\frac{1}{2}\left(-e^{-2\,i\,\pi}\,-e^{2\,i\,\pi}\right) + \frac{1}{2}\left(e^{-2\,\pi\,\sqrt{3}}\,+e^{2\,\pi\,\sqrt{3}}\,\right)\right)\left(4\,\sqrt{3}\,\right)}\right) \end{split}$$

$$\begin{split} &\exp^{16}\!\left(\frac{\left(\sinh\!\left(2\,\pi\,\sqrt{3}\,\right)-\sqrt{3}\,\sin\!\left(2\,\pi\right)\right)\pi}{\left(\cosh\!\left(2\,\pi\,\sqrt{3}\,\right)-\cos\!\left(2\,\pi\right)\right)\left(4\,\sqrt{3}\,\right)}\right) - 29 + \frac{1}{\phi} = \\ &-29 + \frac{1}{\phi} + \exp^{16}\!\left(\frac{\pi\left(\frac{1}{2}\left(-e^{-2\,\pi\,\sqrt{3}}\,+e^{2\,\pi\,\sqrt{3}}\right)-\cos\!\left(-\frac{3\,\pi}{2}\right)\sqrt{3}\,\right)}{\left(\frac{1}{2}\left(-e^{-2\,i\,\pi}\,-e^{2\,i\,\pi}\right) + \frac{1}{2}\left(e^{-2\,\pi\,\sqrt{3}}\,+e^{2\,\pi\,\sqrt{3}}\,\right)\right)\left(4\,\sqrt{3}\,\right)}\right) \end{split}$$

$$\begin{split} \exp^{16} & \left( \frac{\left( \sinh\left(2\,\pi\,\sqrt{3}\,\right) - \sqrt{3}\,\sin(2\,\pi) \right)\pi}{\left( \cosh\left(2\,\pi\,\sqrt{3}\,\right) - \cos(2\,\pi) \right) \left( 4\,\sqrt{3}\,\right)} \right) - 29 + \frac{1}{\phi} = \\ & - 27 - 29\,\sqrt{5}\, + \exp\left( \frac{4\,i\,\pi\,\sum_{k=0}^{\infty} \frac{\left( \frac{1}{2}\left( -i + 4\,\sqrt{3}\,\right)\pi \right)^{2\,k}}{\left( 2\,k \right)!}}{\sqrt{3}\left( -1 + \sum_{k=0}^{\infty} \frac{12^{k}\,\pi^{2\,k}}{(2\,k)!} \right)} \right) + \sqrt{5}\, \exp\left( \frac{4\,i\,\pi\,\sum_{k=0}^{\infty} \frac{\left( \frac{1}{2}\left( -i + 4\,\sqrt{3}\,\right)\pi \right)^{2\,k}}{(2\,k)!}}{\sqrt{3}\left( -1 + \sum_{k=0}^{\infty} \frac{12^{k}\,\pi^{2\,k}}{(2\,k)!} \right)} \right) \right) + \frac{1}{\sqrt{5}} \left( \frac{1}{2}\left( -i + 4\,\sqrt{3}\,\right)\pi \right)^{2\,k}} \right) + \frac{1}{\sqrt{5}} \left( \frac{1}{2}\left( -i + 4\,\sqrt{3}\,\right)\pi \right)^{2\,k}} \left( \frac{1}{2}\left( -i + 4\,\sqrt{3}\,\right)\pi \right)^{2\,k}}{\sqrt{3}\left( -1 + \sum_{k=0}^{\infty} \frac{12^{k}\,\pi^{2\,k}}{(2\,k)!} \right)} \right) + \frac{1}{2}\left( \frac{1}{2}\left( -i + 4\,\sqrt{3}\,\right)\pi \right)^{2\,k}} \left( \frac{1}{2}\left( -i + 4\,\sqrt{3}\,\right)\pi \right)^{2\,k}}{\sqrt{3}\left( -1 + \sum_{k=0}^{\infty} \frac{12^{k}\,\pi^{2\,k}}{(2\,k)!} \right)} \right) + \frac{1}{2}\left( \frac{1}{2}\left( -i + 4\,\sqrt{3}\,\right)\pi \right)^{2\,k}} \right) + \frac{1}{2}\left( \frac{1}{2}\left( -i + 4\,\sqrt{3}\,\right)\pi \right)^{2\,k}} \left( \frac{1}{2}\left( -i + 4\,\sqrt{3}\,\right)\pi \right)^{2\,k}}{\sqrt{3}\left( -1 + \sum_{k=0}^{\infty} \frac{12^{k}\,\pi^{2\,k}}{(2\,k)!} \right)} \right) + \frac{1}{2}\left( \frac{1}{2}\left( -i + 4\,\sqrt{3}\,\right)\pi \right)^{2\,k}}{\sqrt{3}\left( -1 + \sum_{k=0}^{\infty} \frac{12^{k}\,\pi^{2\,k}}{(2\,k)!} \right)} \right) + \frac{1}{2}\left( \frac{1}{2}\left( -i + 4\,\sqrt{3}\,\right)\pi \right)^{2\,k}}{\sqrt{3}\left( -1 + \sum_{k=0}^{\infty} \frac{12^{k}\,\pi^{2\,k}}{(2\,k)!} \right)} \right) + \frac{1}{2}\left( \frac{1}{2}\left( -i + 4\,\sqrt{3}\,\right)\pi \right)^{2\,k}}{\sqrt{3}\left( -1 + \sum_{k=0}^{\infty} \frac{12^{k}\,\pi^{2\,k}}{(2\,k)!} \right)} \right) + \frac{1}{2}\left( \frac{1}{2}\left( -i + 4\,\sqrt{3}\,\right)\pi \right)^{2\,k}}{\sqrt{3}\left( -1 + \sum_{k=0}^{\infty} \frac{12^{k}\,\pi^{2\,k}}{(2\,k)!} \right)} \right) + \frac{1}{2}\left( \frac{1}{2}\left( -i + 4\,\sqrt{3}\,\right)\pi \right)^{2\,k}}{\sqrt{3}\left( -1 + \sum_{k=0}^{\infty} \frac{12^{k}\,\pi^{2\,k}}{(2\,k)!} \right)} \right) + \frac{1}{2}\left( \frac{1}{2}\left( -i + 4\,\sqrt{3}\,\right)\pi \right)^{2\,k}}{\sqrt{3}\left( -1 + \sum_{k=0}^{\infty} \frac{12^{k}\,\pi^{2\,k}}{(2\,k)!} \right)} \right) + \frac{1}{2}\left( \frac{1}{2}\left( -i + 4\,\sqrt{3}\,\right)\pi \right)^{2\,k}}{\sqrt{3}\left( -1 + \sum_{k=0}^{\infty} \frac{12^{k}\,\pi^{2\,k}}{(2\,k)!} \right)} \right) + \frac{1}{2}\left( \frac{1}{2}\left( -i + 4\,\sqrt{3}\,\right)\pi \right)^{2\,k}}{\sqrt{3}\left( -1 + \sum_{k=0}^{\infty} \frac{12^{k}\,\pi^{2\,k}}{(2\,k)!} \right)} \right)$$

$$\exp^{16} \left( \frac{\left( \sinh(2\pi\sqrt{3}) - \sqrt{3} \sin(2\pi) \right) \pi}{\left( \cosh(2\pi\sqrt{3}) - \cos(2\pi) \right) \left( 4\sqrt{3} \right)} \right) - 29 + \frac{1}{\phi} = \\ -27 - 29\sqrt{5} + \exp \left( \frac{4\pi\sum_{k=0}^{\infty} \frac{3^{1/2+k}(2\pi)^{1+2k}}{(1+2k)!}}{\sqrt{3}\left( -1 + \sum_{k=0}^{\infty} \frac{12^k \pi^{2k}}{(2k)!} \right)} \right) + \sqrt{5} \exp \left( \frac{4\pi\sum_{k=0}^{\infty} \frac{3^{1/2+k}(2\pi)^{1+2k}}{(1+2k)!}}{\sqrt{3}\left( -1 + \sum_{k=0}^{\infty} \frac{12^k \pi^{2k}}{(2k)!} \right)} \right) \\ - 1 + \sqrt{5}$$

$$\begin{split} \exp^{16} & \left( \frac{\left( \sinh\left(2\,\pi\,\sqrt{3}\,\right) - \sqrt{3}\,\sin(2\,\pi) \right)\pi}{\left( \cosh\left(2\,\pi\,\sqrt{3}\,\right) - \cos(2\,\pi) \right) \left( 4\,\sqrt{3}\,\right)} \right) - 29 + \frac{1}{\phi} = \frac{1}{1 + \sqrt{5}} \\ & \left( -27 - 29\,\sqrt{5}\, + \exp\left( \frac{4\,\pi\,\sum_{k=0}^{\infty}\,\frac{3^{1/2 + k}\,(2\,\pi)^{1 + 2\,k}}{\left( 1 + 2\,k \right)!}}{\sqrt{3}\,\left( -1 + i\,\sum_{k=0}^{\infty}\,\frac{\left(\frac{1}{2}\left( -i + 4\,\sqrt{3}\,\right)\pi\right)^{1 + 2\,k}}{\left( 1 + 2\,k \right)!}\right)} \right) + \\ & \sqrt{5}\,\exp\left( \frac{4\,\pi\,\sum_{k=0}^{\infty}\,\frac{3^{1/2 + k}\,(2\,\pi)^{1 + 2\,k}}{\left( 1 + 2\,k \right)!}}{\sqrt{3}\,\left( -1 + i\,\sum_{k=0}^{\infty}\,\frac{\left(\frac{1}{2}\left( -i + 4\,\sqrt{3}\,\right)\pi\right)^{1 + 2\,k}}{\left( 1 + 2\,k \right)!}\right)} \right) \right) \end{split}$$

# Multiple-argument formulas:

$$\exp^{16}\!\!\left(\!\frac{\left(\sinh\!\left(2\,\pi\,\sqrt{3}\,\right)-\sqrt{3}\,\sin\!\left(2\,\pi\right)\right)\pi}{\left(\cosh\!\left(2\,\pi\,\sqrt{3}\,\right)-\cos\!\left(2\,\pi\right)\right)\left(4\,\sqrt{3}\,\right)}\right) - 29 + \frac{1}{\phi} = -29 + e^{\left(4\,\pi\,\coth\!\left(\sqrt{3}\,\pi\right)\right)\!\left/\sqrt{3}\,\right.} + \frac{1}{\phi}$$

$$\begin{split} \exp^{16} & \left( \frac{\left( \sinh\left( 2\,\pi\,\sqrt{3}\,\right) - \sqrt{3}\,\sin(2\,\pi) \right) \pi}{\left( \cosh\left( 2\,\pi\,\sqrt{3}\,\right) - \cos(2\,\pi) \right) \left( 4\,\sqrt{3}\,\right)} \right) - 29 + \frac{1}{\phi} = \\ & -29 + \exp \left( \frac{2\,\pi\, \mathrm{csch}^2 \left( \sqrt{3}\,\pi \right) \left( 3\,\sinh\left( \frac{2\,\pi}{\sqrt{3}}\,\right) + 4\,\sinh^3\left( \frac{2\,\pi}{\sqrt{3}}\,\right) \right)}{\sqrt{3}} \right) + \frac{1}{\phi} \end{split}$$

$$\begin{split} \exp^{16} & \left( \frac{\left( \sinh\left( 2\,\pi\,\sqrt{3}\,\right) - \sqrt{3}\,\sin(2\,\pi) \right) \pi}{\left( \cosh\left( 2\,\pi\,\sqrt{3}\,\right) - \cos(2\,\pi) \right) \left( 4\,\sqrt{3}\,\right)} \right) - 29 + \frac{1}{\phi} = \\ & -29 + \exp\!\left( \frac{8\,\pi\,\cosh\!\left( \sqrt{3}\,\pi\right) \sinh\!\left( \sqrt{3}\,\pi\right)}{\sqrt{3}\,\left( -2 + 2\,\cosh^2\!\left( \sqrt{3}\,\pi\right) \right)} \right) + \frac{1}{\phi} \end{split}$$

Where 10 is the number of dimensions in superstring theory. In bosonic string theory, spacetime is 26-dimensional, while in superstring theory it is 10-dimensional, and in M-theory it is 11-dimensional. Note that 26 - 10 = 16

**Input:** 

$$\frac{1}{10} \exp^{16} \left( \frac{\pi}{4\sqrt{3}} \times \frac{\sinh(2\pi\sqrt{3}) - \sqrt{3} \sin(2\pi)}{\cosh(2\pi\sqrt{3}) - \cos(2\pi)} \right) - 2$$

 $\sinh(x)$  is the hyperbolic sine function

cosh(x) is the hyperbolic cosine function

#### **Exact result:**

$$\frac{1}{10} e^{\frac{4\pi \sinh\left(2\sqrt{3}\pi\right)}{\sqrt{3}\left(\cosh\left(2\sqrt{3}\pi\right)-1\right)}} - 2$$

## **Decimal approximation:**

139.5828209597742432903281112938675132697875089223221784253...

139.5828209.... result practically equal to the rest mass of Pion meson 139.57

#### **Alternate forms:**

$$\frac{1}{10} e^{\left(4\pi \coth\left(\sqrt{3}\pi\right)\right)/\sqrt{3}} - 2$$

$$\frac{1}{10} \left( e^{\frac{4\pi \sinh\left(2\sqrt{3}\pi\right)}{\sqrt{3}\left(\cosh\left(2\sqrt{3}\pi\right)-1\right)}} - 20 \right)$$

$$\frac{1}{10} \left( e^{\frac{4\left(1+e^{2\sqrt{3}\pi}\right)\pi}{\sqrt{3}\left(e^{2\sqrt{3}\pi}-1\right)}} - 20 \right)$$

 $\coth(x)$  is the hyperbolic cotangent function

## Alternative representations:

$$\begin{split} &\frac{1}{10} \exp^{16} \! \left( \frac{\pi \left( \sinh \! \left( 2 \, \pi \, \sqrt{3} \, \right) - \sqrt{3} \, \sin \! \left( 2 \, \pi \right) \right)}{\left( 4 \, \sqrt{3} \, \right) \left( \cosh \! \left( 2 \, \pi \, \sqrt{3} \, \right) - \cos \! \left( 2 \, \pi \right) \right)} \right) - 2 = \\ &-2 + \frac{1}{10} \exp^{16} \! \left( \frac{\pi \left( \frac{1}{2} \left( -e^{-2 \, \pi \, \sqrt{3}} \, + e^{2 \, \pi \, \sqrt{3}} \right) - \frac{\left( -e^{-2 \, i \, \pi} + e^{2 \, i \, \pi} \right) \sqrt{3}}{2 \, i} \right)}{\left( -\cosh (-2 \, i \, \pi) + \frac{1}{2} \left( e^{-2 \, \pi \, \sqrt{3}} \, + e^{2 \, \pi \, \sqrt{3}} \right) \right) \left( 4 \, \sqrt{3} \, \right)} \right) \end{split}$$

$$\begin{split} &\frac{1}{10} \exp^{16} \! \left( \frac{\pi \left( \sinh \! \left( 2 \, \pi \, \sqrt{3} \, \right) - \sqrt{3} \, \sin \! \left( 2 \, \pi \right) \right)}{\left( 4 \, \sqrt{3} \, \right) \left( \cosh \! \left( 2 \, \pi \, \sqrt{3} \, \right) - \cos \! \left( 2 \, \pi \right) \right)} \right) - 2 = \\ &-2 + \frac{1}{10} \exp^{16} \! \left( \frac{\pi \left( \frac{1}{2} \left( -e^{-2 \, \pi \, \sqrt{3}} \, + e^{2 \, \pi \, \sqrt{3}} \right) + \cos \! \left( \frac{5 \, \pi}{2} \right) \sqrt{3} \, \right)}{\left( \frac{1}{2} \left( -e^{-2 \, i \, \pi} - e^{2 \, i \, \pi} \right) + \frac{1}{2} \left( e^{-2 \, \pi \, \sqrt{3}} \, + e^{2 \, \pi \, \sqrt{3}} \, \right) \right) \left( 4 \, \sqrt{3} \, \right)} \right) \end{split}$$

$$\begin{split} &\frac{1}{10} \exp^{16} \! \left( \frac{\pi \left( \sinh \! \left( 2 \, \pi \, \sqrt{3} \, \right) - \sqrt{3} \, \sin \! \left( 2 \, \pi \right) \right)}{\left( 4 \, \sqrt{3} \, \right) \left( \cosh \! \left( 2 \, \pi \, \sqrt{3} \, \right) - \cos \! \left( 2 \, \pi \right) \right)} \right) - 2 = \\ &-2 + \frac{1}{10} \exp^{16} \! \left( \frac{\pi \left( \frac{1}{2} \left( -e^{-2 \, \pi \, \sqrt{3}} \, + e^{2 \, \pi \, \sqrt{3}} \right) - \cos \! \left( -\frac{3 \, \pi}{2} \right) \sqrt{3} \, \right)}{\left( \frac{1}{2} \left( -e^{-2 \, i \, \pi} \, - e^{2 \, i \, \pi} \right) + \frac{1}{2} \left( e^{-2 \, \pi \, \sqrt{3}} \, + e^{2 \, \pi \, \sqrt{3}} \, \right) \right) \left( 4 \, \sqrt{3} \, \right)} \right) \end{split}$$

### Series representations:

$$\frac{1}{10} \exp^{16} \left( \frac{\pi \left( \sinh(2\pi\sqrt{3}) - \sqrt{3} \sin(2\pi) \right)}{\left( 4\sqrt{3} \right) \left( \cosh(2\pi\sqrt{3}) - \cos(2\pi) \right)} \right) - 2 =$$

$$\frac{1}{10} \left( -20 + \exp \left( \frac{4\pi \sum_{k=0}^{\infty} \frac{3^{1/2+k} (2\pi)^{1+2k}}{(1+2k)!}}{\sqrt{3} \left( -1 + \sum_{k=0}^{\infty} \frac{12^k \pi^{2k}}{(2k)!} \right)} \right) \right)$$

$$\frac{1}{10} \exp^{16} \left( \frac{\pi \left( \sinh(2\pi\sqrt{3}) - \sqrt{3} \sin(2\pi) \right)}{\left( 4\sqrt{3} \right) \left( \cosh(2\pi\sqrt{3}) - \cos(2\pi) \right)} \right) - 2 =$$

$$\frac{1}{10} \left[ -20 + \exp \left( \frac{4\pi \sum_{k=0}^{\infty} \frac{3^{1/2+k} (2\pi)^{1+2k}}{(1+2k)!}}{\sqrt{3} \left( -1 + i \sum_{k=0}^{\infty} \frac{\left( \frac{1}{2} \left( -i + 4\sqrt{3} \right) \pi \right)^{1+2k}}{(1+2k)!} \right)} \right) \right]$$

$$\frac{1}{10} \exp^{16} \left( \frac{\pi \left( \sinh(2 \pi \sqrt{3} \right) - \sqrt{3} \sin(2 \pi) \right)}{\left( 4 \sqrt{3} \right) \left( \cosh(2 \pi \sqrt{3} \right) - \cos(2 \pi) \right)} \right) - 2 =$$

$$\frac{1}{10} \left[ -20 + \exp \left( \frac{4 \pi^{5/2} \sum_{j=0}^{\infty} \operatorname{Res}_{s=-j} \frac{(-3)^{-s} \pi^{-2 s} \Gamma(s)}{\Gamma\left(\frac{3}{2} - s\right)}}{-1 + \sum_{k=0}^{\infty} \frac{12^k \pi^{2k}}{(2k)!}} \right] \right]$$

# Multiple-argument formulas:

$$\frac{1}{10} \exp^{16} \left( \frac{\pi \left( \sinh(2 \pi \sqrt{3}) - \sqrt{3} \sin(2 \pi) \right)}{(4 \sqrt{3}) \left( \cosh(2 \pi \sqrt{3}) - \cos(2 \pi) \right)} \right) - 2 = -2 + \frac{1}{10} e^{\left( 4 \pi \coth\left( \sqrt{3} \pi\right) \right) / \sqrt{3}}$$

$$\begin{split} &\frac{1}{10} \, \exp^{16}\!\!\left(\!\frac{\pi \left(\sinh\!\left(2\,\pi\,\sqrt{\,3\,}\right) - \sqrt{\,3\,}\,\sin(2\,\pi)\right)}{\left(4\,\sqrt{\,3\,}\right) \left(\cosh\!\left(2\,\pi\,\sqrt{\,3\,}\right) - \cos(2\,\pi)\right)}\right) - 2 = \\ &-2 + \frac{1}{10} \, \exp\!\left(\!\frac{8\,\pi\,\cosh\!\left(\sqrt{\,3\,}\,\pi\right) \sinh\!\left(\sqrt{\,3\,}\,\pi\right)}{\sqrt{\,3\,}\left(-2 + 2\,\cosh^2\!\left(\sqrt{\,3\,}\,\pi\right)\right)}\right) \end{split}$$

$$\begin{split} &\frac{1}{10} \exp^{16}\!\!\left(\frac{\pi \left(\sinh\!\left(2\,\pi\,\sqrt{3}\,\right) - \sqrt{3}\,\sin(2\,\pi)\right)}{\left(4\,\sqrt{3}\,\right) \left(\cosh\!\left(2\,\pi\,\sqrt{3}\,\right) - \cos(2\,\pi)\right)}\right) - 2 = \\ &-2 + \frac{1}{10} \exp\!\!\left(\!\!-\frac{4\,i\,\pi\, \mathrm{csch}^2\!\left(\sqrt{3}\,\pi\right) \prod_{k=0}^1 \sinh\!\left(\!\left(\sqrt{3}\,+\frac{i\,k}{2}\right)\!\pi\right)}{\sqrt{3}}\right) \end{split}$$

Or:

$$1/(1^2+2^2+2^4/1^2)+1/(2^2+2^2+2^4/2^2)+1/(3^2+2^2+2^4/3^2)+...$$

Input interpretation: 
$$\frac{1}{1^2 + 2^2 + \frac{2^4}{1^2}} + \frac{1}{2^2 + 2^2 + \frac{2^4}{2^2}} + \frac{1}{3^2 + 2^2 + \frac{2^4}{3^2}} + \cdots$$

**Infinite sum:** 

$$\sum_{n=1}^{\infty} \frac{1}{n^2 + \frac{16}{n^2} + 4} = -\frac{\pi \sinh(2\sqrt{3} \pi)}{16\sqrt{3} (1 - \cosh(2\sqrt{3} \pi))} - \frac{\sqrt{3} \pi \sinh(2\sqrt{3} \pi)}{16 (1 - \cosh(2\sqrt{3} \pi))}$$

 $\cosh(x)$  is the hyperbolic cosine function

sinh(x) is the hyperbolic sine function

## **Decimal approximation:**

0.453466871624258724623634815745739322304887984526058956146...

0.4534668716242587.....

## **Convergence tests:**

The ratio test is inconclusive.

The root test is inconclusive.

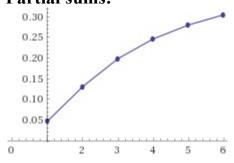
By the comparison test, the series converges.

#### Partial sum formula:

$$\begin{split} \sum_{n=1}^{m} \frac{1}{4 + \frac{16}{n^2} + n^2} &= \\ \left( i \left( i \, m^4 - \sqrt{3} \, \, m^4 \, \psi^{(0)} \! \left( m - i \, \sqrt{3} \, \right) + \sqrt{3} \, \, m^4 \, \psi^{(0)} \! \left( m + i \, \sqrt{3} \, \right) - \sqrt{3} \, \, m^4 \, \psi^{(0)} \! \left( i \, \sqrt{3} \, \right) + \\ \sqrt{3} \, \, m^4 \, \psi^{(0)} \! \left( - i \, \sqrt{3} \, \right) + 8 \, i \, m^3 - 2 \, \sqrt{3} \, \, m^3 \, \psi^{(0)} \! \left( m - i \, \sqrt{3} \, \right) + \\ 2 \, \sqrt{3} \, \, m^3 \, \psi^{(0)} \! \left( m + i \, \sqrt{3} \, \right) - 2 \, \sqrt{3} \, \, m^3 \, \psi^{(0)} \! \left( i \, \sqrt{3} \, \right) + 2 \, \sqrt{3} \, \, m^3 \, \psi^{(0)} \! \left( - i \, \sqrt{3} \, \right) + \\ 10 \, i \, m^2 - 7 \, \sqrt{3} \, \, m^2 \, \psi^{(0)} \! \left( m - i \, \sqrt{3} \, \right) + 7 \, \sqrt{3} \, \, m^2 \, \psi^{(0)} \! \left( m + i \, \sqrt{3} \, \right) - \\ 7 \, \sqrt{3} \, \, m^2 \, \psi^{(0)} \! \left( i \, \sqrt{3} \, \right) + 7 \, \sqrt{3} \, \, m^2 \, \psi^{(0)} \! \left( - i \, \sqrt{3} \, \right) + 21 \, i \, m - \\ 6 \, \sqrt{3} \, \, m \, \psi^{(0)} \! \left( m - i \, \sqrt{3} \, \right) + 6 \, \sqrt{3} \, \, m \, \psi^{(0)} \! \left( m + i \, \sqrt{3} \, \right) - 6 \, \sqrt{3} \, \, m \, \psi^{(0)} \! \left( i \, \sqrt{3} \, \right) + \\ 6 \, \sqrt{3} \, \, m \, \psi^{(0)} \! \left( - i \, \sqrt{3} \, \right) - 12 \, \sqrt{3} \, \, \psi^{(0)} \! \left( m - i \, \sqrt{3} \, \right) + 12 \, \sqrt{3} \, \, \psi^{(0)} \! \left( m + i \, \sqrt{3} \, \right) - \\ 12 \, \sqrt{3} \, \, \psi^{(0)} \! \left( i \, \sqrt{3} \, \right) + 12 \, \sqrt{3} \, \, \psi^{(0)} \! \left( - i \, \sqrt{3} \, \right) \right) \right) / \left( 12 \left( m^2 + 3 \right) \left( m^2 + 2 \, m + 4 \right) \right) \end{split}$$

 $\psi^{(n)}(x)$  is the  $n^{th}$  derivative of the digamma function

#### **Partial sums:**



#### **Alternate forms:**

$$\frac{\pi \coth(\sqrt{3} \pi)}{4\sqrt{3}}$$

$$-\frac{\pi \sinh(2\sqrt{3} \pi)}{16\sqrt{3} (1 - \cosh(2\sqrt{3} \pi))} - \frac{\sqrt{3} \pi \sinh(2\sqrt{3} \pi)}{16 (1 - \cosh(2\sqrt{3} \pi))}$$

$$\frac{\left(e^{-\sqrt{3} \pi} + e^{\sqrt{3} \pi}\right) \pi}{\sqrt{3} \left(\sqrt{3} + -i\right) \left(\sqrt{3} + i\right) \left(e^{\sqrt{3} \pi} - e^{-\sqrt{3} \pi}\right)}$$

### **Series representations:**

$$-\frac{\pi \sinh(2\sqrt{3} \pi)}{16\sqrt{3} (1 - \cosh(2\sqrt{3} \pi))} - \frac{\sqrt{3} \pi \sinh(2\sqrt{3} \pi)}{16 (1 - \cosh(2\sqrt{3} \pi))} = \frac{\pi \sum_{k=0}^{\infty} \frac{3^{1/2+k} (2\pi)^{1+2k}}{(1+2k)!}}{4\sqrt{3} \left(-1 + \sum_{k=0}^{\infty} \frac{12^k \pi^{2k}}{(2k)!}\right)}$$

$$-\frac{\pi \sinh(2\sqrt{3} \pi)}{16\sqrt{3} (1 - \cosh(2\sqrt{3} \pi))} - \frac{\sqrt{3} \pi \sinh(2\sqrt{3} \pi)}{16 (1 - \cosh(2\sqrt{3} \pi))} = \frac{\pi \sum_{k=0}^{\infty} \frac{3^{1/2+k} (2\pi)^{1+2k}}{(1+2k)!}}{4\sqrt{3} \left(-1 + i \sum_{k=0}^{\infty} \frac{\left(\frac{1}{2}\left(-i+4\sqrt{3}\right)\pi\right)^{1+2k}}{(1+2k)!}\right)}$$

$$-\frac{\pi \sinh(2\sqrt{3} \pi)}{16\sqrt{3} (1 - \cosh(2\sqrt{3} \pi))} - \frac{\sqrt{3} \pi \sinh(2\sqrt{3} \pi)}{16 (1 - \cosh(2\sqrt{3} \pi))} = \frac{\pi^{5/2} \sum_{j=0}^{\infty} \operatorname{Res}_{s=-j} \frac{(-3)^{-5} \pi^{-2\cdot 5} \Gamma(s)}{\Gamma(\frac{3}{2} - s)}}{4 \left(-1 + \sum_{k=0}^{\infty} \frac{12^k \pi^{2k}}{(2k)!}\right)}$$

$$-\frac{\pi \sinh(2\sqrt{3} \pi)}{16\sqrt{3} (1 - \cosh(2\sqrt{3} \pi))} - \frac{\sqrt{3} \pi \sinh(2\sqrt{3} \pi)}{16 (1 - \cosh(2\sqrt{3} \pi))} = \frac{\pi^{5/2} \sum_{j=0}^{\infty} \operatorname{Res}_{s=-j} \frac{(-3)^{-5} \pi^{-2\cdot 5} \Gamma(s)}{\Gamma(\frac{1}{2} - s)}}{16 (1 - \cosh(2\sqrt{3} \pi))} = \frac{\pi^{5/2} \sum_{j=0}^{\infty} \operatorname{Res}_{s=-j} \frac{(-3)^{-5} \pi^{-2\cdot 5} \Gamma(s)}{\Gamma(\frac{1}{2} - s)}}{16 (1 - \cosh(2\sqrt{3} \pi))} = \frac{\pi^{5/2} \sum_{j=0}^{\infty} \operatorname{Res}_{s=-j} \frac{(-3)^{-5} \pi^{-2\cdot 5} \Gamma(s)}{\Gamma(\frac{1}{2} - s)}}{16 (1 - \cosh(2\sqrt{3} \pi))} = \frac{\pi^{5/2} \sum_{j=0}^{\infty} \operatorname{Res}_{s=-j} \frac{(-3)^{-5} \pi^{-2\cdot 5} \Gamma(s)}{\Gamma(\frac{1}{2} - s)}}}{16 (1 - \cosh(2\sqrt{3} \pi))} = \frac{\pi^{5/2} \sum_{j=0}^{\infty} \operatorname{Res}_{s=-j} \frac{(-3)^{-5} \pi^{-2\cdot 5} \Gamma(s)}{\Gamma(\frac{1}{2} - s)}}}{16 (1 - \cosh(2\sqrt{3} \pi))} = \frac{\pi^{5/2} \sum_{j=0}^{\infty} \operatorname{Res}_{s=-j} \frac{(-3)^{-5} \pi^{-2\cdot 5} \Gamma(s)}{\Gamma(\frac{1}{2} - s)}}}{16 (1 - \cosh(2\sqrt{3} \pi))} = \frac{\pi^{5/2} \sum_{j=0}^{\infty} \operatorname{Res}_{s=-j} \frac{(-3)^{-5} \pi^{-2\cdot 5} \Gamma(s)}{\Gamma(\frac{1}{2} - s)}}}{16 (1 - \cosh(2\sqrt{3} \pi))} = \frac{\pi^{5/2} \sum_{j=0}^{\infty} \operatorname{Res}_{s=-j} \frac{(-3)^{-5} \pi^{-2\cdot 5} \Gamma(s)}{\Gamma(\frac{1}{2} - s)}}}{16 (1 - \cosh(2\sqrt{3} \pi))}$$

For n = 2, we obtain:

$$1/(12*2^2) + 1/2*(1/(1^2+3*2^2)+1/(2^2+3*2^2)+1/(3^2+3*2^2)+...)$$

Input interpretation: 
$$\frac{1}{12 \times 2^2} + \frac{1}{2} \left( \frac{1}{1^2 + 3 \times 2^2} + \frac{1}{2^2 + 3 \times 2^2} + \frac{1}{3^2 + 3 \times 2^2} + \cdots \right)$$

## **Result:**

$$\frac{1}{48} + \frac{1}{48} \left( 2\sqrt{3} \ \pi \ \text{coth} \left( 2\sqrt{3} \ \pi \right) - 1 \right)$$

coth(x) is the hyperbolic cotangent function

#### **Alternate forms:**

$$\frac{\pi \coth(2\sqrt{3} \pi)}{8\sqrt{3}}$$

$$-\frac{\pi \sinh(4\sqrt{3} \pi)}{8\sqrt{3} (1-\cosh(4\sqrt{3} \pi))}$$

$$\frac{\pi \tanh(\sqrt{3} \pi)}{16\sqrt{3}} + \frac{\pi \coth(\sqrt{3} \pi)}{16\sqrt{3}}$$

$$1/48 + 1/48 (-1 + 2 \operatorname{sqrt}(3) \pi \operatorname{coth}(2 \operatorname{sqrt}(3) \pi))$$

Input: 
$$\frac{1}{48} + \frac{1}{48} \left( -1 + 2\sqrt{3} \pi \coth\left(2\sqrt{3} \pi\right) \right)$$

coth(x) is the hyperbolic cotangent function

## **Decimal approximation:**

0.226724920689178751345059994437316352094407237779531520754...

0.22672492...

#### **Alternate forms:**

$$\frac{\pi \coth(2\sqrt{3} \pi)}{8\sqrt{3}}$$

$$-\frac{\pi \sinh(4\sqrt{3} \pi)}{8\sqrt{3} (1-\cosh(4\sqrt{3} \pi))}$$

$$\frac{\pi \tanh(\sqrt{3} \pi)}{16\sqrt{3}} + \frac{\pi \coth(\sqrt{3} \pi)}{16\sqrt{3}}$$

## Alternative representations:

$$\frac{1}{48} + \frac{1}{48} \left( -1 + 2\sqrt{3} \pi \coth\left(2\sqrt{3} \pi\right) \right) = \frac{1}{48} + \frac{1}{48} \left( -1 - 2i\pi \cot\left(-2i\pi\sqrt{3}\right)\sqrt{3}\right)$$

$$\frac{1}{48} + \frac{1}{48} \left( -1 + 2\sqrt{3} \pi \coth \left( 2\sqrt{3} \pi \right) \right) = \frac{1}{48} + \frac{1}{48} \left( -1 + 2i\pi \cot \left( 2i\pi \sqrt{3} \right) \sqrt{3} \right)$$

$$\frac{1}{48} + \frac{1}{48} \left( -1 + 2\sqrt{3} \pi \coth\left(2\sqrt{3}\pi\right) \right) = \frac{1}{48} + \frac{1}{48} \left( -1 + 2\pi\left(1 + \frac{2}{-1 + e^{4\pi\sqrt{3}}}\right)\sqrt{3}\right)$$

### **Series representations:**

$$\frac{1}{48} + \frac{1}{48} \left( -1 + 2\sqrt{3} \pi \coth\left(2\sqrt{3} \pi\right) \right) = \frac{1}{48} + \frac{1}{2} \sum_{k=1}^{\infty} \frac{1}{12 + k^2}$$

$$\frac{1}{48} + \frac{1}{48} \left( -1 + 2\sqrt{3} \pi \coth\left(2\sqrt{3}\pi\right) \right) = \frac{1}{4} \pi \sum_{k=-\infty}^{\infty} \frac{1}{12\pi + k^2 \pi}$$

$$\frac{1}{48} + \frac{1}{48} \left( -1 + 2\sqrt{3} \pi \coth\left(2\sqrt{3}\pi\right) \right) = \frac{\pi}{8\sqrt{3}} + \frac{\pi \sum_{k=0}^{\infty} e^{-4\sqrt{3}(1+k)\pi}}{4\sqrt{3}}$$

## **Integral representation:**

$$\frac{1}{48} + \frac{1}{48} \left( -1 + 2\sqrt{3} \pi \coth\left(2\sqrt{3}\pi\right) \right) = -\frac{\pi}{8\sqrt{3}} \int_{\frac{i\pi}{2}}^{2\sqrt{3}\pi} \operatorname{csch}^{2}(t) dt$$

 $(((\exp(((1/48 + 1/48 (-1 + 2 \operatorname{sqrt}(3) \pi \operatorname{coth}(2 \operatorname{sqrt}(3) \pi)))))))^3 2 - 29 + 1/\operatorname{golden ratio})$ Where 29 is a Lucas number

### **Input:**

$$\exp^{32}\left(\frac{1}{48} + \frac{1}{48}\left(-1 + 2\sqrt{3} \pi \coth\left(2\sqrt{3} \pi\right)\right)\right) - 29 + \frac{1}{\phi}$$

coth(x) is the hyperbolic cotangent function

φ is the golden ratio

#### **Exact result:**

$$\frac{1}{\phi} - 29 + e^{2/3 + 2/3 \left(2\sqrt{3} \pi \coth\left(2\sqrt{3} \pi\right) - 1\right)}$$

## **Decimal approximation:**

 $1387.060505701553890257491110080389692406376704143735732815\dots \\$ 

1387.0605057.... result practically equal to the rest mass of Sigma baryon 1387.2

#### **Alternate forms:**

$$\frac{1}{2} \left( \sqrt{5} - 59 \right) + e^{\left( 4 \pi \coth \left( 2 \sqrt{3} \pi \right) \right) / \sqrt{3}}$$

$$-29 + \frac{2}{1 + \sqrt{5}} + e^{(4\pi \coth(2\sqrt{3}\pi))/\sqrt{3}}$$

$$\frac{1}{\phi} - 29 + e^{-\frac{4\pi \sinh\left(4\sqrt{3}\pi\right)}{\sqrt{3}\left(1-\cosh\left(4\sqrt{3}\pi\right)\right)}}$$

 $\cosh(x)$  is the hyperbolic cosine function

sinh(x) is the hyperbolic sine function

## **Alternative representations:**

$$\begin{split} \exp^{32} & \left( \frac{1}{48} + \frac{1}{48} \left( -1 + 2\sqrt{3} \pi \coth\left(2\sqrt{3} \pi\right) \right) \right) - 29 + \frac{1}{\phi} = \\ & -29 + \frac{1}{\phi} + \exp^{32} \left( \frac{1}{48} + \frac{1}{48} \left( -1 - 2i\pi \cot\left(-2i\pi\sqrt{3}\right)\sqrt{3}\right) \right) \end{split}$$

$$\exp^{32}\left(\frac{1}{48} + \frac{1}{48}\left(-1 + 2\sqrt{3}\pi\coth\left(2\sqrt{3}\pi\right)\right)\right) - 29 + \frac{1}{\phi} = \\ -29 + \frac{1}{\phi} + \exp^{32}\left(\frac{1}{48} + \frac{1}{48}\left(-1 + 2\pi\left(1 + \frac{2}{-1 + e^{4\pi\sqrt{3}}}\right)\sqrt{3}\right)\right)$$

$$\exp^{32}\left(\frac{1}{48} + \frac{1}{48}\left(-1 + 2\sqrt{3} \pi \coth\left(2\sqrt{3} \pi\right)\right)\right) - 29 + \frac{1}{\phi} = \\ -29 + \frac{1}{\phi} + \exp^{32}\left(\frac{1}{48} + \frac{1}{48}\left(-1 + \frac{2\pi\left(e^{-2\pi\sqrt{3}} + e^{2\pi\sqrt{3}}\right)\sqrt{3}}{-e^{-2\pi\sqrt{3}} + e^{2\pi\sqrt{3}}}\right)\right)$$

### Series representations:

$$\exp^{32}\left(\frac{1}{48} + \frac{1}{48}\left(-1 + 2\sqrt{3} \pi \coth\left(2\sqrt{3} \pi\right)\right)\right) - 29 + \frac{1}{\phi} = \frac{-27 - 29\sqrt{5} + \left(1 + \sqrt{5}\right)e^{2/3 + 16 \times \sum_{k=1}^{\infty} 1/\left(12 + k^2\right)}}{1 + \sqrt{5}}$$

$$\exp^{32}\left(\frac{1}{48} + \frac{1}{48}\left(-1 + 2\sqrt{3} \pi \coth\left(2\sqrt{3} \pi\right)\right)\right) - 29 + \frac{1}{\phi} = -29 + \exp\left(\frac{2}{3} + \frac{2}{3}\left(-1 + 12\pi^2 \sum_{k=-\infty}^{\infty} \frac{1}{(12 + k^2)\pi^2}\right)\right) + \frac{1}{\phi}$$

$$\begin{split} \exp^{32} & \Big( \frac{1}{48} + \frac{1}{48} \left( -1 + 2\sqrt{3} \pi \coth \left( 2\sqrt{3} \pi \right) \right) \Big) - 29 + \frac{1}{\phi} = \\ & - \frac{59}{2} + \frac{\sqrt{5}}{2} + e^{8\pi \sum_{k=-\infty}^{\infty} 1/(12\pi + k^2\pi)} \end{split}$$

## **Integral representation:**

$$\begin{split} \exp^{32} & \left( \frac{1}{48} + \frac{1}{48} \left( -1 + 2\sqrt{3} \pi \coth \left( 2\sqrt{3} \pi \right) \right) \right) - 29 + \frac{1}{\phi} = \\ & -29 + \exp \left( \frac{2}{3} + \frac{2}{3} \left( -1 - 2\sqrt{3} \pi \int_{\frac{i\pi}{2}}^{2\sqrt{3}\pi} \operatorname{csch}^{2}(t) dt \right) \right) + \frac{1}{\phi} \end{split}$$

 $1/10((((((\exp(((1/48 + 1/48 (-1 + 2 \operatorname{sqrt}(3) \pi \operatorname{coth}(2 \operatorname{sqrt}(3) \pi))))))))^32-29+1/\operatorname{golden} \operatorname{ratio})))+1/\operatorname{golden} \operatorname{ratio}))$ 

Where 10 is the numbers of dimensions in superstring theory and 29 is a Lucas number

**Input:** 

$$\frac{1}{10} \left( \exp^{32} \left( \frac{1}{48} + \frac{1}{48} \left( -1 + 2\sqrt{3} \pi \coth \left( 2\sqrt{3} \pi \right) \right) \right) - 29 + \frac{1}{\phi} \right) + \frac{1}{\phi}$$

#### **Exact result:**

$$\frac{1}{\phi} + \frac{1}{10} \left( \frac{1}{\phi} - 29 + e^{2/3 + 2/3 \left( 2\sqrt{3} \pi \coth\left( 2\sqrt{3} \pi\right) - 1 \right)} \right)$$

### **Decimal approximation:**

139.3240845589052838739536978424046073583579795941793361436...

139.32408455.... result practically equal to the rest mass of Pion meson 139.57

Alternate forms:  

$$\frac{11}{10 \phi} - \frac{29}{10} + \frac{1}{10} e^{\left(4\pi \coth\left(2\sqrt{3} \pi\right)\right)/\sqrt{3}}$$

$$\frac{1}{20} \left(-69 + 11\sqrt{5} + 2 e^{\left(4\pi \coth\left(2\sqrt{3} \pi\right)\right)/\sqrt{3}}\right)$$

$$-\frac{29}{10} + \frac{11}{5(1+\sqrt{5})} + \frac{1}{10} e^{\left(4\pi \coth\left(2\sqrt{3} \pi\right)\right)/\sqrt{3}}$$

Expanded form: 
$$\frac{11}{10 \phi} - \frac{29}{10} + \frac{1}{10} e^{2/3 + 2/3 (2\sqrt{3} \pi \coth(2\sqrt{3} \pi) - 1)}$$

## Alternative representations:

$$\frac{1}{\phi} + \frac{1}{10} \left( -29 + \frac{1}{\phi} + \exp^{32} \left( \frac{1}{48} + \frac{1}{48} \left( -1 - 2i\pi \cot \left( -2i\pi \sqrt{3} \right) \sqrt{3} \right) \right) \right)$$

$$\frac{1}{10} \left( \exp^{32} \left( \frac{1}{48} + \frac{1}{48} \left( -1 + 2\sqrt{3}\pi \coth \left( 2\sqrt{3}\pi \right) \right) \right) - 29 + \frac{1}{\phi} \right) + \frac{1}{\phi} =$$

$$\frac{1}{\phi} + \frac{1}{10} \left( -29 + \frac{1}{\phi} + \exp^{32} \left( \frac{1}{48} + \frac{1}{48} \left( -1 + 2\pi \left( 1 + \frac{2}{-1 + e^{4\pi \sqrt{3}}} \right) \sqrt{3} \right) \right) \right)$$

 $\frac{1}{10} \left( \exp^{32} \left( \frac{1}{48} + \frac{1}{48} \left( -1 + 2\sqrt{3} \pi \coth \left( 2\sqrt{3} \pi \right) \right) \right) - 29 + \frac{1}{4} \right) + \frac{1}{4} =$ 

$$\frac{1}{10} \left( \exp^{32} \left( \frac{1}{48} + \frac{1}{48} \left( -1 + 2\sqrt{3} \pi \coth \left( 2\sqrt{3} \pi \right) \right) \right) - 29 + \frac{1}{\phi} \right) + \frac{1}{\phi} =$$

$$\frac{1}{\phi} + \frac{1}{10} \left( -29 + \frac{1}{\phi} + \exp^{32} \left( \frac{1}{48} + \frac{1}{48} \left( -1 + \frac{2\pi \left( e^{-2\pi\sqrt{3}} + e^{2\pi\sqrt{3}} \right) \sqrt{3}}{-e^{-2\pi\sqrt{3}} + e^{2\pi\sqrt{3}}} \right) \right) \right)$$

### Series representations:

$$\frac{1}{10} \left( \exp^{32} \left( \frac{1}{48} + \frac{1}{48} \left( -1 + 2\sqrt{3} \pi \coth \left( 2\sqrt{3} \pi \right) \right) \right) - 29 + \frac{1}{\phi} \right) + \frac{1}{\phi} = \frac{-7 - 29\sqrt{5} + \left( 1 + \sqrt{5} \right) e^{2/3 + 16 \times \sum_{k=1}^{\infty} 1/(12 + k^2)}}{10\left( 1 + \sqrt{5} \right)}$$

$$\begin{split} &\frac{1}{10} \left( \exp^{32} \! \left( \frac{1}{48} + \frac{1}{48} \left( -1 + 2\sqrt{3} \pi \coth \! \left( 2\sqrt{3} \pi \right) \right) \right) - 29 + \frac{1}{\phi} \right) + \frac{1}{\phi} = \\ &- \frac{69}{20} + \frac{11}{4\sqrt{5}} + \frac{1}{10} e^{8\pi \sum_{k=-\infty}^{\infty} 1/(12\pi + k^2\pi)} \end{split}$$

$$\begin{split} &\frac{1}{10} \left( \exp^{32} \! \left( \frac{1}{48} + \frac{1}{48} \left( -1 + 2\sqrt{3} \pi \coth \! \left( 2\sqrt{3} \pi \right) \right) \right) - 29 + \frac{1}{\phi} \right) + \frac{1}{\phi} = \\ &- \frac{29}{10} + \frac{1}{10} \exp \! \left( \frac{2}{3} + \frac{2}{3} \left( -1 + 12\pi^2 \sum_{k=-\infty}^{\infty} \frac{1}{\left( 12 + k^2 \right) \pi^2} \right) \right) + \frac{11}{10 \phi} \end{split}$$

## **Integral representation:**

$$\begin{split} &\frac{1}{10} \left( \exp^{32} \! \left( \frac{1}{48} + \frac{1}{48} \left( -1 + 2\sqrt{3} \pi \coth \! \left( 2\sqrt{3} \pi \right) \right) \right) - 29 + \frac{1}{\phi} \right) + \frac{1}{\phi} = \\ &- \frac{29}{10} + \frac{1}{10} \exp \! \left( \frac{2}{3} + \frac{2}{3} \left( -1 - 2\sqrt{3} \pi \int_{\frac{i\pi}{2}}^{2\sqrt{3}\pi} \operatorname{csch}^2(t) \, dt \right) \right) + \frac{11}{10 \phi} \end{split}$$

Or:

$$\frac{1/(12*2^2) + }{1/2*(1/(1^2+3*2^2)+1/(2^2+3*2^2)+1/(3^2+3*2^2)+1/(4^2+3*2^2)+1/(5^2+3*2^2)}{+1/(6^2+3*2^2)+1/(7^2+3*2^2))}$$

Input:

$$\frac{1}{12 \times 2^{2}} + \frac{1}{2} \left( \frac{1}{1^{2} + 3 \times 2^{2}} + \frac{1}{2^{2} + 3 \times 2^{2}} + \frac{1}{12^{2} + 3 \times$$

## **Exact result:**

## **Decimal approximation:**

0.164338439953194051554707292412210444997330243231882576144...

