

On various Ramanujan equations: mathematical connections with some cosmological parameters and some sectors of Particle physics, in particular the masses of the two Pion mesons.

Michele Nardelli¹, Antonio Nardelli

Abstract

In this research thesis, we have analyzed various Ramanujan equations and described the new possible mathematical connections with some cosmological parameters and some sectors of Particle physics, in particular the masses of the two Pion mesons.

¹ M.Nardelli have studied by Dipartimento di Scienze della Terra Università degli Studi di Napoli Federico II, Largo S. Marcellino, 10 - 80138 Napoli, Dipartimento di Matematica ed Applicazioni “R. Caccioppoli” - Università degli Studi di Napoli “Federico II” – Polo delle Scienze e delle Tecnologie Monte S. Angelo, Via Cintia (Fuorigrotta), 80126 Napoli, Italy



<https://twitter.com/royalsociety/status/1076386910845710337>



<https://www.insidescience.org/news/every-black-hole-contains-new-universe>

Now, we have that:

Page 284-285

if $n-2$ be a multiple of 4,

We take $n-2 = 24$; $n = 26$

(2). $\int_0^{\infty} x^{2n} e^{-x^2} dx = \frac{\sqrt{\pi}}{2} e^{-2a} a^n \left\{ 1 + \frac{n(n+1)}{4a} + \frac{(n-1)n(n+1)(n+2)}{4 \cdot 8 \cdot a^2} + \frac{(n-2)(n-1)n(n+1)(n+2)(n+3)}{4 \cdot 8 \cdot 12 \cdot a^3} + \dots \right\}$

Thence: $a = 1$, $b = 2$, $p = 6$, $q = 3$ and $r = -3$; $n = 26$; $x = 2$

$$\left(\left(\frac{1}{2} \sqrt{\pi} \frac{1}{e^2} \right) \cdot \left(\frac{26 \cdot 27}{4} + \frac{1}{32} (25 \cdot 26 (27 \cdot 28)) + \frac{24 \cdot 25 \cdot 26 (27 \cdot 28 \cdot 29)}{32 \cdot 12} \right) \right)$$

Input:

$$\left(\frac{1}{2} \sqrt{\pi} \times \frac{1}{e^2} \right) \left(\frac{26 \times 27}{4} + \frac{1}{32} (25 \times 26 (27 \times 28)) + \frac{24 \times 25 \times 26 (27 \times 28 \times 29)}{32 \times 12} \right)$$

Exact result:

$$\frac{3624777 \sqrt{\pi}}{8 e^2}$$

Decimal approximation:

108686.9193152684672515923115733778728353477376809511182260...

108686.9193...

$\left(\left(\left(\frac{1}{2} \sqrt{\pi} \frac{1}{e^2}\right)\right)\right) *$

$\left(\left(\left(\frac{26 \times 27}{4} + \frac{25 \times 26 (27 \times 28)}{32} + \frac{24 \times 25 \times 26 (27 \times 28 \times 29)}{32 \times 12}\right)\right)\right) / 64 + (18 + 11 + 2)$

Input:

$$\left(\frac{1}{2} \sqrt{\pi} \times \frac{1}{e^2}\right) \left(\frac{1}{64} \left(\frac{26 \times 27}{4} + \frac{1}{32} (25 \times 26 (27 \times 28)) + \frac{24 \times 25 \times 26 (27 \times 28 \times 29)}{32 \times 12}\right)\right) + (18 + 11 + 2)$$

Exact result:

$$31 + \frac{3624777 \sqrt{\pi}}{512 e^2}$$

Decimal approximation:

1729.233114301069800806129868334029263052308401264861222281...

1729.2331143...

This result is very near to the mass of candidate glueball $f_0(1710)$ meson. Furthermore, 1728 occurs in the algebraic formula for the j -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

Alternate form:

$$\frac{15872 e^2 + 3624777 \sqrt{\pi}}{512 e^2}$$

Series representations:

$$\frac{\left(\frac{26 \times 27}{4} + \frac{25}{32} \times 26 (27 \times 28) + \frac{24 \times 27 (28 \times 29) 25 \times 26}{32 \times 12}\right) \sqrt{\pi}}{64 \times 2 e^2} + (18 + 11 + 2) =$$

$$31 + \frac{3624777 \sqrt{-1 + \pi} \sum_{k=0}^{\infty} (-1 + \pi)^{-k} \binom{\frac{1}{2}}{k}}{512 e^2}$$

$$\frac{\left(\frac{26 \times 27}{4} + \frac{25}{32} \times 26 (27 \times 28) + \frac{24 \times 27 (28 \times 29) 25 \times 26}{32 \times 12}\right) \sqrt{\pi}}{64 \times 2 e^2} + (18 + 11 + 2) =$$

$$31 + \frac{3624777 \sqrt{-1 + \pi} \sum_{k=0}^{\infty} \frac{(-1)^k (-1 + \pi)^{-k} \left(-\frac{1}{2}\right)_k}{k!}}{512 e^2}$$

$$\frac{\left(\frac{26 \times 27}{4} + \frac{25}{32} \times 26 (27 \times 28) + \frac{24 \times 27 (28 \times 29) 25 \times 26}{32 \times 12}\right) \sqrt{\pi}}{64 \times 2 e^2} + (18 + 11 + 2) =$$

$$31 + \frac{3624777 \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (\pi - z_0)^k z_0^{-k}}{k!}}{512 e^2} \text{ for not } ((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$$

(((1/2* sqrt(Pi) 1/e^(2)))) *
 (((((26*27)/4+(25*26(27*28))/32+(24*25*26(27*28*29))/(32*12))))-4096*8-
 (2048+512-128))

Input:

$$\left(\frac{1}{2} \sqrt{\pi} \times \frac{1}{e^2}\right) \left(\frac{26 \times 27}{4} + \frac{1}{32} (25 \times 26 (27 \times 28)) + \frac{24 \times 25 \times 26 (27 \times 28 \times 29)}{32 \times 12}\right) -$$

$$4096 \times 8 - (2048 + 512 - 128)$$

Exact result:

$$\frac{3624777 \sqrt{\pi}}{8 e^2} - 35200$$

Decimal approximation:

73486.91931526846725159231157337787283534773768095111822603...

73486.91931...

Alternate forms:

$$-\frac{281600 e^2 - 3624777 \sqrt{\pi}}{8 e^2}$$

$$\frac{3624777 \sqrt{\pi} - 281600 e^2}{8 e^2}$$

Series representations:

$$\frac{\left(\frac{26 \times 27}{4} + \frac{25}{32} \times 26 (27 \times 28) + \frac{24 \times 27 (28 \times 29) 25 \times 26}{32 \times 12}\right) \sqrt{\pi}}{2 e^2} - 4096 \times 8 - (2048 + 512 - 128) =$$

$$-35200 + \frac{3624777 \sqrt{-1 + \pi} \sum_{k=0}^{\infty} (-1 + \pi)^{-k} \left(\frac{1}{2}\right)_k}{8 e^2}$$

$$\begin{aligned} & \frac{\left(\frac{26 \times 27}{4} + \frac{25}{32} \times 26 (27 \times 28) + \frac{24 \times 27 (28 \times 29) 25 \times 26}{32 \times 12}\right) \sqrt{\pi}}{2 e^2} - 4096 \times 8 - (2048 + 512 - 128) = \\ & -35200 + \frac{3624777 \sqrt{-1+\pi} \sum_{k=0}^{\infty} \frac{(-1)^k (-1+\pi)^{-k} \left(-\frac{1}{2}\right)_k}{k!}}{8 e^2} \\ & \frac{\left(\frac{26 \times 27}{4} + \frac{25}{32} \times 26 (27 \times 28) + \frac{24 \times 27 (28 \times 29) 25 \times 26}{32 \times 12}\right) \sqrt{\pi}}{2 e^2} - 4096 \times 8 - (2048 + 512 - 128) = \\ & -35200 + \frac{3624777 \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (\pi-z_0)^k z_0^{-k}}{k!}}{8 e^2} \\ & \text{for not } ((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0)) \end{aligned}$$

Thence, we have the following mathematical connection:

$$\left(\frac{\left(\frac{1}{2} \sqrt{\pi} \times \frac{1}{e^2}\right) \left(\frac{26 \times 27}{4} + \frac{1}{32} (25 \times 26 (27 \times 28)) + \frac{24 \times 25 \times 26 (27 \times 28 \times 29)}{32 \times 12}\right) - 4096 \times 8 - (2048 + 512 - 128)}{1} \right) = 73486.919 \Rightarrow$$

$$\begin{aligned} & \Rightarrow -3927 + 2 \left(\sqrt[13]{ N \exp \left[\int d\hat{\sigma} \left(-\frac{1}{4u^2} \mathbf{P}_i D \mathbf{P}_i \right) \right] |Bp\rangle_{NS} + \int [d\mathbf{X}^\mu] \exp \left\{ \int d\hat{\sigma} \left(-\frac{1}{4v^2} D \mathbf{X}^\mu D^2 \mathbf{X}^\mu \right) \right\} |X^\mu, X^i = 0\rangle_{NS} } \right) = \\ & -3927 + 2 \sqrt[13]{ 2.2983717437 \times 10^{59} + 2.0823329825883 \times 10^{59} } \\ & = 73490.8437525.... \Rightarrow \end{aligned}$$

$$\Rightarrow \left(A(r) \times \frac{1}{B(r)} \left(-\frac{1}{\phi(r)} \right) \times \frac{1}{e^{\Lambda(r)}} \right) \Rightarrow$$

$$\Rightarrow \left(-0.000029211892 \times \frac{1}{0.0003644621} \left(-\frac{1}{0.0005946833} \right) \times \frac{1}{0.00183393} \right) =$$

$$= 73491.78832548118710549159572042220548025195726563413398700\dots$$

$$= 73491.7883254\dots \Rightarrow$$

$$\left(I_{21} \ll \int_{-\infty}^{+\infty} \exp\left(-\left(\frac{t}{H}\right)^2\right) \left| \sum_{\lambda \leq p^{i-\varepsilon_1}} \frac{a(\lambda)}{\sqrt{\lambda}} B(\lambda) \lambda^{-i(T+t)} \right|^2 dt \ll \right.$$

$$\left. \ll H \left\{ \left(\frac{4}{\varepsilon_2 \log T} \right)^{2r} (\log T) (\log X)^{-2\beta} + (\varepsilon_2^{-2r} (\log T)^{-2r} + \varepsilon_2^{-r} h_1^r (\log T)^{-r} T^{-\varepsilon_1} \right\} \right)$$

$$/(26 \times 4)^2 - 24 = \left(\frac{7.9313976505275 \times 10^8}{(26 \times 4)^2 - 24} \right) = 73493.30662\dots$$

Mathematical connections with the boundary state corresponding to the NSNS-sector of N Dp-branes in the limit of $u \rightarrow \infty$, with the ratio concerning the general asymptotically flat solution of the equations of motion of the p-brane and with the Karatsuba's equation concerning the zeros of a special type of function connected with Dirichlet series.

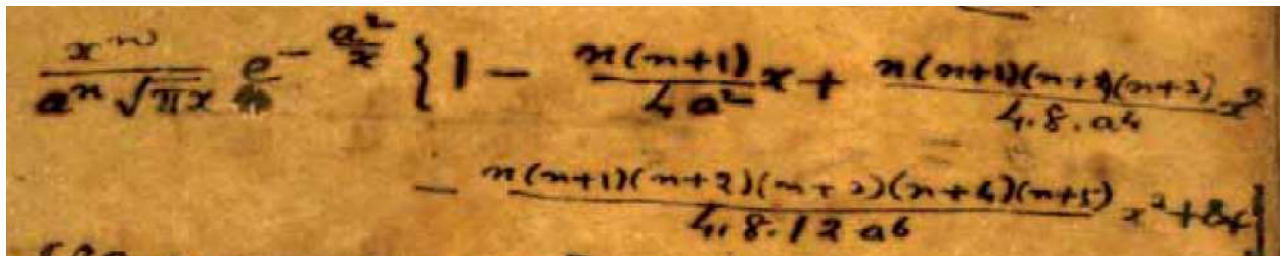
Pag,286

(7) If $\int_0^{\infty} e^{-px} \phi(x) dx = \frac{e^{-2ap}}{p^{n+1}}$, then

$$\phi(x) = \frac{x^n}{\sqrt{\pi x}} e^{-\frac{a^2}{x}} \int_0^{\infty} e^{-az - \frac{xz^2}{4}} \frac{\pi^{n+1}}{\Gamma(n+1)} dz.$$

$$= \frac{x^n}{a^n \sqrt{\pi x}} e^{-\frac{a^2}{x}} \left\{ 1 - \frac{\pi(n+1)x}{4a^2} + \frac{\pi(n+1)(n+3)(n+2)x^2}{4 \cdot 8 \cdot a^4} - \frac{\pi(n+1)(n+3)(n+2)(n+4)(n+1)x^3 + 8a^6}{4 \cdot 8 \cdot 12 \cdot a^6} \right\}$$

Thence: $a = 1$, $b = 2$, $p = 6$, $q = 3$ and $r = -3$; $n = 10$; $x = 2$



$$(2^{10})/(\text{sqrt}(2\text{Pi})) \exp(-1/2) [1-(10*2*(11))/4+(10*4*(11*12*13))/32-(10*8*(11*12*13*14*15))/(4*8*12)]$$

Input:

$$\frac{2^{10}}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}\right) \left(1 - \frac{1}{4} (10 \times 2 \times 11) + \frac{1}{32} (10 \times 4 (11 \times 12 \times 13)) - \frac{10 \times 8 (11 \times 12 \times 13 \times 14 \times 15)}{4 \times 8 \times 12}\right)$$

Exact result:

$$-37367808 \sqrt{\frac{2}{e\pi}}$$

Decimal approximation:

$$-1.8083831150904482039444314932437973843984174194996180... \times 10^7$$

$$-18083831.150904482$$

$$-1.8083831150904482 \times 10^7$$

Series representations:

$$\frac{\left(\exp\left(-\frac{1}{2}\right)\left(1 - \frac{10 \times 2 \times 11}{4} + \frac{10}{32} \times 4 (11 \times 12 \times 13) - \frac{10 \times 8 (11 \times 12 \times 13 \times 14 \times 15)}{4 \times 8 \times 12}\right)\right) 2^{10}}{\sqrt{2\pi}} = \frac{74735616 \exp\left(-\frac{1}{2}\right)}{\sqrt{-1+2\pi} \sum_{k=0}^{\infty} (-1+2\pi)^{-k} \binom{\frac{1}{2}}{k}}$$

$$\frac{\left(\exp\left(-\frac{1}{2}\right)\left(1 - \frac{10 \times 2 \times 11}{4} + \frac{10}{32} \times 4(11 \times 12 \times 13) - \frac{10 \times 8(11 \times 12 \times 13 \times 14 \times 15)}{4 \times 8 \times 12}\right)\right) 2^{10}}{\sqrt{2\pi}} =$$

$$-\frac{74735616 \exp\left(-\frac{1}{2}\right)}{\sqrt{-1+2\pi} \sum_{k=0}^{\infty} \frac{(-1)^k (-1+2\pi)^{-k} \left(-\frac{1}{2}\right)_k}{k!}}$$

$$\frac{\left(\exp\left(-\frac{1}{2}\right)\left(1 - \frac{10 \times 2 \times 11}{4} + \frac{10}{32} \times 4(11 \times 12 \times 13) - \frac{10 \times 8(11 \times 12 \times 13 \times 14 \times 15)}{4 \times 8 \times 12}\right)\right) 2^{10}}{\sqrt{2\pi}} =$$

$$-\frac{74735616 \exp\left(-\frac{1}{2}\right)}{\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2\pi - z_0)^k z_0^{-k}}{k!}} \text{ for not } ((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$$

And:

$$\sqrt{\left(\left(\left(\left(-2^{10}\right)/\left(\sqrt{2\pi}\right)\right) \exp\left(-\frac{1}{2}\right) \left[1 - \frac{10 \times 2 \times 11}{4} + \frac{10 \times 4 \times (11 \times 12 \times 13)}{32} - \frac{10 \times 8 \times (11 \times 12 \times 13 \times 14 \times 15)}{4 \times 8 \times 12}\right]\right)\right) - 144 - 12}$$

Input:

$$\sqrt{\left(-\frac{2^{10}}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}\right) \left(1 - \frac{1}{4} (10 \times 2 \times 11) + \frac{1}{32} (10 \times 4 (11 \times 12 \times 13)) - \frac{10 \times 8 (11 \times 12 \times 13 \times 14 \times 15)}{4 \times 8 \times 12}\right)\right) - 144 - 12}$$

Exact result:

$$64 \sqrt{9123} \sqrt[4]{\frac{2}{e\pi}} - 156$$

Decimal approximation:

4096.508806681590170348462033510757690661965244591610596046...
4096.5088.....

