Unification for Gravity and Electromagnetic Field in Kerr-Newman Solution

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ABSTRACT
Solutions of unified theory equations of gravity and electromagnetism has to satisfy Einstein-Maxwell equation. Specially, solution of the unified theory is generally Kerr-Newman solution in vacuum. We finally found the revised Einstein gravity tensor equation with new term (2-order contravariant metric tensor two times product and the constant matrix) is right in Kerr-Newman solution.

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1. Introduction

This theory’s aim is that we discover the revised Einstein gravity equation had Kerr-Newman solution in vacuum.

First, we know the revised Einstein gravity equation [1].

\[ R_{\mu \nu} - \frac{1}{2} g_{\mu \nu} R + \Lambda g_{\mu \nu} R^{(0)} = - \frac{8\pi G}{c^4} T_{\mu \nu} \]

In this time,

\[ \Lambda = k \frac{G Q^2}{c^4}, \quad I = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \]

If Eq(1) take covariant differential operator,

\[ (R_{\mu \nu} - \frac{1}{2} g_{\mu \nu} R)_{;\mu} + \Lambda g_{\mu \nu} I g^{00} g_{00} = - \frac{8\pi G}{c^4} T_{;\mu \nu} = 0 \] (2-i)

\[ (R_{\mu \nu} - \frac{1}{2} g_{\mu \nu} R)_{;\nu} + \Lambda g_{\mu \nu} I g^{00} g_{00} = - \frac{8\pi G}{c^4} T_{;\mu \nu} = 0 \] (2-ii)

In this time, in Kerr-Newman solution

\[ g_{\theta \theta} = 1 / \hat{g}^{\theta \theta} = \rho^2 = \hat{r} + \hat{a} \cos \hat{\theta} \]

\[ g^{00} = \frac{\partial \hat{g}^{00}}{\partial x^0} + 2\Gamma^0_{\alpha \beta} g^{\alpha \beta} = \frac{\partial \hat{g}^{00}}{\partial r} + 2\Gamma^0_{\theta \phi} \hat{g}^{00} \]

\[ = \frac{\partial}{\partial r} \left( \frac{1}{\rho^2} \right) + 2 \cdot \frac{r}{\rho^2} \cdot \frac{1}{\rho^2} = -2 \frac{1}{\rho^2} \cdot \frac{r}{\rho} + \frac{2r}{\rho^2} = 0 \] (3)

\[ g^{00} = \frac{\partial \hat{g}^{00}}{\partial x^\rho} + 2\Gamma^0_{\theta \phi} g^{\theta \phi} = \frac{\partial \hat{g}^{00}}{\partial \theta} + 2\Gamma^0_{\theta \theta} \hat{g}^{00} \]

\[ = \frac{\partial}{\partial \theta} \left( \frac{1}{\rho^2} \right) - 2 \cdot \frac{1}{\rho^2} \cdot \frac{1}{\rho^2} \cdot \frac{2a^2}{\rho} \cos \theta \sin \theta \cdot \frac{1}{\rho^2} \]

\[ = -2 \cdot \frac{1}{\rho^2} \cdot \frac{2a^2 \cos \theta \sin \theta}{\rho} - \frac{4a^2}{\rho^4} \cos \theta \sin \theta = 0 \] (4)

If \( g^{00}_{;\rho} = V_\rho \), the vector transformation is

\[ 0 = V_\rho = \frac{\partial x^\alpha}{\partial x^\rho} V^\alpha, \quad V^\alpha = 0 \] (5)

Therefore, if the coordinate is not the Kerr-Newman’s coordinate, the covariant differential of
\( g^{\theta\theta} = \frac{1}{\rho^2} \) is still zero in the changed coordinates.

2. The revised Einstein gravity equation and Kerr-Newman solution

In this theory, Eq(1) can change the following equation.

\[
R_{\mu\nu} = -\frac{8\pi G}{c^4} (\mathcal{T}_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T^\lambda_\lambda) + \Lambda g_{\mu\nu} \langle g^{\theta\theta} \rangle^2
\]

(6)

In this time, in vacuum, specially, in Kerr-Newman solution,

\[
T_{\mu\nu} = 0, \quad T^\lambda_\lambda = g^{\mu\nu} T_{\mu\nu} \neq 0
\]

(7)

Therefore, Eq(1) is

\[
T_{\mu\nu} = 0, -\Lambda g_{\mu\nu} \langle g^{\theta\theta} \rangle^2 = \mathcal{R}_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \mathcal{R} = \frac{2G}{c^5} (g_{\mu\nu} F_{uv} F^{uv} - \frac{1}{4} g_{\mu\nu} F_{\rho\sigma} F^{\rho\sigma})
\]

(8)

In this time, According to [2],

\[
E = F_{01} = -F_{10} = \frac{Q (\hat{r} - \hat{a} \cos \theta \hat{\phi})}{r^2 + a^2 \cos \theta \hat{\phi}} = \frac{\rho^2}{Q} \frac{\hat{r} - \hat{a} \cos \theta \hat{\phi}}{\rho^4}
\]

\[
B = F_{23} = -F_{32} = \frac{2Q \arccos \theta}{(r^2 + a^2 \cos \theta \hat{\phi})} = \frac{2Q \arccos \theta}{\rho^4}
\]

(9)

Hence,

\[
\frac{2G}{c^5} (g_{\mu\nu} F_{uv} F^{uv} - \frac{1}{4} g_{\mu\nu} F_{\rho\sigma} F^{\rho\sigma})
\]

\[
= -\frac{G}{c^5} g_{\mu\nu} \langle B^2 + E^2 \rangle, \quad I = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}
\]

\[
= -\frac{G}{c^5} \begin{pmatrix} g_{00} & 0 & 0 & -g_{03} \\ 0 & g_{11} & 0 & 0 \\ 0 & 0 & -g_{22} & 0 \\ g_{30} & 0 & 0 & -g_{33} \end{pmatrix} \langle B^2 + E^2 \rangle, \quad B^2 + E^2 = \frac{Q^2}{\rho^2}
\]

\[
= -\frac{G}{c^5} \begin{pmatrix} g_{00} & 0 & 0 & -g_{03} \\ 0 & g_{11} & 0 & 0 \\ 0 & 0 & -g_{22} & 0 \\ g_{30} & 0 & 0 & -g_{33} \end{pmatrix} \frac{Q^2}{\rho^4} = -\Lambda g_{\mu\nu} \langle g^{\theta\theta} \rangle^2
\]
\[ \Lambda = \kappa \frac{GQ^2}{c^6} \]  

(10)

3. Conclusion

We finally found the revised Einstein equation of unified theory (the gravity and electromagnetic field) is right in Kerr-Newman solution.

References