DISRUPTIVE GRAVITY (corrected)
A mathematically consistent take on gravity

Ramsès BOUNKEU SAFO

May 9th 2019

Abstract

Viewing gravity as a spacetime bending force instead of just a spacetime curvature, we come to the conclusion of rest mass relativity since it yields equivalent equations as General Relativity. A close analysis of the Schwarzschild metric leads us naturally to the Vacuum Apparent Energy Invariance principle from which we derive the metric equation. Applying this theory to cosmology, we can explain galaxies redshift as a delayed gravitational redshift which explains Hubble diagrams with no need for Dark Energy. This theory has the same predictive power as General Relativity for every local experimental tests of the latter since it's based on a slight modification of the Schwarzschild metric.

INTRODUCTION

This theory is a new take on gravity that deserves further investigations. It shows the mathematical consistency of seeing gravity as a spacetime bending force and provides sort of a framework for a consistent theory of gravity even for violations of the weak equivalence principle and non-newtonian gravitational potentials. Seeing gravity as a spacetime bending force has two main advantages: gravity can be easily quantized and it gives an alternative explanation to Dark Energy.

For every classical test of General Relativity, this theory gives the same measurable results since it uses a slightly modified Schwarzschild metric. Some other tests are possible where General Relativity and this theory would give different results. Many such tests are presented in this paper and are a way to either falsify this theory or to ascertain its physical consistency.

We should keep in mind that the overall implications of it could lead to paradoxes as it was the case for Special Relativity and General Relativity whose paradoxes have sometimes been resolved decades after their publication, so only the mathematical consistency is of relevance in this paper.

In this paper, Greek letters range from 0 to 3 (representing spacetime) while roman letters range from 1 to 3 (representing space), the metric signature is $(+ - --)$ and we use Einstein's summation convention.

I - A Light-Speed-Invariance-like Principle

Most tests of General Relativity are based on the Schwarzschild metric [1] below. Let's see if we can give a physical meaning to it.

$$ds^2 = (1 + 2\Phi/c^2)c^2dt^2 - (1 + 2\Phi/c^2)^{-1}dr^2 - r^2(d\theta^2 + sin^2\theta d\psi^2)$$

The determinant of the spatial part of the Schwarzschild metric is:

$$det(g_s) = (1 + 2\Phi/c^2)^{-1}$$

So in weak fields we have: $(1 + \Phi/c^2)\sqrt{det(g_s)} = 1$

Then, considering a hypothetical mass density of vacuum ρ in an infinitely small volume $dxdydz$, multiplying by $\rho c^2 dxdydz$, we have:

$$(\rho c^2 + \rho\Phi)\sqrt{det(g_s)}dxdydz = \rho c^2 dxdydz$$

In other words, analogous to the invariance of the speed of light, we have the following principle:

"The energy of vacuum is invariant".

It seems like the same way speed of light invariance induces time dilation, vacuum energy invariance induces space dilation. It is therefore a strong incentive to searching for a consistent theory of gravity as a spacetime bending force instead of just a spacetime curvature.

That means a complete paradigm shift so we cannot refer to recent papers, hence the few references, most of them being obvious ones.

II - A Spacetime Bending Force

Describing gravity as a spacetime bending force has to produce the same tested predictions as General Relativity which are: Mercury's Orbit Precession, Time Dilation, Light Bending, Shapiro Delay, Lens-Thirring and geodetic effects.

We know Lens-Thirring and geodetic effets are both well discribed by Gravitoelectromagnetism [2] which is a theory of gravity in a flat spacetime analoguous to Maxwell's theory of electromagnetism. So including spacetime curvature in Gravitoelectromagnetism would still make those predictions.

Analogous to electromagnetism in General Relativity, we can consider gravity as some kind of gravitoelectromagnetism in a curved spacetime and see if it makes the same predictions as General Relativity. The lagrangian of an electrically charged body in General Relativity is:

$$L = -mc\sqrt{g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu} - q\dot{x}^\mu A_\mu$$

where A_μ is the electromagnetic four-vector potential, m the rest mass and q the electric charge of the body. The idea is to consider a gravitational four-vector potential G_μ analogous to the electromagnetic four-vector potential A_μ and consider the following lagrangian:

$$L = -m_{rest}c\sqrt{g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu} - m_{gravitational}\dot{x}^\mu G_\mu$$

where m_{rest} is the rest mass of the body and $m_{gravitational}$ is its gravitational mass. We will see, that under the hypothesis of rest mass relativity, this lagrangian gives equivalent results as General Relativity.

For some reason that will become clear in Section III, we define the gravitational mass as:

$$m_{gravitational} = \gamma^{-1} m_{rest}$$

where γ is Lorentz factor of the body defined as $\gamma^{-1} = \sqrt{g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu / c^2}$ and we hypothesize that the rest mass is relative such that:

$$m_{rest} = \alpha(\Phi) m_0$$

where m_0 is the Absolute Mass, defined as the rest mass if the gravitational potential is null: $\alpha(0) = 1$. The lagrangian becomes:

$$\boxed{L = -\alpha(\Phi)m_0 c\sqrt{g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu} - \gamma^{-1}\alpha(\Phi)m_0\dot{x}^\mu G_\mu} \quad (1)$$