0.164338439953194...

$$1/(48) +$$

 $\frac{1/2*(1/(64+12)+1/(81+12)+1/(100+12)+1/(121+12)+1/(144+12)+1/(169+12)+1/(196+12)+1/(225+12)+1/(256+12)+1/(289+12)+1/(324+12)+1/(361+12)+1/(400+12)+1/(441+12)+1/(496)+1/(541))}{441+12)+1/(496)+1/(541))}$ 

**Input:** 

$$\frac{1}{48} + \frac{1}{2} \left( \frac{1}{64 + 12} + \frac{1}{81 + 12} + \frac{1}{100 + 12} + \frac{1}{121 + 12} + \frac{1}{144 + 12} + \frac{1}{144 + 12} + \frac{1}{169 + 12} + \frac{1}{196 + 12} + \frac{1}{225 + 12} + \frac{1}{256 + 12} + \frac{1}{289 + 12} + \frac{1}{324 + 12} + \frac{1}{361 + 12} + \frac{1}{400 + 12} + \frac{1}{441 + 12} + \frac{1}{496} + \frac{1}{541} \right)$$

#### **Exact result:**

2950 867 038 919 393 320 551 47519 195 324 227 082 625 936

#### **Decimal approximation:**

0.062098421885837989402622956253925345596783182155262208075...

0.06209842...

$$0.164338439 + 1/(48) +$$

 $\frac{1}{2}(1/(64+12)+1/(81+12)+1/(100+12)+1/(121+12)+1/(144+12)+1/(169+12)+1/(196+12)+1/(225+12)+1/(256+12)+1/(289+12)+1/(324+12)+1/(361+12)+1/(400+12)+1/(441+12)+1/(496)+1/(541))}{441+12)+1/(496)+1/(541)}$ 

# **Input interpretation:**

$$0.164338439 + \frac{1}{48} + \frac{1}{2} \left( \frac{1}{64 + 12} + \frac{1}{81 + 12} + \frac{1}{100 + 12} + \frac{1}{121 + 12} + \frac{1}{144 + 12} + \frac{1}{169 + 12} + \frac{1}{196 + 12} + \frac{1}{225 + 12} + \frac{1}{256 + 12} + \frac{1}{289 + 12} + \frac{1}{324 + 12} + \frac{1}{361 + 12} + \frac{1}{400 + 12} + \frac{1}{441 + 12} + \frac{1}{496} + \frac{1}{541} \right)$$

#### **Result:**

0.226436860885837989402622956253925345596783182155262208075...

0.22643686...

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$$\frac{15}{e^{4\pi}} + \frac{2^{5}}{e^{4\pi}} + \frac{3^{5}}{e^{6\pi}} + \frac{4^{5}}{e^{7}} + 4c = \frac{1}{304}$$

$$\frac{17}{e^{4\pi}} + \frac{17}{e^{5\pi}} + \frac{17}{e^{6\pi}} + \frac{49}{e^{6\pi}} + 4c = \frac{1}{364}$$

$$\frac{19}{100} + \frac{19}{e^{4\pi}} + \frac{2^{13}}{e^{6\pi}} + \frac{3^{13}}{e^{6\pi}} + \frac{4^{13}}{e^{8\pi}} + 4c = \frac{1}{34}$$

$$1^5/(e^(2Pi)-1)+2^5/(e^(4Pi)-1)+3^5/(e^(6Pi)-1)+4^5/(e^(8Pi)-1)$$

Input: 
$$\frac{1^5}{e^{2\pi}-1} + \frac{2^5}{e^{4\pi}-1} + \frac{3^5}{e^{6\pi}-1} + \frac{4^5}{e^{8\pi}-1}$$

# **Decimal approximation:**

0.001984126912823947830626260402638897891829286043471033054...

0.001984126912...

Property: 
$$\frac{1}{-1 + e^{2\pi}} + \frac{32}{-1 + e^{4\pi}} + \frac{243}{-1 + e^{6\pi}} + \frac{1024}{-1 + e^{8\pi}}$$
 is a transcendental number

Alternate forms: 
$$\frac{1}{2} \left( -1 + \frac{64}{e^{4\pi} - 1} + \frac{486}{e^{6\pi} - 1} + \frac{2048}{e^{8\pi} - 1} + \coth(\pi) \right)$$

$$\frac{177}{e^{\pi} - 1} - \frac{177}{1 + e^{\pi}} - \frac{272}{1 + e^{2\pi}} + \frac{81(e^{\pi} - 2)}{2(1 - e^{\pi} + e^{2\pi})} - \frac{81(2 + e^{\pi})}{2(1 + e^{\pi} + e^{2\pi})} - \frac{512}{1 + e^{4\pi}}$$

$$\frac{1300 + 1301 e^{2\pi} + 1334 e^{4\pi} + 278 e^{6\pi} + 34 e^{8\pi} + e^{10\pi}}{(e^{\pi} - 1)(1 + e^{\pi})(1 + e^{2\pi})(1 - e^{\pi} + e^{2\pi})(1 + e^{\pi} + e^{2\pi})(1 + e^{4\pi})}$$

coth(x) is the hyperbolic cotangent function

$$\frac{1^{5}}{e^{2\pi} - 1} + \frac{2^{5}}{e^{4\pi} - 1} + \frac{3^{5}}{e^{6\pi} - 1} + \frac{4^{5}}{e^{8\pi} - 1} = \frac{1^{5}}{-1 + e^{360^{\circ}}} + \frac{2^{5}}{-1 + e^{720^{\circ}}} + \frac{3^{5}}{-1 + e^{1080^{\circ}}} + \frac{4^{5}}{-1 + e^{1440^{\circ}}}$$

$$\frac{1^{5}}{e^{2\pi} - 1} + \frac{2^{5}}{e^{4\pi} - 1} + \frac{3^{5}}{e^{6\pi} - 1} + \frac{4^{5}}{e^{8\pi} - 1} = \frac{4^{5}}{-1 + e^{-8i\log(-1)}} + \frac{3^{5}}{-1 + e^{-6i\log(-1)}} + \frac{2^{5}}{-1 + e^{-4i\log(-1)}} + \frac{1^{5}}{-1 + e^{-2i\log(-1)}}$$

$$\frac{1^{5}}{e^{2\pi} - 1} + \frac{2^{5}}{e^{4\pi} - 1} + \frac{3^{5}}{e^{6\pi} - 1} + \frac{4^{5}}{e^{8\pi} - 1} = \frac{1^{5}}{e^{2\pi} - 1} + \frac{2^{5}}{e^{4\pi} - 1} + \frac{3^{5}}{e^{6\pi} - 1} + \frac{4^{5}}{e^{8\pi} - 1} = \frac{1^{5}}{\exp^{2\pi}(z) - 1} + \frac{1^{5}}{\exp^{2\pi}(z) - 1} + \frac{1^{5}}{\exp^{4\pi}(z) - 1} + \frac{1^{5}}{\exp^{6\pi}(z) - 1} + \frac{1^{5}}{\exp^{8\pi}(z) - 1}$$
 for  $z = 1$ 

Series representations: 
$$\frac{1^{5}}{e^{2\pi}-1} + \frac{2^{5}}{e^{4\pi}-1} + \frac{3^{5}}{e^{6\pi}-1} + \frac{4^{5}}{e^{8\pi}-1} = \frac{1}{-1+e^{8\sum_{k=0}^{\infty}(-1)^{k}/(1+2k)}} + \frac{32}{-1+e^{16\sum_{k=0}^{\infty}(-1)^{k}/(1+2k)}} + \frac{1024}{-1+e^{24\sum_{k=0}^{\infty}(-1)^{k}/(1+2k)}} + \frac{1024}{-1+e^{32\sum_{k=0}^{\infty}(-1)^{k}/(1+2k)}} + \frac{1024}{-1+e^{32\sum_{k=0}^{\infty}(-1)^{k}/(1+2k)}} + \frac{1024}{-1+e^{32\sum_{k=0}^{\infty}(-1)^{k}/(1+2k)}} + \frac{1024}{-1+e^{32\sum_{k=0}^{\infty}(-1)^{k}/(1+2k)}} + \frac{35}{e^{6\pi}-1} + \frac{4^{5}}{e^{8\pi}-1} = \frac{32}{-1+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{16\sum_{k=0}^{\infty}(-1)^{k}/(1+2k)}} + \frac{1024}{-1+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{24\sum_{k=0}^{\infty}(-1)^{k}/(1+2k)}} + \frac{1024}{-1+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{32\sum_{k=0}^{\infty}(-1)^{k}/(1+2k)}} + \frac{1}{-1+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{32\sum_{k=0}^{\infty}(-1)^{k}/(1+2k)}} + \frac{35}{e^{2\pi}-1} + \frac{4^{5}}{e^{8\pi}-1} = \frac{32}{-1+\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^{k}}{k!}}\right)^{8\sum_{k=0}^{\infty}(-1)^{k}/(1+2k)}} + \frac{35}{-1+\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^{k}}{k!}}\right)^{32\sum_{k=0}^{\infty}(-1)^{k}/(1+2k)}} + \frac{32}{-1+\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^{k}}{k!}}\right)^{32\sum_{k=0}^{\infty}(-1)^{k}/(1+2k)}} + \frac{32}{-1+\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^{k}}{k!}}\right)^{32\sum_{k=0}^{\infty}\frac{(-1)^{k}/(1+2k)}}{k!}} + \frac{32}{-1+\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^{k}}{k!}}\right)^{32\sum_{k=0}^{\infty}\frac{(-1)^{k}/(1+2k)}}{k!}} + \frac{32}{-1+\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^{k}}{k!}}\right)^{32\sum_$$

# **Integral representations:**

$$\frac{1^{5}}{e^{2\pi}-1} + \frac{2^{5}}{e^{4\pi}-1} + \frac{3^{5}}{e^{6\pi}-1} + \frac{4^{5}}{e^{8\pi}-1} = \frac{1}{-1 + e^{4} \int_{0}^{\infty} 1/(1+t^{2})dt} + \frac{32}{-1 + e^{8} \int_{0}^{\infty} 1/(1+t^{2})dt} + \frac{243}{-1 + e^{15} \int_{0}^{\infty} 1/(1+t^{2})dt} + \frac{1024}{-1 + e^{16} \int_{0}^{\infty} 1/(1+t^{2})dt}$$

$$\frac{1^{5}}{e^{2\pi}-1} + \frac{2^{5}}{e^{4\pi}-1} + \frac{3^{5}}{e^{6\pi}-1} + \frac{4^{5}}{e^{8\pi}-1} = \frac{243}{-1 + e^{4} \int_{0}^{\infty} \sin(t)/t \, dt} + \frac{1024}{-1 + e^{8} \int_{0}^{\infty} \sin(t)/t \, dt} + \frac{1024}{-1 + e^{12} \int_{0}^{\infty} \sin(t)/t \, dt} + \frac{1024}{-1 + e^{16} \int_{0}^{\infty} \sin(t)/t \, dt}$$

$$\frac{1^{5}}{e^{2\pi}-1} + \frac{2^{5}}{e^{4\pi}-1} + \frac{3^{5}}{e^{6\pi}-1} + \frac{4^{5}}{e^{8\pi}-1} = \frac{243}{1} + \frac{1024}{1 + e^{8\int_{0}^{1}\sqrt{1-t^{2}}\ dt}} + \frac{1024}{1 + e^{16\int_{0}^{1}\sqrt{1-t^{2}}\ dt}} + \frac{1024}{1 + e^{24\int_{0}^{1}\sqrt{1-t^{2}}\ dt}} + \frac{1024}{1 + e^{32\int_{0}^{1}\sqrt{1-t^{2}}\ dt}}$$

1/504

# Input:

$$\frac{1}{504}$$

# **Exact result:**

$$\frac{1}{504}$$
 (irreducible)

# **Decimal approximation:**

0.001984126984126984126984126984126984126984126984126984126984126... 0.001984126984....

$$1^9/(e^(2Pi)-1)+2^9/(e^(4Pi)-1)+3^9/(e^(6Pi)-1)+4^9/(e^(8Pi)-1)$$

Input:

$$\frac{1^9}{e^{2\pi} - 1} + \frac{2^9}{e^{4\pi} - 1} + \frac{3^9}{e^{6\pi} - 1} + \frac{4^9}{e^{8\pi} - 1}$$

#### **Decimal approximation:**

0.003787833999809716424483550438828375181491636367211553105...

0.0037878339...

Property: 
$$\frac{1}{-1 + e^{2\pi}} + \frac{512}{-1 + e^{4\pi}} + \frac{19683}{-1 + e^{6\pi}} + \frac{262144}{-1 + e^{8\pi}}$$
 is a transcendental number

#### Alternate forms:

After flate forms. 
$$\frac{512}{e^{4\pi} - 1} + \frac{1}{2} \left( -1 + \frac{39366}{e^{6\pi} - 1} + \frac{524288}{e^{8\pi} - 1} + \coth(\pi) \right)$$

$$\frac{36177}{e^{\pi} - 1} - \frac{36177}{1 + e^{\pi}} - \frac{65792}{1 + e^{2\pi}} + \frac{6561(e^{\pi} - 2)}{2(1 - e^{\pi} + e^{2\pi})} - \frac{6561(2 + e^{\pi})}{2(1 + e^{\pi} + e^{2\pi})} - \frac{131072}{1 + e^{4\pi}}$$

$$\frac{282340 + 282341 e^{2\pi} + 282854 e^{4\pi} + 20198 e^{6\pi} + 514 e^{8\pi} + e^{10\pi}}{(e^{\pi} - 1)(1 + e^{\pi})(1 + e^{2\pi})(1 - e^{\pi} + e^{2\pi})(1 + e^{\pi} + e^{2\pi})(1 + e^{4\pi})}$$

coth(x) is the hyperbolic cotangent function

#### **Alternative representations:**

$$\frac{1^{9}}{e^{2\pi} - 1} + \frac{2^{9}}{e^{4\pi} - 1} + \frac{3^{9}}{e^{6\pi} - 1} + \frac{4^{9}}{e^{8\pi} - 1} = \frac{1^{9}}{-1 + e^{360^{\circ}}} + \frac{2^{9}}{-1 + e^{720^{\circ}}} + \frac{3^{9}}{-1 + e^{1080^{\circ}}} + \frac{4^{9}}{-1 + e^{1440^{\circ}}}$$

$$\frac{1^{9}}{e^{2\pi} - 1} + \frac{2^{9}}{e^{4\pi} - 1} + \frac{3^{9}}{e^{6\pi} - 1} + \frac{4^{9}}{e^{8\pi} - 1} = \frac{4^{9}}{-1 + e^{-8i\log(-1)}} + \frac{3^{9}}{-1 + e^{-6i\log(-1)}} + \frac{2^{9}}{-1 + e^{-4i\log(-1)}} + \frac{1^{9}}{-1 + e^{-2i\log(-1)}}$$

$$\frac{1^{9}}{e^{2\pi} - 1} + \frac{2^{9}}{e^{4\pi} - 1} + \frac{3^{9}}{e^{6\pi} - 1} + \frac{4^{9}}{e^{8\pi} - 1} = \frac{1^{9}}{\exp^{2\pi}(z) - 1} + \frac{2^{9}}{\exp^{4\pi}(z) - 1} + \frac{3^{9}}{\exp^{6\pi}(z) - 1} + \frac{4^{9}}{\exp^{8\pi}(z) - 1} \text{ for } z = 1$$

$$\frac{1^9}{e^{2\pi}-1} + \frac{2^9}{e^{4\pi}-1} + \frac{3^9}{e^{6\pi}-1} + \frac{4^9}{e^{8\pi}-1} = \frac{1}{-1 + e^{8\sum_{k=0}^{\infty} (-1)^k / (1+2k)}} + \frac{512}{-1 + e^{16\sum_{k=0}^{\infty} (-1)^k / (1+2k)}} + \frac{19\,683}{-1 + e^{16\sum_{k=0}^{\infty} (-1)^k / (1+2k)}} + \frac{262\,144}{-1 + e^{24\sum_{k=0}^{\infty} (-1)^k / (1+2k)}} + \frac{11}{-1 + e^{22\sum_{k=0}^{\infty} (-1)^k / (1+2k)}} + \frac{11}{-1 + e^{22\sum_{k=0}^{\infty} (-1)^k / (1+2k)}} + \frac{11}{-1 + e^{22\sum_{k=0}^{\infty} (-1)^k / (1+2k)}} + \frac{11}{-1 + e^{2\sum_{k=0}^{\infty} (-1)^k / (1+2k)}} + \frac{11}{-1 + e^{2\sum_{k=0}^{\infty} (-1)^k / (1+2k$$

$$\frac{1^{9}}{e^{2\pi} - 1} + \frac{2^{9}}{e^{4\pi} - 1} + \frac{3^{9}}{e^{6\pi} - 1} + \frac{4^{9}}{e^{8\pi} - 1} = \frac{512}{1 + \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{8} \sum_{k=0}^{\infty} (-1)^{k} / (1+2k)} + \frac{512}{1 + \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{16} \sum_{k=0}^{\infty} (-1)^{k} / (1+2k)} + \frac{19683}{1 + \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{24} \sum_{k=0}^{\infty} (-1)^{k} / (1+2k)} + \frac{2^{9}}{1 + \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{32} \sum_{k=0}^{\infty} (-1)^{k} / (1+2k)}}{\frac{1^{9}}{e^{2\pi} - 1} + \frac{2^{9}}{e^{4\pi} - 1} + \frac{3^{9}}{e^{6\pi} - 1} + \frac{4^{9}}{e^{8\pi} - 1}} = \frac{512}{1 + \left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^{k}}{k!}}\right)^{8} \sum_{k=0}^{\infty} (-1)^{k} / (1+2k)} + \frac{512}{19683} + \frac{512} + \frac{512}{19683} + \frac{512}{19683} + \frac{512}{19683} + \frac{512}{196$$

# **Integral representations:**

$$\frac{1^{\circ}}{e^{2\pi}-1} + \frac{2^{\circ}}{e^{4\pi}-1} + \frac{3^{\circ}}{e^{6\pi}-1} + \frac{4^{\circ}}{e^{8\pi}-1} = \frac{1}{-1 + e^4 \int_0^{\infty} 1/(1+t^2)dt} + \frac{512}{-1 + e^8 \int_0^{\infty} 1/(1+t^2)dt} + \frac{19\,683}{-1 + e^{12} \int_0^{\infty} 1/(1+t^2)dt} + \frac{262\,144}{-1 + e^{16} \int_0^{\infty} 1/(1+t^2)dt}$$

$$\frac{1^9}{e^{2\pi}-1} + \frac{2^9}{e^{4\pi}-1} + \frac{3^9}{e^{6\pi}-1} + \frac{4^9}{e^{8\pi}-1} = \frac{1}{-1 + e^4 \int_0^\infty \sin(t)/t \, dt} + \frac{512}{-1 + e^8 \int_0^\infty \sin(t)/t \, dt} + \frac{19\,683}{-1 + e^{12} \int_0^\infty \sin(t)/t \, dt} + \frac{262\,144}{-1 + e^{16} \int_0^\infty \sin(t)/t \, dt}$$

$$\frac{1^{9}}{e^{2\pi} - 1} + \frac{2^{9}}{e^{4\pi} - 1} + \frac{3^{9}}{e^{6\pi} - 1} + \frac{4^{9}}{e^{8\pi} - 1} = \frac{1}{512} + \frac{19683}{-1 + e^{8} \int_{0}^{1} \sqrt{1 - t^{2}} dt} + \frac{19683}{-1 + e^{16} \int_{0}^{1} \sqrt{1 - t^{2}} dt} + \frac{19683}{-1 + e^{24} \int_{0}^{1} \sqrt{1 - t^{2}} dt} + \frac{262144}{-1 + e^{32} \int_{0}^{1} \sqrt{1 - t^{2}} dt}$$

1/264

# **Input:**

 $\frac{1}{264}$ 

#### **Exact result:**

#### **Decimal approximation:**

0.003787878...

$$1^{13}/(e^{(2Pi)-1})+2^{13}/(e^{(4Pi)-1})+3^{13}/(e^{(6Pi)-1})+4^{13}/(e^{(8Pi)-1})$$

**Input:** 

$$\frac{1^{13}}{e^{2\pi} - 1} + \frac{2^{13}}{e^{4\pi} - 1} + \frac{3^{13}}{e^{6\pi} - 1} + \frac{4^{13}}{e^{8\pi} - 1}$$

#### **Decimal approximation:**

0.041638381585443662182517651348977915286722918403784080981...

0.0416383815854.....

Property: 
$$\frac{1}{-1+e^{2\pi}} + \frac{8192}{-1+e^{4\pi}} + \frac{1594323}{-1+e^{6\pi}} + \frac{67108864}{-1+e^{8\pi}}$$
 is a transcendental number

Alternate forms: 
$$\frac{8192}{e^{4\pi}-1} + \frac{1594323}{e^{6\pi}-1} + \frac{67108864}{e^{8\pi}-1} + \frac{1}{2} \left( \coth(\pi) - 1 \right)$$

$$\frac{8\,656\,377}{e^{\pi}-1}-\frac{8\,656\,377}{1+e^{\pi}}-\frac{16\,781\,312}{1+e^{2\,\pi}}+\\ \frac{531\,441\,(e^{\pi}-2)}{2\left(1-e^{\pi}+e^{2\,\pi}\right)}-\frac{531\,441\,(2+e^{\pi})}{2\left(1+e^{\pi}+e^{2\,\pi}\right)}-\frac{33\,554\,432}{1+e^{4\,\pi}}$$

$$\frac{68\,711\,380+68\,711\,381\,{e^{2\,\pi}}+68\,719\,574\,{e^{4\,\pi}}+1\,602\,518\,{e^{6\,\pi}}+8194\,{e^{8\,\pi}}+{e^{10\,\pi}}}{({e^{\pi}}-1)\,(1+{e^{\pi}})\,\big(1+{e^{2\,\pi}}\big)\,\big(1-{e^{\pi}}+{e^{2\,\pi}}\big)\,\big(1+{e^{\pi}}+{e^{2\,\pi}}\big)\,\big(1+{e^{4\,\pi}}\big)}$$

coth(x) is the hyperbolic cotangent function

$$\frac{1^{13}}{e^{2\pi} - 1} + \frac{2^{13}}{e^{4\pi} - 1} + \frac{3^{13}}{e^{6\pi} - 1} + \frac{4^{13}}{e^{8\pi} - 1} = \frac{1^{13}}{-1 + e^{360^{\circ}}} + \frac{2^{13}}{-1 + e^{720^{\circ}}} + \frac{3^{13}}{-1 + e^{1080^{\circ}}} + \frac{4^{13}}{-1 + e^{1440^{\circ}}}$$

$$\begin{split} &\frac{1^{13}}{e^{2\pi}-1} + \frac{2^{13}}{e^{4\pi}-1} + \frac{3^{13}}{e^{6\pi}-1} + \frac{4^{13}}{e^{8\pi}-1} = \\ &\frac{4^{13}}{-1 + e^{-8i\log(-1)}} + \frac{3^{13}}{-1 + e^{-6i\log(-1)}} + \frac{2^{13}}{-1 + e^{-4i\log(-1)}} + \frac{1^{13}}{-1 + e^{-2i\log(-1)}} \\ &\frac{1^{13}}{e^{2\pi}-1} + \frac{2^{13}}{e^{4\pi}-1} + \frac{3^{13}}{e^{6\pi}-1} + \frac{4^{13}}{e^{8\pi}-1} = \\ &\frac{1^{13}}{\exp^{2\pi}(z)-1} + \frac{2^{13}}{\exp^{4\pi}(z)-1} + \frac{3^{13}}{\exp^{6\pi}(z)-1} + \frac{4^{13}}{\exp^{8\pi}(z)-1} \quad \text{for } z = 1 \end{split}$$

Series representations: 
$$\frac{1^{13}}{e^{2\pi}-1} + \frac{2^{13}}{e^{4\pi}-1} + \frac{3^{13}}{e^{6\pi}-1} + \frac{4^{13}}{e^{8\pi}-1} = \frac{1}{-1 + e^{8} \sum_{k=0}^{\infty} (-1)^{k}/(1+2k)} + \frac{8192}{-1 + e^{16} \sum_{k=0}^{\infty} (-1)^{k}/(1+2k)} + \frac{1594323}{-1 + e^{24} \sum_{k=0}^{\infty} (-1)^{k}/(1+2k)} + \frac{67108864}{-1 + e^{32} \sum_{k=0}^{\infty} (-1)^{k}/(1+2k)} + \frac{113}{e^{2\pi}-1} + \frac{2^{13}}{e^{4\pi}-1} + \frac{3^{13}}{e^{6\pi}-1} + \frac{4^{13}}{e^{8\pi}-1} = \frac{8192}{-1 + \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{8} \sum_{k=0}^{\infty} (-1)^{k}/(1+2k)} + \frac{1594323}{-1 + \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{24} \sum_{k=0}^{\infty} (-1)^{k}/(1+2k)} + \frac{67108864}{-1 + \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{32} \sum_{k=0}^{\infty} (-1)^{k}/(1+2k)} + \frac{113}{e^{2\pi}-1} + \frac{2^{13}}{e^{4\pi}-1} + \frac{3^{13}}{e^{6\pi}-1} + \frac{4^{13}}{e^{8\pi}-1} = \frac{8192}{-1 + \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{32} \sum_{k=0}^{\infty} (-1)^{k}/(1+2k)} + \frac{113}{e^{2\pi}-1} + \frac{2^{13}}{e^{4\pi}-1} + \frac{3^{13}}{e^{6\pi}-1} + \frac{4^{13}}{e^{8\pi}-1} = \frac{8192}{-1 + \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{32} \sum_{k=0}^{\infty} (-1)^{k}/(1+2k)} + \frac{1594323}{-1 + \left(\sum_{k=0}^{\infty} \frac{(-1)^{k}}{k!}\right)^{24} \sum_{k=0}^{\infty} (-1)^{k}/(1+2k)} + \frac{67108864}{-1 + \left(\sum_{k=0}^{\infty} \frac{(-1)^{k}}{k!}\right)^{24} \sum_{k=0}^{\infty} (-1)^{k}/(1+2k)} + \frac{1594323}{-1 + \left(\sum_{k=0}^{\infty} \frac{(-1)^{k}}{k!}\right)^{24} \sum_{k=0}^{\infty} (-1)^{k}/(1+2k)} + \frac{67108864}{-1 + \left(\sum_{k=0}^{\infty} \frac{(-1)^{k}}{k!}\right)^{24} \sum_{k=0}^{\infty} (-1)^{k}/(1+2k)} + \frac{6710864}{24} \sum_{k=0}^{\infty} (-1)^{k}/(1+2k)} + \frac{6710864}{24} \sum_{k=0}^{\infty} (-1$$

# **Integral representations:**

$$\frac{1^{13}}{e^{2\pi} - 1} + \frac{2^{13}}{e^{4\pi} - 1} + \frac{3^{13}}{e^{6\pi} - 1} + \frac{4^{13}}{e^{8\pi} - 1} = \frac{1}{-1 + e^4 \int_0^\infty 1/(1 + t^2) dt} + \frac{8192}{-1 + e^8 \int_0^\infty 1/(1 + t^2) dt} + \frac{1594323}{-1 + e^{12} \int_0^\infty 1/(1 + t^2) dt} + \frac{67108864}{-1 + e^{16} \int_0^\infty 1/(1 + t^2) dt}$$

$$\frac{1^{13}}{e^{2\pi}-1} + \frac{2^{13}}{e^{4\pi}-1} + \frac{3^{13}}{e^{6\pi}-1} + \frac{4^{13}}{e^{8\pi}-1} = \\ \frac{1}{-1 + e^{4} \int_{0}^{\infty} \sin(t)/t \, dt} + \frac{8192}{-1 + e^{8} \int_{0}^{\infty} \sin(t)/t \, dt} + \frac{1594323}{-1 + e^{12} \int_{0}^{\infty} \sin(t)/t \, dt} + \frac{67108864}{-1 + e^{16} \int_{0}^{\infty} \sin(t)/t \, dt}$$

$$\frac{1^{13}}{e^{2\pi}-1} + \frac{2^{13}}{e^{4\pi}-1} + \frac{3^{13}}{e^{6\pi}-1} + \frac{4^{13}}{e^{8\pi}-1} = \\ \frac{1}{-1 + e^{8} \int_{0}^{1} \sqrt{1 - t^{2}} \ dt} + \frac{8192}{-1 + e^{16} \int_{0}^{1} \sqrt{1 - t^{2}} \ dt} + \frac{1594323}{-1 + e^{24} \int_{0}^{1} \sqrt{1 - t^{2}} \ dt} + \frac{67108864}{-1 + e^{32} \int_{0}^{1} \sqrt{1 - t^{2}} \ dt}$$

1/24

#### **Input:**

 $\frac{1}{24}$ 

#### **Exact result:**

$$\frac{1}{24}$$
 (irreducible)

# **Decimal approximation:**

0.0416666666666.....

We note that:

From:

# ${\bf SUPERSYMMETRY\ AND\ STRING\ THEORY\ -\ Beyond\ the\ Standard\ Model}\\ {\it MICHAEL\ DINE}$

University of California, Santa Cruz - First published in print format 2006 - © M. Dine 2007

First, we give a general formula for the normal ordering constant. This is related to the algebra of the energy-momentum tensor we have discussed in Section 21.4. For a left- or right-moving boson, with modes which differ from an integer by  $\eta$  (e.g. modes are  $1 - \eta$ ,  $2 - \eta$ , etc.), the contribution to the normal ordering constant is:

$$\Delta = -\frac{1}{24} + \frac{1}{4}\eta(1-\eta). \tag{22.30}$$

For fermions, the contribution is the opposite. So we can recover some familiar results. In the bosonic string, with 24 transverse degrees of freedom, we see that the normal ordering constant is -1. For the superstring, in the NS-NS sector, we have a contribution of -1/24 for each boson, and 1/24 - 1/16 for each of the eight fermions on the left (and similarly on the right). So the normal ordering constant is -1/2. For the RR sector, the normal ordering vanishes.

Thence 24 can be identified with the number of the transverse degrees of freedom in the bosonic string

We have that:

Input: 
$$\left(\frac{1^{5}}{e^{2\pi}-1}+\frac{2^{5}}{e^{4\pi}-1}+\frac{3^{5}}{e^{6\pi}-1}+\frac{4^{5}}{e^{8\pi}-1}\right)+\left(\frac{1^{9}}{e^{2\pi}-1}+\frac{2^{9}}{e^{4\pi}-1}+\frac{3^{9}}{e^{6\pi}-1}+\frac{4^{9}}{e^{8\pi}-1}\right)$$

Exact result: 
$$\frac{2}{e^{2\pi}-1} + \frac{544}{e^{4\pi}-1} + \frac{19926}{e^{6\pi}-1} + \frac{263168}{e^{8\pi}-1}$$

# **Decimal approximation:**

0.005771960912633664255109810841467273073320922410682586159...

0.0057719609126336... Partial Result

Property: 
$$\frac{2}{-1+e^{2\pi}} + \frac{544}{-1+e^{4\pi}} + \frac{19926}{-1+e^{6\pi}} + \frac{263168}{-1+e^{8\pi}}$$
 is a transcendental number

 $0.0057719609126336642551098 + 1^13/(e^(2Pi)-1) + 2^13/(e^(4Pi)-1) + 3^13/(e^(6Pi)-1) + 4^13/(e^(8Pi)-1)$ 

**Input interpretation:** 

$$0.0057719609126336642551098 + \frac{1^{13}}{e^{2\pi} - 1} + \frac{2^{13}}{e^{4\pi} - 1} + \frac{3^{13}}{e^{6\pi} - 1} + \frac{4^{13}}{e^{8\pi} - 1}$$

#### **Result:**

0.0474103424980773264376275...

0.047410342498.....

# **Alternative representations:**

$$0.00577196091263366425510980000 + \frac{1^{13}}{e^{2\pi} - 1} + \frac{2^{13}}{e^{4\pi} - 1} + \frac{3^{13}}{e^{6\pi} - 1} + \frac{4^{13}}{e^{8\pi} - 1} = 0.00577196091263366425510980000 + \frac{1^{13}}{-1 + e^{360^{\circ}}} + \frac{2^{13}}{-1 + e^{720^{\circ}}} + \frac{3^{13}}{-1 + e^{1080^{\circ}}} + \frac{4^{13}}{-1 + e^{1440^{\circ}}}$$

$$\begin{array}{l} 0.00577196091263366425510980000 + \frac{1^{13}}{e^{2\pi}-1} + \frac{2^{13}}{e^{4\pi}-1} + \frac{3^{13}}{e^{6\pi}-1} + \frac{4^{13}}{e^{8\pi}-1} = \\ 0.00577196091263366425510980000 + \frac{4^{13}}{-1+e^{-8i\log(-1)}} + \frac{3^{13}}{-1+e^{-6i\log(-1)}} + \frac{2^{13}}{-1+e^{-4i\log(-1)}} + \frac{2^{13}}{-1+e^{-4i\log(-1)}} + \frac{1^{13}}{-1+e^{-2i\log(-1)}} \end{array}$$

$$\begin{aligned} 0.00577196091263366425510980000 + \frac{1^{13}}{e^{2\pi}-1} + \frac{2^{13}}{e^{4\pi}-1} + \frac{3^{13}}{e^{6\pi}-1} + \frac{4^{13}}{e^{8\pi}-1} = \\ 0.00577196091263366425510980000 + \frac{1^{13}}{\exp^{2\pi}(z)-1} + \\ \frac{2^{13}}{\exp^{4\pi}(z)-1} + \frac{3^{13}}{\exp^{6\pi}(z)-1} + \frac{4^{13}}{\exp^{8\pi}(z)-1} & \text{for } z=1 \end{aligned}$$

$$\begin{array}{l} 0.00577196091263366425510980000 + \frac{1^{13}}{e^{2\pi}-1} + \frac{2^{13}}{e^{4\pi}-1} + \frac{3^{13}}{e^{6\pi}-1} + \frac{4^{13}}{e^{8\pi}-1} = \\ 0.00577196091263366425510980000 + \\ \hline \frac{1}{-1 + \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{8} \sum_{k=0}^{\infty} (-1)^{k} / (1+2k)} + \frac{8192}{-1 + \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{16} \sum_{k=0}^{\infty} (-1)^{k} / (1+2k)} + \\ \hline \frac{1594 323}{-1 + \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{24} \sum_{k=0}^{\infty} (-1)^{k} / (1+2k)} + \frac{67108 864}{-1 + \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{32} \sum_{k=0}^{\infty} (-1)^{k} / (1+2k)} \\ 0.00577196091263366425510980000 + \frac{1^{13}}{e^{2\pi}-1} + \frac{2^{13}}{e^{4\pi}-1} + \frac{3^{13}}{e^{6\pi}-1} + \frac{4^{13}}{e^{8\pi}-1} = \\ 0.00577196091263366425510980000 + \frac{1}{-1 + \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{8} \sum_{k=1}^{\infty} \tan^{-1}(1/F_{1+2k})} + \frac{8192}{-1 + \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{16} \sum_{k=1}^{\infty} \tan^{-1}(1/F_{1+2k})} + \\ \frac{1594 323}{-1 + \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{24} \sum_{k=1}^{\infty} \tan^{-1}(1/F_{1+2k})} + \frac{67108 864}{-1 + \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{32} \sum_{k=1}^{\infty} \tan^{-1}(1/F_{1+2k})}} \\ 0.00577196091263366425510980000 + \frac{1^{13}}{e^{2\pi}-1} + \frac{2^{13}}{e^{4\pi}-1} + \frac{3^{13}}{e^{6\pi}-1} + \frac{4^{13}}{e^{8\pi}-1} = \\ 0.00577196091263366425510980000 + \frac{1^{13}}{e^{2\pi}-1} + \frac{2^{13}}{e^{4\pi}-1} + \frac{3^{13}}{e^{6\pi}-1} + \frac{4^{13}}{e^{8\pi}-1} = \\ 0.00577196091263366425510980000 + \frac{1^{13}}{e^{2\pi}-1} + \frac{2^{13}}{e^{4\pi}-1} + \frac{3^{13}}{e^{6\pi}-1} + \frac{4^{13}}{e^{8\pi}-1} = \\ 0.00577196091263366425510980000 + \frac{1^{13}}{e^{2\pi}-1} + \frac{2^{13}}{e^{4\pi}-1} + \frac{3^{13}}{e^{6\pi}-1} + \frac{4^{13}}{e^{8\pi}-1} = \\ 0.00577196091263366425510980000 + \frac{1^{13}}{e^{2\pi}-1} + \frac{2^{13}}{e^{4\pi}-1} + \frac{3^{13}}{e^{6\pi}-1} + \frac{4^{13}}{e^{8\pi}-1} = \\ 0.00577196091263366425510980000 + \frac{1^{13}}{e^{2\pi}-1} + \frac{2^{13}}{e^{4\pi}-1} + \frac{3^{13}}{e^{6\pi}-1} + \frac{4^{13}}{e^{8\pi}-1} = \\ 0.00577196091263366425510980000 + \frac{1^{13}}{e^{2\pi}-1} + \frac{2^{13}}{e^{4\pi}-1} + \frac{3^{13}}{e^{6\pi}-1} + \frac{4^{13}}{e^{8\pi}-1} = \\ 0.00577196091263366425510980000 + \frac{1^{13}}{e^{2\pi}-1} + \frac{2^{13}}{e^{4\pi}-1} + \frac{3^{13}}{e^{4\pi}-1} + \frac{4^{13}}{e^{4\pi}-1} + \frac{4^{13}}{e^{4\pi}-1} + \frac{4^{13}}{e^{4\pi}-1} + \frac{4^{13}}{e^{4\pi}-1} + \frac{4^{13}}{e^{4\pi}-1} + \frac{4^{13}}{e^$$

$$\frac{0.00577196091263366425510980000 + \frac{1}{e^{2\pi} - 1} + \frac{1}{e^{4\pi} - 1} + \frac{1}{e^{6\pi} - 1} + \frac{1}{e^{8\pi} - 1} = 0.00577196091263366425510980000 + \frac{1}{e^{2\pi} - 1} + \frac{8192}{e^{4\pi} - 1} + \frac{1}{e^{6\pi} - 1} + \frac{1}{e^{8\pi} - 1} = 0.00577196091263366425510980000 + \frac{1}{e^{2\pi} - 1} + \frac{1}{e^{4\pi} - 1} + \frac{1}{e^{6\pi} - 1} + \frac{1}{e^{8\pi} - 1} = 0.00577196091263366425510980000 + \frac{1}{e^{2\pi} - 1} + \frac{1}{e^{4\pi} - 1} + \frac{1}{e^{6\pi} - 1} + \frac{1}{e^{8\pi} - 1} = 0.00577196091263366425510980000 + \frac{1}{e^{2\pi} - 1} + \frac{1}{e^{4\pi} - 1} + \frac{1}{e^{6\pi} - 1} + \frac{1}{e^{8\pi} - 1} = 0.00577196091263366425510980000 + \frac{1}{e^{2\pi} - 1} + \frac{1}{e^{4\pi} - 1} + \frac{1}{e^{6\pi} - 1} + \frac{1}{e^{8\pi} - 1} = 0.00577196091263366425510980000 + \frac{1}{e^{2\pi} - 1} + \frac{1}{e^{4\pi} - 1} + \frac{1}{e^{6\pi} - 1} + \frac{1}{e^{8\pi} - 1} = 0.00577196091263366425510980000 + \frac{1}{e^{2\pi} - 1} + \frac{1}{e^{4\pi} - 1} + \frac{1}{e^{6\pi} - 1} + \frac{1}{e^{8\pi} - 1} = 0.00577196091263366425510980000 + \frac{1}{e^{2\pi} - 1} + \frac{1}{e^{4\pi} - 1} + \frac{1}{e^{6\pi} - 1} + \frac{1}{e^{6\pi} - 1} + \frac{1}{e^{8\pi} - 1} = 0.00577196091263366425510980000 + \frac{1}{e^{2\pi} - 1} + \frac{1}{e^{4\pi} - 1} + \frac{1}{e^{6\pi} - 1} + \frac{1}{e^{6\pi} - 1} + \frac{1}{e^{8\pi} - 1} = 0.00577196091263366425510980000 + \frac{1}{e^{2\pi} - 1} + \frac{1}{e^{4\pi} - 1} + \frac{1}{e^{6\pi} - 1} + \frac{1}{e^{6\pi}$$

We observe that:

$$1/((((0.00577196091263 + 1^13/(e^(2Pi)-1)+2^13/(e^(4Pi)-1)+3^13/(e^(6Pi)-1)+4^13/(e^(8Pi)-1)))))$$

# Input interpretation:

$$\frac{1}{0.00577196091263 + \frac{1^{13}}{e^{2}\pi_{-1}} + \frac{2^{13}}{e^{4}\pi_{-1}} + \frac{3^{13}}{e^{6}\pi_{-1}} + \frac{4^{13}}{e^{8}\pi_{-1}}}$$

#### **Result:**

21.09244412315...

21.09244412315...

