Proving the Goldbach’s conjecture

Ninh Khac Son

Date Performed: 06 December 2019

Abstract
Goldbach’s conjecture is one of the oldest and best-known unsolved problems in number theory and all of mathematics. It states:

"Every even integer greater than 2 can be expressed as the sum of two primes”.

Manuscript content: Prove that Goldbach’s conjecture is correct.

Key words: Prime numbers, Goldbach’s conjecture, number theory.

1. Notation system
We briefly mention the symbols and theorems in number theory to apply to this manuscript.

1.1. Notation
- Symbol of positive natural number: $N^*$
- Symbol of prime number greater than 2: $P^*$
- Symbol of odd-number greater than 2: $O^*$

1.2. The operations express odd and prime numbers
- For every odd natural number $O$ greater than 2, it can always be expressed as:

$$O = 2n + 1 \ (With: \ O \in O^*, n \in N^*) \quad (1)$$

This deduces the result: For every odd natural number $O'$ greater than 5, it can always be expressed as:

$$O' = 2n' + P \ (With: \ n' \in N^*, P \in P^*, P < O') \quad (2)$$

Preprint submitted to viXra December 14, 2019
This also deduces the result: For every prime number $P$ greater than 5, it can always be expressed as:

$$P = 2m + P' \quad (\text{With} : \ m \in \mathbb{N}^*, P' \in \mathbb{P}^*, P' < P) \quad (3)$$

1.3. **Bertrand’s postulate**

Bertrand’s postulate is a theorem stating that for any integer $n > 3$, there always exists at least one prime number $p$ with

$$n < p < 2n - 2$$

2. **Goldbach’s conjecture**

Goldbach’s conjecture is one of the oldest and best-known unsolved problems in number theory and all of mathematics. It states:

"Every even integer greater than 2 can be expressed as the sum of two primes".

3. **Proving the Goldbach’s conjecture**

3.1. **Consider even integer numbers $2 < N \leq 10$**

- For $N = 4$, represent: $N = 2 + 2$
- For $N = 6$, represent: $N = 3 + 3$
- For $N = 8$, represent: $N = 3 + 5$
- For $N = 10$, represent: $N = 3 + 7$

3.2. **Consider even integer numbers $N > 10$**

Because $N$ is an even integer greater than 10, $N$ can always be expressed as the sum of two odd numbers:

$$N = O_1 + O_2 \quad (\text{With} : \ O_1, O_2 \in \mathbb{O}^*, O_1 < O_2) \quad (4)$$

Because the prime number $P_1$ is greater than 2 in the set $\mathbb{P}^*$, it also belongs to the set $\mathbb{O}^*$. Therefore, $N$ can always be expressed as the sum of a prime number $P_1$ and an odd number $O$:

$$N = P_x + O \quad (\text{With} : \ \forall P_x \in \mathbb{P}^*, O \in \mathbb{O}^*, P_x < N/2) \quad (5)$$
Based on the result of (2), we can express the odd number \( O \) to:

\[
O = P_x + 2n \quad (\text{With: } n \in N^*, P_x \in P^*, P_x < O) \tag{6}
\]

Finally, this is stated as follows: With every even natural number \( N > 10 \), there is always at least a prime number \( P_1 \) in the set \( P^* \) such that

\[
N = P_x + (P_x + 2n) \quad (\text{With: } P_x \in P^*, n \in N^*, P_x, n < N/2) \tag{7}
\]

Example: Any even natural number \( N \) greater than 10, it can be expressed by \( \forall P_x < N/2 \text{ or } N = P_x = \text{const} + (P_x = \text{const} + 2n) \).