How the Gravitational four-vector potential G_μ is calculated is not of relevance in this paper since gravity is not postulated to be newtonian. It should then be subject to further investigations. It depends on the type of gravitational potential. If newtonian, it would be the exact analogous of electromagnetism in curved spacetime as we would just have to replace ϵ_0 by $-1/4\pi\mathcal{G}$ where $\mathcal{G}$ is Newton's constant. And the Gravitational tensor would be defined as:

$$F_{\mu\nu} = \partial_\mu G_\nu - \partial_\nu G_\mu = \begin{pmatrix} 0 & -\frac{1}{c}E^x_{\mathcal{G}} & -\frac{1}{c}E^y_{\mathcal{G}} & -\frac{1}{c}E^z_{\mathcal{G}} \\ \frac{1}{c}E^x_{\mathcal{G}} & 0 & B^z_{\mathcal{G}} & -B^y_{\mathcal{G}} \\ \frac{1}{c}E^y_{\mathcal{G}} & -B^z_{\mathcal{G}} & 0 & B^x_{\mathcal{G}} \\ \frac{1}{c}E^z_{\mathcal{G}} & B^y_{\mathcal{G}} & -B^x_{\mathcal{G}} & 0 \end{pmatrix}$$

Another prediction of General Relativity is Gravitational Waves. It is not mentioned in the tests because it is in fact due to a gauge choice. Whereas viewing gravity as a spacetime bending force, gravitational waves would not be due to a gauge choice since G_μ is lorenzian by definition. Indeed, Lorentz gauge induces a wave equation of the potential.

III - First Order Non-Relativistic Dynamics

As we said in the previous section, we consider the following lagrangian:

$$L = -\alpha(\Phi)m_0c\sqrt{g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu} - \gamma^{-1}\alpha(\Phi)m_0\dot{x}^\mu G_\mu \qquad \text{[i]}$$

Let's demonstrate that this lagrangian gives equivalent equations of motion as General Relativity for a certain choice of α when Lens-Thirring and geodetic effects can be neglected in case of non-relativistic speeds in low gravitational fields.

In this case we have:

$$\dot{x}^\mu G_\mu = \dot{x}^0 G_0 = \dot{x}^0\Phi/c \qquad \text{[ii]}$$

where Φ is the gravitational potential. Cross-terms between space and time are neglected in this case so parametrizing with the body's proper time, for non-relativistic speeds, $c^2 = g_{00}(\dot{x}^0)^2 - g_{ij}\dot{x}^i\dot{x}^j$ yields:

$$\dot{x}^0 = c \cdot (1 + 1/2 \cdot g_{ij}\dot{x}^i\dot{x}^j/c^2)/\sqrt{g_{00}} \qquad \text{[iii]}$$

Since for non-relativistic speeds we have:

$$\gamma^{-1} = \sqrt{g_{00}}(1 - 1/2 \cdot g_{ij}\dot{x}^i\dot{x}^j/c^2) \qquad \text{[iv]}$$

Hence:

$$\gamma^{-1}\dot{x}^0 = c \cdot (1 + 1/2 \cdot g_{ij}\dot{x}^i\dot{x}^j/c^2 - 1/2 \cdot g_{ij}\dot{x}^i\dot{x}^j/c^2 - (1/2 \cdot g_{ij}\dot{x}^i\dot{x}^j/c^2)^2) \qquad \text{[v]}$$

After neglection of higher order terms, it yields: $\gamma^{-1}\dot{x}^\mu G_\mu = \Phi$ [vi]

Introducing Lorentz factor in the definition of the gravitational mass is convienient as it suppresses perturbative terms. Its physical meaning is quite intuive though: the faster a body, the more massive it gets in term of relativistic mass, and the lesser the influence of a force on it. Taking this into account yields the introduction of Lorentz factor in the definition of gravitational mass.

The lagrangian becomes:

$$\boxed{L = -\alpha(\Phi)m_0c\sqrt{g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu} - \alpha(\Phi)m_0\Phi} \ (2) \qquad [vii]$$

For more clarity, let's also write: $\dot{s_0} = \sqrt{g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu}$

We then have: $L = -\alpha(\Phi)m_0c\dot{s_0} - \alpha(\Phi)m_0\Phi$ [viii]

The lagrangian equation is: $\frac{d}{d\tau}\frac{\partial L}{\partial \dot{x}^\kappa} - \frac{\partial L}{\partial x^\kappa} = 0$ [ix]

Since Φ doesn't depend explicitly on $\dot{x}^\kappa$, we have:

$$-\frac{\partial \alpha(\Phi)m_0c\dot{s_0}}{\partial x^\kappa} - \frac{\partial \alpha(\Phi)m_0\Phi}{\partial x^\kappa} + \frac{d}{d\tau}\frac{\partial \alpha(\Phi)m_0c\dot{s_0}}{\partial \dot{x}^\kappa} = 0 \qquad [x]$$

Leading to: $-\frac{\partial \alpha(\Phi)c\dot{s_0}}{\partial x^\kappa} - \frac{\partial \alpha(\Phi)\Phi}{\partial x^\kappa} + \frac{d}{d\tau}(\alpha(\Phi)\frac{\partial c\dot{s_0}}{\partial \dot{x}^\kappa}) = 0$ [xi]

It comes:

$$-\alpha(\Phi)\frac{\partial c\dot{s_0}}{\partial x^\kappa} - \frac{\partial \alpha(\Phi)}{\partial x^\kappa}c\dot{s_0} - \frac{\partial \alpha(\Phi)\Phi}{\partial x_\kappa} + \frac{d\alpha(\Phi)}{d\tau}\frac{\partial c\dot{s_0}}{\partial \dot{x}^\kappa} + \alpha(\Phi)\frac{d}{d\tau}\frac{\partial c\dot{s_0}}{\partial \dot{x}^\kappa} = 0 \qquad [xii]$$