#### **Alternative representations:**

$$\frac{1}{0.005771960912630000 + \frac{1^{13}}{e^{2\pi}-1} + \frac{2^{13}}{e^{4\pi}-1} + \frac{3^{13}}{e^{6\pi}-1} + \frac{4^{13}}{e^{8\pi}-1}} = \frac{1}{0.005771960912630000 + \frac{1^{13}}{-1+e^{360}} + \frac{2^{13}}{-1+e^{720}} + \frac{3^{13}}{-1+e^{1080}} + \frac{4^{13}}{-1+e^{1440}}} = \frac{1}{0.005771960912630000 + \frac{1^{13}}{e^{2\pi}-1} + \frac{2^{13}}{e^{4\pi}-1} + \frac{3^{13}}{e^{6\pi}-1} + \frac{4^{13}}{e^{8\pi}-1}} = \frac{1}{0.005771960912630000 + \frac{1^{13}}{\exp^{2\pi}(z)-1} + \frac{2^{13}}{\exp^{2\pi}(z)-1} + \frac{3^{13}}{\exp^{6\pi}(z)-1} + \frac{4^{13}}{\exp^{6\pi}(z)-1} + \frac{1}{\exp^{8\pi}(z)-1}} = \frac{1}{0.005771960912630000 + \frac{1^{13}}{e^{2\pi}-1} + \frac{2^{13}}{e^{4\pi}-1} + \frac{3^{13}}{e^{6\pi}-1} + \frac{4^{13}}{e^{8\pi}-1}} = \frac{1}{1 / \left(0.005771960912630000 + \frac{1^{13}}{e^{2\pi}-1} + \frac{2^{13}}{e^{4\pi}-1} + \frac{3^{13}}{e^{6\pi}-1} + \frac{4^{13}}{e^{8\pi}-1} + \frac{4^{13}}{e^{8\pi}-1} + \frac{1^{13}}{e^{8\pi}-1} + \frac{3^{13}}{e^{8\pi}-1} + \frac{4^{13}}{e^{8\pi}-1} + \frac{1^{13}}{e^{8\pi}-1} + \frac{3^{13}}{e^{8\pi}-1} + \frac{4^{13}}{e^{8\pi}-1} + \frac{1^{13}}{e^{8\pi}-1} + \frac{3^{13}}{e^{8\pi}-1} + \frac{1^{13}}{e^{8\pi}-1} + \frac{1^{13}}{e^{8\pi}-1}$$

$$\begin{split} \frac{1}{0.005771960912630000 + \frac{1^{13}}{e^{2\pi} - 1} + \frac{2^{13}}{e^{4\pi} - 1} + \frac{3^{13}}{e^{6\pi} - 1} + \frac{4^{13}}{e^{8\pi} - 1}} = \\ 1 / \left( 0.005771960912630000 + \frac{1}{e^{2\pi} - 1} + \frac{8192}{e^{8\pi} - 1} + \frac{1}{e^{8\pi} - 1} + \frac$$

$$\frac{1}{0.005771960912630000 + \frac{1^{13}}{\epsilon^{2}\pi_{-1}} + \frac{2^{13}}{\epsilon^{4}\pi_{-1}} + \frac{3^{13}}{\epsilon^{6}\pi_{-1}} + \frac{4^{13}}{\epsilon^{8}\pi_{-1}}} =$$

$$\frac{1}{\left(0.005771960912630000 + \frac{1}{-1 + \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{8} \sum_{k=1}^{\infty} tan^{-1} (1/F_{1+2})^{k}} + \frac{8192}{1 + \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{16} \sum_{k=1}^{\infty} tan^{-1} (1/F_{1+2})^{k}} + \frac{67108864}{1 + \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{24} \sum_{k=1}^{\infty} tan^{-1} (1/F_{1+2})^{k}} + \frac{1}{-1 + \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{32} \sum_{k=1}^{\infty} tan^{-1} (1/F_{1+2})^{k}} }{\frac{1}{0.005771960912630000} + \frac{1^{13}}{\epsilon^{2}\pi_{-1}} + \frac{2^{13}}{\epsilon^{4}\pi_{-1}} + \frac{3^{13}}{\epsilon^{6}\pi_{-1}} + \frac{4^{13}}{\epsilon^{8}\pi_{-1}}} =$$

$$\frac{1}{\left(0.005771960912630000 + \frac{1^{13}}{\epsilon^{2}\pi_{-1}} + \frac{2^{13}}{\epsilon^{4}\pi_{-1}} + \frac{3^{13}}{\epsilon^{6}\pi_{-1}} + \frac{4^{13}}{\epsilon^{8}\pi_{-1}}\right)^{2}} \right) }{\frac{1}{\left(1 + \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{4} \sum_{k=1}^{\infty} 4^{-k} \left(-1+3^{k}\right) \xi \left(1+k\right)} + \frac{1}{1 + \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{6} \sum_{k=1}^{\infty} 4^{-k} \left(-1+3^{k}\right) \xi \left(1+k\right)} + \frac{1}{1 + \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{6} \sum_{k=1}^{\infty} 4^{-k} \left(-1+3^{k}\right) \xi \left(1+k\right)} + \frac{1}{1 + \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{6} \sum_{k=1}^{\infty} 4^{-k} \left(-1+3^{k}\right) \xi \left(1+k\right)} + \frac{1}{1 + \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{6} \sum_{k=1}^{\infty} 4^{-k} \left(-1+3^{k}\right) \xi \left(1+k\right)} + \frac{1}{1 + \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{6} \sum_{k=1}^{\infty} 4^{-k} \left(-1+3^{k}\right) \xi \left(1+k\right)} + \frac{1}{1 + \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{8} \sum_{k=1}^{\infty} 4^{-k} \left(-1+3^{k}\right) \xi \left(1+k\right)} + \frac{1}{1 + \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{6} \sum_{k=1}^{\infty} 4^{-k} \left(-1+3^{k}\right) \xi \left(1+k\right)} + \frac{1}{1 + \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{8} \sum_{k=1}^{\infty} 4^{-k} \left(-1+3^{k}\right) \xi \left(1+k\right)} + \frac{1}{1 + \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{8} \sum_{k=1}^{\infty} 4^{-k} \left(-1+3^{k}\right) \xi \left(1+k\right)} + \frac{1}{1 + \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{8} \sum_{k=1}^{\infty} 4^{-k} \left(-1+3^{k}\right) \xi \left(1+k\right)} + \frac{1}{1 + \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{8} \sum_{k=1}^{\infty} 4^{-k} \left(-1+3^{k}\right) \xi \left(1+k\right)} + \frac{1}{1 + \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{8} \sum_{k=1}^{\infty} 4^{-k} \left(-1+3^{k}\right) \xi \left(1+k\right)} + \frac{1}{1 + \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{8} \sum_{k=1}^{\infty} 4^{-k} \left(-1+3^{k}\right) \xi \left(1+k\right)} + \frac{1}{1 + \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{8} \sum_{k=1}^{\infty} 4^{-k} \left(-1+3^{k}\right) \xi \left(1+k\right)} + \frac{1}{1 + \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right$$

# **Integral representations:**

$$\frac{1}{0.005771960912630000 + \frac{1^{13}}{e^{2\pi}-1} + \frac{2^{13}}{e^{4\pi}-1} + \frac{3^{13}}{e^{6\pi}-1} + \frac{4^{13}}{e^{8\pi}-1}} = \frac{1}{1/\left(0.005771960912630000 + \frac{1}{-1 + e^{4}} \int_{0}^{\infty} \sin(t)/t \, dt} + \frac{1}{-1 + e^{4}} \int_{0}^{\infty} \sin(t)/t \, dt} + \frac{8192}{-1 + e^{8}} \int_{0}^{\infty} \sin(t)/t \, dt} + \frac{1594323}{-1 + e^{12}} \int_{0}^{\infty} \sin(t)/t \, dt} + \frac{67108864}{-1 + e^{16}} \int_{0}^{\infty} \sin(t)/t \, dt} \right)$$

$$\frac{1}{0.005771960912630000 + \frac{1^{13}}{e^{2\pi}-1} + \frac{2^{13}}{e^{4\pi}-1} + \frac{3^{13}}{e^{6\pi}-1} + \frac{4^{13}}{e^{8\pi}-1}} = \frac{1}{1/\left(0.0057719609126300000 + \frac{1}{-1 + e^{4}} \int_{0}^{\infty} 1/(1+t^{2}) dt} + \frac{67108864}{-1 + e^{8}} \int_{0}^{\infty} 1/(1+t^{2}) dt} + \frac{1594323}{-1 + e^{8}} \int_{0}^{\infty} 1/(1+t^{2}) dt} + \frac{67108864}{-1 + e^{16}} \int_{0}^{\infty} 1/(1+t^{2}) dt} \right)$$

$$\begin{split} \frac{1}{0.005771960912630000 + \frac{1^{13}}{e^{2\pi} - 1} + \frac{2^{13}}{e^{4\pi} - 1} + \frac{3^{13}}{e^{6\pi} - 1} + \frac{4^{13}}{e^{8\pi} - 1}} = \\ 1 / \left[ 0.005771960912630000 + \frac{1}{-1 + e^{8} \int_{0}^{1} \sqrt{1 - t^{2}} \ dt} + \frac{8192}{-1 + e^{16} \int_{0}^{1} \sqrt{1 - t^{2}} \ dt} + \frac{1594323}{-1 + e^{24} \int_{0}^{1} \sqrt{1 - t^{2}} \ dt} + \frac{67108864}{-1 + e^{32} \int_{0}^{1} \sqrt{1 - t^{2}} \ dt} \right] \end{split}$$

$$6/((((0.00577196091263 + 1^13/(e^(2Pi)-1)+2^13/(e^(4Pi)-1)+3^13/(e^(6Pi)-1)+4^13/(e^(8Pi)-1)))))$$
-golden ratio

Input interpretation:  

$$\frac{6}{0.00577196091263 + \frac{1^{13}}{e^2\pi_{-1}} + \frac{2^{13}}{e^4\pi_{-1}} + \frac{3^{13}}{e^6\pi_{-1}} + \frac{4^{13}}{e^8\pi_{-1}}} - \phi$$

ø is the golden ratio

#### **Result:**

124.9366307501...

124.936630.... result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T = 0 and to the Higgs boson mass 125.18

$$\frac{6}{0.005771960912630000 + \frac{1^{13}}{e^{2\pi}-1} + \frac{2^{13}}{e^{4\pi}-1} + \frac{3^{13}}{e^{6\pi}-1} + \frac{4^{13}}{e^{8\pi}-1}} - \phi = \\ -2\cos\left(\frac{\pi}{5}\right) + \frac{6}{0.005771960912630000 + \frac{1^{13}}{-1+e^{360}^{\circ}} + \frac{2^{13}}{-1+e^{720}^{\circ}} + \frac{3^{13}}{-1+e^{1080}^{\circ}} + \frac{4^{13}}{-1+e^{1440}^{\circ}}}$$

$$\frac{6}{0.005771960912630000 + \frac{1^{13}}{e^{2\pi} - 1} + \frac{2^{13}}{e^{4\pi} - 1} + \frac{3^{13}}{e^{6\pi} - 1} + \frac{4^{13}}{e^{8\pi} - 1}} - \phi = \\
-\phi + 6 \left/ 0.005771960912630000 + \frac{1}{-1 + \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^8 \sum_{k=0}^{\infty} (-1)^k / (1+2k)} + \frac{8192}{-1 + \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{16} \sum_{k=0}^{\infty} (-1)^k / (1+2k)} + \frac{1594323}{-1 + \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{24} \sum_{k=0}^{\infty} (-1)^k / (1+2k)} + \frac{67108864}{-1 + \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{32} \sum_{k=0}^{\infty} (-1)^k / (1+2k)} \right)} \\
\frac{6}{0.005771960912630000 + \frac{1^{13}}{e^{2\pi} - 1} + \frac{2^{13}}{e^{4\pi} - 1} + \frac{3^{13}}{e^{6\pi} - 1} + \frac{4^{13}}{e^{8\pi} - 1}} - \phi = \\
-\phi + 6 \left/ 0.005771960912630000 + \frac{1}{e^{2\pi} - 1} + \frac{2^{13}}{e^{4\pi} - 1} + \frac{3^{13}}{e^{6\pi} - 1} + \frac{4^{13}}{e^{8\pi} - 1} - \phi = \\
-\frac{1}{-1 + \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^8} \sum_{k=1}^{\infty} \tanh^{-1} (1/F_{1+2k})} + \frac{8192}{-1 + \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{16} \sum_{k=1}^{\infty} \tanh^{-1} (1/F_{1+2k})} + \\
\frac{1594323}{-1 + \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{24} \sum_{k=1}^{\infty} \tanh^{-1} (1/F_{1+2k})}} + \frac{67108864}{-1 + \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{32} \sum_{k=1}^{\infty} \tanh^{-1} (1/F_{1+2k})}}$$

$$\frac{6}{0.005771960912630000 + \frac{1^{13}}{e^{2\pi}-1} + \frac{2^{13}}{e^{4\pi}-1} + \frac{3^{13}}{e^{6\pi}-1} + \frac{4^{13}}{e^{8\pi}-1}} - \phi = \\ -\phi + 6 / \left( 0.005771960912630000 + \frac{1}{-1 + \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{2 \times \sum_{k=1}^{\infty} 4^{-k} \left(-1+3^{k}\right) \xi(1+k)}} + \frac{8192}{-1 + \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{4 \times \sum_{k=1}^{\infty} 4^{-k} \left(-1+3^{k}\right) \xi(1+k)}} + \frac{1594323}{-1 + \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{6 \times \sum_{k=1}^{\infty} 4^{-k} \left(-1+3^{k}\right) \xi(1+k)}} + \frac{67108864}{-1 + \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{8 \times \sum_{k=1}^{\infty} 4^{-k} \left(-1+3^{k}\right) \xi(1+k)}} \right)$$

#### **Integral representations:**

$$\frac{6}{0.005771960912630000 + \frac{1^{13}}{e^{2\pi}-1} + \frac{2^{13}}{e^{4\pi}-1} + \frac{3^{13}}{e^{6\pi}-1} + \frac{4^{13}}{e^{8\pi}-1}} - \phi = \\ \frac{6}{\left( 0.005771960912630000 + \frac{1}{-1 + e^{4}} \int_{0}^{\infty} \sin(t)/t \ dt} + \frac{1}{-1 + e^{4}} \int_{0}^{\infty} \sin(t)/t \ dt} + \frac{8192}{-1 + e^{15}} \int_{0}^{\infty} \sin(t)/t \ dt} + \frac{1}{-1 + e^{16}} \int_{0}^{\infty} \sin(t)/t \ dt} \right) - \phi} \\ \frac{6}{0.005771960912630000 + \frac{1^{13}}{e^{2\pi}-1} + \frac{2^{13}}{e^{4\pi}-1} + \frac{3^{13}}{e^{6\pi}-1} + \frac{4^{13}}{e^{8\pi}-1}} - \phi = \\ \frac{6}{\left( 0.005771960912630000 + \frac{1}{e^{2\pi}-1} + \frac{2^{13}}{e^{4\pi}-1} + \frac{3^{13}}{e^{6\pi}-1} + \frac{4^{13}}{e^{8\pi}-1} + \frac{67108864}{-1 + e^{16}} \int_{0}^{\infty} 1/(1+t^{2}) dt} + \frac{1}{-1 + e^{4}} \int_{0}^{\infty} 1/(1+t^{2}) dt} + \frac{67108864}{-1 + e^{16}} \int_{0}^{\infty} 1/(1+t^{2}) dt} - \phi = \\ \frac{6}{0.005771960912630000 + \frac{1^{13}}{e^{2\pi}-1} + \frac{2^{13}}{e^{4\pi}-1} + \frac{3^{13}}{e^{6\pi}-1} + \frac{4^{13}}{e^{8\pi}-1}} - \phi = \\ \frac{6}{\left( 0.005771960912630000 + \frac{1^{13}}{e^{2\pi}-1} + \frac{2^{13}}{e^{4\pi}-1} + \frac{3^{13}}{e^{6\pi}-1} + \frac{4^{13}}{e^{8\pi}-1} + \frac{1}{e^{8\pi}-1} + \frac{1}{e^{8\pi}-1}} \right) - \phi = \\ \frac{8192}{-1 + e^{16}} \int_{0}^{1} \sqrt{1-t^{2}} \ dt} + \frac{1594323}{-1 + e^{24}} \int_{0}^{1} \sqrt{1-t^{2}} \ dt} + \frac{67108864}{-1 + e^{32}} \int_{0}^{1} \sqrt{1-t^{2}} \ dt} - \phi = \\ \frac{8192}{-1 + e^{16}} \int_{0}^{1} \sqrt{1-t^{2}} \ dt} + \frac{1594323}{-1 + e^{24}} \int_{0}^{1} \sqrt{1-t^{2}} \ dt} + \frac{67108864}{-1 + e^{32}} \int_{0}^{1} \sqrt{1-t^{2}} \ dt} - \phi = \\ \frac{8192}{-1 + e^{16}} \int_{0}^{1} \sqrt{1-t^{2}} \ dt} + \frac{1594323}{-1 + e^{24}} \int_{0}^{1} \sqrt{1-t^{2}} \ dt} + \frac{67108864}{-1 + e^{32}} \int_{0}^{1} \sqrt{1-t^{2}} \ dt} - \phi = \\ \frac{8192}{-1 + e^{16}} \int_{0}^{1} \sqrt{1-t^{2}} \ dt} + \frac{1594323}{-1 + e^{24}} \int_{0}^{1} \sqrt{1-t^{2}} \ dt} + \frac{67108864}{-1 + e^{32}} \int_{0}^{1} \sqrt{1-t^{2}} \ dt} - \phi = \\ \frac{8192}{-1 + e^{16}} \int_{0}^{1} \sqrt{1-t^{2}} \ dt} + \frac{1594323}{-1 + e^{24}} \int_{0}^{1} \sqrt{1-t^{2}} \ dt} + \frac{67108864}{-1 + e^{32}} \int_{0}^{1} \sqrt{1-t^{2}} \ dt} - \phi = \\ \frac{8192}{-1 + e^{16}} \int_{0}^{1} \sqrt{1-t^{2}} \ dt} + \frac{1594323}{-1 + e^{24}} \int_{0}^{1} \sqrt{1-t^{2}} \ dt} + \frac{1}{-1 + e^{32}} \int_{0}^{1} \sqrt{1-t^{2}} \ dt} + \frac{1}{-1 + e^{32}} \int_{0}^{1} \sqrt{1-t^{2}} \ dt} + \frac{1}{-1 + e^{32}}$$

$$6/((((0.00577196091263 + 1^13/(e^(2Pi)-1)+2^13/(e^(4Pi)-1)+3^13/(e^(6Pi)-1)+4^13/(e^(8Pi)-1)))))+11+golden ratio$$

Where 11 is a Lucas number and are the number of dimensions of bulk in M-theory (hyperspace) and 6 are the extra dimensions (compactified toroidal dimensions) of the superstring theory in 10 D

Input interpretation: 
$$\frac{6}{0.00577196091263 + \frac{1^{13}}{e^2\pi_{-1}} + \frac{2^{13}}{e^4\pi_{-1}} + \frac{3^{13}}{e^6\pi_{-1}} + \frac{4^{13}}{e^8\pi_{-1}}} + 11 + \phi$$

φ is the golden ratio

#### **Result:**

139.1726987276...

139.1726987276.... result practically equal to the rest mass of Pion meson 139.57

Alternative representations: 
$$\frac{6}{0.005771960912630000 + \frac{1^{13}}{e^{2\pi}-1} + \frac{2^{13}}{e^{4\pi}-1} + \frac{3^{13}}{e^{6\pi}-1} + \frac{4^{13}}{e^{8\pi}-1}} + 11 + \phi = 11 + 2\cos\left(\frac{\pi}{5}\right) + \frac{6}{0.005771960912630000 + \frac{1^{13}}{e^{1\pi}-1} + \frac{2^{13}}{e^{4\pi}-1} + \frac{3^{13}}{e^{6\pi}-1} + \frac{4^{13}}{e^{8\pi}-1}} + 11 + \phi = \frac{6}{0.005771960912630000 + \frac{1^{13}}{e^{2\pi}-1} + \frac{2^{13}}{e^{4\pi}-1} + \frac{3^{13}}{e^{6\pi}-1} + \frac{4^{13}}{e^{8\pi}-1}} + 11 + \phi = \frac{11 + \phi}{0.005771960912630000 + \frac{1^{13}}{e^{1\pi}-1} + \frac{2^{13}}{e^{1\pi}-1} + \frac{2^{13}}{e^{1\pi}-1} + \frac{3^{13}}{e^{1\pi}-1} + \frac{4^{13}}{e^{1\pi}-1} + \frac{4^{13}}{e^{1\pi}-1} + \frac{11}{e^{1\pi}-1} + \frac{11}{e^{1\pi$$

$$\frac{6}{0.005771960912630000 + \frac{1^{13}}{e^{2\pi}-1} + \frac{2^{13}}{e^{4\pi}-1} + \frac{3^{13}}{e^{6\pi}-1} + \frac{4^{13}}{e^{8\pi}-1}} + 11 + \phi = \frac{11 + \phi + 6}{0.005771960912630000 + \frac{1}{10}} + \frac{11 + \phi + 6}{10.005771960912630000 + \frac{1}{10}} + \frac{10}{10.005771960912630000 + \frac{1}{10.005771960912630000 + \frac{1}{10.00577196091263000 + \frac$$

# **Integral representations:**

$$\frac{6}{0.005771960912630000 + \frac{1^{13}}{e^{2\pi}-1} + \frac{2^{13}}{e^{4\pi}-1} + \frac{3^{13}}{e^{8\pi}-1} + \frac{4^{13}}{e^{8\pi}-1}} + 11 + \phi = \frac{11 + 6}{0.005771960912630000 + \frac{1}{-1 + e^{4}} \int_{0}^{\infty} \sin(t)/t \, dt} + \frac{8192}{-1 + e^{8} \int_{0}^{\infty} \sin(t)/t \, dt} + \frac{1594323}{-1 + e^{12} \int_{0}^{\infty} \sin(t)/t \, dt} + \frac{67108864}{-1 + e^{16} \int_{0}^{\infty} \sin(t)/t \, dt} + \phi = \frac{6}{0.005771960912630000 + \frac{1^{13}}{e^{2\pi}-1} + \frac{2^{13}}{e^{4\pi}-1} + \frac{3^{13}}{e^{6\pi}-1} + \frac{4^{13}}{e^{8\pi}-1}} + 11 + \phi = \frac{11 + 6}{0.005771960912630000 + \frac{1}{-1 + e^{4}} \int_{0}^{\infty} 1/(1+t^{2}) dt} + \frac{67108864}{-1 + e^{16} \int_{0}^{\infty} 1/(1+t^{2}) dt} + \frac{1594323}{-1 + e^{12} \int_{0}^{\infty} 1/(1+t^{2}) dt} + \frac{67108864}{-1 + e^{16} \int_{0}^{\infty} 1/(1+t^{2}) dt} + \phi = \frac{6}{0.005771960912630000 + \frac{1^{13}}{e^{2\pi}-1} + \frac{2^{13}}{e^{4\pi}-1} + \frac{3^{13}}{e^{6\pi}-1} + \frac{4^{13}}{e^{8\pi}-1}} + 11 + \phi = \frac{6}{0.005771960912630000 + \frac{1^{13}}{e^{2\pi}-1} + \frac{2^{13}}{e^{4\pi}-1} + \frac{3^{13}}{e^{6\pi}-1} + \frac{4^{13}}{e^{8\pi}-1}} + 11 + \phi = \frac{8192}{-1 + e^{16} \int_{0}^{1} \sqrt{1-t^{2}} \, dt} + \frac{1594323}{-1 + e^{24} \int_{0}^{1} \sqrt{1-t^{2}} \, dt} + \frac{67108864}{-1 + e^{32} \int_{0}^{1} \sqrt{1-t^{2}} \, dt} + \phi$$

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For x = 0.5, we obtain:

1-0.5<sup>4</sup>\*4\*Pi(((((coth(Pi)/(1<sup>4</sup>-0.5<sup>4</sup>)+(2coth(2Pi))/(2<sup>4</sup>-0.5<sup>4</sup>)+(3coth(3Pi))/(3<sup>4</sup>-0.5<sup>4</sup>))))))

#### **Input:**

$$1 - 0.5^4 \times 4\,\pi \left( \frac{\coth(\pi)}{1^4 - 0.5^4} + \frac{2\, \coth(2\,\pi)}{2^4 - 0.5^4} + \frac{3\, \coth(3\,\pi)}{3^4 - 0.5^4} \right)$$

#### **Result:**

0.0314354...

0.0314354...

# Alternative representations:

$$1 - 0.5^{4} \times 4\pi \left( \frac{\coth(\pi)}{1^{4} - 0.5^{4}} + \frac{2\coth(2\pi)}{2^{4} - 0.5^{4}} + \frac{3\coth(3\pi)}{3^{4} - 0.5^{4}} \right) = 1 - 4\pi 0.5^{4} \left( \frac{i\cot(i\pi)}{-0.5^{4} + 1^{4}} + \frac{2i\cot(2i\pi)}{-0.5^{4} + 2^{4}} + \frac{3i\cot(3i\pi)}{-0.5^{4} + 3^{4}} \right)$$

$$\begin{aligned} 1 - 0.5^4 \times 4\pi \left( \frac{\coth(\pi)}{1^4 - 0.5^4} + \frac{2\coth(2\pi)}{2^4 - 0.5^4} + \frac{3\coth(3\pi)}{3^4 - 0.5^4} \right) = \\ 1 - 4\pi 0.5^4 \left( -\frac{i\cot(-i\pi)}{-0.5^4 + 1^4} - \frac{2i\cot(-2i\pi)}{-0.5^4 + 2^4} - \frac{3i\cot(-3i\pi)}{-0.5^4 + 3^4} \right) \end{aligned}$$

$$\begin{split} 1 - 0.5^4 \times 4\pi \left( \frac{\coth(\pi)}{1^4 - 0.5^4} + \frac{2\coth(2\pi)}{2^4 - 0.5^4} + \frac{3\coth(3\pi)}{3^4 - 0.5^4} \right) = \\ 1 - 4\pi 0.5^4 \left( \frac{1 + \frac{2}{-1 + e^{2\pi}}}{-0.5^4 + 1^4} + \frac{2\left(1 + \frac{2}{-1 + e^{4\pi}}\right)}{-0.5^4 + 2^4} + \frac{3\left(1 + \frac{2}{-1 + e^{6\pi}}\right)}{-0.5^4 + 3^4} \right) \end{split}$$

$$1 - 0.5^{4} \times 4\pi \left(\frac{\coth(\pi)}{1^{4} - 0.5^{4}} + \frac{2\coth(2\pi)}{2^{4} - 0.5^{4}} + \frac{3\coth(3\pi)}{3^{4} - 0.5^{4}}\right) = 0.714558 + \sum_{k=1}^{\infty} \left(-\frac{0.533333}{1 + k^{2}} - \frac{0.12549}{4 + k^{2}} - \frac{0.0555985}{9 + k^{2}}\right)$$

$$1 - 0.5^{4} \times 4\pi \left( \frac{\coth(\pi)}{1^{4} - 0.5^{4}} + \frac{2\coth(2\pi)}{2^{4} - 0.5^{4}} + \frac{3\coth(3\pi)}{3^{4} - 0.5^{4}} \right) = 1 + \sum_{k=-\infty}^{\infty} \frac{-10.2759 - 4.23311 \, k^{2} - 0.357211 \, k^{4}}{36 + 49 \, k^{2} + 14 \, k^{4} + k^{6}}$$

$$\begin{split} 1 - 0.5^4 \times 4\,\pi \left( \frac{\coth(\pi)}{1^4 - 0.5^4} + \frac{2\,\coth(2\,\pi)}{2^4 - 0.5^4} + \frac{3\,\coth(3\,\pi)}{3^4 - 0.5^4} \right) = \\ 0.714558 + \sum_{k = -\infty}^{\infty} \left\{ \begin{cases} -\frac{(0.357211\,i)\left(5.16175 - (4.6687\,i)k - k^2\right)}{\left(-3 + i\,k\right)\left(-2 + i\,k\right)\left(-1 + i\,k\right)k} & \text{$k \neq 0$} \\ 0 & \text{otherwise} \end{cases} \right. \end{cases}$$

#### **Integral representation:**

$$1 - 0.5^{4} \times 4\pi \left(\frac{\coth(\pi)}{1^{4} - 0.5^{4}} + \frac{2\coth(2\pi)}{2^{4} - 0.5^{4}} + \frac{3\coth(3\pi)}{3^{4} - 0.5^{4}}\right) = 1 + \int_{\frac{i\pi}{2}}^{3\pi} \frac{1}{-6 + i} \pi \left((-0.0555985 + 0.00926641 i) \operatorname{csch}^{2}(t) + (-0.12549 + 0.0313725 i) \operatorname{csch}^{2}\left(\frac{-i\pi - 4t + it}{-6 + i}\right) + (-0.533333 + 0.266667 i) \operatorname{csch}^{2}\left(\frac{-2i\pi - 2t + it}{-6 + i}\right)\right) dt$$

For x = 1/12 = 0.083..., we obtain:

## **Input:**

$$\left(1 - 0.083^4 \times 4\pi \left(\frac{\coth(\pi)}{1^4 - 0.083^4} + \frac{2\coth(2\pi)}{2^4 - 0.083^4} + \frac{3\coth(3\pi)}{3^4 - 0.083^4}\right)\right)^{16}$$

 $\coth(x)$  is the hyperbolic cotangent function

#### **Result:**

0.9889334...

0.9889334.... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

and to the dilaton value **0**. **989117352243** =  $\phi$ 

For x = 12, we obtain:

#### **Input:**

$$1 - 12^4 \times 4\pi \left( \frac{\coth(\pi)}{1^4 - 12^4} + \frac{2\coth(2\pi)}{2^4 - 12^4} + \frac{3\coth(3\pi)}{3^4 - 12^4} \right)$$

coth(x) is the hyperbolic cotangent function

#### **Exact result:**

$$1 - 82\,944\,\pi \left( -\frac{\coth(\pi)}{20\,735} - \frac{\coth(2\,\pi)}{10\,360} - \frac{\coth(3\,\pi)}{6885} \right)$$

#### **Decimal approximation:**

76.61327686396115476033877181540069163017090611360142200794...

76.6132768639...

#### **Alternate forms:**

$$\frac{1}{91\,296\,205}(91\,296\,205 + 365\,202\,432\,\pi\,\coth(\pi) + 730\,933\,632\,\pi\,\coth(2\,\pi) + 1\,099\,850\,752\,\pi\,\coth(3\,\pi))$$

$$1 + \frac{82944 \pi \coth(\pi)}{20735} + \frac{10368 \pi \coth(2\pi)}{1295} + \frac{1024}{85} \pi \coth(3\pi)$$

$$\frac{5\,370\,365 + 21\,482\,496\,\pi\,\coth(\pi) + 42\,996\,096\,\pi\,\coth(2\,\pi)}{5\,370\,365} + \frac{1024}{85}\,\pi\,\coth(3\,\pi)$$

$$1 - 12^{4} \times 4\pi \left( \frac{\coth(\pi)}{1^{4} - 12^{4}} + \frac{2\coth(2\pi)}{2^{4} - 12^{4}} + \frac{3\coth(3\pi)}{3^{4} - 12^{4}} \right) =$$

$$1 - 4\pi 12^{4} \left( \frac{i\cot(i\pi)}{1^{4} - 12^{4}} + \frac{2i\cot(2i\pi)}{2^{4} - 12^{4}} + \frac{3i\cot(3i\pi)}{3^{4} - 12^{4}} \right)$$

$$\begin{split} 1 - 12^4 \times 4\,\pi \left( \frac{\coth(\pi)}{1^4 - 12^4} + \frac{2\,\coth(2\,\pi)}{2^4 - 12^4} + \frac{3\,\coth(3\,\pi)}{3^4 - 12^4} \right) = \\ 1 - 4\,\pi\,12^4 \left( -\frac{i\,\cot(-i\,\pi)}{1^4 - 12^4} - \frac{2\,i\cot(-2\,i\,\pi)}{2^4 - 12^4} - \frac{3\,i\cot(-3\,i\,\pi)}{3^4 - 12^4} \right) \end{split}$$

$$1 - 12^{4} \times 4\pi \left( \frac{\coth(\pi)}{1^{4} - 12^{4}} + \frac{2\coth(2\pi)}{2^{4} - 12^{4}} + \frac{3\coth(3\pi)}{3^{4} - 12^{4}} \right) =$$

$$1 - 4\pi 12^{4} \left( \frac{1 + \frac{2}{-1 + e^{2\pi}}}{1^{4} - 12^{4}} + \frac{2\left(1 + \frac{2}{-1 + e^{4\pi}}\right)}{2^{4} - 12^{4}} + \frac{3\left(1 + \frac{2}{-1 + e^{6\pi}}\right)}{3^{4} - 12^{4}} \right)$$

# Series representations:

$$\begin{split} 1 - 12^4 \times 4\pi \left( \frac{\coth(\pi)}{1^4 - 12^4} + \frac{2\coth(2\pi)}{2^4 - 12^4} + \frac{3\coth(3\pi)}{3^4 - 12^4} \right) = \\ 1 + \sum_{k = -\infty}^{\infty} \frac{768 \left( 51435289 + 46698002 \, k^2 + 6675289 \, k^4 \right)}{91296205 \left( 1 + k^2 \right) \left( 4 + k^2 \right) \left( 9 + k^2 \right)} \end{split}$$

$$\begin{split} 1 - 12^4 \times 4\pi \left( \frac{\coth(\pi)}{1^4 - 12^4} + \frac{2\coth(2\pi)}{2^4 - 12^4} + \frac{3\coth(3\pi)}{3^4 - 12^4} \right) &= \\ \frac{3565747111}{273888615} + \sum_{k=1}^{\infty} \left( \frac{165888}{20735(1+k^2)} + \frac{41472}{1295(4+k^2)} + \frac{6144}{85(9+k^2)} \right) \end{split}$$

$$\begin{split} 1 - 12^4 \times 4\,\pi \left(\frac{\coth(\pi)}{1^4 - 12^4} + \frac{2\coth(2\,\pi)}{2^4 - 12^4} + \frac{3\coth(3\,\pi)}{3^4 - 12^4}\right) &= 1 + \frac{2\,195\,986\,816\,\pi}{91\,296\,205} + \\ \sum_{k=0}^{\infty} \frac{256\,e^{-6\left(1+k\right)\pi} \left(8\,592\,584 + 5\,710\,419\,e^{2\left(1+k\right)\pi} + 2\,853\,144\,e^{4\left(1+k\right)\pi}\right)\pi}{91\,296\,205} \end{split}$$

# Integral representation:

$$1 - 12^{4} \times 4\pi \left(\frac{\coth(\pi)}{1^{4} - 12^{4}} + \frac{2\coth(2\pi)}{2^{4} - 12^{4}} + \frac{3\coth(3\pi)}{3^{4} - 12^{4}}\right) =$$

$$1 + \int_{\frac{i\pi}{2}}^{3\pi} \left(-\frac{1024}{85}\pi \operatorname{csch}^{2}(t) + \left(\frac{13}{37} - \frac{4i}{37}\right)\right)$$

$$\left(-\frac{82944\pi \operatorname{csch}^{2}\left(\frac{\left(\frac{12}{37} + \frac{2i}{37}\right)\left(-i\pi^{2} - \left(1 - \frac{i}{2}\right)\pi t\right)\right)}{\pi}\right) - \left(\frac{93312}{6475} + \frac{20736i}{6475}\right)\pi$$

$$\operatorname{csch}^{2}\left(\frac{\left(\frac{4}{5} + \frac{2i}{5}\right)\left(\frac{i\pi^{2}}{2} + \left(\frac{25}{37} - \frac{2i}{37}\right)\left(-i\pi^{2} - \left(1 - \frac{i}{2}\right)\pi t\right)\right)}{\pi}\right)\right)dt$$

8/5\*(((1-12^4\*4\*Pi((((coth(Pi)/(1^4-12^4)+(2coth(2Pi))/(2^4- $12^4$ +(3coth(3Pi))/(3^4-12^4)))))))+Pi

**Input:** 

$$\frac{8}{5} \left( 1 - 12^4 \times 4\pi \left( \frac{\coth(\pi)}{1^4 - 12^4} + \frac{2 \coth(2\pi)}{2^4 - 12^4} + \frac{3 \coth(3\pi)}{3^4 - 12^4} \right) \right) + \pi$$

 $\coth(x)$  is the hyperbolic cotangent function

#### **Exact result:**

$$\pi + \frac{8}{5} \left( 1 - 82944 \, \pi \left( -\frac{\coth(\pi)}{20735} - \frac{\coth(2\,\pi)}{10360} - \frac{\coth(3\,\pi)}{6885} \right) \right)$$

#### **Decimal approximation:**

125.7228356359276408550046782879206094924706191811373810336...

125.72283563.... result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T = 0 and to the Higgs boson mass 125.18

Alternate forms: 
$$\frac{8}{5} + \pi + \frac{663552 \pi \coth(\pi)}{103675} + \frac{82944 \pi \coth(2 \pi)}{6475} + \frac{8192}{425} \pi \coth(3 \pi)$$

$$\frac{1}{456481025} (730369640 + 456481025 \pi + 2921619456 \pi \coth(\pi) + 456481025 \pi + 2921619456 \pi \coth(\pi) + 456481025 \pi + 2921619456 \pi \cot(\pi) + 456481025 \pi$$

 $5\,847\,469\,056\,\pi\,\coth(2\,\pi) + 8\,798\,806\,016\,\pi\,\coth(3\,\pi)$ 

$$\frac{8}{5} + \pi \left(1 + \frac{663552 \coth(\pi)}{103675} + \frac{82944 \coth(2\pi)}{6475} + \frac{8192}{425} \coth(3\pi)\right)$$

$$\frac{1}{5} \left(1 - 12^4 \times 4\pi \left(\frac{\coth(\pi)}{1^4 - 12^4} + \frac{2\coth(2\pi)}{2^4 - 12^4} + \frac{3\coth(3\pi)}{3^4 - 12^4}\right)\right) 8 + \pi = \pi + \frac{8}{5} \left(1 - 4\pi 12^4 \left(\frac{i\cot(i\pi)}{1^4 - 12^4} + \frac{2i\cot(2i\pi)}{2^4 - 12^4} + \frac{3i\cot(3i\pi)}{3^4 - 12^4}\right)\right)$$

$$\frac{1}{5} \left( 1 - 12^4 \times 4\pi \left( \frac{\coth(\pi)}{1^4 - 12^4} + \frac{2\coth(2\pi)}{2^4 - 12^4} + \frac{3\coth(3\pi)}{3^4 - 12^4} \right) \right) 8 + \pi = \\ \pi + \frac{8}{5} \left( 1 - 4\pi 12^4 \left( -\frac{i\cot(-i\pi)}{1^4 - 12^4} - \frac{2i\cot(-2i\pi)}{2^4 - 12^4} - \frac{3i\cot(-3i\pi)}{3^4 - 12^4} \right) \right)$$

$$\begin{split} &\frac{1}{5} \left(1 - 12^4 \times 4\,\pi \left(\frac{\coth(\pi)}{1^4 - 12^4} + \frac{2\,\coth(2\,\pi)}{2^4 - 12^4} + \frac{3\,\coth(3\,\pi)}{3^4 - 12^4}\right)\right) 8 + \pi = \\ &\pi + \frac{8}{5} \left(1 - 4\,\pi\,12^4 \left(\frac{1 + \frac{2}{-1 + \epsilon^2\pi}}{1^4 - 12^4} + \frac{2\left(1 + \frac{2}{-1 + \epsilon^4\pi}\right)}{2^4 - 12^4} + \frac{3\left(1 + \frac{2}{-1 + \epsilon^6\pi}\right)}{3^4 - 12^4}\right)\right) \end{split}$$

# Series representations:

$$\frac{1}{5} \left( 1 - 12^4 \times 4\pi \left( \frac{\coth(\pi)}{1^4 - 12^4} + \frac{2\coth(2\pi)}{2^4 - 12^4} + \frac{3\coth(3\pi)}{3^4 - 12^4} \right) \right) 8 + \pi = \frac{8}{5} + \pi + \sum_{k=-\infty}^{\infty} \frac{6144 \left( 51435289 + 46698002 \, k^2 + 6675289 \, k^4 \right)}{456481025 \left( 1 + k^2 \right) \left( 4 + k^2 \right) \left( 9 + k^2 \right)}$$

$$\frac{1}{5} \left( 1 - 12^4 \times 4\pi \left( \frac{\coth(\pi)}{1^4 - 12^4} + \frac{2\coth(2\pi)}{2^4 - 12^4} + \frac{3\coth(3\pi)}{3^4 - 12^4} \right) \right) 8 + \pi = \frac{28525976888}{1369443075} + \pi + \sum_{k=1}^{\infty} \left( \frac{1327104}{103675(1+k^2)} + \frac{331776}{6475(4+k^2)} + \frac{49152}{425(9+k^2)} \right)$$

$$\frac{1}{5} \left(1 - 12^4 \times 4\pi \left(\frac{\coth(\pi)}{1^4 - 12^4} + \frac{2\coth(2\pi)}{2^4 - 12^4} + \frac{3\coth(3\pi)}{3^4 - 12^4}\right)\right) 8 + \pi = \frac{8}{5} + \frac{18\,024\,375\,553\,\pi}{456\,481\,025} + \sum_{k=0}^{\infty} \frac{2048\,e^{-6\left(1+k\right)\pi} \left(8\,592\,584 + 5\,710\,419\,e^{2\left(1+k\right)\pi} + 2\,853\,144\,e^{4\left(1+k\right)\pi}\right)\pi}{456\,481\,025} + \frac{18\,024\,375\,553\,\pi}{456\,481\,025} + \frac{18\,024\,375\,553\,\pi}{456\,48$$

# **Integral representation:**

$$\begin{split} \frac{1}{5} \left( 1 - 12^4 \times 4 \, \pi \left( \frac{\coth(\pi)}{1^4 - 12^4} + \frac{2 \coth(2 \, \pi)}{2^4 - 12^4} + \frac{3 \coth(3 \, \pi)}{3^4 - 12^4} \right) \right) 8 + \pi &= \\ \frac{8}{5} + \pi + \int_{\frac{i \pi}{2}}^{3 \, \pi} \left( -\frac{8192}{425} \, \pi \, \operatorname{csch}^2(t) + \left( \frac{13}{37} - \frac{4 \, i}{37} \right) \right) \\ - \frac{663 \, 552 \, \pi \, \operatorname{csch}^2\left( \frac{\left( \frac{12}{37} + \frac{2 \, i}{37} \right) \left( -i \, \pi^2 - \left( 1 - \frac{i}{2} \right) \pi \, t \right)}{\pi} \right)}{103 \, 675} - \left( \frac{746 \, 496}{32 \, 375} + \frac{165 \, 888 \, i}{32 \, 375} \right) \\ \pi \, \operatorname{csch}^2\left( \frac{\left( \frac{4}{5} + \frac{2 \, i}{5} \right) \left( i \, \frac{\pi^2}{2} + \left( \frac{25}{37} - \frac{2 \, i}{37} \right) \left( -i \, \pi^2 - \left( 1 - \frac{i}{2} \right) \pi \, t \right) \right)}{\pi} \right) \right] dt \end{split}$$

Where 11 and 3 are Lucas number (furthermore 11 is also the number of dimensions of M-Theory)

**Input:** 

$$\frac{8}{5} \left( 1 - 12^4 \times 4\pi \left( \frac{\coth(\pi)}{1^4 - 12^4} + \frac{2\coth(2\pi)}{2^4 - 12^4} + \frac{3\coth(3\pi)}{3^4 - 12^4} \right) \right) + \pi + 11 + 3$$

coth(x) is the hyperbolic cotangent function

#### **Exact result:**

$$14 + \pi + \frac{8}{5} \left( 1 - 82944 \pi \left( -\frac{\coth(\pi)}{20735} - \frac{\coth(2\pi)}{10360} - \frac{\coth(3\pi)}{6885} \right) \right)$$

# **Decimal approximation:**

139.7228356359276408550046782879206094924706191811373810336...

139.72283563.... result practically equal to the rest mass of Pion meson 139.57

#### **Alternate forms:**

$$\frac{78}{5} + \pi + \frac{663552 \pi \coth(\pi)}{103675} + \frac{82944 \pi \coth(2 \pi)}{6475} + \frac{8192}{425} \pi \coth(3 \pi)$$

$$\frac{1}{456\,481\,025}(7\,121\,103\,990\,+456\,481\,025\,\pi+2\,921\,619\,456\,\pi\,\coth(\pi)\,+$$

 $5\,847\,469\,056\,\pi\,\coth(2\,\pi) + 8\,798\,806\,016\,\pi\,\coth(3\,\pi)$ 

$$\frac{78}{5} + \pi \left(1 + \frac{663552 \coth(\pi)}{103675} + \frac{82944 \coth(2\pi)}{6475} + \frac{8192}{425} \coth(3\pi)\right)$$

$$\frac{1}{5} \left( 1 - 12^4 \times 4\pi \left( \frac{\coth(\pi)}{1^4 - 12^4} + \frac{2\coth(2\pi)}{2^4 - 12^4} + \frac{3\coth(3\pi)}{3^4 - 12^4} \right) \right) 8 + \pi + 11 + 3 = 14 + \pi + \frac{8}{5} \left( 1 - 4\pi 12^4 \left( \frac{i\cot(i\pi)}{1^4 - 12^4} + \frac{2i\cot(2i\pi)}{2^4 - 12^4} + \frac{3i\cot(3i\pi)}{3^4 - 12^4} \right) \right)$$

$$\frac{1}{5} \left(1 - 12^4 \times 4\pi \left(\frac{\coth(\pi)}{1^4 - 12^4} + \frac{2\coth(2\pi)}{2^4 - 12^4} + \frac{3\coth(3\pi)}{3^4 - 12^4}\right)\right) 8 + \pi + 11 + 3 = \\ 14 + \pi + \frac{8}{5} \left(1 - 4\pi 12^4 \left(-\frac{i\cot(-i\pi)}{1^4 - 12^4} - \frac{2i\cot(-2i\pi)}{2^4 - 12^4} - \frac{3i\cot(-3i\pi)}{3^4 - 12^4}\right)\right)$$

$$\frac{1}{5} \left(1 - 12^4 \times 4\pi \left(\frac{\coth(\pi)}{1^4 - 12^4} + \frac{2\coth(2\pi)}{2^4 - 12^4} + \frac{3\coth(3\pi)}{3^4 - 12^4}\right)\right) 8 + \pi + 11 + 3 = \\ 14 + \pi + \frac{8}{5} \left(1 - 4\pi 12^4 \left(\frac{1 + \frac{2}{-1 + e^{2\pi}}}{1^4 - 12^4} + \frac{2\left(1 + \frac{2}{-1 + e^{4\pi}}\right)}{2^4 - 12^4} + \frac{3\left(1 + \frac{2}{-1 + e^{6\pi}}\right)}{3^4 - 12^4}\right)\right)$$

#### Series representations:

$$\frac{1}{5} \left( 1 - 12^4 \times 4\pi \left( \frac{\coth(\pi)}{1^4 - 12^4} + \frac{2\coth(2\pi)}{2^4 - 12^4} + \frac{3\coth(3\pi)}{3^4 - 12^4} \right) \right) 8 + \pi + 11 + 3 = \frac{78}{5} + \pi + \sum_{k = -\infty}^{\infty} \frac{6144 \left( 51435289 + 46698002 \, k^2 + 6675289 \, k^4 \right)}{456481025 \left( 1 + k^2 \right) \left( 4 + k^2 \right) \left( 9 + k^2 \right)}$$

$$\frac{1}{5} \left(1 - 12^4 \times 4\pi \left(\frac{\coth(\pi)}{1^4 - 12^4} + \frac{2\coth(2\pi)}{2^4 - 12^4} + \frac{3\coth(3\pi)}{3^4 - 12^4}\right)\right) 8 + \pi + 11 + 3 = \\ \frac{47698179938}{1369443075} + \pi + \sum_{k=1}^{\infty} \left(\frac{1327104}{103675(1+k^2)} + \frac{331776}{6475(4+k^2)} + \frac{49152}{425(9+k^2)}\right)$$

$$\frac{1}{5} \left(1 - 12^{4} \times 4\pi \left(\frac{\coth(\pi)}{1^{4} - 12^{4}} + \frac{2\coth(2\pi)}{2^{4} - 12^{4}} + \frac{3\coth(3\pi)}{3^{4} - 12^{4}}\right)\right) 8 + \pi + 11 + 3 = \frac{78}{5} + \frac{18024375553\pi}{456481025} + \sum_{k=0}^{\infty} \frac{2048 e^{-6(1+k)\pi} \left(8592584 + 5710419 e^{2(1+k)\pi} + 2853144 e^{4(1+k)\pi}\right)\pi}{456481025}$$

# **Integral representation:**

$$\begin{split} \frac{1}{5} \left( 1 - 12^4 \times 4 \, \pi \left( \frac{\coth(\pi)}{1^4 - 12^4} + \frac{2 \, \coth(2 \, \pi)}{2^4 - 12^4} + \frac{3 \, \coth(3 \, \pi)}{3^4 - 12^4} \right) \right) 8 + \pi + 11 + 3 = \\ \frac{78}{5} + \pi + \int_{\frac{i \, \pi}{2}}^{3 \, \pi} \left( -\frac{8192}{425} \, \pi \, \operatorname{csch}^2(t) + \left( \frac{13}{37} - \frac{4 \, i}{37} \right) \right) \\ - \frac{663552 \, \pi \, \operatorname{csch}^2\left( \frac{\left( \frac{12}{37} + \frac{2 \, i}{37} \right) \left( -i \, \pi^2 - \left( 1 - \frac{i}{2} \right) \pi \, t \right)}{\pi} \right)}{103675} - \left( \frac{746496}{32375} + \frac{165888 \, i}{32375} \right) \\ \pi \, \operatorname{csch}^2\left( \frac{\left( \frac{4}{5} + \frac{2 \, i}{5} \right) \left( \frac{i \, \pi^2}{2} + \left( \frac{25}{37} - \frac{2 \, i}{37} \right) \left( -i \, \pi^2 - \left( 1 - \frac{i}{2} \right) \pi \, t \right) \right)}{\pi} \right) \right] dt \end{split}$$

Now, we have that:

Cor. 
$$(\pi x)^2 \frac{\cosh \pi x \sqrt{z} + \cos \pi x \sqrt{z}}{\cosh \pi x \sqrt{z} - \cosh \pi x \sqrt{z}}$$
  

$$= 1 + 4 \pi x^4 \left\{ \frac{\cosh \pi}{1^6 + x^4} + \frac{3 \coth 2\pi}{2^4 + x^4} + \frac{3 \coth 8\pi}{3^6 + x^6} + \frac{3c}{3^6 + x^6} \right\}$$

1+12^4\*4\*Pi(((((coth(Pi)/(1^4+12^4)+(2coth(2Pi))/(2^4+12^4)+(3coth(3Pi))/(3^4+12^4))))))

#### **Input:**

$$1 + 12^4 \times 4\pi \left( \frac{\coth(\pi)}{1^4 + 12^4} + \frac{2\coth(2\pi)}{2^4 + 12^4} + \frac{3\coth(3\pi)}{3^4 + 12^4} \right)$$

coth(x) is the hyperbolic cotangent function

#### **Exact result:**

$$1 + 82944 \pi \left( \frac{\coth(\pi)}{20737} + \frac{\coth(2\pi)}{10376} + \frac{\coth(3\pi)}{6939} \right)$$

#### **Decimal approximation:**

76.27874609711877953712482478244915518016178203760089714270...

