

# **On the Ramanujan's equations applied to various sectors of Particle Physics and Cosmology: new possible mathematical connections. VII**

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## **Abstract**

*In this research thesis, we have analyzed further Ramanujan formulas and described new possible mathematical connections with some sectors of Particle Physics and Cosmology*

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<https://www.scientificamerican.com/article/one-of-srinivasa-ramanujans-neglected-manuscripts-has-helped-solve-long-standing-mathematical-mysteries/>

## Summary

In this research thesis, we have analyzed further Ramanujan formulas and described new mathematical connections with some sectors of Particle Physics and Cosmology. We have described, as in previous papers, the possible and new connections between different formulas of Ramanujan's mathematics and some formulas concerning particle physics and cosmology. In the course of the discussion we describe and highlight the connections between some developments of Ramanujan equations and particles type solutions such as the mass of the Higgs boson, those in the range of the mass of candidates "glueball", the scalar meson  $f_0(1710)$  and the hypothetical mass of Gluino ("glueball" =  $1760 \pm 15$  MeV; gluino = 1785.16 GeV) and the masses of proton (or neutron), and other baryons and mesons. Moreover solutions of Ramanujan equations, connected with the masses of the  $\pi$  mesons (139.576 and 134.9766 MeV) have been described and highlighted. We have showed also the mathematical connections between some Ramanujan equations, the boundary state corresponding to the NSNS-sector of N Dp-branes in the limit of  $u \rightarrow \infty$ , the ratio concerning the general asymptotically flat solution of the equations of motion of the p-brane and the Karatsuba's equation concerning the zeros of a special type of function connected with Dirichlet series.

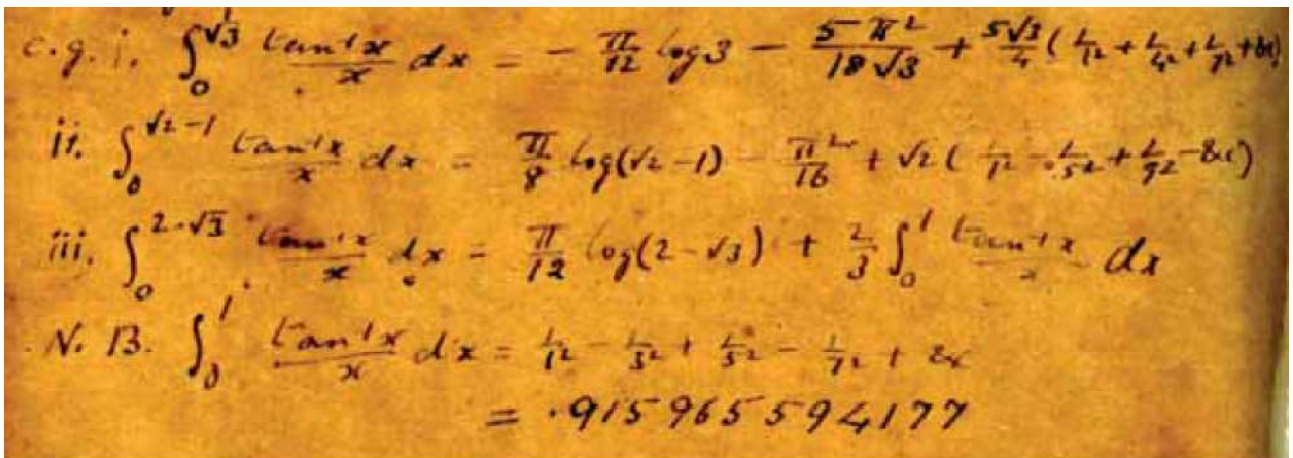
Further, we have described the connections between the mathematics of Ramanujan and different equations concerning some areas of cosmology such as "*Trans-Planckian Censorship and the Swampland*" and the sector that describes the "*similarities between the conditions needed to avoid eternal inflation and several recently-proposed Swampland criteria, which leads us to speculate on the possibility that the de Sitter Swampland conjectures should be viewed as approximate consequences of a No Eternal Inflation principle*". In our opinion, that the possible connections between the mathematical developments of some Rogers-Ramanujan continued fractions, the value of the dilaton and that of "the dilaton mass calculated as a type of Higgs boson that is equal about to 125 GeV", are fundamental. It is interesting to note that particle-type solutions (mass values) also result from the equations of the cosmological sectors explored in this thesis.

All the results of the most important connections are highlighted in blue throughout the drafting of the paper

**From:**

**MANUSCRIPT BOOK 2 OF SRINIVASA RAMANUJAN**

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$$-\frac{\pi}{12} \ln 3 - \frac{5\pi^2}{18\sqrt{3}} + \frac{5\sqrt{3}}{4} \left( \frac{1}{1^2} + \frac{1}{4^2} + \frac{1}{7^2} + \dots \right)$$

**Input:**

$$-\frac{\pi}{12} \log(3) - \frac{5\pi^2}{18\sqrt{3}} + \left( \frac{1}{4} (5\sqrt{3}) \right) \left( \frac{1}{1^2} + \frac{1}{4^2} + \frac{1}{7^2} \right)$$

log(x) is the natural logarithm

**Exact result:**

$$\frac{4245\sqrt{3}}{3136} - \frac{5\pi^2}{18\sqrt{3}} - \frac{1}{12} \pi \log(3)$$

**Decimal approximation:**

0.474110379957971314708360700551730280433508219885955434556...

0.4741103799579713147...

**Alternate forms:**

$$\frac{-114615 + 7840 \pi^2 + 2352 \sqrt{3} \pi \log(3)}{28224 \sqrt{3}}$$

$$\frac{114615 \sqrt{3} - 7840 \sqrt{3} \pi^2 - 7056 \pi \log(3)}{84672}$$

$$\frac{4245 \sqrt{3}}{3136} - \frac{1}{108} \pi (10 \sqrt{3} \pi + 9 \log(3))$$

**Alternative representations:**

$$\begin{aligned} & \frac{1}{12} \log(3) (-\pi) - \frac{5 \pi^2}{18 \sqrt{3}} + \frac{1}{4} \left( \frac{1}{1^2} + \frac{1}{4^2} + \frac{1}{7^2} \right) (5 \sqrt{3}) = \\ & - \frac{\pi \log_e(3)}{12} - \frac{5 \pi^2}{18 \sqrt{3}} + \frac{5}{4} \left( \frac{1}{1} + \frac{1}{4^2} + \frac{1}{7^2} \right) \sqrt{3} \end{aligned}$$

$$\begin{aligned} & \frac{1}{12} \log(3) (-\pi) - \frac{5 \pi^2}{18 \sqrt{3}} + \frac{1}{4} \left( \frac{1}{1^2} + \frac{1}{4^2} + \frac{1}{7^2} \right) (5 \sqrt{3}) = \\ & - \frac{1}{12} \pi \log(a) \log_a(3) - \frac{5 \pi^2}{18 \sqrt{3}} + \frac{5}{4} \left( \frac{1}{1} + \frac{1}{4^2} + \frac{1}{7^2} \right) \sqrt{3} \end{aligned}$$

$$\begin{aligned} & \frac{1}{12} \log(3) (-\pi) - \frac{5 \pi^2}{18 \sqrt{3}} + \frac{1}{4} \left( \frac{1}{1^2} + \frac{1}{4^2} + \frac{1}{7^2} \right) (5 \sqrt{3}) = \\ & \frac{\pi \text{Li}_1(-2)}{12} - \frac{5 \pi^2}{18 \sqrt{3}} + \frac{5}{4} \left( \frac{1}{1} + \frac{1}{4^2} + \frac{1}{7^2} \right) \sqrt{3} \end{aligned}$$

**Series representations:**

$$\begin{aligned} & \frac{1}{12} \log(3) (-\pi) - \frac{5 \pi^2}{18 \sqrt{3}} + \frac{1}{4} \left( \frac{1}{1^2} + \frac{1}{4^2} + \frac{1}{7^2} \right) (5 \sqrt{3}) = \\ & \frac{4245 \sqrt{3}}{3136} - \frac{5 \pi^2}{18 \sqrt{3}} - \frac{1}{12} \pi \log(2) + \frac{1}{12} \pi \sum_{k=1}^{\infty} \frac{(-\frac{1}{2})^k}{k} \end{aligned}$$

$$\begin{aligned} & \frac{1}{12} \log(3) (-\pi) - \frac{5 \pi^2}{18 \sqrt{3}} + \frac{1}{4} \left( \frac{1}{1^2} + \frac{1}{4^2} + \frac{1}{7^2} \right) (5 \sqrt{3}) = \frac{4245 \sqrt{3}}{3136} - \frac{5 \pi^2}{18 \sqrt{3}} - \\ & \frac{1}{6} i \pi^2 \left[ \frac{\arg(3-x)}{2\pi} \right] - \frac{1}{12} \pi \log(x) + \frac{1}{12} \pi \sum_{k=1}^{\infty} \frac{(-1)^k (3-x)^k x^{-k}}{k} \text{ for } x < 0 \end{aligned}$$

$$\frac{1}{12} \log(3) (-\pi) - \frac{5\pi^2}{18\sqrt{3}} + \frac{1}{4} \left( \frac{1}{1^2} + \frac{1}{4^2} + \frac{1}{7^2} \right) (5\sqrt{3}) =$$

$$\frac{4245\sqrt{3}}{3136} - \frac{5\pi^2}{18\sqrt{3}} - \frac{1}{12} \pi \left[ \frac{\arg(3-z_0)}{2\pi} \right] \log\left(\frac{1}{z_0}\right) - \frac{1}{12} \pi \log(z_0) -$$

$$\frac{1}{12} \pi \left[ \frac{\arg(3-z_0)}{2\pi} \right] \log(z_0) + \frac{1}{12} \pi \sum_{k=1}^{\infty} \frac{(-1)^k (3-z_0)^k z_0^{-k}}{k}$$

**Integral representations:**

$$\frac{1}{12} \log(3) (-\pi) - \frac{5\pi^2}{18\sqrt{3}} + \frac{1}{4} \left( \frac{1}{1^2} + \frac{1}{4^2} + \frac{1}{7^2} \right) (5\sqrt{3}) =$$

$$\frac{4245\sqrt{3}}{3136} - \frac{5\pi^2}{18\sqrt{3}} - \frac{\pi}{12} \int_1^3 \frac{1}{t} dt$$

$$\frac{1}{12} \log(3) (-\pi) - \frac{5\pi^2}{18\sqrt{3}} + \frac{1}{4} \left( \frac{1}{1^2} + \frac{1}{4^2} + \frac{1}{7^2} \right) (5\sqrt{3}) =$$

$$\frac{4245\sqrt{3}}{3136} - \frac{5\pi^2}{18\sqrt{3}} + \frac{i}{24} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{2^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \text{ for } -1 < \gamma < 0$$

$$\frac{\pi}{8} \ln(\sqrt{2}-1) - \frac{\pi^2}{16} + (\sqrt{2}) \cdot \left( \frac{1}{1^2} - \frac{1}{5^2} + \frac{1}{9^2} \right)$$

**Input:**

$$\frac{\pi}{8} \log(\sqrt{2}-1) - \frac{\pi^2}{16} + \sqrt{2} \left( \frac{1}{1^2} - \frac{1}{5^2} + \frac{1}{9^2} \right)$$

log(x) is the natural logarithm

**Exact result:**

$$\frac{1969\sqrt{2}}{2025} - \frac{\pi^2}{16} + \frac{1}{8} \pi \log(\sqrt{2}-1)$$

**Decimal approximation:**

0.412139573249965379868443135333525543135133825659917090145...

0.41213957...

**Alternate forms:**

$$\frac{1969\sqrt{2}}{2025} - \frac{1}{16} \pi (\pi + 2 \sinh^{-1}(1))$$

$$\frac{31504\sqrt{2} - 2025\pi^2 + 4050\pi \log(\sqrt{2}-1)}{32400}$$

$\sinh^{-1}(x)$  is the inverse hyperbolic sine function

### Alternative representations:

$$\frac{1}{8} \log(\sqrt{2} - 1) \pi - \frac{\pi^2}{16} + \sqrt{2} \left( \frac{1}{1^2} - \frac{1}{5^2} + \frac{1}{9^2} \right) =$$

$$\frac{1}{8} \pi \log_e(-1 + \sqrt{2}) - \frac{\pi^2}{16} + \left( \frac{1}{1} - \frac{1}{5^2} + \frac{1}{9^2} \right) \sqrt{2}$$

$$\frac{1}{8} \log(\sqrt{2} - 1) \pi - \frac{\pi^2}{16} + \sqrt{2} \left( \frac{1}{1^2} - \frac{1}{5^2} + \frac{1}{9^2} \right) =$$

$$\frac{1}{8} \pi \log(a) \log_a(-1 + \sqrt{2}) - \frac{\pi^2}{16} + \left( \frac{1}{1} - \frac{1}{5^2} + \frac{1}{9^2} \right) \sqrt{2}$$

$$\frac{1}{8} \log(\sqrt{2} - 1) \pi - \frac{\pi^2}{16} + \sqrt{2} \left( \frac{1}{1^2} - \frac{1}{5^2} + \frac{1}{9^2} \right) =$$

$$-\frac{1}{8} \pi \operatorname{Li}_1(2 - \sqrt{2}) - \frac{\pi^2}{16} + \left( \frac{1}{1} - \frac{1}{5^2} + \frac{1}{9^2} \right) \sqrt{2}$$

### Series representations:

$$\frac{1}{8} \log(\sqrt{2} - 1) \pi - \frac{\pi^2}{16} + \sqrt{2} \left( \frac{1}{1^2} - \frac{1}{5^2} + \frac{1}{9^2} \right) =$$

$$\frac{1969 \sqrt{2}}{2025} - \frac{\pi^2}{16} - \frac{1}{8} \pi \sum_{k=1}^{\infty} \frac{(-1)^k (-2 + \sqrt{2})^k}{k}$$

$$\frac{1}{8} \log(\sqrt{2} - 1) \pi - \frac{\pi^2}{16} + \sqrt{2} \left( \frac{1}{1^2} - \frac{1}{5^2} + \frac{1}{9^2} \right) =$$

$$\frac{1969 \sqrt{2}}{2025} - \frac{\pi^2}{16} + \frac{1}{4} i \pi^2 \left[ \frac{\arg(-1 + \sqrt{2} - x)}{2 \pi} \right] +$$

$$\frac{1}{8} \pi \log(x) - \frac{1}{8} \pi \sum_{k=1}^{\infty} \frac{(-1)^k (-1 + \sqrt{2} - x)^k x^{-k}}{k} \quad \text{for } x < 0$$

$$\frac{1}{8} \log(\sqrt{2} - 1) \pi - \frac{\pi^2}{16} + \sqrt{2} \left( \frac{1}{1^2} - \frac{1}{5^2} + \frac{1}{9^2} \right) = \frac{1969 \sqrt{2}}{2025} - \frac{\pi^2}{16} +$$

$$\frac{1}{4} i \pi^2 \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2 \pi} \right] + \frac{1}{8} \pi \log(z_0) - \frac{1}{8} \pi \sum_{k=1}^{\infty} \frac{(-1)^k (-1 + \sqrt{2} - z_0)^k z_0^{-k}}{k}$$

### Integral representation:

$$\frac{1}{8} \log(\sqrt{2} - 1) \pi - \frac{\pi^2}{16} + \sqrt{2} \left( \frac{1}{1^2} - \frac{1}{5^2} + \frac{1}{9^2} \right) = \frac{1969 \sqrt{2}}{2025} - \frac{\pi^2}{16} + \frac{\pi}{8} \int_1^{-1+\sqrt{2}} \frac{1}{t} dt$$

$\frac{\pi}{12} \ln(2-\sqrt{3}) + \frac{2}{3} \int_0^1 \frac{\tan^{-1}(x)}{x} dx$ , [0,1]

**Input:**

$$\frac{\pi}{12} \log(2 - \sqrt{3}) + \frac{2}{3} \int_0^1 \frac{\tan^{-1}(x)}{x} dx$$

$\log(x)$  is the natural logarithm  
 $\tan^{-1}(x)$  is the inverse tangent function

**Result:**

$$\frac{2C}{3} + \frac{1}{12} \pi \log(2 - \sqrt{3}) \approx 0.265865$$

0.265865

$C$  is Catalan's constant

**Computation result:**

$$\frac{1}{12} \pi \log(2 - \sqrt{3}) + \frac{2}{3} \int_0^1 \frac{\tan^{-1}(x)}{x} dx = \frac{2C}{3} + \frac{1}{12} \pi \log(2 - \sqrt{3})$$

**Alternate form:**

$$\frac{1}{12} (8C + \pi \log(2 - \sqrt{3}))$$

$1 - \frac{1}{3^2} + \frac{1}{5^2} - \frac{1}{7^2} + \frac{1}{8^2} - \frac{1}{9^2} + \frac{1}{10^2} - \frac{1}{11^2} + \frac{1}{12^2} - \frac{1}{13^2} + \frac{1}{14^2} - \frac{1}{15^2} + \frac{1}{16^2} - \frac{1}{17^2} + \frac{1}{18^2} - \frac{1}{19^2}$

**Input:**

$$1 - \frac{1}{3^2} + \frac{1}{5^2} - \frac{1}{7^2} + \frac{1}{8^2} - \frac{1}{9^2} + \frac{1}{10^2} - \frac{1}{11^2} + \frac{1}{12^2} - \frac{1}{13^2} + \frac{1}{14^2} - \frac{1}{15^2} + \frac{1}{16^2} - \frac{1}{17^2} + \frac{1}{18^2} - \frac{1}{19^2}$$

**Exact result:**

$$\frac{16545706327603463}{18064125330451200}$$



**Decimal approximation:**

0.915942843892469250756450430033563632608435632956664934875...

0.915942843892469.... result very near to the spectral index  $n_s$ , to the mesonic Regge slope, to the inflaton value at the end of the inflation 0.9402 (see Appendix)

The sum of the results is:

$$0.474110379957 + 0.412139573249 + 0.265865 + 0.91594284389 = 2.068057797096$$

From which

$$322 / (0.474110379957 + 0.412139573249 + 0.265865 + 0.91594284389) - 21 + 5$$

Where 322 is a Lucas number and 21 and 5 are Fibonacci numbers

**Input interpretation:**

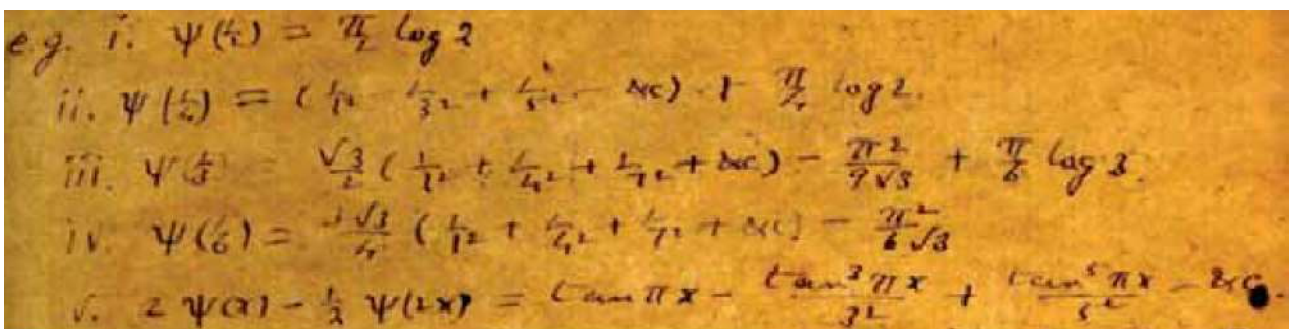
$$\frac{322}{0.474110379957 + 0.412139573249 + 0.265865 + 0.91594284389} - 21 + 5$$

**Result:**

139.7016445343827313375126419600732205125275668641273247650...

139.70164453.... result very near to the rest mass of Pion meson 139.57

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Pi/2 ln 2

**Input:**

$$\frac{\pi}{2} \log(2)$$

log(x) is the natural logarithm

**Exact result:**

$$\frac{1}{2} \pi \log(2)$$

**Decimal approximation:**

- More digits

1.088793045151801065250344449118806973669291850184643147162...

1.0887930451518...

**Alternative representations:**

$$\frac{1}{2} \log(2) \pi = \frac{\pi \log_e(2)}{2}$$

$$\frac{1}{2} \log(2) \pi = \pi \coth^{-1}(3)$$

$$\frac{1}{2} \log(2) \pi = \frac{1}{2} \pi \log(a) \log_a(2)$$

**Series representations:**

$$\frac{1}{2} \log(2) \pi = i \pi^2 \left[ \frac{\arg(2-x)}{2\pi} \right] + \frac{1}{2} \pi \log(x) - \frac{1}{2} \pi \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k} \quad \text{for } x < 0$$

$$\frac{1}{2} \log(2) \pi = i \pi^2 \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] + \frac{1}{2} \pi \log(z_0) - \frac{1}{2} \pi \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}$$

$$\begin{aligned} \frac{1}{2} \log(2) \pi &= \frac{1}{2} \pi \left[ \frac{\arg(2-z_0)}{2\pi} \right] \log\left(\frac{1}{z_0}\right) + \\ &\frac{1}{2} \pi \log(z_0) + \frac{1}{2} \pi \left[ \frac{\arg(2-z_0)}{2\pi} \right] \log(z_0) - \frac{1}{2} \pi \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k} \end{aligned}$$

**Integral representations:**

$$\frac{1}{2} \log(2) \pi = \frac{\pi}{2} \int_1^2 \frac{1}{t} dt$$

$$\frac{1}{2} \log(2) \pi = -\frac{i}{4} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \quad \text{for } -1 < \gamma < 0$$

$$(1/1^2 - 1/3^2 + 1/5^2) + \pi/4 \ln 2$$

**Input:**

$$\left(\frac{1}{1^2} - \frac{1}{3^2} + \frac{1}{5^2}\right) + \frac{\pi}{4} \log(2)$$

$\log(x)$  is the natural logarithm

**Exact result:**

$$\frac{209}{225} + \frac{1}{4} \pi \log(2)$$

**Decimal approximation:**

1.473285411464789421514061113448292375723534813981210462470...