We see the lagrangian equation of General Relativity in the first and last terms of the equation. Let $L_0 = -m_0c\dot{s_0}$, it comes:

$$-\frac{\partial \alpha(\Phi)}{\partial x^\kappa}c\dot{s_0} - \frac{\partial \alpha(\Phi)\Phi}{\partial x^\kappa} + \frac{d\alpha(\Phi)}{d\tau}\frac{\partial c\dot{s_0}}{\partial \dot{x}^\kappa} + \alpha(\Phi)(\frac{\partial L_0}{\partial x^\kappa} - \frac{d}{d\tau}\frac{\partial L_0}{\partial \dot{x}^\kappa})/m_0 = 0 \qquad [xiii]$$

Parametrizing with the body's proper time, we have: $\dot{s_0} = c$. Thus:

$$-\frac{\partial(\alpha(\Phi)c^2 + \alpha(\Phi)\Phi)}{\partial x^\kappa} + \frac{d\alpha(\Phi)}{d\tau}\frac{\partial c\dot{s_0}}{\partial \dot{x}^\kappa} + \alpha(\Phi)(\frac{\partial L_0}{\partial x^\kappa} - \frac{d}{d\tau}\frac{\partial L_0}{\partial \dot{x}^\kappa})/m_0 = 0 \qquad [xiv]$$

Notations can be misleading. We cannot replace $\dot{s_0}$ by c in the expression $\frac{\partial c\dot{s_0}}{\partial \dot{x}^\kappa}$ since it's a partial derivative. We have in fact:

$$\frac{\partial c\dot{s_0}}{\partial \dot{x}^\kappa} = c \cdot \frac{\partial \sqrt{g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu}}{\partial \dot{x}^\kappa} = c \cdot \frac{2 \cdot g_{\mu\kappa}\dot{x}^\mu}{2 \cdot \sqrt{g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu}} = c \cdot \frac{2 \cdot \dot{x}_\kappa}{2 \cdot c} = \dot{x}_\kappa \qquad \text{[xv]}$$

Hence: $\frac{d\alpha(\Phi)}{d\tau}\frac{\partial c\dot{s_0}}{\partial \dot{x}^\kappa} = \frac{\partial \alpha(\Phi)}{\partial \Phi} \cdot \frac{\partial \Phi}{\partial x^\mu}\dot{x}^\mu \cdot \dot{x}_\kappa$ [xvi]

And calculating $(\frac{\partial L_0}{\partial x^\kappa} - \frac{d}{d\tau}\frac{\partial L_0}{\partial \dot{x}^\kappa})/m_0$ gives a known standard result of General Relativity [3] [4] [5] [6]:

$$(\frac{\partial L_0}{\partial x^\kappa} - \frac{d}{d\tau}\frac{\partial L_0}{\partial \dot{x}^\kappa})/m_0 = g_{\mu\kappa}\ddot{x}^\mu + 1/2 \cdot (-\partial^\kappa g_{\mu\nu} + \partial^\mu g_{\nu\kappa} + \partial^\nu g_{\mu\kappa})\dot{x}^\mu \dot{x}^\nu \qquad \text{[xvii]}$$

Thus, after multiplying [xiv] by $g^{\kappa\eta}$, introducing Christoffel symbols $\Gamma^\eta_{\mu\nu}$ we get:

$$-g^{\kappa\eta}\frac{\partial(\alpha(\Phi)c^2 + \alpha(\Phi)\Phi)}{\partial x^\kappa} + \frac{\partial \alpha(\Phi)}{\partial \Phi}\frac{\partial \Phi}{\partial x^\mu}\dot{x}^\mu \dot{x}^\eta + \alpha(\Phi)(\ddot{x}^\eta + \Gamma^\eta_{\mu\nu}\dot{x}^\mu \dot{x}^\nu) = 0 \qquad \text{[xviii]}$$

We see that, for it to give the equations of motion as in the newtonian limit, we necessarily have:

$$\frac{\partial(\alpha(\Phi)c^2 + \alpha(\Phi)\Phi)}{\partial x^\kappa} = 0 \qquad \text{[xix]}$$

That yields: $\alpha(\Phi) = (1 + \Phi/c^2)^{-1}$ [xx]

Then: $\frac{\partial \alpha(\Phi)}{\partial \Phi} = -1/c^2 \cdot (1 + \Phi/c^2)^{-2}$ [xxi]

Hence, recasting in [xviii] we get:

$$-(1 + \Phi/c^2)^{-2} \cdot \frac{\partial \Phi}{\partial x^\mu}\dot{x}^\mu \dot{x}^\eta/c^2 + (1 + \Phi/c^2)^{-1}(\ddot{x}^\eta + \Gamma^\eta_{\mu\nu}\dot{x}^\mu \dot{x}^\nu) = 0 \qquad \text{[xxii]}$$

After neglecting second order terms in Φ/c^2, that yields:

$$\boxed{\ddot{x}^\eta + \Gamma^\eta_{\mu\nu}\dot{x}^\mu \dot{x}^\nu = \frac{\partial \Phi}{\partial x^\mu}\dot{x}^\mu \dot{x}^\eta/c^2} \ (3) \qquad \text{[xxiii]}$$

These equations of motion look like the geodesic equations of General Relativity. For weak fields and low speeds, we trivially get the newtonian limit.

Hence, if rest mass is relative such that:

$\boxed{m_{rest} = (1 + \Phi/c^2)^{-1} m_0}$ (4)

Gravity described as a spacetime bending force instead of a spacetime curvature yields similar results. The small deviation from General Relativity induced by $\dfrac{\partial \Phi}{\partial x^\mu} \dot{x}^\mu \dot{x}^\eta / c^2$ would be a test of the theory.

