Broken geodetics and Dark Matter

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Abstract

Is accepted, that the tidal forces rip falling (into Black Hole) astronaut into a subatomic cloud, and the end-state of this cloud is a singular point. However, is obvious then, that this cloud begins to shrink despite the tidal forces.

Indeed, a falling drop of fluid will not be stretched by tidal forces but will shrink to a point of zero volume and disappear, violating the law of conservation of energy. However, the place of disappearance will be located not necessarily in the singular center of the Black Hole. As solution to this puzzle the Dark Matter can be useful.

Ocean tides on Earth are also described by my formula. Yes, gravity stretches the Earth. But this is all in my formula. But if you fall into a Black Hole, the tidal forces will compress you. At first, the forces will stretch you apart. But then they will compress, even outside the event horizon. A star turning into a black hole is being compressed. Instead of being stretched, objects will be compressed.

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I. MOTIVATION OF THIS STUDY

I follow my "guiding star" in a way that I must be convinced (by me or others) if I have made a mistake. This mistake must be found, and I must be convinced that it is a mistake. This principle is my guiding star. Some journals have rejected some of my papers without even trying to convince me of having done mistakes. Besides logic, the scientific community always uses feelings, and then feelings can be positive or negative, as there are two options in the realm of feelings: scepticism or trust.

There is a historical case about Einstein. After his publication of the logical debunkment against Newton who thought the spacetime is absolute, hundreds of scientists were not accepting his debunkment. Therefore, the feeling of scepticism has stopped the "train" of science for four years! But if the mind of a reader would see that the logic of the paper seems not to be violated, the mind would trust this conclusion and accept the paper.

Considered motion in Kerr-Newman, Kerr, and Reissner-Nordström spacetimes. According to Stephen Hawking's "The Large Scale Structure of Space-Time" the Kerr spacetime is the most common case in our Universe. If you release a test-particle (from any position outside the Kerr Black Hole, but not at its equatorial plane $\theta = \pi/2$) from the rest frame, it will end up in an abrupt-end apart from the point of the spacetime singularity. As solution to this problem the Dark Matter "fix" is used.

I am talking about ripping-compression from the perspective of a coordinate frame co-moving with the falling object. Thus, it is the true effect and not a virtual one. The Strong Equivalence Principle tells us that physics laws are the same in all inertial reference frames; accordingly, the inner physics of the moving object have the known form only in the co-moving frame of coordinates, because only in this case simultaneoucy inside the object is given.

II. NEUTRAL TEST-PARTICLE IN KERR-NEWMAN SPACETIME

Taken from Ref. [1], the velocity components in the Kerr-Newman spacetime are given by

$$u^r \equiv dr/d\tau = -\sqrt{B}/(r^2 + a^2\cos^2\theta)\,, \tag{1}$$

$$u^\theta \equiv d\theta/d\tau = \sqrt{L - \cos^2\theta\,(a^2\,(\mu^2 - E^2) + L_z^2/\sin^2\theta)}/(r^2 + a^2\cos^2\theta)\,, \tag{2}$$

$$u^\phi \equiv d\phi/d\tau = (-(a\,E - L_z/\sin^2\theta) + a\,P/\Delta)/(r^2 + a^2\cos^2\theta)\,, \tag{3}$$

$$u^t \equiv dt/d\tau = (-a\,(a\,E\sin^2\theta - L_z) + (r^2+a^2)\,P/\Delta)/(r^2 + a^2\cos^2\theta)\,, \tag{4}$$

where $B := P^2 - \Delta\,(\mu^2\,r^2 + (L_z - a\,E)^2 + L)$, $P := E\,(r^2+a^2) - L_z\,a - q\,Q\,r$, $\Delta := r^2 - 2\,M\,r + a^2 + Q^2$.