1.473285411464....

**Alternate form:**

$$\frac{1}{900} (836 + 225 \pi \log(2))$$

**Alternative representations:**

$$\left(\frac{1}{1^2} - \frac{1}{3^2} + \frac{1}{5^2}\right) + \frac{1}{4} \log(2) \pi = \frac{1}{1} + \frac{\pi \log_e(2)}{4} - \frac{1}{9} + \frac{1}{5^2}$$

$$\left(\frac{1}{1^2} - \frac{1}{3^2} + \frac{1}{5^2}\right) + \frac{1}{4} \log(2) \pi = \frac{1}{1} + \frac{1}{4} \pi \log(a) \log_a(2) - \frac{1}{9} + \frac{1}{5^2}$$

$$\left(\frac{1}{1^2} - \frac{1}{3^2} + \frac{1}{5^2}\right) + \frac{1}{4} \log(2) \pi = \frac{1}{1} + \frac{2}{4} \pi \coth^{-1}(3) - \frac{1}{9} + \frac{1}{5^2}$$

**Series representations:**

$$\left(\frac{1}{1^2} - \frac{1}{3^2} + \frac{1}{5^2}\right) + \frac{1}{4} \log(2) \pi = \frac{209}{225} + \frac{1}{2} i \pi^2 \left[ \frac{\arg(2-x)}{2\pi} \right] + \frac{1}{4} \pi \log(x) - \frac{1}{4} \pi \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k} \quad \text{for } x < 0$$

$$\left(\frac{1}{1^2} - \frac{1}{3^2} + \frac{1}{5^2}\right) + \frac{1}{4} \log(2) \pi = \frac{209}{225} + \frac{1}{2} i \pi^2 \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] + \frac{1}{4} \pi \log(z_0) - \frac{1}{4} \pi \sum_{k=1}^{\infty} \frac{(-1)^k (2 - z_0)^k z_0^{-k}}{k}$$

$$\left(\frac{1}{1^2} - \frac{1}{3^2} + \frac{1}{5^2}\right) + \frac{1}{4} \log(2) \pi = \frac{209}{225} + \frac{1}{4} \pi \left[ \frac{\arg(2 - z_0)}{2\pi} \right] \log\left(\frac{1}{z_0}\right) + \frac{1}{4} \pi \log(z_0) + \frac{1}{4} \pi \left[ \frac{\arg(2 - z_0)}{2\pi} \right] \log(z_0) - \frac{1}{4} \pi \sum_{k=1}^{\infty} \frac{(-1)^k (2 - z_0)^k z_0^{-k}}{k}$$

### Integral representations:

$$\left(\frac{1}{1^2} - \frac{1}{3^2} + \frac{1}{5^2}\right) + \frac{1}{4} \log(2) \pi = \frac{209}{225} + \frac{\pi}{4} \int_1^2 \frac{1}{t} dt$$

$$\left(\frac{1}{1^2} - \frac{1}{3^2} + \frac{1}{5^2}\right) + \frac{1}{4} \log(2) \pi = \frac{209}{225} - \frac{i}{8} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \quad \text{for } -1 < \gamma < 0$$

$$\frac{\sqrt{3}}{2} \left(\frac{1}{1^2} + \frac{1}{4^2} + \frac{1}{7^2}\right) - \frac{\pi^2}{9\sqrt{3}} + \frac{\pi}{6} \log(3)$$

### Input:

$$\frac{\sqrt{3}}{2} \left(\frac{1}{1^2} + \frac{1}{4^2} + \frac{1}{7^2}\right) - \frac{\pi^2}{9\sqrt{3}} + \frac{\pi}{6} \log(3)$$

$\log(x)$  is the natural logarithm

### Exact result:

$$\frac{849\sqrt{3}}{1568} - \frac{\pi^2}{9\sqrt{3}} + \frac{1}{6} \pi \log(3)$$

### Decimal approximation:

0.879922611027829058170108396984309241203841294159520743783...

0.8799226110278...

### Alternate forms:

$$\frac{-22923 + 1568\pi^2 - 2352\sqrt{3}\pi \log(3)}{14112\sqrt{3}}$$

$$\frac{22923\sqrt{3} - 1568\sqrt{3}\pi^2 + 7056\pi\log(3)}{42336}$$

$$\frac{849\sqrt{3}}{1568} + \frac{1}{54}\pi(9\log(3) - 2\sqrt{3}\pi)$$

### Alternative representations:

$$\frac{1}{2}\left(\frac{1}{1^2} + \frac{1}{4^2} + \frac{1}{7^2}\right)\sqrt{3} - \frac{\pi^2}{9\sqrt{3}} + \frac{1}{6}\log(3)\pi = \frac{\pi\log_e(3)}{6} - \frac{\pi^2}{9\sqrt{3}} + \frac{1}{2}\left(\frac{1}{1} + \frac{1}{4^2} + \frac{1}{7^2}\right)\sqrt{3}$$

$$\begin{aligned} \frac{1}{2}\left(\frac{1}{1^2} + \frac{1}{4^2} + \frac{1}{7^2}\right)\sqrt{3} - \frac{\pi^2}{9\sqrt{3}} + \frac{1}{6}\log(3)\pi = \\ \frac{1}{6}\pi\log(a)\log_a(3) - \frac{\pi^2}{9\sqrt{3}} + \frac{1}{2}\left(\frac{1}{1} + \frac{1}{4^2} + \frac{1}{7^2}\right)\sqrt{3} \end{aligned}$$

$$\begin{aligned} \frac{1}{2}\left(\frac{1}{1^2} + \frac{1}{4^2} + \frac{1}{7^2}\right)\sqrt{3} - \frac{\pi^2}{9\sqrt{3}} + \frac{1}{6}\log(3)\pi = \\ \frac{2}{6}\pi\coth^{-1}(2) - \frac{\pi^2}{9\sqrt{3}} + \frac{1}{2}\left(\frac{1}{1} + \frac{1}{4^2} + \frac{1}{7^2}\right)\sqrt{3} \end{aligned}$$

### Series representations:

$$\begin{aligned} \frac{1}{2}\left(\frac{1}{1^2} + \frac{1}{4^2} + \frac{1}{7^2}\right)\sqrt{3} - \frac{\pi^2}{9\sqrt{3}} + \frac{1}{6}\log(3)\pi = \\ \frac{849\sqrt{3}}{1568} - \frac{\pi^2}{9\sqrt{3}} + \frac{1}{6}\pi\log(2) - \frac{1}{6}\pi\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{2}\right)^k}{k} \end{aligned}$$

$$\begin{aligned} \frac{1}{2}\left(\frac{1}{1^2} + \frac{1}{4^2} + \frac{1}{7^2}\right)\sqrt{3} - \frac{\pi^2}{9\sqrt{3}} + \frac{1}{6}\log(3)\pi = \frac{849\sqrt{3}}{1568} - \frac{\pi^2}{9\sqrt{3}} + \\ \frac{1}{3}i\pi^2\left[\frac{\arg(3-x)}{2\pi}\right] + \frac{1}{6}\pi\log(x) - \frac{1}{6}\pi\sum_{k=1}^{\infty}\frac{(-1)^k(3-x)^kx^{-k}}{k} \text{ for } x < 0 \end{aligned}$$

$$\begin{aligned} \frac{1}{2}\left(\frac{1}{1^2} + \frac{1}{4^2} + \frac{1}{7^2}\right)\sqrt{3} - \frac{\pi^2}{9\sqrt{3}} + \frac{1}{6}\log(3)\pi = \\ \frac{849\sqrt{3}}{1568} - \frac{\pi^2}{9\sqrt{3}} + \frac{1}{6}\pi\left[\frac{\arg(3-z_0)}{2\pi}\right]\log\left(\frac{1}{z_0}\right) + \frac{1}{6}\pi\log(z_0) + \\ \frac{1}{6}\pi\left[\frac{\arg(3-z_0)}{2\pi}\right]\log(z_0) - \frac{1}{6}\pi\sum_{k=1}^{\infty}\frac{(-1)^k(3-z_0)^kz_0^{-k}}{k} \end{aligned}$$

**Integral representations:**

$$\frac{1}{2} \left( \frac{1}{1^2} + \frac{1}{4^2} + \frac{1}{7^2} \right) \sqrt{3} - \frac{\pi^2}{9\sqrt{3}} + \frac{1}{6} \log(3)\pi = \frac{849\sqrt{3}}{1568} - \frac{\pi^2}{9\sqrt{3}} + \frac{\pi}{6} \int_1^3 \frac{1}{t} dt$$

$$\frac{1}{2} \left( \frac{1}{1^2} + \frac{1}{4^2} + \frac{1}{7^2} \right) \sqrt{3} - \frac{\pi^2}{9\sqrt{3}} + \frac{1}{6} \log(3)\pi = \frac{849\sqrt{3}}{1568} - \frac{\pi^2}{9\sqrt{3}} - \frac{i}{12} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{2^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \text{ for } -1 < \gamma < 0$$

$$3\sqrt{3}/(4) (1/1^2+1/4^2+1/7^2)-\pi^2/(6\sqrt{3})$$

**Input:**

$$3 \times \frac{\sqrt{3}}{4} \left( \frac{1}{1^2} + \frac{1}{4^2} + \frac{1}{7^2} \right) - \frac{\pi^2}{6\sqrt{3}}$$

**Result:**

$$\frac{2547\sqrt{3}}{3136} - \frac{\pi^2}{6\sqrt{3}}$$

**Decimal approximation:**

0.457035842735942921896707449521942450517714433482857903224...

0.4570358427359429...

**Property:**

$$\frac{2547\sqrt{3}}{3136} - \frac{\pi^2}{6\sqrt{3}} \text{ is a transcendental number}$$

**Alternate forms:**

$$-\frac{\sqrt{3} (1568 \pi^2 - 22923)}{28224}$$

$$\frac{22923 - 1568 \pi^2}{9408 \sqrt{3}}$$

$$-\frac{1568 \pi^2 - 22923}{9408 \sqrt{3}}$$

**Series representations:**

$$\frac{1}{4} \left( 3 \left( \frac{1}{1^2} + \frac{1}{4^2} + \frac{1}{7^2} \right) \right) \sqrt{3} - \frac{\pi^2}{6\sqrt{3}} = - \frac{1568 \pi^2 - 7641 \sqrt{2}^2 \left( \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k} \right)^2}{9408 \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k}}$$

$$\frac{1}{4} \left( 3 \left( \frac{1}{1^2} + \frac{1}{4^2} + \frac{1}{7^2} \right) \right) \sqrt{3} - \frac{\pi^2}{6\sqrt{3}} = - \frac{1568 \pi^2 - 7641 \sqrt{2}^2 \left( \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right)^2}{9408 \sqrt{2} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!}}$$

$$\frac{1}{4} \left( 3 \left( \frac{1}{1^2} + \frac{1}{4^2} + \frac{1}{7^2} \right) \right) \sqrt{3} - \frac{\pi^2}{6\sqrt{3}} = \frac{6272 \pi^2 \sqrt{\pi}^2 - 7641 \left( \sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 2^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s) \right)^2}{18816 \sqrt{\pi} \sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 2^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}$$

$$\tan(\pi \cdot 2) - \tan^3\left(\frac{\pi \cdot 2}{3^2}\right) + \tan^5\left(\frac{\pi \cdot 2}{5^2}\right)$$

**Input:**

$$\tan(\pi \times 2) - \tan^3\left(\frac{\pi \times 2}{3^2}\right) + \tan^5\left(\frac{\pi \times 2}{5^2}\right)$$

**Exact result:**

$$\tan^5\left(\frac{2\pi}{25}\right) - \tan^3\left(\frac{2\pi}{9}\right)$$

**Decimal approximation:**

-0.58968429081324040112391641696729911087166749799619960612...

-0.58968429081324....

**Alternate forms:**

$$\frac{\sin^5\left(\frac{2\pi}{25}\right) - \sin^3\left(\frac{2\pi}{9}\right)}{\cos^5\left(\frac{2\pi}{25}\right) - \cos^3\left(\frac{2\pi}{9}\right)}$$

$$\sec^5\left(\frac{2\pi}{25}\right) \sec^3\left(\frac{2\pi}{9}\right) \left( \sin^5\left(\frac{2\pi}{25}\right) \cos^3\left(\frac{2\pi}{9}\right) - \sin^3\left(\frac{2\pi}{9}\right) \cos^5\left(\frac{2\pi}{25}\right) \right)$$

$$\frac{i(e^{-2i\pi/25} - e^{2i\pi/25})^5}{(e^{-2i\pi/25} + e^{2i\pi/25})^5} + \frac{i(e^{-2i\pi/9} - e^{2i\pi/9})^3}{(e^{-2i\pi/9} + e^{2i\pi/9})^3}$$

$\sec(x)$  is the secant function

### Alternative representations:

$$\tan(\pi/2) - \tan^3\left(\frac{\pi/2}{3^2}\right) + \tan^5\left(\frac{\pi/2}{5^2}\right) = \frac{1}{\cot(2\pi)} - \left(\frac{1}{\cot\left(\frac{2\pi}{9}\right)}\right)^3 + \left(\frac{1}{\cot\left(\frac{2\pi}{25}\right)}\right)^5$$

$$\tan(\pi/2) - \tan^3\left(\frac{\pi/2}{3^2}\right) + \tan^5\left(\frac{\pi/2}{5^2}\right) = \cot\left(-\frac{3\pi}{2}\right) - \cot^3\left(\frac{\pi}{2} - \frac{2\pi}{9}\right) + \cot^5\left(\frac{\pi}{2} - \frac{2\pi}{25}\right)$$

$$\tan(\pi/2) - \tan^3\left(\frac{\pi/2}{3^2}\right) + \tan^5\left(\frac{\pi/2}{5^2}\right) = -\cot\left(\frac{5\pi}{2}\right) - \left(-\cot\left(\frac{\pi}{2} + \frac{2\pi}{9}\right)\right)^3 + \left(-\cot\left(\frac{\pi}{2} + \frac{2\pi}{25}\right)\right)^5$$

### Multiple-argument formulas:

$$\tan(\pi/2) - \tan^3\left(\frac{\pi/2}{3^2}\right) + \tan^5\left(\frac{\pi/2}{5^2}\right) = \frac{32 \tan^5\left(\frac{\pi}{25}\right)}{\left(1 - \tan^2\left(\frac{\pi}{25}\right)\right)^5} - \frac{8 \tan^3\left(\frac{\pi}{9}\right)}{\left(1 - \tan^2\left(\frac{\pi}{9}\right)\right)^3}$$

$$\tan(\pi/2) - \tan^3\left(\frac{\pi/2}{3^2}\right) + \tan^5\left(\frac{\pi/2}{5^2}\right) = \frac{\left(3 \tan\left(\frac{2\pi}{75}\right) - \tan^3\left(\frac{2\pi}{75}\right)\right)^5}{\left(1 - 3 \tan^2\left(\frac{2\pi}{75}\right)\right)^5} - \frac{\left(3 \tan\left(\frac{2\pi}{27}\right) - \tan^3\left(\frac{2\pi}{27}\right)\right)^3}{\left(1 - 3 \tan^2\left(\frac{2\pi}{27}\right)\right)^3}$$

Now, we have, from the algebraic sum of these results, multiply by 1/2:

$$(1.08879304515180106525034 + 1.47328541146478942151406 + 0.87992261102782905817010 + 0.45703584273594292189670 - 0.5896842908132404011239)/2$$

### Input interpretation:



$$\frac{1}{2} (1.08879304515180106525034 + 1.47328541146478942151406 + 0.87992261102782905817010 + 0.45703584273594292189670 - 0.5896842908132404011239)$$

**Result:**

1.65467630978356103285365  
1.65467630....

$$10^3 * \frac{1}{2}(1.08879304515180106525034 + 1.47328541146478942151406 + 0.87992261102782905817010 + 0.45703584273594292189670 - 0.5896842908132404011239) + 18$$

Where 18 is a Lucas number

**Input interpretation:**

$$10^3 \times \frac{1}{2} (1.08879304515180106525034 + 1.47328541146478942151406 + 0.87992261102782905817010 + 0.45703584273594292189670 - 0.5896842908132404011239) + 18$$

**Result:**

1672.67630978356103285365  
1672.6763.... result practically equal to the rest mass of Omega baryon 1672.45

And:

$$10^3 * \frac{1}{2}(1.08879304515180106525034 + 1.47328541146478942151406 + 0.87992261102782905817010 + 0.45703584273594292189670 - 0.5896842908132404011239) + 18 + 47 + 7 + 2$$

**Input interpretation:**

$$10^3 \times \frac{1}{2} (1.08879304515180106525034 + 1.47328541146478942151406 + 0.87992261102782905817010 + 0.45703584273594292189670 - 0.5896842908132404011239) + 18 + 47 + 7 + 2$$

**Result:**

1728.67630978356103285365  
1728.6763...

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson. Furthermore, 1728 occurs in the algebraic formula for the  $j$ -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

And again:

$$10^3 * \frac{1}{2}(1.08879304515180106525034 + 1.47328541146478942151406 + 0.87992261102782905817010 + 0.45703584273594292189670 - 0.5896842908132404011239) + 123 + 4$$

**Input interpretation:**

$$10^3 \times \frac{1}{2} (1.08879304515180106525034 + 1.47328541146478942151406 + 0.87992261102782905817010 + 0.45703584273594292189670 - 0.5896842908132404011239) + 123 + 4$$

**Result:**

1781.67630978356103285365

1781.6763097...result in the range of the hypothetical mass of Gluino (gluino = 1785.16 GeV).

We obtain also:

$$1/((1.08879304515 + 1.47328541146 + 0.879922611 + 0.4570358427 - 0.5896842908)^{1/128})$$

**Input interpretation:**

$$1/((1.08879304515 + 1.47328541146 + 0.879922611 + 0.4570358427 - 0.5896842908)^{(1/128)})$$

**Result:**

0.990693942301...

0.990693942301.... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

$$\frac{1 + \sqrt[5]{\sqrt{\phi^5 \sqrt[4]{5^3}} - 1}}{\sqrt{5}} - \phi + 1$$

and to the dilaton value **0.989117352243 =  $\phi$**  (see Appendix)

From which:

log base 0.990693942301 (((1/(1.08879304515 + 1.47328541146 + 0.879922611 + 0.4570358427 - 0.5896842908)))) - Pi + 1/golden ratio

**Input interpretation:**

$$\log_{0.990693942301}(1 / (1.08879304515 + 1.47328541146 + 0.879922611 + 0.4570358427 - 0.5896842908)) - \pi + \frac{1}{\phi}$$

$\log_b(x)$  is the base- $b$  logarithm

$\phi$  is the golden ratio

**Result:**

125.476441...

125.476441.... result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T = 0 (see Appendix)

**Alternative representation:**

$$\log_{0.9906939423010000}(1 / (1.088793045150000 + 1.473285411460000 + 0.879923 + 0.457036 - 0.589684)) - \pi + \frac{1}{\phi} = -\pi + \frac{1}{\phi} + \frac{\log\left(\frac{1}{3.30935}\right)}{\log(0.9906939423010000)}$$



$$-\pi \ln(3)/(3\sqrt{3}) - (10\pi^2)/27 + 5(1/1^2 + 1/4^2 + 1/7^2)$$

**Input:**

$$-\pi \times \frac{\log(3)}{3\sqrt{3}} - \frac{1}{27} (10\pi^2) + 5 \left( \frac{1}{1^2} + \frac{1}{4^2} + \frac{1}{7^2} \right)$$

log(x) is the natural logarithm

**Exact result:**

$$\frac{4245}{784} - \frac{10\pi^2}{27} - \frac{\pi \log(3)}{3\sqrt{3}}$$

**Decimal approximation:**

1.094911021977321962009636108932592188128029561000019424565...

1.094911021977321....