For more clarity, let's write: $m_0 \dfrac{\partial \Phi}{\partial x^\mu} \dot{x}^\mu \dot{x}^\eta / c^2 = -(\vec{F} \cdot \vec{v}) \cdot \vec{v} / c^2$

where $\vec{F}$ is the gravitational force and $\vec{v}$ the speed of the body. We can interpret it as an anomalous thrust unexpected from General Relativity. Such an anomaly is to be expected in the recently lauched Parker Solar Probe if solar wind and pressure radiation can be neglected so close to the Sun and would be a test of this theory.

In case of an orbital motion, we see that its net value over a revolution period is aligned with the great axis of the trajectory, thus not influencing orbital precession (contrary to what a net pull or push would have done). That gives the same value for Mercury's Orbital Precession as General Relativity.

However, the effect of this net force is a kind of translation of the trajectory along the great axis, so contributes to either increasing or decreasing the eccentricity of the trajectory. It can be shown that the corresponding net force is directed toward the aphelion of the trajectory rather than its perihelion and increases in magnitude with the eccentricity (indeed, it is trivially null in case of a circular motion). Thus, it contributes to increasing the eccentricity of the trajectory over time. That might be the main reason why Mercury's eccentricity is so high although tidal circularization would tend to make it null.

That is a good argument in favor of gravity as a spacetime bending force instead of just a spacetime curvature, but is not a test of the theory per se.

IV - Physical Implications

Rest mass relativity has physical consequences. Indeed, for conservation of energy to remain true, the rest mass-energy formula should be:

$E = mc^2 (1 + \Phi/c^2)$

Generalized to a Relativistic body, we have:

$E = \gamma mc^2 (1 + \Phi/c^2)$ where $\gamma = 1/\sqrt{1 - v^2/c^2}$ is Lorentz factor [8].

We can then give a meaning to the following quantity: $E_\Phi = \gamma m_0 c^2 (1 + \Phi/c^2)$

Indeed, let's rewrite it as: $E_\Phi = \sqrt{m_0^2c^4 + p_0^2c^2}(1 + \Phi/c^2)$

Or rather, for brevity : $\boxed{E_\Phi = E_0 \cdot (1 + \Phi/c^2)}$ (5)

Applied to photons of energy $E_0 = h\nu_0$, with $E_\Phi = h\nu_\Phi$ we have:

$\nu_\Phi = \nu_0(1 + \Phi/c^2)$

That looks a lot like General Relativity's formula of gravitational redshift. Thus we define E_Φ as the Apparent Energy of the body.

Writing it as $E_\Phi = E_0\sqrt{g_{00}}$, it's as if the energy of a body could be redshifted. It's as if a body was also a wave which we know accurate since De Broglie's hypothesis of wave-particle duality.

Apparent Energy is nothing new. When a wave is Doppler-shifted for a moving observer, the shifted frequency is said to be apparent frequency. Analogously, the energy of a photon for a moving observer doesn't change, but since its frequency is Doppler-shifted, the change in energy is in fact Apparent Energy.

This physical meaning implies the dilation factor be:

$g_{00} = (1 + \Phi/c^2)^2$

This provides another testable deviation from General Relativity. Indeed in General Relativity we have:

$g_{00,schwarzschild} = 1 + 2\Phi/c^2$

The difference is $(\Phi/c^2)^2$. It's really small but measurable so this theory is falsifiable.

V - Vacuum Apparent Energy Invariance

We can now give a more coherent meaning to the analysis in Section I replacing mass density by energy density.

$\mathcal{E}_0(1 + \Phi/c^2)\sqrt{det(g_s)}dxdydz = \mathcal{E}_0 dxdydz$

We get the following principle:

"The apparent energy of vacuum is invariant."

What we called energy in Section I was from a classical perspective which is in fact apparent energy.

We consider Vacuum Apparent Energy Invariance principle (VAEI) as a fundamental postulate.

VI - Metric Derivation (part I)

We showed in Section II and III that gravity can be coherently described as a spacetime bending force if the rest mass is relative. We are left with how the metric can be derived such that the Schwarzschild metric is a particular case.

We naturally postulate that the metric $g_{\mu\nu}$ is of the form:

$$g = \begin{pmatrix} g_{00}(\Phi) & 0 \\ 0 & -g_s(\Phi) \end{pmatrix}$$

Indeed, in General Relativity, cross terms between space and time are responsible for Lens-Thirring and geodetic effects but since these are already accounted for by considering gravity as spacetime bending force, we can postulate that space and time curvature are disjoint.

We then consider that space and time are independently dilated by VAEI.

Let's derive both $det(g_s)$ and g_{00} thanks to VAEI principle.

At a given point in time t, in an infinitely small volume $dx_1dx_2dx_3$, under zero gravity (flat space) with vacuum energy density $\mathcal{E}_0$, we have:

$$dE_0 = \mathcal{E}_0 dx_1dx_2dx_3$$

and under Φ-gravity potential, we have:

$$dE_\Phi = \mathcal{E}_0(1 + \Phi/c^2)\sqrt{det(g_s)}dx_1dx_2dx_3$$

Applying VAEI, we have: $dE_0 = dE_\Phi$.

It comes:

$$\boxed{det(g_s) = (1 + \Phi/c^2)^{-2}} \quad (6)$$

Let's apply VAEI in time domain to have a more rigorous way to find g_{00}.

The reasoning is a bit similar to the one for the derivation of the gravitational redshift. We reason in terms of observational events.