Alternate form:

$$4 \left(16 \sqrt{9123} \sqrt[4]{\frac{2}{e\pi}} - 39 \right)$$

Series representations:

$$\sqrt{\frac{\left(\exp\left(-\frac{1}{2}\right)\left(1 - \frac{10 \times 2 \times 11}{4} + \frac{10}{32} \times 4(11 \times 12 \times 13) - \frac{10 \times 8(11 \times 12 \times 13 \times 14 \times 15)}{4 \times 8 \times 12}\right)\right)(-2^{10})}{\sqrt{2\pi}}}$$

$$144 - 12 =$$

$$-156 + \sqrt{-1 + \frac{74735616 \exp\left(-\frac{1}{2}\right)}{\sqrt{2\pi}} \sum_{k=0}^{\infty} \binom{\frac{1}{2}}{k} \left(-1 + \frac{74735616 \exp\left(-\frac{1}{2}\right)}{\sqrt{2\pi}}\right)^{-k}}$$

$$\sqrt{\frac{\left(\exp\left(-\frac{1}{2}\right)\left(1 - \frac{10 \times 2 \times 11}{4} + \frac{10}{32} \times 4(11 \times 12 \times 13) - \frac{10 \times 8(11 \times 12 \times 13 \times 14 \times 15)}{4 \times 8 \times 12}\right)\right)(-2^{10})}{\sqrt{2\pi}}}$$

$$144 - 12 =$$

$$-156 + \sqrt{-1 + \frac{74735616 \exp\left(-\frac{1}{2}\right)}{\sqrt{2\pi}} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(-1 + \frac{74735616 \exp\left(-\frac{1}{2}\right)}{\sqrt{2\pi}}\right)^{-k}}{k!}}$$

$$\sqrt{\frac{\left(\exp\left(-\frac{1}{2}\right)\left(1 - \frac{10 \times 2 \times 11}{4} + \frac{10}{32} \times 4(11 \times 12 \times 13) - \frac{10 \times 8(11 \times 12 \times 13 \times 14 \times 15)}{4 \times 8 \times 12}\right)\right)(-2^{10})}{\sqrt{2\pi}}}$$

$$144 - 12 = -156 + \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{74735616 \exp\left(-\frac{1}{2}\right)}{\sqrt{2\pi}} - z_0\right)^k}{k!} z_0^{-k}$$

for not $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

And:

$$\left(\sqrt{\left(\left(\left(\left(\left(-2^{10}\right)/\left(\sqrt{2\pi}\right)\right) \exp\left(-\frac{1}{2}\right) \left[1 - \frac{10 \times 2 \times 11}{4} + \frac{10 \times 4 \times (11 \times 12 \times 13)}{32} - \frac{10 \times 8 \times (11 \times 12 \times 13 \times 14 \times 15)}{4 \times 8 \times 12}\right]\right)\right)\right)\right)\right) + 24 \times 5$$

Input:

$$\sqrt{\left(-\frac{2^{10}}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}\right) \left(1 - \frac{1}{4}(10 \times 2 \times 11) + \frac{1}{32}(10 \times 4(11 \times 12 \times 13)) - \frac{10 \times 8(11 \times 12 \times 13 \times 14 \times 15)}{4 \times 8 \times 12}\right)\right) + 24 \times 5}$$

Exact result:

$$120 + 64 \sqrt{9123} \sqrt[4]{\frac{2}{e\pi}}$$

Decimal approximation:

4372.508806681590170348462033510757690661965244591610596046...

4372.5088....

Alternate form:

$$8 \left(15 + 8 \sqrt{9123} \sqrt[4]{\frac{2}{e \pi}} \right)$$

Series representations:

$$\sqrt{\frac{\left(\exp\left(-\frac{1}{2}\right)\left(1 - \frac{10 \times 2 \times 11}{4} + \frac{10}{32} \times 4 (11 \times 12 \times 13) - \frac{10 \times 8 (11 \times 12 \times 13 \times 14 \times 15)}{4 \times 8 \times 12}\right)\right)(-2^{10})}{\sqrt{2 \pi}}} +$$

$$24 \times 5 = 120 + \sqrt{-1 + \frac{74735616 \exp\left(-\frac{1}{2}\right)}{\sqrt{2 \pi}} \sum_{k=0}^{\infty} \binom{\frac{1}{2}}{k} \left(-1 + \frac{74735616 \exp\left(-\frac{1}{2}\right)}{\sqrt{2 \pi}}\right)^{-k}}$$

$$\sqrt{\frac{\left(\exp\left(-\frac{1}{2}\right)\left(1 - \frac{10 \times 2 \times 11}{4} + \frac{10}{32} \times 4 (11 \times 12 \times 13) - \frac{10 \times 8 (11 \times 12 \times 13 \times 14 \times 15)}{4 \times 8 \times 12}\right)\right)(-2^{10})}{\sqrt{2 \pi}}} +$$

$$24 \times 5 = 120 + \sqrt{-1 + \frac{74735616 \exp\left(-\frac{1}{2}\right)}{\sqrt{2 \pi}} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(-1 + \frac{74735616 \exp\left(-\frac{1}{2}\right)}{\sqrt{2 \pi}}\right)^{-k}}{k!}}$$

$$\sqrt{\frac{\left(\exp\left(-\frac{1}{2}\right)\left(1 - \frac{10 \times 2 \times 11}{4} + \frac{10}{32} \times 4 (11 \times 12 \times 13) - \frac{10 \times 8 (11 \times 12 \times 13 \times 14 \times 15)}{4 \times 8 \times 12}\right)\right)(-2^{10})}{\sqrt{2 \pi}}} +$$

$$24 \times 5 = 120 + \sqrt{z_0 \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{74735616 \exp\left(-\frac{1}{2}\right)}{\sqrt{2 \pi}} - z_0\right)^k}{k!} z_0^{-k}}$$

for not $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

$$1/\left(\left(1 + 1/\left(\left(\left(\left(\sqrt{\left(\left(\left(-2^{10}\right)/\left(\sqrt{2\pi}\right)\right)\right)\right)\right)\right)\right)\right)\right)\left[1 - \left(\frac{10 \times 2 \times (11)}{4} + \frac{10 \times 4 \times (11 \times 12 \times 13)}{32} - \frac{10 \times 8 \times (11 \times 12 \times 13 \times 14 \times 15)}{4 \times 8 \times 12}\right)\right]\right)\right)\right)$$

Input:

$$1 + \frac{1}{\sqrt{-\frac{2^{10}}{\sqrt{2 \pi}} \exp\left(-\frac{1}{2}\right)\left(1 - \frac{1}{4} (10 \times 2 \times 11) + \frac{1}{32} (10 \times 4 (11 \times 12 \times 13)) - \frac{10 \times 8 (11 \times 12 \times 13 \times 14 \times 15)}{4 \times 8 \times 12}\right)}}$$

Exact result:

$$\frac{1}{1 + \frac{\sqrt[4]{\frac{e\pi}{2}}}{64\sqrt{9123}}}$$

Decimal approximation:

0.999764899981297991430982364499774708115031635025582195985...

0.9997648999812979..... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}}}{1 + \sqrt[5]{\sqrt{\varphi^5 4\sqrt{5^3} - 1}} - \varphi + 1} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

and to the dilaton value **0.989117352243 = ϕ**

Alternate forms:

$$\frac{1167744}{1167744 + 2^{3/4} \sqrt{9123} \sqrt[4]{e\pi}}$$

$$\frac{64 \sqrt[4]{2} \sqrt{9123}}{64 \sqrt[4]{2} \sqrt{9123} + \sqrt[4]{e\pi}}$$

Series representations:

$$1 + \frac{1}{\sqrt{\frac{1}{\left(\exp\left(-\frac{1}{2}\right)\left(1 - \frac{10 \times 2 \times 11}{4} + \frac{10}{32} \times 4 (11 \times 12 \times 13) - \frac{10 \times 8 (11 \times 12 \times 13 \times 14 \times 15)}{4 \times 8 \times 12}\right)\right)} (-2^{10})}} \sqrt{2\pi}}$$

$$1 + \frac{1}{\sqrt{-1 + \frac{74735616 \exp\left(-\frac{1}{2}\right)}{\sqrt{2\pi}} \sum_{k=0}^{\infty} \binom{\frac{1}{2}}{k} \left(-1 + \frac{74735616 \exp\left(-\frac{1}{2}\right)}{\sqrt{2\pi}}\right)^{-k}}}$$

$$\begin{aligned}
& \frac{1}{1 + \frac{1}{\sqrt{\frac{(\exp(-\frac{1}{2}) (1 - \frac{10 \times 2 \times 11}{4} + \frac{10}{32} \times 4 (11 \times 12 \times 13) - \frac{10 \times 8 (11 \times 12 \times 13 \times 14 \times 15))}{4 \times 8 \times 12})) (-2^{10})}{\sqrt{2\pi}}}}} = \\
& \frac{1}{1 + \frac{1}{\sqrt{-1 + \frac{74\,735\,616 \exp(-\frac{1}{2})}{\sqrt{2\pi}} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(-1 + \frac{74\,735\,616 \exp(-\frac{1}{2})}{\sqrt{2\pi}}\right)^{-k}}{k!}}}}} = \\
& \frac{1}{1 + \frac{1}{\sqrt{\frac{(\exp(-\frac{1}{2}) (1 - \frac{10 \times 2 \times 11}{4} + \frac{10}{32} \times 4 (11 \times 12 \times 13) - \frac{10 \times 8 (11 \times 12 \times 13 \times 14 \times 15))}{4 \times 8 \times 12})) (-2^{10})}{\sqrt{2\pi}}}}} = \\
& \frac{1}{1 + \frac{1}{\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{74\,735\,616 \exp(-\frac{1}{2})}{\sqrt{2\pi}}\right)^k z_0^{-k}}{k!}}} \quad \text{for not } ((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))
\end{aligned}$$

$$\begin{aligned}
& (((((1/4(((\sqrt{\frac{(-2^{10})}{\sqrt{2\pi}} \exp(-1/2) [1 - \\
& (10 \times 2 \times (11))/4 + (10 \times 4 \times (11 \times 12 \times 13))/32 - \\
& (10 \times 8 \times (11 \times 12 \times 13 \times 14 \times 15))/(4 \times 8 \times 12)]))))))))))^{1/14}
\end{aligned}$$

Input:

$$\left(\frac{1}{4} \sqrt{\left(-\frac{2^{10}}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}\right) \left(1 - \frac{1}{4} (10 \times 2 \times 11) + \frac{1}{32} (10 \times 4 (11 \times 12 \times 13)) - \frac{10 \times 8 (11 \times 12 \times 13 \times 14 \times 15)}{4 \times 8 \times 12}\right)\right)} \right)^{(1/14)}$$

Exact result:

$$\frac{2^{17/56} \sqrt[28]{9123}}{\sqrt[56]{e\pi}}$$

Decimal approximation:

$$1.645071048502345545475653905732866906056802385803957545462\dots$$

$$1.645071048\dots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934\dots$$

All 14th roots of 16 sqrt(9123) (2/(e pi))^(1/4):

$$\frac{2^{17/56} \sqrt[28]{9123} e^0}{\sqrt[56]{e\pi}} \approx 1.64507 \quad (\text{real, principal root})$$

$$\frac{2^{17/56} \sqrt[28]{9123} e^{(i\pi)/7}}{\sqrt[56]{e\pi}} \approx 1.4822 + 0.7138 i$$

$$\frac{2^{17/56} \sqrt[28]{9123} e^{(2i\pi)/7}}{\sqrt[56]{e\pi}} \approx 1.0257 + 1.2862 i$$

$$\frac{2^{17/56} \sqrt[28]{9123} e^{(3i\pi)/7}}{\sqrt[56]{e\pi}} \approx 0.36606 + 1.60383 i$$

$$\frac{2^{17/56} \sqrt[28]{9123} e^{(4i\pi)/7}}{\sqrt[56]{e\pi}} \approx -0.3661 + 1.60383 i$$

Series representations:

$$\frac{\sqrt[14]{\frac{1}{4} \sqrt{-\frac{2^{10} \exp(-\frac{1}{2}) \left(1 - \frac{10 \times 2 \times 11}{4} + \frac{10}{32} \times 4 (11 \times 12 \times 13) - \frac{10 \times 8 (11 \times 12 \times 13 \times 14 \times 15)}{4 \times 8 \times 12}\right)}{\sqrt{2\pi}}}}}{2^{15/56} \sqrt[28]{9123}} = \frac{\sqrt[56]{e} \sqrt[56]{\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}}}{\sqrt[56]{e\pi} \sqrt[56]{\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}}}$$

$$\frac{\sqrt[14]{\frac{1}{4} \sqrt{-\frac{2^{10} \exp(-\frac{1}{2}) \left(1 - \frac{10 \times 2 \times 11}{4} + \frac{10}{32} \times 4 (11 \times 12 \times 13) - \frac{10 \times 8 (11 \times 12 \times 13 \times 14 \times 15)}{4 \times 8 \times 12}\right)}{\sqrt{2\pi}}}}}{2^{17/56} \sqrt[28]{9123}} = \frac{\sqrt[56]{\pi} \sqrt[56]{\sum_{k=0}^{\infty} \frac{1}{k!}}}{\sqrt[56]{\pi} \sqrt[56]{\sum_{k=0}^{\infty} \frac{1}{k!}}}$$

$$\frac{\sqrt[14]{\frac{1}{4} \sqrt{-\frac{2^{10} \exp(-\frac{1}{2}) \left(1 - \frac{10 \times 2 \times 11}{4} + \frac{10}{32} \times 4 (11 \times 12 \times 13) - \frac{10 \times 8 (11 \times 12 \times 13 \times 14 \times 15)}{4 \times 8 \times 12}\right)}{\sqrt{2\pi}}}}}{2^{17/56} \sqrt[28]{9123}} = \frac{\sqrt[56]{\pi} \sqrt[56]{\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}}}}{\sqrt[56]{\pi} \sqrt[56]{\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}}}}$$

Integral representations:

$$\sqrt[14]{\frac{\frac{1}{4} \sqrt{-\frac{2^{10} \exp(-\frac{1}{2}) \left(1 - \frac{10 \times 2 \times 11}{4} + \frac{10}{32} \times 4(11 \times 12 \times 13) - \frac{10 \times 8(11 \times 12 \times 13 \times 14 \times 15)}{4 \times 8 \times 12}\right)}{\sqrt{2\pi}}}}{2^{15/56} \sqrt[28]{9123}}} = \frac{56\sqrt{e} \sqrt[56]{\int_0^1 \sqrt{1-t^2} dt}}{2^{15/56} \sqrt[28]{9123}}$$

$$\sqrt[14]{\frac{\frac{1}{4} \sqrt{-\frac{2^{10} \exp(-\frac{1}{2}) \left(1 - \frac{10 \times 2 \times 11}{4} + \frac{10}{32} \times 4(11 \times 12 \times 13) - \frac{10 \times 8(11 \times 12 \times 13 \times 14 \times 15)}{4 \times 8 \times 12}\right)}{\sqrt{2\pi}}}}{2^{2/7} \sqrt[28]{9123}}} = \frac{56\sqrt{e} \sqrt[56]{\int_0^\infty \frac{1}{1+t^2} dt}}{2^{2/7} \sqrt[28]{9123}}$$

$$\sqrt[14]{\frac{\frac{1}{4} \sqrt{-\frac{2^{10} \exp(-\frac{1}{2}) \left(1 - \frac{10 \times 2 \times 11}{4} + \frac{10}{32} \times 4(11 \times 12 \times 13) - \frac{10 \times 8(11 \times 12 \times 13 \times 14 \times 15)}{4 \times 8 \times 12}\right)}{\sqrt{2\pi}}}}{2^{2/7} \sqrt[28]{9123}}} = \frac{56\sqrt{e} \sqrt[56]{\int_0^1 \frac{1}{\sqrt{1-t^2}} dt}}{2^{2/7} \sqrt[28]{9123}}$$

$$1/10^{27} * (((((21+5)/10^3+(((1/4(((sqrt(((((-2^10)/(sqrt(2Pi)) exp(-1/2) [1-(10*2*(11))/4+(10*4*(11*12*13))/32-(10*8*(11*12*13*14*15))/(4*8*12)))))))))))))))))^1/14))))$$