Here $\mu = 0$ for null-geodetics, and $\mu = 1$ for time-like ones. A neutral test-particle has zero charge $q = 0$. L_z, L and E are constants of geodetic motion; M, Q and a are the three parameters of the Black Hole. τ is the geodetic parameter, e.g. the proper time of falling particle. t, r, θ and ϕ are spacetime coordinates. The physical curvature singularity is placed at $r = 0$, and requires unknown "Quantum Gravity". The General Relativity is applicable at spacetime points with $r \neq 0$, however there is no convincing inclusion of Dark Matter and Dark Energy into General Relativity [2]. I am adding to this the following effect never seen before: the abrupt-end-geodetics. My own solution to the problem is found in section VII.

III. THE FIRST SIGHT OF ABRUPT-END-GEODETICS

The velocity component of a test-particle is given by Eq. (1),

$$u^r \equiv \frac{d\,r}{d\tau} = -\frac{1}{r^2}\sqrt{B}\,, \tag{5}$$

where $B = E^2\,r^4 - (r^2 - 2\,M\,r + Q^2)\,r^2$.

In "geometrized" units (Q, M, r in meters) let us choose $Q = 1/5$ and $M = 1/2$. Zero initial velocity ($B = 0$ at $r = r_0 = 20$) requires a trajectory with

$$E = \frac{\sqrt{9501}}{100}\,. \tag{6}$$

Therefore

$$B = -\frac{499}{10000}\,r^4 + r^3 - \frac{1}{25}\,r^2\,, \tag{7}$$

which is negative in $r < r_m = 20/499$. Thus, at r_m one has $u^r = 0$.

This can mean a termination of the falling body. My study in Section V.B. below shows that photons are being terminated as well; the terminations are present also in a Kerr spacetime (see Section IV) as well as in naked singularity regimes (Section V). Such termination was never found yet. Indeed, it is not reported in Refs. [1, 3].

The value of E in Eq. (6) is slightly different for either slightly different parameter of spacetime (Q or M) or for slightly different initial velocities of the test particle. Therefore, my effect holds not for very specific parameters, but has a wide range of physically allowed parameters.

IV. THE ABRUPT-END IN NEUTRAL ($Q = 0$) KERR SPACETIME

The radial coordinate velocity of a test-body (falling from a large distance $r_0 = 20$ with zero initial velocity and $\theta_0 = \pi/4$) into a Kerr Black Hole with mass $M = 1/2$ and rotation $a = 1/4$) is given by Eq. (1),

$$u^r \equiv \frac{dr}{d\tau} = -\frac{\sqrt{B}}{r^2 + (1/16)\cos^2\theta} \,. \tag{8}$$

$$B = -\frac{640}{12801}\, r^4 + r^3 - \frac{742460}{155672961}\, r^2 + \frac{12481}{194576}\, r - \frac{62405}{622691844} \,. \tag{9}$$

Therefore, one must have $B \geq 0$, but for $r < r_m = 1/640$ one has $B < 0$. Therefore, there is no falling body in $0 \leq r < r_m$.

In addition to $a \neq 0$, in a more realistic scenario one also has $Q \neq 0$. Therefore, the singular state will be reached way before the curvature singularity. The remainings of this crushed body will never reach $r = 0$ because the trajectory is impossible in $r < r_m$. Note that the Black Hole tidal forces do not stretch the body apart but do compress to perfect point-size (see Section VI).

A. Kinetics at the point of abrupt-end

What are the derivatives of the trajectory (the worldline), what are the values of space coordinates, and what is the time for reaching the abrupt-end finite? Such curious bold questions are subjects in this section of the paper.