Let E_0 be the total vacuum energy and N be the number of observationnal events.

The apparent total vacuum energy by time unit for an observer under a global 0-potential is:

$$P_0 = \frac{d(NE_0)}{dt}$$

The apparent total vacuum energy by time unit for the same observer under a global Φ-potential is:

$$P_\phi = \frac{d(NE_0(1+\Phi/c^2))}{d\tau}$$

Applying VAEI, we have: $P_0 = P_\Phi$

It comes: $E_0 dN d\tau = E_0(1+\Phi/c^2)dNdt$

With $d\tau^2 = g_{00}dt^2$ it eventually comes:

$$\boxed{g_{00} = (1+\Phi/c^2)^2} \ (7)$$

The equation of motion [xxii] of Section III, for non-relativistic speeds becomes:

$$\ddot{x}^\kappa + \Gamma^\kappa_{00}\dot{x}^0\dot{x}^0 = \frac{\partial\Phi}{\partial x^\mu}\dot{x}^\mu\dot{x}^\kappa/c^2$$

In weak fields, standard result of linearized General Relativity yields :

$\Gamma^\kappa_{00} = -1/2\eta^{\kappa\nu}\partial_\nu h_{00}$ where $h_{\mu\nu}$ is the perturbation of the linearized metric.

In this case, $h_{00} = 2\Phi/c^2$ and $\dfrac{\partial\Phi}{\partial x^\mu}\dot{x}^\mu\dot{x}^\kappa/c^2$ can be neglected and yields Newton's law.

VII - Metric Derivation (part 2)

We still don't fully know g_s. Any g_s formula predicting a correct Light Deflection and reproducing the Schwarzschild metric for the Sun's mass distribution works to account for every experimental tests.

Considering gravity as a spacetime bending force would give us a space metric g_s different from General Relativity. It doesn't change anything to the newtonian limit since in that case only g_{00} is relevant for the equations of motion. The idea is to aggregate the contributions of every mass of the distribution to the space deformation. In case of a compact spherical distribution, close to the sphere, space dilation wouldn't be purely radial as in the Schwarzshild metric whereas it would be the case far from the mass distribution.

Space deformations induced by a single punctual mass must be radial for trivial physical reasons. Then in a cartesian coordinate system $(\vec{e_r}, \vec{e_u}, \vec{e_v})$ where $\vec{e_r}$ is radial, space metric is $-g_{s,ruv}$ of the form:

$$g_{s,ruv} = \begin{pmatrix} \beta^{-2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = I + (\beta^{-2} - 1) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Applying VAEI yields: $\beta = 1 + \Phi/c^2$.

Let M^T be the change of basis orthogonal matrix from $(\vec{e_r}, \vec{e_u}, \vec{e_v})$ to $(\vec{e_1}, \vec{e_2}, \vec{e_3})$. So with $\vec{e_r} = r_i \vec{e_i}$, $\vec{e_u} = u_i \vec{e_i}$ and $\vec{e_v} = v_i \vec{e_i}$, changing coordinates we have:

$$g_s = M^T g_{s,ruv} M \text{ with } M^T = \begin{pmatrix} r_1 & u_1 & v_1 \\ r_2 & u_2 & v_2 \\ r_3 & u_3 & v_3 \end{pmatrix}$$

Since $M^T M = I$, it comes: $g_s = I + (\beta^{-2} - 1) M^T \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} M$

Eventually:

$$g_s = I + (\beta^{-2} - 1) \begin{pmatrix} r_1^2 & r_1 r_2 & r_1 r_3 \\ r_2 r_1 & r_2^2 & r_2 r_3 \\ r_3 r_1 & r_3 r_2 & r_3^2 \end{pmatrix} \text{ or } g_{s,ij} = \delta_{ij} + (\beta^{-2} - 1) r_i r_j$$

In low fields, this is equivalent to the Schwarzschild metric written in cartesian coordinates.

For an infinitely small potential $d\Phi$, we have $\beta^{-2} - 1 = -2d\Phi/c^2$ and the metric becomes when integrating over every infinitely small potential:

$g_{s,ij} = \delta_{ij} + \lambda \cdot \int -2r_i r_j d\Phi/c^2$ with λ such that $det(g_s) = (1 + \Phi/c^2)^{-2}$

Space being curved there might not be a unique choice of r_i. Therefore we introduce the potential angular distribution $\phi(\vec{\sigma})$, where $\vec{\sigma}$ is the observed direction. Leading to the following metric equation:

$$\boxed{g_{s,ij} = \delta_{ij} + \lambda \cdot \int -2\phi(\vec{\sigma})/c^2 \cdot r_i(\vec{\sigma}) r_j(\vec{\sigma}) d\sigma} \quad (8)$$

With: $\boxed{B_{ij} = \int -2\phi(\vec{\sigma})/c^2 \cdot r_i(\vec{\sigma}) r_j(\vec{\sigma}) d\sigma}$ (9)

We have: $g_{s,ij} = \delta_{ij} + \lambda B_{ij}$

In fact, for any 3x3 matricial function f such that $f(P^{-1}MP) = P^{-1}f(M)P$ and $f(M) = I + M$ if M is small, $g_s = f(\lambda B)$ would also be valid. For physical reasons, rather than summing the infinitely small pertubations, we should multiply the metrics induced by each infinitely small perturbations. That would yield:

$$\boxed{g_s = e^{\lambda B}} \quad (10)$$

Deriving λ is then straightforward since B being symmetric, it is diagonal in a certain base, and $e^{\lambda B}$ would be a diagonal matrix in such a base whose determinant is the exponential of the sum of its eigenvalues. The sum of the eigenvalues being the trace of λB, we have:

$$det(e^{\lambda B}) = e^{Tr(\lambda B)}$$

Applying VAEI principle we then have : $e^{\lambda Tr(B)} = (1 + \Phi/c^2)^{-2}$

Hence : $\boxed{\lambda = -2 \cdot ln(1 + \Phi/c^2)/Tr(B)}$ (11)