Input:

$$\frac{1}{10^{27}} \left(\frac{21+5}{10^3} + \left(\frac{1}{4} \sqrt{\left(-\frac{2^{10}}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}\right) \left(1 - \frac{1}{4} (10 \times 2 \times 11) + \frac{1}{32} (10 \times 4 (11 \times 12 \times 13)) - \frac{10 \times 8 (11 \times 12 \times 13 \times 14 \times 15)}{4 \times 8 \times 12} \right) \right)} \right)^{(1/14)} \right)$$

Exact result:

$$\frac{13}{500} + \frac{2^{17/56} \sqrt[28]{9123}}{56\sqrt{e\pi}}$$

1 000 000 000 000 000 000 000 000 000

Decimal approximation:

$$1.6710710485023455454756539057328669060568023858039575... \times 10^{-27}$$

$$1.6710710485... * 10^{-27}$$

result practically equal to the value of the formula:

$$m_{p'} = 2 \times \frac{\eta}{R} m_P = 1.6714213 \times 10^{-24} \text{ gm}$$

that is the holographic proton mass (N. Hamein)

Alternate forms:

$$\frac{13}{500\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000} + \frac{500\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000 \times 2^{39/56} \sqrt[56]{e\pi}}{500 \times 2^{17/56} \sqrt[28]{9123} + 13 \sqrt[56]{e\pi}} \frac{1}{500\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000 \sqrt[56]{e\pi}}$$

Series representations:

$$\frac{\frac{21+5}{10^3} + 14 \sqrt{\frac{1}{4} \sqrt{-\frac{2^{10} \exp(-\frac{1}{2}) (1 - \frac{10 \times 2 \times 11}{4} + \frac{10}{32} \times 4 (11 \times 12 \times 13) - \frac{10 \times 8 (11 \times 12 \times 13 \times 14 \times 15)}{4 \times 8 \times 12})}{\sqrt{2\pi}}}}}{13} \frac{10^{27}}{500\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000} + \frac{500\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000 \times 2^{41/56} \sqrt[56]{e} \sqrt[56]{\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}}}{\frac{21+5}{10^3} + 14 \sqrt{\frac{1}{4} \sqrt{-\frac{2^{10} \exp(-\frac{1}{2}) (1 - \frac{10 \times 2 \times 11}{4} + \frac{10}{32} \times 4 (11 \times 12 \times 13) - \frac{10 \times 8 (11 \times 12 \times 13 \times 14 \times 15)}{4 \times 8 \times 12})}{\sqrt{2\pi}}}}}{13} \frac{10^{27}}{500\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000} + \frac{500\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000 \times 2^{39/56} \sqrt[56]{\pi} \sqrt[56]{\sum_{k=0}^{\infty} \frac{1}{k!}}}{\frac{21+5}{10^3} + 14 \sqrt{\frac{1}{4} \sqrt{-\frac{2^{10} \exp(-\frac{1}{2}) (1 - \frac{10 \times 2 \times 11}{4} + \frac{10}{32} \times 4 (11 \times 12 \times 13) - \frac{10 \times 8 (11 \times 12 \times 13 \times 14 \times 15)}{4 \times 8 \times 12})}{\sqrt{2\pi}}}}}{13} \frac{10^{27}}{500\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000} + \frac{500\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000 \times 2^{39/56} \sqrt[56]{\pi} \sqrt[56]{\sum_{k=0}^{\infty} \frac{1}{k!}}}{\frac{21+5}{10^3} + 14 \sqrt{\frac{1}{4} \sqrt{-\frac{2^{10} \exp(-\frac{1}{2}) (1 - \frac{10 \times 2 \times 11}{4} + \frac{10}{32} \times 4 (11 \times 12 \times 13) - \frac{10 \times 8 (11 \times 12 \times 13 \times 14 \times 15)}{4 \times 8 \times 12})}{\sqrt{2\pi}}}}}{13} \frac{10^{27}}{500\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000}$$

$$\frac{\frac{21+5}{10^3} + 14 \sqrt{\frac{1}{4} \sqrt{-\frac{2^{10} \exp(-\frac{1}{2}) (1 - \frac{10 \times 2 \times 11}{4} + \frac{10}{32} \times 4 (11 \times 12 \times 13) - \frac{10 \times 8 (11 \times 12 \times 13 \times 14 \times 15)}{4 \times 8 \times 12})}{\sqrt{2\pi}}}}}{13} \frac{10^{27}}{500\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000} + \frac{500\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000 \times 2^{39/56} 56\sqrt{\pi}}{28\sqrt{9123}} \sqrt{\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}}}$$

Integral representations:

$$\frac{\frac{21+5}{10^3} + 14 \sqrt{\frac{1}{4} \sqrt{-\frac{2^{10} \exp(-\frac{1}{2}) (1 - \frac{10 \times 2 \times 11}{4} + \frac{10}{32} \times 4 (11 \times 12 \times 13) - \frac{10 \times 8 (11 \times 12 \times 13 \times 14 \times 15)}{4 \times 8 \times 12})}{\sqrt{2\pi}}}}}{13} \frac{10^{27}}{500\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000} + \frac{500\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000 \times 2^{5/7} 56\sqrt{e}}{28\sqrt{9123}} \sqrt{\int_0^{\infty} \frac{1}{1+t^2} dt}$$

$$\frac{\frac{21+5}{10^3} + 14 \sqrt{\frac{1}{4} \sqrt{-\frac{2^{10} \exp(-\frac{1}{2}) (1 - \frac{10 \times 2 \times 11}{4} + \frac{10}{32} \times 4 (11 \times 12 \times 13) - \frac{10 \times 8 (11 \times 12 \times 13 \times 14 \times 15)}{4 \times 8 \times 12})}{\sqrt{2\pi}}}}}{13} \frac{10^{27}}{500\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000} + \frac{500\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000 \times 2^{5/7} 56\sqrt{e}}{28\sqrt{9123}} \sqrt{\int_0^1 \frac{1}{\sqrt{1-t^2}} dt}$$

$$\frac{\frac{21+5}{10^3} + 14 \sqrt{\frac{1}{4} \sqrt{-\frac{2^{10} \exp(-\frac{1}{2}) (1 - \frac{10 \times 2 \times 11}{4} + \frac{10}{32} \times 4 (11 \times 12 \times 13) - \frac{10 \times 8 (11 \times 12 \times 13 \times 14 \times 15)}{4 \times 8 \times 12})}{\sqrt{2\pi}}}}}{13} \frac{10^{27}}{500\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000} + \frac{500\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000 \times 2^{41/56} 56\sqrt{e}}{28\sqrt{9123}} \sqrt{\int_0^1 \sqrt{1-t^2} dt}$$

$$\left(\left(\left(-\frac{29-2}{10^3} + \left(\frac{1}{4} \sqrt{\left(-\frac{2^{10}}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}\right) \left(1 - \frac{1}{4} (10 \times 2 \times 11) + \frac{1}{32} (10 \times 4 (11 \times 12 \times 13)) - \frac{10 \times 8 (11 \times 12 \times 13 \times 14 \times 15)}{4 \times 8 \times 12}\right)\right)}\right)}\right)\right)^{1/14}\right)$$

Input:

$$-\frac{29-2}{10^3} + \left(\frac{1}{4} \sqrt{\left(-\frac{2^{10}}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}\right) \left(1 - \frac{1}{4} (10 \times 2 \times 11) + \frac{1}{32} (10 \times 4 (11 \times 12 \times 13)) - \frac{10 \times 8 (11 \times 12 \times 13 \times 14 \times 15)}{4 \times 8 \times 12}\right)\right)}\right)^{(1/14)}$$

Exact result:

$$\frac{2^{17/56} \sqrt[28]{9123}}{\sqrt[56]{e\pi}} - \frac{27}{1000}$$

Decimal approximation:

1.618071048502345545475653905732866906056802385803957545462...

1.6180710485.....

This result is a very good approximation to the value of the golden ratio
1,618033988749...

Alternate forms:

$$\frac{1000 \times 2^{17/56} \sqrt[28]{9123} - 27 \sqrt[56]{e\pi}}{1000 \sqrt[56]{e\pi}}$$

$$-\frac{27 \sqrt[56]{e\pi} - 1000 \times 2^{17/56} \sqrt[28]{9123}}{1000 \sqrt[56]{e\pi}}$$

Continued fraction:

$$-\frac{29-2}{10^3} + \left(\frac{1}{4} \sqrt{\left(-\frac{1}{\sqrt{2\pi}} 2^{10} \exp\left(-\frac{1}{2}\right) \left(1 - \frac{10 \times 2 \times 11}{4} + \frac{10}{32} \times 4(11 \times 12 \times 13) - \frac{10 \times 8(11 \times 12 \times 13 \times 14 \times 15)}{4 \times 8 \times 12} \right) \right)} \right)^{(1/14)} =$$

$$\frac{-27 \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} + 1000 \times 2^{15/56} \sqrt[28]{9123} \left(\frac{\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} \right)^{55/56} \sum_{k=0}^{\infty} \frac{(-1)^k}{k!}}{1000 \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}}$$

$$-\frac{29-2}{10^3} + \left(\frac{1}{4} \sqrt{\left(-\frac{1}{\sqrt{2\pi}} 2^{10} \exp\left(-\frac{1}{2}\right) \left(1 - \frac{10 \times 2 \times 11}{4} + \frac{10}{32} \times 4(11 \times 12 \times 13) - \frac{10 \times 8(11 \times 12 \times 13 \times 14 \times 15)}{4 \times 8 \times 12} \right) \right)} \right)^{(1/14)} =$$

$$\frac{1000 \times 2^{15/56} \sqrt[28]{9123} \left(\sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1}}{k_2!(1+2k_1)} \right)^{55/56} - 27 \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1}}{k_2!(1+2k_1)}}{1000 \left(\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} \right) \sum_{k=0}^{\infty} \frac{1}{k!}}$$

Integral representations:

$$-\frac{29-2}{10^3} + \left(\frac{1}{4} \sqrt{\left(-\frac{1}{\sqrt{2\pi}} 2^{10} \exp\left(-\frac{1}{2}\right) \left(1 - \frac{10 \times 2 \times 11}{4} + \frac{10}{32} \times 4(11 \times 12 \times 13) - \frac{10 \times 8(11 \times 12 \times 13 \times 14 \times 15)}{4 \times 8 \times 12} \right) \right)} \right)^{(1/14)} = -\frac{27}{1000} + \frac{2^{15/56} \sqrt[28]{9123}}{\sqrt[56]{e} \sqrt[56]{\int_0^1 \sqrt{1-t^2} dt}}$$

$$-\frac{29-2}{10^3} + \left(\frac{1}{4} \sqrt{\left(-\frac{1}{\sqrt{2\pi}} 2^{10} \exp\left(-\frac{1}{2}\right) \left(1 - \frac{10 \times 2 \times 11}{4} + \frac{10}{32} \times 4(11 \times 12 \times 13) - \frac{10 \times 8(11 \times 12 \times 13 \times 14 \times 15)}{4 \times 8 \times 12} \right) \right)} \right)^{(1/14)} = -\frac{27}{1000} + \frac{2^{2/7} \sqrt[28]{9123}}{\sqrt[56]{e} \sqrt[56]{\int_0^{\infty} \frac{1}{1+t^2} dt}}$$

$$-\frac{29-2}{10^3} + \left(\frac{1}{4} \sqrt{\left(-\frac{1}{\sqrt{2\pi}} 2^{10} \exp\left(-\frac{1}{2}\right) \left(1 - \frac{10 \times 2 \times 11}{4} + \frac{10}{32} \times 4(11 \times 12 \times 13) - \frac{10 \times 8(11 \times 12 \times 13 \times 14 \times 15)}{4 \times 8 \times 12} \right) \right)} \right)^{(1/14)} = -\frac{27}{1000} + \frac{2^{2/7} \sqrt[28]{9123}}{\sqrt[56]{e} \sqrt[56]{\int_0^1 \frac{1}{\sqrt{1-t^2}} dt}}$$

$$-\frac{29-2}{10^3} + \left(\frac{1}{4} \sqrt{\left(-\frac{1}{\sqrt{2}\pi} 2^{10} \exp\left(-\frac{1}{2}\right) \left(1 - \frac{10 \times 2 \times 11}{4} + \frac{10}{32} \times 4 (11 \times 12 \times 13) - \frac{10 \times 8 (11 \times 12 \times 13 \times 14 \times 15)}{4 \times 8 \times 12} \right) \right)} \right) \wedge$$

$$(1/14) = -\frac{27}{1000} + \frac{2^{2/7} \sqrt[28]{9123}}{\sqrt[56]{e} \sqrt[56]{\int_0^\infty \frac{\sin(t)}{t} dt}}$$

$$-\frac{29-2}{10^3} + \left(\frac{1}{4} \sqrt{\left(-\frac{1}{\sqrt{2}\pi} 2^{10} \exp\left(-\frac{1}{2}\right) \left(1 - \frac{10 \times 2 \times 11}{4} + \frac{10}{32} \times 4 (11 \times 12 \times 13) - \frac{10 \times 8 (11 \times 12 \times 13 \times 14 \times 15)}{4 \times 8 \times 12} \right) \right)} \right) \wedge$$

$$(1/14) = -\frac{27}{1000} + \frac{2^{2/7} \sqrt[28]{9123}}{\sqrt[56]{e} \sqrt[56]{\int_0^\infty \frac{\sin^2(t)}{t^2} dt}}$$

$$-\frac{29-2}{10^3} + \left(\frac{1}{4} \sqrt{\left(-\frac{1}{\sqrt{2}\pi} 2^{10} \exp\left(-\frac{1}{2}\right) \left(1 - \frac{10 \times 2 \times 11}{4} + \frac{10}{32} \times 4 (11 \times 12 \times 13) - \frac{10 \times 8 (11 \times 12 \times 13 \times 14 \times 15)}{4 \times 8 \times 12} \right) \right)} \right) \wedge$$

