The coupled Einstein equations

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Abstract

The coupled Einstein equations are defined for a manifold with two Riemannian metrics.

1 The Einstein equations

Let \((M, g)\) be a Riemannian manifold with Riemannian curvature \(R\).

\[
    r_g(x, y, z, t) = g(R(x, y)z, t)
\]

\[
    \text{Ric}(g)(x, y) = \sum_i r_g(x, e_i, y, e_i)
\]

The Einstein equations are then [Be]:

\[
    \text{Ric}(g) = \lambda g
\]

2 The coupled Einstein equations

Let \((g, g')\) be two metrics over the manifold \(M\), then the coupled Einstein equations are:

\[
    \text{Ric}(g) = \lambda g'
\]

\[
    \text{Ric}(g') = \lambda' g
\]

If \(g = g'\), these equations are obviously the Einstein equations.

References