The Energetics of the an ExB propulsor That avoids the Hidden Momentum pitfall

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Abstract

This paper updates earlier thoughts by the author on a putative electromagnetic propulsion system, to perform a more detailed energy balance. The previous paper demonstrated how momentum could be dumped to the ground state of the electromagnetic field but a claim was left somewhat hanging at the end of the previous paper, that the work done in changing the craft's velocity would effectively shift the centre of mass of the field – although that would be an infinitesimal shift in practice. The craft must always supply work to change velocity, such as by an accelerate/de-accelerate cycle and superficially this looks to violate the conservation of energy; we prove that this isn't so.

1. Introduction

The Feynman Disk[1-3] "paradox" arose by the literal interpretation of the Poynting Vector applied to static electric and magnetic fields; whereas for the case of dynamic fields the interpretation is easy, given Maxwell's wave equation: energy and momentum propagate in the direction of the wave-vector; the static case implies momenergy circulating in the space around the static fields. This has indeed been experimentally verified[3] and the author utilising, this effect for a putative linear propulsor[4], has identified this circulating momenergy to the zeropoint of the field, where before its provenance, classical or quantum, was uncertain.

It is of note that the author's argument appears to get around the hidden momentum, relativistic current carrying fluid argument, which is the bane of linear propulsors that seek to utilise crossed static fields: apparently the linear momentum developed by the craft can be countered by "dumping" momentum to the field ground state by a cancellation mechanism[4] on the second half of the cycle. Nethertheless criticism was made of the proposal and the contentious statement that was left hanging towards the end of the paper, which didn't account for where the work required (always positive) and converted into kinetic energy by the craft went on a closed cycle (i.e. to destination and then back) – the implication being that this was, too, somehow dumped to the field ground state. This paper seeks to justify this claim.

The continuity equation[1, 2] considering the energy of the field leads to the familiar the terms for the rate of work of charges in a field, the field energy and the Poynting expression:

$$\int_V \frac{\partial u}{\partial t} dV = -\int_V (\nabla\cdot\mathbf{S})\,dV - \int_V (\mathbf{E}\cdot\mathbf{j})\,dV$$

$$\therefore \frac{\varepsilon_0}{2}\frac{\partial}{\partial t}\int_V \left(c^2\mathbf{B}\cdot\mathbf{B}+\mathbf{E}\cdot\mathbf{E}\right)dV = -\varepsilon_0 c^2\int_V \nabla\cdot(\mathbf{E}\times\mathbf{B})\,dV - \int_V (\mathbf{E}\cdot\mathbf{j})\,dV \qquad \text{eqn. 1}$$

We can identify the terms involved in generating and then distributing power to the putative propulsion device (fig. 1), which depicts a capacitor of charge q and electric field **E** as the power source:

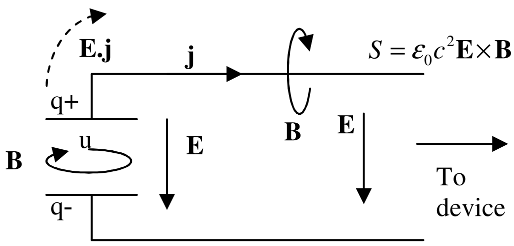


Figure 1 - Power from the power source to the device

As an aside, it is moot whether the power from the source that moves through the wires, in the low-frequency steady-state limit, is described by an **E.j** (minus the work against resistance) term or a Poynting term $\varepsilon_0 c^2$**ExB**. Hence in the latter case, the power is not travelling through the wires but the fields in the space between; the terms in this case are equal. However the description for a waveguide *is* via the Poynting term with dynamic fields.

At the device, the power from the source establishes the magnetic field that acts on the device's electret, so there is a flow of electromagnetic energy into the region for the magnetic field energy. Figure 2 shows the Feynman Disk[1], upon which the propulsion device is based: the coil in the centre establishes an axial magnetic field (B field), around the periphery are charged balls (E field). Once the electromagnet's field has been established, there is no more flow of energy into the region, save only the flow of power from the source against resistive dissipation. The literal interpretation of the Poynting term and the crossing of the static E and B fields means that there is a circulatory flow of energy in this static situation and a concomitant flow of angular momentum too. Necessarily, because of the magnetic field admitting no sources, the field lines are closed, the momentum is then angular and not linear.

E_B is electrical field from the solenoid as its flux changes. It acts first one way then the other when the current is switched on, then off. It reacts against the charged spheres, sending the disk spinning one way first, then the other.