From expression (7), we transform to produce the result:

\[
n = \frac{N}{2} - P_x \quad (\text{With: } P_x \in P^*, n \in N^*, P_x < N/2) \tag{8}
\]

In particular, \( N \) has a given value, so the value of \( n \) will vary with \( P_x \). While \( P_x \in P^* \), we convert \( P_x \) into the set \( N^* \) to construct a function \( f \):

\[
f = \frac{N}{2} - x \quad (\text{With: } x \in N^*, x < N/2) \tag{9}
\]

Thus, the value of \( f \) contains the value of \( n \) and the value of \( x \) contains the value of \( P_x \). This means that the values of \( n \) and \( P_x \) always belong the graph of the function \( f \).

On the other hand, based on the expressions (5) (6), since \( N/2 < P_x + 2n < N \), this results in the value of \( n \) also in the graph of the function \( g \):

\[
g = x - \frac{N}{2} \quad (\text{With: } x \in N^*, N/2 < x < N) \tag{10}
\]

Thus, the value of \( g \) contains the value of \( n \) and the value of \( x \) contains the value of \( 2n + P_x \). This means that the values of \( n \) and \( 2n + P_x \) always belong the graph of the function \( g \).

Purpose of functions: We construct two functions \( f \) and \( g \) to refer to the general method, then apply Bertrand’s theorem to find the value \( x \) of the function \( g \). In addition, it also determines the graph of the variation of \( n \) and two values \( P_x \), \( 2n + P_x \) on the same coordinate system.
Graphing two functions $f$ and $g$ on the same coordinate system, we have:

- Based on the results of (5)(6)(7), we rewrite: $N = P_x + (P_x + 2n)$.

- Where $O = (P_x + 2n)$ with $O \in O^*$, and $O \in (N/2, N - 1)$. This leads to if any of the prime exists $P_y \in (N/2, N - 1)$, then it is also the value of $O \in O^*$.

- On the other hand, applying Bertrand’s theorem to the value of $x$ of the function $g = x - \frac{N}{2}$, we have: There is always at least a prime number $x = P_y$ such that $N/2 < x = P_y < N - 1$.

- Assuming $P_y = P_x + 2m$, and $N = P_x + P_y$. This satisfies the problem.

- Assuming $P_y = P_x + 2m$, and $N \neq P_x + P_y$ with $\forall P_x < N/2$, this means:

  \[
  N = O' + P_{y=\max} \quad (\text{With} : \quad O' \in O^*)
  \]

  \[
  = P_{x=\min} + (2n' + P_{y=\max}) \quad (\text{With} : \quad n' \in N^*)
  \]
- In which, \( P_{y = \text{max}} \) is the largest prime number of the set \((N/2, N - 1)\); and \( P_{x = \text{min}} \) is the smallest prime number of the set \((2, N/2)\).

- Thus, in order not to conflict with (7), the following system of equations always has solutions with \( n, n' \in N^* \):

\[
\begin{align*}
N &= P_{x = \text{min}} + (P_{y = \text{max}} + 2n') \\
N &= P_{x = \text{min}} + (P_{x = \text{min}} + 2n)
\end{align*}
\]

\[\Rightarrow P_{y = \text{max}} + 2n' = P_{x = \text{min}} + 2n \quad (\text{With : } n, n' \in N^*) \quad (11)\]

- Consider function \( K(n) = P_{x = \text{min}} + 2n \) with \( n \in N^*, K(n) \in (N/2, N - 1) \).

- Hence there exists at least a prime \( P_z \in (N/2, N - 1) \) such that \( K(n) = P_z \). Because from of the Bertrand’s theorem, we have proved: If \( N > 10 \), there are at least 2 primes \( P_y, P_z \in (N/2, N - 1) \). But this leads to a conflict with Equation (11), because if \( K(n) = P_z = P_{y = \text{max}} + 2n' \) then \( n' \leq 0 \neq N^* \).

Therefore: For every even natural number \( N > 10 \), it can always be expressed as the sum of two primes, with \( P_x, P_y \in P^* \), and \( P_y = P_x + 2n \).

Combining with even natural numbers \( 2 < N \leq 10 \) has been expressed as the sum of the two primes in section 3.1, leading us to prove that the Goldbach’s conjecture is correct.

Proving end.

References