Take a look at the start of the paper, the calculation with the rotating Kerr Black Hole, at $r = r_m = 1/640$, $\theta = \theta_m = 3\,\pi/4$ the four-velocity space components vanish, $u^r = u^\theta = u^\phi = 0$. At this abrupt end the body is not moving. The time component is positive, finite and non-zero, $u^t = (1/12161)\sqrt{155672961}$, so the body is future-directed. The proper time for reaching the abrupt end is finite,

$$\Delta\tau = -\int \frac{dr}{u^r} < -2 \int \frac{dr}{u^r} < -2 \int_{1/640+0.02}^{1/640} 1/\sqrt{B}\, dr \,. \tag{10}$$

The latter inequalities hold because inside the Black Hole one has $r < 1$. In addition,

$$B > \left(\frac{319475839}{4981145600} - 2\,\frac{8228483}{99630695040}\,(r - 1/640)\right)(r - 1/640) \tag{11}$$

at least when $1/640 < r < 1/640 + 0.02$. So, there holds

$$B > \left(\frac{319475839}{4981145600} - 2\,\frac{8228483}{99630695040}\,(1/640 + 0.02 - 1/640)\right)(r - 1/640) > 0.05\,(r - 1/640)\,.$$

. Thus,

$$\Delta\tau < -(2/\sqrt{0.05})\int_{1/640+0.02}^{1/640} 1/\sqrt{r - 1/640}\,dr \approx 2.5298\,. \tag{12}$$

Even though the instant velocity is zero in $r = r_m$, the displacement velocity from $r = r_m + 0.02$ into the abrupt end at r_m is less than $2.5298/c \approx 84.32666667$ microseconds. By simple division one obtains $(0.02\,meters/(84.33\,mks) \approx 2.4$ million meters per second.

The position $\phi = \phi_m$ is the position of the abrupt end. Note that $|\phi_m| < \infty$, because $|d\phi/d\tau| < N =$ fixed holds always, so that

$$\phi_m = \int_0^{\tau_m} \frac{d\phi}{d\tau}\,d\tau < N\,\tau_m < \infty, \tag{13}$$

where τ_m is the proper time at the abrupt end.

V. RANGE OF r_m

Next we analyse the function $r_m = r_m(r_0, \theta_0, a, M)$ as the radial position of the abrupt end. The central singularity is found in $r = 0 < r_m$.

The r_m is the higher the closer the θ_0 is to the axis of rotation. Also, the r_m is the higher the higher is the a. a is critical if $a = M$. But if would be the naked singularity. For $a > M$, then the abrupt end will be at the distance $r_m \gg 0$.

Let us study the most dramatic case for r_m. It is obtained by taking the limits $\theta_0 \to 0+$ in the formulas for L_z,L and E, the latter given as solutions of the equations $u^r = u^\theta = u^\phi = 0$ at the starting point $(\theta_0 = 0,\ \phi_0,\ r_0)$. It turned out that

$$\begin{aligned} L_z &= 0\,, \qquad L = a^2\,\frac{2\,M\,r_0 - Q^2}{r_0^2 + a^2}\,, \\ E &= \frac{\sqrt{-2\,r_0^3\,M + r_0^4 + 2\,r_0^2\,a^2 + r_0^2\,Q^2 - 2\,M\,r_0\,a^2 + a^4 + a^2\,Q^2}}{r_0^2 + a^2}\,. \end{aligned} \tag{14}$$