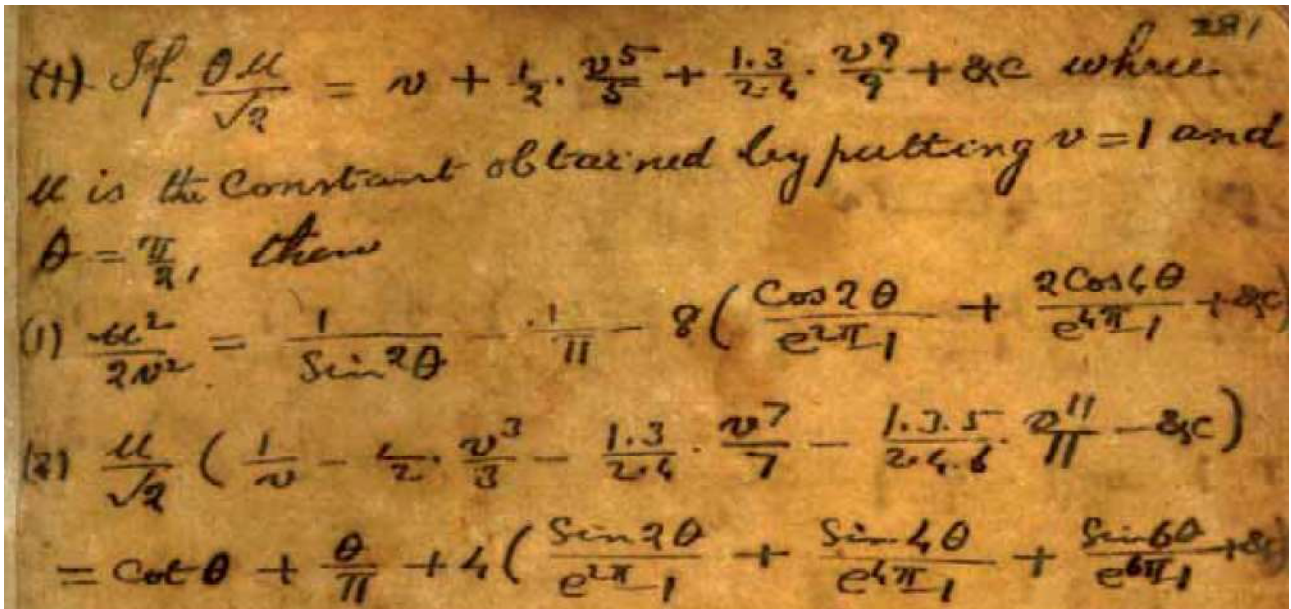
$$(1/14) = -\frac{27}{1000} + \frac{\sqrt[4]{2} 3^{3/56} \sqrt[28]{3041}}{\sqrt[56]{e} \sqrt[56]{\int_0^\infty \frac{\sin^3(t)}{t^3} dt}}$$

$$-\frac{29-2}{10^3} + \left(\frac{1}{4} \sqrt{\left(-\frac{1}{\sqrt{2}\pi} 2^{10} \exp\left(-\frac{1}{2}\right) \left(1 - \frac{10 \times 2 \times 11}{4} + \frac{10}{32} \times 4 (11 \times 12 \times 13) - \frac{10 \times 8 (11 \times 12 \times 13 \times 14 \times 15)}{4 \times 8 \times 12} \right) \right)} \right) \wedge$$

$$(1/14) = -\frac{27}{1000} + \frac{2^{17/56} \sqrt[28]{3041} \sqrt[56]{\frac{3}{e}}}{\sqrt[56]{\int_0^\infty \frac{\sin^4(t)}{t^4} dt}}$$

$$-\frac{29-2}{10^3} + \left(\frac{1}{4} \sqrt{\left(-\frac{1}{\sqrt{2}\pi} 2^{10} \exp\left(-\frac{1}{2}\right) \left(1 - \frac{10 \times 2 \times 11}{4} + \frac{10}{32} \times 4 (11 \times 12 \times 13) - \frac{10 \times 8 (11 \times 12 \times 13 \times 14 \times 15)}{4 \times 8 \times 12} \right) \right)} \right) \wedge$$

$$(1/14) = -\frac{27}{1000} + \frac{2^{17/56} \sqrt[28]{9123}}{\sqrt[56]{e} \sqrt[56]{\frac{3\sqrt{3}}{4} + 24 \int_0^1 \sqrt{t-t^2} dt}}$$



$$\cot(\pi/2) + (\pi/2 * 1/\pi) + 4 \left(\left(\left(\left(\left(\frac{\sin(2 * \pi/2)}{e^{2\pi} - 1} \right) + \frac{\sin(4 * \pi/2)}{e^{4\pi} - 1} \right) + \frac{\sin(6 * \pi/2)}{e^{6\pi} - 1} \right) \right) \right)$$

Input:

$$\cot\left(\frac{\pi}{2}\right) + \frac{\pi}{2} \times \frac{1}{\pi} + 4 \left(\frac{\sin\left(2 \times \frac{\pi}{2}\right)}{e^{2\pi} - 1} + \frac{\sin\left(4 \times \frac{\pi}{2}\right)}{e^{4\pi} - 1} + \frac{\sin\left(6 \times \frac{\pi}{2}\right)}{e^{6\pi} - 1} \right)$$

cot(x) is the cotangent function

Exact result:

$$\frac{1}{2}$$

$$1/2$$

Decimal form:

$$0.5$$

Alternative representations:

$$\cot\left(\frac{\pi}{2}\right) + \frac{\pi}{\pi 2} + 4 \left(\frac{\sin\left(\frac{2\pi}{2}\right)}{e^{2\pi} - 1} + \frac{\sin\left(\frac{4\pi}{2}\right)}{e^{4\pi} - 1} + \frac{\sin\left(\frac{6\pi}{2}\right)}{e^{6\pi} - 1} \right) =$$

$$i + \frac{\pi}{2\pi} + 4 \left(-\frac{\cos\left(\frac{3\pi}{2}\right)}{-1 + e^{2\pi}} - \frac{\cos\left(\frac{5\pi}{2}\right)}{-1 + e^{4\pi}} - \frac{\cos\left(\frac{7\pi}{2}\right)}{-1 + e^{6\pi}} \right) + \frac{2i}{-1 + e^{i\pi}}$$

$$\cot\left(\frac{\pi}{2}\right) + \frac{\pi}{\pi 2} + 4 \left(\frac{\sin\left(\frac{2\pi}{2}\right)}{e^{2\pi} - 1} + \frac{\sin\left(\frac{4\pi}{2}\right)}{e^{4\pi} - 1} + \frac{\sin\left(\frac{6\pi}{2}\right)}{e^{6\pi} - 1} \right) =$$

$$i + \frac{\pi}{2\pi} + 4 \left(\frac{\cos\left(-\frac{\pi}{2}\right)}{-1 + e^{2\pi}} + \frac{\cos\left(-\frac{3\pi}{2}\right)}{-1 + e^{4\pi}} + \frac{\cos\left(-\frac{5\pi}{2}\right)}{-1 + e^{6\pi}} \right) + \frac{2i}{-1 + e^{i\pi}}$$

$$\cot\left(\frac{\pi}{2}\right) + \frac{\pi}{\pi 2} + 4 \left(\frac{\sin\left(\frac{2\pi}{2}\right)}{e^{2\pi} - 1} + \frac{\sin\left(\frac{4\pi}{2}\right)}{e^{4\pi} - 1} + \frac{\sin\left(\frac{6\pi}{2}\right)}{e^{6\pi} - 1} \right) =$$

$$-i \coth\left(-\frac{i\pi}{2}\right) + \frac{\pi}{2\pi} + 4 \left(\frac{\cos\left(-\frac{\pi}{2}\right)}{-1 + e^{2\pi}} + \frac{\cos\left(-\frac{3\pi}{2}\right)}{-1 + e^{4\pi}} + \frac{\cos\left(-\frac{5\pi}{2}\right)}{-1 + e^{6\pi}} \right)$$

Series representations:

$$\cot\left(\frac{\pi}{2}\right) + \frac{\pi}{\pi 2} + 4 \left(\frac{\sin\left(\frac{2\pi}{2}\right)}{e^{2\pi} - 1} + \frac{\sin\left(\frac{4\pi}{2}\right)}{e^{4\pi} - 1} + \frac{\sin\left(\frac{6\pi}{2}\right)}{e^{6\pi} - 1} \right) =$$

$$\frac{1}{2} \left(1 - 2i - 4i \sum_{k=1}^{\infty} q^{2k} + 2 \sum_{k=0}^{\infty} \frac{4(-1)^k \left(\frac{1}{-1+e^{2\pi}} + \frac{2^{1+2k}}{-1+e^{4\pi}} + \frac{3^{1+2k}}{-1+e^{6\pi}} \right) \pi^{1+2k}}{(1+2k)!} \right) \text{ for } q = i$$

$$\cot\left(\frac{\pi}{2}\right) + \frac{\pi}{\pi 2} + 4 \left(\frac{\sin\left(\frac{2\pi}{2}\right)}{e^{2\pi} - 1} + \frac{\sin\left(\frac{4\pi}{2}\right)}{e^{4\pi} - 1} + \frac{\sin\left(\frac{6\pi}{2}\right)}{e^{6\pi} - 1} \right) =$$

$$\frac{1}{2} \left(1 + 2 \sum_{k=0}^{\infty} \left(\frac{(-1)^k 2^{2-2k} \pi^{2k}}{(-1 + e^{2\pi}) (2k)!} + \frac{(-1)^k 2^{2-2k} \times 3^{2k} \pi^{2k}}{(-1 + e^{4\pi}) (2k)!} + \frac{(-1)^k 2^{2-2k} \times 5^{2k} \pi^{2k}}{(-1 + e^{6\pi}) (2k)!} \right) - \right.$$

$$\left. 2i \sum_{k=-\infty}^{\infty} \mathcal{A}^{ik\pi} \operatorname{sgn}(k) \right)$$

$$\cot\left(\frac{\pi}{2}\right) + \frac{\pi}{\pi 2} + 4 \left(\frac{\sin\left(\frac{2\pi}{2}\right)}{e^{2\pi} - 1} + \frac{\sin\left(\frac{4\pi}{2}\right)}{e^{4\pi} - 1} + \frac{\sin\left(\frac{6\pi}{2}\right)}{e^{6\pi} - 1} \right) =$$

$$\frac{1}{2} \left(1 + 2 \sum_{k=0}^{\infty} \left(\frac{4(-1)^k \pi^{1+2k}}{(-1 + e^{2\pi}) (1+2k)!} + \frac{(-1)^k 2^{3+2k} \pi^{1+2k}}{(-1 + e^{4\pi}) (1+2k)!} + \frac{4(-1)^k 3^{1+2k} \pi^{1+2k}}{(-1 + e^{6\pi}) (1+2k)!} \right) - \right.$$

$$\left. 2i \sum_{k=-\infty}^{\infty} \mathcal{A}^{ik\pi} \operatorname{sgn}(k) \right)$$

Integral representations:

$$\cot\left(\frac{\pi}{2}\right) + \frac{\pi}{\pi 2} + 4 \left(\frac{\sin\left(\frac{2\pi}{2}\right)}{e^{2\pi} - 1} + \frac{\sin\left(\frac{4\pi}{2}\right)}{e^{4\pi} - 1} + \frac{\sin\left(\frac{6\pi}{2}\right)}{e^{6\pi} - 1} \right) =$$

$$\frac{1}{2} + \int_0^1 4\pi \left(\frac{\cos(\pi t)}{-1 + e^{2\pi}} + \frac{2 \cos(2\pi t)}{-1 + e^{4\pi}} + \frac{3 \cos(3\pi t)}{-1 + e^{6\pi}} \right) dt$$

$$\cot\left(\frac{\pi}{2}\right) + \frac{\pi}{\pi 2} + 4 \left(\frac{\sin\left(\frac{2\pi}{2}\right)}{e^{2\pi} - 1} + \frac{\sin\left(\frac{4\pi}{2}\right)}{e^{4\pi} - 1} + \frac{\sin\left(\frac{6\pi}{2}\right)}{e^{6\pi} - 1} \right) =$$

$$\frac{1}{2} \left(1 - 2 \int_{\frac{\pi}{2}}^{\pi} \csc^2(t) dt + 2 \int_{-i\infty+\gamma}^{i\infty+\gamma} \left(\frac{3 \mathcal{A}^{-\pi^2/(4s)+s} \sqrt{\pi}}{(-1 + e^{6\pi}) i s^{3/2}} + \frac{2 \mathcal{A}^{-\pi^2/s+s} \sqrt{\pi}}{(-1 + e^{4\pi}) i s^{3/2}} + \frac{\mathcal{A}^{-\pi^2/(4s)+s} \sqrt{\pi}}{(-1 + e^{2\pi}) i s^{3/2}} \right) ds \right) \text{ for } \gamma > 0$$

$$\cot\left(\frac{\pi}{2}\right) + \frac{\pi}{\pi 2} + 4 \left(\frac{\sin\left(\frac{2\pi}{2}\right)}{e^{2\pi} - 1} + \frac{\sin\left(\frac{4\pi}{2}\right)}{e^{4\pi} - 1} + \frac{\sin\left(\frac{6\pi}{2}\right)}{e^{6\pi} - 1} \right) =$$

$$\frac{1}{2} \left(1 - 2 \int_{\frac{\pi}{2}}^{\pi} \csc^2(t) dt + 2 \int_{-i\infty+\gamma}^{i\infty+\gamma} \left(\frac{2^{2s} \pi^{-2s} \Gamma(s) \sqrt{\pi}}{(-1 + e^{2\pi}) i \Gamma\left(\frac{3}{2} - s\right)} + \frac{2 \pi^{-2s} \Gamma(s) \sqrt{\pi}}{(-1 + e^{4\pi}) i \Gamma\left(\frac{3}{2} - s\right)} + \frac{2^{2s} \times 3^{1-2s} \pi^{-2s} \Gamma(s) \sqrt{\pi}}{(-1 + e^{6\pi}) i \Gamma\left(\frac{3}{2} - s\right)} \right) ds \right) \text{ for } 0 < \gamma < 1$$

Multiple-argument formulas:

$$\cot\left(\frac{\pi}{2}\right) + \frac{\pi}{\pi 2} + 4 \left(\frac{\sin\left(\frac{2\pi}{2}\right)}{e^{2\pi} - 1} + \frac{\sin\left(\frac{4\pi}{2}\right)}{e^{4\pi} - 1} + \frac{\sin\left(\frac{6\pi}{2}\right)}{e^{6\pi} - 1} \right) =$$

$$\frac{1}{2} + 4 \left(\frac{\sin(\pi)}{-1 + e^{2\pi}} + \frac{U_1(\cos(\pi)) \sin(\pi)}{-1 + e^{4\pi}} + \frac{U_2(\cos(\pi)) \sin(\pi)}{-1 + e^{6\pi}} \right) + \frac{1}{2} \left(\cot\left(\frac{\pi}{4}\right) - \tan\left(\frac{\pi}{4}\right) \right)$$

$$\cot\left(\frac{\pi}{2}\right) + \frac{\pi}{\pi 2} + 4 \left(\frac{\sin\left(\frac{2\pi}{2}\right)}{e^{2\pi} - 1} + \frac{\sin\left(\frac{4\pi}{2}\right)}{e^{4\pi} - 1} + \frac{\sin\left(\frac{6\pi}{2}\right)}{e^{6\pi} - 1} \right) =$$

$$\frac{1}{2} + 4 \left(\frac{3 \sin\left(\frac{\pi}{3}\right) - 4 \sin^3\left(\frac{\pi}{3}\right)}{-1 + e^{2\pi}} + \frac{3 \sin\left(\frac{2\pi}{3}\right) - 4 \sin^3\left(\frac{2\pi}{3}\right)}{-1 + e^{4\pi}} + \frac{3 \sin(\pi) - 4 \sin^3(\pi)}{-1 + e^{6\pi}} \right) +$$

$$\frac{1}{2} \left(\cot\left(\frac{\pi}{4}\right) - \tan\left(\frac{\pi}{4}\right) \right)$$

$$\cot\left(\frac{\pi}{2}\right) + \frac{\pi}{\pi 2} + 4 \left(\frac{\sin\left(\frac{2\pi}{2}\right)}{e^{2\pi} - 1} + \frac{\sin\left(\frac{4\pi}{2}\right)}{e^{4\pi} - 1} + \frac{\sin\left(\frac{6\pi}{2}\right)}{e^{6\pi} - 1} \right) =$$

$$\frac{1}{2} + 4 \left(\frac{2 \cos\left(\frac{\pi}{2}\right) \sin\left(\frac{\pi}{2}\right)}{-1 + e^{2\pi}} + \frac{2 \cos(\pi) \sin(\pi)}{-1 + e^{4\pi}} + \frac{2 \cos\left(\frac{3\pi}{2}\right) \sin\left(\frac{3\pi}{2}\right)}{-1 + e^{6\pi}} \right) + \frac{1}{2} \left(\cot\left(\frac{\pi}{4}\right) - \tan\left(\frac{\pi}{4}\right) \right)$$

And we obtain also:

$$\sqrt{\left(\left(\left(\left(\left(\cot\left(\frac{\pi}{2}\right)+\left(\frac{\pi}{2}\times\frac{1}{\pi}\right)+4\left(\left(\left(\left(\frac{\sin\left(2\times\frac{\pi}{2}\right)}{e^{2\pi}-1}\right)+\left(\frac{\sin\left(4\times\frac{\pi}{2}\right)}{e^{4\pi}-1}\right)+\left(\frac{\sin\left(6\times\frac{\pi}{2}\right)}{e^{6\pi}-1}\right)\right)\right)\right)\right)\right)\right)\right)$$

Input:

$$\sqrt{\cot\left(\frac{\pi}{2}\right)+\frac{\pi}{2}\times\frac{1}{\pi}+4\left(\frac{\sin\left(2\times\frac{\pi}{2}\right)}{e^{2\pi}-1}+\frac{\sin\left(4\times\frac{\pi}{2}\right)}{e^{4\pi}-1}+\frac{\sin\left(6\times\frac{\pi}{2}\right)}{e^{6\pi}-1}\right)}$$

$\cot(x)$ is the cotangent function

Exact result:

$$\frac{1}{\sqrt{2}}$$

Decimal approximation:

0.707106781186547524400844362104849039284835937688474036588...