**Alternate forms:**

$$\frac{4245}{784} - \frac{1}{27} \pi (10\pi + \sqrt{3} \log(27))$$

$$\frac{114615 - 7840\pi^2 - 2352\sqrt{3}\pi \log(3)}{21168}$$

$$-\frac{-114615\sqrt{3} + 7840\sqrt{3}\pi^2 + 7056\pi \log(3)}{21168\sqrt{3}}$$

**Alternative representations:**

$$-\frac{\pi \log(3)}{3\sqrt{3}} - \frac{10\pi^2}{27} + 5 \left( \frac{1}{1^2} + \frac{1}{4^2} + \frac{1}{7^2} \right) = -\frac{10\pi^2}{27} + 5 \left( \frac{1}{1} + \frac{1}{4^2} + \frac{1}{7^2} \right) - \frac{\pi \log_e(3)}{3\sqrt{3}}$$

$$-\frac{\pi \log(3)}{3\sqrt{3}} - \frac{10\pi^2}{27} + 5 \left( \frac{1}{1^2} + \frac{1}{4^2} + \frac{1}{7^2} \right) = -\frac{10\pi^2}{27} + 5 \left( \frac{1}{1} + \frac{1}{4^2} + \frac{1}{7^2} \right) - \frac{\pi \log(a) \log_a(3)}{3\sqrt{3}}$$

$$-\frac{\pi \log(3)}{3\sqrt{3}} - \frac{10\pi^2}{27} + 5 \left( \frac{1}{1^2} + \frac{1}{4^2} + \frac{1}{7^2} \right) = -\frac{10\pi^2}{27} + 5 \left( \frac{1}{1} + \frac{1}{4^2} + \frac{1}{7^2} \right) + \frac{\pi \text{Li}_1(-2)}{3\sqrt{3}}$$

**Series representations:**

$$-\frac{\pi \log(3)}{3\sqrt{3}} - \frac{10\pi^2}{27} + 5\left(\frac{1}{1^2} + \frac{1}{4^2} + \frac{1}{7^2}\right) = \frac{4245}{784} - \frac{10\pi^2}{27} - \frac{\pi \log(2)}{3\sqrt{3}} + \frac{\pi \sum_{k=1}^{\infty} \left(\frac{-1}{2}\right)^k}{3\sqrt{3}}$$

$$-\frac{\pi \log(3)}{3\sqrt{3}} - \frac{10\pi^2}{27} + 5\left(\frac{1}{1^2} + \frac{1}{4^2} + \frac{1}{7^2}\right) =$$

$$\frac{4245}{784} - \frac{10\pi^2}{27} - \frac{\pi \left( \log(z_0) + \left\lfloor \frac{\arg(3-z_0)}{2\pi} \right\rfloor \left( \log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (3-z_0)^k z_0^{-k}}{k} \right)}{3\sqrt{3}}$$

$$-\frac{\pi \log(3)}{3\sqrt{3}} - \frac{10\pi^2}{27} + 5\left(\frac{1}{1^2} + \frac{1}{4^2} + \frac{1}{7^2}\right) =$$

$$\frac{4245}{784} - \frac{10\pi^2}{27} - \frac{2i\pi^2 \left\lfloor \frac{\arg(3-x)}{2\pi} \right\rfloor}{3\sqrt{3}} - \frac{\pi \log(x)}{3\sqrt{3}} + \frac{\pi \sum_{k=1}^{\infty} \frac{(-1)^k (3-x)^k x^{-k}}{k}}{3\sqrt{3}} \quad \text{for } x < 0$$

**Integral representations:**

$$-\frac{\pi \log(3)}{3\sqrt{3}} - \frac{10\pi^2}{27} + 5\left(\frac{1}{1^2} + \frac{1}{4^2} + \frac{1}{7^2}\right) = \frac{4245}{784} - \frac{10\pi^2}{27} - \frac{\pi}{3\sqrt{3}} \int_1^3 \frac{1}{t} dt$$

$$-\frac{\pi \log(3)}{3\sqrt{3}} - \frac{10\pi^2}{27} + 5\left(\frac{1}{1^2} + \frac{1}{4^2} + \frac{1}{7^2}\right) =$$

$$\frac{4245}{784} - \frac{10\pi^2}{27} + \frac{i}{6\sqrt{3}} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{2^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \quad \text{for } -1 < \gamma < 0$$

$$-\frac{\pi}{6} \ln(2+\sqrt{3}) + \frac{4}{3}\left(\frac{1}{1^2} - \frac{1}{3^2} + \frac{1}{5^2}\right)$$

**Input:**

$$-\frac{\pi}{6} \log(2 + \sqrt{3}) + \frac{4}{3} \left( \frac{1}{1^2} - \frac{1}{3^2} + \frac{1}{5^2} \right)$$

$\log(x)$  is the natural logarithm

**Exact result:**

$$\frac{836}{675} - \frac{1}{6} \pi \log(2 + \sqrt{3})$$

**Decimal approximation:**

0.548960976174173797162797338069581079438553667426567004924...

0.54896097617417....

**Alternate form:**

$$\frac{1672 - 225 \pi \log(2 + \sqrt{3})}{1350}$$

**Alternative representations:**

$$\frac{1}{6} \log(2 + \sqrt{3}) (-\pi) + \frac{1}{3} \left( \frac{1}{1^2} - \frac{1}{3^2} + \frac{1}{5^2} \right) 4 = -\frac{1}{6} \pi \log_e(2 + \sqrt{3}) + \frac{4}{3} \left( \frac{1}{1} - \frac{1}{9} + \frac{1}{5^2} \right)$$

$$\frac{1}{6} \log(2 + \sqrt{3}) (-\pi) + \frac{1}{3} \left( \frac{1}{1^2} - \frac{1}{3^2} + \frac{1}{5^2} \right) 4 = -\frac{1}{6} \pi \log(a) \log_a(2 + \sqrt{3}) + \frac{4}{3} \left( \frac{1}{1} - \frac{1}{9} + \frac{1}{5^2} \right)$$

$$\frac{1}{6} \log(2 + \sqrt{3}) (-\pi) + \frac{1}{3} \left( \frac{1}{1^2} - \frac{1}{3^2} + \frac{1}{5^2} \right) 4 = \frac{1}{6} \pi \operatorname{Li}_1(-1 - \sqrt{3}) + \frac{4}{3} \left( \frac{1}{1} - \frac{1}{9} + \frac{1}{5^2} \right)$$

**Series representations:**

$$\frac{1}{6} \log(2 + \sqrt{3}) (-\pi) + \frac{1}{3} \left( \frac{1}{1^2} - \frac{1}{3^2} + \frac{1}{5^2} \right) 4 = \frac{836}{675} - \frac{1}{6} \pi \log(1 + \sqrt{3}) + \frac{1}{6} \pi \sum_{k=1}^{\infty} \frac{\left( -\frac{1}{1+\sqrt{3}} \right)^k}{k}$$

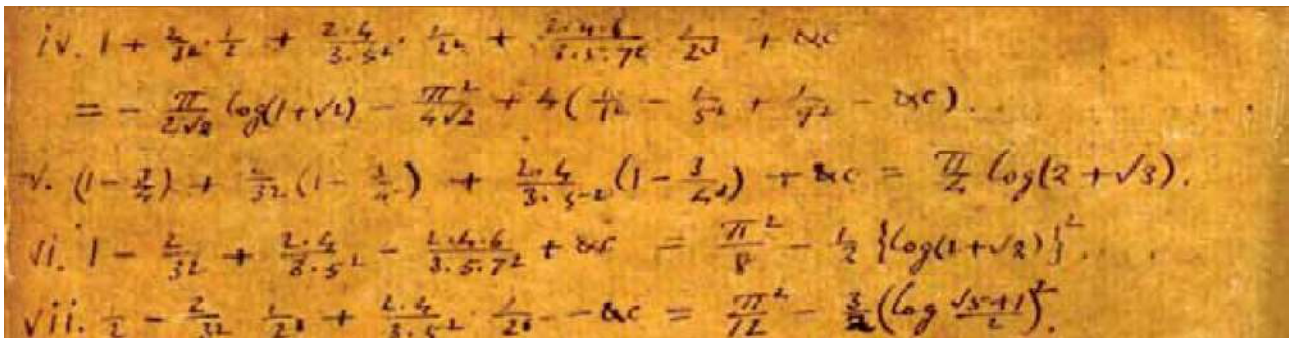
$$\frac{1}{6} \log(2 + \sqrt{3}) (-\pi) + \frac{1}{3} \left( \frac{1}{1^2} - \frac{1}{3^2} + \frac{1}{5^2} \right) 4 = \frac{836}{675} - \frac{1}{3} i \pi^2 \left[ \frac{\arg(2 + \sqrt{3} - x)}{2\pi} \right] - \frac{1}{6} \pi \log(x) + \frac{1}{6} \pi \sum_{k=1}^{\infty} \frac{(-1)^k (2 + \sqrt{3} - x)^k x^{-k}}{k} \text{ for } x < 0$$

$$\frac{1}{6} \log(2 + \sqrt{3}) (-\pi) + \frac{1}{3} \left( \frac{1}{1^2} - \frac{1}{3^2} + \frac{1}{5^2} \right) 4 = \frac{836}{675} - \frac{1}{6} \pi \left[ \frac{\arg(2 + \sqrt{3} - z_0)}{2\pi} \right] \log\left(\frac{1}{z_0}\right) - \frac{1}{6} \pi \log(z_0) - \frac{1}{6} \pi \left[ \frac{\arg(2 + \sqrt{3} - z_0)}{2\pi} \right] \log(z_0) + \frac{1}{6} \pi \sum_{k=1}^{\infty} \frac{(-1)^k (2 + \sqrt{3} - z_0)^k z_0^{-k}}{k}$$

**Integral representations:**

$$\frac{1}{6} \log(2 + \sqrt{3}) (-\pi) + \frac{1}{3} \left( \frac{1}{1^2} - \frac{1}{3^2} + \frac{1}{5^2} \right) 4 = \frac{836}{675} - \frac{\pi}{6} \int_1^{2+\sqrt{3}} \frac{1}{t} dt$$

$$\frac{1}{6} \log(2 + \sqrt{3}) (-\pi) + \frac{1}{3} \left( \frac{1}{1^2} - \frac{1}{3^2} + \frac{1}{5^2} \right) 4 = \frac{836}{675} + \frac{i}{12} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{(1 + \sqrt{3})^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \text{ for } -1 < \gamma < 0$$



$$-\frac{\pi}{2\sqrt{2}} \ln(1+\sqrt{2}) - \frac{\pi^2}{4\sqrt{2}} + 4\left(\frac{1}{1^2} - \frac{1}{5^2} + \frac{1}{9^2} - \dots\right)$$

**Input:**

$$-\frac{\pi}{2\sqrt{2}} \log(1 + \sqrt{2}) - \frac{\pi^2}{4\sqrt{2}} + 4\left(\frac{1}{1^2} - \frac{1}{5^2} + \frac{1}{9^2}\right)$$

$\log(x)$  is the natural logarithm

**Exact result:**

$$\frac{7876}{2025} - \frac{\pi^2}{4\sqrt{2}} - \frac{\pi \log(1 + \sqrt{2})}{2\sqrt{2}}$$

**Decimal approximation:**

1.165706748161521380870474111779844793363208411651671677860...

1.16570674816152138.....

**Alternate forms:**

$$\frac{7876}{2025} - \frac{\pi(\pi + 2 \sinh^{-1}(1))}{4\sqrt{2}}$$

$$\frac{63008 - 2025\sqrt{2}\pi^2 - 4050\sqrt{2}\pi \log(1 + \sqrt{2})}{16200}$$

$$-\frac{31504\sqrt{2} + 2025\pi^2 + 4050\pi \log(1 + \sqrt{2})}{8100\sqrt{2}}$$

$\sinh^{-1}(x)$  is the inverse hyperbolic sine function

**Alternative representations:**



$$\frac{\log(1+\sqrt{2})(-\pi)}{2\sqrt{2}} - \frac{\pi^2}{4\sqrt{2}} + 4\left(\frac{1}{1^2} - \frac{1}{5^2} + \frac{1}{9^2}\right) =$$

$$4\left(\frac{1}{1} - \frac{1}{5^2} + \frac{1}{9^2}\right) - \frac{\pi \log_e(1+\sqrt{2})}{2\sqrt{2}} - \frac{\pi^2}{4\sqrt{2}}$$

$$\frac{\log(1+\sqrt{2})(-\pi)}{2\sqrt{2}} - \frac{\pi^2}{4\sqrt{2}} + 4\left(\frac{1}{1^2} - \frac{1}{5^2} + \frac{1}{9^2}\right) =$$

$$4\left(\frac{1}{1} - \frac{1}{5^2} + \frac{1}{9^2}\right) - \frac{\pi \log(a) \log_a(1+\sqrt{2})}{2\sqrt{2}} - \frac{\pi^2}{4\sqrt{2}}$$

$$\frac{\log(1+\sqrt{2})(-\pi)}{2\sqrt{2}} - \frac{\pi^2}{4\sqrt{2}} + 4\left(\frac{1}{1^2} - \frac{1}{5^2} + \frac{1}{9^2}\right) = 4\left(\frac{1}{1} - \frac{1}{5^2} + \frac{1}{9^2}\right) + \frac{\pi \operatorname{Li}_1(-\sqrt{2})}{2\sqrt{2}} - \frac{\pi^2}{4\sqrt{2}}$$

### Series representations:

$$\frac{\log(1+\sqrt{2})(-\pi)}{2\sqrt{2}} - \frac{\pi^2}{4\sqrt{2}} + 4\left(\frac{1}{1^2} - \frac{1}{5^2} + \frac{1}{9^2}\right) =$$

$$\frac{7876}{2025} - \frac{\pi^2}{4\sqrt{2}} - \frac{\pi \log(2)}{4\sqrt{2}} + \frac{\pi \sum_{k=1}^{\infty} \frac{(-1)^k 2^{-k/2}}{k}}{2\sqrt{2}}$$

$$\frac{\log(1+\sqrt{2})(-\pi)}{2\sqrt{2}} - \frac{\pi^2}{4\sqrt{2}} + 4\left(\frac{1}{1^2} - \frac{1}{5^2} + \frac{1}{9^2}\right) =$$

$$\frac{7876}{2025} - \frac{\pi^2}{4\sqrt{2}} - \frac{i\pi^2 \left[ \frac{\operatorname{arg}(1+\sqrt{2}-x)}{2\pi} \right]}{\sqrt{2}} - \frac{\pi \log(x)}{2\sqrt{2}} + \frac{\pi \sum_{k=1}^{\infty} \frac{(-1)^k (1+\sqrt{2}-x)^k x^{-k}}{k}}{2\sqrt{2}} \quad \text{for } x < 0$$

$$\frac{\log(1+\sqrt{2})(-\pi)}{2\sqrt{2}} - \frac{\pi^2}{4\sqrt{2}} + 4\left(\frac{1}{1^2} - \frac{1}{5^2} + \frac{1}{9^2}\right) = \frac{7876}{2025} - \frac{\pi^2}{4\sqrt{2}} -$$

$$\frac{\pi \left( \log(z_0) + \left[ \frac{\operatorname{arg}(1+\sqrt{2}-z_0)}{2\pi} \right] \right) \left( \log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (1+\sqrt{2}-z_0)^k z_0^{-k}}{k}}{2\sqrt{2}}$$

### Integral representations:

$$\frac{\log(1+\sqrt{2})(-\pi)}{2\sqrt{2}} - \frac{\pi^2}{4\sqrt{2}} + 4\left(\frac{1}{1^2} - \frac{1}{5^2} + \frac{1}{9^2}\right) = \frac{7876}{2025} - \frac{\pi^2}{4\sqrt{2}} - \frac{\pi}{2\sqrt{2}} \int_1^{1+\sqrt{2}} \frac{1}{t} dt$$

$$\frac{\log(1 + \sqrt{2}) (-\pi)}{2\sqrt{2}} - \frac{\pi^2}{4\sqrt{2}} + 4 \left( \frac{1}{1^2} - \frac{1}{5^2} + \frac{1}{9^2} \right) =$$

$$\frac{7876}{2025} - \frac{\pi^2}{4\sqrt{2}} + \frac{i}{4\sqrt{2}} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{2^{-s/2} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \quad \text{for } -1 < \gamma < 0$$

$$\text{Pi}/4 * \ln(2 + \text{sqrt}3)$$

**Input:**

$$\frac{\pi}{4} \log(2 + \sqrt{3})$$

$\log(x)$  is the natural logarithm

**Exact result:**

$$\frac{1}{4} \pi \log(2 + \sqrt{3})$$

**Decimal approximation:**

1.034336313516517082033581770673406158619947276637927270391...

1.034336313516517....

**Alternative representations:**

$$\frac{1}{4} \log(2 + \sqrt{3}) \pi = \frac{1}{4} \pi \log_e(2 + \sqrt{3})$$

$$\frac{1}{4} \log(2 + \sqrt{3}) \pi = \frac{1}{4} \pi \log(a) \log_a(2 + \sqrt{3})$$

$$\frac{1}{4} \log(2 + \sqrt{3}) \pi = -\frac{1}{4} \pi \text{Li}_1(-1 - \sqrt{3})$$

**Series representations:**

$$\frac{1}{4} \log(2 + \sqrt{3}) \pi = \frac{1}{4} \pi \log(1 + \sqrt{3}) - \frac{1}{4} \pi \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{1+\sqrt{3}}\right)^k}{k}$$

$$\frac{1}{4} \log(2 + \sqrt{3}) \pi = \frac{1}{2} i \pi^2 \left[ \frac{\arg(2 + \sqrt{3} - x)}{2\pi} \right] + \frac{1}{4} \pi \log(x) - \frac{1}{4} \pi \sum_{k=1}^{\infty} \frac{(-1)^k (2 + \sqrt{3} - x)^k x^{-k}}{k} \quad \text{for } x < 0$$

$$\frac{1}{4} \log(2 + \sqrt{3}) \pi = \frac{1}{2} i \pi^2 \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] + \frac{1}{4} \pi \log(z_0) - \frac{1}{4} \pi \sum_{k=1}^{\infty} \frac{(-1)^k (2 + \sqrt{3} - z_0)^k z_0^{-k}}{k}$$

### Integral representations:

$$\frac{1}{4} \log(2 + \sqrt{3}) \pi = \frac{\pi}{4} \int_1^{2+\sqrt{3}} \frac{1}{t} dt$$

$$\frac{1}{4} \log(2 + \sqrt{3}) \pi = -\frac{i}{8} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{(1 + \sqrt{3})^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \quad \text{for } -1 < \gamma < 0$$

$$\pi^2/8 - 1/2((\ln(1+\sqrt{2}))^2)$$

### Input:

$$\frac{\pi^2}{8} - \frac{1}{2} \log^2(1 + \sqrt{2})$$

$\log(x)$  is the natural logarithm

### Decimal approximation:

0.845290850188321836604024019939809439610683518750121373697...

0.8452908501883218366...

### Alternate forms:

$$\frac{1}{8} (\pi^2 - 4 \sinh^{-1}(1)^2)$$

$$\frac{1}{8} (\pi^2 - 4 \log^2(1 + \sqrt{2}))$$

$$\frac{1}{8} (\pi - 2 \log(1 + \sqrt{2})) (\pi + 2 \log(1 + \sqrt{2}))$$

$\sinh^{-1}(x)$  is the inverse hyperbolic sine function

### Alternative representations:

$$\frac{\pi^2}{8} - \frac{1}{2} \log^2(1 + \sqrt{2}) = \frac{\pi^2}{8} - \frac{1}{2} \log_e^2(1 + \sqrt{2})$$

$$\frac{\pi^2}{8} - \frac{1}{2} \log^2(1 + \sqrt{2}) = \frac{\pi^2}{8} - \frac{1}{2} \left( \log(a) \log_a(1 + \sqrt{2}) \right)^2$$

$$\frac{\pi^2}{8} - \frac{1}{2} \log^2(1 + \sqrt{2}) = \frac{\pi^2}{8} - \frac{1}{2} \left( -\text{Li}_1(-\sqrt{2}) \right)^2$$

### Series representations:

$$\frac{\pi^2}{8} - \frac{1}{2} \log^2(1 + \sqrt{2}) = \frac{1}{8} \left( \pi^2 - \left( \log(2) - 2 \sum_{k=1}^{\infty} \frac{(-1)^k 2^{-k/2}}{k} \right)^2 \right)$$

$$\frac{\pi^2}{8} - \frac{1}{2} \log^2(1 + \sqrt{2}) = \frac{1}{8} \left( \pi^2 - 4 \left( 2i\pi \left[ \frac{\arg(1 + \sqrt{2} - x)}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (1 + \sqrt{2} - x)^k x^{-k}}{k} \right)^2 \right) \text{ for } x < 0$$

$$\frac{\pi^2}{8} - \frac{1}{2} \log^2(1 + \sqrt{2}) = \frac{\pi^2}{8} - \frac{1}{2} \left( 2i\pi \left[ \frac{\arg(1 + \sqrt{2} - x)}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (1 + \sqrt{2} - x)^k x^{-k}}{k} \right)^2 \text{ for } x < 0$$

### Integral representations:

$$\frac{\pi^2}{8} - \frac{1}{2} \log^2(1 + \sqrt{2}) = \frac{\pi^2}{8} - \frac{1}{2} \left( \int_1^{1+\sqrt{2}} \frac{1}{t} dt \right)^2$$

$$\frac{\pi^2}{8} - \frac{1}{2} \log^2(1 + \sqrt{2}) = \frac{\pi^2}{8} + \frac{\left( \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{2^{-s/2} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right)^2}{8\pi^2} \text{ for } -1 < \gamma < 0$$

$$\text{Pi}^2/(12)-3/2 \left( \left( \ln \left( \frac{\sqrt{5}+1}{2} \right) \right)^2 \right)$$

**Input:**

$$\frac{\pi^2}{12} - \frac{3}{2} \log^2\left(\frac{1}{2}(\sqrt{5} + 1)\right)$$

$\log(x)$  is the natural logarithm

### Decimal approximation:

0.475119802558321629490813976475713454584865425529450912644...