76.278746097....

#### **Alternate forms:**

$$\frac{1}{6\,912\,243\,473}(6\,912\,243\,473+27\,647\,640\,576\,\pi\,\coth(\pi)+55\,255\,312\,512\,\pi\,\coth(2\,\pi)+82\,624\,171\,008\,\pi\,\coth(3\,\pi))$$

$$1 + \frac{82\,944\,\pi\,\coth(\pi)}{20\,737} + \frac{10\,368\,\pi\,\coth(2\,\pi)}{1297} + \frac{3072}{257}\,\pi\,\coth(3\,\pi)$$

$$\frac{26\,895\,889 + 107\,578\,368\,\pi\,\coth(\pi) + 215\,001\,216\,\pi\,\coth(2\,\pi)}{26\,895\,889} + \frac{3072}{257}\,\pi\,\coth(3\,\pi)$$

$$1 + 12^{4} \times 4\pi \left( \frac{\coth(\pi)}{1^{4} + 12^{4}} + \frac{2\coth(2\pi)}{2^{4} + 12^{4}} + \frac{3\coth(3\pi)}{3^{4} + 12^{4}} \right) = 1 + 4\pi 12^{4} \left( \frac{i\cot(i\pi)}{1^{4} + 12^{4}} + \frac{2i\cot(2i\pi)}{2^{4} + 12^{4}} + \frac{3i\cot(3i\pi)}{3^{4} + 12^{4}} \right)$$

$$1 + 12^{4} \times 4\pi \left( \frac{\coth(\pi)}{1^{4} + 12^{4}} + \frac{2\coth(2\pi)}{2^{4} + 12^{4}} + \frac{3\coth(3\pi)}{3^{4} + 12^{4}} \right) =$$

$$1 + 4\pi 12^{4} \left( -\frac{i\cot(-i\pi)}{1^{4} + 12^{4}} - \frac{2i\cot(-2i\pi)}{2^{4} + 12^{4}} - \frac{3i\cot(-3i\pi)}{3^{4} + 12^{4}} \right)$$

$$\begin{split} 1 + 12^4 \times 4\,\pi \left( \frac{\coth(\pi)}{1^4 + 12^4} + \frac{2\,\coth(2\,\pi)}{2^4 + 12^4} + \frac{3\,\coth(3\,\pi)}{3^4 + 12^4} \right) = \\ 1 + 4\,\pi\,12^4 \left( \frac{1 + \frac{2}{-1 + e^{2\,\pi}}}{1^4 + 12^4} + \frac{2\left(1 + \frac{2}{-1 + e^{4\,\pi}}\right)}{2^4 + 12^4} + \frac{3\left(1 + \frac{2}{-1 + e^{6\,\pi}}\right)}{3^4 + 12^4} \right) \end{split}$$

#### Series representations:

$$\begin{aligned} 1 + 12^{4} \times 4\pi \left( \frac{\coth(\pi)}{1^{4} + 12^{4}} + \frac{2\coth(2\pi)}{2^{4} + 12^{4}} + \frac{3\coth(3\pi)}{3^{4} + 12^{4}} \right) &= \\ 1 + \sum_{k = -\infty}^{\infty} \left( \frac{82\,944}{20\,737\left(1 + k^{2}\right)} + \frac{20\,736}{1297\left(4 + k^{2}\right)} + \frac{9216}{257\left(9 + k^{2}\right)} \right) \end{aligned}$$

$$\begin{aligned} &1 + 12^{4} \times 4\,\pi \left(\frac{\coth(\pi)}{1^{4} + 12^{4}} + \frac{2\,\coth(2\,\pi)}{2^{4} + 12^{4}} + \frac{3\,\coth(3\,\pi)}{3^{4} + 12^{4}}\right) = \\ &\frac{89\,728\,930\,641}{6\,912\,243\,473} + \sum_{k=1}^{\infty} \left(\frac{165\,888}{20\,737\left(1 + k^{2}\right)} + \frac{41\,472}{1297\left(4 + k^{2}\right)} + \frac{18\,432}{257\left(9 + k^{2}\right)}\right) \end{aligned}$$

$$\begin{split} 1 + 12^4 \times 4\,\pi \left(\frac{\coth(\pi)}{1^4 + 12^4} + \frac{2\,\coth(2\,\pi)}{2^4 + 12^4} + \frac{3\,\coth(3\,\pi)}{3^4 + 12^4}\right) &= 1 + \frac{165\,527\,124\,096\,\pi}{6\,912\,243\,473} + \\ \sum_{k=0}^{\infty} \left(\frac{6144}{257}\,e^{-6\left(1+k\right)\pi}\,\pi + \frac{20\,736\,e^{-4\left(1+k\right)\pi}\,\pi}{1297} + \frac{165\,888\,e^{-2\left(1+k\right)\pi}\,\pi}{20\,737}\right) \end{split}$$

# Integral representation:

$$\begin{split} 1 + 12^4 \times 4\pi \left( \frac{\coth(\pi)}{1^4 + 12^4} + \frac{2\coth(2\pi)}{2^4 + 12^4} + \frac{3\coth(3\pi)}{3^4 + 12^4} \right) = \\ 1 + \int_{\frac{i\pi}{2}}^{3\pi} \left( -\frac{3072}{257} \pi \operatorname{csch}^2(t) + \left( \frac{13}{37} - \frac{4i}{37} \right) \right. \\ \left. \left( -\frac{82\,944 \,\pi \operatorname{csch}^2\left( \frac{\left( \frac{12}{37} + \frac{2i}{37} \right) \left( -i\,\pi^2 - \left( 1 - \frac{i}{2} \right)\pi\,t \right)}{\pi} \right)}{20\,737} - \left( \frac{93\,312}{6485} + \frac{20\,736\,i}{6485} \right) \pi \right. \\ \left. \operatorname{csch}^2\left( \frac{\left( \frac{4}{5} + \frac{2\,i}{5} \right) \left( \frac{i\,\pi^2}{2} + \left( \frac{25}{37} - \frac{2\,i}{37} \right) \left( -i\,\pi^2 - \left( 1 - \frac{i}{2} \right)\pi\,t \right)}{\pi} \right) \right] \right] dt \end{split}$$

(((((1+12^4\*4\*Pi(((((coth(Pi)/(1^4+12^4)+(2coth(2Pi))/(2^4+12^4)+(3coth(3Pi))/(3^4+12^4))))))))))+47+golden ratio

Where 47 is a Lucas number

## **Input:**

$$\left(1+12^{4}\times 4\pi\left(\frac{\coth(\pi)}{1^{4}+12^{4}}+\frac{2\coth(2\pi)}{2^{4}+12^{4}}+\frac{3\coth(3\pi)}{3^{4}+12^{4}}\right)\right)+47+\phi$$

coth(x) is the hyperbolic cotangent function

ø is the golden ratio

#### **Exact result:**

$$\phi + 48 + 82944 \pi \left( \frac{\coth(\pi)}{20737} + \frac{\coth(2\pi)}{10376} + \frac{\coth(3\pi)}{6939} \right)$$

## **Decimal approximation:**

124.8967800858686743853294116168147932978820912174066600048...

124.89678008586... result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T = 0 and to the Higgs boson mass 125.18

#### **Alternate forms:**

$$\frac{1}{13824486946} \left(670487616881 + 6912243473\sqrt{5} + 55295281152\pi \coth(\pi) + 10510625024\pi \coth(2\pi) + 165248342016\pi \coth(3\pi)\right)$$

$$\frac{97}{2} + \frac{\sqrt{5}}{2} + \frac{82\,944\,\pi\,\coth(\pi)}{20\,737} + \frac{10\,368\,\pi\,\coth(2\,\pi)}{1297} + \frac{3072}{257}\,\pi\,\coth(3\,\pi)$$

$$\frac{1}{2} \left(97 + \sqrt{5}\right) + \\ \frac{384 \pi \left(71\,999\,064\, \text{coth}(\pi) + 143\,894\,043\, \text{coth}(2\,\pi) + 215\,167\,112\, \text{coth}(3\,\pi)\right)}{6\,912\,243\,473}$$

$$\left(1+12^{4}\times4\pi\left(\frac{\coth(\pi)}{1^{4}+12^{4}}+\frac{2\coth(2\pi)}{2^{4}+12^{4}}+\frac{3\coth(3\pi)}{3^{4}+12^{4}}\right)\right)+47+\phi=\\48+\phi+4\pi12^{4}\left(\frac{i\cot(i\pi)}{1^{4}+12^{4}}+\frac{2i\cot(2i\pi)}{2^{4}+12^{4}}+\frac{3i\cot(3i\pi)}{3^{4}+12^{4}}\right)$$

$$\left(1 + 12^4 \times 4\pi \left(\frac{\coth(\pi)}{1^4 + 12^4} + \frac{2\coth(2\pi)}{2^4 + 12^4} + \frac{3\coth(3\pi)}{3^4 + 12^4}\right)\right) + 47 + \phi = 48 + \phi + 4\pi 12^4 \left(-\frac{i\cot(-i\pi)}{1^4 + 12^4} - \frac{2i\cot(-2i\pi)}{2^4 + 12^4} - \frac{3i\cot(-3i\pi)}{3^4 + 12^4}\right)$$

$$\left(1+12^{4}\times4\pi\left(\frac{\coth(\pi)}{1^{4}+12^{4}}+\frac{2\coth(2\pi)}{2^{4}+12^{4}}+\frac{3\coth(3\pi)}{3^{4}+12^{4}}\right)\right)+47+\phi= \\ 48+\phi+4\pi12^{4}\left(\frac{1+\frac{2}{-1+e^{2\pi}}}{1^{4}+12^{4}}+\frac{2\left(1+\frac{2}{-1+e^{4\pi}}\right)}{2^{4}+12^{4}}+\frac{3\left(1+\frac{2}{-1+e^{6\pi}}\right)}{3^{4}+12^{4}}\right)$$

#### **Series representations:**

$$\left(1 + 12^{4} \times 4\pi \left(\frac{\coth(\pi)}{1^{4} + 12^{4}} + \frac{2\coth(2\pi)}{2^{4} + 12^{4}} + \frac{3\coth(3\pi)}{3^{4} + 12^{4}}\right)\right) + 47 + \phi =$$

$$48 + \phi + \sum_{k=-\infty}^{\infty} \frac{2304 \left(1294010737 + 1173562562 k^{2} + 167548081 k^{4}\right)}{6912243473 \left(1 + k^{2}\right) \left(4 + k^{2}\right) \left(9 + k^{2}\right)}$$

$$\left(1 + 12^4 \times 4\pi \left(\frac{\coth(\pi)}{1^4 + 12^4} + \frac{2\coth(2\pi)}{2^4 + 12^4} + \frac{3\coth(3\pi)}{3^4 + 12^4}\right)\right) + 47 + \phi = \\ \frac{414604373872}{6912243473} + \phi + \sum_{k=1}^{\infty} \left(\frac{165888}{20737(1+k^2)} + \frac{41472}{1297(4+k^2)} + \frac{18432}{257(9+k^2)}\right)$$

$$\begin{split} &\left(1+12^4\times 4\,\pi\left(\frac{\coth(\pi)}{1^4+12^4}+\frac{2\coth(2\,\pi)}{2^4+12^4}+\frac{3\coth(3\,\pi)}{3^4+12^4}\right)\right)+47+\phi=\\ &48+\phi+\frac{165\,527\,124\,096\,\pi}{6\,912\,243\,473}+\\ &\sum_{k=0}^{\infty}\left(\frac{6144}{257}\,e^{-6\left(1+k\right)\pi}\,\pi+\frac{20\,736\,e^{-4\left(1+k\right)\pi}\,\pi}{1297}+\frac{165\,888\,e^{-2\left(1+k\right)\pi}\,\pi}{20\,737}\right) \end{split}$$

# **Integral representation:**

$$\left(1 + 12^{4} \times 4\pi \left(\frac{\coth(\pi)}{1^{4} + 12^{4}} + \frac{2\coth(2\pi)}{2^{4} + 12^{4}} + \frac{3\coth(3\pi)}{3^{4} + 12^{4}}\right)\right) + 47 + \phi =$$

$$48 + \phi + \int_{\frac{i\pi}{2}}^{3\pi} \left(-\frac{3072}{257}\pi \operatorname{csch}^{2}(t) + \left(\frac{13}{37} - \frac{4i}{37}\right) \left(-\frac{82944\pi \operatorname{csch}^{2}\left(\frac{\left(\frac{12}{37} + \frac{2i}{37}\right)\left(-i\pi^{2} - \left(1 - \frac{i}{2}\right)\pi t\right)\right)}{\pi}\right) - \left(\frac{93312}{6485} + \frac{20736i}{6485}\right)$$

$$\pi \operatorname{csch}^{2}\left(\frac{\left(\frac{4}{5} + \frac{2i}{5}\right)\left(\frac{i\pi^{2}}{2} + \left(\frac{25}{37} - \frac{2i}{37}\right)\left(-i\pi^{2} - \left(1 - \frac{i}{2}\right)\pi t\right)\right)}{\pi}\right) \right) dt$$

Where 13 is a Fibonacci number

#### **Input:**

$$\left(1+12^{4}\times4\pi\left(\frac{\coth(\pi)}{1^{4}+12^{4}}+\frac{2\coth(2\pi)}{2^{4}+12^{4}}+\frac{3\coth(3\pi)}{3^{4}+12^{4}}\right)\right)\times2-13$$

coth(x) is the hyperbolic cotangent function

#### **Exact result:**

$$2 \left(1 + 82\,944\,\pi \left(\frac{\coth(\pi)}{20\,737} + \frac{\coth(2\,\pi)}{10\,376} + \frac{\coth(3\,\pi)}{6939}\right)\right) - 13$$

# **Decimal approximation:**

139.5574921942375590742496495648983103603235640752017942854...

139.557492.... result practically equal to the rest mass of Pion meson 139.57

#### **Alternate forms:**

$$\frac{1}{6\,912\,243\,473}(-76\,034\,678\,203 + 55\,295\,281\,152\,\pi\,\coth(\pi) + \\110\,510\,625\,024\,\pi\,\coth(2\,\pi) + 165\,248\,342\,016\,\pi\,\coth(3\,\pi))$$

$$-11 + \frac{165\,888\,\pi\,\coth(\pi)}{20\,737} + \frac{20\,736\,\pi\,\coth(2\,\pi)}{1297} + \frac{6144}{257}\,\pi\,\coth(3\,\pi)$$

$$-\frac{295\,854\,779 + 215\,156\,736\,\pi\,\coth(\pi) + 430\,002\,432\,\pi\,\coth(2\,\pi)}{26\,895\,889} + \frac{6144}{257}\,\pi\,\coth(3\,\pi)$$

#### **Alternative representations:**

$$\left(1 + 12^4 \times 4\pi \left(\frac{\coth(\pi)}{1^4 + 12^4} + \frac{2\coth(2\pi)}{2^4 + 12^4} + \frac{3\coth(3\pi)}{3^4 + 12^4}\right)\right) 2 - 13 = \\ -13 + 2\left(1 + 4\pi 12^4 \left(\frac{i\cot(i\pi)}{1^4 + 12^4} + \frac{2i\cot(2i\pi)}{2^4 + 12^4} + \frac{3i\cot(3i\pi)}{3^4 + 12^4}\right)\right)$$

$$\left(1 + 12^4 \times 4\pi \left(\frac{\coth(\pi)}{1^4 + 12^4} + \frac{2\coth(2\pi)}{2^4 + 12^4} + \frac{3\coth(3\pi)}{3^4 + 12^4}\right)\right) 2 - 13 = \\ -13 + 2\left(1 + 4\pi 12^4 \left(-\frac{i\cot(-i\pi)}{1^4 + 12^4} - \frac{2i\cot(-2i\pi)}{2^4 + 12^4} - \frac{3i\cot(-3i\pi)}{3^4 + 12^4}\right)\right)$$

$$\left(1 + 12^4 \times 4\pi \left(\frac{\coth(\pi)}{1^4 + 12^4} + \frac{2\coth(2\pi)}{2^4 + 12^4} + \frac{3\coth(3\pi)}{3^4 + 12^4}\right)\right) 2 - 13 = \\ -13 + 2\left(1 + 4\pi 12^4 \left(\frac{1 + \frac{2}{-1 + e^{2\pi}}}{1^4 + 12^4} + \frac{2\left(1 + \frac{2}{-1 + e^{4\pi}}\right)}{2^4 + 12^4} + \frac{3\left(1 + \frac{2}{-1 + e^{6\pi}}\right)}{3^4 + 12^4}\right)\right)$$

$$\left(1 + 12^4 \times 4\pi \left(\frac{\coth(\pi)}{1^4 + 12^4} + \frac{2\coth(2\pi)}{2^4 + 12^4} + \frac{3\coth(3\pi)}{3^4 + 12^4}\right)\right) 2 - 13 = \\ -11 + \sum_{k = -\infty}^{\infty} \left(\frac{165\,888}{20\,737\left(1 + k^2\right)} + \frac{41\,472}{1297\left(4 + k^2\right)} + \frac{18\,432}{257\left(9 + k^2\right)}\right)$$

$$\left(1 + 12^4 \times 4\pi \left(\frac{\coth(\pi)}{1^4 + 12^4} + \frac{2\coth(2\pi)}{2^4 + 12^4} + \frac{3\coth(3\pi)}{3^4 + 12^4}\right)\right)2 - 13 = \\ \frac{89598696133}{6912243473} + \sum_{k=1}^{\infty} \left(\frac{331776}{20737(1+k^2)} + \frac{82944}{1297(4+k^2)} + \frac{36864}{257(9+k^2)}\right)$$

$$\begin{split} &\left(1+12^4\times 4\,\pi\left(\frac{\coth(\pi)}{1^4+12^4}+\frac{2\coth(2\,\pi)}{2^4+12^4}+\frac{3\coth(3\,\pi)}{3^4+12^4}\right)\right)2-13=\\ &-11+\frac{331\,054\,248\,192\,\pi}{6\,912\,243\,473}+\\ &\sum_{k=0}^{\infty}\left(\frac{12\,288}{257}\,e^{-6\left(1+k\right)\pi}\,\pi+\frac{41\,472\,e^{-4\left(1+k\right)\pi}\,\pi}{1297}+\frac{331\,776\,e^{-2\left(1+k\right)\pi}\,\pi}{20\,737}\right) \end{split}$$

#### **Integral representation:**

$$\left(1 + 12^{4} \times 4\pi \left(\frac{\coth(\pi)}{1^{4} + 12^{4}} + \frac{2\coth(2\pi)}{2^{4} + 12^{4}} + \frac{3\coth(3\pi)}{3^{4} + 12^{4}}\right)\right) 2 - 13 =$$

$$-11 + \int_{\frac{i\pi}{2}}^{3\pi} \left(-\frac{6144}{257}\pi \operatorname{csch}^{2}(t) + \left(\frac{13}{37} - \frac{4i}{37}\right)\right)$$

$$\left(-\frac{165888\pi \operatorname{csch}^{2}\left(\frac{\left(\frac{12}{37} + \frac{2i}{37}\right)\left(-i\pi^{2} - \left(1 - \frac{i}{2}\right)\pi t\right)\right)}{\pi}\right) - \left(\frac{186624}{6485} + \frac{41472i}{6485}\right)$$

$$\pi \operatorname{csch}^{2}\left(\frac{\left(\frac{4}{5} + \frac{2i}{5}\right)\left(i\pi^{2} + \left(\frac{25}{37} - \frac{2i}{37}\right)\left(-i\pi^{2} - \left(1 - \frac{i}{2}\right)\pi t\right)\right)}{\pi}\right)\right) dt$$

And:

ii. 
$$\pi^{\nu} x^{\nu} \operatorname{Cosec} \pi x \operatorname{cosech} \pi x$$
  
=  $1 + 4 \pi x^{4} \left\{ \frac{\operatorname{Cosech} \pi}{1^{4} - x^{4}} - \frac{2 \operatorname{Cosech} 2 \pi}{2^{4} - x^{4}} + \frac{3 \operatorname{Cosech} 3 \pi}{3^{4} - x^{4}} - 84 \right\}$ 

1+12^4\*4\*Pi(((((cosech(Pi)/(1^4-12^4)-(2cosech(2Pi))/(2^4-12^4)+(3cosech(3Pi))/(3^4-12^4))))))

#### **Input:**

$$1 + 12^{4} \times 4\pi \left( \frac{\operatorname{csch}(\pi)}{1^{4} - 12^{4}} - \frac{2\operatorname{csch}(2\pi)}{2^{4} - 12^{4}} + \frac{3\operatorname{csch}(3\pi)}{3^{4} - 12^{4}} \right)$$

 $\operatorname{csch}(x)$  is the hyperbolic cosecant function

#### **Exact result:**

$$1 + 82944 \pi \left( -\frac{\operatorname{csch}(\pi)}{20735} + \frac{\operatorname{csch}(2\pi)}{10360} - \frac{\operatorname{csch}(3\pi)}{6885} \right)$$

# **Decimal approximation:**

-0.00033643634739567899698155811973395443437640261934855899...

-0.000336436347...

#### **Alternate forms:**

$$\frac{1}{91\,296\,205}$$
 (91 296 205 – 365 202 432  $\pi$  csch( $\pi$ ) + 730 933 632  $\pi$  csch(2  $\pi$ ) – 1 099 850 752  $\pi$  csch(3  $\pi$ ))

$$1 - \frac{82\,944\,\pi\,\operatorname{csch}(\pi)}{20\,735} + \frac{10\,368\,\pi\,\operatorname{csch}(2\,\pi)}{1295} - \frac{1024}{85}\,\pi\,\operatorname{csch}(3\,\pi)$$

$$\frac{5\,370\,365 - 21\,482\,496\,\pi\,\operatorname{csch}(\pi) + 42\,996\,096\,\pi\,\operatorname{csch}(2\,\pi)}{5\,370\,365} - \frac{1024}{85}\,\pi\,\operatorname{csch}(3\,\pi)$$

#### **Alternative representations:**

$$1 + 12^{4} \times 4\pi \left( \frac{\operatorname{csch}(\pi)}{1^{4} - 12^{4}} - \frac{2\operatorname{csch}(2\pi)}{2^{4} - 12^{4}} + \frac{3\operatorname{csch}(3\pi)}{3^{4} - 12^{4}} \right) = 1 + 4\pi 12^{4} \left( \frac{i\operatorname{csc}(i\pi)}{1^{4} - 12^{4}} - \frac{2i\operatorname{csc}(2i\pi)}{2^{4} - 12^{4}} + \frac{3i\operatorname{csc}(3i\pi)}{3^{4} - 12^{4}} \right)$$

$$\begin{aligned} 1 + 12^4 \times 4\pi \left( \frac{\operatorname{csch}(\pi)}{1^4 - 12^4} - \frac{2\operatorname{csch}(2\pi)}{2^4 - 12^4} + \frac{3\operatorname{csch}(3\pi)}{3^4 - 12^4} \right) &= \\ 1 + 4\pi 12^4 \left( -\frac{i\operatorname{csc}(-i\pi)}{1^4 - 12^4} + \frac{2i\operatorname{csc}(-2i\pi)}{2^4 - 12^4} - \frac{3i\operatorname{csc}(-3i\pi)}{3^4 - 12^4} \right) \end{aligned}$$

$$\begin{split} 1 + 12^{4} \times 4\pi \left( \frac{\operatorname{csch}(\pi)}{1^{4} - 12^{4}} - \frac{2\operatorname{csch}(2\pi)}{2^{4} - 12^{4}} + \frac{3\operatorname{csch}(3\pi)}{3^{4} - 12^{4}} \right) = \\ 1 + 4\pi 12^{4} \left( \frac{2e^{\pi}}{\left(1^{4} - 12^{4}\right)\left(-1 + e^{2\pi}\right)} - \frac{4e^{2\pi}}{\left(2^{4} - 12^{4}\right)\left(-1 + e^{4\pi}\right)} + \frac{6e^{3\pi}}{\left(3^{4} - 12^{4}\right)\left(-1 + e^{6\pi}\right)} \right) \end{split}$$

$$\begin{split} 1 + 12^4 \times 4\pi \left( \frac{\operatorname{csch}(\pi)}{1^4 - 12^4} - \frac{2\operatorname{csch}(2\pi)}{2^4 - 12^4} + \frac{3\operatorname{csch}(3\pi)}{3^4 - 12^4} \right) = \\ 1 + \sum_{k = -\infty}^{\infty} - \frac{768\left(-1\right)^k \left(17172775 + 8628542k^2 + 2868343k^4\right)}{91296205\left(1 + k^2\right)\left(4 + k^2\right)\left(9 + k^2\right)} \end{split}$$

$$\begin{aligned} 1 + 12^4 \times 4\pi \left( \frac{\operatorname{csch}(\pi)}{1^4 - 12^4} - \frac{2\operatorname{csch}(2\pi)}{2^4 - 12^4} + \frac{3\operatorname{csch}(3\pi)}{3^4 - 12^4} \right) &= \\ -\frac{165033797}{54777723} + \sum_{k=1}^{\infty} \frac{1536(-1)^k \left( -\frac{475524}{1+k^2} + \frac{1903473}{4+k^2} - \frac{4296292}{9+k^2} \right)}{91296205} \end{aligned}$$

$$\begin{aligned} 1 + 12^4 \times 4\pi \left( \frac{\operatorname{csch}(\pi)}{1^4 - 12^4} - \frac{2\operatorname{csch}(2\pi)}{2^4 - 12^4} + \frac{3\operatorname{csch}(3\pi)}{3^4 - 12^4} \right) &= \\ 1 + \sum_{k=0}^{\infty} -\frac{256\,e^{-3\left(\pi + 2\,k\,\pi\right)} \left( 8\,592\,584 - 5\,710\,419\,e^{\pi + 2\,k\,\pi} + 2\,853\,144\,e^{2\,\pi + 4\,k\,\pi} \right)\pi}{91\,296\,205} \end{aligned}$$

Where 11 is a Lucas number and the number of dimensions of M-Theory

### Input:

$$-\frac{1}{1+12^4\times 4\,\pi\left(\frac{\cosh(\pi)}{1^4-12^4}-\frac{2\cosh(2\,\pi)}{2^4-12^4}+\frac{3\cosh(3\,\pi)}{3^4-12^4}\right)}+11$$

csch(x) is the hyperbolic cosecant function

### **Exact result:**

11 - 
$$\frac{1}{1 + 82944 \pi \left(-\frac{\operatorname{csch}(\pi)}{20735} + \frac{\operatorname{csch}(2\pi)}{10360} - \frac{\operatorname{csch}(3\pi)}{6885}\right)}$$

#### **Decimal approximation:**

2983.330450443011345236626372998106078723434944496179850604...

2983.330450443... result very near to the rest mass of Charmed eta meson 2980.3

#### **Alternate forms:**

 $11 + 91296205 / (-91296205 + 365202432 \pi \operatorname{csch}(\pi) - 730933632 \pi \operatorname{csch}(2\pi) +$  $1099850752 \pi \operatorname{csch}(3\pi)$ 

$$11 - \frac{1}{1 + 82\,944\,\pi \left( csch(\pi) \left( \frac{sech(\pi)}{20\,720} - \frac{1}{20\,735} \right) - \frac{csch(3\,\pi)}{6885} \right)}$$

$$11 - \frac{1}{1 - \frac{82944 \ \pi \ csch(\pi)}{20\ 735} - \frac{1024 \ \pi}{85 \left( \sinh^3(\pi) + 3 \sinh(\pi) \cosh^2(\pi) \right)} + \frac{5184 \ \pi \ csch(\pi) \ sech(\pi)}{1295}}$$

$$-\frac{1}{1+12^{4}\times 4\pi\left(\frac{\cosh(\pi)}{1^{4}-12^{4}}-\frac{2\cosh(2\pi)}{2^{4}-12^{4}}+\frac{3\cosh(3\pi)}{3^{4}-12^{4}}\right)}+11=$$

$$11-\frac{1}{1+4\pi 12^{4}\left(\frac{i\csc(i\pi)}{1^{4}-12^{4}}-\frac{2i\csc(2i\pi)}{2^{4}-12^{4}}+\frac{3i\csc(3i\pi)}{3^{4}-12^{4}}\right)}$$

$$-\frac{1}{1+12^{4}\times 4\pi\left(\frac{\cosh(\pi)}{1^{4}-12^{4}}-\frac{2\cosh(2\pi)}{2^{4}-12^{4}}+\frac{3\cosh(3\pi)}{3^{4}-12^{4}}\right)}+11=\\11-\frac{1}{1+4\pi 12^{4}\left(-\frac{i\csc(-i\pi)}{1^{4}-12^{4}}+\frac{2i\csc(-2i\pi)}{2^{4}-12^{4}}-\frac{3i\csc(-3i\pi)}{3^{4}-12^{4}}\right)}$$

$$\begin{aligned} &-\frac{1}{1+12^4\times 4\pi\left(\frac{\cosh(\pi)}{1^4-12^4}-\frac{2\cosh(2\pi)}{2^4-12^4}+\frac{3\cosh(3\pi)}{3^4-12^4}\right)}+11=\\ &11-\frac{1}{1+4\pi\ 12^4\left(\frac{2\,e^\pi}{(1^4-12^4)(-1+e^{2\,\pi})}-\frac{4\,e^{2\,\pi}}{(2^4-12^4)(-1+e^{4\,\pi})}+\frac{6\,e^{3\,\pi}}{(3^4-12^4)(-1+e^{6\,\pi})}\right)} \end{aligned}$$

$$-\frac{1}{1+12^{4}\times4\pi\left(\frac{\cosh(\pi)}{1^{4}-12^{4}}-\frac{2\cosh(2\pi)}{2^{4}-12^{4}}+\frac{3\cosh(3\pi)}{3^{4}-12^{4}}\right)}+11=\\ 11-\frac{1}{1+82944\pi\sum_{k=-\infty}^{\infty}-\frac{(-1)^{k}\left(17172775+8628542k^{2}+2868343k^{4}\right)}{9859990140\left(1+k^{2}\right)\left(9+k^{2}\right)\pi}}\\ -\frac{1}{1+12^{4}\times4\pi\left(\frac{\cosh(\pi)}{1^{4}-12^{4}}-\frac{2\cosh(2\pi)}{2^{4}-12^{4}}+\frac{3\cosh(3\pi)}{3^{4}-12^{4}}\right)}+11=\\ 11-\frac{1}{1+82944\pi\sum_{k=0}^{\infty}\left(-\frac{2e^{-3\pi-6k\pi}}{6885}+\frac{e^{-2\pi-4k\pi}}{5180}-\frac{2e^{-\pi-2k\pi}}{20735}\right)}\\ -\frac{1}{1+12^{4}\times4\pi\left(\frac{\cosh(\pi)}{1^{4}-12^{4}}-\frac{2\cosh(2\pi)}{2^{4}-12^{4}}+\frac{3\cosh(3\pi)}{3^{4}-12^{4}}\right)}+11=\\ 11-\frac{1}{1+12^{4}\times4\pi\left(\frac{\cosh(\pi)}{1^{4}-12^{4}}-\frac{2\cosh(2\pi)}{2^{4}-12^{4}}+\frac{3\cosh(3\pi)}{3^{4}-12^{4}}\right)}+11=\\ 11-1/\left(1+82944\pi\sum_{k=0}^{\infty}-\left(\left(\left(\text{Li}_{-k}\left(-e^{z_{0}}\right)-\text{Li}_{-k}\left(e^{z_{0}}\right)\right)\left(2853144\left(\pi-z_{0}\right)^{k}-5710419\right)\right)\right)\\ (59159940840k!)\right)\right) \text{ for } \frac{i\,z_{0}}{\pi}\notin\mathbb{Z}$$

Where 11 is a Lucas number and the number of dimensions of M-Theory and 24 can be identified with the number of the transverse degrees of freedom in the bosonic string

$$\begin{array}{l} \textbf{Input:} \\ -\frac{1}{24} \times \frac{1}{1 + 12^4 \times 4 \, \pi \left( \frac{\cosh(\pi)}{1^4 - 12^4} - \frac{2 \cosh(2 \, \pi)}{2^4 - 12^4} + \frac{3 \cosh(3 \, \pi)}{3^4 - 12^4} \right)} + 11 \end{array}$$

#### **Exact result:**

$$11 - \frac{1}{24 \left(1 + 82944 \pi \left(-\frac{\cosh(\pi)}{20735} + \frac{\cosh(2\pi)}{10360} - \frac{\cosh(3\pi)}{6885}\right)\right)}$$

## **Decimal approximation:**

134.8471021017921393848594322082544199468097893540074937751...

134.847102101... result practically equal to the rest mass of Pion meson 134.9766

#### **Alternate forms:**

$$11 + 91296205 / (24(-91296205 + 365202432 \pi \operatorname{csch}(\pi) - 730933632 \pi \operatorname{csch}(2\pi) + 1099850752 \pi \operatorname{csch}(3\pi)))$$

$$11 - \frac{1}{24 \left(1 + 82\,944\,\pi \left( csch(\pi) \left( \frac{sech(\pi)}{20\,720} - \frac{1}{20\,735} \right) - \frac{csch(3\,\pi)}{6885} \right) \right)}$$

$$11 - \frac{1}{24\left(1 - \frac{82944 \pi \operatorname{csch}(\pi)}{20735} - \frac{1024 \pi}{85\left(\sinh^3(\pi) + 3\sinh(\pi)\cosh^2(\pi)\right)} + \frac{5184 \pi \operatorname{csch}(\pi)\operatorname{sech}(\pi)}{1295}\right)}$$

$$-\frac{1}{\left(1+12^{4}\times4\pi\left(\frac{\cosh(\pi)}{1^{4}-12^{4}}-\frac{2\cosh(2\pi)}{2^{4}-12^{4}}+\frac{3\cosh(3\pi)}{3^{4}-12^{4}}\right)\right)24}+11=$$

$$11-\frac{1}{24\left(1+4\pi12^{4}\left(\frac{i\csc(i\pi)}{1^{4}-12^{4}}-\frac{2i\csc(2i\pi)}{2^{4}-12^{4}}+\frac{3i\csc(3i\pi)}{3^{4}-12^{4}}\right)\right)}$$

$$-\frac{1}{\left(1+12^{4}\times4\pi\left(\frac{\cosh(\pi)}{1^{4}-12^{4}}-\frac{2\cosh(2\pi)}{2^{4}-12^{4}}+\frac{3\cosh(3\pi)}{3^{4}-12^{4}}\right)\right)24}+11=$$

$$11-\frac{1}{24\left(1+4\pi12^{4}\left(-\frac{i\csc(-i\pi)}{1^{4}-12^{4}}+\frac{2i\csc(-2i\pi)}{2^{4}-12^{4}}-\frac{3i\csc(-3i\pi)}{3^{4}-12^{4}}\right)\right)}$$

$$-\frac{1}{\left(1+12^{4}\times4\pi\left(\frac{\cosh(\pi)}{1^{4}-12^{4}}-\frac{2\cosh(2\pi)}{2^{4}-12^{4}}+\frac{3\cosh(3\pi)}{3^{4}-12^{4}}\right)\right)24}+11=\\11-\frac{1}{24\left(1+4\pi12^{4}\left(\frac{2e^{\pi}}{\left(1^{4}-12^{4}\right)\left(-1+e^{2\pi}\right)}-\frac{4e^{2\pi}}{\left(2^{4}-12^{4}\right)\left(-1+e^{4\pi}\right)}+\frac{6e^{3\pi}}{\left(3^{4}-12^{4}\right)\left(-1+e^{6\pi}\right)}\right)\right)}$$

$$-\frac{1}{\left(1+12^{4}\times4\pi\left(\frac{\cosh(\pi)}{1^{4}-12^{4}}-\frac{2\cosh(2\pi)}{2^{4}-12^{4}}+\frac{3\cosh(3\pi)}{3^{4}-12^{4}}\right)\right)24}+11=\\ 11+\frac{1}{8\left(825\,168\,985+\sum_{k=1}^{\infty}\frac{4608\,(-1)^{k}\left(17\,172\,775+8\,628\,542\,k^{2}+2\,868\,343\,k^{4}\right)}{\left(1+k^{2}\right)\left(4+k^{2}\right)\left(9+k^{2}\right)}\right)}-\frac{1}{\left(1+12^{4}\times4\pi\left(\frac{\cosh(\pi)}{1^{4}-12^{4}}-\frac{2\cosh(2\pi)}{2^{4}-12^{4}}+\frac{3\cosh(3\pi)}{3^{4}-12^{4}}\right)\right)24}+11=\\ 11-\frac{1}{24\left(1+82\,944\,\pi\sum_{k=-\infty}^{\infty}-\frac{(-1)^{k}\left(17\,172\,775+8\,628\,542\,k^{2}+2\,868\,343\,k^{4}\right)}{9859\,990\,140\,\left(1+k^{2}\right)\left(4+k^{2}\right)\left(9+k^{2}\right)\pi}\right)}-\frac{1}{\left(1+12^{4}\times4\pi\left(\frac{\cosh(\pi)}{1^{4}-12^{4}}-\frac{2\cosh(2\pi)}{2^{4}-12^{4}}+\frac{3\cosh(3\pi)}{3^{4}-12^{4}}\right)\right)24}+11=\\ 11-\frac{1}{24\left(1+82\,944\,\pi\sum_{k=0}^{\infty}\left(-\frac{2\cosh(2\pi)}{6885}+\frac{3\cosh(3\pi)}{2^{4}-12^{4}}\right)\right)24}}$$

And:

Cor. 
$$2\pi^{4}x^{4}$$

Cor.  $2\pi^{4}x^{4}$ 

Cor.  $2\pi^{4}x^{4}$ 
 $= 1 - 4\pi x^{4}$ 
 $= \frac{2\cos ch \pi}{1^{4} + x^{4}} - \frac{2\cos ch \pi}{2^{4} + x^{4}} + \frac{3\cos ch \pi}{3^{4} + x^{4}} - 8c$ 

## **Input:**

$$1 - 12^4 \times 4\pi \left( \frac{\operatorname{csch}(\pi)}{1^4 + 12^4} - \frac{2\operatorname{csch}(2\pi)}{2^4 + 12^4} + \frac{3\operatorname{csch}(3\pi)}{3^4 + 12^4} \right)$$

csch(x) is the hyperbolic cosecant function

#### **Exact result:**

$$1 - 82944 \pi \left( \frac{\cosh(\pi)}{20737} - \frac{\operatorname{csch}(2\pi)}{10376} + \frac{\operatorname{csch}(3\pi)}{6939} \right)$$

## **Decimal approximation:**

-0.00032880867775530301888073140480179229202636145261724743...

-0.000328808677...

#### **Alternate forms:**

 $\frac{1}{6912243473}$  (6912243473 – 27647640576  $\pi$  csch( $\pi$ ) + 55255312512  $\pi$  csch( $2\pi$ ) – 82624171008  $\pi$  csch( $3\pi$ ))

$$1 - \frac{82\,944\,\pi\,\operatorname{csch}(\pi)}{20\,737} + \frac{10\,368\,\pi\,\operatorname{csch}(2\,\pi)}{1297} - \frac{3072}{257}\,\pi\,\operatorname{csch}(3\,\pi)$$

$$\frac{26\,895\,889 - 107\,578\,368\,\pi\,\operatorname{csch}(\pi) + 215\,001\,216\,\pi\,\operatorname{csch}(2\,\pi)}{26\,895\,889} - \frac{3072}{257}\,\pi\,\operatorname{csch}(3\,\pi)$$

#### **Alternative representations:**

$$1 - 12^{4} \times 4\pi \left( \frac{\operatorname{csch}(\pi)}{1^{4} + 12^{4}} - \frac{2\operatorname{csch}(2\pi)}{2^{4} + 12^{4}} + \frac{3\operatorname{csch}(3\pi)}{3^{4} + 12^{4}} \right) = 1 - 4\pi 12^{4} \left( \frac{i\operatorname{csc}(i\pi)}{1^{4} + 12^{4}} - \frac{2i\operatorname{csc}(2i\pi)}{2^{4} + 12^{4}} + \frac{3i\operatorname{csc}(3i\pi)}{3^{4} + 12^{4}} \right)$$

$$1 - 12^{4} \times 4\pi \left( \frac{\operatorname{csch}(\pi)}{1^{4} + 12^{4}} - \frac{2\operatorname{csch}(2\pi)}{2^{4} + 12^{4}} + \frac{3\operatorname{csch}(3\pi)}{3^{4} + 12^{4}} \right) = 1 - 4\pi 12^{4} \left( -\frac{i\operatorname{csc}(-i\pi)}{1^{4} + 12^{4}} + \frac{2i\operatorname{csc}(-2i\pi)}{2^{4} + 12^{4}} - \frac{3i\operatorname{csc}(-3i\pi)}{3^{4} + 12^{4}} \right)$$

$$\begin{split} 1 - 12^{4} \times 4\,\pi \left( \frac{\operatorname{csch}(\pi)}{1^{4} + 12^{4}} - \frac{2\,\operatorname{csch}(2\,\pi)}{2^{4} + 12^{4}} + \frac{3\,\operatorname{csch}(3\,\pi)}{3^{4} + 12^{4}} \right) = \\ 1 - 4\,\pi\,12^{4} \left( \frac{2\,e^{\pi}}{\left(1^{4} + 12^{4}\right)\left(-1 + e^{2\,\pi}\right)} - \frac{4\,e^{2\,\pi}}{\left(2^{4} + 12^{4}\right)\left(-1 + e^{4\,\pi}\right)} + \frac{6\,e^{3\,\pi}}{\left(3^{4} + 12^{4}\right)\left(-1 + e^{6\,\pi}\right)} \right) \end{split}$$

$$\begin{split} 1 - 12^4 \times 4\pi \left( \frac{\operatorname{csch}(\pi)}{1^4 + 12^4} - \frac{2\operatorname{csch}(2\pi)}{2^4 + 12^4} + \frac{3\operatorname{csch}(3\pi)}{3^4 + 12^4} \right) = \\ 1 + \sum_{k = -\infty}^{\infty} \left( -\frac{82\,944\,(-1)^k}{20\,737\,\big(1 + k^2\big)} + \frac{20\,736\,(-1)^k}{1297\,\big(4 + k^2\big)} - \frac{9216\,(-1)^k}{257\,\big(9 + k^2\big)} \right) \end{split}$$

$$\begin{split} &1 - 12^{4} \times 4\,\pi \left(\frac{\cosh(\pi)}{1^{4} + 12^{4}} - \frac{2\,\cosh(2\,\pi)}{2^{4} + 12^{4}} + \frac{3\,\cosh(3\,\pi)}{3^{4} + 12^{4}}\right) = \\ &- \frac{20\,649\,131\,183}{6\,912\,243\,473} + \sum_{k=1}^{\infty} \left( -\frac{165\,888\,(-1)^{k}}{20\,737\,\big(1 + k^{2}\big)} + \frac{41\,472\,(-1)^{k}}{1297\,\big(4 + k^{2}\big)} - \frac{18\,432\,(-1)^{k}}{257\,\big(9 + k^{2}\big)} \right) \end{split}$$

$$\begin{split} 1 - 12^4 \times 4\,\pi \left( \frac{\mathrm{csch}(\pi)}{1^4 + 12^4} - \frac{2\,\mathrm{csch}(2\,\pi)}{2^4 + 12^4} + \frac{3\,\mathrm{csch}(3\,\pi)}{3^4 + 12^4} \right) = \\ 1 + \sum_{k=0}^{\infty} \left( -\frac{6144}{257}\,e^{-3\,\pi - 6\,k\,\pi}\,\pi + \frac{20\,736\,e^{-2\,\pi - 4\,k\,\pi}\,\pi}{1297} - \frac{165\,888\,e^{-\pi - 2\,k\,\pi}\,\pi}{20\,737} \right) \end{split}$$

-1/((((1-12^4\*4\*Pi(((((cosech(Pi)/(1^4+12^4)-(2cosech(2Pi))/(2^4+12^4)+(3cosech(3Pi))/(3^4+12^4))))))))))+47+7+golden ratio

## **Input:**

$$-\frac{1}{1-12^4\times 4\pi\left(\frac{\cosh(\pi)}{1^4+12^4}-\frac{2\cosh(2\pi)}{2^4+12^4}+\frac{3\cosh(3\pi)}{3^4+12^4}\right)}+47+7+\phi$$

csch(x) is the hyperbolic cosecant function

ø is the golden ratio

#### **Exact result:**

$$\phi + 54 - \frac{1}{1 - 82944 \pi \left(\frac{\cosh(\pi)}{20737} - \frac{\cosh(2\pi)}{10376} + \frac{\cosh(3\pi)}{6939}\right)}$$

## **Decimal approximation:**

3096.900298273126807801702180739848133876304011281727955975...