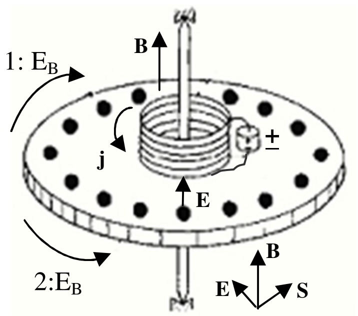


Figure 2 – The Feynman Disk

As a consequence of conservation of momentum, the disk must begin to rotate when the current is switched on: the momentum of the circulating Poynting vector is cancelled by the mechanical momentum. When the current is switched off, the disk stops rotating. The electrical work from the battery (neglecting resistive losses) can then be calculated as the sum of the magnetic field energy (just an inductor), the energy in the static Poynting flow and the work done on accelerating the disk (its rotational energy), which is obtained in the Newtonian limit from the angular momentum and moment of inertia as $L^2/2I_D$:

$$\begin{aligned}&\int_{t,V}(\mathbf{E}_{bat}\cdot\mathbf{j})\,dtdV=\\&+\frac{\varepsilon_0 c^2}{2}\int_V(\mathbf{B}\cdot\mathbf{B})\,dV\\&+\varepsilon_0 c^2\int_V\nabla\cdot(\mathbf{E}_{electret}\times\mathbf{B})\,dV\\&+\frac{\left(\varepsilon_0\int_V\nabla\cdot(\mathbf{r}\times\mathbf{E}_{electret}\times\mathbf{B})\,dV\right)^2}{2I_D}\end{aligned}\qquad\text{eqn. 2}$$

It is more informative to write for the last term involving the rotational kinetic energy, the rate of work as the product of the torque and angular velocity:

$$\frac{dW_{mech}}{dt}=\tau\omega\qquad\text{eqn. 3}$$

Whereupon:

$$\frac{dW_{mech}}{dt}=\frac{dL}{dt}\cdot\frac{1}{I_D}\int\frac{dL}{dt}dt$$

$$\Rightarrow W_{mech}=\frac{1}{I_D}\int L\frac{dL}{dt}dt$$

This is by noting that the angular acceleration is the torque (dL/dt) divided by the moment of inertia and that the angular velocity is the time integral of this. Obviously this leads to the same result in eqn. 2 for the mechanical work done by the electrical system of $L^2/2I_D$. It is then an easy matter to substitute the angular momentum term coming from the static Poynting field.

$$\begin{aligned}\frac{dW_{mech}}{dt}&=\frac{1}{I_D}L\frac{dL}{dt}\\&\propto\frac{(\varepsilon_0 rE_{electret}V)^2}{I_D}B\frac{\partial B}{\partial t}\end{aligned}\qquad\text{eqn. 4}$$

It is easy to see that this work is reversible over a cycle by noting the term dB/dt.

2. The Feynman Disk in several different scenarios

There now follows a number of scenarios that put the disk in different circumstances and are relevant to discussions on the energetics of the disk and the linear thruster. The electrical engineering means of "current ramps-up" and then "ramps-down" are not the issue here, save to say it can be done in a regenerative manner with a flyback converter[5]. Here are the scenarios:-

1. The current is ramped-up and the disk spins-up and translates angularly. The current is ramped-down and the disk comes to a halt. This has already been discussed in the introduction regarding eqn. 2 where the (ideal) energy requirements are reversible (the B-field, the static Poynting field and Kinetic Energy of the disk).
2. Current is ramped-on the disk spins-up. We brake to zero angular velocity and

the K.E. of rotation goes to heat energy in the brake. The mechanical angular momentum is taken up by the brake too. As the current is still on, the static E.M. angular momentum of the Poynting field is still circulating. If the brake is left on and the current is ramped down, only the energy of the B-field and static Poynting field (eqn. 2) is returned to the source. No additional mechanical work is done as the disk, even though acted upon by a force/torque, does not translate.

3. Current ramped-on, the disk spins-up and then is braked to zero angular velocity; the K.E. of rotation obviously becomes heat energy. The static angular momentum is still circulating as the current is on, however if the brake is released and the current is ramped down, the disk will now rotate in the opposite direction. Electrical power is once again taken from the battery. If the disk is braked again, this will become heat energy again. Overall twice the K.E. of rotation ends up as heat energy. This may seem paradoxical, since the last term of eqn. 2 is squared and doesn't seem to permit scenarios (1) or (3), until we look at eqn. 3 and figure 2 in this cyclical scenario and see that this is so: the torque and angular velocity are negative, so the work is positive, too, on the second step.