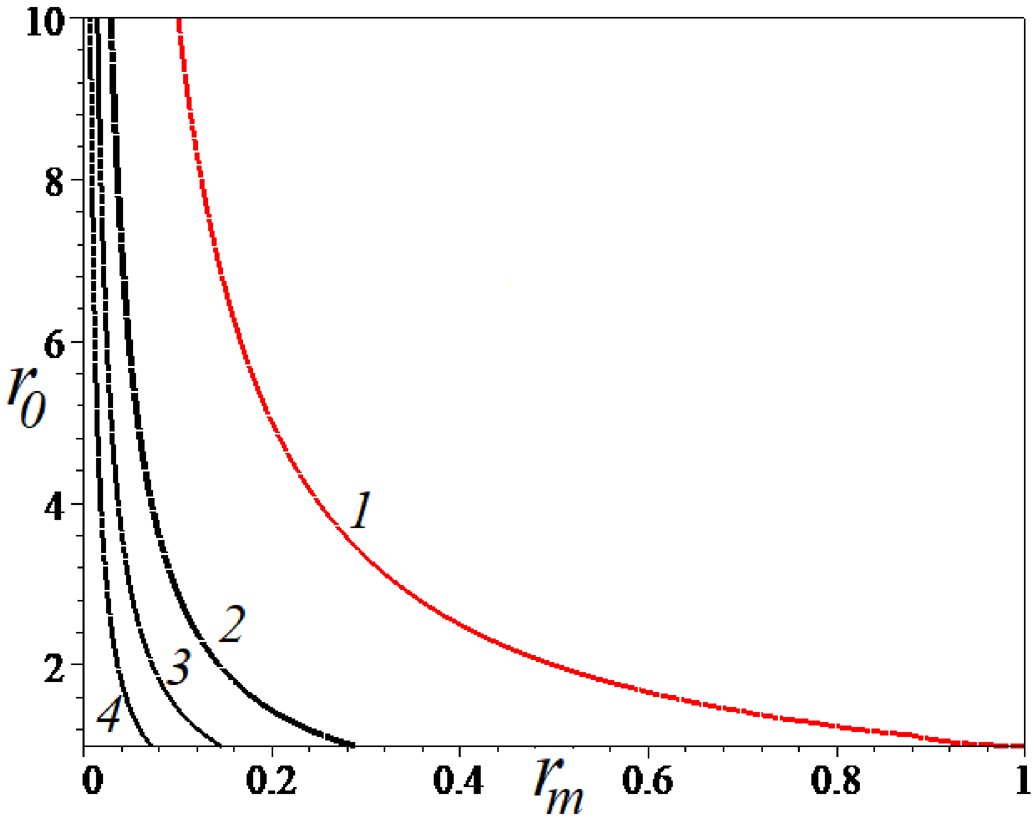

FIG. 1: The plot of r_m, curves (1): $\delta = 6/6$, (2): $\delta = 5/6$, (3): $\delta = 4/6$, (4): $\delta = 3/6$. The radius of BH is adjusted (through variation of M) to be the same for all curves: one meter.

Now, let us work with $Q = 0$. Then $B = 0$, if

$$a^4\, r_m - a^4\, r_0 + a^2\, r_m^3 + a^2\, r_m\, r_0^2 - 2\, a^2\, r_0\, r_m^2 + r_m^3\, r_0^2 - r_0\, r_m^4 = 0\,. \tag{15}$$

Obviously, r_m is independent of M.

Because the outer event horizon is placed at $r_s = M\,(1 + \sqrt{1-\delta^2})$ with $a = \delta\, M$, then holding $r_s = 1$ we can adjust M for any $0 \leq \delta \leq 1$ by

$$M = \frac{1}{1 + \sqrt{1-\delta^2}}\,. \tag{16}$$

In this way we can produce the convincing Fig. 1.

A. Reissner-Nordström

Now we consider the collapse of a dust cloud where each dust particle has its own electric charge. We expect that the Reissner–Norström (R.-N.) spacetime will be produced. Therefore, the R.-N. solution is physical. In such spacetime the r_m is practically independent of

r_0, i.e. independent from the parameter of the test object. Thus, the abrupt end in the R.-N. case can have the status of an intrinsic parameter of the Black Hole. The higher r_m, the higher is the charge Q. We can read $Q^2/(2M) < r_m < M - \sqrt{M^2 - Q^2}$. However, the faster r_m tends to $Q^2/(2M)$ the smaller Q/M is. If r_0 is at the outer event horizon $r_0 \to M + \sqrt{M^2 - Q^2}$, the r_m is at the inner event horizon: $r_m \to M - \sqrt{M^2 - Q^2}$. There is always $Q^2/(2M) < M - \sqrt{M^2 - Q^2}$.