3096.90029827... result practically equal to the rest mass of J/Psi meson 3096.916

## **Alternate forms:**

$$\phi + 54 - \frac{1}{1 - 82944 \pi \left(\frac{\operatorname{csch}(3\pi)}{6939} + \operatorname{csch}(\pi) \left(\frac{1}{20737} - \frac{\operatorname{sech}(\pi)}{20752}\right)\right)}$$

$$\frac{1}{2} \left( 109 + \sqrt{5} \right) + 6912243473 / (-6912243473 + 27647640576 \pi \operatorname{csch}(\pi) - 55255312512 \pi \operatorname{csch}(2\pi) + 82624171008 \pi \operatorname{csch}(3\pi) \right)$$

$$54 + \phi - \frac{1}{1 - 82944 \pi \left(\frac{1}{20737 \sinh(\pi)} - \frac{1}{10376 \sinh(2\pi)} + \frac{1}{6939 \sinh(3\pi)}\right)}$$
(2)

$$-\frac{1}{1-12^{4}\times 4\pi\left(\frac{\cosh(\pi)}{1^{4}+12^{4}}-\frac{2\cosh(2\pi)}{2^{4}+12^{4}}+\frac{3\cosh(3\pi)}{3^{4}+12^{4}}\right)}+47+7+\phi=$$

$$54+\phi-\frac{1}{1-4\pi 12^{4}\left(\frac{i\csc(i\pi)}{1^{4}+12^{4}}-\frac{2i\csc(2i\pi)}{2^{4}+12^{4}}+\frac{3i\csc(3i\pi)}{3^{4}+12^{4}}\right)}$$

$$-\frac{1}{1-12^{4}\times 4\pi\left(\frac{\cosh(\pi)}{1^{4}+12^{4}}-\frac{2\cosh(2\pi)}{2^{4}+12^{4}}+\frac{3\cosh(3\pi)}{3^{4}+12^{4}}\right)}+47+7+\phi=$$

$$54+\phi-\frac{1}{1-4\pi 12^{4}\left(-\frac{i\csc(-i\pi)}{1^{4}+12^{4}}+\frac{2i\csc(-2i\pi)}{2^{4}+12^{4}}-\frac{3i\csc(-3i\pi)}{3^{4}+12^{4}}\right)}$$

$$-\frac{1}{1-12^{4}\times 4\pi\left(\frac{\cosh(\pi)}{1^{4}+12^{4}}-\frac{2\cosh(2\pi)}{2^{4}+12^{4}}+\frac{3\cosh(3\pi)}{3^{4}+12^{4}}\right)}+47+7+\phi=$$

$$54+\phi-\frac{1}{1-4\pi 12^{4}\left(\frac{2e^{\pi}}{(1^{4}+12^{4})(-1+e^{2\pi})}-\frac{4e^{2\pi}}{(2^{4}+12^{4})(-1+e^{4\pi})}+\frac{6e^{3\pi}}{(3^{4}+12^{4})(-1+e^{6\pi})}\right)}$$

$$-\frac{1}{1-12^{4}\times 4\pi\left(\frac{\operatorname{csch}(\pi)}{1^{4}+12^{4}}-\frac{2\operatorname{csch}(2\pi)}{2^{4}+12^{4}}+\frac{3\operatorname{csch}(3\pi)}{3^{4}+12^{4}}\right)}+47+7+\phi=\\ 54+\phi-\frac{1}{1-82944\pi\sum_{k=-\infty}^{\infty}\frac{\left(-1\right)^{k}\left(430646479+214268942\,k^{2}+71618719\,k^{4}\right)}{248840765028\left(1+k^{2}\right)\left(4+k^{2}\right)\left(9+k^{2}\right)\pi}}\\ -\frac{1}{1-12^{4}\times 4\pi\left(\frac{\operatorname{csch}(\pi)}{1^{4}+12^{4}}-\frac{2\operatorname{csch}(2\pi)}{2^{4}+12^{4}}+\frac{3\operatorname{csch}(3\pi)}{3^{4}+12^{4}}\right)}+47+7+\phi=\\ 54+\phi-\frac{1}{1-82944\pi\sum_{k=0}^{\infty}\left(\frac{2\,e^{-3\pi-6\,k\pi}}{6939}-\frac{e^{-2\pi-4\,k\pi}}{5188}+\frac{2\,e^{-\pi-2\,k\pi}}{20737}\right)}\\ -\frac{1}{1-12^{4}\times 4\pi\left(\frac{\operatorname{csch}(\pi)}{1^{4}+12^{4}}-\frac{2\operatorname{csch}(2\pi)}{2^{4}+12^{4}}+\frac{3\operatorname{csch}(3\pi)}{3^{4}+12^{4}}\right)}+47+7+\phi=\\ 54+\phi-1\left/\left(1-82944\pi\sum_{k=0}^{\infty}\left(\left(\operatorname{Li}_{-k}\left(-e^{z_{0}}\right)-\operatorname{Li}_{-k}\left(e^{z_{0}}\right)\right)\left(71999064\left(\pi-z_{0}\right)^{k}-143894043\left(2\pi-z_{0}\right)^{k}+215167112\left(3\pi-z_{0}\right)^{k}\right)\right)\right/(1493044590168\,k!)\right)\ \text{for}\ \frac{i\,z_{0}}{\pi}\notin\mathbb{Z}$$

**Input:** 

$$-\frac{1}{24} \times \frac{1}{1 - 12^4 \times 4 \pi \left(\frac{\cosh(\pi)}{1^4 + 12^4} - \frac{2 \cosh(2 \pi)}{2^4 + 12^4} + \frac{3 \cosh(3 \pi)}{3^4 + 12^4}\right)} - 1$$

#### **Exact result:**

$$-1 - \frac{1}{24 \left(1 - 82944 \pi \left(\frac{\csc h(\pi)}{20737} - \frac{\csc h(2\pi)}{10376} + \frac{\csc h(3\pi)}{6939}\right)\right)}$$

#### **Decimal approximation:**

125.7200943451823713730623997460617706566076542542467580463...

125.7200943451... result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T = 0 and to the Higgs boson mass 125.18

#### **Alternate forms:**

 $6\,912\,243\,473\,/ \\ (24\,(-6\,912\,243\,473\,+27\,647\,640\,576\,\pi\,\operatorname{csch}(\pi)\,-55\,255\,312\,512\,\pi\,\operatorname{csch}(2\,\pi)\,+ \\ 82\,624\,171\,008\,\pi\,\operatorname{csch}(3\,\pi))) - 1 \\ -1 - \frac{1}{24\left(1-82\,944\,\pi\left(\frac{\operatorname{csch}(3\,\pi)}{6939} + \operatorname{csch}(\pi)\left(\frac{1}{20\,737} - \frac{\operatorname{sech}(\pi)}{20\,752}\right)\right)\right)}$ 

$$-1 - \frac{1}{24 \left(1 - \frac{82944 \,\pi \, \text{csch}(\pi)}{20\,737} - \frac{3072 \,\pi}{257 \left(\sinh^3(\pi) + 3 \,\sinh(\pi) \,\cosh^2(\pi)\right)} + \frac{5184 \,\pi \, \text{csch}(\pi) \,\text{sech}(\pi)}{1297}\right)}$$

$$-\frac{1}{\left(1-12^{4}\times4\pi\left(\frac{\cosh(\pi)}{1^{4}+12^{4}}-\frac{2\cosh(2\pi)}{2^{4}+12^{4}}+\frac{3\cosh(3\pi)}{3^{4}+12^{4}}\right)\right)24}-1=$$

$$-1-\frac{1}{24\left(1-4\pi12^{4}\left(\frac{i\csc(i\pi)}{1^{4}+12^{4}}-\frac{2i\csc(2i\pi)}{2^{4}+12^{4}}+\frac{3i\csc(3i\pi)}{3^{4}+12^{4}}\right)\right)}$$

$$-\frac{1}{\left(1-12^{4}\times4\pi\left(\frac{\cosh(\pi)}{1^{4}+12^{4}}-\frac{2\cosh(2\pi)}{2^{4}+12^{4}}+\frac{3\cosh(3\pi)}{3^{4}+12^{4}}\right)\right)24}-1=$$

$$-1-\frac{1}{24\left(1-4\pi12^{4}\left(-\frac{i\csc(-i\pi)}{1^{4}+12^{4}}+\frac{2i\csc(-2i\pi)}{2^{4}+12^{4}}-\frac{3i\csc(-3i\pi)}{3^{4}+12^{4}}\right)\right)}$$

$$-\frac{1}{\left(1-12^{4}\times4\pi\left(\frac{\cosh(\pi)}{1^{4}+12^{4}}-\frac{2\cosh(2\pi)}{2^{4}+12^{4}}+\frac{3\csc(3\pi)}{2^{4}+12^{4}}-\frac{3i\csc(-3i\pi)}{3^{4}+12^{4}}\right)\right)}$$

$$-\frac{1}{24\left(1-4\pi12^{4}\left(\frac{2e^{\pi}}{1^{4}+12^{4}}-\frac{2\cosh(2\pi)}{3^{4}+12^{4}}+\frac{3\csc(3\pi)}{3^{4}+12^{4}}\right)\right)24}-1=$$

$$-1-\frac{1}{24\left(1-4\pi12^{4}\left(\frac{2e^{\pi}}{1^{4}+12^{4}}-\frac{4e^{2\pi}}{2^{4}+12^{4}}+\frac{6e^{3\pi}}{(3^{4}+12^{4})(-1+e^{6\pi})}\right)\right)}$$

$$-\frac{1}{\left(1-12^{4}\times4\pi\left(\frac{\cosh(\pi)}{1^{4}+12^{4}}-\frac{2\cosh(2\pi)}{2^{4}+12^{4}}+\frac{3\cosh(3\pi)}{3^{4}+12^{4}}\right)\right)24}-1=$$

$$-1-\frac{24\left(1-82944\pi\sum_{k=-\infty}^{\infty}\frac{\left(-1\right)^{k}\left(430646479+214268942\,k^{2}+71618719\,k^{4}\right)}{248840765028\left(1+k^{2}\right)\left(4+k^{2}\right)\left(9+k^{2}\right)\pi}\right)}{1}$$

$$-\frac{1}{\left(1-12^{4}\times4\pi\left(\frac{\cosh(\pi)}{1^{4}+12^{4}}-\frac{2\cosh(2\pi)}{2^{4}+12^{4}}+\frac{3\cosh(3\pi)}{3^{4}+12^{4}}\right)\right)24}-1=$$

$$-1-\frac{1}{24\left(1-82944\pi\sum_{k=0}^{\infty}\left(\frac{2\,e^{-3}\pi-6\,k\pi}{6939}-\frac{e^{-2}\pi-4\,k\pi}{5188}+\frac{2\,e^{-\pi-2\,k\pi}}{20737}\right)\right)}$$

$$-\frac{1}{\left(1-12^{4}\times4\pi\left(\frac{\cosh(\pi)}{1^{4}+12^{4}}-\frac{2\cosh(2\pi)}{2^{4}+12^{4}}+\frac{3\cosh(3\pi)}{3^{4}+12^{4}}\right)\right)24}-1=$$

$$-1-\frac{1}{24\left(1-82944\pi\sum_{k=0}^{\infty}\left(\left(\text{Li}_{-k}\left(-e^{20}\right)-\text{Li}_{-k}\left(e^{20}\right)\right)\left(71999064\left(\pi-z_{0}\right)^{k}-143894043\left(2\pi-z_{0}\right)^{k}+215167112\left(3\pi-z_{0}\right)^{k}\right)\right)}$$

$$-\frac{143894043\left(2\pi-z_{0}\right)^{k}+215167112\left(3\pi-z_{0}\right)^{k}\right)}{1493044590168\,k!}$$

**Input:** 

$$-\frac{1}{24} \times \frac{1}{1 - 12^4 \times 4\pi \left(\frac{\cosh(\pi)}{1^4 + 12^4} - \frac{2\cosh(2\pi)}{2^4 + 12^4} + \frac{3\cosh(3\pi)}{3^4 + 12^4}\right)} + 13$$

csch(x) is the hyperbolic cosecant function

#### **Exact result:**

$$13 - \frac{1}{24 \left(1 - 82944 \pi \left(\frac{\cosh(\pi)}{20737} - \frac{\cosh(2\pi)}{10376} + \frac{\cosh(3\pi)}{6939}\right)\right)}$$

### **Decimal approximation:**

139.7200943451823713730623997460617706566076542542467580463...

139.7200943451... result practically equal to the rest mass of Pion meson 139.57

#### **Alternate forms:**

$$13 + 6912243473 /$$
 $(24(-6912243473 + 27647640576 \pi \operatorname{csch}(\pi) - 55255312512 \pi \operatorname{csch}(2\pi) + 82624171008 \pi \operatorname{csch}(3\pi)))$ 

$$13 - \frac{1}{24 \left(1 - 82944 \pi \left(\frac{\operatorname{csch}(3\pi)}{6939} + \operatorname{csch}(\pi) \left(\frac{1}{20737} - \frac{\operatorname{sech}(\pi)}{20752}\right)\right)\right)}$$

$$13 - \frac{1}{24 \left(1 - \frac{82944 \pi \operatorname{csch}(\pi)}{20737} - \frac{3072 \pi}{257 \left(\sinh^3(\pi) + 3 \sinh(\pi) \cosh^2(\pi)\right)} + \frac{5184 \pi \operatorname{csch}(\pi) \operatorname{sech}(\pi)}{1297}\right)}$$

# Alternative representations:

$$-\frac{1}{\left(1-12^{4}\times4\pi\left(\frac{\cosh(\pi)}{1^{4}+12^{4}}-\frac{2\cosh(2\pi)}{2^{4}+12^{4}}+\frac{3\cosh(3\pi)}{3^{4}+12^{4}}\right)\right)24}+13=$$

$$13-\frac{1}{24\left(1-4\pi12^{4}\left(\frac{i\csc(i\pi)}{1^{4}+12^{4}}-\frac{2i\csc(2i\pi)}{2^{4}+12^{4}}+\frac{3i\csc(3i\pi)}{3^{4}+12^{4}}\right)\right)}$$

$$-\frac{1}{\left(1-12^{4}\times4\pi\left(\frac{\cosh(\pi)}{1^{4}+12^{4}}-\frac{2\cosh(2\pi)}{2^{4}+12^{4}}+\frac{3\csc(3\pi)}{3^{4}+12^{4}}\right)\right)24}+13=$$

$$13-\frac{1}{24\left(1-4\pi12^{4}\left(-\frac{i\csc(-i\pi)}{1^{4}+12^{4}}+\frac{2i\csc(-2i\pi)}{2^{4}+12^{4}}-\frac{3i\csc(-3i\pi)}{3^{4}+12^{4}}\right)\right)}$$

$$-\frac{1}{\left(1-12^{4}\times4\pi\left(\frac{\cosh(\pi)}{1^{4}+12^{4}}-\frac{2\cosh(2\pi)}{2^{4}+12^{4}}+\frac{3\cosh(3\pi)}{3^{4}+12^{4}}\right)\right)24}+13=$$

$$13-\frac{1}{24\left(1-4\pi12^{4}\left(\frac{2e^{\pi}}{(1^{4}+12^{4})(-1+e^{2\pi})}-\frac{4e^{2\pi}}{(2^{4}+12^{4})(-1+e^{4\pi})}+\frac{6e^{3\pi}}{(3^{4}+12^{4})(-1+e^{6\pi})}\right)\right)}$$

$$-\frac{1}{\left(1-12^{4}\times4\pi\left(\frac{\cosh(\pi)}{1^{4}+12^{4}}-\frac{2\cosh(2\pi)}{2^{4}+12^{4}}+\frac{3\cosh(3\pi)}{3^{4}+12^{4}}\right)\right)24}+13=$$

$$13-\frac{24\left(1-82944\pi\sum_{k=-\infty}^{\infty}\frac{(-1)^{k}\left(430646479+214268942k^{2}+71618719k^{4}\right)}{248840765028\left(1+k^{2}\right)\left(4+k^{2}\right)\left(9+k^{2}\right)\pi}\right)}{1}$$

$$-\frac{1}{\left(1-12^{4}\times4\pi\left(\frac{\cosh(\pi)}{1^{4}+12^{4}}-\frac{2\cosh(2\pi)}{2^{4}+12^{4}}+\frac{3\cosh(3\pi)}{3^{4}+12^{4}}\right)\right)24}+13=$$

$$13-\frac{1}{24\left(1-82944\pi\sum_{k=0}^{\infty}\left(\frac{2e^{-3\pi-6k\pi}}{6939}-\frac{e^{-2\pi-4k\pi}}{5188}+\frac{2e^{-\pi-2k\pi}}{20737}\right)\right)}$$

$$-\frac{1}{\left(1-12^{4}\times4\pi\left(\frac{\cosh(\pi)}{1^{4}+12^{4}}-\frac{2\cosh(2\pi)}{2^{4}+12^{4}}+\frac{3\cosh(3\pi)}{3^{4}+12^{4}}\right)\right)24}+13=$$

$$13-1\left/\left(24\left(1-82944\pi\sum_{k=0}^{\infty}\left(\left(\text{Li}_{-k}\left(-e^{z_{0}}\right)-\text{Li}_{-k}\left(e^{z_{0}}\right)\right)\left(71999064\left(\pi-z_{0}\right)^{k}-143894043\left(2\pi-z_{0}\right)^{k}+215167112\left(3\pi-z_{0}\right)^{k}\right)\right)\right/$$

$$\left.\left(1493044590168\,k!\right)\right)\right)\ \text{for}\ \frac{i\,z_{0}}{\pi}\notin\mathbb{Z}$$

((((((tanh(Pi/2)/(1^4-12^4)+((3tanh(3Pi)/2))/(3^4-12^4)+((5tanh(5Pi)/2))/(5^4-12^4))))

**Input:** 

$$\frac{\tanh\left(\frac{\pi}{2}\right)}{1^4 - 12^4} + \frac{3\left(\frac{1}{2}\tanh(3\pi)\right)}{3^4 - 12^4} + \frac{5\left(\frac{1}{2}\tanh(5\pi)\right)}{5^4 - 12^4}$$

tanh(x) is the hyperbolic tangent function

#### **Exact result:**

$$-\frac{\tanh\left(\frac{\pi}{2}\right)}{20735} - \frac{\tanh(3\pi)}{13770} - \frac{5\tanh(5\pi)}{40222}$$

### **Decimal approximation:**

-0.00024116380692975031845195053018781805637144094896405406...

-0.0002411638...

## **Property:**

$$-\frac{\tanh\left(\frac{\pi}{2}\right)}{20735} - \frac{\tanh(3\pi)}{13770} - \frac{5\tanh(5\pi)}{40222}$$
 is a transcendental number

#### **Alternate forms:**

$$\frac{-250614 \tanh\left(\frac{\pi}{2}\right) - 377377 \tanh(3\pi) - 645975 \tanh(5\pi)}{5196481290}$$

$$\begin{aligned} &\frac{-2754 \tanh \left(\frac{\pi}{2}\right) - 4147 \tanh (3\,\pi)}{57\,104\,190} - \frac{5\tanh (5\,\pi)}{40\,222} \\ &-\frac{\sinh (\pi)}{20\,735\,(1+\cosh (\pi))} - \frac{\sinh (6\,\pi)}{13\,770\,(1+\cosh (6\,\pi))} - \frac{5\sinh (10\,\pi)}{40\,222\,(1+\cosh (10\,\pi))} \end{aligned}$$

## **Alternative representations:**

$$\begin{split} \frac{\tanh\left(\frac{\pi}{2}\right)}{1^4-12^4} + \frac{3\tanh(3\pi)}{\left(3^4-12^4\right)2} + \frac{5\tanh(5\pi)}{\left(5^4-12^4\right)2} &= \\ \frac{5\left(-1+\frac{2}{1+e^{-10\pi}}\right)}{2\left(5^4-12^4\right)} + \frac{3\left(-1+\frac{2}{1+e^{-6\pi}}\right)}{2\left(3^4-12^4\right)} + \frac{-1+\frac{2}{1+e^{-\pi}}}{1^4-12^4} \end{split}$$

$$\frac{\tanh\left(\frac{\pi}{2}\right)}{1^{4}-12^{4}} + \frac{3\tanh(3\pi)}{\left(3^{4}-12^{4}\right)2} + \frac{5\tanh(5\pi)}{\left(5^{4}-12^{4}\right)2} = \frac{1}{\coth\left(\frac{\pi}{2}\right)\left(1^{4}-12^{4}\right)} + \frac{3}{2\coth(3\pi)\left(3^{4}-12^{4}\right)} + \frac{5}{2\coth(5\pi)\left(5^{4}-12^{4}\right)}$$

$$\frac{\tanh\left(\frac{\pi}{2}\right)}{1^4 - 12^4} + \frac{3\tanh(3\pi)}{\left(3^4 - 12^4\right)2} + \frac{5\tanh(5\pi)}{\left(5^4 - 12^4\right)2} = \frac{\coth\left(\frac{\pi}{2} - \frac{i\pi}{2}\right)}{1^4 - 12^4} + \frac{3\coth\left(3\pi - \frac{i\pi}{2}\right)}{2\left(3^4 - 12^4\right)} + \frac{5\coth\left(5\pi - \frac{i\pi}{2}\right)}{2\left(5^4 - 12^4\right)}$$

$$\frac{\tanh\left(\frac{\pi}{2}\right)}{1^4 - 12^4} + \frac{3\tanh(3\pi)}{\left(3^4 - 12^4\right)2} + \frac{5\tanh(5\pi)}{\left(5^4 - 12^4\right)2} = \sum_{k=1}^{\infty} -\frac{\frac{4}{20735\left(1+\left(1-2k\right)^2\right)} + \frac{5}{2295\left(37-4k+4k^2\right)} + \frac{100}{20111\left(101-4k+4k^2\right)}}{\pi}$$

$$\frac{\tanh\left(\frac{\pi}{2}\right)}{1^4 - 12^4} + \frac{3\tanh(3\pi)}{\left(3^4 - 12^4\right)2} + \frac{5\tanh(5\pi)}{\left(5^4 - 12^4\right)2} = \\
-\frac{636983}{2598240645} + \sum_{k=0}^{\infty} \left(\frac{e^{\left(-6 - (6-i)k\right)\pi}}{6885} + \frac{2e^{\left(-1 - (1-i)k\right)\pi}}{20735} + \frac{5(-1)^k e^{-10\left(1+k\right)\pi}}{20111}\right)$$

$$\begin{split} \frac{\tanh\left(\frac{\pi}{2}\right)}{1^4 - 12^4} + \frac{3\tanh(3\pi)}{\left(3^4 - 12^4\right)2} + \frac{5\tanh(5\pi)}{\left(5^4 - 12^4\right)2} &= \\ \sum_{k=0}^{\infty} \left( \frac{\left(\delta_k + \frac{2^{1+k} \operatorname{Li}_{-k}\left(-e^{2z_0}\right)}{k!}\right) \left(\frac{\pi}{2} - z_0\right)^k}{20735} + \frac{\left(\delta_k + \frac{2^{1+k} \operatorname{Li}_{-k}\left(-e^{2z_0}\right)}{k!}\right) (3\pi - z_0)^k}{13770} + \\ \frac{5\left(\delta_k + \frac{2^{1+k} \operatorname{Li}_{-k}\left(-e^{2z_0}\right)}{k!}\right) (5\pi - z_0)^k}{40222} \right) \operatorname{for} \frac{1}{2} + \frac{iz_0}{\pi} \notin \mathbb{Z} \end{split}$$

## **Integral representation:**

$$\frac{\tanh\left(\frac{\pi}{2}\right)}{1^4 - 12^4} + \frac{3\tanh(3\pi)}{\left(3^4 - 12^4\right)2} + \frac{5\tanh(5\pi)}{\left(5^4 - 12^4\right)2} = \int_0^{5\pi} \left(\frac{1}{10} \left(-\frac{\operatorname{sech}^2\left(\frac{t}{10}\right)}{20735} - \frac{\operatorname{sech}^2\left(\frac{3t}{5}\right)}{2295}\right) - \frac{5\operatorname{sech}^2(t)}{40222}\right) dt$$

Where 521 and 4 are Lucas numbers. Note that 521 = 496 + 25, where 496 is the dimension of Lie's Group E<sub>8</sub> X E<sub>8</sub> and 25 corresponding to the dimensions of a D-25 brane

Input: 
$$-\frac{1}{\frac{\tanh(\frac{\pi}{2})}{1^4-12^4} + \frac{3\left(\frac{1}{2}\tanh(3\pi)\right)}{3^4-12^4} + \frac{5\left(\frac{1}{2}\tanh(5\pi)\right)}{5^4-12^4}} -521 - 4$$

tanh(x) is the hyperbolic tangent function

#### **Exact result:**

$$-525 - \frac{1}{-\frac{\tanh(\frac{\pi}{2})}{20.735} - \frac{\tanh(3\pi)}{13.770} - \frac{5\tanh(5\pi)}{40.222}}$$

## **Decimal approximation:**

3621.559190331965785566481981872280066466747509278894151676...

3621.55919... result practically equal to the rest mass of double charmed Xi baryon 3621.40

#### **Property:**

$$-525 - \frac{1}{-\frac{\tanh(\frac{\pi}{2})}{20.735} - \frac{\tanh(3\pi)}{13.770} - \frac{5\tanh(5\pi)}{40.222}}$$
 is a transcendental number

#### **Alternate forms:**

$$\frac{5\,196\,481\,290}{250\,614\,\tanh\left(\frac{\pi}{2}\right) + 377\,377\,\tanh(3\,\pi) + 645\,975\,\tanh(5\,\pi)} - 525$$

$$\frac{1}{\frac{\tanh\left(\frac{\pi}{2}\right)}{20\,735} + \frac{\tanh(3\,\pi)}{13\,770} + \frac{5\,\tanh(5\,\pi)}{40\,222}} - 525$$

$$-\frac{105\left(-49\,490\,298 + 1\,253\,070\,\tanh\left(\frac{\pi}{2}\right) + 1\,886\,885\,\tanh(3\,\pi) + 3\,229\,875\,\tanh(5\,\pi)\right)}{250\,614\,\tanh\left(\frac{\pi}{2}\right) + 377\,377\,\tanh(3\,\pi) + 645\,975\,\tanh(5\,\pi)}$$

$$-\frac{1}{\frac{\tanh(\frac{\pi}{2})}{1^4-12^4}} + \frac{3\tanh(3\pi)}{(3^4-12^4)^2} + \frac{5\tanh(5\pi)}{(5^4-12^4)^2} - 521 - 4 = \frac{1}{-525} - \frac{1}{\frac{1}{\coth(\frac{\pi}{2})(1^4-12^4)}} + \frac{5\tanh(5\pi)}{2\coth(3\pi)(3^4-12^4)} + \frac{5}{2\coth(5\pi)(5^4-12^4)} - \frac{1}{\frac{\tanh(\frac{\pi}{2})}{1^4-12^4}} + \frac{3\tanh(3\pi)}{(3^4-12^4)^2} + \frac{5\tanh(5\pi)}{(5^4-12^4)^2} - 525 - \frac{1}{\frac{5\left(-1+\frac{2}{1+e^{-10}\pi}\right)}{2\left(5^4-12^4\right)}} + \frac{3\left(-1+\frac{2}{1+e^{-6}\pi}\right)}{2\left(3^4-12^4\right)} + \frac{-1+\frac{2}{1+e^{-\pi}}}{1^4-12^4} - \frac{1}{\frac{\tanh(\frac{\pi}{2})}{1^4-12^4}} - 521 - 4 = \frac{1}{\frac{\tanh(\frac{\pi}{2})}{1^4-12^4}} - \frac{1}{\frac{\coth(\frac{\pi}{2}-\frac{i\pi}{2})}{1^4-12^4}} + \frac{3\tanh(3\pi)}{(3^4-12^4)^2} + \frac{5\tanh(5\pi)}{(5^4-12^4)^2} - \frac{1}{\frac{\coth(\frac{\pi}{2}-\frac{i\pi}{2})}{1^4-12^4}} + \frac{3\coth(3\pi-\frac{i\pi}{2})}{2\left(3^4-12^4\right)} + \frac{5\coth(5\pi-\frac{i\pi}{2})}{2\left(5^4-12^4\right)}$$

 $1/golden\ ratio + 1/29*((((-1/((((tanh(Pi/2)/(1^4-12^4)+((3tanh(3Pi)/2))/(3^4-12^4)+((5tanh(5Pi)/2))/(5^4-12^4)))))))))-521-4))$ 

Where 29 is a Lucas numbers

#### **Input:**

$$\frac{1}{\phi} + \frac{1}{29} \left[ -\frac{1}{\frac{\tanh(\frac{\pi}{2})}{1^4 - 12^4} + \frac{3(\frac{1}{2}\tanh(3\pi))}{3^4 - 12^4} + \frac{5(\frac{1}{2}\tanh(5\pi))}{5^4 - 12^4}} - 521 - 4 \right]$$

tanh(x) is the hyperbolic tangent function  $\phi$  is the golden ratio

#### **Exact result:**

$$\frac{1}{\phi} + \frac{1}{29} \left[ -525 - \frac{1}{-\frac{\tanh(\frac{\pi}{2})}{20\ 735} - \frac{\tanh(3\pi)}{13\ 770} - \frac{5\tanh(5\pi)}{40\ 222}} \right]$$

#### **Decimal approximation:**

125.4993853795073357298074137954787438579529819135607336096...

125.49938537... result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T=0 and to the Higgs boson mass 125.18

#### **Property:**

$$\frac{1}{\phi} + \frac{1}{29} \left( -525 - \frac{1}{-\frac{\tanh(\frac{\pi}{2})}{20.735} - \frac{\tanh(3\pi)}{13.770} - \frac{5\tanh(5\pi)}{40.222}} \right) \text{ is a transcendental number}$$

Where 8 and 2 are Fibonacci numbers

#### **Input:**

$$8 + 2 + \frac{1}{29} \left[ -\frac{1}{\frac{\tanh(\frac{\pi}{2})}{1^4 - 12^4} + \frac{3(\frac{1}{2}\tanh(3\pi))}{3^4 - 12^4} + \frac{5(\frac{1}{2}\tanh(5\pi))}{5^4 - 12^4}} - 521 - 4 \right]$$

## **Exact result:**

$$10 + \frac{1}{29} \left[ -525 - \frac{1}{-\frac{\tanh(\frac{\pi}{2})}{20735} - \frac{\tanh(3\pi)}{13770} - \frac{5\tanh(5\pi)}{40222}} \right]$$

## **Decimal approximation:**

134.8813513907574408816028269611131057402326727337549707474...

134.88135139... result practically equal to the rest mass of Pion meson 134.9766

## **Property:**

$$10 + \frac{1}{29} \left( -525 - \frac{1}{-\frac{\tanh(\frac{\pi}{2})}{20735} - \frac{\tanh(3\pi)}{13770} - \frac{5\tanh(5\pi)}{40222}} \right) \text{ is a transcendental number}$$

$$\begin{array}{lll} \text{Cot.} & \frac{\pi}{9 \, \pi^2} & \frac{\text{Cosh} \, \pi_{\chi_2}^{\times} - \text{cos} \, \pi_{\chi_2}^{\times}}{\text{Cosh} \, \pi_{\chi_2}^{\times} + \text{cos} \, \pi_{\chi_2}^{\times}} \\ & = \frac{\text{Cosh} \, \frac{\pi}{\sqrt{2}} \, + \frac{3 \, \text{tanh} \, \frac{2 \, \pi}{2}}{1^6 + x^6} \, + \, \frac{5 \, \text{tanh} \, \frac{5 \, \pi}{2}}{3^6 + x^6} \, + \, \frac{5 \, \text{tanh} \, \frac{5 \, \pi}{2}}{5^6 + x^6} \, + \, \frac{8 \, \text{cosh} \, \frac{5 \, \pi}{2}}{5^6 + x^6} \, + \, \frac{5 \, \text{tanh} \, \frac{5 \, \pi}{2}}{5^6 + x^6} \, + \, \frac{8 \, \text{cosh} \, \frac{5 \, \pi}{2}}{5^6 + x^6} \, + \, \frac{3 \, \text{tanh} \, \frac{5 \, \pi}{2}}{5^6 + x^6} \, + \, \frac{3 \, \text{tanh} \, \frac{5 \, \pi}{2}}{5^6 + x^6} \, + \, \frac{3 \, \text{tanh} \, \frac{5 \, \pi}{2}}{5^6 + x^6} \, + \, \frac{3 \, \text{tanh} \, \frac{5 \, \pi}{2}}{5^6 + x^6} \, + \, \frac{3 \, \text{tanh} \, \frac{5 \, \pi}{2}}{5^6 + x^6} \, + \, \frac{3 \, \text{tanh} \, \frac{5 \, \pi}{2}}{5^6 + x^6} \, + \, \frac{3 \, \text{tanh} \, \frac{5 \, \pi}{2}}{5^6 + x^6} \, + \, \frac{3 \, \text{tanh} \, \frac{5 \, \pi}{2}}{5^6 + x^6} \, + \, \frac{3 \, \text{tanh} \, \frac{5 \, \pi}{2}}{5^6 + x^6} \, + \, \frac{3 \, \text{tanh} \, \frac{5 \, \pi}{2}}{5^6 + x^6} \, + \, \frac{3 \, \text{tanh} \, \frac{5 \, \pi}{2}}{5^6 + x^6} \, + \, \frac{3 \, \text{tanh} \, \frac{5 \, \pi}{2}}{5^6 + x^6} \, + \, \frac{3 \, \text{tanh} \, \frac{5 \, \pi}{2}}{5^6 + x^6} \, + \, \frac{3 \, \text{tanh} \, \frac{5 \, \pi}{2}}{5^6 + x^6} \, + \, \frac{3 \, \text{tanh} \, \frac{5 \, \pi}{2}}{5^6 + x^6} \, + \, \frac{3 \, \text{tanh} \, \frac{5 \, \pi}{2}}{5^6 + x^6} \, + \, \frac{3 \, \text{tanh} \, \frac{5 \, \pi}{2}}{5^6 + x^6} \, + \, \frac{3 \, \text{tanh} \, \frac{5 \, \pi}{2}}{5^6 + x^6} \, + \, \frac{3 \, \text{tanh} \, \frac{5 \, \pi}{2}}{5^6 + x^6} \, + \, \frac{3 \, \text{tanh} \, \frac{5 \, \pi}{2}}{5^6 + x^6} \, + \, \frac{3 \, \text{tanh} \, \frac{5 \, \pi}{2}}{5^6 + x^6} \, + \, \frac{3 \, \text{tanh} \, \frac{5 \, \pi}{2}}{5^6 + x^6} \, + \, \frac{3 \, \text{tanh} \, \frac{5 \, \pi}{2}}{5^6 + x^6} \, + \, \frac{3 \, \text{tanh} \, \frac{5 \, \pi}{2}}{5^6 + x^6} \, + \, \frac{3 \, \text{tanh} \, \frac{5 \, \pi}{2}}{5^6 + x^6}} \, + \, \frac{3 \, \text{tanh} \, \frac{5 \, \pi}{2}}{5^6 + x^6}} \, + \, \frac{3 \, \text{tanh} \, \frac{5 \, \pi}{2}}{5^6 + x^6}} \, + \, \frac{3 \, \text{tanh} \, \frac{5 \, \pi}{2}}{5^6 + x^6}} \, + \, \frac{3 \, \text{tanh} \, \frac{5 \, \pi}{2}}{5^6 + x^6}} \, + \, \frac{3 \, \text{tanh} \, \frac{5 \, \pi}{2}}{5^6 + x^6}} \, + \, \frac{3 \, \text{tanh} \, \frac{5 \, \pi}{2}}{5^6 + x^6}} \, + \, \frac{3 \, \text{tanh} \, \frac{5 \, \pi}{2}}{5^6 + x^6}} \, + \, \frac{3 \, \text{tanh} \, \frac{5 \, \pi}{2}}{5^6 + x^6}} \, + \, \frac{3 \, \text{tanh} \, \frac{5 \, \pi}{2}}{5^6 + x^6}} \, + \, \frac{3 \, \text{tanh} \, \frac{5 \, \pi}{2}}{5^6 +$$

((((((tanh(Pi/2)/(1^4+12^4)+((3tanh(3Pi)/2))/(3^4+12^4)+((5tanh(5Pi)/2))/(5^4+12^4)
)))

#### **Input:**

$$\frac{\tanh\left(\frac{\pi}{2}\right)}{1^4 + 12^4} + \frac{3\left(\frac{1}{2}\tanh(3\pi)\right)}{3^4 + 12^4} + \frac{5\left(\frac{1}{2}\tanh(5\pi)\right)}{5^4 + 12^4}$$

tanh(x) is the hyperbolic tangent function

#### **Exact result:**

$$\frac{\tanh\left(\frac{\pi}{2}\right)}{20737} + \frac{\tanh(3\pi)}{13878} + \frac{5\tanh(5\pi)}{42722}$$

## **Decimal approximation:**

 $0.000233320032211875296176516082527934356176673416489630365\dots \\$ 

0.0002333200322...

## **Property:**

$$\frac{\tanh\left(\frac{\pi}{2}\right)}{20737} + \frac{\tanh(3\pi)}{13878} + \frac{5\tanh(5\pi)}{42722} \text{ is a transcendental number}$$

#### **Alternate forms:**

$$\frac{296\,447\,958\,\tanh\left(\frac{\pi}{2}\right) + 442\,963\,057\,\tanh(3\,\pi) + 719\,470\,215\,\tanh(5\,\pi)}{6\,147\,441\,305\,046}$$

$$\frac{13878 \tanh \left(\frac{\pi}{2}\right) + 20737 \tanh (3\pi)}{287788086} + \frac{5 \tanh (5\pi)}{42722}$$

$$\frac{\sinh(\pi)}{20\,737\,(1+\cosh(\pi))} + \frac{\sinh(6\,\pi)}{13\,878\,(1+\cosh(6\,\pi))} + \frac{5\,\sinh(10\,\pi)}{42\,722\,(1+\cosh(10\,\pi))}$$

$$\frac{\tanh\left(\frac{\pi}{2}\right)}{1^4 + 12^4} + \frac{3\tanh(3\pi)}{\left(3^4 + 12^4\right)2} + \frac{5\tanh(5\pi)}{\left(5^4 + 12^4\right)2} = \frac{5\left(-1 + \frac{2}{1+e^{-10\pi}}\right)}{2\left(5^4 + 12^4\right)} + \frac{3\left(-1 + \frac{2}{1+e^{-6\pi}}\right)}{2\left(3^4 + 12^4\right)} + \frac{-1 + \frac{2}{1+e^{-\pi}}}{1^4 + 12^4}$$

$$\frac{\tanh\left(\frac{\pi}{2}\right)}{1^{4}+12^{4}} + \frac{3\tanh(3\pi)}{\left(3^{4}+12^{4}\right)2} + \frac{5\tanh(5\pi)}{\left(5^{4}+12^{4}\right)2} = \frac{1}{\coth\left(\frac{\pi}{2}\right)\left(1^{4}+12^{4}\right)} + \frac{3}{2\coth(3\pi)\left(3^{4}+12^{4}\right)} + \frac{5}{2\coth(5\pi)\left(5^{4}+12^{4}\right)}$$

$$\begin{split} \frac{\tanh\left(\frac{\pi}{2}\right)}{1^4+12^4} + \frac{3\tanh(3\pi)}{\left(3^4+12^4\right)2} + \frac{5\tanh(5\pi)}{\left(5^4+12^4\right)2} &= \\ \frac{\coth\left(\frac{\pi}{2}-\frac{i\pi}{2}\right)}{1^4+12^4} + \frac{3\coth\left(3\pi-\frac{i\pi}{2}\right)}{2\left(3^4+12^4\right)} + \frac{5\coth\left(5\pi-\frac{i\pi}{2}\right)}{2\left(5^4+12^4\right)} \end{split}$$

$$\frac{\tanh\left(\frac{\pi}{2}\right)}{1^4 + 12^4} + \frac{3\tanh(3\pi)}{\left(3^4 + 12^4\right)2} + \frac{5\tanh(5\pi)}{\left(5^4 + 12^4\right)2} = \sum_{k=1}^{\infty} \frac{\frac{4}{20737\left(1+\left(1-2k\right)^2\right)} + \frac{4}{2313\left(37-4k+4k^2\right)} + \frac{100}{21361\left(101-4k+4k^2\right)}}{\pi}$$

$$\frac{\tanh\left(\frac{\pi}{2}\right)}{1^4 + 12^4} + \frac{3\tanh(3\pi)}{\left(3^4 + 12^4\right)2} + \frac{5\tanh(5\pi)}{\left(5^4 + 12^4\right)2} = \frac{729440615}{3073720652523} + \sum_{k=0}^{\infty} \left(-\frac{e^{\left(-6 - (6 - i)k\right)\pi}}{6939} - \frac{2e^{\left(-1 - (1 - i)k\right)\pi}}{20737} - \frac{5(-1)^k e^{-10(1 + k)\pi}}{21361}\right)$$

$$\frac{\tanh\left(\frac{\pi}{2}\right)}{1^{4} + 12^{4}} + \frac{3\tanh(3\pi)}{(3^{4} + 12^{4})2} + \frac{5\tanh(5\pi)}{(5^{4} + 12^{4})2} =$$

$$\sum_{k=0}^{\infty} \left( -\frac{\left(\delta_{k} + \frac{2^{1+k} \operatorname{Li}_{-k}\left(-e^{2z_{0}}\right)}{k!}\right)\left(\frac{\pi}{2} - z_{0}\right)^{k}}{20737} - \frac{\left(\delta_{k} + \frac{2^{1+k} \operatorname{Li}_{-k}\left(-e^{2z_{0}}\right)}{k!}\right)(3\pi - z_{0})^{k}}{13878} - \frac{5\left(\delta_{k} + \frac{2^{1+k} \operatorname{Li}_{-k}\left(-e^{2z_{0}}\right)}{k!}\right)(5\pi - z_{0})^{k}}{42722} \right) \int \operatorname{for} \frac{1}{2} + \frac{iz_{0}}{\pi} \notin \mathbb{Z}$$

## Integral representation:

$$\frac{\tanh\left(\frac{\pi}{2}\right)}{1^4 + 12^4} + \frac{3\tanh(3\pi)}{\left(3^4 + 12^4\right)2} + \frac{5\tanh(5\pi)}{\left(5^4 + 12^4\right)2} = \int_0^{5\pi} \left(\frac{1}{10} \left(\frac{\operatorname{sech}^2\left(\frac{t}{10}\right)}{20737} + \frac{\operatorname{sech}^2\left(\frac{3t}{5}\right)}{2313}\right) + \frac{5\operatorname{sech}^2(t)}{42722}\right) dt$$

Where 18 is a Lucas number and  $0.256 = (64*4)/10^3$ 

#### **Input:**

$$\frac{0.256}{\frac{\tanh(\frac{\pi}{2})}{1^4+12^4} + \frac{3(\frac{1}{2}\tanh(3\pi))}{3^4+12^4} + \frac{5(\frac{1}{2}\tanh(5\pi))}{5^4+12^4}} + 18$$

#### **Result:**

1115.21...

#### 1115.21... result practically equal to the rest mass of Lambda baryon 1115.683

## **Alternative representations:**

$$\frac{0.256}{\frac{\tanh(\frac{\pi}{2})}{1^4 + 12^4} + \frac{3\tanh(3\pi)}{(3^4 + 12^4)^2} + \frac{5\tanh(5\pi)}{(5^4 + 12^4)^2}}{0.256}$$

$$18 + \frac{0.256}{\frac{1}{\coth(\frac{\pi}{2})(1^4 + 12^4)} + \frac{3}{2\coth(3\pi)(3^4 + 12^4)} + \frac{5}{2\coth(5\pi)(5^4 + 12^4)}}$$

$$\frac{0.256}{\frac{\tanh(\frac{\pi}{2})}{1^4 + 12^4} + \frac{3\tanh(3\,\pi)}{(3^4 + 12^4)^2} + \frac{5\tanh(5\,\pi)}{(5^4 + 12^4)^2}} + 18 = 18 + \frac{0.256}{\frac{5\left(-1 + \frac{2}{1 + e^{-10\,\pi}}\right)}{2\left(5^4 + 12^4\right)} + \frac{3\left(-1 + \frac{2}{1 + e^{-6\,\pi}}\right)}{2\left(3^4 + 12^4\right)} + \frac{-1 + \frac{2}{1 + e^{-\pi}}}{1^4 + 12^4}}$$

$$\frac{0.256}{\frac{\tanh(\frac{\pi}{2})}{1^4+12^4} + \frac{3\tanh(3\pi)}{\left(3^4+12^4\right)2} + \frac{5\tanh(5\pi)}{\left(5^4+12^4\right)2}} + 18 = 18 + \frac{0.256}{\frac{\coth\left(\frac{\pi}{2} - \frac{i\pi}{2}\right)}{1^4+12^4} + \frac{3\coth\left(3\pi - \frac{i\pi}{2}\right)}{2\left(3^4+12^4\right)} + \frac{5\coth\left(5\pi - \frac{i\pi}{2}\right)}{2\left(5^4+12^4\right)}}$$

$$\frac{0.256}{\frac{\tanh(\frac{\pi}{2})}{1^4+12^4} + \frac{3\tanh(3\pi)}{(3^4+12^4)^2} + \frac{5\tanh(5\pi)}{(5^4+12^4)^2}} + 18 = \frac{1093.68}{-1.01386 + \sum_{k=0}^{\infty} (-1)^k e^{-10(1+k)\pi} \left(1 + 0.615679 e^{4(1+k)\pi} + 0.412036 e^{9(1+k)\pi}\right)}$$

$$\frac{0.256}{\frac{\tanh(\frac{\pi}{2})}{1^4 + 12^4} + \frac{3\tanh(3\pi)}{\left(3^4 + 12^4\right)2} + \frac{5\tanh(5\pi)}{\left(5^4 + 12^4\right)2}} + 18 = 18 + \frac{1327.17}{\pi \sum_{k=1}^{\infty} \frac{\frac{1}{1 + (1 - 2k)^2} + \frac{6.06742}{25.25 - k + k^2} + \frac{8.96541}{37 - 4k + 4k^2}}{\pi^2}}$$

$$\frac{0.256}{\frac{\tanh(\frac{\pi}{2})}{1^4 + 12^4} + \frac{3\tanh(3\pi)}{(3^4 + 12^4)^2} + \frac{5\tanh(5\pi)}{(5^4 + 12^4)^2}} + 18 = 18 +$$

$$\frac{0.256}{\sum_{k=0}^{\infty} -\frac{\left(k! \delta_{k} + 2^{1+k} \operatorname{Li}_{-k} \left(-e^{2z_{0}}\right)\right) \left(296447958 \left(\frac{\pi}{2} - z_{0}\right)^{k} + 442963057 \left(3\pi - z_{0}\right)^{k} + 719470215 \left(5\pi - z_{0}\right)^{k}\right)}{6147441305046k!}}$$
for  $\frac{1}{2} + \frac{i z_{0}}{\pi} \notin \mathbb{Z}$ 

## **Integral representation:**

$$\frac{18 + \frac{18 + \frac{1}{2}}{1^4 + 12^4} + \frac{3 \tanh(3\pi)}{(3^4 + 12^4)^2} + \frac{5 \tanh(5\pi)}{(5^4 + 12^4)^2}}{5308.67} + 18 = \frac{5308.67}{16 + \frac{1}{2}} + \frac{5 \tanh(5\pi)}{(5^4 + 12^4)^2} + \frac{5308.67}{(5^5 \pi)^2 (0.1 \operatorname{sech}^2\left(\frac{t}{10}\right) + 0.896541 \operatorname{sech}^2\left(\frac{3t}{5}\right) + 2.42697 \operatorname{sech}^2(t)\right) dt}$$

Where 7, 76, 2 and 11 are Lucas numbers (11 is also the number of dimensions of M-Theory)

#### **Input:**

$$\frac{1}{7} \left[ \frac{0.256}{\frac{\tanh(\frac{\pi}{2})}{1^4 + 12^4} + \frac{3(\frac{1}{2}\tanh(3\pi))}{3^4 + 12^4} + \frac{5(\frac{1}{2}\tanh(5\pi))}{5^4 + 12^4}} - 76 - 2 \right] - 11$$

tanh(x) is the hyperbolic tangent function

#### **Result:**

134.601...