3. Momenergy analysis

The previous sections looked at the Feynman disk in various scenarios and showed how power from the source coupled to the angular of the disk. The argument is now moved on to the linear propulsor discussed in the author's previous paper[4]. In the appendix of the said paper, a standard method of quantisation of the electromagnetic field is set out that models the Fourier modes of the field as harmonic oscillators. What is interesting is the inevitable zero-point energy of such a system and its huge though contested size[6-8]. In the realm of quantum electrodynamics it can be interpreted as explaining the Lamb Shift, The Casmir Effect or Spontaneous Emission[9] but if so massive, why doesn't it gravitate, why don't we feel its effects more? Part of the answer to this is its isotropy and Lorentz invariance[8†].

† The author seeks to link the magnitude of zero-point to the cosmological constant in a future paper and provide a reason why one is so massive and the other is so small.

It may be that the linear propulsor, devised from the Feynman disk, may provide literal evidence of the sheer size of the zero-point, as there is a step in the cycle of "dumping" angular momentum on the second half of the cycle such that the propelled craft acts like a conveyor belt between rollers[4]. Here we seek to do the momenergy analysis to show the feasibility of this.

We shall analyse a round trip where a craft accelerates to some velocity, to then de-accelerate and return to the original frame. The rest-energy of the craft will be seen to be converted into kinetic energy each time it accelerates. This begs the question, on a round trip where the craft returns to the original frame with no kinetic energy, just where did the missing rest-energy go? The conclusion in the author's first paper on this matter was handled somewhat glibly (or seemed obvious to the author) - this missing energy went into the kinetic energy of translating the field, specifically the massive zero-point. This assertion shall be proven.

Maxwell's equations are relativistically invariant (namely the transformation of the E and B-fields to any frame) and the treatment given to quantisation in the appendix of the author's previous paper will always yield a Lorentz invariant zero-point at any velocity. Thus a craft always finds mass-energy to push reactively against at any velocity:

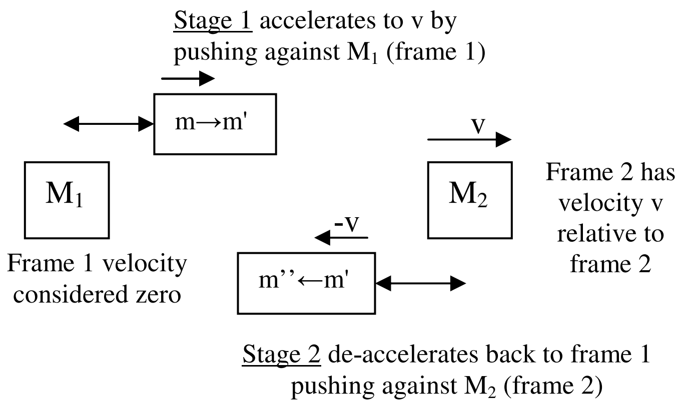


Figure 3 – The Linear Propulsor's round trip between two frames

Figure 3 shows a craft utilising the linear propulsor scheme based on the Feynman Disk accelerating between two frames of, respectively, mass-energy M_1c^2 and M_2c^2; the mass-energy is of course in our argument the zero-point energy of the electromagnetic field.

The craft initially starts in frame 1 with mass-energy mc^2 but converts some of this into velocity of the remaining mass energy $m'c^2$ moving at velocity v to the initial frame 1. Finally the craft "returns to home" by pushing against frame 2 whilst converting some of its rest energy $m'c^2$, such that the craft of rest energy $m''c^2$, now, achieves a velocity of –v relative to frame 2. The frames 1 and 2 will be shown, in the limit of infinite mass-energy, to show negligible change in their relative velocities. The loss in mass-energy of the craft will be seen to be taken by kinetic energy of the frames by virtue of the momentum imparted to them, from any frame's perspective.