B. The case of null-geodetics (worldline of photons)

The effect is possible only for the Kerr-Newman Black Hole, hereby (from Eq. (1) with $\mu = 0$)

$$r_m^4 + r_m^2\, a^2 + 2\, a^2\, M\, r_m - a^2\, Q^2 = 0\,.$$

For simplicity we have chosen $L_z = 0$, $L = 0$, $\theta_0 = 0$.

VI. SMALL DROP OF PERFECT FLUID

A small drop of perfec fluid has nearly the same velocity vector throughout the drop. The density ρ obbeys

$$d\rho/d\tau = -(\rho + p(\rho))\, u^\mu_{;\mu} \tag{17}$$

(cf. pp. 226–227 in Ref. [1]), where $p = p(\rho)$ is the pressure. Here the covariant four-divergence $u^\mu_{;\mu}$ uses the Christoffel symbols. Despite of our intuition the Black Hole can compress the drop to a zero size, i.e. $u^\mu_{;\mu} \sim 1/u^r \to -\infty$ at the abrupt end. In case $M = 1/2$, $Q = a = 0$ the

$$\frac{d\rho}{d\tau} = (\rho + p(\rho))\, \frac{3\, r_0 - 4\, r}{2\, \sqrt{r_0\, r^3\, (r_0 - r)}} \tag{18}$$

The $d\rho/d\tau = 0$ at $r = r_f = 3\, r_0/4$, it is the start of compression. Here $r_m = 0$, but generally $r_m \neq 0$

The r_m is independent of $p(\rho)$ – the resistance of the falling matter. But to acknowledge the resistance, the following idea can be useful: if the spatial density of energy-mass is $\rho < \rho_c$, usual physics is applicable. However, for $\rho \geq \rho_c$ the matter part vanishes.

By the way, the compression begins even outside the Black Hole – even in the area of weak gravity (in case if $r_f \gg 2M$) where Newton's formulas should be true. I have calculated that the famous Geodetics Deviation Equation gives the same result as the one presented here. [4]

VII. HOW TO INCLUDE DARK MATTER IN GENERAL RELATIVITY?

Einstein's known equation

$$G^{\mu\nu} + \Lambda\, g^{\mu\nu} = 8\pi\, T^{\mu\nu}$$

runs into a problem, because of the abrupt end motion and the vanishing. So, the following mathematical extension of it is possible,

$$G^{\mu\nu} + \Lambda\, g^{\mu\nu} = 8\pi (T^{\mu\nu} + K^{\mu\nu})\,,$$

where $K^{\mu\nu}$ cannot be detected in our particle detectors, because it is just mathematical fixing to General Relativity, i.e. not a new kind of "actual" matter. I call it virtual matter. In case of the vanishing of covariant divergence $K^{\mu\nu}_{;\mu} = 0$ I call it Dark Matter. In my opinion, the Dark Energy above $\Lambda\, g^{\mu\nu}$ is also a kind of mathematical fixing, not matter.

Indeed, in the task we have only a falling drop of liquid which vanishes. Thus, the fix to this problem is not a new kind of matter, because we have only one kind of matter in the task – a drop of liquid.

One should include such a concept as Virtual Terms – mathematical insertions into the equations and laws of nature which are made not from fundamental premises but "by hand" in order to fit the theory under observation. An example of such insertions is Dark Matter and Dark Energy. Therefore, these cannot be directly detected, but it is possible to measure their effect on nature. As a prime example, the Dark Matter anomaly has acted on the space-time grid so much that it created an additional force of attraction of stars to the center of their galaxy. By the way, the proton radius measured by many experimenters was different in different years. This riddle did not find yet a solution (see e.g. Jean-Philippe Karr, Dominique Marchand, "Progress on the proton-radius puzzle", Nature 2019). I solve this problem with a virtual insert VT into the radius value: $r = R + VT$.

The demand of "energy conservation" is not applicable to Virtual Matter, because it is not subject to measurements. So, one would not measure the negative energy.

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