So in the weak field limit we have: $\boxed{g_{s,ij} = \delta_{ij} - 2\Phi/c^2 \cdot B_{ij}/B_{kk}}$ (12)

Applying it to a punctual mass, space deformation being radial, in spherical coordinates we trivially obtain a modified Schwarzschild metric:

$$\boxed{ds^2 = (1 + \Phi/c^2)^2 c^2 dt^2 - (1 + \Phi/c^2)^{-2} dr^2 - r^2(d\theta^2 + sin^2\theta d\psi^2)} \quad (13)$$

So this predicts Light Deflection by the Sun since its mass is concentrated in its core. But in case of a homogenous spherical mass distribution like the Earth, the radial dilation would be smaller than the one predicted by the Schwarzschild metric because the deformation is fairly distributed according to the influence of every part of the mass distribution. This could be measured through interferometry and provide another test of the theory.

VIII - Summary

The formalism could be enhanced but is not necessary to show the mathematical consistency of this theory. G_μ being Lorentzian, $G_0 = \Phi/c$ depends on the referential frame. So space dilation through VAEI would be relative. There is a paradox there that we won't adress and suppose that a better formalism would erase it. In last resort, General Covariance could be dropped.

The theory can be summarized by the equations below:

$\Phi_0 = \Phi$

$L = -\alpha m_0 c\sqrt{g_{\mu\nu}\dot{x^\mu}\dot{x^\nu}} - \gamma^{-1}\alpha m_0 \dot{x^\mu} G_\mu$

$\alpha = (1 + \Phi_0/c^2)^{-1}$

$\gamma^{-1} = \sqrt{g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu/c^2}$

$g = \begin{pmatrix} g_{00}(\Phi_0) & 0 \\ 0 & -g_s(\Phi_0) \end{pmatrix}$

$g_{00} = (1 + \Phi_0/c^2)^2$

$g_s = e^{\lambda B}$

$B_{ij} = \int -2\phi_0(\vec{\sigma})/c^2 \cdot r_i(\vec{\sigma}) r_j(\vec{\sigma}) d\sigma$

$det(g_s) = (1 + \Phi_0/c^2)^{-2}$

This can be easily adapted to any violation of Weak Equivalence principle by separating vacuum gravitational potential from the bodies' gravitational potential: $\Phi_0 \neq \Phi$

IX - Cosmology

Standard model of cosmology has emerged as the best explanation to both Cosmological Redshift and Cosmic Microwave Background. The Steady State Universe model and models based on tired light have been ruled out. The first one because of gravitational instability that would make an eternal universe unstable and the second one because of cosmological time dilation and the modified power spectrum of a black body which doesn't match CMB perfect black body measurements. In this section we present another alternative that hasn't been ruled out yet.

Cosmological Redshift could be seen as a delayed gravitational redshift if we postulate that the universe is homogeneous and isotropic and has a beginning. This wouldn't be an extension of General Relativity but an alternative coherent model that doesn't imply an expanding universe. Indeed, if gravity is a force, gravitational potential propagates at the speed of light. The older the universe, the more propagated the gravitational potential. That means locally, the gravitational potential would decrease with time. But seeing a galaxy far away is seeing it as it was in past, thus under a higher gravitational potential. Hence a delayed gravitational redshift which is an alternative explanation to the cosmological redshift that wouldn't require an expanding universe. In every cosmological models built upon General Relativity, we have to postulate the existence of Dark Energy to account for what we interpret as an accelerating expanding universe. We will see that there is no need for Dark Energy to explain the Cosmological Redshift measurements if we consider it to be due to a Delayed Gravitational Redshift instead of an expansion. So this model is not a Friedmann-Lemaitre-Roberson-Walker cosmological model but an alternative non-standart cosmological model worth exploring since it is mathematically consistent. In this approach, the weak field approximation can be applyied.

Let's see how global vacuum gravitational potential evolves in a homogeneous and isotropic universe from its creation. The potential is induced by the mass in a cT radius sphere where T is the age of the universe. Space dilation can be neglected in weak field approximation. We have:

$$\Phi_0 = \int_0^{cT} \phi(r)\rho \cdot 4\pi r^2 dr$$

Where ρ is the universe matter density and $\phi(r)$ is the potential of the gravitational field by mass unit at a distance r. In the special case of Newton's law [9], it would be $-\mathcal{G}/r$. Time dilation is neglected in the integral in weak field approximation and $\rho \cdot 4\pi r^2 dr$ is obviously not dependent on space dilation all the more justifing its neglection.

Let Gal_1 and Gal_2 be two galaxies at a distance D away from each other. An observer in Gal_1 at time T would see Gal_2 as it was in the past at time $T - D/c$.