Let us consider the first process, the craft pushing off against M_1. We know that momentum is conserved and so write the sum of the four-momentum before and after the process as:

$$\left(P_{M_1}^{\mu} + P_m^{\mu}\right)\Big|_{start} = \left(P_{M_1}^{\mu} + P_m^{\mu}\right)\Big|_{end} \qquad \text{eqn. 5}$$

Let us consider frame M_1 stationary and write,

$$\begin{aligned} \left(P_{M_1}^{\mu} + P_m^{\mu}\right)\Big|_{start} &= \left[\frac{E_{M_1}}{c},0,0,0\right] + \left[\frac{E_m}{c},0,0,0\right] \\ \left(P_{M_1}^{\mu} + P_m^{\mu}\right)\Big|_{end} &= \left[\frac{E_{M_1}}{c},0,0,0\right] + \left[\frac{\gamma E'_m}{c},p_x,0,0\right] \end{aligned} \qquad \text{eqns. 6}$$

So the final vector shows how some of the craft's rest energy is converted into K.E. of the craft giving it momentum. Noting the following from Relativity theory:

The norm of a four vector is invariant, in this case with the signature [+ - - -], the norm of a four-momentum is:

$$\left\|P_m^{\mu}\right\| = \frac{E^2}{c^2} - |p|^2 = m_0^2 c^2 \qquad \text{eqn. 7}$$

And the four-momentum transforms between the rest frame and other frames as a four-vector, so:

$$\begin{aligned} E &= \gamma E_0 \\ \mathbf{p} &= \gamma m_0 \mathbf{v} \\ \Rightarrow \mathbf{p} &= \gamma \frac{E_0}{c^2}\mathbf{v} \end{aligned} \qquad \text{eqns. 8}$$

The norm of eqn. 5 (with eqns. 6) can be written:

$$\left\|\left[\frac{E_{M_1}}{c},0,0,0\right] + \left[\frac{E_m}{c},0,0,0\right]\right\| = \left\|\left[\frac{E_{M_1}}{c},0,0,0\right] + \left[\frac{\gamma E'_m}{c},\frac{\gamma E'_m}{c^2}v,0,0\right]\right\|$$

This leads by eqn. 5 to,

$$\begin{aligned} &\frac{E_{M_1}^2 + 2E_{M_1}E_m + E_m^2}{c^2} \\ &= \left(\frac{E_{M_1}}{c} + \frac{\gamma E'_m}{c}\right)^2 - \left(\frac{\gamma E'_m}{c^2}v\right)^2 \\ &= \frac{E'^2_m\left(\cancel{\gamma^2 - \gamma^2\frac{v^2}{c^2}}\right) + 2\gamma E_{M_1}E'_m + E_{M_1}^2}{c^2} \end{aligned}$$

Hence,

$$\Rightarrow 2E_{M_1}E_m + E_m^2 = 2\gamma E_{M_1}E'_m + E'^2_m \qquad \text{eqn. 9}$$

And if the zero-point mass-energy is much, much greater than the mass-energy of the craft, we find:

$$E_m \Rightarrow \gamma E'_m \qquad \text{eqn. 10}$$

And this simply states that all the mass-energy converted has gone into the kinetic energy and momentum of the remaining mass-energy of the craft.

A similar calculation considers the craft's frame as stationary with these momentum vectors:

$$\begin{aligned} \left(P_{M_1}^{\mu} + P_m^{\mu}\right)\Big|_{start} &= \left[\frac{E_{M_1}}{c},0,0,0\right] + \left[\frac{E_m}{c},0,0,0\right] \\ \left(P_{M_1}^{\mu} + P_m^{\mu}\right)\Big|_{end} &= \left[\frac{\gamma E_{M_1}}{c},-\gamma p_x,0,0\right] + \left[\frac{\gamma E'_m}{c},0,0,0\right] \end{aligned} \qquad \text{eqns. 11}$$

Gives the norm for the final 4-vector,

$$\begin{aligned} &\left(\frac{\gamma E_{M_1}}{c} + \frac{E'_m}{c}\right)^2 - \left(\frac{-\gamma E_{M_1}}{c^2}v\right)^2 \\ &= \frac{E_{M_1}^2\left(\cancel{\gamma^2 - \gamma^2\frac{v^2}{c^2}}\right) + 2\gamma E_{M_1}E'_m + E'^2_m}{c^2} \end{aligned}$$