0.475119802558321.....

### Alternate forms:

$$\frac{1}{12} (\pi^2 - 18 \operatorname{csch}^{-1}(2)^2)$$

$$\frac{\pi^2}{12} - \frac{3}{2} \operatorname{csch}^{-1}(2)^2$$

$$\frac{1}{12} \left( \pi^2 - 18 \log^2\left(\frac{1}{2}(1 + \sqrt{5})\right) \right)$$

$\operatorname{csch}^{-1}(x)$  is the inverse hyperbolic cosecant function

### Alternative representations:

$$\frac{\pi^2}{12} - \frac{1}{2} \log^2\left(\frac{1}{2}(\sqrt{5} + 1)\right) 3 = \frac{\pi^2}{12} - \frac{3}{2} \log_e^2\left(\frac{1}{2}(1 + \sqrt{5})\right)$$

$$\frac{\pi^2}{12} - \frac{1}{2} \log^2\left(\frac{1}{2}(\sqrt{5} + 1)\right) 3 = \frac{\pi^2}{12} - \frac{3}{2} \left( \log(a) \log_a\left(\frac{1}{2}(1 + \sqrt{5})\right) \right)^2$$

$$\frac{\pi^2}{12} - \frac{1}{2} \log^2\left(\frac{1}{2}(\sqrt{5} + 1)\right) 3 = \frac{\pi^2}{12} - \frac{3}{2} \left( -\operatorname{Li}_1\left(1 + \frac{1}{2}(-1 - \sqrt{5})\right) \right)^2$$

### Series representations:

$$\frac{\pi^2}{12} - \frac{1}{2} \log^2\left(\frac{1}{2}(\sqrt{5} + 1)\right) 3 = \frac{\pi^2}{12} - \frac{3}{2} \left( \sum_{k=1}^{\infty} \frac{\left(\frac{1}{2}(1 - \sqrt{5})\right)^k}{k} \right)^2$$

$$\frac{\pi^2}{12} - \frac{1}{2} \log^2\left(\frac{1}{2}(\sqrt{5} + 1)\right) 3 = \frac{1}{12} \left( \pi^2 - 18 \left[ 2i\pi \left\lfloor \frac{\arg(1 + \sqrt{5} - 2x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{2}\right)^k (1 + \sqrt{5} - 2x)^k x^{-k}}{k} \right]^2 \right)$$

for  $x < 0$

$$\frac{\pi^2}{12} - \frac{1}{2} \log^2\left(\frac{1}{2}(\sqrt{5} + 1)\right) 3 = \frac{\pi^2}{12} - \frac{3}{2} \left( 2i\pi \left| \frac{\arg\left(\frac{1}{2}(1 + \sqrt{5}) - x\right)}{2\pi} \right| + \log(x) - \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{2}\right)^k (1 + \sqrt{5} - 2x)^k x^{-k}}{k} \right)^2 \text{ for } x < 0$$

**Integral representations:**

$$\frac{\pi^2}{12} - \frac{1}{2} \log^2\left(\frac{1}{2}(\sqrt{5} + 1)\right) 3 = \frac{\pi^2}{12} - \frac{3}{2} \left( \int_1^{\frac{1}{2}(1+\sqrt{5})} \frac{1}{t} dt \right)^2$$

$$\frac{\pi^2}{12} - \frac{1}{2} \log^2\left(\frac{1}{2}(\sqrt{5} + 1)\right) 3 = \frac{\pi^2}{12} + \frac{3 \left( \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\left(-1+\frac{1}{2}(1+\sqrt{5})\right)^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right)^2}{8\pi^2}$$

for  $-1 < \gamma < 0$

From the sum of the previous seven results, we obtain:

$$1.857777777777 + 1.094911021977321 + 0.54896097617417 + 1.16570674816152138 + 1.034336313516517 + 0.8452908501883218 + 0.475119802558321 = 7.02210349035317218$$

And:

$$\exp(1.857777777777 + 1.094911021977321 + 0.54896097617417 + 1.16570674816152138 + 1.034336313516517 + 0.8452908501883218 + 0.475119802558321) + 76$$

where 76 is a Lucas number

**Input interpretation:**

$$\exp(1.857777777777 + 1.094911021977321 + 0.54896097617417 + 1.16570674816152138 + 1.034336313516517 + 0.8452908501883218 + 0.475119802558321) + 76$$

**Result:**

1197.142451472...

1197.142451472.... result practically equal to the rest mass of Sigma baryon

1197.449

From the multiplication of the previous results, we obtain:

$$1 - \ln(1.857777777777 * 1.094911021977321 * 0.54896097617417 * 1.16570674816152138 * 1.034336313516517 * 0.8452908501883218 * 0.475119802558321)$$

**Input interpretation:**

$$1 - \log(1.857777777777 * 1.094911021977321 * 0.54896097617417 * 1.16570674816152138 * 1.034336313516517 * 0.8452908501883218 * 0.475119802558321)$$

log(x) is the natural logarithm

**Result:**

1.614849058193...

1.614849058193... result that is a good approximation to the value of the golden ratio 1,618033988749...

And from the division of the results, we obtain:

$$\exp(1.857777777777 * 1/1.094911021977321 * 1/0.54896097617417 * 1/1.16570674816152138 * 1/1.034336313516517 * 1/0.8452908501883218 * 1/0.475119802558321) - 47 + 3$$

where 47 and 3 are Lucas numbers

**Input interpretation:**

$$\exp\left(1.857777777777 * \frac{1}{1.094911021977321} * \frac{1}{0.54896097617417} * \frac{1}{1.16570674816152138} * \frac{1}{1.034336313516517} * \frac{1}{0.8452908501883218} * \frac{1}{0.475119802558321}\right) - 47 + 3$$

**Result:**

547.597853899...

547.597853899.... result practically equal to the rest mass of Eta meson 547.862

From the sum, we have also:

$$1/(1.857777777777 + 1.094911021977321 + 0.54896097617417 + 1.16570674816152138 + 1.034336313516517 + 0.8452908501883218 + 0.475119802558321)^{1/256}$$

**Input interpretation:**

$$1 / ((1.8577777777777 + 1.094911021977321 + 0.54896097617417 + 1.16570674816152138 + 1.034336313516517 + 0.8452908501883218 + 0.475119802558321) ^ (1 / 256))$$

**Result:**

0.9924153828559520...

0.9924153828559520.... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

and to the dilaton value **0.989117352243 =  $\phi$**  (see Appendix)

And:

$$1/2 \log_{0.9924153828559520} (((1/(1.8577777777777 + 1.094911021977321 + 0.54896097617417 + 1.16570674816152138 + 1.034336313516517 + 0.8452908501883218 + 0.475119802558321)))) - \pi + 1/\text{golden ratio}$$

**Input interpretation:**

$$\frac{1}{2} \log_{0.9924153828559520} (1 / (1.8577777777777 + 1.094911021977321 + 0.54896097617417 + 1.16570674816152138 + 1.034336313516517 + 0.8452908501883218 + 0.475119802558321)) - \pi + \frac{1}{\phi}$$

$\log_b(x)$  is the base-  $b$  logarithm

$\phi$  is the golden ratio



**Result:**

125.47644133516...

125.47644133516.... result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for  $T = 0$  (see Appendix)

**Alternative representation:**

$$\frac{1}{2} \log_{0.99241538285595200000} (1 / (1.8577777777770000 + 1.0949110219773210000 + 0.548960976174170000 + 1.165706748161521380000 + 1.0343363135165170000 + 0.84529085018832180000 + 0.4751198025583210000)) - \pi + \frac{1}{\phi} = -\pi + \frac{1}{\phi} + \frac{\log\left(\frac{1}{7.0221034903531722}\right)}{2 \log(0.99241538285595200000)}$$

**Series representations:**

$$\frac{1}{2} \log_{0.99241538285595200000} (1 / (1.8577777777770000 + 1.0949110219773210000 + 0.548960976174170000 + 1.165706748161521380000 + 1.0343363135165170000 + 0.84529085018832180000 + 0.4751198025583210000)) - \pi + \frac{1}{\phi} = \frac{1}{\phi} - \pi - \frac{\sum_{k=1}^{\infty} \frac{(-1)^k (-0.857592528880586799)^k}{k}}{2 \log(0.99241538285595200000)}$$

$$\frac{1}{2} \log_{0.99241538285595200000} (1 / (1.8577777777770000 + 1.0949110219773210000 + 0.548960976174170000 + 1.165706748161521380000 + 1.0343363135165170000 + 0.84529085018832180000 + 0.4751198025583210000)) - \pi + \frac{1}{\phi} = \frac{1}{\phi} - \pi - 65.6729056001031158 \log(0.142407471119413201) - \frac{1}{2} \log(0.142407471119413201) - \sum_{k=0}^{\infty} (-0.00758461714404800000)^k G(k)$$

for  $\left( G(0) = 0 \text{ and } G(k) = \frac{(-1)^{1+k} k}{2(1+k)(2+k)} + \sum_{j=1}^k \frac{(-1)^{1+j} G(-j+k)}{1+j} \right)$



$$\frac{\pi^2}{3} - 3$$

### Alternative representations:

$$\frac{1}{3} (\pi^2 - 9) = \frac{1}{3} (-9 + (180^\circ)^2)$$

$$\frac{1}{3} (\pi^2 - 9) = \frac{1}{3} (-9 + (-i \log(-1))^2)$$

$$\frac{1}{3} (\pi^2 - 9) = \frac{1}{3} (-9 + 6 \zeta(2))$$

### Series representations:

$$\frac{1}{3} (\pi^2 - 9) = -3 + 2 \sum_{k=1}^{\infty} \frac{1}{k^2}$$

$$\frac{1}{3} (\pi^2 - 9) = -3 - 4 \sum_{k=1}^{\infty} \frac{(-1)^k}{k^2}$$

$$\frac{1}{3} (\pi^2 - 9) = -3 + \frac{8}{3} \sum_{k=0}^{\infty} \frac{1}{(1+2k)^2}$$

### Integral representations:

$$\frac{1}{3} (\pi^2 - 9) = -3 + \frac{16}{3} \left( \int_0^1 \sqrt{1-t^2} dt \right)^2$$

$$\frac{1}{3} (\pi^2 - 9) = -3 + \frac{4}{3} \left( \int_0^{\infty} \frac{1}{1+t^2} dt \right)^2$$

$$\frac{1}{3} (\pi^2 - 9) = -3 + \frac{4}{3} \left( \int_0^1 \frac{1}{\sqrt{1-t^2}} dt \right)^2$$



$$10 - \pi^2 = 10 - 8 \sum_{k=0}^{\infty} \frac{1}{(1+2k)^2}$$

**Integral representations:**

$$10 - \pi^2 = 10 - 16 \left( \int_0^1 \sqrt{1-t^2} dt \right)^2$$

$$10 - \pi^2 = 10 - 4 \left( \int_0^{\infty} \frac{1}{1+t^2} dt \right)^2$$

$$10 - \pi^2 = 10 - 4 \left( \int_0^1 \frac{1}{\sqrt{1-t^2}} dt \right)^2$$

$$(1/2^4 + 1/6^4 + 1/12^4)$$

**Input:**

$$\frac{1}{2^4} + \frac{1}{6^4} + \frac{1}{12^4}$$

**Exact result:**

$$\frac{1313}{20736}$$

**Decimal approximation:**

0.063319830246913580246913580246913580246913580246913580246...

0.0633198302469.....

**Repeating decimal:**

0.06331983024691358 (period 9)

$$\pi^4/(45)+10\pi^2/(3)-35$$

**Input:**

$$\frac{\pi^4}{45} + 10 \times \frac{\pi^2}{3} - 35$$

**Result:**

$$-35 + \frac{10\pi^2}{3} + \frac{\pi^4}{45}$$

**Decimal approximation:**

0.063327804386805112480310726002839589928499927973422570077...

0.0633278043868.....

**Property:**

$-35 + \frac{10\pi^2}{3} + \frac{\pi^4}{45}$  is a transcendental number

**Alternate forms:**

$$\frac{1}{45} (\pi^4 + 150\pi^2 - 1575)$$

$$\frac{1}{45} \pi^2 (150 + \pi^2) - 35$$

**Alternative representations:**

$$\frac{\pi^4}{45} + \frac{10\pi^2}{3} - 35 = -35 + \frac{10}{3} (180^\circ)^2 + \frac{1}{45} (180^\circ)^4$$

$$\frac{\pi^4}{45} + \frac{10\pi^2}{3} - 35 = -35 + \frac{\pi^4}{45} + 20\zeta(2)$$

$$\frac{\pi^4}{45} + \frac{10\pi^2}{3} - 35 = -35 + \frac{10}{3} \cos^{-1}(-1)^2 + \frac{1}{45} \cos^{-1}(-1)^4$$

**Series representations:**

$$\frac{\pi^4}{45} + \frac{10\pi^2}{3} - 35 = -35 + \frac{10\pi^2}{3} + 2 \sum_{k=1}^{\infty} \frac{1}{k^4}$$

$$\frac{\pi^4}{45} + \frac{10\pi^2}{3} - 35 = -35 + \frac{\pi^4}{45} + 20 \sum_{k=1}^{\infty} \frac{1}{k^2}$$

$$\frac{\pi^4}{45} + \frac{10\pi^2}{3} - 35 = -35 + \frac{\pi^4}{45} - 40 \sum_{k=1}^{\infty} \frac{(-1)^k}{k^2}$$

**Integral representations:**

$$\frac{\pi^4}{45} + \frac{10\pi^2}{3} - 35 = \frac{1}{45} \left( -1575 + 600 \left( \int_0^{\infty} \frac{1}{1+t^2} dt \right)^2 + 16 \left( \int_0^{\infty} \frac{1}{1+t^2} dt \right)^4 \right)$$

$$\frac{\pi^4}{45} + \frac{10\pi^2}{3} - 35 = \frac{1}{45} \left( -1575 + 2400 \left( \int_0^1 \sqrt{1-t^2} dt \right)^2 + 256 \left( \int_0^1 \sqrt{1-t^2} dt \right)^4 \right)$$

$$\frac{\pi^4}{45} + \frac{10\pi^2}{3} - 35 = \frac{1}{45} \left( -1575 + 600 \left( \int_0^\infty \frac{\sin(t)}{t} dt \right)^2 + 16 \left( \int_0^\infty \frac{\sin(t)}{t} dt \right)^4 \right)$$

$$(1/2^5 + 1/6^5 + 1/12^5)$$

**Input:**

$$\frac{1}{2^5} + \frac{1}{6^5} + \frac{1}{12^5}$$

**Exact result:**

$$\frac{2603}{82944}$$

**Decimal approximation:**

0.031382619598765432098765432098765432098765432098765432098...

0.031382619598765432...

**Repeating decimal:**

0.0313826195987654320 (period 9)

$$126 - 35\pi^2/(3) - \pi^4/(9)$$

**Input:**

$$126 - 35 \times \frac{\pi^2}{3} - \frac{\pi^4}{9}$$

**Result:**

$$126 - \frac{35\pi^2}{3} - \frac{\pi^4}{9}$$

**Decimal approximation:**

0.031382983512767531770901369366557726925997396336840281681...

0.0313829835127675.....

**Property:**

$126 - \frac{35\pi^2}{3} - \frac{\pi^4}{9}$  is a transcendental number

**Alternate forms:**

$$-\frac{1}{9}(-1134 + 105\pi^2 + \pi^4)$$

$$126 - \frac{1}{9}\pi^2(105 + \pi^2)$$

$$\frac{1}{9}(1134 - 105\pi^2 - \pi^4)$$

**Alternative representations:**

$$126 - \frac{35\pi^2}{3} - \frac{\pi^4}{9} = 126 - \frac{35}{3}(180^\circ)^2 - \frac{1}{9}(180^\circ)^4$$

$$126 - \frac{35\pi^2}{3} - \frac{\pi^4}{9} = 126 - \frac{\pi^4}{9} - 70\zeta(2)$$

$$126 - \frac{35\pi^2}{3} - \frac{\pi^4}{9} = 126 - \frac{35}{3}\cos^{-1}(-1)^2 - \frac{1}{9}\cos^{-1}(-1)^4$$

**Series representations:**

$$126 - \frac{35\pi^2}{3} - \frac{\pi^4}{9} = 126 - \frac{\pi^4}{9} - 70 \sum_{k=1}^{\infty} \frac{1}{k^2}$$

$$126 - \frac{35\pi^2}{3} - \frac{\pi^4}{9} = 126 - \frac{35\pi^2}{3} - 10 \sum_{k=1}^{\infty} \frac{1}{k^4}$$

$$126 - \frac{35\pi^2}{3} - \frac{\pi^4}{9} = 126 - \frac{\pi^4}{9} + 140 \sum_{k=1}^{\infty} \frac{(-1)^k}{k^2}$$

**Integral representations:**

$$126 - \frac{35\pi^2}{3} - \frac{\pi^4}{9} = -\frac{2}{9} \left( -567 + 210 \left( \int_0^\infty \frac{1}{1+t^2} dt \right)^2 + 8 \left( \int_0^\infty \frac{1}{1+t^2} dt \right)^4 \right)$$

$$126 - \frac{35\pi^2}{3} - \frac{\pi^4}{9} = -\frac{2}{9} \left( -567 + 840 \left( \int_0^1 \sqrt{1-t^2} dt \right)^2 + 128 \left( \int_0^1 \sqrt{1-t^2} dt \right)^4 \right)$$

$$126 - \frac{35\pi^2}{3} - \frac{\pi^4}{9} = -\frac{2}{9} \left( -567 + 210 \left( \int_0^\infty \frac{\sin(t)}{t} dt \right)^2 + 8 \left( \int_0^\infty \frac{\sin(t)}{t} dt \right)^4 \right)$$



We note that from this last equation, we can to obtain a result near to the Higgs boson mass. Indeed:

$$0.0313829835127675317 + (35 \pi^2)/3 + \pi^4/9$$

**Input interpretation:**

$$0.0313829835127675317 + \frac{1}{3} (35 \pi^2) + \frac{\pi^4}{9}$$

**Result:**

125.999999999999999999999999999999...

125.999999999..... result very near also to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T = 0 (see Appendix)

**Alternative representations:**

$$0.03138298351276753170000 + \frac{35 \pi^2}{3} + \frac{\pi^4}{9} =$$

$$0.03138298351276753170000 + \frac{35}{3} (180^\circ)^2 + \frac{1}{9} (180^\circ)^4$$

$$0.03138298351276753170000 + \frac{35 \pi^2}{3} + \frac{\pi^4}{9} =$$

$$0.03138298351276753170000 + \frac{\pi^4}{9} + 70 \zeta(2)$$

$$0.03138298351276753170000 + \frac{35 \pi^2}{3} + \frac{\pi^4}{9} =$$

$$0.03138298351276753170000 + \frac{35}{3} \cos^{-1}(-1)^2 + \frac{1}{9} \cos^{-1}(-1)^4$$

**Integral representations:**



**Result:**

$$1.6726802... \times 10^{-27}$$

1.6726802... \* 10<sup>-27</sup> result practically equal to the proton mass

And:

$$1/10^{27} * (((1 - \ln(((0.284722222 + 0.130208333 + 0.06331983 + 0.0313826195))))))$$

**Input interpretation:**

$$\frac{1}{10^{27}} (1 - \log(0.284722222 + 0.130208333 + 0.06331983 + 0.0313826195))$$

log(x) is the natural logarithm

**Result:**

$$1.6740644... \times 10^{-27}$$

1.6740644... \* 10<sup>-27</sup> result practically equal to the neutron mass

**Alternative representations:**

$$\frac{1 - \log(0.284722 + 0.130208 + 0.0633198 + 0.0313826)}{10^{27}} = \frac{1 - \log_e(0.509633)}{10^{27}}$$

$$\frac{1 - \log(0.284722 + 0.130208 + 0.0633198 + 0.0313826)}{10^{27}} = \frac{1 - \log(a) \log_a(0.509633)}{10^{27}}$$

$$\frac{1 - \log(0.284722 + 0.130208 + 0.0633198 + 0.0313826)}{10^{27}} = \frac{1 + \text{Li}_1(0.490367)}{10^{27}}$$

**Series representations:**

$$\frac{1 - \log(0.284722 + 0.130208 + 0.0633198 + 0.0313826)}{10^{27}} = \frac{1}{1\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000} + \frac{\sum_{k=1}^{\infty} \frac{(-1)^k (-0.490367)^k}{k}}{1\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000}$$

$$\frac{1 - \log(0.284722 + 0.130208 + 0.0633198 + 0.0313826)}{10^{27}} = \frac{1}{1000000000000000000000000000} - \frac{i\pi \left\lfloor \frac{\arg(0.509633-x)}{2\pi} \right\rfloor}{500000000000000000000000000} - \frac{\log(x)}{1000000000000000000000000000} + \frac{\sum_{k=1}^{\infty} \frac{(-1)^k (0.509633-x)^k x^{-k}}{k}}{1000000000000000000000000000} \text{ for } x < 0$$

$$\frac{1 - \log(0.284722 + 0.130208 + 0.0633198 + 0.0313826)}{10^{27}} = \frac{1}{1000000000000000000000000000} - \frac{\left\lfloor \frac{\arg(0.509633-z_0)}{2\pi} \right\rfloor \log\left(\frac{1}{z_0}\right)}{1000000000000000000000000000} - \frac{\log(z_0)}{1000000000000000000000000000} - \frac{\left\lfloor \frac{\arg(0.509633-z_0)}{2\pi} \right\rfloor \log(z_0)}{1000000000000000000000000000} + \frac{\sum_{k=1}^{\infty} \frac{(-1)^k (0.509633-z_0)^k z_0^{-k}}{k}}{1000000000000000000000000000}$$

**Integral representation:**

$$\frac{1 - \log(0.284722 + 0.130208 + 0.0633198 + 0.0313826)}{10^{27}} = \frac{1}{1000000000000000000000000000} - \frac{1}{1000000000000000000000000000} \int_1^{0.509633} \frac{1}{t} dt$$

From the multiplication of the results, we obtain:

$$(2e)/((((0.284722222 * 0.130208333 * 0.06331983 * 0.0313826195))))-322+18$$

Where 322 and 18 are Lucas numbers

**Input interpretation:**

$$\frac{2e}{0.284722222 \times 0.130208333 \times 0.06331983 \times 0.0313826195} - 322 + 18$$

**Result:**

73492.44...