0.707106781....

Alternate form:

$$\frac{\sqrt{2}}{2}$$

All 2nd roots of 1/2:

$$\frac{e^0}{\sqrt{2}} \approx 0.70711 \quad (\text{real, principal root})$$

$$\frac{e^{i\pi}}{\sqrt{2}} \approx -0.7071 \quad (\text{real root})$$

Alternative representations:

$$\sqrt{\cot\left(\frac{\pi}{2}\right)+\frac{\pi}{\pi 2}+4\left(\frac{\sin\left(\frac{2\pi}{2}\right)}{e^{2\pi}-1}+\frac{\sin\left(\frac{4\pi}{2}\right)}{e^{4\pi}-1}+\frac{\sin\left(\frac{6\pi}{2}\right)}{e^{6\pi}-1}\right)} = \sqrt{i+\frac{\pi}{2\pi}+4\left(-\frac{\cos\left(\frac{3\pi}{2}\right)}{-1+e^{2\pi}}-\frac{\cos\left(\frac{5\pi}{2}\right)}{-1+e^{4\pi}}-\frac{\cos\left(\frac{7\pi}{2}\right)}{-1+e^{6\pi}}\right)+\frac{2i}{-1+e^{i\pi}}}$$

$$\sqrt{\cot\left(\frac{\pi}{2}\right)+\frac{\pi}{\pi 2}+4\left(\frac{\sin\left(\frac{2\pi}{2}\right)}{e^{2\pi}-1}+\frac{\sin\left(\frac{4\pi}{2}\right)}{e^{4\pi}-1}+\frac{\sin\left(\frac{6\pi}{2}\right)}{e^{6\pi}-1}\right)} = \sqrt{i+\frac{\pi}{2\pi}+4\left(\frac{\cos\left(-\frac{\pi}{2}\right)}{-1+e^{2\pi}}+\frac{\cos\left(-\frac{3\pi}{2}\right)}{-1+e^{4\pi}}+\frac{\cos\left(-\frac{5\pi}{2}\right)}{-1+e^{6\pi}}\right)+\frac{2i}{-1+e^{i\pi}}}$$

$$\sqrt{\cot\left(\frac{\pi}{2}\right) + \frac{\pi}{2} + 4\left(\frac{\sin\left(\frac{2\pi}{2}\right)}{e^{2\pi} - 1} + \frac{\sin\left(\frac{4\pi}{2}\right)}{e^{4\pi} - 1} + \frac{\sin\left(\frac{6\pi}{2}\right)}{e^{6\pi} - 1}\right)} =$$

$$\sqrt{-i \coth\left(-\frac{i\pi}{2}\right) + \frac{\pi}{2\pi} + 4\left(\frac{\cos\left(-\frac{\pi}{2}\right)}{-1 + e^{2\pi}} + \frac{\cos\left(-\frac{3\pi}{2}\right)}{-1 + e^{4\pi}} + \frac{\cos\left(-\frac{5\pi}{2}\right)}{-1 + e^{6\pi}}\right)}$$

Series representations:

$$\sqrt{\cot\left(\frac{\pi}{2}\right) + \frac{\pi}{2} + 4\left(\frac{\sin\left(\frac{2\pi}{2}\right)}{e^{2\pi} - 1} + \frac{\sin\left(\frac{4\pi}{2}\right)}{e^{4\pi} - 1} + \frac{\sin\left(\frac{6\pi}{2}\right)}{e^{6\pi} - 1}\right)} =$$

$$\sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(-\frac{1}{2} + \cot\left(\frac{\pi}{2}\right) + 4\left(\frac{\sin(\pi)}{-1+e^{2\pi}} + \frac{\sin(2\pi)}{-1+e^{4\pi}} + \frac{\sin(3\pi)}{-1+e^{6\pi}}\right)\right)^k}{k!}$$

$$\sqrt{\cot\left(\frac{\pi}{2}\right) + \frac{\pi}{2} + 4\left(\frac{\sin\left(\frac{2\pi}{2}\right)}{e^{2\pi} - 1} + \frac{\sin\left(\frac{4\pi}{2}\right)}{e^{4\pi} - 1} + \frac{\sin\left(\frac{6\pi}{2}\right)}{e^{6\pi} - 1}\right)} =$$

$$\exp\left(i\pi \left[\frac{\arg\left(\frac{1}{2} - x + \cot\left(\frac{\pi}{2}\right) + 4\left(\frac{\sin(\pi)}{-1+e^{2\pi}} + \frac{\sin(2\pi)}{-1+e^{4\pi}} + \frac{\sin(3\pi)}{-1+e^{6\pi}}\right)\right)}{2\pi} \right]\right) \sqrt{x}$$

$$\sum_{k=0}^{\infty} \frac{(-1)^k x^{-k} \left(-\frac{1}{2}\right)_k \left(\frac{1}{2} - x + \cot\left(\frac{\pi}{2}\right) + 4\left(\frac{\sin(\pi)}{-1+e^{2\pi}} + \frac{\sin(2\pi)}{-1+e^{4\pi}} + \frac{\sin(3\pi)}{-1+e^{6\pi}}\right)\right)^k}{k!}$$

for ($x \in \mathbb{R}$ and $x < 0$)

$$\sqrt{\cot\left(\frac{\pi}{2}\right) + \frac{\pi}{2} + 4\left(\frac{\sin\left(\frac{2\pi}{2}\right)}{e^{2\pi} - 1} + \frac{\sin\left(\frac{4\pi}{2}\right)}{e^{4\pi} - 1} + \frac{\sin\left(\frac{6\pi}{2}\right)}{e^{6\pi} - 1}\right)} =$$

$$\left(\frac{1}{z_0}\right)^{1/2} \left[\arg\left(\frac{1}{2} + \cot\left(\frac{\pi}{2}\right) + 4\left(\frac{\sin(\pi)}{-1+e^{2\pi}} + \frac{\sin(2\pi)}{-1+e^{4\pi}} + \frac{\sin(3\pi)}{-1+e^{6\pi}}\right)\right) - z_0 \right] / (2\pi)$$

$$z_0^{1/2} \left[1 + \arg\left(\frac{1}{2} + \cot\left(\frac{\pi}{2}\right) + 4\left(\frac{\sin(\pi)}{-1+e^{2\pi}} + \frac{\sin(2\pi)}{-1+e^{4\pi}} + \frac{\sin(3\pi)}{-1+e^{6\pi}}\right)\right) - z_0 \right] / (2\pi)$$

$$\sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{1}{2} + \cot\left(\frac{\pi}{2}\right) + 4\left(\frac{\sin(\pi)}{-1+e^{2\pi}} + \frac{\sin(2\pi)}{-1+e^{4\pi}} + \frac{\sin(3\pi)}{-1+e^{6\pi}}\right) - z_0\right)^k z_0^{-k}}{k!}$$

Integral representations:

$$\sqrt{\cot\left(\frac{\pi}{2}\right) + \frac{\pi}{2} + 4\left(\frac{\sin\left(\frac{2\pi}{2}\right)}{e^{2\pi} - 1} + \frac{\sin\left(\frac{4\pi}{2}\right)}{e^{4\pi} - 1} + \frac{\sin\left(\frac{6\pi}{2}\right)}{e^{6\pi} - 1}\right)} =$$

$$\sqrt{\frac{1}{2} + \int_0^1 4\pi \left(\frac{\cos(\pi t)}{-1 + e^{2\pi}} + \frac{2 \cos(2\pi t)}{-1 + e^{4\pi}} + \frac{3 \cos(3\pi t)}{-1 + e^{6\pi}}\right) dt}$$

$$\sqrt{\cot\left(\frac{\pi}{2}\right) + \frac{\pi}{\pi 2} + 4\left(\frac{\sin\left(\frac{2\pi}{2}\right)}{e^{2\pi} - 1} + \frac{\sin\left(\frac{4\pi}{2}\right)}{e^{4\pi} - 1} + \frac{\sin\left(\frac{6\pi}{2}\right)}{e^{6\pi} - 1}\right)} =$$

$$\sqrt{\left(\frac{1}{2} - \int_{\frac{\pi}{2}}^{\pi} \csc^2(t) dt + 4 \int_{-i\infty+\gamma}^{i\infty+\gamma} \left(\frac{2^{-2+2s} \pi^{-2s} \Gamma(s) \sqrt{\pi}}{(-1 + e^{2\pi}) i \Gamma\left(\frac{3}{2} - s\right)} + \frac{\pi^{-2s} \Gamma(s) \sqrt{\pi}}{2(-1 + e^{4\pi}) i \Gamma\left(\frac{3}{2} - s\right)} + \frac{2^{-2+2s} \times 3^{1-2s} \pi^{-2s} \Gamma(s) \sqrt{\pi}}{(-1 + e^{6\pi}) i \Gamma\left(\frac{3}{2} - s\right)}\right) ds\right)} \text{ for } 0 < \gamma < 1$$

$$\sqrt{\cot\left(\frac{\pi}{2}\right) + \frac{\pi}{\pi 2} + 4\left(\frac{\sin\left(\frac{2\pi}{2}\right)}{e^{2\pi} - 1} + \frac{\sin\left(\frac{4\pi}{2}\right)}{e^{4\pi} - 1} + \frac{\sin\left(\frac{6\pi}{2}\right)}{e^{6\pi} - 1}\right)} =$$

$$\sqrt{\left(\frac{1}{2} - \int_{\frac{\pi}{2}}^{\pi} \csc^2(t) dt + 4 \int_{-i\infty+\gamma}^{i\infty+\gamma} \left(\frac{3 \mathcal{A}^{-(9\pi^2)/(4s+s)} \sqrt{\pi}}{4(-1 + e^{6\pi}) i s^{3/2}} + \frac{\mathcal{A}^{-\pi^2/s+s} \sqrt{\pi}}{2(-1 + e^{4\pi}) i s^{3/2}} + \frac{\mathcal{A}^{-\pi^2/(4s+s)} \sqrt{\pi}}{4(-1 + e^{2\pi}) i s^{3/2}}\right) ds\right)} \text{ for } \gamma > 0$$

Thence, with regard this solution, we can to obtain the following interesting mathematical connection:

We know that (From: Anomaly Inflow and the η -Invariant - Edward Witten, Kazuya Yonekura (*Submitted on 19 Sep 2019 (v1), last revised 7 Oct 2019 (this version, v2)*) see previous paper part III):

From:

$$\cos \theta_a \rightarrow \lambda_a / (2|m|)$$

$$\prod_a \left(\frac{\lambda_a}{2|m|}\right)_{\text{reg}} = |\text{Det}(\mathcal{D}_W^+)|$$

$$\cos \theta_a \rightarrow 1/\sqrt{2},$$

and

$$\mathcal{Z}(Y, L) = |\text{Det}(\mathcal{D}_W^+)| \exp(-\pi i \eta_D)$$

we obtain:

$$1/\sqrt{2} * \exp(-\pi * i * 0.004827949)$$

$$\frac{1}{\sqrt{2}} \exp(-\pi(i \times 0.004827949))$$

Result:

$$0.707025447... - \\ 0.0107245949... i$$

Polar coordinates:

$$r = 0.707107 \text{ (radius)}, \quad \theta = -0.869031^\circ \text{ (angle)}$$

$$0.707107$$

And:

$$\exp(-i\pi\eta_Y/2) = \exp\left(-i \int_Y \Phi\right),$$

$$\exp(-i\pi\eta_Y/2) \neq 1.$$

$$\text{sqrt}(2)/2 * \exp(-(-i*Pi*0.00482794999383144)*1/2)$$

$$\frac{1}{\sqrt{2}} \exp\left(-(-i(\pi \times 0.00482794999383144)) \times \frac{1}{2}\right)$$

Result:

$$0.70708644740256586... + \\ 0.0053624527614207895... i$$

Polar coordinates:

$$r = 0.7071067811865475244 \text{ (radius)}, \quad \theta = 0.434515499444830^\circ \text{ (angle)}$$

$$0.7071067811865475244$$

Or:

Input interpretation:

$$\frac{\sqrt{2}}{2} \exp(-(-i(\pi \times 0.00482794999383144)))$$

Result:

$$0.70702544722007017... + \\ 0.010724597114100597... i$$

Polar coordinates:

$$r = 0.707106781186547524 \text{ (radius)}, \quad \theta = 0.869030998889659^\circ \text{ (angle)}$$

$$0.707106781186547524$$

Thence, we have the following mathematical connection:

$$\left(\sqrt{\cot\left(\frac{\pi}{2}\right) + \frac{\pi}{2} \times \frac{1}{\pi} + 4 \left(\frac{\sin\left(2 \times \frac{\pi}{2}\right)}{e^{2\pi} - 1} + \frac{\sin\left(4 \times \frac{\pi}{2}\right)}{e^{4\pi} - 1} + \frac{\sin\left(6 \times \frac{\pi}{2}\right)}{e^{6\pi} - 1} \right)} \right) = 0.707106781\dots \Rightarrow$$

$$\Rightarrow \left(\frac{1}{\sqrt{2}} \exp\left(-(-i(\pi \times 0.00482794999383144)) \times \frac{1}{2}\right) \right) = 0.7071067811865475244$$

$$= 1/\sqrt{2}$$

We have also:

$$\left(\left(\left(\left(\sqrt{\cot\left(\frac{\pi}{2}\right) + \frac{\pi}{2} \times \frac{1}{\pi} + 4 \left(\frac{\sin\left(2 \times \frac{\pi}{2}\right)}{e^{2\pi} - 1} + \frac{\sin\left(4 \times \frac{\pi}{2}\right)}{e^{4\pi} - 1} + \frac{\sin\left(6 \times \frac{\pi}{2}\right)}{e^{6\pi} - 1} \right)} \right) \right) \right)^{1/32}$$

Input:

$$\sqrt[32]{\sqrt{\cot\left(\frac{\pi}{2}\right) + \frac{\pi}{2} \times \frac{1}{\pi} + 4 \left(\frac{\sin\left(2 \times \frac{\pi}{2}\right)}{e^{2\pi} - 1} + \frac{\sin\left(4 \times \frac{\pi}{2}\right)}{e^{4\pi} - 1} + \frac{\sin\left(6 \times \frac{\pi}{2}\right)}{e^{6\pi} - 1} \right)}}$$

cot(x) is the cotangent function

Exact result:

$$\frac{1}{\sqrt[64]{2}}$$

Decimal approximation:

0.989228013193975484129124959065583667774674335384985164716...