And then $2E_{M_1}E_m + E_{M_1}^2 = 2\gamma E_{M_1}E'_m + E'^2_m$ and the same result as eqn. 10. This can be immediately applied to the second leg of the mission as it returns home from frame 2. What then is peculiar is how the craft returns to the base of frame 1 with a smaller rest energy, which is now,

$$E''_m = \frac{E_m}{\gamma^2} \qquad \text{eqn. 12}$$

Where has this "missing" energy gone? In solely looking at the craft we have neglected the total energy of the system and have failed to take into account the momentum (hence kinetic energy) given to frames 1 and 2. Let us consider the craft already in motion ready to exchange momentum with the second frame <u>as seen from the first frame</u>. The sum of the four-momenta before is:

$$P^{\mu}_{start}=\left[\frac{\gamma E'_m}{c},\frac{\gamma E'_m}{c^2}v,0,0\right]+\left[\frac{\gamma E_{M_2}}{c},\frac{\gamma E_{M_2}}{c^2}v,0,0\right] \qquad \text{eqn. 13}$$
$$=\left[\frac{E_m}{c},\frac{E_m}{c^2}v,0,0\right]+\left[\frac{\gamma E_{M_2}}{c},\frac{\gamma E_{M_2}}{c^2}v,0,0\right]$$

This arises from the reduced rest-energy (given by eqn. 10) of the craft $\gamma E'_m$ being accelerated to velocity v after converting some of its original rest-energy E_m. Frame 2's rest-energy E_{M_2} is viewed as Lorentz transformed.

After the exchange of momentum the craft "returns to base" with rest-energy E''_m and we obtain the final sum of the four-momenta:

$$P^{\mu}_{end}=\left[\frac{E_m}{\gamma^2 c},0,0,0\right]+\left[\frac{\gamma E_{M_2}}{c},\left(\frac{\gamma E_{M_2}}{c^2}+\frac{E_m}{c^2}\right)v,0,0\right] \qquad \text{eqn. 14}$$

Once again the norms of these four-momenta are equal, that is,

$$\left\|P^{\mu}_{start}\right\|=\left\|P^{\mu}_{end}\right\|$$

And this just leads to:

$$\frac{\left(E_m+\gamma E_{M_1}\right)^2}{c^2}\left[1-\cancel{\frac{v^2}{c^2}}\right]=\frac{\left(E_m+\gamma^3 E_{M_1}\right)^2}{\gamma^2 c^2}-\cancel{\frac{v^2}{c^2}}\cdot\cancel{\frac{\left(E_m+\gamma E_{M_1}\right)^2}{c^2}}$$

$$\Rightarrow \gamma^2\left(E_m+\gamma E_{M_1}\right)=E_m+\gamma^3 E_{M_1}$$
$$\Rightarrow \gamma^2=1$$

From which we conclude,

$$\beta=\frac{v}{c}=0 \qquad \text{eqn. 15}$$

What does this mean? Frame 2 is identical to frame 1, only that it is moving with velocity v relative to it. The craft is able to exchange momentum with it, yet it appears identical to the original frame the craft set out from - whose velocity was zero. The only conclusion is that frame 1 and 2 *are* identical and that the craft converted some of its mass-energy to kinetic energy and loses momentum to the zero-point (the mass-energy of the frames) every time it accelerates.

QED.

<u>4. Comparing the linear thruster to devices that reject reaction mass</u>

According to eqn. 10 the linear thruster based on the Feynman Disk[4] converts some of its rest-energy into kinetic energy of the remaining rest-energy. Obviously this is an ideal situation and the conversion process may generate heat that could radiate away. The conversion of rest-energy could be via a battery/capacitor, a chemical reaction or even nuclear processes. It may even be possible to convert the heat energy from such processes into electricity needed for the propulsor in an highly efficient manner[10-12], however such concerns aside, eqn. 10 is always valid.

We might then compare this loss of rest-energy/rest-mass to achieve a set velocity with the specific impulse of current technologies. We can write by eqn. 10,

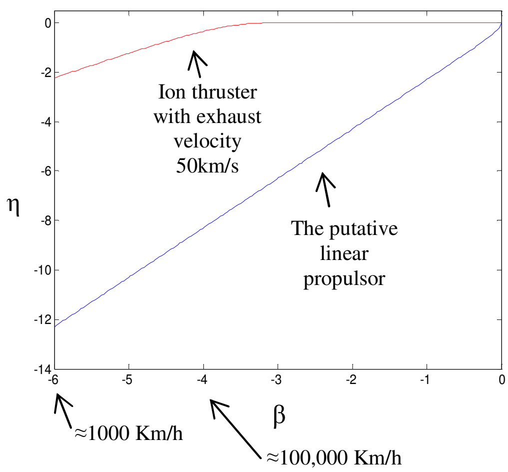
$$\left(\frac{E'_m}{E_m}\right)^2=1-\frac{v^2}{c^2}$$