The gravitational potential of Gal_1 and the gravitational potential of Gal_2 at the time it's being observed are then:

$\Phi_{0,Gal_1} = \int_0^{cT} \phi(r)\rho \cdot 4\pi r^2 dr$

$\Phi_{0,Gal_2} = \int_0^{cT-D} \phi(r)\rho \cdot 4\pi r^2 dr$

With the time dilation factor the observed redshifted frequency is:

$\omega = \dfrac{\sqrt{g_{00,Gal_1}}}{\sqrt{g_{00,Gal_2}}}\omega_0$

It comes: $\omega = (1 + \dfrac{\Phi_{0,Gal_1} - \Phi_{0,Gal_2}}{c^2})\omega_0$

Thus: $\omega/\omega_0 - 1 = \int_{ct_0-D}^{ct_0} \phi(r)\rho/c^2 \cdot 4\pi r^2 dr$

And eventually:

$$\boxed{\omega/\omega_0 - 1 = 4\pi\rho/c^2 \cdot \int_0^D \phi(cT-r)(cT-r)^2 dr} \quad (14)$$

For small distances, with $h(r) = \phi(r) \cdot r^2$, we then have the following formula:

$$\boxed{\omega/\omega_0 - 1 = 4\pi T^2 \rho\phi(cT) \cdot D - 4\pi\rho h'(cT)/c^2 \cdot D^2/2} \quad (15)$$

This is equivalent to Hubble's law with an acceleration term. From Hubble diagrams we deduce: $h'(cT) < 0$, which is verified for a newtonian potential. So we don't need Dark Energy to explain the observations.

Redshift is related to Hubble constant as follows: $\omega/\omega_0 - 1 = -H_0 D/c$

Identifing it to our formula, we have: $\boxed{H_0 = -4\pi\rho c T^2 \phi(cT)}$ (16)

In case of newtonian gravity, we have $\phi(r) = -\mathcal{G}/r$, it comes:

$\omega/\omega_0 - 1 = -4\pi T\mathcal{G}\rho/c \cdot (D - D^2/2cT)$

Hubble's constant would then be $H_0 = 4\pi T\mathcal{G}\rho$ (if weak field approximation still holds that is).

What is very important there is to notice that Hubble constant is not a cosmological parameter. It can be derived from other parameters depending on gravitational models used, not necessarily newtonian. It is a good way to test if gravity is still newtonian for distances compared to the size of the universe.

X - Apparent Energy and Cosmological Redshift Analysis

On could be curious about how to view it from an apparent energy perspective. Indeed, what we observe are hydrogen and other elements absorbation lines. What would be the energy of a photon emitted from Gal_2 and observed on Gal_1 relatively to a photon emitted in Gal_1 and observed in Gal_1?

In the case of hydrogen (that can be generalized to other cases), photon energy is given by Balmer series. Hydrogen energy states are -E_I/n^2 with $E_I = \dfrac{\mu e^4}{8\epsilon_0^2 h^2}$ where $\mu = m_e m_p/(m_e + m_p)$ is the reduced mass of an electron and a proton, h is Planck constant and ϵ_0 is dielectric constant. Rest mass being relative, reduced mass in Gal_2 at the time it is being observed is:

$$\mu = \mu_0/(1 + \Phi_{0,Gal_2}/c^2)$$

Hence the Balmer serie frequencies, with $\nu_0 = \dfrac{\mu_0 e^4}{8\epsilon_0^2 h^3}$:

$$\nu_{Gal_2} = E_{Gal_2}/h = \nu_0/(1 + \Phi_{0,Gal_2}/c^2) \cdot (\frac{1}{n_1} - \frac{1}{n_2})$$

And the apparent frequency under Φ_{0,Gal_1} potential is then:

$$\nu_{app,Gal_2} = \nu_{Gal_2} \cdot (1 + \Phi_{0,Gal_1}/c^2) = \nu_0 \cdot (\frac{1}{n_1} - \frac{1}{n_2}) \cdot \frac{1 + \Phi_{0,Gal_1}/c^2}{1 + \Phi_{0,Gal_2}/c^2}$$

Similarly, apparent energy of hydrogen in Gal_1 under Φ_{0,Gal_1} potential is:

$$\nu_{app,Gal_1} = \nu_0 \cdot (\frac{1}{n_1} - \frac{1}{n_2}) \cdot \frac{1 + \Phi_{0,Gal_1}/c^2}{1 + \Phi_{0,Gal_1}/c^2} = \nu_0 \cdot (\frac{1}{n_1} - \frac{1}{n_2})$$

Hence the redshift: $\nu_{app,Gal_2}/\nu_{app,Gal_1} = \dfrac{1 + \Phi_{0,Gal_1}/c^2}{1 + \Phi_{0,Gal_2}/c^2} = \dfrac{\sqrt{g_{00,Gal_1}}}{\sqrt{g_{00,Gal_2}}}$

Which is the same formula as derived in the previous section.

XI - Conformity to Cosmological Observations

Lambda-CDM Model has been granted a lot of means so no alternative model can really compete. This theory provides a possible alternative to Big Bang Cosmology that is yet to be ruled out. Lambda-CDM model is known to be the only model to account for the cosmological observations so far. This theory can provide an alternative that also accounts for most observations that are considered as evidence of the Big Bang model as The Expansion of The Universe or The Cosmic Microwave Background. Indeed, The Expansion of The Universe is in fact the observation of a Cosmological Redshift, gravity as a spacetime bending force can explain it as a Delayed Gravitational Redshift as we just saw and The Cosmic Microwave Background can be explained as a cold primordial universe black body radiation.

Other observations like Nucleosynthesis of Light Elements or Growth or Structure are not to be adressed since they can be considered not conclusive. Indeed, the Cosmological Lithium Problem in the Nucleosynthesis of Light Elements means that it is possible for the Standard Model to have a major flaw. On the other hand, the Growth of Structures needs Non-Baryonic Dark Matter with different gravitational properties to be consistent whereas this theory would rather resort to non-newtonian potentials to obtain similar simulation results.