0.9892280131939..... result very near to the dilaton value **0.989117352243 = ϕ**

Alternate form:

$$\frac{2^{63/64}}{2}$$

All 32nd roots of 1/sqrt(2):

$$\frac{e^0}{\sqrt[64]{2}} \approx 0.989228 \text{ (real, principal root)}$$

$$\frac{e^{(i\pi)/16}}{\sqrt[64]{2}} \approx 0.970220 + 0.19299i$$

$$\frac{e^{(i\pi)/8}}{\sqrt[64]{2}} \approx 0.91393 + 0.37856i$$

$$\frac{e^{(3i\pi)/16}}{64\sqrt{2}} \approx 0.82251 + 0.54959i$$

$$\frac{e^{(i\pi)/4}}{64\sqrt{2}} \approx 0.69949 + 0.69949i$$

Alternative representations:

$$\sqrt[32]{\sqrt{\cot\left(\frac{\pi}{2}\right) + \frac{\pi}{2} + 4\left(\frac{\sin\left(\frac{2\pi}{2}\right)}{e^{2\pi} - 1} + \frac{\sin\left(\frac{4\pi}{2}\right)}{e^{4\pi} - 1} + \frac{\sin\left(\frac{6\pi}{2}\right)}{e^{6\pi} - 1}\right)}} =$$

$$\sqrt[32]{\sqrt{-i \coth\left(-\frac{i\pi}{2}\right) + \frac{\pi}{2\pi} + 4\left(\frac{\cos\left(-\frac{\pi}{2}\right)}{-1 + e^{2\pi}} + \frac{\cos\left(-\frac{3\pi}{2}\right)}{-1 + e^{4\pi}} + \frac{\cos\left(-\frac{5\pi}{2}\right)}{-1 + e^{6\pi}}\right)}}$$

$$\sqrt[32]{\sqrt{\cot\left(\frac{\pi}{2}\right) + \frac{\pi}{2} + 4\left(\frac{\sin\left(\frac{2\pi}{2}\right)}{e^{2\pi} - 1} + \frac{\sin\left(\frac{4\pi}{2}\right)}{e^{4\pi} - 1} + \frac{\sin\left(\frac{6\pi}{2}\right)}{e^{6\pi} - 1}\right)}} =$$

$$\sqrt[32]{\sqrt{i + \frac{\pi}{2\pi} + 4\left(\frac{\cos\left(-\frac{\pi}{2}\right)}{-1 + e^{2\pi}} + \frac{\cos\left(-\frac{3\pi}{2}\right)}{-1 + e^{4\pi}} + \frac{\cos\left(-\frac{5\pi}{2}\right)}{-1 + e^{6\pi}}\right) + \frac{2i}{-1 + e^{i\pi}}}}$$

$$\sqrt[32]{\sqrt{\cot\left(\frac{\pi}{2}\right) + \frac{\pi}{2} + 4\left(\frac{\sin\left(\frac{2\pi}{2}\right)}{e^{2\pi} - 1} + \frac{\sin\left(\frac{4\pi}{2}\right)}{e^{4\pi} - 1} + \frac{\sin\left(\frac{6\pi}{2}\right)}{e^{6\pi} - 1}\right)}} =$$

$$\sqrt[32]{\sqrt{i + \frac{\pi}{2\pi} + 4\left(-\frac{\cos\left(\frac{3\pi}{2}\right)}{-1 + e^{2\pi}} - \frac{\cos\left(\frac{5\pi}{2}\right)}{-1 + e^{4\pi}} - \frac{\cos\left(\frac{7\pi}{2}\right)}{-1 + e^{6\pi}}\right) + \frac{2i}{-1 + e^{i\pi}}}}$$

Series representations:

$$\sqrt[32]{\sqrt{\cot\left(\frac{\pi}{2}\right) + \frac{\pi}{2} + 4\left(\frac{\sin\left(\frac{2\pi}{2}\right)}{e^{2\pi} - 1} + \frac{\sin\left(\frac{4\pi}{2}\right)}{e^{4\pi} - 1} + \frac{\sin\left(\frac{6\pi}{2}\right)}{e^{6\pi} - 1}\right)}} =$$

$$\sqrt[32]{\sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(-\frac{1}{2} + \cot\left(\frac{\pi}{2}\right) + 4\left(\frac{\sin(\pi)}{-1 + e^{2\pi}} + \frac{\sin(2\pi)}{-1 + e^{4\pi}} + \frac{\sin(3\pi)}{-1 + e^{6\pi}}\right)\right)^k}{k!}}$$

$$\begin{aligned}
& \sqrt[32]{\sqrt{\cot\left(\frac{\pi}{2}\right) + \frac{\pi}{2} + 4\left(\frac{\sin\left(\frac{2\pi}{2}\right)}{e^{2\pi} - 1} + \frac{\sin\left(\frac{4\pi}{2}\right)}{e^{4\pi} - 1} + \frac{\sin\left(\frac{6\pi}{2}\right)}{e^{6\pi} - 1}\right)}} = \\
& \left(\exp\left(i\pi \left[\frac{\arg\left(\frac{1}{2} - x + \cot\left(\frac{\pi}{2}\right) + 4\left(\frac{\sin(\pi)}{-1+e^{2\pi}} + \frac{\sin(2\pi)}{-1+e^{4\pi}} + \frac{\sin(3\pi)}{-1+e^{6\pi}}\right)\right)}{2\pi} \right] \right) \sqrt{x} \right. \\
& \left. \sum_{k=0}^{\infty} \frac{(-1)^k x^{-k} \left(-\frac{1}{2}\right)_k \left(\frac{1}{2} - x + \cot\left(\frac{\pi}{2}\right) + 4\left(\frac{\sin(\pi)}{-1+e^{2\pi}} + \frac{\sin(2\pi)}{-1+e^{4\pi}} + \frac{\sin(3\pi)}{-1+e^{6\pi}}\right)\right)^k}{k!} \right)^{\wedge} \\
& (1/32) \text{ for } (x \in \mathbb{R} \text{ and } x < 0)
\end{aligned}$$

$$\begin{aligned}
& \sqrt[32]{\sqrt{\cot\left(\frac{\pi}{2}\right) + \frac{\pi}{2} + 4\left(\frac{\sin\left(\frac{2\pi}{2}\right)}{e^{2\pi} - 1} + \frac{\sin\left(\frac{4\pi}{2}\right)}{e^{4\pi} - 1} + \frac{\sin\left(\frac{6\pi}{2}\right)}{e^{6\pi} - 1}\right)}} = \\
& \left(\left(\frac{1}{z_0}\right)^{1/2} \left[\arg\left(\frac{1}{2} + \cot\left(\frac{\pi}{2}\right) + 4\left(\frac{\sin(\pi)}{-1+e^{2\pi}} + \frac{\sin(2\pi)}{-1+e^{4\pi}} + \frac{\sin(3\pi)}{-1+e^{6\pi}}\right) - z_0\right) / (2\pi) \right] \right. \\
& \left. z_0^{1/2+1/2} \left[\arg\left(\frac{1}{2} + \cot\left(\frac{\pi}{2}\right) + 4\left(\frac{\sin(\pi)}{-1+e^{2\pi}} + \frac{\sin(2\pi)}{-1+e^{4\pi}} + \frac{\sin(3\pi)}{-1+e^{6\pi}}\right) - z_0\right) / (2\pi) \right] \right. \\
& \left. \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{1}{2} + \cot\left(\frac{\pi}{2}\right) + 4\left(\frac{\sin(\pi)}{-1+e^{2\pi}} + \frac{\sin(2\pi)}{-1+e^{4\pi}} + \frac{\sin(3\pi)}{-1+e^{6\pi}}\right) - z_0\right)^k z_0^{-k}}{k!} \right)^{\wedge} (1/32)
\end{aligned}$$

Integral representations:

$$\begin{aligned}
& \sqrt[32]{\sqrt{\cot\left(\frac{\pi}{2}\right) + \frac{\pi}{2} + 4\left(\frac{\sin\left(\frac{2\pi}{2}\right)}{e^{2\pi} - 1} + \frac{\sin\left(\frac{4\pi}{2}\right)}{e^{4\pi} - 1} + \frac{\sin\left(\frac{6\pi}{2}\right)}{e^{6\pi} - 1}\right)}} = \\
& \sqrt[32]{\sqrt{\frac{1}{2} + \int_0^1 4\pi \left(\frac{\cos(\pi t)}{-1+e^{2\pi}} + \frac{2\cos(2\pi t)}{-1+e^{4\pi}} + \frac{3\cos(3\pi t)}{-1+e^{6\pi}}\right) dt}} \\
& \sqrt[32]{\sqrt{\cot\left(\frac{\pi}{2}\right) + \frac{\pi}{2} + 4\left(\frac{\sin\left(\frac{2\pi}{2}\right)}{e^{2\pi} - 1} + \frac{\sin\left(\frac{4\pi}{2}\right)}{e^{4\pi} - 1} + \frac{\sin\left(\frac{6\pi}{2}\right)}{e^{6\pi} - 1}\right)}} = \\
& \left(\sqrt{\left(\frac{1}{2} - \int_{\frac{\pi}{2}}^{\pi} \csc^2(t) dt + 4 \int_{-i\infty+\gamma}^{i\infty+\gamma} \left(\frac{3\mathcal{A}^{-(9\pi^2)/(4s)+s} \sqrt{\pi}}{4(-1+e^{6\pi}) i s^{3/2}} + \frac{\mathcal{A}^{-\pi^2/s+s} \sqrt{\pi}}{2(-1+e^{4\pi}) i s^{3/2}} + \right. \right. \right. \\
& \left. \left. \left. \frac{\mathcal{A}^{-\pi^2/(4s)+s} \sqrt{\pi}}{4(-1+e^{2\pi}) i s^{3/2}} \right) ds \right)} \right)^{\wedge} (1/32) \text{ for } \gamma > 0
\end{aligned}$$

$$\sqrt[32]{\sqrt{\cot\left(\frac{\pi}{2}\right) + \frac{\pi}{2} + 4\left(\frac{\sin\left(\frac{2\pi}{2}\right)}{e^{2\pi} - 1} + \frac{\sin\left(\frac{4\pi}{2}\right)}{e^{4\pi} - 1} + \frac{\sin\left(\frac{6\pi}{2}\right)}{e^{6\pi} - 1}\right)} = \left(\sqrt{\left(\frac{1}{2} - \int_{\frac{\pi}{2}}^{\pi} \csc^2(t) dt + 4 \int_{-i\infty+\gamma}^{i\infty+\gamma} \left(\frac{2^{-2+2s} \pi^{-2s} \Gamma(s) \sqrt{\pi}}{(-1+e^{2\pi})^i \Gamma\left(\frac{3}{2}-s\right)} + \frac{\pi^{-2s} \Gamma(s) \sqrt{\pi}}{2(-1+e^{4\pi})^i \Gamma\left(\frac{3}{2}-s\right)} + \frac{2^{-2+2s} \times 3^{1-2s} \pi^{-2s} \Gamma(s) \sqrt{\pi}}{(-1+e^{6\pi})^i \Gamma\left(\frac{3}{2}-s\right)}\right) ds}\right)\right)^{(1/32)} \text{ for } 0 < \gamma < 1$$

And:

((((sqrt((((cot(Pi/2)+(Pi/2*1/Pi)+4((((sin(2*Pi/2))/(e^(2Pi) - 1)+(sin(4*Pi/2))/(e^(4Pi) - 1)+ (sin(6*Pi/2))/(e^(6Pi) - 1))))))))))))))^(1/1024

Input:

$$1024\sqrt{\sqrt{\cot\left(\frac{\pi}{2}\right) + \frac{\pi}{2} \times \frac{1}{\pi} + 4\left(\frac{\sin\left(2 \times \frac{\pi}{2}\right)}{e^{2\pi} - 1} + \frac{\sin\left(4 \times \frac{\pi}{2}\right)}{e^{4\pi} - 1} + \frac{\sin\left(6 \times \frac{\pi}{2}\right)}{e^{6\pi} - 1}\right)}}$$

cot(x) is the cotangent function

Exact result:

$$\frac{1}{2048\sqrt[2]{2}}$$

Decimal approximation:

0.999661606496243683942196868762815655612113089806646993243...

0.9996616064962..... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} - \phi + 1 \approx 0.9991104684$$

and to the dilaton value **0.989117352243 = ϕ**

All 1024th roots of 1/sqrt(2):

$$\frac{e^0}{\sqrt[2048]{2}} \approx 0.99966161 \text{ (real, principal root)}$$

$$\frac{e^{(i\pi)/512}}{\sqrt[2048]{2}} \approx 0.99964279 + 0.006134 i$$

$$\frac{e^{(i\pi)/256}}{\sqrt[2048]{2}} \approx 0.99958633 + 0.012267 i$$

$$\frac{e^{(3i\pi)/512}}{\sqrt[2048]{2}} \approx 0.9994922 + 0.018401 i$$

$$\frac{e^{(i\pi)/128}}{\sqrt[2048]{2}} \approx 0.9993605 + 0.024533 i$$

Alternative representations:

$$\sqrt[1024]{\sqrt{\cot\left(\frac{\pi}{2}\right) + \frac{\pi}{2\pi} + 4\left(\frac{\sin\left(\frac{2\pi}{2}\right)}{e^{2\pi} - 1} + \frac{\sin\left(\frac{4\pi}{2}\right)}{e^{4\pi} - 1} + \frac{\sin\left(\frac{6\pi}{2}\right)}{e^{6\pi} - 1}\right)}} =$$

$$\sqrt[1024]{\sqrt{-i \coth\left(-\frac{i\pi}{2}\right) + \frac{\pi}{2\pi} + 4\left(\frac{\cos\left(-\frac{\pi}{2}\right)}{-1 + e^{2\pi}} + \frac{\cos\left(-\frac{3\pi}{2}\right)}{-1 + e^{4\pi}} + \frac{\cos\left(-\frac{5\pi}{2}\right)}{-1 + e^{6\pi}}\right)}}}$$

$$\sqrt[1024]{\sqrt{\cot\left(\frac{\pi}{2}\right) + \frac{\pi}{2\pi} + 4\left(\frac{\sin\left(\frac{2\pi}{2}\right)}{e^{2\pi} - 1} + \frac{\sin\left(\frac{4\pi}{2}\right)}{e^{4\pi} - 1} + \frac{\sin\left(\frac{6\pi}{2}\right)}{e^{6\pi} - 1}\right)}} =$$

$$\sqrt[1024]{\sqrt{i + \frac{\pi}{2\pi} + 4\left(\frac{\cos\left(-\frac{\pi}{2}\right)}{-1 + e^{2\pi}} + \frac{\cos\left(-\frac{3\pi}{2}\right)}{-1 + e^{4\pi}} + \frac{\cos\left(-\frac{5\pi}{2}\right)}{-1 + e^{6\pi}}\right)} + \frac{2i}{-1 + e^{i\pi}}}$$

$$\sqrt[1024]{\sqrt{\cot\left(\frac{\pi}{2}\right) + \frac{\pi}{2\pi} + 4\left(\frac{\sin\left(\frac{2\pi}{2}\right)}{e^{2\pi} - 1} + \frac{\sin\left(\frac{4\pi}{2}\right)}{e^{4\pi} - 1} + \frac{\sin\left(\frac{6\pi}{2}\right)}{e^{6\pi} - 1}\right)}} =$$

$$\sqrt[1024]{\sqrt{i + \frac{\pi}{2\pi} + 4\left(-\frac{\cos\left(\frac{3\pi}{2}\right)}{-1 + e^{2\pi}} - \frac{\cos\left(\frac{5\pi}{2}\right)}{-1 + e^{4\pi}} - \frac{\cos\left(\frac{7\pi}{2}\right)}{-1 + e^{6\pi}}\right)} + \frac{2i}{-1 + e^{i\pi}}}$$

$\coth(x)$ is the hyperbolic cotangent function

i is the imaginary unit

Series representations:

$$\begin{aligned}
& \sqrt[1024]{\sqrt{\cot\left(\frac{\pi}{2}\right) + \frac{\pi}{2} + 4\left(\frac{\sin\left(\frac{2\pi}{2}\right)}{e^{2\pi} - 1} + \frac{\sin\left(\frac{4\pi}{2}\right)}{e^{4\pi} - 1} + \frac{\sin\left(\frac{6\pi}{2}\right)}{e^{6\pi} - 1}\right)}} = \\
& \sqrt[1024]{\sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(-\frac{1}{2} + \cot\left(\frac{\pi}{2}\right) + 4\left(\frac{\sin(\pi)}{-1+e^{2\pi}} + \frac{\sin(2\pi)}{-1+e^{4\pi}} + \frac{\sin(3\pi)}{-1+e^{6\pi}}\right)\right)^k}{k!}} \\
& \sqrt[1024]{\sqrt{\cot\left(\frac{\pi}{2}\right) + \frac{\pi}{2} + 4\left(\frac{\sin\left(\frac{2\pi}{2}\right)}{e^{2\pi} - 1} + \frac{\sin\left(\frac{4\pi}{2}\right)}{e^{4\pi} - 1} + \frac{\sin\left(\frac{6\pi}{2}\right)}{e^{6\pi} - 1}\right)}} = \\
& \left(\exp\left(i\pi \left[\frac{\arg\left(\frac{1}{2} - x + \cot\left(\frac{\pi}{2}\right) + 4\left(\frac{\sin(\pi)}{-1+e^{2\pi}} + \frac{\sin(2\pi)}{-1+e^{4\pi}} + \frac{\sin(3\pi)}{-1+e^{6\pi}}\right)\right)}{2\pi} \right] \right) \sqrt{x} \right) \\
& \sum_{k=0}^{\infty} \frac{(-1)^k x^{-k} \left(-\frac{1}{2}\right)_k \left(\frac{1}{2} - x + \cot\left(\frac{\pi}{2}\right) + 4\left(\frac{\sin(\pi)}{-1+e^{2\pi}} + \frac{\sin(2\pi)}{-1+e^{4\pi}} + \frac{\sin(3\pi)}{-1+e^{6\pi}}\right)\right)^k}{k!} \Bigg)^{\wedge} \\
& (1/1024) \text{ for } (x \in \mathbb{R} \text{ and } x < 0)
\end{aligned}$$

$$\begin{aligned}
& \sqrt[1024]{\sqrt{\cot\left(\frac{\pi}{2}\right) + \frac{\pi}{2} + 4\left(\frac{\sin\left(\frac{2\pi}{2}\right)}{e^{2\pi} - 1} + \frac{\sin\left(\frac{4\pi}{2}\right)}{e^{4\pi} - 1} + \frac{\sin\left(\frac{6\pi}{2}\right)}{e^{6\pi} - 1}\right)}} = \\
& \left(\left(\frac{1}{z_0}\right)^{1/2} \left[\arg\left(\frac{1}{2} + \cot\left(\frac{\pi}{2}\right) + 4\left(\frac{\sin(\pi)}{-1+e^{2\pi}} + \frac{\sin(2\pi)}{-1+e^{4\pi}} + \frac{\sin(3\pi)}{-1+e^{6\pi}}\right) - z_0\right) / (2\pi) \right] \right) \\
& \left(\frac{1/2 + 1/2}{z_0} \left[\arg\left(\frac{1}{2} + \cot\left(\frac{\pi}{2}\right) + 4\left(\frac{\sin(\pi)}{-1+e^{2\pi}} + \frac{\sin(2\pi)}{-1+e^{4\pi}} + \frac{\sin(3\pi)}{-1+e^{6\pi}}\right) - z_0\right) / (2\pi) \right] \right) \\
& \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{1}{2} + \cot\left(\frac{\pi}{2}\right) + 4\left(\frac{\sin(\pi)}{-1+e^{2\pi}} + \frac{\sin(2\pi)}{-1+e^{4\pi}} + \frac{\sin(3\pi)}{-1+e^{6\pi}}\right) - z_0\right)^k z_0^{-k}}{k!} \Bigg)^{\wedge} (1/ \\
& 1024)
\end{aligned}$$