73492.44....

**Alternative representation:**

$$\frac{2e}{0.284722 \times 0.130208 \times 0.0633198 \times 0.0313826} - 322 + 18 =$$

$$\frac{2 \exp(z)}{0.284722 \times 0.130208 \times 0.0633198 \times 0.0313826} - 322 + 18 \text{ for } z = 1$$

**Series representations:**

$$\frac{2e}{0.284722 \times 0.130208 \times 0.0633198 \times 0.0313826} - 322 + 18 = -304 + 27148.2 \sum_{k=0}^{\infty} \frac{1}{k!}$$

$$\frac{2e}{0.284722 \times 0.130208 \times 0.0633198 \times 0.0313826} - 322 + 18 = -304 + 13574.1 \sum_{k=0}^{\infty} \frac{1+k}{k!}$$

$$\frac{2e}{0.284722 \times 0.130208 \times 0.0633198 \times 0.0313826} - 322 + 18 =$$

$$-304 + \frac{27148.2 \sum_{k=0}^{\infty} \frac{-1+k+z}{k!}}{z}$$

Thence, we obtain the following mathematical connections:

$$\left( \frac{2e}{0.284722222 \times 0.130208333 \times 0.06331983 \times 0.0313826195} - 322 + 18 \right) = 73492.44 \Rightarrow$$

$$\Rightarrow -3927 + 2 \left( \sqrt[13]{ N \exp \left[ \int d\hat{\sigma} \left( -\frac{1}{4u^2} P_i D P_i \right) \right] |Bp\rangle_{NS} + \int [dX^\mu] \exp \left\{ \int d\hat{\sigma} \left( -\frac{1}{4v^2} D X^\mu D^2 X^\mu \right) \right\} |X^\mu, X^i = 0\rangle_{NS} } \right) =$$

$$-3927 + 2 \sqrt[13]{ 2.2983717437 \times 10^{59} + 2.0823329825883 \times 10^{59} }$$

$$= 73490.8437525.... \Rightarrow$$

$$\Rightarrow \left( A(r) \times \frac{1}{B(r)} \left( -\frac{1}{\phi(r)} \right) \times \frac{1}{e^{\Lambda(r)}} \right) \Rightarrow$$

$$\Rightarrow \left( -0.000029211892 \times \frac{1}{0.0003644621} \left( -\frac{1}{0.0005946833} \right) \times \frac{1}{0.00183393} \right) =$$

$$= 73491.78832548118710549159572042220548025195726563413398700...$$

$$= 73491.7883254... \Rightarrow$$

$$\left( I_{21} \ll \int_{-\infty}^{+\infty} \exp \left( -\left( \frac{t}{H} \right)^2 \right) \left| \sum_{\lambda \ll p^{1-\varepsilon_1}} \frac{\alpha(\lambda)}{\sqrt{\lambda}} B(\lambda) \lambda^{-i(T+t)} \right|^2 dt \ll \right.$$

$$\left. \ll H \left\{ \left( \frac{4}{\varepsilon_2 \log T} \right)^{2r} (\log T) (\log X)^{-2\beta} + (\varepsilon_2^{-2r} (\log T)^{-2r} + \varepsilon_2^{-r} h_1^r (\log T)^{-r} \right) T^{-\varepsilon_1} \right\} \right)$$

$$/(26 \times 4)^2 - 24 = \left( \frac{7.9313976505275 \times 10^8}{(26 \times 4)^2 - 24} \right) = 73493.30662...$$

Mathematical connections with the boundary state corresponding to the NSNS-sector of N Dp-branes in the limit of  $u \rightarrow \infty$ , with the ratio concerning the general asymptotically flat solution of the equations of motion of the p-brane and with the Karatsuba's equation concerning the zeros of a special type of function connected with Dirichlet series.

From the division of the result, we obtain:

$$((((0.284722222 * 1/0.130208333 * 1/0.06331983 * 1/0.0313826195))))-322-3$$

Where 322 and 3 are Lucas numbers

**Input interpretation:**

$$0.284722222 \times \frac{1}{0.130208333} \times \frac{1}{0.06331983} \times \frac{1}{0.0313826195} - 322 - 3$$

**Result:**

775.4077844663391593483052537907051283391936401514108819184...

775.407784466339.... result practically equal to the rest mass of Charged rho meson  
775.11



**Result:**

139.67200...

139.67200.... result very near to the rest mass of Pion meson 139.57

**Alternative representations:**

$$\log\left(\sqrt{\frac{3 \times 2}{\frac{2 \times 2.888 \times 5}{10^{122}}}}\right) = \log_e\left(\sqrt{\frac{6}{\frac{28.88}{10^{122}}}}\right)$$

$$\log\left(\sqrt{\frac{3 \times 2}{\frac{2 \times 2.888 \times 5}{10^{122}}}}\right) = \log(a) \log_a\left(\sqrt{\frac{6}{\frac{28.88}{10^{122}}}}\right)$$

$$\log\left(\sqrt{\frac{3 \times 2}{\frac{2 \times 2.888 \times 5}{10^{122}}}}\right) = -\text{Li}_1\left(1 - \sqrt{\frac{6}{\frac{28.88}{10^{122}}}}\right)$$

**Series representations:**

$$\log\left(\sqrt{\frac{3 \times 2}{\frac{2 \times 2.888 \times 5}{10^{122}}}}\right) = \log\left(\sqrt{2.07756 \times 10^{121}} \sum_{k=0}^{\infty} e^{-279.344 k} \binom{\frac{1}{2}}{k}\right)$$

$$\log\left(\sqrt{\frac{3 \times 2}{\frac{2 \times 2.888 \times 5}{10^{122}}}}\right) =$$

$$\log\left(-1 + \sqrt{2.07756 \times 10^{121}}\right) - \sum_{k=1}^{\infty} \frac{(-1)^k \left(-1 + \sqrt{2.07756 \times 10^{121}}\right)^k}{k}$$

$$\log\left(\sqrt{\frac{3 \times 2}{\frac{2 \times 2.888 \times 5}{10^{122}}}}\right) = \log\left(\sqrt{2.07756 \times 10^{121}} \sum_{k=0}^{\infty} \frac{(-4.81333 \times 10^{-122})^k \left(-\frac{1}{2}\right)_k}{k!}\right)$$

**Integral representations:**

$$\log\left(\sqrt{\frac{3 \times 2}{\frac{2 \times 2.888 \times 5}{10^{122}}}}\right) = \int_1^{\sqrt{2.07756 \times 10^{121}}} \frac{1}{t} dt$$



$$\log\left(\sqrt{\frac{3 \times 2}{\frac{2 \times 2.888 \times 5}{10^{122}}}}\right) = \frac{1}{2 i \pi} \int_{-i \infty + \gamma}^{i \infty + \gamma} \frac{\Gamma(-s)^2 \Gamma(1+s) \left(-1 + \sqrt{2.07756 \times 10^{121}}\right)^{-s}}{\Gamma(1-s)} ds$$

for  $-1 < \gamma < 0$

Now, for:

$$\ln\left(\sqrt{\frac{(d-1)(d-2)}{2V(\phi)}}\right) \geq -\ln(H)$$

We have that, the right hand-side is:

$$-\ln(2.888e-122)$$

**Input:**

$$-\log\left(\frac{2.888}{10^{122}}\right)$$

$\log(x)$  is the natural logarithm

**Result:**

$$279.85482\dots$$

$$279.85482\dots$$

**Alternative representations:**

$$-\log\left(\frac{2.888}{10^{122}}\right) = -\log_e\left(\frac{2.888}{10^{122}}\right)$$

$$-\log\left(\frac{2.888}{10^{122}}\right) = -\log(a) \log_a\left(\frac{2.888}{10^{122}}\right)$$

$$-\log\left(\frac{2.888}{10^{122}}\right) = \left(\text{Li}_1\left(1 - \frac{2.888}{10^{122}}\right)\right) = \text{Li}_1(1)$$

**Series representations:**

$$-\log\left(\frac{2.888}{10^{122}}\right) = -2 i \pi \left[ \frac{\arg(2.888 \times 10^{-122} - x)}{2 \pi} \right] - \log(x) + \sum_{k=1}^{\infty} \frac{(-1)^k (2.888 \times 10^{-122} - x)^k x^{-k}}{k} \quad \text{for } x < 0$$

$$-\log\left(\frac{2.888}{10^{122}}\right) = -\left[\frac{\arg(2.888 \times 10^{-122} - z_0)}{2\pi}\right] \log\left(\frac{1}{z_0}\right) - \log(z_0) - \left[\frac{\arg(2.888 \times 10^{-122} - z_0)}{2\pi}\right] \log(z_0) + \sum_{k=1}^{\infty} \frac{(-1)^k (2.888 \times 10^{-122} - z_0)^k z_0^{-k}}{k}$$

$$-\log\left(\frac{2.888}{10^{122}}\right) = -2i\pi \left[ -\frac{-\pi + \arg\left(\frac{2.888 \times 10^{-122}}{z_0}\right) + \arg(z_0)}{2\pi} \right] - \log(z_0) + \sum_{k=1}^{\infty} \frac{(-1)^k (2.888 \times 10^{-122} - z_0)^k z_0^{-k}}{k}$$

### Integral representation:

$$-\log\left(\frac{2.888}{10^{122}}\right) = -\int_1^{2.888 \times 10^{-122}} \frac{1}{t} dt$$

Furthermore, we have that:

$$1/2((-\ln(2.888e-122)))$$

### Input:

$$\frac{1}{2} \left( -\log\left(\frac{2.888}{10^{122}}\right) \right)$$

$\log(x)$  is the natural logarithm

### Result:

139.92741...

139.92471.... result about equal to the previous: 139.67200

### Alternative representations:

$$-\frac{1}{2} \log\left(\frac{2.888}{10^{122}}\right) = -\frac{1}{2} \log_e\left(\frac{2.888}{10^{122}}\right)$$

$$-\frac{1}{2} \log\left(\frac{2.888}{10^{122}}\right) = -\frac{1}{2} \log(a) \log_a\left(\frac{2.888}{10^{122}}\right)$$

$$-\frac{1}{2} \log\left(\frac{2.888}{10^{122}}\right) = \left(\frac{1}{2} \operatorname{Li}_1\left(1 - \frac{2.888}{10^{122}}\right) = \frac{\operatorname{Li}_1(1)}{2}\right)$$

**Series representations:**

$$-\frac{1}{2} \log\left(\frac{2.888}{10^{122}}\right) = -i \left( \pi \left[ \frac{\arg(2.888 \times 10^{-122} - x)}{2\pi} \right] \right) - \frac{\log(x)}{2} + \frac{1}{2} \sum_{k=1}^{\infty} \frac{(-1)^k (2.888 \times 10^{-122} - x)^k x^{-k}}{k} \quad \text{for } x < 0$$

$$-\frac{1}{2} \log\left(\frac{2.888}{10^{122}}\right) = -\frac{1}{2} \left[ \frac{\arg(2.888 \times 10^{-122} - z_0)}{2\pi} \right] \log\left(\frac{1}{z_0}\right) - \frac{\log(z_0)}{2} - \frac{1}{2} \left[ \frac{\arg(2.888 \times 10^{-122} - z_0)}{2\pi} \right] \log(z_0) + \frac{1}{2} \sum_{k=1}^{\infty} \frac{(-1)^k (2.888 \times 10^{-122} - z_0)^k z_0^{-k}}{k}$$

$$-\frac{1}{2} \log\left(\frac{2.888}{10^{122}}\right) = -i \left( \pi \left[ \frac{-\pi + \arg\left(\frac{2.888 \times 10^{-122}}{z_0}\right) + \arg(z_0)}{2\pi} \right] \right) - \frac{\log(z_0)}{2} + \frac{1}{2} \sum_{k=1}^{\infty} \frac{(-1)^k (2.888 \times 10^{-122} - z_0)^k z_0^{-k}}{k}$$

**Integral representation:**

$$-\frac{1}{2} \log\left(\frac{2.888}{10^{122}}\right) = -\frac{1}{2} \int_1^{2.888 \times 10^{-122}} \frac{1}{t} dt$$

Now, we have that:

$$\left(\frac{|V'|_{\max}}{V_{\max}}\right) > \frac{(\phi_f - \phi)}{4(d-1)(d-2)} \ln \left( \sqrt{\frac{(d-1)(d-2)}{2V(\phi_f)}} \right)^{-2}. \quad (3.40)$$

$$(5-3)/(4(3*2)) * \ln (((\sqrt{((3*2)/(2*5*2.888e-122))}))))^2$$

**Input:**

$$\frac{\frac{5-3}{4(3 \times 2)}}{\log^2 \left( \sqrt{\frac{3 \times 2}{2 \times 5 \times \frac{2.888}{10^{122}}}} \right)}$$

$\log(x)$  is the natural logarithm

**Result:**

$$4.2716934... \times 10^{-6}$$

$$4.2716934... * 10^{-6}$$

**Alternative representations:**

$$\frac{5-3}{\log^2 \left( \sqrt{\frac{3 \times 2}{2 \times 5 \times \frac{2.888}{10^{122}}}} \right) (4(3 \times 2))} = \frac{2}{24 \log_e^2 \left( \sqrt{\frac{6}{\frac{28.88}{10^{122}}}} \right)}$$

$$\frac{5-3}{\log^2 \left( \sqrt{\frac{3 \times 2}{2 \times 5 \times \frac{2.888}{10^{122}}}} \right) (4(3 \times 2))} = \frac{2}{24 \left( \log(a) \log_a \left( \sqrt{\frac{6}{\frac{28.88}{10^{122}}}} \right) \right)^2}$$

$$\frac{5-3}{\log^2 \left( \sqrt{\frac{3 \times 2}{2 \times 5 \times \frac{2.888}{10^{122}}}} \right) (4(3 \times 2))} = \frac{2}{24 \left( -\text{Li}_1 \left( 1 - \sqrt{\frac{6}{\frac{28.88}{10^{122}}}} \right) \right)^2}$$

**Series representations:**

$$\frac{5-3}{\log^2 \left( \sqrt{\frac{3 \times 2}{2 \times 5 \times \frac{2.888}{10^{122}}}} \right) (4(3 \times 2))} = \frac{1}{12 \log^2 \left( \sqrt{2.07756 \times 10^{121}} \sum_{k=0}^{\infty} e^{-279.344 k} \binom{\frac{1}{2}}{k} \right)}$$

$$\frac{5-3}{\log^2 \left( \sqrt{\frac{3 \times 2}{2 \times 5 \times 2.888}} \right) (4(3 \times 2))} = \frac{1}{12 \left( \log \left( -1 + \sqrt{2.07756 \times 10^{121}} \right) - \sum_{k=1}^{\infty} \frac{(-1)^k \left( -1 + \sqrt{2.07756 \times 10^{121}} \right)^{-k}}{k} \right)^2}$$

$$\frac{5-3}{\log^2 \left( \sqrt{\frac{3 \times 2}{2 \times 5 \times 2.888}} \right) (4(3 \times 2))} = \frac{1}{12 \log^2 \left( \sqrt{2.07756 \times 10^{121}} \sum_{k=0}^{\infty} \frac{(-4.81333 \times 10^{-122})^k \left( -\frac{1}{2} \right)^k}{k!} \right)}$$

### Integral representations:

$$\frac{5-3}{\log^2 \left( \sqrt{\frac{3 \times 2}{2 \times 5 \times 2.888}} \right) (4(3 \times 2))} = \frac{1}{12 \left( \int_1^{\sqrt{2.07756 \times 10^{121}}} \frac{1}{t} dt \right)^2}$$

$$\frac{5-3}{\log^2 \left( \sqrt{\frac{3 \times 2}{2 \times 5 \times 2.888}} \right) (4(3 \times 2))} = \frac{i^2 \pi^2}{3 \left( \int_{-i \infty + \gamma}^{i \infty + \gamma} \frac{\Gamma(-s)^2 \Gamma(1+s) \left( -1 + \sqrt{2.07756 \times 10^{121}} \right)^{-s}}{\Gamma(1-s)} ds \right)^2}$$

for  $-1 < \gamma < 0$

For:

$$V_{max} = V(\phi_0) \text{ and } V_{min} = V(\phi_0 + \Delta\phi)$$

from:

$$\left( \frac{|V'|_{max}}{V_{max}} \right)$$

we have that:  $V_{max} = 2.888e-122 * 3$  and  $V'_{max} = 3.70099516176e-127$

From (3.39)

$$\sqrt{\frac{V(\phi_i)\Delta\phi}{4(d-1)(d-2)|V'|_{\max}}}$$

$$\text{sqrt}(\frac{(2.888e-122 * 3 * 2)}{(4*(3*2)* 3.70099516176e-127)})$$

**Input interpretation:**

$$\sqrt{\frac{\frac{2.888}{10^{122}} \times 3 \times 2}{4(3 \times 2) \times \frac{3.70099516176}{10^{127}}}}$$

**Result:**

139.672...

139.672.... result very near to the rest mass of Pion meson 139.57

Note that:

$$\text{sqrt}(\frac{(2.888e-122 * 3 * 2)}{(4*(3*2)* 3.70099516176e-127)})) - 18 + 4$$

where 18 and 4 are Lucas numbers

**Input interpretation:**

$$\sqrt{\frac{\frac{2.888}{10^{122}} \times 3 \times 2}{4(3 \times 2) \times \frac{3.70099516176}{10^{127}}}} - 18 + 4$$

**Result:**

125.672...

125.672.... result very near to the dilaton mass calculated as a type of Higgs boson:

125 GeV for T = 0 (see Appendix)

We have also that:

$$[1/\text{sqrt}(\frac{(2.888e-122 * 3 * 2)}{(4*(3*2)* 3.70099516176e-127)})]^{1/4096}$$

**Input interpretation:**

$$4096 \sqrt{\sqrt{\frac{1}{\frac{\frac{2.888 \times 3 \times 2}{10^{122}}}{4(3 \times 2) \times \frac{3.70099516176}{10^{127}}}}}}$$

**Result:**

0.9987948438...

0.9987948438.... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

$$1 + \sqrt[5]{\sqrt{\varphi^5 \sqrt[4]{5^3} - 1}} - \varphi + 1$$

and to the dilaton value **0.989117352243 =  $\phi$**  (see Appendix)

And again:

$$\text{sqrt}(\frac{(2.888e-122 * 3 * 2)}{(4*(3*2)* 3.70099516176e-127)})^{11+199-7}$$

where 11, 199 and 7 are Lucas numbers

**Input interpretation:**

$$\sqrt{\frac{\frac{2.888 \times 3 \times 2}{10^{122}}}{4(3 \times 2) \times \frac{3.70099516176}{10^{127}}}} \times 11 + 199 - 7$$

**Result:**

1728.39...

1728.39...