The paradigm shift induced by this theory trivially resolves the Flatness Problem. The same goes for the Monopole Problem, since there is no more need for a dense and hot primordial universe.

The Horizon Problem however, cannot be resolved by a purely gravitational theory. Additionnal hypothesis need to be added analogous to how Inflation Theory [7] has been added to General Relativity. Here is an hypothesis that one could come up with that elegantly solves this problem:

The Horizon Problem could be explained by random matter generation in the primordial universe since Law of Large Numbers would give a homogeneous matter density. That would result in a homogeneous temperature distribution of the early universe and tiny fluctuations due to statistical standard deviation with no need for causal connection. The randomness of this phenomenon could lead to the same CMB fluctuation spectrum equations as those derived from Inflation Theory as it happens often in science that two different phenomena could lead to the same equations.

Far from saying this theory solves every problem, it just shows that it should be taken seriously. Even more so since it can also provide possible explanations to some unsolved problems of Standard Cosmology as Dark Energy as we saw in the previous section. And since there is no critical density in this model, there is no need for most of the Dark Matter needed in lambda-CDM model, especially Non-Baryonic Dark Matter.

XII - Tests and Predictions

This theory could be tested by measuring the anomalous thrust predicted by this theory in Section II. It is expected to be observed in the recently launched Parker Solar Probe.

Another test of this theory would be precise measurements of time dilation factor. The difference is really small but measurable. This theory could be falsified if the difference from the time dilation factor of the Schwarzschild metric is not observed. Here are two ways this could be experimentally done: precise measurement of time dilation on a satellite with high eccentricity would provide high enough precision or precise measurements of the Sun's solar crown redshift. Sun's strong gravity can make this difference easier to spot.

This theory predicts that radial space dilation would have a measurable deviation from the Schwarzschild metric on Earth since space dilation is not expected to be radial near a compact object. That could be measured with a sensitive interferometer, making it rotate and measuring radial dilation and dilation in other directions.

Another kind of space dilation can be predicted. Since gravitational potential dilates space, it influences the way it is derived. There is a feedback loop between space dilation and gravitational potential derivation that is easy to derive in case of a spherical compact mass distribution as the Sun. That induces a different formula of the potential as a function of the radial coordinate. This could be measured with highly precise Shapiro delays measurements. It is important to mention that this phenomenon doesn't change anything to Mercury's Orbital Precession as further analysis would show. This phenomenon might not be neglectable near Earth, affecting the calculation of gravitational potential and make time dilation and space dilation on Earth as a function of the radial coordinate formula slightly more different than what's expected from the Schwarzschild metric.

A prediction of Vacuum Apparent Energy Invariance would be the possibility of magnetically-induced space dilation. Assuming Vacuum and Quantum Vacuum are the same, Vacuum is filled with particles with a magnetic dipole moment. They would then tend to line up with magnetic field lines and have negative magnetic energy. Such magnetic energy by mass unit would have the same effect as a gravitational potential and thus dilate space in orders of magnitude of a gravitational field. Gravitational potential Φ would be equivalent to $-\mu B/m$ where μ is the dipole moment of vacuum particules, m is the mass of vacuum particles and B is the magnetic field. That could be measurable with high precision interferometers for magnetic fields of a few Teslas across a few meters.

That is five different tests of this theory that could highlight measurable differences from General Relativity. This theory is highly testable then falsifiable.

There is an other important thing to mention. In Section X, we derived the Delayed Gravitational Redshift in case of a universe with constant matter density from its creation and that yielded Hubble's law and an explanation of Dark Energy. This was an approximation to simplify the model. In fact, we don't know how and when matter appeared from this cosmological model but there is a way to probe it. Indeed, sending a signal and making it bounce back to where it has been emitted, by the time it came back, the global gravitational potential would have evolved while the local one due to Earth would have stayed the same. The Delayed Gravitational Redshift in this case would depend on the gravitational potential induced by matter from the first instants of the universe. If matter always existed at the same mass density, the Delayed Gravitational Redshift would still follow some kind of Hubble's law, else, it would drift away from it, giving information on primordial matter creation if any. That could be a probe of primordial matter. A precise study of this phenomenon is necessary but is off-topic.

CONCLUSION

Vacuum Apparent Energy Invariance is a principle analogous to speed of light invariance that is the fundamental postulate of this theory, just like Strong Equivalence principle is the fundamental postulate of General Relativity. From it we can derive the Schwarzschild metric in weak fields and equations of space dilation similar to linearized General Relativity in weak fields. We demonstrated that in every experimental tests done so far, viewing gravity as a spacetime bending force gives the same results as viewing gravity as a spacetime curvature. The only new concept introduced is rest mass relativity which is physically acceptable since the concept of rest mass and relative mass already exist. It also takes into account any possible violations of Weak Equivalence Principle and non-newtonian potentials which is an open door to further studies. It provides an alternative explanation to Dark Energy and other cosmological observations.

Above all, more than being mathematically consistent, it is testable. Many predicted deviations from General Relativity are described in the last section and could ascertain its physical consistency.

Gravity as a spacetime bending force instead of a spacetime curvature is trivially quantizable as a force in a curved spacetime analogous to electromagnetism (see V. Fock, Z. Phys. 57, 261 (1929) and H. Tetrode, Z. Phys. 50, 336 (1928)). It is then a possible alternative to General Relativity that is worth investigating further even though it could very well be ruled out as many other theories.

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Ramsès BOUNKEU SAFO
CentraleSupélec (FRANCE), May 9th 2019, update on December 6th 2019

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