The Dirac-Ricci operator

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Abstract

We define a Dirac type operators called the Dirac-Ricci operator with help of the Ricci curvature.

1 The Dirac operator

Let \((M, g)\) be a spin manifold, then we can define the Dirac operator \(D\) with help of the Levi-Civita connection \(\nabla\).

\[ D(\psi) = \sum_i e_i \cdot \nabla_{e_i}(\psi) \]

\((e_i)\) is an orthonormal basis.

\[ D = \mu \circ \nabla \]

with \(\mu\) the Clifford multiplication.

2 The Dirac-Ricci operator

The Dirac-Ricci operator can be defined by the following formula:

\[ DR(\psi) = \sum_i Ric(e_i) \cdot \nabla_{e_i}(\psi) \]

\(Ric\) is the Ricci curvature [GHL] viewed as an endomorphism of the tangent bundle.

\[ DR = \mu \circ (Ric \otimes 1) \circ \nabla \]

If \(M\) is an Einstein manifold [Be] then the Dirac-Ricci operator is reduced to the Dirac operator.

3 The Dirac-Lichnerowicz-Ricci formula

The Dirac-Ricci operator is symmetric with respect of the spinor product and we have the Dirac-Lichnerowicz-Ricci formula:

\[ DR^2 = -\Delta_R + \alpha \]
with:

$$\Delta R = \sum_{i,j} g(Ric(e_i), Ric(e_j)) \nabla e_i \nabla e_j$$

and $\alpha$ is a lower term; it is a scalar if $\nabla Ric = 0$.

References