134.601... result practically equal to the rest mass of Pion meson 134.9766

#### **Percent decrease:**

$$\frac{1}{7} \left( \frac{0.256}{\frac{\tanh(\frac{\pi}{2})}{1^4 + 12^4} + \frac{3\tanh(3\pi)}{2(3^4 + 12^4)} + \frac{5\tanh(5\pi)}{2(5^4 + 12^4)}} - 76 - 2 \right) - 11 = 134.601 \text{ is } 7.55491$$
% smaller than 
$$\frac{1}{7} \left( \frac{0.256}{\frac{\tanh(\frac{\pi}{2})}{1^4 + 12^4} + \frac{3\tanh(3\pi)}{2(3^4 + 12^4)}} - 76 - 2 \right) = 145.601.$$

# Alternative representations:

$$\begin{split} &\frac{1}{7} \left( \frac{0.256}{\frac{\tanh(\frac{\pi}{2})}{1^4 + 12^4} + \frac{3\tanh(3\pi)}{2\left(3^4 + 12^4\right)} + \frac{5\tanh(5\pi)}{2\left(5^4 + 12^4\right)}} - 76 - 2 \right) - 11 = \\ &-11 + \frac{1}{7} \left( -78 + \frac{0.256}{\frac{1}{\coth(\frac{\pi}{2})\left(1^4 + 12^4\right)} + \frac{3}{2\coth(3\pi)\left(3^4 + 12^4\right)} + \frac{5}{2\coth(5\pi)\left(5^4 + 12^4\right)}} \right) \end{split}$$

$$\frac{1}{7} \left( \frac{0.256}{\frac{\tanh(\frac{\pi}{2})}{1^4 + 12^4} + \frac{3\tanh(3\pi)}{2(3^4 + 12^4)} + \frac{5\tanh(5\pi)}{2(5^4 + 12^4)}} - 76 - 2 \right) - 11 =$$

$$-11 + \frac{1}{7} \left( -78 + \frac{0.256}{\frac{5\left(-1 + \frac{2}{1 + e^{-10\pi}}\right)}{2(5^4 + 12^4)} + \frac{3\left(-1 + \frac{2}{1 + e^{-6\pi}}\right)}{2(3^4 + 12^4)} + \frac{-1 + \frac{2}{1 + e^{-\pi}}}{1^4 + 12^4} \right)$$

$$\frac{1}{7} \left( \frac{0.256}{\frac{\tanh(\frac{\pi}{2})}{1^4 + 12^4} + \frac{3\tanh(3\pi)}{2(3^4 + 12^4)} + \frac{5\tanh(5\pi)}{2(5^4 + 12^4)}} - 76 - 2 \right) - 11 =$$

$$-11 + \frac{1}{7} \left( -78 + \frac{0.256}{\frac{\coth(\frac{\pi}{2} - i\pi)}{1^4 + 12^4} + \frac{3\coth(3\pi - i\pi)}{2(3^4 + 12^4)}} + \frac{5\coth(5\pi - i\pi)}{2(5^4 + 12^4)} \right)$$

$$\frac{1}{7} \left( \frac{0.256}{\frac{\tanh(\frac{\pi}{2})}{1^4 + 12^4} + \frac{3\tanh(3\pi)}{2(3^4 + 12^4)} + \frac{5\tanh(5\pi)}{2(5^4 + 12^4)}} - 76 - 2 \right) - 11 = -\frac{155}{7} - \frac{156.24}{-1.01386 + \sum_{k=0}^{\infty} (-1)^k e^{-10(1+k)\pi} \left( 1 + 0.615679 e^{4(1+k)\pi} + 0.412036 e^{9(1+k)\pi} \right)}$$

$$\frac{1}{7} \left( \frac{0.256}{\frac{\tanh(\frac{\pi}{2})}{1^4 + 12^4}} + \frac{3 \tanh(3\pi)}{2(3^4 + 12^4)} + \frac{5 \tanh(5\pi)}{2(5^4 + 12^4)} - 76 - 2 \right) - 11 = \frac{155}{7} + \frac{0.0365714}{\sum_{k=1}^{\infty} \frac{2}{20 737(1 - 2k + 2k^2)} + \frac{4}{2313(37 - 4k + 4k^2)} + \frac{100}{21 361(101 - 4k + 4k^2)}}{\pi} \right)$$

$$\frac{1}{7} \left( \frac{0.256}{\frac{\tanh(\frac{\pi}{2})}{1^4 + 12^4} + \frac{3 \tanh(3\pi)}{2(3^4 + 12^4)}} - 76 - 2 \right) - 11 = -\frac{155}{7} + \frac{100}{21 361(101 - 4k + 4k^2)} - \frac{11}{7} + \frac{100}{21 361(101 - 4k + 4k^2)} - \frac{100}{21 361(101 - 4k + 4k^2)} - \frac{100}{21 361(101 - 4k + 4k^2)} - \frac$$

## **Integral representation:**

$$\begin{split} &\frac{1}{7} \left( \frac{0.256}{\frac{\tanh(\frac{\pi}{2})}{1^4 + 12^4} + \frac{3\tanh(3\pi)}{2\left(3^4 + 12^4\right)} + \frac{5\tanh(5\pi)}{2\left(5^4 + 12^4\right)}} - 76 - 2 \right) - 11 = \\ &- \frac{155}{7} + \frac{758.382}{\int_0^{5\pi} \left(0.1 \operatorname{sech}^2\left(\frac{t}{10}\right) + 0.896541 \operatorname{sech}^2\left(\frac{3t}{5}\right) + 2.42697 \operatorname{sech}^2(t)\right) dt} \end{split}$$

## Now, we have that:

$$iV. \frac{\pi}{g} Sec \frac{\pi x}{L} Sech \frac{\pi x}{L}$$

$$= \frac{1^{3} S \cdot ch \pi}{1^{4} \times 1} - \frac{3^{3} Sech 3\pi}{3^{5} \times 1} + \frac{5^{3} Sech 5\pi}{5^{4} \times 1} - 8c$$

$$Cor. \frac{\pi \chi}{4}$$

$$= \frac{1^{3} Sech \pi}{1^{4} + x^{4}} - \frac{3^{3} Sech 3\pi}{5^{4} + x^{4}} + \frac{5^{3} Sech 5\pi}{5^{4} + x^{4}} - 8c$$

$$= \frac{1^{3} Sech \pi}{1^{4} + x^{4}} - \frac{3^{3} Sech 3\pi}{3^{4} + x^{4}} + \frac{5^{3} Sech 5\pi}{5^{4} + x^{4}} - 8c$$

((((((1^3sech(Pi/2)/(1^4-12^4)-((3^3sech(3Pi)/2))/(3^4-12^4)+((5^3sech(5Pi)/2))/(5^4-12^4))))))

**Input:** 

$$1^{3} \times \frac{\operatorname{sech}\left(\frac{\pi}{2}\right)}{1^{4} - 12^{4}} - \frac{3^{3}\left(\frac{1}{2}\operatorname{sech}(3\pi)\right)}{3^{4} - 12^{4}} + \frac{5^{3}\left(\frac{1}{2}\operatorname{sech}(5\pi)\right)}{5^{4} - 12^{4}}$$

 $\operatorname{sech}(x)$  is the hyperbolic secant function

**Exact result:** 

$$-\frac{\operatorname{sech}\left(\frac{\pi}{2}\right)}{20735} + \frac{\operatorname{sech}(3\pi)}{1530} - \frac{125\operatorname{sech}(5\pi)}{40222}$$

## **Decimal approximation:**

-0.00001911593496126075908503136511058224125590211372808061...

-0.00001911593496126....

**Property:** 

$$-\frac{\operatorname{sech}\left(\frac{\pi}{2}\right)}{20.735} + \frac{\operatorname{sech}(3\pi)}{1530} - \frac{125\operatorname{sech}(5\pi)}{40.222}$$
 is a transcendental number

#### **Alternate forms:**

$$\frac{-27\,846\,\operatorname{sech}\left(\frac{\pi}{2}\right) + 377\,377\,\operatorname{sech}(3\,\pi) - 1\,794\,375\,\operatorname{sech}(5\,\pi)}{577\,386\,810}$$

$$\frac{4147\,\operatorname{sech}(3\,\pi) - 306\,\operatorname{sech}\left(\frac{\pi}{2}\right)}{6\,344\,910} - \frac{125\,\operatorname{sech}(5\,\pi)}{40\,222}$$

$$-\frac{1}{20\,735\,\cosh\left(\frac{\pi}{2}\right)} + \frac{1}{1530\,\cosh(3\,\pi)} - \frac{125}{40\,222\,\cosh(5\,\pi)}$$

cosh(x) is the hyperbolic cosine function

$$\begin{split} &\frac{1^{3} \operatorname{sech}\left(\frac{\pi}{2}\right)}{1^{4}-12^{4}}-\frac{3^{3} \operatorname{sech}(3 \, \pi)}{2 \left(3^{4}-12^{4}\right)}+\frac{5^{3} \operatorname{sech}(5 \, \pi)}{\left(5^{4}-12^{4}\right) 2}=\\ &\frac{1}{\cosh\left(\frac{\pi}{2}\right) \left(1^{4}-12^{4}\right)}-\frac{27}{2 \cosh(3 \, \pi) \left(3^{4}-12^{4}\right)}+\frac{5^{3}}{2 \cosh(5 \, \pi) \left(5^{4}-12^{4}\right)} \end{split}$$

$$\frac{1^{3} \operatorname{sech}\left(\frac{\pi}{2}\right)}{1^{4} - 12^{4}} - \frac{3^{3} \operatorname{sech}(3\pi)}{2\left(3^{4} - 12^{4}\right)} + \frac{5^{3} \operatorname{sech}(5\pi)}{\left(5^{4} - 12^{4}\right)2} = \frac{1}{\cos\left(\frac{i\pi}{2}\right)\left(1^{4} - 12^{4}\right)} - \frac{27}{2\cos(3i\pi)\left(3^{4} - 12^{4}\right)} + \frac{5^{3}}{2\cos(5i\pi)\left(5^{4} - 12^{4}\right)}$$

$$\begin{split} &\frac{1^3 \operatorname{sech}\left(\frac{\pi}{2}\right)}{1^4 - 12^4} - \frac{3^3 \operatorname{sech}(3\,\pi)}{2\left(3^4 - 12^4\right)} + \frac{5^3 \operatorname{sech}(5\,\pi)}{\left(5^4 - 12^4\right)2} = \\ &\frac{\csc\left(\frac{\pi}{2} + \frac{i\,\pi}{2}\right)}{1^4 - 12^4} - \frac{27 \csc\left(\frac{\pi}{2} + 3\,i\,\pi\right)}{2\left(3^4 - 12^4\right)} + \frac{\csc\left(\frac{\pi}{2} + 5\,i\,\pi\right)5^3}{2\left(5^4 - 12^4\right)} \end{split}$$

$$\begin{split} \frac{1^3 \operatorname{sech}\left(\frac{\pi}{2}\right)}{1^4 - 12^4} - \frac{3^3 \operatorname{sech}(3\pi)}{2\left(3^4 - 12^4\right)} + \frac{5^3 \operatorname{sech}(5\pi)}{\left(5^4 - 12^4\right)2} = \\ \sum_{k=0}^{\infty} \frac{2\left(-1\right)^k \left(1 + 2k\right) \left(-\frac{13923}{1+2\,k+2\,k^2} + \frac{377\,377}{37+4\,k+4\,k^2} - \frac{1\,794375}{10\,1+4\,k+4\,k^2}\right)}{288\,693\,405\,\pi} \end{split}$$

$$\frac{1^{3} \operatorname{sech}\left(\frac{\pi}{2}\right)}{1^{4} - 12^{4}} - \frac{3^{3} \operatorname{sech}(3\pi)}{2\left(3^{4} - 12^{4}\right)} + \frac{5^{3} \operatorname{sech}(5\pi)}{\left(5^{4} - 12^{4}\right)2} = \\ \sum_{k=0}^{\infty} -\frac{e^{\left(-5 - (10 - i)k\right)\pi} \left(1794375 - 377377 e^{2\pi + 4k\pi} + 27846 e^{(9\pi)/2 + 9k\pi}\right)}{288693405}$$

$$\begin{split} \frac{1^{3} \operatorname{sech}\left(\frac{\pi}{2}\right)}{1^{4} - 12^{4}} - \frac{3^{3} \operatorname{sech}(3\pi)}{2\left(3^{4} - 12^{4}\right)} + \frac{5^{3} \operatorname{sech}(5\pi)}{\left(5^{4} - 12^{4}\right)2} = \\ \sum_{k=0}^{\infty} -\frac{1}{577386810 \, k!} \, i \left( \operatorname{Li}_{-k}\left(-i \, e^{z_{0}}\right) - \operatorname{Li}_{-k}\left(i \, e^{z_{0}}\right) \right) \\ \left( 27846 \left(\frac{\pi}{2} - z_{0}\right)^{k} - 377377 \left(3\pi - z_{0}\right)^{k} + 1794375 \left(5\pi - z_{0}\right)^{k} \right) \, \operatorname{for} \, \frac{1}{2} + \frac{i \, z_{0}}{\pi} \notin \mathbb{Z} \end{split}$$

# **Integral representation:**

$$\frac{1^{3} \operatorname{sech}\left(\frac{\pi}{2}\right)}{1^{4} - 12^{4}} - \frac{3^{3} \operatorname{sech}(3\pi)}{2\left(3^{4} - 12^{4}\right)} + \frac{5^{3} \operatorname{sech}(5\pi)}{\left(5^{4} - 12^{4}\right)2} = \\ \int_{0}^{\infty} -\frac{\left(27\,846 - 377\,377\,t^{5\,i} + 1\,794\,375\,t^{9\,i}\right)t^{i}}{288\,693\,405\,\pi\left(1 + t^{2}\right)}\,dt$$

**Input:** 

$$1^{3} \times \frac{\operatorname{sech}\left(\frac{\pi}{2}\right)}{1^{4} + 12^{4}} - \frac{3^{3}\left(\frac{1}{2}\operatorname{sech}(3\pi)\right)}{3^{4} + 12^{4}} + \frac{5^{3}\left(\frac{1}{2}\operatorname{sech}(5\pi)\right)}{5^{4} + 12^{4}}$$

sech(x) is the hyperbolic secant function

#### **Exact result:**

$$\frac{\operatorname{sech}\left(\frac{\pi}{2}\right)}{20737} - \frac{\operatorname{sech}(3\pi)}{1542} + \frac{125\operatorname{sech}(5\pi)}{42722}$$

## **Decimal approximation:**

0.000019114847340277282671102750452267872320911492891002346...

0.00001911484734027.....

## **Property:**

$$\frac{\operatorname{sech}\left(\frac{\pi}{2}\right)}{20737} - \frac{\operatorname{sech}(3\pi)}{1542} + \frac{125\operatorname{sech}(5\pi)}{42722} \text{ is a transcendental number}$$

#### **Alternate forms:**

$$\frac{32\,938\,662\,\mathrm{sech}\left(\frac{\pi}{2}\right)-442\,963\,057\,\mathrm{sech}(3\,\pi)+1\,998\,528\,375\,\mathrm{sech}(5\,\pi)}{683\,049\,033\,894}$$

$$\frac{1542\,\text{sech}\left(\frac{\pi}{2}\right) - 20\,737\,\text{sech}(3\,\pi)}{31\,976\,454} + \frac{125\,\text{sech}(5\,\pi)}{42\,722}$$

$$\frac{1}{20737\cosh(\frac{\pi}{2})} - \frac{1}{1542\cosh(3\pi)} + \frac{125}{42722\cosh(5\pi)}$$

 $\cosh(x)$  is the hyperbolic cosine function

## Alternative representations:

$$\begin{split} &\frac{1^{3} \operatorname{sech}\left(\frac{\pi}{2}\right)}{1^{4} + 12^{4}} - \frac{3^{3} \operatorname{sech}(3 \, \pi)}{2 \left(3^{4} + 12^{4}\right)} + \frac{5^{3} \operatorname{sech}(5 \, \pi)}{\left(5^{4} + 12^{4}\right) 2} = \\ &\frac{1}{\cosh\left(\frac{\pi}{2}\right)\left(1^{4} + 12^{4}\right)} - \frac{27}{2 \cosh(3 \, \pi)\left(3^{4} + 12^{4}\right)} + \frac{5^{3}}{2 \cosh(5 \, \pi)\left(5^{4} + 12^{4}\right)} \end{split}$$

$$\begin{split} &\frac{1^{3} \operatorname{sech}\left(\frac{\pi}{2}\right)}{1^{4}+12^{4}}-\frac{3^{3} \operatorname{sech}(3 \, \pi)}{2 \left(3^{4}+12^{4}\right)}+\frac{5^{3} \operatorname{sech}(5 \, \pi)}{\left(5^{4}+12^{4}\right) 2}=\\ &\frac{1}{\cos\left(\frac{i \, \pi}{2}\right) \left(1^{4}+12^{4}\right)}-\frac{27}{2 \cos(3 \, i \, \pi) \left(3^{4}+12^{4}\right)}+\frac{5^{3}}{2 \cos(5 \, i \, \pi) \left(5^{4}+12^{4}\right)} \end{split}$$

$$\begin{split} &\frac{1^3 \operatorname{sech}\left(\frac{\pi}{2}\right)}{1^4 + 12^4} - \frac{3^3 \operatorname{sech}(3\,\pi)}{2\left(3^4 + 12^4\right)} + \frac{5^3 \operatorname{sech}(5\,\pi)}{\left(5^4 + 12^4\right)2} = \\ &\frac{\csc\left(\frac{\pi}{2} + \frac{i\,\pi}{2}\right)}{1^4 + 12^4} - \frac{27 \csc\left(\frac{\pi}{2} + 3\,i\,\pi\right)}{2\left(3^4 + 12^4\right)} + \frac{\csc\left(\frac{\pi}{2} + 5\,i\,\pi\right)5^3}{2\left(5^4 + 12^4\right)} \end{split}$$

$$\frac{1^{3} \operatorname{sech}\left(\frac{\pi}{2}\right)}{1^{4} + 12^{4}} - \frac{3^{3} \operatorname{sech}(3\pi)}{2\left(3^{4} + 12^{4}\right)} + \frac{5^{3} \operatorname{sech}(5\pi)}{\left(5^{4} + 12^{4}\right)2} = \sum_{k=0}^{\infty} \left(\frac{125 e^{\left(-5 - (10 - i)k\right)\pi}}{21361} - \frac{1}{771} e^{\left(-3 - (6 - i)k\right)\pi} + \frac{2 e^{\left(-1/2 - (1 - i)k\right)\pi}}{20737}\right)$$

$$\frac{1^{3} \operatorname{sech}\left(\frac{\pi}{2}\right)}{1^{4} + 12^{4}} - \frac{3^{3} \operatorname{sech}(3\pi)}{2\left(3^{4} + 12^{4}\right)} + \frac{5^{3} \operatorname{sech}(5\pi)}{\left(5^{4} + 12^{4}\right)2} = \\ \sum_{k=0}^{\infty} \frac{2\left(-1\right)^{k} \left(1 + 2k\right) \left(\frac{16469331}{1 + 2k + 2k^{2}} - \frac{442963057}{37 + 4k + 4k^{2}} + \frac{1998528375}{101 + 4k + 4k^{2}}\right)}{341524516947\pi}$$

$$\begin{split} \frac{1^3 \operatorname{sech}\left(\frac{\pi}{2}\right)}{1^4 + 12^4} - \frac{3^3 \operatorname{sech}(3\pi)}{2\left(3^4 + 12^4\right)} + \frac{5^3 \operatorname{sech}(5\pi)}{\left(5^4 + 12^4\right)2} = \\ \sum_{k=0}^{\infty} \frac{1}{683\,049\,033\,894\,k!} \, i \left(\operatorname{Li}_{-k}\left(-i\,e^{z_0}\right) - \operatorname{Li}_{-k}\left(i\,e^{z_0}\right)\right) \left(32\,938\,662\left(\frac{\pi}{2} - z_0\right)^k - 442\,963\,057\,(3\pi - z_0)^k + 1\,998\,528\,375\,(5\pi - z_0)^k\right) \, \operatorname{for} \, \frac{1}{2} + \frac{i\,z_0}{\pi} \notin \mathbb{Z} \end{split}$$

## **Integral representation:**

$$\frac{1^{3} \operatorname{sech}\left(\frac{\pi}{2}\right)}{1^{4} + 12^{4}} - \frac{3^{3} \operatorname{sech}(3\pi)}{2\left(3^{4} + 12^{4}\right)} + \frac{5^{3} \operatorname{sech}(5\pi)}{\left(5^{4} + 12^{4}\right)2} = \\ \int_{0}^{\infty} \frac{\left(32938662 - 442963057t^{5i} + 1998528375t^{9i}\right)t^{i}}{341524516947\pi\left(1 + t^{2}\right)} dt$$

Where 24 can be identified with the number of the transverse degrees of freedom in the bosonic string

## Input:

$$\frac{1}{2^{4}} \times \frac{1}{1^{3} \times \frac{\operatorname{sech}\left(\frac{\pi}{2}\right)}{1^{4} + 12^{4}} - \frac{3^{3}\left(\frac{1}{2}\operatorname{sech}(3\pi)\right)}{3^{4} + 12^{4}} + \frac{5^{3}\left(\frac{1}{2}\operatorname{sech}(5\pi)\right)}{5^{4} + 12^{4}}} - 64 - \pi}$$

 $\operatorname{sech}(x)$  is the hyperbolic secant function

#### **Exact result:**

$$-64 - \pi + \frac{1}{24 \left(\frac{\operatorname{sech}(\frac{\pi}{2})}{20\ 737} - \frac{\operatorname{sech}(3\ \pi)}{1542} + \frac{125\ \operatorname{sech}(5\ \pi)}{42\ 722}\right)}$$

#### **Decimal approximation:**

2112.664812541705066184005071570661311862410928268875704164...

2112.66481254..... result practically equal to the rest mass of strange D meson 2112.3

#### **Alternate forms:**

$$-64 - \pi + \frac{113\,841\,505\,649}{4\left(32\,938\,662\,\operatorname{sech}\left(\frac{\pi}{2}\right) - 442\,963\,057\,\operatorname{sech}(3\,\pi) + 1\,998\,528\,375\,\operatorname{sech}(5\,\pi)\right)}$$
$$-64 - \pi + \frac{1}{\frac{24\,\operatorname{sech}\left(\frac{\pi}{2}\right)}{20\,737} - \frac{4}{257}\,\operatorname{sech}(3\,\pi) + \frac{1500\,\operatorname{sech}(5\,\pi)}{21\,361}}$$

$$-64 - \pi + \frac{1}{24 \left(\frac{1}{20\,737\,\cosh(\frac{\pi}{2})} - \frac{1}{1542\,\cosh(3\,\pi)} + \frac{125}{42\,722\,\cosh(5\,\pi)}\right)}$$

 $\cosh(x)$  is the hyperbolic cosine function

## **Alternative representations:**

$$\frac{1}{\left(\frac{1^{3} \operatorname{sech}(\frac{\pi}{2})}{1^{4}+12^{4}} - \frac{3^{3} \operatorname{sech}(3\pi)}{2\left(3^{4}+12^{4}\right)} + \frac{5^{3} \operatorname{sech}(5\pi)}{2\left(5^{4}+12^{4}\right)}\right) 24} - 64 - \pi = \frac{1}{24\left(\frac{1}{\cosh(\frac{\pi}{2})\left(1^{4}+12^{4}\right)} - \frac{27}{2\cosh(3\pi)\left(3^{4}+12^{4}\right)} + \frac{5^{3}}{2\cosh(5\pi)\left(5^{4}+12^{4}\right)}\right)}$$

$$\frac{1}{\left(\frac{1^{3} \operatorname{sech}(\frac{\pi}{2})}{1^{4}+12^{4}} - \frac{3^{3} \operatorname{sech}(3\pi)}{2\left(3^{4}+12^{4}\right)} + \frac{5^{3} \operatorname{sech}(5\pi)}{2\left(5^{4}+12^{4}\right)}\right) 24} - 64 - \pi = \frac{1}{24\left(\frac{1}{\cos(\frac{i\pi}{2})\left(1^{4}+12^{4}\right)} - \frac{27}{2\cos(3i\pi)\left(3^{4}+12^{4}\right)} + \frac{5^{3}}{2\cos(5i\pi)\left(5^{4}+12^{4}\right)}\right)}$$

$$\frac{1}{\left(\frac{1^{3} \operatorname{sech}(\frac{\pi}{2})}{1^{4}+12^{4}} - \frac{3^{3} \operatorname{sech}(3\pi)}{2\left(3^{4}+12^{4}\right)} + \frac{5^{3} \operatorname{sech}(5\pi)}{2\left(5^{4}+12^{4}\right)}\right) 24} - 64 - \pi = \frac{1}{24\left(\frac{1}{\cos(-\frac{i\pi}{2})\left(1^{4}+12^{4}\right)} - \frac{27}{2\cos(-3i\pi)\left(3^{4}+12^{4}\right)} + \frac{5^{3}}{2\cos(-5i\pi)\left(5^{4}+12^{4}\right)}\right)}$$

$$\frac{1}{\left(\frac{1^{3} \operatorname{sech}(\frac{\pi}{2})}{1^{4}+12^{4}} - \frac{3^{3} \operatorname{sech}(3\pi)}{2\left(3^{4}+12^{4}\right)} + \frac{5^{3} \operatorname{sech}(5\pi)}{2\left(5^{4}+12^{4}\right)}\right) 24} - 64 - \pi = \frac{1}{24 \sum_{k=0}^{\infty} \frac{2\left(-1\right)^{k} \left(1+2k\right) \left(\frac{16469\ 331}{1+2\ k+2\ k^{2}} - \frac{442\ 963057}{37+4\ k+4\ k^{2}} + \frac{1998\ 528\ 375}{101+4\ k+4\ k^{2}}\right)}{341524516947\ \pi}} - 64 - \pi = \frac{1}{24 \sum_{k=0}^{\infty} \left(\frac{1^{3} \operatorname{sech}(\frac{\pi}{2})}{2\left(3^{4}+12^{4}\right)} + \frac{5^{3} \operatorname{sech}(5\pi)}{2\left(5^{4}+12^{4}\right)}\right) 24} - 64 - \pi + \frac{1}{24 \sum_{k=0}^{\infty} \left(\frac{125\ (-1)^{k}\ e^{-5\ \pi-10\ k\pi}}{21361} - \frac{1}{771}\ (-1)^{k}\ e^{-3\ \pi-6\ k\pi} + \frac{2\ (-1)^{k}\ e^{-\pi/2-k\pi}}{20737}\right)}$$

$$\frac{1}{\left(\frac{1^{3}\operatorname{sexh}(\frac{\pi}{2})}{1^{4}+12^{4}} - \frac{3^{3}\operatorname{sexh}(3\pi)}{2(3^{4}+12^{4})} + \frac{5^{3}\operatorname{sexh}(5\pi)}{2(5^{4}+12^{4})}\right)24} - 64 - \pi =$$

$$-64 - \pi + 1 / \left(24 \sum_{k=0}^{\infty} \left(i\left(\operatorname{Li}_{-k}(-ie^{z_{0}}) - \operatorname{Li}_{-k}(ie^{z_{0}})\right)\right)\left(32938662\left(\frac{\pi}{2} - z_{0}\right)^{k} - 442963057\left(3\pi - z_{0}\right)^{k} + 1998528375\left(5\pi - z_{0}\right)^{k}\right)\right) /$$

$$(683049033894 k!) \int \operatorname{for} \frac{1}{2} + \frac{iz_{0}}{\pi} \notin \mathbb{Z}$$

## **Integral representation:**

$$\frac{1}{\left(\frac{1^{3}\operatorname{sech}(\frac{\pi}{2})}{1^{4}+12^{4}} - \frac{3^{3}\operatorname{sech}(3\pi)}{2\left(3^{4}+12^{4}\right)} + \frac{5^{3}\operatorname{sech}(5\pi)}{2\left(5^{4}+12^{4}\right)}\right)24} - 64 - \pi = \frac{1}{24\int_{0}^{\infty} \frac{\left(32\,938\,662 - 442\,963057\,t^{5\,i} + 1\,998\,528\,375\,t^{9\,i}\right)t^{i}}{341524\,516947\,\pi\left(1+t^{2}\right)}}\,dt$$

$$\frac{1}{256} \times \frac{1}{1^3 \times \frac{\mathrm{sech}(\frac{\pi}{2})}{1^4 + 12^4} - \frac{3^3 \left(\frac{1}{2} \operatorname{sech}(3 \pi)\right)}{3^4 + 12^4} + \frac{5^3 \left(\frac{1}{2} \operatorname{sech}(5 \pi)\right)}{5^4 + 12^4}} - 64 - \frac{1}{\phi}$$

sech(x) is the hyperbolic secant function

φ is the golden ratio

#### **Exact result:**

$$-\frac{1}{\phi} - 64 + \frac{1}{256 \left(\frac{\operatorname{sech}(\frac{\pi}{2})}{20737} - \frac{\operatorname{sech}(3\pi)}{1542} + \frac{125\operatorname{sech}(5\pi)}{42722}\right)}$$

## **Decimal approximation:**

139.7388164983089982226517614425663132647741999765927505739...

139.738816498... result practically equal to the rest mass of Pion meson 139.57

Property: 
$$-64 - \frac{1}{\phi} + \frac{1}{256 \left( \frac{\operatorname{sech}(\frac{\pi}{2})}{20737} - \frac{\operatorname{sech}(3\pi)}{1542} + \frac{125\operatorname{sech}(5\pi)}{42722} \right)} \text{ is a transcendental number}$$

Alternate forms: 
$$-\frac{1}{\phi} - 64 + \frac{1}{\frac{256 \operatorname{sech}(\frac{\pi}{2})}{20737} - \frac{128}{771} \operatorname{sech}(3\pi) + \frac{16000 \operatorname{sech}(5\pi)}{21361}}$$

$$\frac{1}{2} \left(-127 - \sqrt{5}\right) + \frac{1}{256 \left(\frac{\operatorname{sech}(\frac{\pi}{2})}{20737} - \frac{\operatorname{sech}(3\pi)}{1542} + \frac{125 \operatorname{sech}(5\pi)}{42722}\right)}$$

$$-64 - \frac{1}{\phi} + \frac{1}{256 \left(\frac{1}{20737 \operatorname{cosh}(\frac{\pi}{2})} - \frac{1}{1542 \operatorname{cosh}(3\pi)} + \frac{125}{42722 \operatorname{cosh}(5\pi)}\right)}$$

 $\cosh(x)$  is the hyperbolic cosine function

$$\frac{1}{\left(\frac{1^3 \operatorname{sech}(\frac{\pi}{2})}{1^4 + 12^4} - \frac{3^3 \operatorname{sech}(3\pi)}{2\left(3^4 + 12^4\right)} + \frac{5^3 \operatorname{sech}(5\pi)}{2\left(5^4 + 12^4\right)}\right) 256} - 64 - \frac{1}{\phi} = \frac{1}{\left(\frac{1^3 \operatorname{sech}(\frac{\pi}{2})}{1^4 + 12^4} - \frac{3^3 \operatorname{sech}(3\pi)}{2\left(5^4 + 12^4\right)} + \frac{5^3 \operatorname{sech}(5\pi)}{2 \operatorname{cosh}(3\pi)\left(3^4 + 12^4\right)} + \frac{5^3}{2 \operatorname{cosh}(5\pi)\left(5^4 + 12^4\right)}\right) }$$
 
$$\frac{1}{\left(\frac{1^3 \operatorname{sech}(\frac{\pi}{2})}{1^4 + 12^4} - \frac{3^3 \operatorname{sech}(3\pi)}{2\left(3^4 + 12^4\right)} + \frac{5^3 \operatorname{sech}(5\pi)}{2\left(5^4 + 12^4\right)}\right) 256} - 64 - \frac{1}{\phi} = \frac{1}{\left(\frac{1^3 \operatorname{sech}(\frac{\pi}{2})}{1^4 + 12^4} - \frac{3^3 \operatorname{sech}(3\pi)}{2\left(3^4 + 12^4\right)} + \frac{5^3 \operatorname{sech}(5\pi)}{2\left(5^4 + 12^4\right)}\right) }{\left(\frac{1^3 \operatorname{sech}(\frac{\pi}{2})}{1^4 + 12^4} - \frac{3^3 \operatorname{sech}(3\pi)}{2\left(3^4 + 12^4\right)} + \frac{5^3 \operatorname{sech}(5\pi)}{2\left(5^4 + 12^4\right)}\right) }$$
 
$$\frac{1}{\left(\frac{1^3 \operatorname{sech}(\frac{\pi}{2})}{1^4 + 12^4} - \frac{3^3 \operatorname{sech}(3\pi)}{2\left(3^4 + 12^4\right)} + \frac{5^3 \operatorname{sech}(5\pi)}{2\left(5^4 + 12^4\right)}\right) } 256} - 64 - \frac{1}{\phi} = \frac{1}{\left(\frac{1^3 \operatorname{sech}(\frac{\pi}{2})}{1^4 + 12^4} - \frac{3^3 \operatorname{sech}(3\pi)}{2\left(3^4 + 12^4\right)} + \frac{5^3 \operatorname{sech}(5\pi)}{2\left(5^4 + 12^4\right)}\right) }{256} - 64 - \frac{1}{\phi} + \frac{1}{256} \left(\frac{1}{\operatorname{cos}(\frac{i\pi}{2})(1^4 + 12^4)} - \frac{27}{2 \operatorname{cos}(-3 i\pi)(3^4 + 12^4)} + \frac{5^3}{2 \operatorname{cos}(-5 i\pi)(5^4 + 12^4)}\right) }$$

$$\frac{1}{\left(\frac{1^{3} \operatorname{sech}(\frac{\pi}{2})}{1^{4}+12^{4}} - \frac{3^{3} \operatorname{sech}(3\pi)}{2(3^{4}+12^{4})} + \frac{5^{3} \operatorname{sech}(5\pi)}{2(5^{4}+12^{4})}\right) 256} - 64 - \frac{1}{\phi} = \frac{1}{256 \sum_{k=0}^{\infty} \frac{2(-1)^{k} \left(1+2k\right) \left(\frac{16 \cdot 469 \cdot 331}{1+2 \cdot k+2 \cdot k^{2}} - \frac{442963 \cdot 057}{3744 \cdot k+4 \cdot k^{2}} + \frac{1998528 \cdot 375}{10144 \cdot k+4 \cdot k^{2}}\right)}{341 \cdot 524516947 \pi}$$

$$\frac{1}{\left(\frac{1^{3} \operatorname{sech}(\frac{\pi}{2})}{1^{4}+12^{4}} - \frac{3^{3} \operatorname{sech}(3\pi)}{2(3^{4}+12^{4})} + \frac{5^{3} \operatorname{sech}(5\pi)}{2(5^{4}+12^{4})}\right) 256} - 64 - \frac{1}{\phi} = \frac{1}{256 \sum_{k=0}^{\infty} \left(\frac{125(-1)^{k} e^{-5\pi-10 \cdot k\pi}}{21361} - \frac{1}{771} \left(-1\right)^{k} e^{-3\pi-6k\pi} + \frac{2(-1)^{k} e^{-\pi/2-k\pi}}{20737}\right)}{\frac{1}{1^{4}+12^{4}} - \frac{3^{3} \operatorname{sech}(3\pi)}{2(3^{4}+12^{4})} + \frac{5^{3} \operatorname{sech}(5\pi)}{2(5^{4}+12^{4})}\right) 256} - 64 - \frac{1}{\phi} = \frac{1}{64 - \frac{1}{\phi}} + \frac{1}{256 \sum_{k=0}^{\infty} \left(i \left(\text{Li}_{-k}(-i e^{2u}) - \text{Li}_{-k}(i e^{2u})\right)\right) \left(32938662 \left(\frac{\pi}{2} - z_{0}\right)^{k} - 442963057 \left(3\pi - z_{0}\right)^{k} + 1998528375 \left(5\pi - z_{0}\right)^{k}\right)\right)}{442963057 \left(3\pi - z_{0}\right)^{k} + 1998528375 \left(5\pi - z_{0}\right)^{k}\right)\right)}$$

$$(683049033894 k!) for \frac{1}{2} + \frac{i z_{0}}{\pi} \notin \mathbb{Z}$$

## **Integral representation:**

Integral representation:  

$$\frac{1}{\left(\frac{1^{3}\operatorname{sech}(\frac{\pi}{2})}{1^{4}+12^{4}} - \frac{3^{3}\operatorname{sech}(3\pi)}{2\left(3^{4}+12^{4}\right)} + \frac{5^{3}\operatorname{sech}(5\pi)}{2\left(5^{4}+12^{4}\right)}\right)256} - 64 - \frac{1}{\phi} = \frac{1}{256\int_{0}^{\infty} \frac{\left(32938662 - 442963057t^{5}i + 1988528375t^{9}i\right)t^{i}}{341524516947\pi\left(1+t^{2}\right)}} dt$$

From the sum of the results, we obtain:

(76.6132768639 + 76.278746097 - 0.000336436347 - 0.000328808677 - 0.0002411638)+0.0002333200322 -0.00001911593496126 +0.00001911484734027)

#### **Input interpretation:**

76.6132768639 + 76.278746097 - 0.000336436347 - 0.000328808677 - 0.0002411638 + 0.0002333200322 - 0.00001911593496126 + 0.00001911484734027

#### **Result:**

152.89134987102057901 152.891349871.....

 $(76.6132768639 + 76.278746097 - 0.000336436347 - 0.000328808677 - 0.0002411638 + 0.0002333200322 - 0.00001911593496126 + 0.00001911484734027) - 18-7-golden \\ ratio^2$ 

Where 18 and 7 are Lucas numbers

#### **Input interpretation:**

```
 \begin{array}{l} (76.6132768639 + 76.278746097 - 0.000336436347 - \\ 0.000328808677 - 0.0002411638 + 0.0002333200322 - \\ 0.00001911593496126 + 0.00001911484734027) - 18 - 7 - \phi^2 \end{array}
```

φ is the golden ratio

#### **Result:**

125.27331588...

125.27331588... result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T=0 and to the Higgs boson mass 125.18

```
 (76.61327686390000 + 76.2787460970000 - 0.000336436 - 0.000328809 - 0.000241164 + 0.00023332 - 0.000019115934961260000 + 0.000019114847340270000) - 18 - 7 - <math>\phi^2 = 127.891 - (2\sin(54°))^2   (76.61327686390000 + 76.2787460970000 - 0.000336436 - 0.000328809 - 0.000241164 + 0.00023332 - 0.000019115934961260000 + 0.000019114847340270000) - 18 - 7 - <math>\phi^2 = 127.891 - (-2\cos(216°))^2   (76.61327686390000 + 76.2787460970000 - 0.000336436 - 0.000328809 - 0.000241164 + 0.00023332 - 0.000019115934961260000 + 0.000241164 + 0.00023332 - 0.000019115934961260000 + 0.000019114847340270000) - 18 - 7 - <math>\phi^2 = 127.891 - (-2\sin(666°))^2
```

(76.6132768639 + 76.278746097 - 0.000336436347 - 0.000328808677 - 0.0002411638+0.0002333200322 -0.00001911593496126 +0.00001911484734027)-11-golden ratio^2

## **Input interpretation:**

```
(76.6132768639 + 76.278746097 - 0.000336436347 -
     0.000328808677 - 0.0002411638 + 0.0002333200322 -
     0.00001911593496126 + 0.00001911484734027) - 11 - \phi^{2}
```

ø is the golden ratio

#### **Result:**

139.27331588...

139.27331588... result practically equal to the rest mass of Pion meson 139.57

#### **Alternative representations:**

```
(76.61327686390000 + 76.2787460970000 - 0.000336436 - 0.000328809 -
       0.000241164 + 0.00023332 - 0.000019115934961260000 +
       0.000019114847340270000) -11-\phi^2=141.891-(2\sin(54^\circ))^2
(76.61327686390000 + 76.2787460970000 - 0.000336436 - 0.000328809 -
        0.000241164 + 0.00023332 - 0.000019115934961260000 +
       0.000019114847340270000) -11 - \phi^2 = 141.891 - (-2\cos(216^\circ))^2
(76.61327686390000 + 76.2787460970000 - 0.000336436 - 0.000328809 -
       0.000241164 + 0.00023332 - 0.000019115934961260000 +
       0.000019114847340270000) -11 - \phi^2 = 141.891 - (-2\sin(666^\circ))^2
```

(sqrt10-3)(1/76.6132768639 \*1/76.278746097 \*1/-0.000336436347 \*1/-0.000328808677 \*1/ -0.0002411638 \*1/ 0.0002333200322 \*1/ -0.00001911593496126 \* 1/0.00001911484734027)

## **Input interpretation:**

Input interpretation: 
$$(\sqrt{10} - 3) \left( \frac{1}{76.6132768639} \times \frac{1}{76.278746097} \left( -\frac{1}{0.000336436347} \right) \right. \\ \left. \left( -\frac{1}{0.000328808677} \right) \left( -\frac{1}{0.0002411638} \right) \times \frac{1}{0.0002333200322} \right. \\ \left. \left( -\frac{1}{0.00001911593496126} \right) \times \frac{1}{0.00001911484734027} \right)$$

#### **Result:**

 $1.220884... \times 10^{19}$ 

 $1.220884...*10^{19} \approx 1.2209*10^{19}$  GeV that is the value of Planck energy

#### Example of Ramanujan mathematics applied to the physics

From:

## Replica Wormholes and the Entropy of Hawking Radiation

Ahmed Almheiri, Thomas Hartman, Juan Maldacena, Edgar Shaghoulian and Amirhossein Tajdini - arXiv:1911.12333v1 [hep-th] 27 Nov 2019

From chapter "Geometry of the black hole", is described the following formula:

$$S_{\text{gen}}([-a,b]) = S_0 + \frac{2\pi\phi_r}{\beta} \frac{1}{\tanh\left(\frac{2\pi a}{\beta}\right)} + \frac{c}{6}\log\left(\frac{2\beta\sinh^2\left(\frac{n}{\beta}(a+b)\right)}{\pi\epsilon\sinh\left(\frac{2\pi a}{\beta}\right)}\right)$$
(3.10)

From the previous Ramanujan expressions

$$54 + \phi - \frac{1}{1 - 82\,944\,\pi \left(\frac{1}{20\,737\,\sinh(\pi)} - \frac{1}{10\,376\,\sinh(2\,\pi)} + \frac{1}{6939\,\sinh(3\,\pi)}\right)}$$

$$\frac{296\,447\,958\,\tanh\!\left(\frac{\pi}{2}\right)+442\,963\,057\,\tanh(3\,\pi)+719\,470\,215\,\tanh(5\,\pi)}{6\,147\,441\,305\,046}$$

We obtain:

 $1/\tanh((((296447958 (\pi/2) + 442963057 (3 \pi) + 719470215 (5 \pi))/6147441305046)))$ 

#### **Input:**

 $296447958 \tanh \left(\frac{\pi}{2}\right) + 442963057 \tanh \left(3\pi\right) + 719470215 \tanh \left(5\pi\right)$ 

tanh(x) is the hyperbolic tangent function

#### **Exact result:**

296 447 958  $\tanh\left(\frac{\pi}{2}\right)$  + 442 963 057  $\tanh(3\pi)$  + 719 470 215  $\tanh(5\pi)$ 

## **Decimal approximation:**

4285.958605954208213734361862548850123878070152765655347630...

4285.9586

### **Property:**

 $\frac{6\,147\,441\,305\,046}{296\,447\,958\,\tanh\left(\frac{\pi}{2}\right) + 442\,963\,057\,\tanh(3\,\pi) + 719\,470\,215\,\tanh(5\,\pi)}$ is a transcendental number

#### **Alternate forms:**

296 447 958  $\tanh\left(\frac{\pi}{2}\right)$  + 20 737 (21 361  $\tanh(3\pi)$  + 34 695  $\tanh(5\pi)$ )

$$\frac{296447958 \sinh(\pi)}{1 + \cosh(\pi)} + \frac{442963057 \sinh(6\pi)}{1 + \cosh(6\pi)} + \frac{719470215 \sinh(10\pi)}{1 + \cosh(10\pi)}$$

$$\frac{6\ 147\ 441\ 305\ 046}{\frac{296447958\ \text{sinh}\left(\frac{\pi}{2}\right)}{\cosh\left(\frac{\pi}{2}\right)} + \frac{442\ 963057\ \text{sinh}\left(3\ \pi\right)}{\cosh(3\ \pi)} + \frac{719\ 470\ 215\ \text{sinh}\left(5\ \pi\right)}{\cosh(5\ \pi)}$$

 $\cosh(x)$  is the hyperbolic cosine function

sinh(x) is the hyperbolic sine function

$$\frac{1}{\frac{296447958 \tanh(\frac{\pi}{2}) + 442963057 \tanh(3\pi) + 719470215 \tanh(5\pi)}{6147441305046}} = \frac{1}{\frac{296447958 + 442963057 + 719470215}{\coth(\frac{\pi}{2})} + \frac{296447958 + 442963057 + 719470215}{\coth(3\pi)}}{6147441305046}}$$

$$\frac{1}{296447958 \tanh(\frac{\pi}{2}) + 442963057 \tanh(3\pi) + 719470215 \tanh(5\pi)}} = \frac{1}{6147441305046}$$

$$\frac{719470215\left(-1 + \frac{2}{1 + e^{-10\pi}}\right) + 442963057\left(-1 + \frac{2}{1 + e^{-6\pi}}\right) + 296447958\left(-1 + \frac{2}{1 + e^{-\pi}}\right)}{6147441305046}$$

$$\frac{1}{296447958 \tanh(\frac{\pi}{2}) + 442963057 \tanh(3\pi) + 719470215 \tanh(5\pi)}} = \frac{6147441305046}{\cot(\frac{\pi}{2})}$$

$$\frac{-\frac{296447958i}{\cot(\frac{\pi}{2})} - \frac{442963057i}{\cot(3i\pi)} - \frac{719470215i}{\cot(5i\pi)}}{\cot(5i\pi)}}{6147441305046}$$

$$\frac{1}{\frac{296447958 \tanh(\frac{\pi}{2})+442963057 \tanh(3\pi)+719470215 \tanh(5\pi)}{6147441305046}} = \frac{6147441305046}{-\left(3073720652523\left/\left(-729440615 + \sum_{k=0}^{\infty} \left(442963057 e^{\left(-6-(6-i)k\right)\pi} + 296447958 e^{\left(-1-(1-i)k\right)\pi} + 719470215(-1)^k e^{-10\left(1+k\right)\pi}\right)\right)\right)}$$

$$\frac{296447958 \tanh \binom{\pi}{2} + 442963057 \tanh (3\pi) + 719470215 \tanh (5\pi)}{1024573550841} \\ 4\pi \sum_{k=1}^{\infty} \frac{\frac{49407993}{1 + (1 - 2k)^2} + 20737 \left(\frac{21361}{37 - 4k + 4k^2} + \frac{57825}{101 - 4k + 4k^2}\right)}{\pi^2}$$

$$\frac{1}{296447958 \tanh(\frac{\pi}{2}) + 442963057 \tanh(3\pi) + 719470215 \tanh(5\pi)} =$$

$$6 147441305046 \left/ \left( \sum_{k=0}^{\infty} \left( -296447958 \left( \delta_{k} + \frac{2^{1+k} \operatorname{Li}_{-k} \left( -e^{2z_{0}} \right)}{k!} \right) \left( \frac{\pi}{2} - z_{0} \right)^{k} - 442963057 \left( \delta_{k} + \frac{2^{1+k} \operatorname{Li}_{-k} \left( -e^{2z_{0}} \right)}{k!} \right) (3\pi - z_{0})^{k} - 719470215 \left( \delta_{k} + \frac{2^{1+k} \operatorname{Li}_{-k} \left( -e^{2z_{0}} \right)}{k!} \right) (5\pi - z_{0})^{k} \right) \right\} \text{ for } \frac{1}{2} + \frac{i z_{0}}{\pi} \notin \mathbb{Z}$$

$$\frac{1}{\frac{296447958 \tanh(\frac{\pi}{2}) + 442963057 \tanh(3\pi) + 719470215 \tanh(5\pi)}{6147441305046}} = \frac{1}{\int_{0}^{\frac{\pi}{2}} (296447958 \operatorname{sech}^{2}(t) + 124422 (21361 \operatorname{sech}^{2}(6t) + 57825 \operatorname{sech}^{2}(10t))) dt}$$

sech(x) is the hyperbolic secant function

 $54 + \text{golden ratio} - \frac{1}{(1 - 82944 \pi (\frac{1}{(20737 \sinh(\pi))} - \frac{1}{(10376 \sinh(2 \pi))} + \frac{1}{(6939 + 1)^2})} + \frac{1}{(6939 + 1)^2}$  $sinh(3 \pi))))$ 

#### Input:

54 + 
$$\phi$$
 -  $\frac{1}{1 - 82944 \pi \left(\frac{1}{20737 \sinh(\pi)} - \frac{1}{10376 \sinh(2\pi)} + \frac{1}{6939 \sinh(3\pi)}\right)}$ 

 $\sinh(x)$  is the hyperbolic sine function

ø is the golden ratio

#### **Exact result:**

$$\phi + 54 - \frac{1}{1 - 82944 \pi \left(\frac{\cosh(\pi)}{20737} - \frac{\cosh(2\pi)}{10376} + \frac{\cosh(3\pi)}{6939}\right)}$$

csch(x) is the hyperbolic cosecant function

# **Decimal approximation:**

3096.900298273126807801702180739848133876304011281727955975...

