The coupled Einstein equations (II)

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Abstract

The coupled Einstein equations are defined for a manifold with two riemannian metrics. We make use of the mixed Riemann curvature.

1 The Einstein equations

Let $(M, g)$ be a riemannian manifold with riemannian curvature $R$.

\[ r_g(x, y, z, t) = g(R(x, y)z, t) \]

\[ Ric_g(x, y) = \sum_i r_g(x, e_i, y, e_i) \]

The Einstein equations are then [Be]:

\[ Ric_g = \lambda g \]

2 The coupled Einstein equations

Let $(g, g')$ be two metrics over the manifold $M$, the two Levi-Civita connections are $(\nabla, \nabla')$. Then the mixed Riemann curvature is:

\[ R_{g,g'}(X, Y) = \nabla_X \nabla'_Y + \nabla'_X \nabla_Y - \nabla_Y \nabla'_X - \nabla'_X \nabla_Y - \nabla_{[X,Y]} - \nabla'_{[X,Y]} \]

The mixed Ricci curvature is:

\[ Ric_{g,g'}(X, Y) = tr(Z \to R_{g,g'}(X, Z)Y) \]

The coupled Einstein equations are:

\[ Ric_{g,g'} = \mu (g + g') \]

\[ Ric_g = \lambda g \]

\[ Ric_{g'} = \lambda' g' \]

References