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson. Furthermore, 1728 occurs in the algebraic formula for the  $j$ -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

From the formula for the coefficients of the 5<sup>th</sup> order Ramanujan mock theta function  $\psi_1(q)$  – sequence A053261 OEIS,

**Input:**

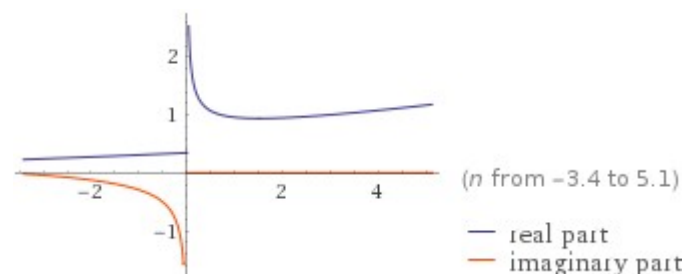
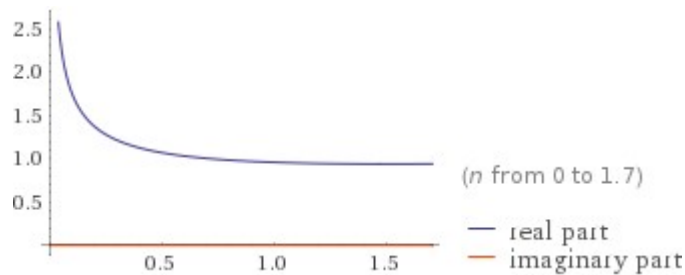
$$\sqrt{\phi} \times \frac{\exp\left(\pi \sqrt{\frac{n}{15}}\right)}{2 \sqrt[4]{5} \sqrt{n}}$$

$\phi$  is the golden ratio

**Exact result:**

$$\frac{e^{(\pi \sqrt{n})/\sqrt{15}} \sqrt{\phi}}{2 \sqrt[4]{5} \sqrt{n}}$$

**Plots:**



**Alternate form:**

$$\frac{\sqrt{1+\sqrt{5}} e^{(\pi \sqrt{n})/\sqrt{15}}}{2 \sqrt{2} \sqrt[4]{5} \sqrt{n}}$$



**Series expansion at n = 0:**

$$\frac{\sqrt{\phi}}{2\sqrt[4]{5}\sqrt{n}} + \frac{\pi\sqrt{\frac{\phi}{3}}}{2\times 5^{3/4}} + \frac{\pi^2\sqrt{n}\sqrt{\phi}}{60\sqrt[4]{5}} + \frac{\pi^3 n\sqrt{\frac{\phi}{3}}}{180\times 5^{3/4}} +$$

$$\frac{\pi^4 n^{3/2}\sqrt{\phi}}{10800\sqrt[4]{5}} + \frac{\pi^5 n^2\sqrt{\frac{\phi}{3}}}{54000\times 5^{3/4}} + \frac{\pi^6 n^{5/2}\sqrt{\phi}}{4860000\sqrt[4]{5}} + O(n^3)$$

(Puiseux series)

**Derivative:**

$$\frac{d}{dn} \left( \frac{\sqrt{\phi} \exp\left(\pi\sqrt{\frac{n}{15}}\right)}{2\sqrt[4]{5}\sqrt{n}} \right) = \frac{\sqrt{\frac{1}{2}(1+\sqrt{5})} e^{(\pi\sqrt{n})/\sqrt{15}} (\sqrt{3}\pi\sqrt{n} - 3\sqrt{5})}{12\times 5^{3/4} n^{3/2}}$$

**Indefinite integral:**

$$\int \frac{\sqrt{\phi} \exp\left(\pi\sqrt{\frac{n}{15}}\right)}{2\sqrt[4]{5}\sqrt{n}} dn = \frac{\sqrt[4]{5} e^{(\pi\sqrt{n})/\sqrt{15}} \sqrt{3}\phi}{\pi} + \text{constant}$$

**Global minimum:**

$$\min\left\{ \frac{\sqrt{\phi} \exp\left(\pi\sqrt{\frac{n}{15}}\right)}{2\sqrt[4]{5}\sqrt{n}} \right\} = \frac{e\pi\sqrt{\frac{\phi}{3}}}{2\times 5^{3/4}} \text{ at } n = \frac{15}{\pi^2}$$

**Limit:**

$$\lim_{n \rightarrow -\infty} \frac{e^{(\sqrt{n}\pi)/\sqrt{15}} \sqrt{\phi}}{2\sqrt[4]{5}\sqrt{n}} = 0$$

**Series representations:**

$$\frac{\sqrt{\phi} \exp\left(\pi\sqrt{\frac{n}{15}}\right)}{2\sqrt[4]{5}\sqrt{n}} = \frac{\sqrt{\frac{1}{2}(1+\sqrt{5})} \sum_{k=0}^{\infty} \frac{15^{-k/2} n^{k/2} \pi^k}{k!}}{2\sqrt[4]{5}\sqrt{n}}$$

$$\frac{\sqrt{\phi} \exp\left(\pi\sqrt{\frac{n}{15}}\right)}{2\sqrt[4]{5}\sqrt{n}} = \frac{\sqrt{\frac{1}{2}(1+\sqrt{5})} \sum_{k=-\infty}^{\infty} I_k\left(\frac{\sqrt{n}\pi}{\sqrt{15}}\right)}{2\sqrt[4]{5}\sqrt{n}}$$

$$\frac{\sqrt{\phi} \exp\left(\pi \sqrt{\frac{n}{15}}\right)}{2 \sqrt[4]{5} \sqrt{n}} = \frac{\sqrt{\frac{1}{2}(1+\sqrt{5})} \sum_{k=0}^{\infty} \frac{15^{-k} n^k \pi^{2k} \left(1+2k+\frac{\sqrt{n}\pi}{\sqrt{15}}\right)}{(1+2k)!}}{2 \sqrt[4]{5} \sqrt{n}}$$

We obtain, for  $n = 99$ :

$$\text{sqrt(golden ratio)} * \exp(\text{Pi} * \text{sqrt}(99/15)) / (2 * 5^{(1/4)} * \text{sqrt}(99))$$

**Input:**

$$\sqrt{\phi} \times \frac{\exp\left(\pi \sqrt{\frac{99}{15}}\right)}{2 \sqrt[4]{5} \sqrt{99}}$$

$\phi$  is the golden ratio

**Exact result:**

$$\frac{e^{\sqrt{33/5} \pi} \sqrt{\frac{\phi}{11}}}{6 \sqrt[4]{5}}$$

**Decimal approximation:**

136.7886439048612082042916291006292653937420527314549866843...

136.7886439...  $\approx$  138, in according to the OEIS list (see above) and very near to the rest mass of Pion meson 134.9766

**Property:**

$$\frac{e^{\sqrt{33/5} \pi} \sqrt{\frac{\phi}{11}}}{6 \sqrt[4]{5}} \text{ is a transcendental number}$$

**Alternate forms:**

$$\frac{1}{6} \sqrt{\frac{1}{110} (5 + \sqrt{5})} e^{\sqrt{33/5} \pi}$$

$$\frac{\sqrt{\frac{1}{22} (1 + \sqrt{5})} e^{\sqrt{33/5} \pi}}{6 \sqrt[4]{5}}$$

### Series representations:

$$\frac{\sqrt{\phi} \exp\left(\pi \sqrt{\frac{\infty}{15}}\right)}{2 \sqrt[4]{5} \sqrt{99}} = \frac{\exp\left(\pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{33}{5} - z_0\right)^k z_0^{-k}}{k!}\right) \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (\phi - z_0)^k z_0^{-k}}{k!}}{2 \sqrt[4]{5} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (99 - z_0)^k z_0^{-k}}{k!}}$$

for not  $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

$$\frac{\sqrt{\phi} \exp\left(\pi \sqrt{\frac{\infty}{15}}\right)}{2 \sqrt[4]{5} \sqrt{99}} = \frac{\left(\exp\left(i\pi \left\lfloor \frac{\arg(\phi - x)}{2\pi} \right\rfloor\right) \exp\left(\pi \exp\left(i\pi \left\lfloor \frac{\arg\left(\frac{33}{5} - x\right)}{2\pi} \right\rfloor\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{33}{5} - x\right)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}\right) \sum_{k=0}^{\infty} \frac{(-1)^k (\phi - x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}\right)}{\left(2 \sqrt[4]{5} \exp\left(i\pi \left\lfloor \frac{\arg(99 - x)}{2\pi} \right\rfloor\right) \sum_{k=0}^{\infty} \frac{(-1)^k (99 - x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}\right)} \text{ for } (x \in \mathbb{R} \text{ and } x < 0)$$

$$\frac{\sqrt{\phi} \exp\left(\pi \sqrt{\frac{\infty}{15}}\right)}{2 \sqrt[4]{5} \sqrt{99}} = \frac{\left(\exp\left(\pi \left(\frac{1}{z_0}\right)^{1/2 \lfloor \arg\left(\frac{33}{5} - z_0\right) / (2\pi) \rfloor} z_0^{1/2 (1 + \lfloor \arg\left(\frac{33}{5} - z_0\right) / (2\pi) \rfloor)} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{33}{5} - z_0\right)^k z_0^{-k}}{k!}\right) \left(\frac{1}{z_0}\right)^{-1/2 \lfloor \arg(99 - z_0) / (2\pi) \rfloor + 1/2 \lfloor \arg(\phi - z_0) / (2\pi) \rfloor} z_0^{-1/2 \lfloor \arg(99 - z_0) / (2\pi) \rfloor + 1/2 \lfloor \arg(\phi - z_0) / (2\pi) \rfloor} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (\phi - z_0)^k z_0^{-k}}{k!}\right)}{\left(2 \sqrt[4]{5} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (99 - z_0)^k z_0^{-k}}{k!}\right)}$$

And , for n = 99.58:

$$\text{sqrt(golden ratio) * exp(Pi*sqrt(99.58/15)) / (2*5^(1/4)*sqrt(99.58))}$$

**Input:**

$$\sqrt{\phi} \times \frac{\exp\left(\pi \sqrt{\frac{99.58}{15}}\right)}{2 \sqrt[4]{5} \sqrt{99.58}}$$

$\phi$  is the golden ratio

**Result:**

139.648...

139.648... result very near to the rest mass of Pion meson 139.57

**Series representations:**

$$\frac{\sqrt{\phi} \exp\left(\pi \sqrt{\frac{99.58}{15}}\right)}{2 \sqrt[4]{5} \sqrt{99.58}} = \frac{\exp\left(\pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (6.63867 - z_0)^k z_0^{-k}}{k!}\right) \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (\phi - z_0)^k z_0^{-k}}{k!}}{2 \sqrt[4]{5} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (99.58 - z_0)^k z_0^{-k}}{k!}}$$

for not  $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$ 

$$\frac{\sqrt{\phi} \exp\left(\pi \sqrt{\frac{99.58}{15}}\right)}{2 \sqrt[4]{5} \sqrt{99.58}} = \left( \exp\left(i \pi \left\lfloor \frac{\arg(\phi - x)}{2 \pi} \right\rfloor\right) \exp\left(\pi \exp\left(i \pi \left\lfloor \frac{\arg(6.63867 - x)}{2 \pi} \right\rfloor\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (6.63867 - x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}\right) \sum_{k=0}^{\infty} \frac{(-1)^k (\phi - x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) / \left( 2 \sqrt[4]{5} \exp\left(i \pi \left\lfloor \frac{\arg(99.58 - x)}{2 \pi} \right\rfloor\right) \sum_{k=0}^{\infty} \frac{(-1)^k (99.58 - x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right)$$

for  $(x \in \mathbb{R} \text{ and } x < 0)$



$$\log\left(\sqrt{\frac{3 \times 2}{\frac{2 \times 2.888}{10^{122}}}}\right) = \log_e\left(\sqrt{\frac{6}{\frac{5.776}{10^{122}}}}\right)$$

$$\log\left(\sqrt{\frac{3 \times 2}{\frac{2 \times 2.888}{10^{122}}}}\right) = \log(a) \log_a\left(\sqrt{\frac{6}{\frac{5.776}{10^{122}}}}\right)$$

$$\log\left(\sqrt{\frac{3 \times 2}{\frac{2 \times 2.888}{10^{122}}}}\right) = -\text{Li}_1\left(1 - \sqrt{\frac{6}{\frac{5.776}{10^{122}}}}\right)$$

### Series representations:

$$\log\left(\sqrt{\frac{3 \times 2}{\frac{2 \times 2.888}{10^{122}}}}\right) = \log\left(\sqrt{1.03878 \times 10^{122} \sum_{k=0}^{\infty} e^{-280.953 k} \binom{\frac{1}{2}}{k}}\right)$$

$$\log\left(\sqrt{\frac{3 \times 2}{\frac{2 \times 2.888}{10^{122}}}}\right) = \log\left(-1 + \sqrt{1.03878 \times 10^{122}}\right) - \sum_{k=1}^{\infty} \frac{(-1)^k \left(-1 + \sqrt{1.03878 \times 10^{122}}\right)^k}{k}$$

$$\log\left(\sqrt{\frac{3 \times 2}{\frac{2 \times 2.888}{10^{122}}}}\right) = \log\left(\sqrt{1.03878 \times 10^{122} \sum_{k=0}^{\infty} \frac{(-9.62667 \times 10^{-123})^k \binom{-\frac{1}{2}}{k}}{k!}}\right)$$

### Integral representations:

$$\log\left(\sqrt{\frac{3 \times 2}{\frac{2 \times 2.888}{10^{122}}}}\right) = \int_1^{\sqrt{1.03878 \times 10^{122}}} \frac{1}{t} dt$$

$$\log\left(\sqrt{\frac{3 \times 2}{\frac{2 \times 2.888}{10^{122}}}}\right) = \frac{1}{2 i \pi} \int_{-i \infty + \gamma}^{i \infty + \gamma} \frac{\Gamma(-s)^2 \Gamma(1+s) \left(-1 + \sqrt{1.03878 \times 10^{122}}\right)^{-s}}{\Gamma(1-s)} ds$$

for  $-1 < \gamma < 0$

And, adding the golden ratio conjugate, we obtain:

$\ln \left( \left( \sqrt{\frac{3 \times 2}{2 \times \frac{2.888}{10^{122}}}} \right) - \frac{1}{\phi} \right)$

**Input:**

$$\log \left( \sqrt{\frac{3 \times 2}{2 \times \frac{2.888}{10^{122}}}} \right) - \frac{1}{\phi}$$

$\log(x)$  is the natural logarithm

$\phi$  is the golden ratio

**Result:**

139.85868...

139.85868... result very near to the rest mass of Pion meson 139.57

**Alternative representations:**

$$\log \left( \sqrt{\frac{3 \times 2}{2 \times \frac{2.888}{10^{122}}}} \right) - \frac{1}{\phi} = \log_e \left( \sqrt{\frac{6}{\frac{5.776}{10^{122}}}} \right) - \frac{1}{\phi}$$

$$\log \left( \sqrt{\frac{3 \times 2}{2 \times \frac{2.888}{10^{122}}}} \right) - \frac{1}{\phi} = \log(a) \log_a \left( \sqrt{\frac{6}{\frac{5.776}{10^{122}}}} \right) - \frac{1}{\phi}$$

$$\log \left( \sqrt{\frac{3 \times 2}{2 \times \frac{2.888}{10^{122}}}} \right) - \frac{1}{\phi} = -\text{Li}_1 \left( 1 - \sqrt{\frac{6}{\frac{5.776}{10^{122}}}} \right) - \frac{1}{\phi}$$

**Series representations:**

$$\log \left( \sqrt{\frac{3 \times 2}{2 \times \frac{2.888}{10^{122}}}} \right) - \frac{1}{\phi} = -\frac{1}{\phi} + \log \left( \sqrt{1.03878 \times 10^{122} \sum_{k=0}^{\infty} e^{-280.953 k} \left( \frac{1}{2} \right)^k} \right)$$

$$\log\left(\sqrt{\frac{3 \times 2}{\frac{2 \times 2.888}{10^{122}}}}\right) - \frac{1}{\phi} =$$

$$-\frac{1}{\phi} + \log\left(-1 + \sqrt{1.03878 \times 10^{122}}\right) - \sum_{k=1}^{\infty} \frac{(-1)^k \left(-1 + \sqrt{1.03878 \times 10^{122}}\right)^{-k}}{k}$$

$$\log\left(\sqrt{\frac{3 \times 2}{\frac{2 \times 2.888}{10^{122}}}}\right) - \frac{1}{\phi} = -\frac{1}{\phi} + \log\left(\sqrt{1.03878 \times 10^{122}} \sum_{k=0}^{\infty} \frac{(-9.62667 \times 10^{-123})^k \left(-\frac{1}{2}\right)_k}{k!}\right)$$

**Integral representations:**

$$\log\left(\sqrt{\frac{3 \times 2}{\frac{2 \times 2.888}{10^{122}}}}\right) - \frac{1}{\phi} = -\frac{1}{\phi} + \int_1^{\sqrt{1.03878 \times 10^{122}}} \frac{1}{t} dt$$

Now, we have:

$$\frac{|V''(\phi_0)|}{V(\phi_0)} \geq \frac{2}{d-2} \ln\left(\sqrt{\frac{(d-1)(d-2)}{2V}}\right)^{-1}. \quad (4.5)$$

$$\ln(\sqrt{\frac{6}{2 \times 2.888 \times 10^{-122}}})^{-1}$$

**Input:**

$$\log\left(\frac{1}{\sqrt{\frac{6}{\frac{2 \times 2.888}{10^{122}}}}}\right)$$

$\log(x)$  is the natural logarithm

**Result:**

-140.47671...

-140.47671...



### Alternative representations:

$$\log \left( \frac{1}{\sqrt{\frac{6}{\frac{2 \cdot 2.888}{10^{122}}}}} \right) = \log_e \left( \frac{1}{\sqrt{\frac{6}{\frac{5.776}{10^{122}}}}} \right)$$

$$\log \left( \frac{1}{\sqrt{\frac{6}{\frac{2 \cdot 2.888}{10^{122}}}}} \right) = \log(a) \log_a \left( \frac{1}{\sqrt{\frac{6}{\frac{5.776}{10^{122}}}}} \right)$$

$$\log \left( \frac{1}{\sqrt{\frac{6}{\frac{2 \cdot 2.888}{10^{122}}}}} \right) = \left( -\text{Li}_1 \left( 1 - \frac{1}{\sqrt{\frac{6}{\frac{5.776}{10^{122}}}}} \right) = -\text{Li}_1 \left( 1 - \frac{1}{\sqrt{1.03878 \times 10^{122}}} \right) \right)$$

### Series representations:

$$\log \left( \frac{1}{\sqrt{\frac{6}{\frac{2 \cdot 2.888}{10^{122}}}}} \right) = \log \left( \frac{1}{\sqrt{1.03878 \times 10^{122}} \sum_{k=0}^{\infty} e^{-280.953 k} \binom{\frac{1}{2}}{k}} \right)$$

$$\log \left( \frac{1}{\sqrt{\frac{6}{\frac{2 \cdot 2.888}{10^{122}}}}} \right) = \log \left( \frac{1}{\sqrt{1.03878 \times 10^{122}} \sum_{k=0}^{\infty} \frac{(-9.62667 \times 10^{-123})^k \binom{-\frac{1}{2}}{k}}{k!}} \right)$$

$$\log\left(\frac{1}{\sqrt{\frac{6}{\frac{2 \cdot 2.888}{10^{122}}}}}\right) = 2i\pi \left[ \frac{\arg\left(-x + \frac{1}{\sqrt{1.03878 \times 10^{122}}}\right)}{2\pi} \right] +$$

$$\log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k x^{-k} \left(-x + \frac{1}{\sqrt{1.03878 \times 10^{122}}}\right)^k}{k} \quad \text{for } x < 0$$

### Integral representation:

$$\log\left(\frac{1}{\sqrt{\frac{6}{\frac{2 \cdot 2.888}{10^{122}}}}}\right) = \int_1^{\frac{1}{\sqrt{1.03878 \times 10^{122}}}} \frac{1}{t} dt$$

And again:

$$\ln\left(\left(\left(\left(\sqrt{6/(2 \cdot 2.888 \times 10^{-122})} - 1\right)\right)\right)\right) \times 18 + 843 - 47 + 4$$

where 18, 843, 47 and 4 are Lucas numbers

### Input:

$$\log\left(\frac{1}{\sqrt{\frac{6}{\frac{2 \cdot 2.888}{10^{122}}}}}\right) \times 18 + 843 - 47 + 4$$

$\log(x)$  is the natural logarithm

### Result:

-1728.5809...

-1728.5809...

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson. Furthermore, 1728 occurs in the algebraic formula for the  $j$ -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–

Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

**Alternative representations:**

$$\log \left( \frac{1}{\sqrt{\frac{6}{\frac{2 \cdot 2.888}{10^{122}}}}} \right) 18 + 843 - 47 + 4 = 800 + 18 \log_e \left( \frac{1}{\sqrt{\frac{6}{\frac{5.776}{10^{122}}}}} \right)$$

$$\log \left( \frac{1}{\sqrt{\frac{6}{\frac{2 \cdot 2.888}{10^{122}}}}} \right) 18 + 843 - 47 + 4 = 800 + 18 \log(a) \log_a \left( \frac{1}{\sqrt{\frac{6}{\frac{5.776}{10^{122}}}}} \right)$$

$$\log \left( \frac{1}{\sqrt{\frac{6}{\frac{2 \cdot 2.888}{10^{122}}}}} \right) 18 + 843 - 47 + 4 = \left( 800 - 18 \operatorname{Li}_1 \left( 1 - \frac{1}{\sqrt{\frac{6}{\frac{5.776}{10^{122}}}}} \right) = 800 - 18 \operatorname{Li}_1 \left( 1 - \frac{1}{\sqrt{1.03878 \times 10^{122}}} \right) \right)$$

**Series representations:**



**Result:**

0.998793442894...

0.998793442894... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

and to the dilaton value **0.989117352243 =  $\phi$**  (see Appendix)

$\ln(\frac{1}{\sqrt{\frac{6}{2 \times 2.888 \times 10^{-122}}}}) + 1/\text{golden ratio}$

**Input:**

$$\log \left( \frac{1}{\sqrt{\frac{6}{2 \times \frac{2.888}{10^{122}}}}} \right) + \frac{1}{\phi}$$

$\log(x)$  is the natural logarithm

$\phi$  is the golden ratio

**Result:**

-139.85868...