3096.9002982... result practically equal to the rest mass of J/Psi meson 3096.916

#### **Alternate forms:**

Alternate forms:  

$$\phi + 54 - \frac{1}{1 - 82944 \pi \left(\frac{\operatorname{csch}(3\pi)}{6939} + \operatorname{csch}(\pi) \left(\frac{1}{20737} - \frac{\operatorname{sech}(\pi)}{20752}\right)\right)}$$

$$\frac{1}{2} \left( 109 + \sqrt{5} \right) + 6912243473 / (-6912243473 + 27647640576 \pi \operatorname{csch}(\pi) - 55255312512 \pi \operatorname{csch}(2\pi) + 82624171008 \pi \operatorname{csch}(3\pi) \right)$$

$$54 + \frac{1}{2} \left( 1 + \sqrt{5} \right) - \frac{1}{1 - 82944 \pi \left( \frac{\operatorname{csch}(\pi)}{20737} - \frac{\operatorname{csch}(2\pi)}{10376} + \frac{\operatorname{csch}(3\pi)}{6939} \right)}$$

sech(x) is the hyperbolic secant function

### **Alternative representations:**

$$54 + \phi - \frac{1}{1 - 82944 \pi \left(\frac{1}{20737 \sinh(\pi)} - \frac{1}{10376 \sinh(2\pi)} + \frac{1}{6939 \sinh(3\pi)}\right)} =$$

$$54 + \phi - \frac{1}{1 - 82944 \pi \left(\frac{1}{\frac{20737}{\cosh(\pi)}} - \frac{1}{\frac{10376}{\cosh(2\pi)}} + \frac{1}{\frac{6939}{\cosh(3\pi)}}\right)}$$

$$54 + \phi - \frac{1}{1 - 82944 \pi \left(\frac{1}{\frac{20737}{\cos(\pi)}} - \frac{1}{10376 \sinh(2\pi)} + \frac{1}{6939 \sinh(3\pi)}\right)} =$$

$$54 + \phi - \frac{1}{1 - 82944 \pi \left(-\frac{1}{\frac{20737i}{\csc(i\pi)}} - -\frac{1}{\frac{10376i}{\csc(2i\pi)}} + -\frac{1}{\frac{6939i}{\csc(3i\pi)}}\right)}$$

$$54 + \phi - \frac{1}{1 - 82944 \pi \left(\frac{1}{\frac{1}{20737 \sinh(\pi)}} - \frac{1}{\frac{10376 \sinh(2\pi)}{\csc(2i\pi)}} + \frac{1}{\frac{6939 \sinh(3\pi)}{\csc(3i\pi)}}\right)} =$$

$$54 + \phi - \frac{1}{1 - 82944 \pi \left(\frac{1}{\frac{1}{20737 \sinh(\pi)}} - \frac{1}{\frac{1}{10376 \sinh(2\pi)}} + \frac{1}{\frac{1}{6939 \sinh(3\pi)}}\right)} =$$

$$54 + \phi - \frac{1}{1 - 82944 \pi \left(\frac{1}{\frac{1}{20737 \sinh(\pi)}} - \frac{1}{\frac{1}{10376 \sinh(2\pi)}} + \frac{1}{\frac{1}{6939 \sinh(3\pi)}}\right)} =$$

$$54 + \phi - \frac{1}{1 - 82944 \pi \left(\frac{1}{20737 \sinh(\pi)} - \frac{1}{10376 \sinh(2\pi)} + \frac{1}{6939 \sinh(3\pi)}\right)} = 54 + \phi - \frac{1}{1 - 82944 \pi \left(\frac{1}{20737 \left(-e^{-\pi} + e^{\pi}\right)} - \frac{1}{5188 \left(-e^{-2\pi} + e^{2\pi}\right)} + \frac{1}{\frac{6939}{2} \left(-e^{-3\pi} + e^{3\pi}\right)}\right)}$$

$$\begin{aligned} 54 + \phi - \frac{1}{1 - 82\,944\,\pi \left(\frac{1}{20\,737\,\sinh(\pi)} - \frac{1}{10\,376\,\sinh(2\,\pi)} + \frac{1}{6939\,\sinh(3\,\pi)}\right)} = \\ 54 + \phi - \frac{1}{1 - 82\,944\,\pi \sum_{k=0}^{\infty} \left(\frac{2\,e^{-3\,\pi - 6\,k\pi}}{6939} - \frac{e^{-2\,\pi - 4\,k\pi}}{5188} + \frac{2\,e^{-\pi - 2\,k\pi}}{20\,737}\right)} \end{aligned}$$

$$54 + \phi - \frac{1}{1 - 82944 \pi \left(\frac{1}{20737 \sinh(\pi)} - \frac{1}{10376 \sinh(2\pi)} + \frac{1}{6939 \sinh(3\pi)}\right)} =$$

$$54 + \phi - 1 / \left(1 - 82944 \pi \sum_{k=0}^{\infty} \left(\left(\text{Li}_{-k}\left(-e^{z_0}\right) - \text{Li}_{-k}\left(e^{z_0}\right)\right)\left(71999064 \left(\pi - z_0\right)^k - 143894043 \left(2\pi - z_0\right)^k + 215167112 \left(3\pi - z_0\right)^k\right)\right) /$$

$$(1493044590168 k!) \int \text{for } \frac{i z_0}{\pi} \notin \mathbb{Z}$$

$$54 + \phi - \frac{1}{1 - 82944 \pi \left(\frac{1}{20737 \sinh(\pi)} - \frac{1}{10376 \sinh(2\pi)} + \frac{1}{6939 \sinh(3\pi)}\right)} = 54 + \phi - \frac{1}{1 - 82944 \pi \left(\frac{1}{20737 \pi \int_{0}^{1} \cosh(\pi t) dt} - \frac{1}{20752 \pi \int_{0}^{1} \cosh(2\pi t) dt} + \frac{1}{20817 \pi \int_{0}^{1} \cosh(3\pi t) dt}\right)}$$

$$54 + \phi - \frac{1}{1 - 82944 \pi \left(\frac{1}{20737 \sinh(\pi)} - \frac{1}{10376 \sinh(2\pi)} + \frac{1}{6939 \sinh(3\pi)}\right)} = 54 + \phi - \frac{1}{1 - 82944 \pi \left(\frac{1}{20737 \sinh(\pi)} - \frac{1}{10376 \sinh(2\pi)} + \frac{1}{6939 \sinh(3\pi)}\right)} = 54 + \phi - \frac{i}{1 - 82944 \pi \left(\frac{1}{20737 \sinh(\pi)} - \frac{1}{10376 \sinh(2\pi)} + \frac{1}{6939 \sinh(3\pi)}\right)} = \frac{1}{1 - 82944 \pi \left(\frac{1}{20737 \sinh(\pi)} - \frac{1}{10376 \sinh(2\pi)} + \frac{1}{6939 \sinh(3\pi)}\right)} = \frac{1}{1 - 82944 \pi \left(\frac{1}{20737 \sinh(\pi)} - \frac{1}{10376 \sinh(2\pi)} + \frac{1}{6939 \sinh(3\pi)}\right)} = \frac{1}{1 - 82944 \pi \left(\frac{1}{20737 \sinh(\pi)} - \frac{1}{10376 \sinh(2\pi)} + \frac{1}{6939 \sinh(3\pi)}\right)} = \frac{1}{1 - 82944 \pi \left(\frac{1}{20737 \sinh(\pi)} - \frac{1}{10376 \sinh(2\pi)} + \frac{1}{6939 \sinh(3\pi)}\right)} = \frac{1}{1 - 82944 \pi \left(\frac{1}{20737 \sinh(\pi)} - \frac{1}{10376 \sinh(2\pi)} + \frac{1}{6939 \sinh(3\pi)}\right)} = \frac{1}{1 - 82944 \pi \left(\frac{1}{20737 \sinh(\pi)} - \frac{1}{10376 \sinh(2\pi)} + \frac{1}{6939 \sinh(3\pi)}\right)} = \frac{1}{1 - 82944 \pi \left(\frac{1}{20737 \sinh(\pi)} - \frac{1}{10376 \sinh(2\pi)} + \frac{1}{6939 \sinh(3\pi)}\right)} = \frac{1}{1 - 82944 \pi \left(\frac{1}{20737 \sinh(\pi)} - \frac{1}{10376 \sinh(2\pi)} + \frac{1}{6939 \sinh(3\pi)}\right)} = \frac{1}{1 - 82944 \pi \left(\frac{1}{20737 \sinh(\pi)} - \frac{1}{10376 \sinh(2\pi)} + \frac{1}{6939 \sinh(3\pi)}\right)} = \frac{1}{1 - 82944 \pi \left(\frac{1}{20737 \sinh(\pi)} - \frac{1}{10376 \sinh(2\pi)} + \frac{1}{6939 \sinh(3\pi)}\right)} = \frac{1}{1 - 82944 \pi \left(\frac{1}{20737 \sinh(\pi)} - \frac{1}{10376 \sinh(2\pi)} + \frac{1}{6939 \sinh(3\pi)}\right)} = \frac{1}{1 - 82944 \pi \left(\frac{1}{20737 \sinh(\pi)} - \frac{1}{10376 \sinh(2\pi)} + \frac{1}{6939 \sinh(3\pi)}\right)} = \frac{1}{1 - 82944 \pi \left(\frac{1}{20737 \sinh(\pi)} - \frac{1}{10376 \sinh(2\pi)} + \frac{1}{6939 \sinh(3\pi)}\right)} = \frac{1}{1 - 82944 \pi \left(\frac{1}{20737 \sinh(\pi)} - \frac{1}{10376 \sinh(2\pi)} + \frac{1}{6939 \sinh(3\pi)}\right)} = \frac{1}{1 - 82944 \pi \left(\frac{1}{20737 \sinh(\pi)} - \frac{1}{10376 \sinh(2\pi)} + \frac{1}{6939 \sinh(3\pi)}\right)} = \frac{1}{1 - 82944 \pi \left(\frac{1}{20737 \sinh(\pi)} - \frac{1}{10376 \sinh(2\pi)} + \frac{1}{6939 \sinh(3\pi)}\right)} = \frac{1}{1 - 82944 \pi \left(\frac{1}{20737 \sinh(\pi)} - \frac{1}{10376 \sinh(\pi)} + \frac{1}{6939 \sinh(3\pi)}\right)} = \frac{1}{1 - 82944 \pi \left(\frac{1}{20737 \sinh(\pi)} - \frac{1}{10376 \sinh(\pi)} + \frac{1}{10376 \sinh(\pi)}$$

If we put:

$$\left(\frac{1}{\tanh\left(\frac{2\pi a}{\beta}\right)}\right) = \left(\frac{1}{\frac{296447958 \tanh\left(\frac{\pi}{2}\right) + 442963057 \tanh\left(3\pi\right) + 719470215 \tanh\left(5\pi\right)}{6147441305046}}\right) = 4285.9586$$

and

$$\left(\frac{2\beta\sinh^2\left(\frac{\pi}{\beta}(a+b)\right)}{\pi\epsilon\sinh\left(\frac{2\pi a}{\beta}\right)}\right) = \left(\phi + 54 - \frac{1}{1 - 82944\pi\left(\frac{\operatorname{csch}(\pi)}{20737} - \frac{\operatorname{csch}(2\pi)}{10376} + \frac{\operatorname{csch}(3\pi)}{6939}\right)}\right) = \left(\phi + 54 - \frac{1}{1 - 82944\pi\left(\frac{\operatorname{csch}(\pi)}{20737} - \frac{\operatorname{csch}(2\pi)}{10376} + \frac{\operatorname{csch}(3\pi)}{6939}\right)}\right) = \left(\phi + 54 - \frac{1}{1 - 82944\pi\left(\frac{\operatorname{csch}(\pi)}{20737} - \frac{\operatorname{csch}(2\pi)}{10376} + \frac{\operatorname{csch}(3\pi)}{6939}\right)}\right) = \left(\phi + 54 - \frac{1}{1 - 82944\pi\left(\frac{\operatorname{csch}(\pi)}{20737} - \frac{\operatorname{csch}(2\pi)}{10376} + \frac{\operatorname{csch}(3\pi)}{6939}\right)}\right) = \left(\phi + 54 - \frac{1}{1 - 82944\pi\left(\frac{\operatorname{csch}(\pi)}{20737} - \frac{\operatorname{csch}(2\pi)}{10376} + \frac{\operatorname{csch}(3\pi)}{6939}\right)}\right) = \left(\phi + 54 - \frac{1}{1 - 82944\pi\left(\frac{\operatorname{csch}(\pi)}{20737} - \frac{\operatorname{csch}(2\pi)}{10376} + \frac{\operatorname{csch}(3\pi)}{6939}\right)}\right) = \left(\phi + 54 - \frac{1}{1 - 82944\pi\left(\frac{\operatorname{csch}(\pi)}{20737} - \frac{\operatorname{csch}(2\pi)}{10376} + \frac{\operatorname{csch}(3\pi)}{6939}\right)}\right) = \left(\phi + 54 - \frac{1}{1 - 82944\pi\left(\frac{\operatorname{csch}(\pi)}{20737} - \frac{\operatorname{csch}(2\pi)}{10376} + \frac{\operatorname{csch}(3\pi)}{6939}\right)}\right) = \left(\phi + 54 - \frac{1}{1 - 82944\pi\left(\frac{\operatorname{csch}(\pi)}{20737} - \frac{\operatorname{csch}(2\pi)}{10376} + \frac{\operatorname{csch}(3\pi)}{6939}\right)}\right)$$

=3096.9002982...

We obtain from

$$S_0 + \frac{2\pi\phi_r}{\beta} \frac{1}{\tanh\left(\frac{2\pi a}{\beta}\right)} + \frac{c}{6}\log\left(\frac{2\beta\sinh^2\left(\frac{\pi}{\beta}(a+b)\right)}{\pi\epsilon\sinh\left(\frac{2\pi a}{\beta}\right)}\right)$$

For

$$\phi_r/(\beta c) \gtrsim 1 = 0.98911$$
 or  $1.0864055$ 

$$\frac{\text{Area}}{4G_N} = S_0 + \phi.$$

$$4G_N = 1$$

$$S_0 = 4Pi - 0.98911$$

c = 1

$$S_0 + 2Pi*0.98911*4285.9586 + \frac{c}{6} \ln (3096.9002982)$$

4Pi-0.98911+2Pi\*0.98911\*4285.9586+1/6\*ln(3096.9002982)

# **Input interpretation:**

$$4\pi - 0.98911 + 2\pi \times 0.98911 \times 4285.9586 + \frac{1}{6}\log(3096.9002982)$$

#### **Result:**

26649.1...

26649.1...

### **Alternative representations:**

$$4\pi - 0.98911 + 2\pi 0.98911 \times 4285.96 + \frac{\log(3096.90029820000)}{6} = -0.98911 + 8482.57\pi + \frac{\log_e(3096.90029820000)}{6}$$

$$4\pi - 0.98911 + 2\pi 0.98911 \times 4285.96 + \frac{\log(3096.90029820000)}{6} = -0.98911 + 8482.57\pi + \frac{1}{6}\log(a)\log_a(3096.90029820000)$$

$$4\pi - 0.98911 + 2\pi 0.98911 \times 4285.96 + \frac{\log(3096.90029820000)}{6} = -0.98911 + 8482.57\pi - \frac{\text{Li}_1(-3095.90029820000)}{6}$$

## Series representations:

$$4\pi - 0.98911 + 2\pi 0.98911 \times 4285.96 + \frac{\log(3096.90029820000)}{6} = \\ -0.98911 + 8482.57\pi + \frac{\log(3095.90029820000)}{6} - \frac{1}{6} \sum_{k=1}^{\infty} \frac{(-1)^k e^{-8.03783403076730 k}}{k}$$

$$\begin{array}{l} 4\pi - 0.98911 + 2\pi 0.98911 \times 4285.96 + \frac{\log(3096.90029820000)}{6} = \\ -0.98911 + 8482.57\pi + \frac{1}{3}i\pi \left[\frac{\arg(3096.90029820000 - x)}{2\pi}\right] + \\ \frac{\log(x)}{6} - \frac{1}{6}\sum_{k=1}^{\infty} \frac{(-1)^k (3096.90029820000 - x)^k x^{-k}}{k} \quad \text{for } x < 0 \end{array}$$

$$\begin{array}{l} 4\,\pi - 0.98911 + 2\,\pi\,0.98911 \times 4285.96 + \frac{\log(3096.90029820000)}{6} = \\ -0.98911 + 8482.57\,\pi + \frac{1}{6}\left\lfloor\frac{\arg(3096.90029820000-z_0)}{2\,\pi}\right\rfloor\log\left(\frac{1}{z_0}\right) + \\ \frac{\log(z_0)}{6} + \frac{1}{6}\left\lfloor\frac{\arg(3096.90029820000-z_0)}{2\,\pi}\right\rfloor\log(z_0) - \\ \frac{1}{6}\sum_{k=1}^{\infty}\frac{(-1)^k\,(3096.90029820000-z_0)^k\,z_0^{-k}}{k} \end{array}$$

# **Integral representations:**

$$4\pi - 0.98911 + 2\pi 0.98911 \times 4285.96 + \frac{\log(3096.90029820000)}{6} = -0.98911 + 8482.57\pi + \frac{1}{6} \int_{1}^{3096.90029820000} \frac{1}{t} dt$$

$$\begin{split} 4\,\pi - 0.98911 + 2\,\pi\,0.98911 \times 4285.96 + \frac{\log(3096.90029820000)}{6} &= -0.98911 + \\ 8482.57\,\pi + \frac{1}{12\,i\,\pi} \int_{-i\,\infty + \gamma}^{i\,\infty + \gamma} \frac{e^{-8.03783403076730\,s}}{\Gamma(1-s)} \frac{\Gamma(1-s)}{\Gamma(1-s)} \,ds \quad \text{for } -1 < \gamma < 0 \end{split}$$

Inserting the entropy value 26649.1 in the Hawking radiation calculator, we obtain:

Mass = 0.00000100229

Radius = 1.48856E-33

Temperature = 1.22416E29

Entropy = 26649.1

From the Ramanujan-Nardelli mock formula, we obtain:

$$sqrt[[[[1/((((((4*1.962364415e+19)/(5*0.0864055^2)))*1/(0.00000100229)* sqrt[[-((((1.22416e+29*4*Pi*(1.48856e-33)^3-(1.48856e-33)^2)))))/((6.67*10^-11))]]]]]$$

### Input interpretation:

$$\sqrt{\left(1/\left(\frac{4\times1.962364415\times10^{19}}{5\times0.0864055^2}\times\frac{1}{1.00229\times10^{-6}}\right)}$$

$$\sqrt{-\frac{1.22416\times10^{29}\times4\pi\left(1.48856\times10^{-33}\right)^3-\left(1.48856\times10^{-33}\right)^2}{6.67\times10^{-11}}}\right)}$$

#### **Result:**

1.618081735392146230436561397898828941494902451109365297284...

1.61808173539...

And:

$$1/sqrt[[[[1/((((((4*1.962364415e+19)/(5*0.0864055^2)))*1/(0.00000100229)*sqrt[[-((((1.22416e+29*4*Pi*(1.48856e-33)^3-(1.48856e-33)^2)))))/((6.67*10^-11))]]]]]$$

# **Input interpretation:**

1

$$\sqrt{\frac{1}{\frac{4\times1.962364415\times10^{19}}{5\times0.0864055^2}\times\frac{1}{1.00229\times10^{-6}}}\sqrt{\frac{1.22416\times10^{29}\times4\pi\left(1.48856\times10^{-33}\right)^3-\left(1.48856\times10^{-33}\right)^2}{6.67\times10^{-11}}}$$

#### **Result:**

 $0.618015751693561668331267490642891547545081526820311348060... \\ 0.61801575169...$ 

Practically we obtain the values of the golden ratio and his conjugate

Or:

# **Input interpretation:**

$$4\pi - 0.98911 + 2\pi \times 1.0864055 \times 4285.9586 + \frac{1}{6} \log(3096.9002982)$$

log(x) is the natural logarithm

#### **Result:**

29269.244...

29269.244...

# Alternative representations:

$$4\pi - 0.98911 + 2\pi 1.08641 \times 4285.96 + \frac{\log(3096.90029820000)}{6} = -0.98911 + 9316.58\pi + \frac{\log_e(3096.90029820000)}{6}$$

$$4\pi - 0.98911 + 2\pi 1.08641 \times 4285.96 + \frac{\log(3096.90029820000)}{6} = -0.98911 + 9316.58\pi + \frac{1}{6}\log(a)\log_a(3096.90029820000)$$

$$4\pi - 0.98911 + 2\pi 1.08641 \times 4285.96 + \frac{\log(3096.90029820000)}{6} = -0.98911 + 9316.58\pi - \frac{\text{Li}_1(-3095.90029820000)}{6}$$

### **Series representations:**

$$4\pi - 0.98911 + 2\pi 1.08641 \times 4285.96 + \frac{\log(3096.90029820000)}{6} = \\ -0.98911 + 9316.58\pi + \frac{\log(3095.90029820000)}{6} - \frac{1}{6} \sum_{k=1}^{\infty} \frac{(-1)^k e^{-8.03783403076730 k}}{k}$$

$$\begin{array}{l} 4\pi - 0.98911 + 2\pi 1.08641 \times 4285.96 + \frac{\log(3096.90029820000)}{6} = \\ -0.98911 + 9316.58\pi + \frac{1}{3}i\pi \left[\frac{\arg(3096.90029820000 - x)}{2\pi}\right] + \\ \frac{\log(x)}{6} - \frac{1}{6}\sum_{k=1}^{\infty} \frac{(-1)^k (3096.90029820000 - x)^k x^{-k}}{k} \quad \text{for } x < 0 \end{array}$$

$$\begin{array}{l} 4\pi - 0.98911 + 2\pi 1.08641 \times 4285.96 + \frac{\log(3096.90029820000)}{6} = \\ -0.98911 + 9316.58\pi + \frac{1}{6} \left\lfloor \frac{\arg(3096.90029820000 - z_0)}{2\pi} \right\rfloor \log \left( \frac{1}{z_0} \right) + \\ \frac{\log(z_0)}{6} + \frac{1}{6} \left\lfloor \frac{\arg(3096.90029820000 - z_0)}{2\pi} \right\rfloor \log(z_0) - \\ \frac{1}{6} \sum_{k=1}^{\infty} \frac{(-1)^k (3096.90029820000 - z_0)^k z_0^{-k}}{k} \end{array}$$

## **Integral representations:**

$$4\pi - 0.98911 + 2\pi 1.08641 \times 4285.96 + \frac{\log(3096.90029820000)}{6} = -0.98911 + 9316.58\pi + \frac{1}{6} \int_{1}^{3096.90029820000} \frac{1}{t} dt$$

$$4\pi - 0.98911 + 2\pi 1.08641 \times 4285.96 + \frac{\log(3096.90029820000)}{6} = -0.98911 + 9316.58\pi + \frac{1}{12i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-8.03783403076730s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \text{ for } -1 < \gamma < 0$$

Inserting the entropy value 29269.244 in the Hawking radiation calculator, we obtain:

$$Mass = 0.00000105040$$

Radius = 
$$1.56002e-33$$

Temperature = 
$$1.16808e+29$$

From the Ramanujan-Nardelli mock formula, we obtain:

 $sqrt[[[[1/(((((4*1.962364415e+19)/(5*0.0864055^2)))*1/(0.00000105040)* \ sqrt[[-((((1.16808e+29*4*Pi*(1.56002e-33)^3-(1.56002e-33)^2)))))/((6.67*10^-11))]]]]]$ 

### Input interpretation:

$$\sqrt{\left(1 / \left(\frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^{2}} \times \frac{1}{1.05040 \times 10^{-6}} \right) - \frac{1.16808 \times 10^{29} \times 4 \pi \left(1.56002 \times 10^{-33}\right)^{3} - \left(1.56002 \times 10^{-33}\right)^{2}}{6.67 \times 10^{-11}}\right)}$$

#### **Result:**

1.618077063491289140603706176247888824149668700084618992874... 1.618077063...

We have also that:

 $(((4Pi\text{-}0.98911 + 2Pi*1.0864055*4285.9586 + 1/6*ln(3096.9002982))))^1/2$  - 29 - golden ratio^2

**Input interpretation:** 

$$\sqrt{4\pi - 0.98911 + 2\pi \times 1.0864055 \times 4285.9586 + \frac{1}{6}\log(3096.9002982)} - 29 - \phi^2}$$

log(x) is the natural logarithm

φ is the golden ratio

#### **Result:**

139.46453...

139.46453... result practically equal to the rest mass of Pion meson 139.57

# **Alternative representations:**

$$\sqrt{4\pi - 0.98911 + 2\pi 1.08641 \times 4285.96 + \frac{\log(3096.90029820000)}{6}} - 29 - \phi^2 = -29 - \phi^2 + \sqrt{-0.98911 + 9316.58\pi + \frac{\log_e(3096.90029820000)}{6}}$$

$$\sqrt{4\pi - 0.98911 + 2\pi 1.08641 \times 4285.96 + \frac{\log(3096.90029820000)}{6}} - 29 - \phi^2 = -29 - \phi^2 + \sqrt{-0.98911 + 9316.58\pi + \frac{1}{6}\log(a)\log_a(3096.90029820000)}}$$

$$\sqrt{4\pi - 0.98911 + 2\pi 1.08641 \times 4285.96 + \frac{\log(3096.90029820000)}{6}} - 29 - \phi^2 = -29 - \phi^2 + \sqrt{-0.98911 + 9316.58\pi - \frac{\text{Li}_1(-3095.90029820000)}{6}}$$

$$\sqrt{4\pi - 0.98911 + 2\pi 1.08641 \times 4285.96 + \frac{\log(3096.90029820000)}{6} - 29 - \phi^2 = \frac{-29 - \phi^2 + \sqrt{-5.93466 + 55899.5\pi + \log(3095.90029820000) - \sum_{k=1}^{\infty} \frac{(-1)^k e^{-8.03783403076730 k}}{k}}{\sqrt{6}}$$

$$\sqrt{4\pi - 0.98911 + 2\pi 1.08641 \times 4285.96 + \frac{\log(3096.90029820000)}{6} - 29 - \phi^2} = -29 - \phi^2 + \sqrt{\left(-0.98911 + 9316.58\pi + \frac{1}{6}\left(2i\pi\left\lfloor\frac{\arg(3096.90029820000 - x)}{2\pi}\right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (3096.90029820000 - x)^k x^{-k}}{k}\right)\right)} \text{ for } x < 0$$

$$\sqrt{4\pi - 0.98911 + 2\pi 1.08641 \times 4285.96 + \frac{\log(3096.90029820000)}{6} - 29 - \phi^2} = -29 - \phi^2 + \sqrt{\left(-0.98911 + 9316.58\pi + \frac{1}{6}\left[\log(z_0) + \left[\frac{\arg(3096.90029820000 - z_0)}{2\pi}\right]\right] + \left(\log\left(\frac{1}{z_0}\right) + \log(z_0)\right) - \sum_{k=1}^{\infty} \frac{(-1)^k (3096.90029820000 - z_0)^k z_0^{-k}}{k}\right)\right)}$$

$$\sqrt{4\pi - 0.98911 + 2\pi 1.08641 \times 4285.96 + \frac{\log(3096.90029820000)}{6}} - 29 - \phi^2 = -29 - \phi^2 + \sqrt{-0.98911 + 9316.58\pi + \frac{1}{6} \int_{1}^{3096.90029820000} \frac{1}{t} dt}$$

$$\sqrt{4\pi - 0.98911 + 2\pi 1.08641 \times 4285.96 + \frac{\log(3096.90029820000)}{6}} - 29 - \phi^2 = \frac{-29 - \phi^2 + \sqrt{-0.98911 + 9316.58\pi + \frac{1}{12i\pi} \int_{-i\,\infty+\gamma}^{i\,\infty+\gamma} \frac{e^{-8.03783403076730\,s} \, \Gamma(-s)^2 \, \Gamma(1+s)}{\Gamma(1-s)} \, ds}$$
for  $-1 < \gamma < 0$ 

solve this equation, we must impose the same condition on the right-hand side. The k=1 mode requires

$$\int_0^{2\pi} d\tau e^{-i\tau} \left( \frac{c}{12\phi_r} \mathcal{F} - \partial_\tau R(\tau) \right) = 0 . \tag{3.29}$$

Doing the integrals, this gives the condition

$$\frac{c}{6\phi_r} \frac{\sinh\frac{a-b}{2}}{\sinh\frac{b+a}{2}} = \frac{1}{\sinh a} \ . \tag{3.30}$$

For

a = 5, b = 2, c = 1 we obtain, from (3.30):

$$(((1/(\sinh(5)))))/(((1/(6*0.98911)*(\sinh(3/2)/\sinh(7/2)))))$$

#### Input:

$$\frac{\frac{1}{\sinh(5)}}{\frac{1}{6\times0.98911}\times\frac{\sinh\left(\frac{3}{2}\right)}{\sinh\left(\frac{7}{2}\right)}$$

sinh(x) is the hyperbolic sine function

#### **Result:**

0.621362...

0.621362...

# Alternative representations:

$$\frac{1}{\frac{\sinh\left(\frac{3}{2}\right)\sinh(5)}{(6\times0.98911)\sinh\left(\frac{7}{2}\right)}} = \frac{1}{\frac{1}{\frac{\operatorname{csch}(5)\left(5.93466\,\operatorname{csch}\left(\frac{3}{2}\right)\right)}{\operatorname{csch}\left(\frac{7}{2}\right)}}}$$

$$\frac{1}{\frac{\sinh\left(\frac{3}{2}\right)\sinh(5)}{(6\times0.98911)\sinh\left(\frac{7}{2}\right)}} = \frac{1}{\frac{\left(-\frac{1}{e^5} + e^5\right)\left(-\frac{1}{e^{3/2}} + e^{3/2}\right)}{\frac{2}{2}\left(2\times5.93466\left(-\frac{1}{e^{7/2}} + e^{7/2}\right)\right)}}$$

$$\frac{1}{\frac{\sinh\left(\frac{3}{2}\right)\sinh(5)}{(6\times0.98911)\sinh\left(\frac{7}{2}\right)}} = -\frac{1}{\frac{i\left(-i\right)}{\csc(5i)\left(5.93466\csc\left(\frac{3i}{2}\right)\left(-i\right)\right)}}$$

## **Series representations:**

$$\frac{1}{\frac{\sinh\left(\frac{3}{2}\right)\sinh(5)}{(6\times0.98911)\sinh\left(\frac{7}{2}\right)}} = \frac{5.93466\sum_{k=0}^{\infty} \frac{\left(\frac{7}{2}\right)^{1+2}k}{\left(1+2k\right)!}}{\left(\sum_{k=0}^{\infty} \frac{\left(\frac{3}{2}\right)^{1+2}k}{\left(1+2k\right)!}\right)\sum_{k=0}^{\infty} \frac{5^{1+2}k}{\left(1+2k\right)!}}$$

$$\frac{1}{\frac{\sinh\left(\frac{3}{2}\right)\sinh(5)}{(6\times0.98911)\sinh\left(\frac{7}{2}\right)}} = \frac{2.96733\sum_{k=0}^{\infty}I_{1+2\,k}\!\left(\frac{7}{2}\right)}{\left(\sum_{k=0}^{\infty}I_{1+2\,k}\!\left(\frac{3}{2}\right)\right)\sum_{k=0}^{\infty}I_{1+2\,k}(5)}$$

$$\frac{1}{\frac{\sinh\left(\frac{3}{2}\right)\sinh(5)}{(6\times0.98911)\sinh\left(\frac{7}{2}\right)}} = \frac{5.93466\sum_{k=0}^{\infty}\frac{\left(\frac{7}{2}-\frac{i\pi}{2}\right)^{2}k}{\left(2\,k\right)!}}{i\left(\sum_{k=0}^{\infty}\frac{\left(\frac{3}{2}-\frac{i\pi}{2}\right)^{2}k}{(2\,k)!}\right)\sum_{k=0}^{\infty}\frac{\left(5-\frac{i\pi}{2}\right)^{2}k}{(2\,k)!}}$$

# **Integral representations:**

$$\frac{1}{\frac{\sinh\left(\frac{3}{2}\right)\sinh(5)}{(6\times0.98911)\sinh\left(\frac{7}{2}\right)}} = \frac{2.76951\int_{0}^{1}\cosh\left(\frac{7t}{2}\right)dt}{\left(\int_{0}^{1}\cosh\left(\frac{3t}{2}\right)dt\right)\int_{0}^{1}\cosh(5t)dt}$$

$$\frac{1}{\frac{\sinh\left(\frac{3}{2}\right)\sinh(5)}{(6\times0.98911)\sinh\left(\frac{7}{2}\right)}} = \frac{11.078 \ i \ \pi \int_{-i \ \infty+\gamma}^{i \ \infty+\gamma} \frac{e^{49/(16 \ s)+s}}{s^{3/2}} \ ds}{\left(\int_{-i \ \infty+\gamma}^{i \ \infty+\gamma} \frac{e^{9/(16 \ s)+s}}{s^{3/2}} \ ds\right) \left(\int_{-i \ \infty+\gamma}^{i \ \infty+\gamma} \frac{e^{25/(4 \ s)+s}}{s^{3/2}} \ ds\right) \sqrt{\pi}} \quad \text{for } \gamma > 0$$

0.62136239751766\*(((1/(6\*0.98911)\*(sinh(3/2)/sinh(7/2)))))

## Input interpretation:

$$0.62136239751766 \left( \frac{1}{6 \times 0.98911} \times \frac{\sinh(\frac{3}{2})}{\sinh(\frac{7}{2})} \right)$$

 $\sinh(x)$  is the hyperbolic sine function

#### **Result:**

0.0134765...

0.0134765...

## Alternative representations:

$$\frac{0.621362397517660000 \sinh\left(\frac{3}{2}\right)}{(6 \times 0.98911) \sinh\left(\frac{7}{2}\right)} = \frac{0.621362397517660000}{\frac{5.93466 \cosh\left(\frac{3}{2}\right)}{\cosh\left(\frac{7}{2}\right)}}$$

$$\frac{0.621362397517660000 \sinh\left(\frac{3}{2}\right)}{(6 \times 0.98911) \sinh\left(\frac{7}{2}\right)} = \frac{0.310681198758830000 \left(-\frac{1}{e^{3/2}} + e^{3/2}\right)}{\frac{1}{2} \times 5.93466 \left(-\frac{1}{e^{7/2}} + e^{7/2}\right)}$$

$$\frac{0.621362397517660000 \sinh\left(\frac{3}{2}\right)}{(6 \times 0.98911) \sinh\left(\frac{7}{2}\right)} = -\frac{0.621362397517660000 i}{\frac{5.93466 \csc\left(\frac{3i}{2}\right)(-i)}{\csc\left(\frac{7i}{2}\right)}}$$

$$\frac{0.621362397517660000 \sinh\left(\frac{3}{2}\right)}{(6 \times 0.98911) \sinh\left(\frac{7}{2}\right)} = \frac{0.104701 \sum_{k=0}^{\infty} \frac{\left(\frac{3}{2}\right)^{1+2} k}{(1+2 k)!}}{\sum_{k=0}^{\infty} \frac{\left(\frac{7}{2}\right)^{1+2} k}{(1+2 k)!}}$$

$$\frac{0.621362397517660000 \sinh\left(\frac{3}{2}\right)}{(6 \times 0.98911) \sinh\left(\frac{7}{2}\right)} = \frac{0.104701 \sum_{k=0}^{\infty} I_{1+2k}\left(\frac{3}{2}\right)}{\sum_{k=0}^{\infty} I_{1+2k}\left(\frac{7}{2}\right)}$$

$$\frac{0.621362397517660000 \sinh\left(\frac{3}{2}\right)}{(6 \times 0.98911) \sinh\left(\frac{7}{2}\right)} = \frac{0.104701 \sum_{k=0}^{\infty} \frac{\left(\frac{3}{2} - \frac{i\pi}{2}\right)^{2k}}{(2k)!}}{\sum_{k=0}^{\infty} \frac{\left(\frac{7}{2} - \frac{i\pi}{2}\right)^{2k}}{(2k)!}}$$

$$\frac{0.621362397517660000 \sinh \left(\frac{3}{2}\right)}{(6 \times 0.98911) \sinh \left(\frac{7}{2}\right)} = \frac{0.0448717 \int_{0}^{1} \cosh \left(\frac{3t}{2}\right) dt}{\int_{0}^{1} \cosh \left(\frac{7t}{2}\right) dt}$$

$$\frac{0.621362397517660000 \sinh \left(\frac{3}{2}\right)}{(6 \times 0.98911) \sinh \left(\frac{7}{2}\right)} = \frac{0.0448717 \int_{-i \, \infty + \gamma}^{i \, \infty + \gamma} \frac{e^{9/(16 \, s) + s}}{s^{3/2}} \, ds}{\int_{-i \, \infty + \gamma}^{i \, \infty + \gamma} \frac{e^{49/(16 \, s) + s}}{s^{3/2}} \, ds} \quad \text{for } \gamma > 0$$

 $(((1/(\sinh(5)))))$ 

# **Input:**

sinh(5)

 $\sinh(x)$  is the hyperbolic sine function

#### **Exact result:**

csch(5)

 $\operatorname{csch}(x)$  is the hyperbolic cosecant function

## **Decimal approximation:**

0.013476505830589086655381881284337964618035455336483814697...

0.013476505...

#### **Property:**

csch(5) is a transcendental number

## **Alternate forms:**

$$\frac{2 e^5}{e^{10} - 1}$$

$$\frac{2}{e^5 - \frac{1}{e^5}}$$

$$-\frac{2\sinh(5)}{1-\cosh(10)}$$

 $\cosh(x)$  is the hyperbolic cosine function

# **Alternative representations:**

$$\frac{1}{sinh(5)} = \frac{1}{\frac{1}{csch(5)}}$$

$$\frac{1}{\sinh(5)} = \frac{1}{\frac{1}{2} \left( -\frac{1}{e^5} + e^5 \right)}$$

$$\frac{1}{\sinh(5)} = -\frac{1}{\frac{i}{\csc(5\,i)}}$$

## **Series representations:**

$$\frac{1}{\sinh(5)} = \frac{2\sum_{k=0}^{\infty} e^{-10k}}{e^5}$$

$$\frac{1}{\sinh(5)} = -2\sum_{k=1}^{\infty} q^{-1+2k} \text{ for } q = e^{5}$$

$$\frac{1}{\sinh(5)} = 5 \sum_{k=-\infty}^{\infty} \frac{(-1)^k}{25 + k^2 \pi^2}$$

# **Integral representations:**

$$\frac{1}{\sinh(5)} = \frac{1}{5 \int_0^1 \cosh(5t) \, dt}$$

$$\frac{1}{\sinh(5)} = \frac{4 i \pi}{5 \sqrt{\pi} \int_{-i + \gamma}^{i + \gamma} \frac{e^{25/(4 s) + s}}{s^{3/2}} ds} \quad \text{for } \gamma > 0$$

The fundamental result in this expression is 0.62136239751766. Note that the inverse of this value is 1.6093667785417...: these are "golden numbers"

at t = 0. The generalized entropy, including the island, is

$$S_{\text{gen}}(I \cup R) = \frac{\phi_r}{a} + \frac{c}{6} \log \frac{(a+b)^2}{a}$$
 (4.3)

Setting  $\partial_a S_{\text{gen}} = 0$  gives the position of the QES,

$$a = \frac{1}{2}(k + b + \sqrt{b^2 + 6bk + k^2}) , \quad k \equiv \frac{6\phi_r}{c} .$$
 (4.4)

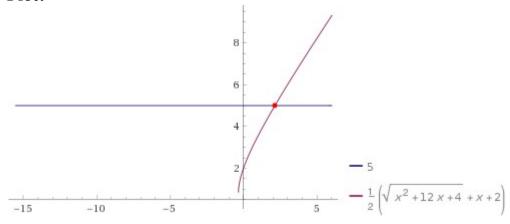
For a = 5, b = 2

$$5 = 1/2(x+2+sqrt(4+12x+x^2))$$

#### **Input:**

$$5 = \frac{1}{2} \left( x + 2 + \sqrt{4 + 12 x + x^2} \right)$$

#### **Plot:**



#### **Alternate forms:**

$$\sqrt{x^2 + 12x + 4} + x = 8$$

$$5 = \frac{1}{2} \left( x + \sqrt{x(x+12) + 4} + 2 \right)$$

# Alternate form assuming x is positive:

$$x + \sqrt{x(x+12) + 4} = 8$$

Expanded form:  

$$5 = \frac{1}{2} \sqrt{x^2 + 12x + 4} + \frac{x}{2} + 1$$

## **Solution:**

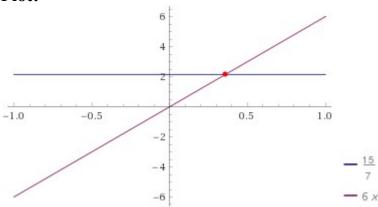
$$x = \frac{15}{7}$$

$$15/7 = k$$

$$15/7 = 6x$$

Input: 
$$\frac{15}{7} = 6 x$$

# **Plot:**



Alternate form: 
$$\frac{15}{7} - 6x = 0$$

# Solution:

$$x = \frac{5}{14}$$

$$5/14 = \phi_r$$

$$a = 5, b = 2$$

$$S_{\mathrm{gen}}(I \cup R) = \frac{\phi_r}{a} + \frac{c}{6}\log\frac{(a+b)^2}{a}$$

$$5/14*1/5 + 1/6 *ln(49/5)$$

Input: 
$$\frac{5}{14} \times \frac{1}{5} + \frac{1}{6} \log \left( \frac{49}{5} \right)$$

log(x) is the natural logarithm

**Exact result:** 

$$\frac{1}{14} + \frac{1}{6} \log \left( \frac{49}{5} \right)$$

## **Decimal approximation:**

0.451825635707992467839752930371933398529523255577772204405...

0.451825635...

Property: 
$$\frac{1}{14} + \frac{1}{6} \log \left( \frac{49}{5} \right)$$
 is a transcendental number

**Alternate forms:** 

$$\frac{1}{42} \left( 3 + 7 \log \left( \frac{49}{5} \right) \right)$$

$$\frac{1}{14} - \frac{\log(5)}{6} + \frac{\log(7)}{3}$$

$$\frac{1}{42}$$
 (3 – 7 log(5) + 14 log(7))

# **Alternative representations:**

$$\frac{5}{5 \times 14} + \frac{1}{6} \log \left( \frac{49}{5} \right) = \frac{\log_e \left( \frac{49}{5} \right)}{6} + \frac{5}{5 \times 14}$$

$$\frac{5}{5 \times 14} + \frac{1}{6} \log \left( \frac{49}{5} \right) = \frac{1}{6} \log(a) \log_a \left( \frac{49}{5} \right) + \frac{5}{5 \times 14}$$

$$\frac{5}{5 \times 14} + \frac{1}{6} \log \left( \frac{49}{5} \right) = -\frac{1}{6} \operatorname{Li}_1 \left( 1 - \frac{49}{5} \right) + \frac{5}{5 \times 14}$$

$$\frac{5}{5 \times 14} + \frac{1}{6} \log \left( \frac{49}{5} \right) = \frac{1}{14} + \frac{1}{6} \log \left( \frac{44}{5} \right) - \frac{1}{6} \sum_{k=1}^{\infty} \frac{\left( -\frac{5}{44} \right)^k}{k}$$

$$\frac{5}{5 \times 14} + \frac{1}{6} \log \left( \frac{49}{5} \right) = \frac{1}{14} + \frac{1}{3} i \pi \left[ \frac{\arg \left( \frac{49}{5} - x \right)}{2 \pi} \right] + \frac{\log(x)}{6} - \frac{1}{6} \sum_{k=1}^{\infty} \frac{(-1)^k \left( \frac{49}{5} - x \right)^k x^{-k}}{k}$$
for  $x < 0$ 

$$\frac{5}{5 \times 14} + \frac{1}{6} \log \left( \frac{49}{5} \right) = \frac{1}{14} + \frac{1}{6} \left[ \frac{\arg \left( \frac{49}{5} - z_0 \right)}{2 \pi} \right] \log \left( \frac{1}{z_0} \right) + \frac{\log(z_0)}{6} + \frac{1}{6} \left[ \frac{\arg \left( \frac{49}{5} - z_0 \right)}{2 \pi} \right] \log(z_0) - \frac{1}{6} \sum_{k=1}^{\infty} \frac{(-1)^k \left( \frac{49}{5} - z_0 \right)^k z_0^{-k}}{k}$$

$$\frac{5}{5\times14} + \frac{1}{6}\log\left(\frac{49}{5}\right) = \frac{1}{14} + \frac{1}{6}\int_{1}^{\frac{49}{5}} \frac{1}{t} dt$$

$$\frac{5}{5 \times 14} + \frac{1}{6} \log \left( \frac{49}{5} \right) = \frac{1}{14} - \frac{i}{12 \pi} \int_{-i \infty + \gamma}^{i \infty + \gamma} \frac{\left( \frac{5}{44} \right)^s \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \quad \text{for } -1 < \gamma < 0$$

Note that:

$$64/(((5/14*1/5 + 1/6*ln(49/5))))-16$$

Input: 
$$\frac{64}{\frac{5}{14} \times \frac{1}{5} + \frac{1}{6} \log(\frac{49}{5})} - 16$$

log(x) is the natural logarithm

#### **Exact result:**

$$\frac{64}{\frac{1}{14} + \frac{1}{6}\log(\frac{49}{5})} - 16$$

## **Decimal approximation:**

125.6475625596466933973543735598565493271424256496263802118...

