In the following I talk about functions in particular I study the behavior of this near infinity assuming the notion of function, limit and derivate is know.

Let a function $f: \mathbb{R} \to \mathbb{R}$ such that $\lim_{x \to +\infty} f(x) = +\infty$ if $\lim_{x \to +\infty} \frac{f(x)}{x} > 1$ than exist $a(\infty) \in \mathbb{R}$ such that $\forall x_0 \geq a(\infty) \lim_{x \to x_0} f(x) = +\infty$ so the function $f$ is not bijective.

**Example 0.1.** Let function $f: \mathbb{R} \to \mathbb{R}$ defined by $x \mapsto x^2$. $\lim_{x \to +\infty} \frac{x^2}{x} = +\infty > 1$ so the function is bijective for $x \in [0,a(\infty)] \subset \mathbb{R}$ and $f(x) \in [0,\infty]$ so $f^{-1}(x): [0,\infty] \to [0,a(\infty)]$ with $\lim_{x \to +\infty} f^{-1}(x) = a(\infty)$.

An important result is the following

**Theorem 0.2.** Let $f$ a function, $f$ have an oblique asymptote if and only if $f'$ have an horizontal asymptote.

**Proof.** we prove the Theorem for $x$ tends $+\infty$ (idem for $x$ tends $-\infty$)

$\Rightarrow$) $f$ have asymptote $y = mx + q$ for $x$ tends $+\infty$ so $q = \lim_{x \to +\infty} (f(x) - mx)$. \[ \frac{d}{dx}(q) = 0 = \lim_{x \to +\infty} \frac{d}{dx} f - m \Rightarrow \lim_{x \to +\infty} f' = m. \]

$\Leftarrow$) $f'$ have an asymptote $y = m$ that is $\lim_{x \to +\infty} f' - m = 0 \Rightarrow \int 0 \, dx = c_1 = \lim_{x \to +\infty} \int (f' - m) \, dx = \lim_{x \to +\infty} (f - mx + c_2)$ if $q = c_1 - c_2$ so $q = \lim_{x \to +\infty} (f - mx)$. \hfill \Box

**Example 0.3.** Let the function $y = \ln(x)$ $y' = \frac{1}{x}$ $\lim_{x \to +\infty} \frac{1}{x} = 0$ so $y'$ have an orizzontal asymptote $y = 0$ so the function have asymptote $y = q$ where $q$ is the real $a(\infty)$. 

1