-139.85868... result very near to the rest mass of Pion meson 139.57 with minus sign (can be the anti-particle of the Pion)

**Alternative representations:**

$$\log \left( \frac{1}{\sqrt{\frac{6}{\frac{2 \times 2.888}{10^{122}}}}} \right) + \frac{1}{\phi} = \log_e \left( \frac{1}{\sqrt{\frac{6}{\frac{5.776}{10^{122}}}}} \right) + \frac{1}{\phi}$$

$$\log \left( \frac{1}{\sqrt{\frac{6}{\frac{2 \times 2.888}{10^{122}}}}} \right) + \frac{1}{\phi} = \log(a) \log_a \left( \frac{1}{\sqrt{\frac{6}{\frac{5.776}{10^{122}}}}} \right) + \frac{1}{\phi}$$

$$\log \left( \frac{1}{\sqrt{\frac{6}{\frac{2 \times 2.888}{10^{122}}}}} \right) + \frac{1}{\phi} = \left( -\text{Li}_1 \left( 1 - \frac{1}{\sqrt{\frac{6}{\frac{5.776}{10^{122}}}}} \right) + \frac{1}{\phi} = \frac{1}{\phi} - \text{Li}_1 \left( 1 - \frac{1}{\sqrt{1.03878 \times 10^{122}}} \right) \right)$$

### Series representations:

$$\log \left( \frac{1}{\sqrt{\frac{6}{\frac{2 \times 2.888}{10^{122}}}}} \right) + \frac{1}{\phi} = \frac{1}{\phi} + \log \left( \frac{1}{\sqrt{1.03878 \times 10^{122}} \sum_{k=0}^{\infty} e^{-280.953 k} \binom{\frac{1}{2}}{k}} \right)$$

$$\log \left( \frac{1}{\sqrt{\frac{6}{\frac{2 \times 2.888}{10^{122}}}}} \right) + \frac{1}{\phi} = \frac{1}{\phi} + \log \left( \frac{1}{\sqrt{1.03878 \times 10^{122}} \sum_{k=0}^{\infty} \frac{(-0.62667 \times 10^{-123})^k \binom{-\frac{1}{2}}{k}}{k!}} \right)$$

$$\log \left( \frac{1}{\sqrt{\frac{6}{\frac{2 \cdot 2.888}{10^{122}}}}} \right) + \frac{1}{\phi} = \frac{1}{\phi} + 2i\pi \left[ \frac{\arg \left( -x + \frac{1}{\sqrt{1.03878 \times 10^{122}}} \right)}{2\pi} \right] +$$

$$\log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k x^{-k} \left( -x + \frac{1}{\sqrt{1.03878 \times 10^{122}}} \right)^k}{k} \quad \text{for } x < 0$$

### Integral representation:

$$\log \left( \frac{1}{\sqrt{\frac{6}{\frac{2 \cdot 2.888}{10^{122}}}}} \right) + \frac{1}{\phi} = \frac{1}{\phi} + \int_1^{\frac{1}{\sqrt{1.03878 \times 10^{122}}}} \frac{1}{t} dt$$

and  $W$  is the superpotential. It was argued that by fine-tuning the coefficients  $A$ ,  $B$ , and  $C$ , we can have a scenario in which the above potential has a positive local minimum with the energy of the order of  $\Lambda \approx \mathcal{V}^{-3}$  and lifetime of the order of  $\epsilon^{\frac{1}{\Lambda}}$  [23]. This lifetime is similar to the lifetime computed in the KKLТ scenario which we studied in the previous subsection and is likewise in contradiction with TCC.

Note that at large volumes, the potential (5.2) decays like  $\exp\left(-3\sqrt{\frac{3}{2}}\hat{\phi}\right)$  where  $\hat{\phi} = \sqrt{2/3}\ln(\mathcal{V})$  is the canonical radial modulus. This decay rate is greater than  $\sqrt{2/3}$  and hence is consistent with the inequality (3.36) which was a consequence of TCC.

For

$$2.888e-122 = 1/x^3$$

**Input:**

$$\frac{2.888}{10^{122}} = \frac{1}{x^3}$$

**Result:**

$$2.888 \times 10^{-122} = \frac{1}{x^3}$$

**Alternate form assuming x is positive:**

$$3.06807 \times 10^{-41} x = 1. \quad (\text{for } x \neq 0)$$

**Real solution:**

$$x = 32593745516583443207701121536522130030592$$

**Complex solutions:**

$$x = -1.62969 \times 10^{40} - 2.8227 \times 10^{40} i$$

$$x = -1.62969 \times 10^{40} + 2.8227 \times 10^{40} i$$

**Integer solution:**

$$x = 32593745516583443207701121536522130030592$$

And:

$$(\sqrt{2/3}) \ln 32593745516583443207701121536522130030592$$

**Input:**

$$\sqrt{\frac{2}{3}} \log(32593745516583443207701121536522130030592)$$

$\log(x)$  is the natural logarithm

**Decimal approximation:**

$$76.16683377936614622583425231971652416990963931274255888909\dots$$

$$76.16683377\dots$$

**Property:**

$$\sqrt{\frac{2}{3}} \log(32593745516583443207701121536522130030592)$$

is a transcendental number

**Alternate forms:**

$$\sqrt{\frac{2}{3}} (86 \log(2) + \log(421264287215703))$$

$$\frac{1}{3} (86 \sqrt{6} \log(2) + 3 \sqrt{6} \log(3) + \sqrt{6} \log(17) + \sqrt{6} (\log(193) + \log(4755373669)))$$

$$86 \sqrt{\frac{2}{3}} \log(2) + \sqrt{6} \log(3) + \sqrt{\frac{2}{3}} \log(17) + \sqrt{\frac{2}{3}} \log(193) + \sqrt{\frac{2}{3}} \log(4755373669)$$



**Alternative representations:**

$$\sqrt{\frac{2}{3}} \log(32593745516583443207701121536522130030592) = \log_e(32593745516583443207701121536522130030592) \sqrt{\frac{2}{3}}$$

$$\sqrt{\frac{2}{3}} \log(32593745516583443207701121536522130030592) = \log(a) \log_a(32593745516583443207701121536522130030592) \sqrt{\frac{2}{3}}$$

$$\sqrt{\frac{2}{3}} \log(32593745516583443207701121536522130030592) = -\text{Li}_1(-32593745516583443207701121536522130030591) \sqrt{\frac{2}{3}}$$

**Series representations:**

$$\sqrt{\frac{2}{3}} \log(32593745516583443207701121536522130030592) = \sqrt{\frac{2}{3}} \log(32593745516583443207701121536522130030591) - \sqrt{\frac{2}{3}} \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{32593745516583443207701121536522130030591}\right)^k}{k}$$

$$\sqrt{\frac{2}{3}} \log(32593745516583443207701121536522130030592) = 2i \sqrt{\frac{2}{3}} \pi \left\lfloor \frac{\arg(32593745516583443207701121536522130030592 - x)}{2\pi} \right\rfloor + \sqrt{\frac{2}{3}} \log(x) - \sqrt{\frac{2}{3}} \sum_{k=1}^{\infty} \frac{(-1)^k (32593745516583443207701121536522130030592 - x)^k x^{-k}}{k}$$

for  $x < 0$

$$\sqrt{\frac{2}{3}} \log(32593745516583443207701121536522130030592) =$$

$$\sqrt{\frac{2}{3}} \left( \log(z_0) + \left[ \frac{\arg(32593745516583443207701121536522130030592 - z_0)}{2\pi} \right] \right)$$

$$\left( \log\left(\frac{1}{z_0}\right) + \log(z_0) \right) -$$

$$\sum_{k=1}^{\infty} \frac{(-1)^k (32593745516583443207701121536522130030592 - z_0)^k z_0^{-k}}{k}$$

And:

$$\exp((-3\sqrt{3/2}) * 76.166833779366)$$

**Input interpretation:**

$$\exp\left(-3 \sqrt{\frac{3}{2}} \times 76.166833779366\right)$$

**Result:**

$$2.888000000000... \times 10^{-122}$$

$$2.888... * 10^{-122}$$

From which:

$$\text{colog}(\exp((-3\sqrt{3/2}) * 76.166833779366)) - \pi - 1/\text{golden ratio}$$

**Input interpretation:**

$$-\log\left(\exp\left(-3 \sqrt{\frac{3}{2}} \times 76.166833779366\right)\right) - \pi - \frac{1}{\phi}$$

$\log(x)$  is the natural logarithm

$\phi$  is the golden ratio

**Result:**

$$276.09519048190...$$

$$276.09519...$$

**Alternative representations:**

$$-\log\left(\exp\left(-3\sqrt{\frac{3}{2}} \cdot 76.1668337793660000\right)\right) - \pi - \frac{1}{\phi} =$$

$$-\pi - \log_e\left(\exp\left(-228.500501338098000 \sqrt{\frac{3}{2}}\right)\right) - \frac{1}{\phi}$$

$$-\log\left(\exp\left(-3\sqrt{\frac{3}{2}} \cdot 76.1668337793660000\right)\right) - \pi - \frac{1}{\phi} =$$

$$-\pi - \log(\alpha) \log_\alpha\left(\exp\left(-228.500501338098000 \sqrt{\frac{3}{2}}\right)\right) - \frac{1}{\phi}$$

### Series representation:

$$-\log\left(\exp\left(-3\sqrt{\frac{3}{2}} \cdot 76.1668337793660000\right)\right) - \pi - \frac{1}{\phi} =$$

$$-\frac{1}{\phi} - \pi + \sum_{k=1}^{\infty} \frac{(-1)^k \left(-1 + \exp\left(-228.500501338098000 \sqrt{\frac{3}{2}}\right)\right)^k}{k}$$

$\left(\left(\left(\frac{1}{64} * \left(\left(\log\left(\left(\exp\left(-3\sqrt{\frac{3}{2}} * 76.166833779366\right)\right)\right)\right) - \pi - \frac{1}{\phi}\right)\right)\right) - (11 - 2) * \frac{1}{10^3}\right)^{\frac{1}{3}}$

Where 11 and 2 are Lucas numbers

### Input interpretation:

$$\sqrt[3]{\frac{1}{64} \left( -\log\left(\exp\left(-3\sqrt{\frac{3}{2}} \times 76.166833779366\right)\right) - \pi - \frac{1}{\phi} \right) - (11 - 2) \times \frac{1}{10^3}}$$

$\log(x)$  is the natural logarithm

$\phi$  is the golden ratio

### Result:

1.6188946247096...

1.6188946247096... result that is a very good approximation to the value of the golden ratio 1,618033988749...

### Alternative representations:

$$\sqrt[3]{\frac{1}{64} \left( -\log \left( \exp \left( -3 \sqrt{\frac{3}{2}} \cdot 76.1668337793660000 \right) \right) - \pi - \frac{1}{\phi} \right) - \frac{11-2}{10^3} =$$

$$-\frac{9}{10^3} + \sqrt[3]{\frac{1}{64} \left( -\pi - \log_e \left( \exp \left( -228.500501338098000 \sqrt{\frac{3}{2}} \right) \right) - \frac{1}{\phi} \right)}$$

$$\sqrt[3]{\frac{1}{64} \left( -\log \left( \exp \left( -3 \sqrt{\frac{3}{2}} \cdot 76.1668337793660000 \right) \right) - \pi - \frac{1}{\phi} \right) - \frac{11-2}{10^3} =$$

$$-\frac{9}{10^3} + \sqrt[3]{\frac{1}{64} \left( -\pi - \log(a) \log_a \left( \exp \left( -228.500501338098000 \sqrt{\frac{3}{2}} \right) \right) - \frac{1}{\phi} \right)}$$

**Series representation:**

$$\sqrt[3]{\frac{1}{64} \left( -\log \left( \exp \left( -3 \sqrt{\frac{3}{2}} \cdot 76.1668337793660000 \right) \right) - \pi - \frac{1}{\phi} \right) - \frac{11-2}{10^3} =$$

$$-\frac{9}{1000} + \frac{1}{4} \sqrt[3]{-\frac{1 + \phi \pi - \phi \sum_{k=1}^{\infty} \frac{(-1)^k \left( -1 + \exp \left( -228.500501338098000 \sqrt{\frac{3}{2}} \right) \right)^k}{k}}{\phi}}$$

**Integral representation:**

$$\sqrt[3]{\frac{1}{64} \left( -\log \left( \exp \left( -3 \sqrt{\frac{3}{2}} \cdot 76.1668337793660000 \right) \right) - \pi - \frac{1}{\phi} \right) - \frac{11-2}{10^3} =$$

$$-\frac{9}{1000} + \frac{1}{4} \sqrt[3]{-\frac{1 + \phi \pi + \phi \int_1^{\infty} \frac{\exp \left( -228.500501338098000 \sqrt{\frac{3}{2}} \right)}{t} dt}{\phi}}$$

$1/10^{27} * ((((((1/64 * (((colog(((exp((-3sqrt(3/2)*76.166833779366)))))-Pi-1/golden ratio))))))^{1/3} + (47-2)*1/10^3))$

**Input interpretation:**

$$\frac{1}{10^{27}} \left( \sqrt[3]{\frac{1}{64} \left( -\log \left( \exp \left( -3 \sqrt{\frac{3}{2}} \times 76.166833779366 \right) \right) - \pi - \frac{1}{\phi} \right)} + (47-2) \times \frac{1}{10^3} \right)$$



From:

**Conditions for (No) Eternal Inflation**

*Tom Rudelius*

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- arXiv:1905.05198v3 [hep-th] 13 Aug 2019

We have that:

Next, we consider models of inflection point inflation, which have a potential of the form

$$V = V_0 + \alpha\phi + \frac{1}{6}\lambda\phi^3 + \dots \quad (4.13)$$

Inflation occurs near the inflection point at  $\phi = 0$ . The phenomenology of these models was considered in [50, 53], and they arise with regularity in models of random inflation [51, 52] as well as string theory models of D-brane inflation on the conifold [54]. Like with quadratic hilltop models, inflection point models can be either small-field or large-field. But unlike quadratic hilltop inflation, the spectral index in the small-field case is actually in good agreement with observation. The spectral index given by [50]

$$n_s \approx 1 - \frac{4\pi}{N_{\text{tot}}} \cot\left(\frac{\pi N_e}{N_{\text{tot}}}\right), \quad (4.14)$$

where  $N_e \lesssim 60$  and  $N_{\text{tot}}$  is the total number of  $e$ -folds, which is given by

$$N_{\text{tot}} \approx \pi\sqrt{2}\frac{V_0}{\sqrt{\alpha\lambda}}. \quad (4.15)$$

The range of this function is complementary to the one for quadratic hilltop inflation (4.10): here,  $n_s > 0.933$  for  $N_e = 60$ , with  $n_s$  approaching the lower bound 0.933 as  $N_{\text{tot}} \rightarrow \infty$  and growing larger as  $N_{\text{tot}} \rightarrow 60$ . Agreement with experiment occurs for  $120 \lesssim N_{\text{tot}} \lesssim 200$ , allowing  $N_e$  to vary between 50 and 60.

Phenomenologically-viable, small-field models of inflection point inflation are not eternal. This follows from equation (4.2):

$$\frac{(V'(\phi_*))^2}{V_*^3} \approx \frac{5 \times 10^{10}}{24\pi^2} \frac{1}{M_{\text{Pl}}^6}, \quad (4.16)$$

with  $\phi_*$  close to the inflection point. This clearly violates (3.9), which means that the potential is not sufficiently flat at the inflection point to generate eternal inflation.

On the other hand, large-field inflection point models can be eternal provided that  $\phi_*$ , the field value  $60$   $e$ -folds before the end of inflation, is a sufficiently-large distance away from the inflection point,  $\phi_{\text{inf}}$ . In this situation, the potential might be much flatter at the inflection point than it is at  $\phi_*$ , so (3.9) is obeyed at the former while (4.2) is satisfied at the latter.

As a concrete example, consider an inflection point model (4.13) with  $V_0 = 2.8 \times 10^{-10} M_{\text{Pl}}^4$ ,

$\alpha = 1.6 \times 10^{-26} M_{\text{Pl}}^3$ ,  $\lambda = 1.0 \times 10^{-12} M_{\text{Pl}}$ . This gives  $r_* \approx .009$ ,  $n_{s,*} \approx 0.964$ ,  $A_s \approx 2 \times 10^{-9}$ , which is in good agreement with observation. Note that a naïve application of (4.14) would indicate  $n_{s,*} = 0.933$ , which is not correct. This discrepancy is due to the fact that  $\phi_*$  is located a super-Planckian distance away from the inflection point (to be precise,  $\phi_* - \phi_{\text{inf}} \approx 4.3 M_p$ ), and the field rolls a distance of larger than  $5 M_p$  during its last  $60$   $e$ -folds. The approximation used to compute (4.14), namely, that the potential  $V(\phi)$  is roughly constant during slow-roll, is not valid in this large-field context. As a result, we have  $V'(\phi_{\text{inf}})/V^{3/2}(\phi_{\text{inf}}) \approx 3 \times 10^{-10} M_{\text{Pl}}^{-3}$  and  $V'''(\phi_{\text{inf}})/V^{1/2}(\phi_{\text{inf}}) \approx 6 \times 10^{-8} M_{\text{Pl}}^{-1}$ , so (3.9) and (3.27) are both satisfied by many orders of magnitude, and eternal inflation occurs at the inflection point. Similar examples of eternal inflation in inflection point models can be found in [55].





**Result:**

$$5.831239... \times 10^{46}$$

$$5.831239... * 10^{46} = N_{\text{tot}}$$

From

$$n_s \approx 1 - \frac{4\pi}{N_{\text{tot}}} \cot\left(\frac{\pi N_e}{N_{\text{tot}}}\right) \quad (4.14)$$

We obtain, for  $N_e = 55, 60$  and  $64$ :

$$1 - 4\pi / (5.831239e+46) \cot(\pi * 55 / 5.831239e+46)$$

**Input interpretation:**

$$1 - \left(4 \times \frac{\pi}{5.831239 \times 10^{46}}\right) \cot\left(\pi \times \frac{55}{5.831239 \times 10^{46}}\right)$$

$\cot(x)$  is the cotangent function

**Result:**

$$0.92727273...$$

$$0.92727273...$$

$$1 - 4\pi / (5.831239e+46) \cot(\pi * 60 / 5.831239e+46)$$

**Input interpretation:**

$$1 - \left(4 \times \frac{\pi}{5.831239 \times 10^{46}}\right) \cot\left(\pi \times \frac{60}{5.831239 \times 10^{46}}\right)$$

$\cot(x)$  is the cotangent function

**Result:**

$$0.93333333...$$

$$0.93333333...$$

$$1 - 4\pi / (5.831239e+46) \cot(\pi * 64 / 5.831239e+46)$$

**Input interpretation:**

$$1 - \left( 4 \times \frac{\pi}{5.831239 \times 10^{46}} \right) \cot \left( \pi \times \frac{64}{5.831239 \times 10^{46}} \right)$$

$\cot(x)$  is the cotangent function

**Result:**

0.93750000...

0.93750000... We know that  $\alpha'$  is the Regge slope (string tension). With regard the Omega mesons, the values are:

$$\omega \quad | \quad 6 \quad | \quad m_{u/d} = 0 - 60 \quad | \quad 0.910 - 0.918$$

$$\omega/\omega_3 \quad | \quad 5 + 3 \quad | \quad m_{u/d} = 255 - 390 \quad | \quad 0.988 - 1.18$$

$$\omega/\omega_3 \quad | \quad 5 + 3 \quad | \quad m_{u/d} = 240 - 345 \quad | \quad 0.937 - 1.000$$

Now, from:

$$\frac{(V'(\phi_*))^2}{V_*^3} \approx \frac{5 \times 10^{10}}{24\pi^2} \frac{1}{M_{Pl}^6}, \quad (4.16)$$

We obtain:

$$5e+10 / (24\pi^2) * 1/(2.435e+18)^6$$

**Input interpretation:**

$$\frac{5 \times 10^{10}}{24\pi^2} \times \frac{1}{(2.435 \times 10^{18})^6}$$

**Result:**

$$1.01266365667212988... \times 10^{-102}$$

$$1.01266365667212988... * 10^{-102}$$

Now, we have:

$$\alpha H^{p-4} = \frac{(12\pi^2 \times 2 \times 10^{-9})^{(p-2)/2}}{3^{(p-4)/2} [(p-2)N_e]^{p-1}} \ll 1, \quad (4.22)$$

For  $p = 6$  and  $N_e = 60$ , we obtain:

$$(((12*\text{Pi}^2*2*1\text{e-}9)^2)) / ((3(4*60)^5))$$

**Input interpretation:**

$$\frac{(12\pi^2 \times 2 \times 1 \times 10^{-9})^2}{3(4 \times 60)^5}$$

**Result:**

$$\frac{\pi^4}{4147200000000000000000000000000}$$

**Decimal approximation:**

$$2.3487917398245186447829941331188539556743727255180705... \times 10^{-26}$$

$$2.3487917398245... * 10^{-26}$$

**Property:**

$$\frac{\pi^4}{4147200000000000000000000000000} \text{ is a transcendental number}$$

We have:

Super-Planckian decay constants would also violate the bound (5.5) and produce eternal inflation (under the assumption that the minimum of the axion potential occurs at  $V = 0$ ). To see this, we simply expand the natural inflation potential around a local maximum as in (4.11) and plug this into (5.5) to find the condition for no eternal inflation,

$$f < \frac{1}{\sqrt{6}} M_{\text{Pl}} \approx 0.41 M_{\text{Pl}}. \quad (5.13)$$

That is:

$$1/(\text{sqrt}6) * 2.435\text{e}+18$$

**Input interpretation:**

$$\frac{1}{\sqrt{6}} \times 2.435 \times 10^{18}$$

**Result:**

$$9.940845872795064449... \times 10^{17}$$

$$9.940845872795... * 10^{17}$$

And:

so the No Eternal Inflation bound (5.5) implies  $f < 1/\sqrt{6}M_{Pl}$ , which allows for  $f_{eff} \lesssim \sqrt{N/6}M_{Pl}$ . In contrast, the Weak Gravity Conjecture for multiple axions [108] implies

Where, for  $N = 4$ , we obtain:

$$(\text{sqrt}(4/6)) * 2.435e+18$$

**Input interpretation:**

$$\sqrt{\frac{4}{6}} \times 2.435 \times 10^{18}$$

**Result:**

$$1.988169174559012890... \times 10^{18}$$

$$1.988169174559... * 10^{18}$$

From the ratio of the two results, we obtain:

$$((((\text{sqrt}(4/6)) * 2.435e+18)))) / (((1/(\text{sqrt}6) * 2.435e+18)))$$

**Input interpretation:**

$$\frac{\sqrt{\frac{4}{6}} \times 2.435 \times 10^{18}}{\frac{1}{\sqrt{6}} \times 2.435 \times 10^{18}}$$

**Result:**

2

2

And:

$$(((1/(\text{sqrt}6) * 2.435e+18))) / (((\text{sqrt}(4/6)) * 2.435e+18))))$$

**Input interpretation:**

$$\frac{\frac{1}{\sqrt{6}} \times 2.435 \times 10^{18}}{\sqrt{\frac{4}{6}} \times 2.435 \times 10^{18}}$$

**Result:**

0.5

**Rational form:**

$$\frac{1}{2}$$

1/2

Where the two results, 2 and 1/2 , are respectively the spin of the graviton and the electron

We have also:

Indeed, has previously been noted that for suitable values of the constants  $c, c'$  the RdSC bounds (5.1) and (5.2) are incompatible with the bounds (5.4) and (5.5), so eternal inflation is incompatible with the RdSC [31, 32, 28, 29]. To see this, we simply use the fact that  $V < M_{\text{Pl}}^4$  and the fact that  $|\sum_i \nabla_i \nabla_i V| \geq |\min \nabla_i \nabla_j V|$ , in which case (5.1) implies (5.4) provided  $c > \sqrt{2}/2\pi M_{\text{Pl}}^{-1}$ , and (5.2) implies (5.5) provided  $c' > 3M_{\text{Pl}}^{-2}$ .