Now, we have that:

Page 291

(6) If $\alpha/\beta = \frac{\pi^2}{2}$, then $\frac{\pi}{8} = \frac{\pi^3}{32\alpha^2} +$
 $\frac{\cosh \alpha}{\cosh \alpha - \cos \alpha} = \frac{\cosh 3\alpha}{3(\cosh 3\alpha - \cos 3\alpha)} + 8, e =$
 $\frac{\sinh \beta \sinh 3\beta}{\cosh 3\beta + \cos 3\beta} \cdot \frac{\cot \frac{\pi}{2}}{1} + \frac{\sinh 2\beta \sinh 4\beta}{\cosh 4\beta + \cos 4\beta} \cdot \frac{\cot \frac{\pi}{2}}{2}$

If:

$$\pi \times \frac{\pi}{2} = \frac{\pi^2}{2} \quad \text{True}$$

Therefore: $\alpha = \pi$; $\beta = \frac{\pi}{2}$

$$\left[\frac{\sin(\pi/2) \sinh(\pi/2)}{\cosh(2 \times \pi/2) + \cos(2 \times \pi/2)} \right] * \coth(\pi) / 1 + \left[\frac{\sin(2 \times \pi/2) \sinh(2 \times \pi/2)}{\cosh(4 \times \pi/2) + \cos(4 \times \pi/2)} \right] * \coth(2\pi) / 2$$

Input:

$$\frac{\sin\left(\frac{\pi}{2}\right) \sinh\left(\frac{\pi}{2}\right)}{\cosh\left(2 \times \frac{\pi}{2}\right) + \cos\left(2 \times \frac{\pi}{2}\right)} \times \frac{\coth(\pi)}{1} + \frac{\sin\left(2 \times \frac{\pi}{2}\right) \sinh\left(2 \times \frac{\pi}{2}\right)}{\cosh\left(4 \times \frac{\pi}{2}\right) + \cos\left(4 \times \frac{\pi}{2}\right)} \left(\frac{1}{2} \coth(2\pi) \right)$$

$\sinh(x)$ is the hyperbolic sine function
 $\cosh(x)$ is the hyperbolic cosine function
 $\coth(x)$ is the hyperbolic cotangent function

Exact result:

$$\frac{\sinh\left(\frac{\pi}{2}\right) \coth(\pi)}{\cosh(\pi) - 1}$$

Decimal approximation:

0.218081595613452079842917651766652016864922914746107422727...

0.218081595613....

Property:

$$\frac{\coth(\pi) \sinh\left(\frac{\pi}{2}\right)}{-1 + \cosh(\pi)} \text{ is a transcendental number}$$

Alternate forms:

$$\frac{e^{\pi/2} \coth(\pi)}{e^{\pi} - 1}$$

$$\frac{\cosh(\pi) \sinh\left(\frac{\pi}{2}\right)}{(-1 + \cosh(\pi)) \sinh(\pi)}$$

$$\frac{1}{4} \operatorname{sech}\left(\frac{\pi}{2}\right) + \frac{1}{4} \coth\left(\frac{\pi}{2}\right) \operatorname{csch}\left(\frac{\pi}{2}\right)$$

$\operatorname{csch}(x)$ is the hyperbolic cosecant function
 $\operatorname{sech}(x)$ is the hyperbolic secant function

Alternative representations:

$$\begin{aligned}
& \frac{\coth(\pi) \left(\sin\left(\frac{\pi}{2}\right) \sinh\left(\frac{\pi}{2}\right) \right)}{\cosh\left(\frac{2\pi}{2}\right) + \cos\left(\frac{2\pi}{2}\right)} + \frac{\coth(2\pi) \left(\sin\left(\frac{2\pi}{2}\right) \sinh\left(\frac{2\pi}{2}\right) \right)}{2 \left(\cosh\left(\frac{4\pi}{2}\right) + \cos\left(\frac{4\pi}{2}\right) \right)} = \\
& \frac{\cos(0) \left(-e^{-\pi/2} + e^{\pi/2} \right) \left(1 + \frac{2}{-1+e^{2\pi}} \right)}{2 \left(\frac{1}{2} (e^{-\pi} + e^{\pi}) + \frac{1}{2} (e^{-i\pi} + e^{i\pi}) \right)} + \frac{\cos\left(-\frac{\pi}{2}\right) (-e^{-\pi} + e^{\pi}) \left(1 + \frac{2}{-1+e^{4\pi}} \right)}{4 \left(\frac{1}{2} (e^{-2\pi} + e^{2\pi}) + \frac{1}{2} (e^{-2i\pi} + e^{2i\pi}) \right)} \\
& \frac{\coth(\pi) \left(\sin\left(\frac{\pi}{2}\right) \sinh\left(\frac{\pi}{2}\right) \right)}{\cosh\left(\frac{2\pi}{2}\right) + \cos\left(\frac{2\pi}{2}\right)} + \frac{\coth(2\pi) \left(\sin\left(\frac{2\pi}{2}\right) \sinh\left(\frac{2\pi}{2}\right) \right)}{2 \left(\cosh\left(\frac{4\pi}{2}\right) + \cos\left(\frac{4\pi}{2}\right) \right)} = \\
& \frac{(-e^{-\pi/2} + e^{\pi/2}) (-e^{-i\pi/2} + e^{i\pi/2}) \left(1 + \frac{2}{-1+e^{2\pi}} \right)}{2 (2i) \left(\cosh(-i\pi) + \frac{1}{2} (e^{-\pi} + e^{\pi}) \right)} + \frac{(-e^{-\pi} + e^{\pi}) (-e^{-i\pi} + e^{i\pi}) \left(1 + \frac{2}{-1+e^{4\pi}} \right)}{4 (2i) \left(\cosh(-2i\pi) + \frac{1}{2} (e^{-2\pi} + e^{2\pi}) \right)} \\
& \frac{\coth(\pi) \left(\sin\left(\frac{\pi}{2}\right) \sinh\left(\frac{\pi}{2}\right) \right)}{\cosh\left(\frac{2\pi}{2}\right) + \cos\left(\frac{2\pi}{2}\right)} + \frac{\coth(2\pi) \left(\sin\left(\frac{2\pi}{2}\right) \sinh\left(\frac{2\pi}{2}\right) \right)}{2 \left(\cosh\left(\frac{4\pi}{2}\right) + \cos\left(\frac{4\pi}{2}\right) \right)} = \\
& \frac{i \left(\cos(0) \cos\left(\frac{\pi}{2} - \frac{i\pi}{2}\right) \left(1 + \frac{2}{-1+e^{2\pi}} \right) \right)}{\frac{1}{2} (e^{-\pi} + e^{\pi}) + \frac{1}{2} (e^{-i\pi} + e^{i\pi})} - \frac{i \cos\left(-\frac{\pi}{2}\right) \cos\left(\frac{\pi}{2} - i\pi\right) \left(1 + \frac{2}{-1+e^{4\pi}} \right)}{2 \left(\frac{1}{2} (e^{-2\pi} + e^{2\pi}) + \frac{1}{2} (e^{-2i\pi} + e^{2i\pi}) \right)}
\end{aligned}$$

Series representations:

$$\begin{aligned}
& \frac{\coth(\pi) \left(\sin\left(\frac{\pi}{2}\right) \sinh\left(\frac{\pi}{2}\right) \right)}{\cosh\left(\frac{2\pi}{2}\right) + \cos\left(\frac{2\pi}{2}\right)} + \frac{\coth(2\pi) \left(\sin\left(\frac{2\pi}{2}\right) \sinh\left(\frac{2\pi}{2}\right) \right)}{2 \left(\cosh\left(\frac{4\pi}{2}\right) + \cos\left(\frac{4\pi}{2}\right) \right)} = \\
& \frac{\left(1 + 2 \sum_{k=1}^{\infty} q^{2k} \right) \sum_{k=0}^{\infty} \frac{\left(\frac{\pi}{2}\right)^{-1-2k}}{(1+2k)!}}{-1 + \sum_{k=0}^{\infty} \frac{\pi^{2k}}{(2k)!}} \quad \text{for } q = e^{\pi} \\
& \frac{\coth(\pi) \left(\sin\left(\frac{\pi}{2}\right) \sinh\left(\frac{\pi}{2}\right) \right)}{\cosh\left(\frac{2\pi}{2}\right) + \cos\left(\frac{2\pi}{2}\right)} + \frac{\coth(2\pi) \left(\sin\left(\frac{2\pi}{2}\right) \sinh\left(\frac{2\pi}{2}\right) \right)}{2 \left(\cosh\left(\frac{4\pi}{2}\right) + \cos\left(\frac{4\pi}{2}\right) \right)} = \\
& \frac{\pi^{3/2} \sum_{k_1=-\infty}^{\infty} \sum_{k_2=0}^{\infty} \frac{\text{Res}_{s=-k_2} \frac{\left(-\frac{1}{16}\right)^{-s} \pi^{-2s} \Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}}{\pi + \pi k_1^2}}{4 \left(-1 + \sum_{k=0}^{\infty} \frac{\pi^{2k}}{(2k)!} \right)} \\
& \frac{\coth(\pi) \left(\sin\left(\frac{\pi}{2}\right) \sinh\left(\frac{\pi}{2}\right) \right)}{\cosh\left(\frac{2\pi}{2}\right) + \cos\left(\frac{2\pi}{2}\right)} + \frac{\coth(2\pi) \left(\sin\left(\frac{2\pi}{2}\right) \sinh\left(\frac{2\pi}{2}\right) \right)}{2 \left(\cosh\left(\frac{4\pi}{2}\right) + \cos\left(\frac{4\pi}{2}\right) \right)} = \\
& \frac{\pi^{3/2} \left(1 + 2 \sum_{k=1}^{\infty} q^{2k} \right) \sum_{j=0}^{\infty} \text{Res}_{s=-j} \frac{\left(-\frac{1}{16}\right)^{-s} \pi^{-2s} \Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}}{4 \left(-1 + \sum_{k=0}^{\infty} \frac{\pi^{2k}}{(2k)!} \right)} \quad \text{for } q = e^{\pi}
\end{aligned}$$

$$\begin{aligned}
& \frac{\coth(\pi) \left(\sin\left(\frac{\pi}{2}\right) \sinh\left(\frac{\pi}{2}\right) \right)}{\cosh\left(\frac{2\pi}{2}\right) + \cos\left(\frac{2\pi}{2}\right)} + \frac{\coth(2\pi) \left(\sin\left(\frac{2\pi}{2}\right) \sinh\left(\frac{2\pi}{2}\right) \right)}{2 \left(\cosh\left(\frac{4\pi}{2}\right) + \cos\left(\frac{4\pi}{2}\right) \right)} = \\
& \frac{\sum_{k_1=-\infty}^{\infty} \sum_{k_2=0}^{\infty} \frac{\left(\frac{2}{\pi}\right)^{-1-2k_2}}{(1+2k_2)! (\pi+\pi k_1^2)}}{-1 + \sqrt{\pi} \sum_{j=0}^{\infty} \operatorname{Res}_{s=-j} \frac{\left(-\frac{1}{4}\right)^{-s} \pi^{-2s} \Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)}} \\
& \frac{\coth(\pi) \left(\sin\left(\frac{\pi}{2}\right) \sinh\left(\frac{\pi}{2}\right) \right)}{\cosh\left(\frac{2\pi}{2}\right) + \cos\left(\frac{2\pi}{2}\right)} + \frac{\coth(2\pi) \left(\sin\left(\frac{2\pi}{2}\right) \sinh\left(\frac{2\pi}{2}\right) \right)}{2 \left(\cosh\left(\frac{4\pi}{2}\right) + \cos\left(\frac{4\pi}{2}\right) \right)} = \\
& \frac{\left(1 + 2 \sum_{k=1}^{\infty} q^{2k}\right) \sum_{k=0}^{\infty} \frac{\left(\frac{2}{\pi}\right)^{-1-2k}}{(1+2k)!}}{-1 + \sqrt{\pi} \sum_{j=0}^{\infty} \operatorname{Res}_{s=-j} \frac{\left(-\frac{1}{4}\right)^{-s} \pi^{-2s} \Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)}} \quad \text{for } q = e^{\pi} \\
& \frac{\coth(\pi) \left(\sin\left(\frac{\pi}{2}\right) \sinh\left(\frac{\pi}{2}\right) \right)}{\cosh\left(\frac{2\pi}{2}\right) + \cos\left(\frac{2\pi}{2}\right)} + \frac{\coth(2\pi) \left(\sin\left(\frac{2\pi}{2}\right) \sinh\left(\frac{2\pi}{2}\right) \right)}{2 \left(\cosh\left(\frac{4\pi}{2}\right) + \cos\left(\frac{4\pi}{2}\right) \right)} = \\
& \frac{i \pi^{3/2} \left(1 + 2 \sum_{k=1}^{\infty} q^{2k}\right) \sum_{j=0}^{\infty} \operatorname{Res}_{s=-j} \frac{\left(-\frac{1}{16}\right)^{-s} \pi^{-2s} \Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}}{4 \left(i + \sum_{k=0}^{\infty} \frac{\left(\left(1-\frac{i}{2}\right)\pi\right)^{1+2k}}{(1+2k)!} \right)} \quad \text{for } q = e^{\pi} \\
& \frac{\coth(\pi) \left(\sin\left(\frac{\pi}{2}\right) \sinh\left(\frac{\pi}{2}\right) \right)}{\cosh\left(\frac{2\pi}{2}\right) + \cos\left(\frac{2\pi}{2}\right)} + \frac{\coth(2\pi) \left(\sin\left(\frac{2\pi}{2}\right) \sinh\left(\frac{2\pi}{2}\right) \right)}{2 \left(\cosh\left(\frac{4\pi}{2}\right) + \cos\left(\frac{4\pi}{2}\right) \right)} = \\
& \frac{\pi^{3/2} \sum_{k_1=-\infty}^{\infty} \sum_{k_2=0}^{\infty} \frac{\operatorname{Res}_{s=-k_2} \frac{\left(-\frac{1}{16}\right)^{-s} \pi^{-2s} \Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}}{\pi+\pi k_1^2}}{4 \left(-1 + \sqrt{\pi} \sum_{j=0}^{\infty} \operatorname{Res}_{s=-j} \frac{\left(-\frac{1}{4}\right)^{-s} \pi^{-2s} \Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)} \right)} \\
& \frac{\coth(\pi) \left(\sin\left(\frac{\pi}{2}\right) \sinh\left(\frac{\pi}{2}\right) \right)}{\cosh\left(\frac{2\pi}{2}\right) + \cos\left(\frac{2\pi}{2}\right)} + \frac{\coth(2\pi) \left(\sin\left(\frac{2\pi}{2}\right) \sinh\left(\frac{2\pi}{2}\right) \right)}{2 \left(\cosh\left(\frac{4\pi}{2}\right) + \cos\left(\frac{4\pi}{2}\right) \right)} = \\
& \frac{\pi^{3/2} \left(1 + 2 \sum_{k=1}^{\infty} q^{2k}\right) \sum_{j=0}^{\infty} \operatorname{Res}_{s=-j} \frac{\left(-\frac{1}{16}\right)^{-s} \pi^{-2s} \Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}}{4 \left(-1 + \sqrt{\pi} \sum_{j=0}^{\infty} \operatorname{Res}_{s=-j} \frac{\left(-\frac{1}{4}\right)^{-s} \pi^{-2s} \Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)} \right)} \quad \text{for } q = e^{\pi}
\end{aligned}$$