125.64756255... result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T = 0 and to the Higgs boson mass 125.18

Property: 
$$-16 + \frac{64}{\frac{1}{14} + \frac{1}{6} \log(\frac{49}{5})}$$
 is a transcendental number

## **Alternate forms:**

$$\frac{2688}{3+7\log\left(\frac{49}{5}\right)} - 16$$

$$-\frac{16\left(7\log\left(\frac{49}{5}\right) - 165\right)}{3+7\log\left(\frac{49}{5}\right)}$$

$$-\frac{16\left(165+7\log(5) - 14\log(7)\right)}{-3+7\log(5) - 14\log(7)}$$

### **Alternative representations:**

$$\frac{64}{\frac{5}{5\times14} + \frac{1}{6}\log(\frac{49}{5})} - 16 = -16 + \frac{64}{\frac{\log_e(\frac{49}{5})}{6} + \frac{5}{5\times14}}$$

$$\frac{64}{\frac{5}{5\times14} + \frac{1}{6}\log\left(\frac{49}{5}\right)} - 16 = -16 + \frac{64}{\frac{1}{6}\log(a)\log_a\left(\frac{49}{5}\right) + \frac{5}{5\times14}}$$

$$\frac{64}{\frac{5}{5\times14} + \frac{1}{6}\log\left(\frac{49}{5}\right)} - 16 = -16 + \frac{64}{-\frac{1}{6}\operatorname{Li}_1\left(1 - \frac{49}{5}\right) + \frac{5}{5\times14}}$$

$$\frac{64}{\frac{5}{5\times14}+\frac{1}{6}\log\!\left(\frac{49}{5}\right)}-16=-16+\frac{2688}{3+7\log\!\left(\frac{44}{5}\right)-7\sum_{k=1}^{\infty}\frac{\left(-\frac{5}{44}\right)^{k}}{k}}$$

$$\frac{\frac{64}{\frac{5}{5 \times 14} + \frac{1}{6} \log\left(\frac{49}{5}\right)} - 16 =$$

$$-16 + \frac{64}{\frac{1}{14} + \frac{1}{6} \left(2 i \pi \left[\frac{\arg\left(\frac{49}{5} - x\right)}{2\pi}\right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{49}{5} - x\right)^k x^{-k}}{k}\right)}{} \text{ for } x < 0$$

$$\frac{\frac{64}{\frac{5}{5\times14} + \frac{1}{6}\log\left(\frac{49}{5}\right)} - 16 = \frac{64}{-16 + \frac{1}{14} + \frac{1}{6}\left[\log(z_0) + \left[\frac{\arg\left(\frac{49}{5} - z_0\right)}{2\pi}\right] \left(\log\left(\frac{1}{z_0}\right) + \log(z_0)\right) - \sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{49}{5} - z_0\right)^k z_0^{-k}}{k}\right)}{}$$

$$\frac{64}{\frac{5}{5\times14} + \frac{1}{6}\log\left(\frac{49}{5}\right)} - 16 = -16 + \frac{2688}{3 + 7\int_{1}^{49} \frac{1}{t} dt}$$

$$\frac{64}{\frac{5}{5\times 14} + \frac{1}{6}\log\left(\frac{49}{5}\right)} - 16 = -16 + \frac{5376\,\pi}{6\,\pi - 7\,i\int_{-i\,\infty + \gamma}^{i\,\infty + \gamma} \frac{\left(\frac{5}{44}\right)^5\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds} \quad \text{for } -1 < \gamma < 0$$

And:

$$64/(((5/14*1/5 + 1/6*ln(49/5))))$$
-sqrt5

Input: 
$$\frac{64}{\frac{5}{14} \times \frac{1}{5} + \frac{1}{6} \log(\frac{49}{5})} - \sqrt{5}$$

log(x) is the natural logarithm

## **Exact result:**

$$\frac{64}{\frac{1}{14} + \frac{1}{6}\log(\frac{49}{5})} - \sqrt{5}$$

### **Decimal approximation:**

139.4114945821469037009451998911252730917018072900148544875...

139.41149458... result practically equal to the rest mass of Pion meson 139.57

Property: 
$$-\sqrt{5} + \frac{64}{\frac{1}{14} + \frac{1}{6} \log(\frac{49}{5})}$$
 is a transcendental number

## **Alternate forms:**

$$\frac{2688}{3+7\log\left(\frac{49}{5}\right)} - \sqrt{5}$$

$$-\frac{2688+3\sqrt{5}+7\sqrt{5}\log\left(\frac{49}{5}\right)}{3+7\log\left(\frac{49}{5}\right)}$$

$$-\frac{2688-3\sqrt{5}+7\sqrt{5}\log(5)-14\sqrt{5}\log(7)}{-3+7\log(5)-14\log(7)}$$

## Alternative representations:

$$\frac{64}{\frac{5}{5\times14} + \frac{1}{6}\log\left(\frac{49}{5}\right)} - \sqrt{5} = \frac{64}{\frac{\log_e\left(\frac{49}{5}\right)}{6} + \frac{5}{5\times14}} - \sqrt{5}$$

$$\frac{64}{\frac{5}{5\times14} + \frac{1}{6}\log\left(\frac{49}{5}\right)} - \sqrt{5} = \frac{64}{\frac{1}{6}\log(a)\log_a\left(\frac{49}{5}\right) + \frac{5}{5\times14}} - \sqrt{5}$$

$$\frac{64}{\frac{5}{5\times14} + \frac{1}{6}\log\left(\frac{49}{5}\right)} - \sqrt{5} = \frac{64}{-\frac{1}{6}\operatorname{Li}_1\left(1 - \frac{49}{5}\right) + \frac{5}{5\times14}} - \sqrt{5}$$

$$\frac{64}{\frac{5}{5\times14} + \frac{1}{6}\log\left(\frac{49}{5}\right)} - \sqrt{5} = -\sqrt{5} + \frac{2688}{3 + 7\log\left(\frac{44}{5}\right) - 7\sum_{k=1}^{\infty} \frac{\left(-\frac{5}{44}\right)^k}{k}}$$

$$\frac{64}{\frac{5}{5\times14} + \frac{1}{6}\log\left(\frac{49}{5}\right)} - \sqrt{5} =$$

$$-\sqrt{5} + \frac{64}{\frac{1}{14} + \frac{1}{6}\left(2i\pi\left[\frac{\arg\left(\frac{49}{5} - x\right)}{2\pi}\right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k\left(\frac{49}{5} - x\right)^k x^{-k}}{k}\right)}{\frac{64}{2\pi}} \quad \text{for } x < 0$$

$$\begin{split} \frac{64}{\frac{5}{5\times14} + \frac{1}{6}\log\left(\frac{49}{5}\right)} - \sqrt{5} &= \\ -\sqrt{5} &+ \frac{2688}{3 + 7\log(z_0) + 7\left[\frac{\arg\left(\frac{49}{5} - z_0\right)}{2\pi}\right] \left(\log\left(\frac{1}{z_0}\right) + \log(z_0)\right) - 7\sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{49}{5} - z_0\right)^k z_0^{-k}}{k}}{2\pi} \end{split}$$

$$\frac{64}{\frac{5}{5\times14} + \frac{1}{6}\log(\frac{49}{5})} - \sqrt{5} = -\sqrt{5} + \frac{2688}{3 + 7\int_{1}^{\frac{49}{5}} \frac{1}{t} dt}$$

$$\frac{64}{\frac{5}{5\times 14} + \frac{1}{6}\log\left(\frac{49}{5}\right)} - \sqrt{5} = -\sqrt{5} + \frac{5376\,\pi}{6\,\pi - 7\,i\int_{-i\,\infty + \gamma}^{i\,\infty + \gamma} \frac{\left(\frac{5}{44}\right)^s\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds} \quad \text{for } -1 < \gamma < 0$$

Inserting the entropy value 0.451826 in the Hawking radiation calculator, we obtain:

Mass = 4.12701e-9

Radius = 6.12930e-36

Temperature = 2.97299e+31

From the Ramanujan-Nardelli mock formula, we obtain:

$$sqrt[[[[1/(((((4*1.962364415e+19)/(5*0.0864055^2)))*1/(4.12701e-9)* sqrt[[-((((2.97299e+31*4*Pi*(6.12930e-36)^3-(6.12930e-36)^2)))))/((6.67*10^-11))]]]]]$$

#### **Input interpretation:**

$$\sqrt{\left(1 / \left(\frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^{2}} \times \frac{1}{4.12701 \times 10^{-9}} \right) - \frac{2.97299 \times 10^{31} \times 4 \pi \left(6.12930 \times 10^{-36}\right)^{3} - \left(6.12930 \times 10^{-36}\right)^{2}}{6.67 \times 10^{-11}}\right)}$$

#### **Result:**

1.618077245318552386950716639328104478879882410161156440606...

1.618077245...

And:

$$1/sqrt[[[1/((((((4*1.962364415e+19)/(5*0.0864055^2)))*1/(4.12701e-9)* sqrt[[-((((2.97299e+31*4*Pi*(6.12930e-36)^3-(6.12930e-36)^2)))))/((6.67*10^-11))]]]]]$$

## **Input interpretation:**

$$\sqrt{\frac{\frac{1}{4 \times 1.962364415 \times 10^{19}} \times \frac{1}{4.12701 \times 10^{-9}} \sqrt{-\frac{2.97299 \times 10^{31} \times 4 \pi \left(6.12930 \times 10^{-36}\right)^{3} - \left(6.12930 \times 10^{-36}\right)^{2}}{6.67 \times 10^{-11}}}$$

#### **Result:**

0.618017466652606600879908700049928924823645848704609289180...

0.61801746...

From:

$$S_{\text{fermions}}(I \cup R) = \frac{c}{3} \log \left[ \frac{2 \cosh t_a \cosh t_b \left| \cosh(t_a - t_b) - \cosh(a + b) \right|}{\sinh a \cosh(\frac{a + b - t_a - t_b}{2}) \cosh(\frac{a + b + t_a + t_b}{2})} \right]$$

we obtain:

$$1/3 \ln (((2-\cosh(5+2))/(\sinh(5)\cosh(7/2)\cosh(7/2))))$$

#### **Input:**

$$\frac{1}{3} \log \left( \frac{2 - \cosh(5 + 2)}{\sinh(5) \cosh\left(\frac{7}{2}\right) \cosh\left(\frac{7}{2}\right)} \right)$$

#### **Exact result:**

$$\frac{1}{3} \left( \log \left( -(2 - \cosh(7)) \operatorname{csch}(5) \operatorname{sech}^2\left(\frac{7}{2}\right) \right) + i \pi \right)$$

# **Decimal approximation:**

- 1.2063788441890901037158798352081118020154307200752687721... + 1.0471975511965977461542144610931676280657231331250352736... i

#### **Polar coordinates:**

$$r \approx 1.59749$$
 (radius),  $\theta \approx 139.04^{\circ}$  (angle)

1.59749

#### **Alternate forms:**

$$\frac{1}{3} \left( \log \left( (\cosh(7) - 2) \operatorname{csch}(5) \operatorname{sech}^2 \left( \frac{7}{2} \right) \right) + i \pi \right)$$

$$\frac{1}{3} \log \left( \cosh(7) - 2 \right) \operatorname{csch}(5) \operatorname{sech}^2 \left( \frac{7}{2} \right) \right) + \frac{i \pi}{3}$$

$$\frac{1}{3}\left(i\pi + 2\log\left(\operatorname{sech}\left(\frac{7}{2}\right)\right) + \log(\cosh(7) - 2) + \log(\operatorname{csch}(5))\right)$$

#### **Alternative representations:**

$$\frac{1}{3}\log\left(\frac{2-\cosh(5+2)}{\sinh(5)\cosh\left(\frac{7}{2}\right)\cosh\left(\frac{7}{2}\right)}\right) = \frac{1}{3}\log_{\epsilon}\left(\frac{2-\cosh(7)}{\cosh^{2}\left(\frac{7}{2}\right)\sinh(5)}\right)$$

$$\frac{1}{3} \log \left( \frac{2 - \cosh(5 + 2)}{\sinh(5) \cosh\left(\frac{7}{2}\right) \cosh\left(\frac{7}{2}\right)} \right) = \frac{1}{3} \log(a) \log_a \left( \frac{2 - \cosh(7)}{\cosh^2\left(\frac{7}{2}\right) \sinh(5)} \right)$$

$$\frac{1}{3} \log \left( \frac{2 - \cosh(5+2)}{\sinh(5) \cosh\left(\frac{7}{2}\right) \cosh\left(\frac{7}{2}\right)} \right) = \frac{1}{3} \log \left( \frac{2 - \cos(7i)}{\frac{1}{2} \cos^2\left(\frac{7i}{2}\right)\left(-\frac{1}{e^5} + e^5\right)} \right)$$

### **Series representation:**

$$\frac{1}{3} \log \left( \frac{2 - \cosh(5 + 2)}{\sinh(5) \cosh\left(\frac{7}{2}\right) \cosh\left(\frac{7}{2}\right)} \right) =$$

$$\frac{i\pi}{3} - \frac{1}{3} \sum_{k=1}^{\infty} \frac{(-1)^k \left(-1 + (-2 + \cosh(7)) \operatorname{csch}(5) \operatorname{sech}^2\left(\frac{7}{2}\right)\right)^k}{k}$$

## **Integral representation**:

$$\frac{1}{3} \log \left( \frac{2 - \cosh(5 + 2)}{\sinh(5) \cosh(\frac{7}{2}) \cosh(\frac{7}{2})} \right) = \frac{i\pi}{3} + \frac{1}{3} \int_{1}^{(-2 + \cosh(7)) \operatorname{csch}(5) \operatorname{sech}^{2}(\frac{7}{2})} \frac{1}{t} dt$$

We have that:

$$S_{\text{gen}}^{\text{island}} = 2S_0 + \frac{2\phi_r}{\tanh a} + \frac{c}{3} \log \left( \frac{4 \tanh^2 \frac{a+b}{2}}{\sinh a} \right) . \tag{5.10}$$

$$2*(4Pi-0.98911) + (((2*0.98911)/(tanh(5)))) + 1/3*ln((((4tanh^2(7/2))/(sinh(5))))$$

#### **Input:**

$$2(4\pi - 0.98911) + \frac{2 \times 0.98911}{\tanh(5)} + \frac{1}{3} \log \left( \frac{4 \tanh^2 \left( \frac{7}{2} \right)}{\sinh(5)} \right)$$

#### **Result:**

24.15820...

24.15820... result very near to the black hole entropy 24.2477 (see Table)

## Alternative representations:

$$2(4\pi - 0.98911) + \frac{2 \times 0.98911}{\tanh(5)} + \frac{1}{3} \log \left( \frac{4 \tanh^2(\frac{7}{2})}{\sinh(5)} \right) =$$

$$2(-0.98911 + 4\pi) + \frac{1}{3} \log_e \left( \frac{4 \tanh^2(\frac{7}{2})}{\sinh(5)} \right) + \frac{1.97822}{-1 + \frac{2}{1 + \frac{1}{e^{10}}}}$$

$$2 (4\pi - 0.98911) + \frac{2 \times 0.98911}{\tanh(5)} + \frac{1}{3} \log \left( \frac{4 \tanh^2(\frac{7}{2})}{\sinh(5)} \right) =$$

$$2 (-0.98911 + 4\pi) + \frac{1}{3} \log(a) \log_a \left( \frac{4 \tanh^2(\frac{7}{2})}{\sinh(5)} \right) + \frac{1.97822}{-1 + \frac{2}{l + \frac{1}{e^{10}}}}$$

$$2(4\pi - 0.98911) + \frac{2 \times 0.98911}{\tanh(5)} + \frac{1}{3} \log \left( \frac{4 \tanh^2(\frac{7}{2})}{\sinh(5)} \right) =$$

$$2(-0.98911 + 4\pi) + \frac{1}{3} \log \left( \frac{4\left[-1 + \frac{2}{1 + \frac{1}{e^7}}\right]^2}{\frac{1}{2}\left(-\frac{1}{e^5} + e^5\right)} \right) + \frac{1.97822}{-1 + \frac{2}{1 + \frac{1}{e^{10}}}}$$

$$2 (4 \pi - 0.98911) + \frac{2 \times 0.98911}{\tanh(5)} + \frac{1}{3} \log \left( \frac{4 \tanh^2(\frac{7}{2})}{\sinh(5)} \right) =$$

$$\frac{1}{\sum_{k=1}^{\infty} \frac{1}{100 + (1 - 2k)^2 \pi^2}} 8 \left( 0.00618194 - 0.247278 \sum_{k=1}^{\infty} \frac{1}{100 + (1 - 2k)^2 \pi^2} + \frac{1}{100 + (1 - 2k)^2 \pi^2} \right) + \frac{1}{100 + (1 - 2k)^2 \pi^2} + \frac{1}{100 + (1 - 2k)^2 \pi^2} - 0.0416667 \sum_{k_1=1}^{\infty} \sum_{k_2=1}^{\infty} \frac{(-1)^{k_2} \left( -1 + \frac{4 \tanh^2(\frac{7}{2})}{\sinh(5)} \right)^{k_2}}{\left( 100 + \pi^2 (1 - 2k_1)^2 \right) k_2}$$

$$2 (4\pi - 0.98911) + \frac{2 \times 0.98911}{\tanh(5)} + \frac{1}{3} \log \left( \frac{4 \tanh^2(\frac{7}{2})}{\sinh(5)} \right) =$$

$$\left\{ 8 \left[ -0.247278 + 0.5 \pi - 0.247278 \sum_{k=1}^{\infty} (-1)^k q^{2k} + \frac{1}{\sinh(5)} \right] + \frac{1}{3} \log \left( \frac{-1}{2} \left( \frac{-1}{2} \right)^k \left( -1 + \frac{4 \tanh^2(\frac{7}{2})}{\sinh(5)} \right)^k - \frac{1}{2} \left( -1 \right)^k q^{2k} \right) \right\}$$

$$- \frac{1}{3} \left[ -\frac{1}{2} \left( -1 \right)^k q^{2k} \right] + \frac{1}{3} \log \left( \frac{-1}{2} \left( -1 + \frac{4 \tanh^2(\frac{7}{2})}{\sinh(5)} \right)^k - \frac{1}{2} \left( -1 + \frac{4 \tanh^2(\frac{7}{2})}{\sinh(5)} \right)^k \right]$$

$$\left[ -\frac{1}{3} \left( -1 \right)^k q^{2k} \right] + \frac{1}{3} \log \left( \frac{4 \tanh^2(\frac{7}{2})}{\sinh(5)} \right) =$$

$$\left[ 8 \left[ -0.247278 - 0.247278 \sum_{k=0}^{\infty} \left( \delta_k + \frac{2^{1+k} \operatorname{Li}_{-k}(-e^{2z_0})}{k!} \right) (5 - z_0)^k + \frac{1}{2} \left( -1 + \frac{4 \tanh^2(\frac{7}{2})}{\sinh(5)} \right) \right]$$

$$- \frac{1}{3} \left[ -\frac{1}{3} \left( -\frac{1}{3} \right)^k \left( -\frac{1}{3} \right) \left( -\frac{1}{3} \right) \left( -\frac{1}{3} \right)^k \left( -\frac{1}{3} \right) \left( -\frac{$$

#### From:

## Three-dimensional AdS gravity and extremal CFTs at c = 8m

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m	$L_0$	d	S	$S_{BH}$	m	$L_0$	d	S	$S_{BH}$
	1	196883	12.1904	12.5664		1	42987519	17.5764	17.7715
3	2	21296876	16.8741	17.7715	6	2	40448921875	24.4233	25.1327
	3	842609326	20.5520	21.7656		3	8463511703277	29.7668	30.7812
	2/3	139503	11.8458	11.8477		2/3	7402775	15.8174	15.6730
4	5/3	69193488	18.0524	18.7328	7	5/3	33934039437	24.2477	24.7812
	8/3	6928824200	22.6589	23.6954		8/3	16953652012291	30.4615	31.3460
	1/3	20619	9.9340	9.3664		1/3	278511	12.5372	11.8477
5	4/3	86645620	18.2773	18.7328	8	4/3	13996384631	23.3621	23.6954
	7/3	24157197490	23.9078	24.7812		7/3	19400406113385	30.5963	31.3460

Table 1: Degeneracies, microscopic entropies and semiclassical entropies for the first few values of m and  $L_0$ .

$$5*((2*(4Pi-0.98911) + (((2*0.98911)/(tanh(5)))) + 1/3 * ln((((4tanh^2(7/2))/(sinh(5))))))+18+1/golden ratio$$

#### **Input:**

$$5\left(2(4\pi - 0.98911) + \frac{2 \times 0.98911}{\tanh(5)} + \frac{1}{3}\log\left(\frac{4\tanh^2\left(\frac{7}{2}\right)}{\sinh(5)}\right)\right) + 18 + \frac{1}{\phi}$$

ø is the golden ratio

#### **Result:**

139.4090...

139.4090... result practically equal to the rest mass of Pion meson 139.57

## **Alternative representations:**

$$\begin{split} 5\left(2\left(4\pi-0.98911\right)+\frac{2\times0.98911}{\tanh(5)}+\frac{1}{3}\log\left(\frac{4\tanh^2\left(\frac{7}{2}\right)}{\sinh(5)}\right)\right)+18+\frac{1}{\phi}=\\ 18+\frac{1}{\phi}+5\left(2\left(-0.98911+4\pi\right)+\frac{1}{3}\log_e\left(\frac{4\tanh^2\left(\frac{7}{2}\right)}{\sinh(5)}\right)+\frac{1.97822}{-1+\frac{2}{1+\frac{1}{e^{10}}}}\right) \end{split}$$

$$5\left[2\left(4\pi - 0.98911\right) + \frac{2\times 0.98911}{\tanh(5)} + \frac{1}{3}\log\left(\frac{4\tanh^2\left(\frac{7}{2}\right)}{\sinh(5)}\right)\right] + 18 + \frac{1}{\phi} = \\ 18 + \frac{1}{\phi} + 5\left[2\left(-0.98911 + 4\pi\right) + \frac{1}{3}\log(a)\log_a\left(\frac{4\tanh^2\left(\frac{7}{2}\right)}{\sinh(5)}\right) + \frac{1.97822}{-1 + \frac{2}{1 + \frac{1}{e^{10}}}}\right]$$

$$5\left[2\left(4\pi - 0.98911\right) + \frac{2\times 0.98911}{\tanh(5)} + \frac{1}{3}\log\left(\frac{4\tanh^2\left(\frac{7}{2}\right)}{\sinh(5)}\right)\right] + 18 + \frac{1}{\phi} = \\ 18 + \frac{1}{\phi} + 5\left[2\left(-0.98911 + 4\pi\right) + \frac{1}{3}\log\left(\frac{4\left(-1 + \frac{2}{1 + \frac{1}{e^7}}\right)}{\frac{1}{2}\left(-\frac{1}{e^5} + e^5\right)}\right] + \frac{1.97822}{-1 + \frac{2}{1 + \frac{1}{e^{10}}}}$$

$$\begin{split} 5\left(2\left(4\pi-0.98911\right) + \frac{2\times0.98911}{\tanh(5)} + \frac{1}{3}\log\left(\frac{4\tanh^2\left(\frac{7}{2}\right)}{\sinh(5)}\right)\right) + 18 + \frac{1}{\phi} = \\ \left(40\left(0.00618194\phi + 0.025\sum_{k=1}^{\infty}\frac{1}{100 + (1-2k)^2\pi^2} + 0.202723\phi\sum_{k=1}^{\infty}\frac{1}{100 + (1-2k)^2\pi^2} + \phi\pi\sum_{k=1}^{\infty}\frac{1}{100 + (1-2k)^2\pi^2} - 0.0416667\right) \\ \phi \sum_{k_1=1}^{\infty}\sum_{k_2=1}^{\infty}\frac{\left(-1\right)^{k_2}\left(-1 + \frac{4\tanh^2\left(\frac{7}{2}\right)}{\sinh(5)}\right)^{k_2}}{\left(100 + \pi^2\left(1 - 2k_1\right)^2\right)k_2}\right) \middle/\left(\phi\sum_{k=1}^{\infty}\frac{1}{100 + (1-2k)^2\pi^2}\right) \end{split}$$

$$\begin{split} & \left[2\left(4\pi-0.98911\right) + \frac{2\times0.98911}{\tanh(5)} + \frac{1}{3}\log\left(\frac{4\tanh^2\left(\frac{7}{2}\right)}{\sinh(5)}\right)\right) + 18 + \frac{1}{\phi} = \\ & \left[40\left[0.0125 - 0.0222775\,\phi + 0.5\,\phi\pi + 0.025\sum_{k=1}^{\infty}\left(-1\right)^kq^{2\,k} + \right. \\ & \left.0.202723\,\phi\sum_{k=1}^{\infty}\left(-1\right)^kq^{2\,k} + \phi\pi\sum_{k=1}^{\infty}\left(-1\right)^kq^{2\,k} - \right. \\ & \left.0.0208333\,\phi\sum_{k=1}^{\infty}\frac{\left(-1\right)^k\left[-1 + \frac{4\tanh^2\left(\frac{7}{2}\right)^k}{\sinh(5)}\right]^k}{k} - \right. \\ & \left.0.0416667\,\phi\sum_{k=1}^{\infty}\sum_{k=1}^{\infty}\frac{\left(-1\right)^k1^{+k2}\,q^{2\,k1}\left[-1 + \frac{4\tanh^2\left(\frac{7}{2}\right)^k}{\sinh(5)}\right]^k}{k_2}\right] \\ & \left[\phi\left(0.5 + \sum_{k=1}^{\infty}\left(-1\right)^kq^{2\,k}\right)\right] \text{ for } q = e^5 \end{split}$$

$$& 5\left[2\left(4\pi - 0.98911\right) + \frac{2\times0.98911}{\tanh(5)} + \frac{1}{3}\log\left(\frac{4\tanh^2\left(\frac{7}{2}\right)}{\sinh(5)}\right)\right] + 18 + \frac{1}{\phi} = \\ & \left[40\left[-0.247277\,\phi + 0.025\sum_{k=0}^{\infty}\left(\delta_k + \frac{2^{1+k}\operatorname{Li}_{-k}\left(-e^{2\,z_0}\right)}{k!}\right)\left(5 - z_0\right)^k + \right. \\ & \left.0.202723\,\phi\sum_{k=0}^{\infty}\left(\delta_k + \frac{2^{1+k}\operatorname{Li}_{-k}\left(-e^{2\,z_0}\right)}{k!}\right)\left(5 - z_0\right)^k + \right. \\ & \left.\phi\pi\sum_{k=0}^{\infty}\left(\delta_k + \frac{2^{1+k}\operatorname{Li}_{-k}\left(-e^{2\,z_0}\right)}{k!}\right)\left(5 - z_0\right)^k - 0.0416667\,\phi \right. \\ & \left.\sum_{k=0}^{\infty}\sum_{k=0}^{\infty}\frac{\left(-1\right)^k2\left(\delta_{k_1} + \frac{2^{1+k}\operatorname{Li}_{-k_1}\left(-e^{2\,z_0}\right)}{k!}\right)\left(5 - z_0\right)^{k_1}\left(-1 + \frac{4\tanh^2\left(\frac{7}{2}\right)^k}{\sinh(5)}\right)^{k_2}}\right] \\ & \left.\int\left[\phi\sum_{k=0}^{\infty}\left(\delta_k + \frac{2^{1+k}\operatorname{Li}_{-k}\left(-e^{2\,z_0}\right)}{k!}\right)\left(5 - z_0\right)^k\right] \text{ for }\frac{1}{2} + \frac{iz_0}{\pi}\notin\mathbb{Z} \right] \right. \end{aligned}$$

Now, we have that:

$$S_{\text{matter}}(I \cup R) \approx 2S_{\text{matter}}([P_1, P_2]) - \frac{c}{3} \log \left( \frac{2|\cosh(a+b) - \cosh(t_c - t_b)|}{\sinh a} \right)$$

 $1/3 \ln ((((2 \cosh(5+2) - \cosh(0)))/((\sinh(5)))))$ 

#### **Input:**

$$\frac{1}{3} \log \left( \frac{2 \cosh(5+2) - \cosh(0)}{\sinh(5)} \right)$$

#### **Exact result:**

$$\frac{1}{3}\log((2\cosh(7)-1)\cosh(5))$$

csch(x) is the hyperbolic cosecant function

## **Decimal approximation:**

0.897427038608265865479582877913152494054097509045630356825...

0.8974270386082....

#### **Alternate forms:**

$$\frac{1}{3} \left( \log(2\cosh(7) - 1) + \log(\cosh(5)) \right)$$

$$\frac{1}{3} \log \left( \frac{2 \left( -1 + \frac{1}{e^7} + e^7 \right)}{e^5 - \frac{1}{e^5}} \right)$$

$$\frac{1}{3} \left(-2 + \log(2) - \log(e^{10} - 1) + \log(1 - e^7 + e^{14})\right)$$

## **Alternative representations:**

$$\frac{1}{3} \log \left( \frac{2 \cosh(5+2) - \cosh(0)}{\sinh(5)} \right) = \frac{1}{3} \log \left( \frac{-1 + \frac{1}{e^7} + e^7}{\frac{1}{2} \left( -\frac{1}{e^5} + e^5 \right)} \right)$$

$$\frac{1}{3} \log \left( \frac{2 \cosh(5+2) - \cosh(0)}{\sinh(5)} \right) = \frac{1}{3} \log_e \left( \frac{-\cosh(0) + 2 \cosh(7)}{\sinh(5)} \right)$$

$$\frac{1}{3}\log\left(\frac{2\cosh(5+2)-\cosh(0)}{\sinh(5)}\right) = \frac{1}{3}\log(a)\log_a\left(\frac{-\cosh(0)+2\cosh(7)}{\sinh(5)}\right)$$

### Series representation:

$$\begin{split} &\frac{1}{3} \log \left( \frac{2 \cosh(5+2) - \cosh(0)}{\sinh(5)} \right) = \\ &\frac{1}{3} \log(-1 + (-1 + 2 \cosh(7)) \operatorname{csch}(5)) - \frac{1}{3} \sum_{k=1}^{\infty} \frac{\left( -\frac{1}{-1 + (-1 + 2 \cosh(7)) \operatorname{csch}(5)} \right)^k}{k} \end{split}$$

### **Integral representations:**

$$\frac{1}{3} \log \left( \frac{2 \cosh(5+2) - \cosh(0)}{\sinh(5)} \right) = \frac{1}{3} \int_{1}^{(-1+2 \cosh(7)) \operatorname{csch}(5)} \frac{1}{t} dt$$

$$\begin{split} &\frac{1}{3} \log \left( \frac{2 \cosh(5+2) - \cosh(0)}{\sinh(5)} \right) = \\ &- \frac{i}{6 \pi} \int_{-i \infty + \gamma}^{i \infty + \gamma} \frac{(-1 + (-1 + 2 \cosh(7)) \operatorname{csch}(5))^{-s} \, \Gamma(-s)^2 \, \Gamma(1+s)}{\Gamma(1-s)} \, ds \, \text{ for } -1 < \gamma < 0 \end{split}$$

 $((((1/3 \ln ((((2 \cosh(5+2)-\cosh(0)))/((\sinh (5)))))))^1/16$ 

### **Input:**

$$16\sqrt{\frac{1}{3}\log\left(\frac{2\cosh(5+2)-\cosh(0)}{\sinh(5)}\right)}$$

#### **Exact result:**

$$\sqrt[16]{\frac{1}{3} \log((2 \cosh(7) - 1) \operatorname{csch}(5))}$$

 $\operatorname{csch}(x)$  is the hyperbolic cosecant function

# **Decimal approximation:**

0.993258858131342001248394167369224755984632041799723686055...

0.993258858131342..... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{\sqrt{5}}} \approx 0.9991104684$$

$$1 + \frac{\sqrt[5]{\sqrt{\phi^5 \sqrt[4]{5^3}} - 1}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}$$

# and to the dilaton value $0.989117352243 = \phi$

#### **Alternate forms:**

$$16\sqrt{\frac{1}{3}} (\log(2\cosh(7) - 1) + \log(\cosh(5)))$$

$$16 \frac{1}{3} \log \left( \frac{2\left(-1 + \frac{1}{e^7} + e^7\right)}{e^5 - \frac{1}{e^5}} \right)$$

$$\frac{1}{16\sqrt{\frac{\frac{3}{-2-\log\left(\frac{e^{10}-1}{2\left(1-e^{7}+e^{14}\right)}\right)}}}$$

# All 16th roots of $1/3 \log((2 \cosh(7) - 1) \operatorname{csch}(5))$ :

$$e^{0.16} \sqrt{\frac{1}{3} \log((2 \cosh(7) - 1) \operatorname{csch}(5))} \approx 0.99326$$
 (real, principal root)

$$e^{(i\,\pi)/8}\, \sqrt[16]{\frac{1}{3}\, \log((2\,\cosh(7)-1)\, \mathrm{csch}(5))} \approx 0.91765 + 0.38010\, i$$

$$e^{(i\,\pi)/4} \sqrt[16]{\frac{1}{3}\,\log((2\cosh(7)-1)\cosh(5))} \approx 0.70234 + 0.70234\,i$$

$$e^{(3 i \pi)/8} \sqrt[16]{\frac{1}{3} \log((2 \cosh(7) - 1) \operatorname{csch}(5))} \approx 0.38010 + 0.91765 i$$

$$e^{(i\pi)/2} \sqrt[16]{\frac{1}{3} \log((2\cosh(7) - 1) \operatorname{csch}(5))} \approx 0.99326 i$$

### **Alternative representations:**

$${}^{16}\!\!\sqrt{\frac{1}{3}\,\log\!\!\left(\!\frac{2\cosh(5+2)-\cosh(0)}{\sinh(5)}\right)} = {}^{16}\!\!\sqrt{\frac{1}{3}\,\log\!\!\left(\!\frac{-1+\frac{1}{e^7}+e^7}{\frac{1}{2}\left(-\frac{1}{e^5}+e^5\right)}\right)}$$

$$16\sqrt{\frac{1}{3}\log\!\left(\frac{2\cosh(5+2)-\cosh(0)}{\sinh(5)}\right)} = 16\sqrt{\frac{1}{3}\log_{\ell}\!\left(\frac{-\cosh(0)+2\cosh(7)}{\sinh(5)}\right)}$$

$$16\sqrt{\frac{1}{3}\log\biggl(\frac{2\cosh(5+2)-\cosh(0)}{\sinh(5)}\biggr)}=16\sqrt{\frac{1}{3}\log(a)\log_a\biggl(\frac{-\cosh(0)+2\cosh(7)}{\sinh(5)}\biggr)}$$

## **Series representation:**

$$\frac{16\sqrt{\frac{1}{3}\log\left(\frac{2\cosh(5+2)-\cosh(0)}{\sinh(5)}\right)}}{\log(-1+(-1+2\cosh(7))\cosh(5))-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{-1+(-1+2\cosh(7))\cosh(5)}\right)^k}{k}}{\frac{16\sqrt{3}}{}}$$

## **Integral representations:**

$$16\sqrt{\frac{1}{3}\log\left(\frac{2\cosh(5+2)-\cosh(0)}{\sinh(5)}\right)} = \frac{16\sqrt{\int_{1}^{(-1+2\cosh(7))\operatorname{csch}(5)}\frac{1}{t}} dt}{16\sqrt{3}}$$

8 log base 0.993258858131342((((1/3 ln ((((2 cosh(5+2)-cosh(0)))/((sinh (5))))))))Pi+1/golden ratio

Where 8 is a Fibonacci number

### **Input interpretation:**

$$8 \log_{0.993258858131342} \left( \frac{1}{3} \log \left( \frac{2 \cosh(5+2) - \cosh(0)}{\sinh(5)} \right) \right) - \pi + \frac{1}{\phi}$$

#### **Result:**

125.4764413352...

125.4764413352... result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T = 0 and to the Higgs boson mass 125.18

## **Alternative representations:**

$$8 \log_{0.0032588581313420000} \left( \frac{1}{3} \log \left( \frac{2 \cosh(5+2) - \cosh(0)}{\sinh(5)} \right) \right) - \pi + \frac{1}{\phi} = -\pi + 8 \log_{0.0032588581313420000} \left( \frac{1}{3} \log \left( \frac{-1 + \frac{1}{e^7} + e^7}{\frac{1}{2} \left( -\frac{1}{e^5} + e^5 \right)} \right) \right) + \frac{1}{\phi}$$

$$8 \log_{0.9932588581313420000} \left( \frac{1}{3} \log \left( \frac{2 \cosh(5+2) - \cosh(0)}{\sinh(5)} \right) \right) - \pi + \frac{1}{\phi} = -\pi + 8 \log_{0.9932588581313420000} \left( \frac{1}{3} \log_{e} \left( \frac{-\cosh(0) + 2 \cosh(7)}{\sinh(5)} \right) \right) + \frac{1}{\phi}$$

$$\begin{split} &8\log_{0.9932588581313420000} \left(\frac{1}{3}\log \left(\frac{2\cosh(5+2)-\cosh(0)}{\sinh(5)}\right)\right) - \pi + \frac{1}{\phi} = \\ &-\pi + \frac{1}{\phi} + \frac{8\log \left(\frac{1}{3}\log \left(\frac{-\cosh(0)+2\cosh(7)}{\sinh(5)}\right)\right)}{\log(0.9932588581313420000)} \end{split}$$

$$\begin{split} &8 \log_{0.9932588581313420000} \left( \frac{1}{3} \log \left( \frac{2 \cosh(5+2) - \cosh(0)}{\sinh(5)} \right) \right) - \pi + \frac{1}{\phi} = \\ &\frac{1}{\phi} - \pi - \frac{8 \sum_{k=1}^{\infty} \frac{\left( -\frac{1}{3} \right)^k \left( -3 + \log \left( -\frac{\cosh(0) - 2 \cosh(7)}{\sinh(5)} \right) \right)^k}{k}}{\log(0.9932588581313420000)} \end{split}$$

$$\begin{split} 8 \log_{0.9932588581313420000} & \left( \frac{1}{3} \log \left( \frac{2 \cosh(5+2) - \cosh(0)}{\sinh(5)} \right) \right) - \pi + \frac{1}{\phi} = \\ & \frac{1}{\phi} - \pi + 8 \log_{0.9932588581313420000} \left( \\ & \frac{1}{3} \left( \log \left( -\frac{\cosh(0) - 2 \cosh(7) + \sinh(5)}{\sinh(5)} \right) - \sum_{k=1}^{\infty} \frac{(-1)^k \left( -\frac{\cosh(0) - 2 \cosh(7) + \sinh(5)}{\sinh(5)} \right)^{-k}}{k} \right) \right) \end{split}$$

$$\begin{split} &8 \log_{0.9932588581313420000} \left( \frac{1}{3} \log \left( \frac{2 \cosh(5+2) - \cosh(0)}{\sinh(5)} \right) \right) - \pi + \frac{1}{\phi} = \\ &\frac{1}{\phi} - \pi + 8 \log_{0.9932588581313420000} \left( \frac{1}{3} \int_{1}^{-\frac{\cosh(0) - 2 \cosh(7)}{\sinh(5)}} \frac{1}{t} \, dt \right) \end{split}$$

$$8 \log_{0.9932588581313420000} \left( \frac{1}{3} \log \left( \frac{2 \cosh(5+2) - \cosh(0)}{\sinh(5)} \right) \right) - \pi + \frac{1}{\phi} =$$

$$-1 + \phi \pi - 8 \phi \log_{0.9932588581313420000} \left( \frac{1}{3} \log \left( \frac{1+14 \int_{0}^{1} \sinh(7t) dt}{5 \int_{0}^{1} \cosh(5t) dt} \right) \right)$$

$$- \frac{\phi}{\phi}$$

$$\begin{split} 8 \log_{0.9932588581313420000} & \left( \frac{1}{3} \log \left( \frac{2 \cosh(5+2) - \cosh(0)}{\sinh(5)} \right) \right) - \pi + \frac{1}{\phi} = \\ & -1 + \phi \pi - 8 \phi \log_{0.9932588581313420000} \left( \frac{1}{3} \log \left( -\frac{\sqrt{\pi} \int_{-i \infty + \gamma}^{i \infty + \gamma} \frac{e^{s} - 2 e^{49/(4 s) + s}}{\sqrt{s}} \frac{ds}{\sqrt{s}} \right) \right) \\ & - \frac{\phi}{\phi} \end{split}$$
 for

8 log base 0.993258858131342((((1/3 ln ((((2 cosh(5+2)-cosh(0)))/((sinh (5))))))))+11+1/golden ratio

Where 8 is a Fibonacci number and 11 is a Lucas number

# **Input interpretation:**

$$8 \log_{0.993258858131342} \left( \frac{1}{3} \log \left( \frac{2 \cosh(5+2) - \cosh(0)}{\sinh(5)} \right) \right) + 11 + \frac{1}{\phi}$$

#### **Result:**

139.6180339887...

139.61803398... result practically equal to the rest mass of Pion meson 139.57

## **Alternative representations:**

$$8 \log_{0.9932588581313420000} \left( \frac{1}{3} \log \left( \frac{2 \cosh(5+2) - \cosh(0)}{\sinh(5)} \right) \right) + 11 + \frac{1}{\phi} = 11 + 8 \log_{0.9932588581313420000} \left( \frac{1}{3} \log \left( \frac{-1 + \frac{1}{e^7} + e^7}{\frac{1}{2} \left( -\frac{1}{e^5} + e^5 \right)} \right) \right) + \frac{1}{\phi}$$

$$8 \log_{0.9932588581313420000} \left( \frac{1}{3} \log \left( \frac{2 \cosh(5+2) - \cosh(0)}{\sinh(5)} \right) \right) + 11 + \frac{1}{\phi} = 11 + 8 \log_{0.9932588581313420000} \left( \frac{1}{3} \log_e \left( \frac{-\cosh(0) + 2 \cosh(7)}{\sinh(5)} \right) \right) + \frac{1}{\phi}$$

$$\begin{split} &8\log_{0.9932588581313420000} \left(\frac{1}{3}\log \left(\frac{2\cosh(5+2)-\cosh(0)}{\sinh(5)}\right)\right) + 11 + \frac{1}{\phi} = \\ &11 + \frac{1}{\phi} + \frac{8\log \left(\frac{1}{3}\log \left(\frac{-\cosh(0)+2\cosh(7)}{\sinh(5)}\right)\right)}{\log(0.9932588581313420000)} \end{split}$$

$$8 \log_{0.0032588581313420000} \left( \frac{1}{3} \log \left( \frac{2 \cosh(5+2) - \cosh(0)}{\sinh(5)} \right) \right) + 11 + \frac{1}{\phi} = 11 + \frac{1}{\phi} - \frac{8 \sum_{k=1}^{\infty} \frac{\left( -\frac{1}{3} \right)^k \left( -3 + \log \left( -\frac{\cosh(0) - 2 \cosh(7)}{\sinh(5)} \right) \right)^k}{k}}{\log(0.9932588581313420000)}$$

$$8 \log_{0.9932588581313420000} \left( \frac{1}{3} \log \left( \frac{2 \cosh(5+2) - \cosh(0)}{\sinh(5)} \right) \right) + 11 + \frac{1}{\phi} = 11 + \frac{1}{\phi} + 8 \log_{0.9932588581313420000} \left( \frac{1}{3} \left( \log \left( -\frac{\cosh(0) - 2 \cosh(7) + \sinh(5)}{\sinh(5)} \right) - \sum_{k=1}^{\infty} \frac{(-1)^k \left( -\frac{\cosh(0) - 2 \cosh(7) + \sinh(5)}{\sinh(5)} \right)^{-k}}{k} \right) \right) \right)$$

$$8 \log_{0.9932588581313420000} \left( \frac{1}{3} \log \left( \frac{2 \cosh(5+2) - \cosh(0)}{\sinh(5)} \right) \right) + 11 + \frac{1}{\phi} = \\ 11 + \frac{1}{\phi} + 8 \log_{0.9932588581313420000} \left( \frac{1}{3} \int_{1}^{-\frac{\cosh(0) - 2 \cosh(7)}{\sinh(5)}} \frac{1}{t} \, dt \right)$$

$$8 \log_{0.9932588581313420000} \left( \frac{1}{3} \log \left( \frac{2 \cosh(5+2) - \cosh(0)}{\sinh(5)} \right) \right) + 11 + \frac{1}{\phi} = \frac{1 + 11 \phi + 8 \phi \log_{0.9932588581313420000} \left( \frac{1}{3} \log \left( \frac{1 + 14 \int_{0}^{1} \sinh(7t) dt}{5 \int_{0}^{1} \cosh(5t) dt} \right) \right)}{\phi}$$

$$8 \log_{0.9932588581313420000} \left( \frac{1}{3} \log \left( \frac{2 \cosh(5+2) - \cosh(0)}{\sinh(5)} \right) \right) + 11 + \frac{1}{\phi} = \\ \frac{1 + 11 \phi + 8 \phi \log_{0.9932588581313420000} \left( \frac{1}{3} \log \left( -\frac{\sqrt{\pi} \int_{-i \, \infty + \gamma}^{i \, \infty + \gamma} \frac{e^{s} - 2 \, e^{49/(4 \, s) + s}}{\sqrt{s}} \, ds}{10 \, i \, \pi \int_{0}^{1} \cosh(5 \, t) \, dt} \right) \right) }{\phi} \quad \text{for } \gamma > 0$$

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