It is also worth pointing out that the RdSC\* bound (5.3) for  $q = 2$  and suitable values of  $a$  and  $b$  implies the bound (5.6). To see this, we multiply both sides of the RdSC\* bound by  $2\pi^2 V/M_{\text{Pl}}^4$ , then set  $V < M_{\text{Pl}}^4$  and  $|\sum_i \nabla_i \nabla_i V| > |\min \nabla_i \nabla_j V|$  to get

$$2\pi^2 \left( M_{\text{Pl}} \frac{|\nabla V|}{V} \right)^2 - 2\pi^2 a \frac{V}{M_{\text{Pl}}^2} \frac{\sum_i \nabla_i \nabla_i V}{V} \geq 2\pi^2 b \frac{V}{M_{\text{Pl}}^4}. \quad (5.12)$$

This implies (5.6) provided  $2\pi^2 a > 1/3$ ,  $2\pi^2 b < 1$ , which is indeed consistent with  $a + b = 1$ .

For  $2\pi^2 b < 1$ ;  $2\pi^2 b = 1/12$ ;  $M_{\text{Pl}} = 2.435 \times 10^{18}$  GeV;  $V = 9.843598548175 \times 10^{63}$ , from

$$2\pi^2 \left( M_{\text{Pl}} \frac{|\nabla V|}{V} \right)^2 - 2\pi^2 a \frac{V}{M_{\text{Pl}}^2} \frac{\sum_i \nabla_i \nabla_i V}{V} \geq 2\pi^2 b \frac{V}{M_{\text{Pl}}^4}.$$

We obtain:



$$(64 + 8) \times (-1) \log \left( \frac{1}{12} \times \frac{9.843598548175 \times 10^{63}}{(2.435 \times 10^{18})^4} \right) + 24 - \phi$$

$\log(x)$  is the natural logarithm

$\phi$  is the golden ratio

**Result:**

1785.0239137147...

1785.0239137147... result in the range of the hypothetical mass of Gluino (gluino = 1785.16 GeV).

And again:

$$\left( \left( \frac{1}{12} * (9.843598548175e+63) / (2.435e+18)^4 \right) \right)^{1/4096}$$

**Input interpretation:**

$$\sqrt[4096]{\frac{1}{12} \times \frac{9.843598548175 \times 10^{63}}{(2.435 \times 10^{18})^4}}$$

**Result:**

0.99404098540707268...

0.99404098540707268... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

$$\frac{1 + \sqrt[5]{\sqrt{\phi^5 4\sqrt{5^3} - 1}}}{\sqrt{5}} - \phi + 1$$

and to the dilaton value **0.989117352243 =  $\phi$**  (see Appendix)

And:

$$2 * \sqrt{\left( \left( \log \text{ base } 0.99404098540707268 \left( \left( \frac{1}{12} * (9.843598548175e+63) / (2.435e+18)^4 \right) \right) \right) \right) - \pi + 1 / \text{golden ratio}}$$

### Input interpretation:

$$2 \sqrt{\log_{0.99404098540707268} \left( \frac{1}{12} \times \frac{9.843598548175 \times 10^{63}}{(2.435 \times 10^{18})^4} \right) - \pi + \frac{1}{\phi}}$$

$\log_b(x)$  is the base- $b$  logarithm

$\phi$  is the golden ratio

### Result:

125.476441335160...

125.476441335160.... result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for  $T = 0$  (see Appendix)

### Conclusion

From what has been described and from the connections between various Ramanujan formulas and equations with physical parameters, such as the mass of the particles and the solutions inherent some sectors of Cosmology, we can conclude that this mathematics could potentially be used to unify sectors of physics and cosmology apparently distant from each other. Especially the "Rogers-Ramanujan continued fractions" and the sequences of Lucas and Fibonacci, together with  $\pi$ , at the value of the golden ratio and its conjugate, play a key role in the development of the equations that provide the new mathematical connections described here.



## Appendix

From:

### Rotating strings confronting PDG mesons

*Jacob Sonnenschein and Dorin Weissman - arXiv:1402.5603v1 [hep-ph] 23 Feb 2014*

*c* $\bar{c}$ . **The  $\Psi$  trajectory:** The left side of figure (15) depicts the  $\Psi$  trajectory. Here we use the states  $J/\Psi(1S)(3097)1^{--}$ ,  $\chi_{c1}(1P)(3510)1^{++}$ , and  $\Psi(3770)1^{--}$ . Since no  $J = 3$  state has been observed, we use three states with  $J = 1$ , but with increasing orbital angular momentum ( $L = 0, 1, 2$ ) and do the fit to  $L$  instead of  $J$ . To give an idea of the shifts in mass involved, the  $J^{PC} = 2^{++}$  state  $\chi_{c2}$  has a mass of 3556 MeV, and the  $J^{PC} = 3^{--}$  state is expected to lie 30 – 60 MeV above the  $\Psi(3770)$ [23].

The best linear fit is

$$\alpha' = 0.418, a = -4.04$$

with  $\chi_l^2 = 3.41 \times 10^{-4}$ , but the optimal fit is far from the linear, with endpoint masses in the range of the constituent  $c$  quark mass:

$$m_c = 1500, \alpha' = 0.979, a = -0.09$$

with  $\chi_m^2 = 5 \times 10^{-7}$  ( $\chi_m^2/\chi_l^2 = 0.002$ ). Aside from the improvement in  $\chi^2$ , by adding the mass we also get a value for the slope (and to a lesser extent, the intercept) that is much closer to that obtained in fits for the light meson trajectories.

where  $\alpha'$  is the Regge slope (string tension)

We know also that:

$$\omega \quad | \quad 6 \quad | \quad m_{u/d} = 0 - 60 \quad | \quad 0.910 - 0.918$$

$$\omega/\omega_3 \quad | \quad 5 + 3 \quad | \quad m_{u/d} = 255 - 390 \quad | \quad 0.988 - 1.18$$

$$\omega/\omega_3 \quad | \quad 5 + 3 \quad | \quad m_{u/d} = 240 - 345 \quad | \quad 0.937 - 1.000$$

The average of the various Regge slope of Omega mesons are:

$$1/7 * (0.979 + 0.910 + 0.918 + 0.988 + 0.937 + 1.18 + 1) = 0.987428571$$

result very near to the value of dilaton and to the solution 0.987516007... of the above expression.

From:

Astronomy & Astrophysics manuscript no. ms c ESO 2019 - September 24, 2019  
**Planck 2018 results. VI. Cosmological parameters**

*The primordial fluctuations are consistent with Gaussian purely adiabatic scalar perturbations characterized by a power spectrum with a spectral index  $n_s = 0.965 \pm 0.004$ , consistent with the predictions of slow-roll, single-field, inflation.*

from:

**Modular equations and approximations to  $\pi$**  - Srinivasa Ramanujan  
Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

We have that:

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \dots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \dots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \dots = 64\{(1 + \sqrt{2})^{12} + (1 - \sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\dots$$

Again

$$G_{37} = (6 + \sqrt{37})^{\frac{1}{4}},$$

$$\begin{aligned} 64G_{37}^{24} &= e^{\pi\sqrt{37}} + 24 + 276e^{-\pi\sqrt{37}} + \dots, \\ 64G_{37}^{-24} &= 4096e^{-\pi\sqrt{37}} - \dots, \end{aligned}$$

so that

$$64(G_{37}^{24} + G_{37}^{-24}) = e^{\pi\sqrt{37}} + 24 + 4372e^{-\pi\sqrt{37}} - \dots = 64\{(6 + \sqrt{37})^6 + (6 - \sqrt{37})^6\}.$$

Hence

$$e^{\pi\sqrt{37}} = 199148647.999978\dots$$

Similarly, from

$$g_{58} = \sqrt{\left(\frac{5 + \sqrt{29}}{2}\right)},$$

we obtain

$$64(g_{58}^{24} + g_{58}^{-24}) = e^{\pi\sqrt{58}} - 24 + 4372e^{-\pi\sqrt{58}} - \dots = 64\left\{\left(\frac{5 + \sqrt{29}}{2}\right)^{12} + \left(\frac{5 - \sqrt{29}}{2}\right)^{12}\right\}.$$

Hence

$$e^{\pi\sqrt{58}} = 24501257751.99999982\dots$$

From:

## An Update on Brane Supersymmetry Breaking

*J. Mourad and A. Sagnotti* - arXiv:1711.11494v1 [hep-th] 30 Nov 2017

From the following vacuum equations:

$$T e^{\gamma_E \phi} = - \frac{\beta_E^{(p)} h^2}{\gamma_E} e^{-2(8-p)C + 2\beta_E^{(p)} \phi}$$

$$16 k' e^{2C} = \frac{h^2 \left( p + 1 - \frac{2\beta_E^{(p)}}{\gamma_E} \right) e^{-2(8-p)C + 2\beta_E^{(p)} \phi}}{(7-p)}$$

$$(A')^2 = k e^{-2A} + \frac{h^2}{16(p+1)} \left( 7 - p + \frac{2\beta_E^{(p)}}{\gamma_E} \right) e^{-2(8-p)C + 2\beta_E^{(p)} \phi}$$

we have obtained, from the results almost equals of the equations, putting

$4096 e^{-\pi\sqrt{18}}$  instead of

$$e^{-2(8-p)C + 2\beta_E^{(p)} \phi}$$

a new possible mathematical connection between the two exponentials. Thence, also the values concerning  $p$ ,  $C$ ,  $\beta_E$  and  $\phi$  correspond to the exponents of  $e$  (i.e. of exp). Thence we obtain for  $p = 5$  and  $\beta_E = 1/2$ :

$$e^{-6C + \phi} = 4096 e^{-\pi\sqrt{18}}$$

Therefore, with respect to the exponentials of the vacuum equations, the Ramanujan's exponential has a coefficient of 4096 which is equal to  $64^2$ , while  $-6C + \phi$  is equal to  $-\pi\sqrt{18}$ . From this it follows that it is possible to establish mathematically, the dilaton value.

For

$\exp(-\pi\sqrt{18})$  we obtain:

**Input:**

$$\exp(-\pi\sqrt{18})$$

**Exact result:**

$$e^{-3\sqrt{2}\pi}$$

**Decimal approximation:**

$$1.6272016226072509292942156739117979541838581136954016... \times 10^{-6}$$

$$1.6272016... * 10^{-6}$$

**Property:**

$e^{-3\sqrt{2}\pi}$  is a transcendental number

**Series representations:**

$$e^{-\pi\sqrt{18}} = e^{-\pi\sqrt{17} \sum_{k=0}^{\infty} 17^{-k} \binom{1/2}{k}}$$

$$e^{-\pi\sqrt{18}} = \exp\left(-\pi\sqrt{17} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{17}\right)^k \binom{-\frac{1}{2}}{k}}{k!}\right)$$

$$e^{-\pi\sqrt{18}} = \exp\left(-\frac{\pi \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 17^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{2\sqrt{\pi}}\right)$$

Now, we have the following calculations:

$$e^{-6C+\phi} = 4096e^{-\pi\sqrt{18}}$$

$$e^{-\pi\sqrt{18}} = 1.6272016... * 10^{-6}$$

from which:

$$\frac{1}{4096} e^{-6C+\phi} = 1.6272016... * 10^{-6}$$

$$0.000244140625 e^{-6C+\phi} = e^{-\pi\sqrt{18}} = 1.6272016... * 10^{-6}$$

Now:

$$\ln\left(e^{-\pi\sqrt{18}}\right) = -13.328648814475 = -\pi\sqrt{18}$$

And:

$$(1.6272016 * 10^{-6}) * 1 / (0.000244140625)$$

**Input interpretation:**

$$\frac{1.6272016}{10^6} \times \frac{1}{0.000244140625}$$

**Result:**

0.0066650177536

0.006665017...

Thence:

$$0.000244140625 e^{-6C+\phi} = e^{-\pi\sqrt{18}}$$

Dividing both sides by 0.000244140625, we obtain:

$$\frac{0.000244140625}{0.000244140625} e^{-6C+\phi} = \frac{1}{0.000244140625} e^{-\pi\sqrt{18}}$$

$$e^{-6C+\phi} = 0.0066650177536$$

$$(((\exp((-Pi*\sqrt{18})))))) * 1 / 0.000244140625$$

**Input interpretation:**

$$\exp(-\pi\sqrt{18}) \times \frac{1}{0.000244140625}$$

**Result:**

0.00666501785...

0.00666501785...

### Series representations:

$$\frac{\exp(-\pi \sqrt{18})}{0.000244141} = 4096 \exp\left(-\pi \sqrt{17} \sum_{k=0}^{\infty} 17^{-k} \binom{\frac{1}{2}}{k}\right)$$

$$\frac{\exp(-\pi \sqrt{18})}{0.000244141} = 4096 \exp\left(-\pi \sqrt{17} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{17}\right)^k \left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$\frac{\exp(-\pi \sqrt{18})}{0.000244141} = 4096 \exp\left(-\frac{\pi \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 17^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{2\sqrt{\pi}}\right)$$

Now:

$$e^{-6C+\phi} = 0.0066650177536$$

$$\exp(-\pi \sqrt{18}) \times \frac{1}{0.000244140625} =$$

$$e^{-\pi \sqrt{18}} \times \frac{1}{0.000244140625}$$

$$= 0.00666501785\dots$$

From:

$$\ln(0.00666501784619)$$

### Input interpretation:

$$\log(0.00666501784619)$$

### Result:

$$-5.010882647757\dots$$

$$-5.010882647757\dots$$

### Alternative representations:

$$\log(0.006665017846190000) = \log_e(0.006665017846190000)$$

$$\log(0.006665017846190000) = \log(a) \log_a(0.006665017846190000)$$

$$\log(0.006665017846190000) = -\text{Li}_1(0.993334982153810000)$$

### Series representations:

$$\log(0.006665017846190000) = -\sum_{k=1}^{\infty} \frac{(-1)^k (-0.993334982153810000)^k}{k}$$

$$\log(0.006665017846190000) = 2i\pi \left[ \frac{\arg(0.006665017846190000 - x)}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (0.006665017846190000 - x)^k x^{-k}}{k} \quad \text{for } x < 0$$

$$\log(0.006665017846190000) = \left[ \frac{\arg(0.006665017846190000 - z_0)}{2\pi} \right] \log\left(\frac{1}{z_0}\right) + \log(z_0) + \left[ \frac{\arg(0.006665017846190000 - z_0)}{2\pi} \right] \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (0.006665017846190000 - z_0)^k z_0^{-k}}{k}$$

### Integral representation:

$$\log(0.006665017846190000) = \int_1^{0.006665017846190000} \frac{1}{t} dt$$

In conclusion:

$$-6C + \phi = -5.010882647757 \dots$$

and for  $C = 1$ , we obtain:



$$\phi = -5.010882647757 + 6 = 0.989117352243 = \phi$$

Note that the values of  $n_s$  (spectral index) 0.965, of the average of the Omega mesons Regge slope 0.987428571 and of the dilaton 0.989117352243, are also connected to the following two Rogers-Ramanujan continued fractions:

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\phi-1)\sqrt{5}} - \phi + 1} = 1 - \frac{e^{-\pi}}{1 + \frac{e^{-2\pi}}{1 + \frac{e^{-3\pi}}{1 + \frac{e^{-4\pi}}{1 + \dots}}}} \approx 0.9568666373$$

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

(<http://www.bitman.name/math/article/102/109/>)

Also performing the 512<sup>th</sup> root of the inverse value of the Pion meson rest mass 139.57, we obtain:

$$((1/(139.57)))^{1/512}$$

**Input interpretation:**

$$\sqrt[512]{\frac{1}{139.57}}$$

**Result:**

0.990400732708644027550973755713301415460732796178555551684...

0.99040073.... result very near to the dilaton value **0.989117352243 =  $\phi$**  and to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} - \phi + 1 \approx 0.9991104684$$

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**Generalized dilaton–axion models of inflation, de Sitter vacua and spontaneous SUSY breaking in supergravity**

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**Table 1** The predictions for the inflationary parameters ( $n_s, r$ ), and the values of  $\varphi$  at the horizon crossing ( $\varphi_i$ ) and at the end of inflation ( $\varphi_f$ ), in the case  $3 \leq \alpha \leq \alpha_*$  with both signs of  $\omega_1$ . The  $\alpha$  parameter is taken to be integer, except of the upper limit  $\alpha_* \equiv (7 + \sqrt{33})/2$

| $\alpha$               | 3      | 4      | 5      | 6      | $\alpha_*$ |        |
|------------------------|--------|--------|--------|--------|------------|--------|
| $\text{sgn}(\omega_1)$ | –      | +      | –      | +/-    | +          | –      |
| $n_s$                  | 0.9650 | 0.9649 | 0.9640 | 0.9639 | 0.9634     | 0.9637 |
| $r$                    | 0.0035 | 0.0010 | 0.0013 | 0.0007 | 0.0005     | 0.0004 |
| $-\kappa\varphi_i$     | 5.3529 | 3.5542 | 3.9899 | 3.2657 | 3.0215     | 2.7427 |
| $-\kappa\varphi_f$     | 0.9402 | 0.7426 | 0.8067 | 0.7163 | 0.6935     | 0.6488 |

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**Gravitational waves from walking technicolor**

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The phase transition dynamics is modified via the shift of  $(2f_2/N_f)(s^0)^2 \rightarrow (\Delta m_s)^2 + (2f_2/N_f)(s^0)^2$  in  $m_{s^i}^2$  with finite  $\Delta m_s$ . The details of the mass spectra at one loop with  $(\Delta m_s)^2$  are summarized in appendix A. Using eq. (4.18), the total effective potential becomes,

$$V_{\text{eff}}(s^0, \Delta m_p, \Delta m_s, T) = \frac{N_f^2 - 1}{64\pi^2} \mathcal{M}_{s^i}^4(s^0, \Delta m_p, \Delta m_s, T) \left( \ln \frac{\mathcal{M}_{s^i}^2(s^0, \Delta m_p, \Delta m_s, T)}{\mu_{\text{GW}}^2} - \frac{3}{2} \right), \\ + \frac{T^4}{2\pi^2} (N_f^2 - 1) J_B(\mathcal{M}_{s^i}^2(s^0, \Delta m_p, \Delta m_s, T)/T^2) + C(T), \quad (4.19)$$

with,

$$\mathcal{M}_{s^i}^2(s^0, \Delta m_p, \Delta m_s, T) = m_{s^i}^2(s^0, \Delta m_p, \Delta m_s) + \Pi(T), \quad (4.20)$$

where the thermal mass  $\Pi(T)$  is given in eq. (3.3). We require that the following properties remain intact for arbitrary  $\Delta m_s$ ; (1) the vev  $\langle s^0 \rangle(T=0)$  determined by the minimum of the potential eq. (4.19) is identified with the dilaton decay constant favored by the walking technicolor model,  $F_\phi = 1.25 \text{ TeV}$  or  $1 \text{ TeV}$ , (2) the dilaton mass given by the potential curvature at the vacuum is identified with the observed SM Higgs mass,  $m_{s^0} = 125 \text{ GeV}$ .

Thence  $F_\phi = 1.25 \text{ TeV}$

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