And consider the fraction of rest-mass used:

$$\eta=1-\frac{E'_m}{E_m} \qquad \text{eqn. 16}$$

So,

$$\eta=1-\sqrt{1-\frac{v^2}{c^2}} \qquad \text{eqn. 17}$$



<u>Figure 4 – Log-Log plot of Fraction, η of rest-mass used vs. Fraction of light-speed, β obtained</u>

Let us compare this to the relativistic Tsiolkovsky rocket equation[13]

$$\Delta v=-c\tanh\left(\frac{v_{exhaust}}{c}\ln\frac{m_{final}}{m_{initial}}\right) \qquad \text{eqn. 18}$$

And much as eqn. 16, we shall use η as our fractional efficiency to obtain:

$$\eta = 1 - e^{\frac{c}{v_{exhaust}} \cdot \frac{1}{2} \tanh^{-1} \frac{-\Delta v}{c}} \qquad \text{eqn. 19}$$

Figure 4 compares eqn. 17 and eqn. 19 and one can immediately see the utility, if feasible, of the linear thruster based on the Feynman Disk.

5. Conclusion

The author's earlier paper on the exposition of the putative device elucidated a mechanism for the dumping of momentum to the zero-point of the electromagnetic field and it also answered quite a long standing mystery as to the nature of just what momenergy was circulating in the static **E**x**B** scenario (the zeropoint energy of the field). However the conclusion indicated that the on continual round trips that the kinetic energy was somehow dumped to the zeropoint. This was left hanging and thus was unsatisfactory.

This paper looked more into the energetics of the putative propulsor based on static field momentum concepts and linked the electrical work to the mechanical work and furthermore proved the assertion, in the earlier paper, of momentum and kinetic energy being dumped to the field ground state. This shows that the device is not in abeyance of the 1st Law.

A comparison was made to the efficiency of rockets based on the use of propellant to achieve a set speed, or in the case of the putative propulsor, the amount of rest-mass converted to kinetic energy. It was found that putative propulsor reaches the ideal conversion efficiency as dictated by Special Relativity.

References

1. Feynman Leighton Sands, *The Feynman Lectures on Physics*. Addison-Wesley, Reading, Massachusetts. Vol. 2 1989.

2. Jackson, J. D., *Classical Electrodynamics*. 2nd ed. 1975: Wiley.

3. Graham Lahoz, *Observation of static electromagnetic angular momentum in vacuo*. Nature, 1980. **285**(154).

4. Cornwall, R. O., *A Mechanism for Propulsion without the Reactive Ejection of Matter or Energy*. Preprints, 2019 (https://www.academia.edu/38062548/A_Mechanism_for_Propulsion_without_The_Reactive_Ejection_of_Matter_or_Energy).

5. Horowitz P., Hill W., *The Art of Electronics*. Cambridge University Press. 1989.

6. Baez, J. *What's the Energy Density of the Vacuum?* 2011; Available from: http://math.ucr.edu/home/baez/vacuum.html.

7. Saunders, S. *Is the Zero-Point Energy Real?* Ontological Aspects of Quantum Field Theory 2002 Available from: http://users.ox.ac.uk/~lina0174/vacuum.pdf.

8. Misner, C.; Thorne, K.; Wheeler, J., *Gravitation*. 21st ed. 1998: W. H. Freeman and Co.

9. Landau, Lifshitz, *A Course in Theoretical Physics: Quantum Electrodynamics*. Butterworth-Heinemann. Vol. 4. 1984.

10. Cornwall R. O., *Heat Engines of Extraordinary Efficiency and the General Principle of their Operation.* Preprints, 2017 (https://www.academia.edu/31637706/Heat_engines_of_extraordinary_efficiency_And_the_general_principle_of_their_operation).

11. Cornwall, R. O., *How to build a Maxwell Demon from a 2nd order Phase Change System.* 2013 (https://www.academia.edu/11587638/How_to_build_a_Maxwell_Demon_from_a_2nd_order_Phase_Change_System).

12. Cornwall, R. O., *A Novel Temporary Magnetic Remanence Thermodynamic Cycle.* 2013 (http://vixra.org/abs/1311.0089).

13. Sutton Biblarz, *Rocket Propulsion Elements*. 9th ed. 2017: Wiley.