Integral representations:

$$\frac{\coth(\pi) \left(\sin\left(\frac{\pi}{2}\right) \sinh\left(\frac{\pi}{2}\right) \right)}{\cosh\left(\frac{2\pi}{2}\right) + \cos\left(\frac{2\pi}{2}\right)} + \frac{\coth(2\pi) \left(\sin\left(\frac{2\pi}{2}\right) \sinh\left(\frac{2\pi}{2}\right) \right)}{2 \left(\cosh\left(\frac{4\pi}{2}\right) + \cos\left(\frac{4\pi}{2}\right) \right)} =$$

$$- \int_0^1 \int_0^1 \cosh\left(\frac{\pi t_1}{2}\right) \sec^2\left(\left(\frac{1}{2} + i\right)\pi t_2\right) dt_2 dt_1$$

$$\frac{\coth(\pi) \left(\sin\left(\frac{\pi}{2}\right) \sinh\left(\frac{\pi}{2}\right) \right)}{\cosh\left(\frac{2\pi}{2}\right) + \cos\left(\frac{2\pi}{2}\right)} + \frac{\coth(2\pi) \left(\sin\left(\frac{2\pi}{2}\right) \sinh\left(\frac{2\pi}{2}\right) \right)}{2 \left(\cosh\left(\frac{4\pi}{2}\right) + \cos\left(\frac{4\pi}{2}\right) \right)} =$$

$$- \int_0^1 \int_0^1 \cosh\left(\frac{\pi t_1}{2}\right) \sec^2\left(\left(\frac{1}{2} + i\right)\pi t_2\right) dt_2 dt_1 \quad \text{for } \gamma > 0$$

$$\frac{\coth(\pi) \left(\sin\left(\frac{\pi}{2}\right) \sinh\left(\frac{\pi}{2}\right) \right)}{\cosh\left(\frac{2\pi}{2}\right) + \cos\left(\frac{2\pi}{2}\right)} + \frac{\coth(2\pi) \left(\sin\left(\frac{2\pi}{2}\right) \sinh\left(\frac{2\pi}{2}\right) \right)}{2 \left(\cosh\left(\frac{4\pi}{2}\right) + \cos\left(\frac{4\pi}{2}\right) \right)} =$$

$$\frac{i \left(\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{\pi^2/(16s)+s}}{s^{3/2}} ds \right) \int_{i\pi/2}^{\pi} \operatorname{csch}^2(t) dt}{8 \sqrt{\pi} \int_0^1 \sinh(\pi t) dt} \quad \text{for } \gamma > 0$$

$$\frac{\coth(\pi) \left(\sin\left(\frac{\pi}{2}\right) \sinh\left(\frac{\pi}{2}\right) \right)}{\cosh\left(\frac{2\pi}{2}\right) + \cos\left(\frac{2\pi}{2}\right)} + \frac{\coth(2\pi) \left(\sin\left(\frac{2\pi}{2}\right) \sinh\left(\frac{2\pi}{2}\right) \right)}{2 \left(\cosh\left(\frac{4\pi}{2}\right) + \cos\left(\frac{4\pi}{2}\right) \right)} =$$

$$\frac{i \sqrt{\pi} \left(\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{\pi^2/(16s)+s}}{s^{3/2}} ds \right) \int_{i\pi/2}^{\pi} \operatorname{csch}^2(t) dt}{8 \left(-1 + \int_{i\pi/2}^{\pi} \sinh(t) dt \right)} \quad \text{for } \gamma > 0$$

$$\frac{\coth(\pi) \left(\sin\left(\frac{\pi}{2}\right) \sinh\left(\frac{\pi}{2}\right) \right)}{\cosh\left(\frac{2\pi}{2}\right) + \cos\left(\frac{2\pi}{2}\right)} + \frac{\coth(2\pi) \left(\sin\left(\frac{2\pi}{2}\right) \sinh\left(\frac{2\pi}{2}\right) \right)}{2 \left(\cosh\left(\frac{4\pi}{2}\right) + \cos\left(\frac{4\pi}{2}\right) \right)} =$$

$$\frac{\pi \left(\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{\pi^2/(16s)+s}}{s^{3/2}} ds \right) \int_{i\pi/2}^{\pi} \operatorname{csch}^2(t) dt}{4 \left(2 i \sqrt{\pi} - \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{\pi^2/(4s)+s}}{\sqrt{s}} ds \right)} \quad \text{for } \gamma > 0$$

$$\left(\left(\left(\left(\left(\left(\frac{\sin(\pi/2) \sinh(\pi/2)}{\cosh(2\pi/2) + \cos(2\pi/2)} \right) \right) \right) \right) \right) \right) \cdot \coth(\pi) + \left(\left(\left(\left(\left(\frac{\sin(2\pi/2) \sinh(2\pi/2)}{\cosh(4\pi/2) + \cos(4\pi/2)} \right) \right) \right) \right) \right) \cdot \coth(2\pi) \right)^{1/64}$$

Input:

$$\sqrt[64]{ \frac{\sin\left(\frac{\pi}{2}\right) \sinh\left(\frac{\pi}{2}\right)}{\cosh\left(2 \times \frac{\pi}{2}\right) + \cos\left(2 \times \frac{\pi}{2}\right)} \times \frac{\coth(\pi)}{1} + \frac{\sin\left(2 \times \frac{\pi}{2}\right) \sinh\left(2 \times \frac{\pi}{2}\right)}{\cosh\left(4 \times \frac{\pi}{2}\right) + \cos\left(4 \times \frac{\pi}{2}\right)} \left(\frac{1}{2} \coth(2\pi) \right) }$$

$\sinh(x)$ is the hyperbolic sine function
 $\cosh(x)$ is the hyperbolic cosine function

Exact result:

$$\sqrt[64]{\frac{\sinh\left(\frac{\pi}{2}\right) \coth(\pi)}{\cosh(\pi) - 1}}$$

Decimal approximation:

0.976485777383638125794730580784990874431636342239055165323...

0.976485777383638.... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}-\varphi+1}} = 1 - \frac{e^{-\pi}}{1 + \frac{e^{-2\pi}}{1 + \frac{e^{-3\pi}}{1 + \frac{e^{-4\pi}}{1 + \dots}}}} \approx 0.9568666373$$

and very near to the spectral index n_s and to the mesonic Regge slope (see Appendix) and to the inflaton value at the end of the inflation 0.9402

Property:

$$\sqrt[64]{\frac{\coth(\pi) \sinh\left(\frac{\pi}{2}\right)}{-1 + \cosh(\pi)}} \text{ is a transcendental number}$$

Alternate forms:

$$e^{\pi/128} \sqrt[64]{\frac{\coth(\pi)}{e^\pi - 1}}$$

$$\sqrt[64]{\frac{\cosh(\pi)}{\cosh\left(\frac{3\pi}{2}\right) - \cosh\left(\frac{\pi}{2}\right)}}$$

$$\sqrt[64]{\frac{\cosh(\pi) \sinh\left(\frac{\pi}{2}\right)}{(-1 + \cosh(\pi)) \sinh(\pi)}}$$

All 64th roots of $(\sinh(\pi/2) \coth(\pi))/(\cosh(\pi) - 1)$:

$$e^0 \sqrt[64]{\frac{\sinh\left(\frac{\pi}{2}\right) \coth(\pi)}{\cosh(\pi) - 1}} \approx 0.97649 \text{ (real, principal root)}$$

$$e^{(i\pi)/32} \sqrt[64]{\frac{\sinh\left(\frac{\pi}{2}\right) \coth(\pi)}{\cosh(\pi) - 1}} \approx 0.97178 + 0.09571 i$$

$$e^{(i\pi)/16} \sqrt[64]{\frac{\sinh\left(\frac{\pi}{2}\right) \coth(\pi)}{\cosh(\pi) - 1}} \approx 0.95772 + 0.19050 i$$

$$e^{(3i\pi)/32} \sqrt[64]{\frac{\sinh\left(\frac{\pi}{2}\right) \coth(\pi)}{\cosh(\pi) - 1}} \approx 0.93444 + 0.28346 i$$

$$e^{(i\pi)/8} \sqrt[64]{\frac{\sinh\left(\frac{\pi}{2}\right) \coth(\pi)}{\cosh(\pi) - 1}} \approx 0.90216 + 0.37368 i$$

Alternative representations:

$$\sqrt[64]{\frac{\coth(\pi) \left(\sin\left(\frac{\pi}{2}\right) \sinh\left(\frac{\pi}{2}\right)\right)}{\cosh\left(\frac{2\pi}{2}\right) + \cos\left(\frac{2\pi}{2}\right)} + \frac{\coth(2\pi) \left(\sin\left(\frac{2\pi}{2}\right) \sinh\left(\frac{2\pi}{2}\right)\right)}{2 \left(\cosh\left(\frac{4\pi}{2}\right) + \cos\left(\frac{4\pi}{2}\right)\right)} =}$$

$$\sqrt[64]{\frac{\cos(0) \left(-e^{-\pi/2} + e^{\pi/2}\right) \left(1 + \frac{2}{-1+e^{2\pi}}\right)}{2 \left(\frac{1}{2} (e^{-\pi} + e^{\pi}) + \frac{1}{2} (e^{-i\pi} + e^{i\pi})\right)} + \frac{\cos\left(-\frac{\pi}{2}\right) \left(-e^{-\pi} + e^{\pi}\right) \left(1 + \frac{2}{-1+e^{4\pi}}\right)}{4 \left(\frac{1}{2} (e^{-2\pi} + e^{2\pi}) + \frac{1}{2} (e^{-2i\pi} + e^{2i\pi})\right)}}$$

$$\sqrt[64]{\frac{\coth(\pi) \left(\sin\left(\frac{\pi}{2}\right) \sinh\left(\frac{\pi}{2}\right)\right)}{\cosh\left(\frac{2\pi}{2}\right) + \cos\left(\frac{2\pi}{2}\right)} + \frac{\coth(2\pi) \left(\sin\left(\frac{2\pi}{2}\right) \sinh\left(\frac{2\pi}{2}\right)\right)}{2 \left(\cosh\left(\frac{4\pi}{2}\right) + \cos\left(\frac{4\pi}{2}\right)\right)} =}$$

$$\sqrt[64]{-\frac{i \left(\cos(0) \cos\left(\frac{\pi}{2} - \frac{i\pi}{2}\right) \left(1 + \frac{2}{-1+e^{2\pi}}\right)\right)}{\frac{1}{2} (e^{-\pi} + e^{\pi}) + \frac{1}{2} (e^{-i\pi} + e^{i\pi})} - \frac{i \cos\left(-\frac{\pi}{2}\right) \cos\left(\frac{\pi}{2} - i\pi\right) \left(1 + \frac{2}{-1+e^{4\pi}}\right)}{2 \left(\frac{1}{2} (e^{-2\pi} + e^{2\pi}) + \frac{1}{2} (e^{-2i\pi} + e^{2i\pi})\right)}}$$

$$\sqrt[64]{\frac{\coth(\pi) \left(\sin\left(\frac{\pi}{2}\right) \sinh\left(\frac{\pi}{2}\right)\right)}{\cosh\left(\frac{2\pi}{2}\right) + \cos\left(\frac{2\pi}{2}\right)} + \frac{\coth(2\pi) \left(\sin\left(\frac{2\pi}{2}\right) \sinh\left(\frac{2\pi}{2}\right)\right)}{2 \left(\cosh\left(\frac{4\pi}{2}\right) + \cos\left(\frac{4\pi}{2}\right)\right)} =}$$

$$\left(\frac{\left(-e^{-\pi/2} + e^{\pi/2}\right) \left(-e^{-i\pi/2} + e^{i\pi/2}\right) \left(1 + \frac{2}{-1+e^{2\pi}}\right)}{2 (2i) \left(\cosh(-i\pi) + \frac{1}{2} (e^{-\pi} + e^{\pi})\right)} + \frac{\left(-e^{-\pi} + e^{\pi}\right) \left(-e^{-i\pi} + e^{i\pi}\right) \left(1 + \frac{2}{-1+e^{4\pi}}\right)}{4 (2i) \left(\cosh(-2i\pi) + \frac{1}{2} (e^{-2\pi} + e^{2\pi})\right)} \right)^{\wedge} (1/64)$$

Series representations:

$$\sqrt[64]{\frac{\coth(\pi) \left(\sin\left(\frac{\pi}{2}\right) \sinh\left(\frac{\pi}{2}\right)\right)}{\cosh\left(\frac{2\pi}{2}\right) + \cos\left(\frac{2\pi}{2}\right)} + \frac{\coth(2\pi) \left(\sin\left(\frac{2\pi}{2}\right) \sinh\left(\frac{2\pi}{2}\right)\right)}{2 \left(\cosh\left(\frac{4\pi}{2}\right) + \cos\left(\frac{4\pi}{2}\right)\right)} =}$$

$$\sqrt[64]{\frac{\left(1 + 2 \sum_{k=1}^{\infty} q^{2k}\right) \sum_{k=0}^{\infty} \frac{\left(\frac{2}{\pi}\right)^{-1-2k}}{(1+2k)!}}{-1 + \sum_{k=0}^{\infty} \frac{\pi^{2k}}{(2k)!}} \quad \text{for } q = e^{\pi}$$

$$\sqrt[64]{\frac{\coth(\pi)\left(\sin\left(\frac{\pi}{2}\right)\sinh\left(\frac{\pi}{2}\right)\right)}{\cosh\left(\frac{2\pi}{2}\right)+\cos\left(\frac{2\pi}{2}\right)}+\frac{\coth(2\pi)\left(\sin\left(\frac{2\pi}{2}\right)\sinh\left(\frac{2\pi}{2}\right)\right)}{2\left(\cosh\left(\frac{4\pi}{2}\right)+\cos\left(\frac{4\pi}{2}\right)\right)}} =$$

$$\frac{\pi^{3/128}\sqrt[64]{\frac{\sum_{k_1=-\infty}^{\infty}\sum_{k_2=0}^{\infty}\frac{\operatorname{Res}_{s=-k_2}\left(\frac{\left(-\frac{1}{16}\right)^{-s}\pi^{-2s}\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\right)}{\pi+\pi k_1^2}}}{-1+\sum_{k=0}^{\infty}\frac{\pi^{2k}}{(2k)!}}}{\sqrt[32]{2}}$$

$$\sqrt[64]{\frac{\coth(\pi)\left(\sin\left(\frac{\pi}{2}\right)\sinh\left(\frac{\pi}{2}\right)\right)}{\cosh\left(\frac{2\pi}{2}\right)+\cos\left(\frac{2\pi}{2}\right)}+\frac{\coth(2\pi)\left(\sin\left(\frac{2\pi}{2}\right)\sinh\left(\frac{2\pi}{2}\right)\right)}{2\left(\cosh\left(\frac{4\pi}{2}\right)+\cos\left(\frac{4\pi}{2}\right)\right)}} =$$

$$\frac{\pi^{3/128}\sqrt[64]{\frac{\left(1+2\sum_{k=1}^{\infty}q^{2k}\right)\sum_{j=0}^{\infty}\operatorname{Res}_{s=-j}\left(\frac{\left(-\frac{1}{16}\right)^{-s}\pi^{-2s}\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\right)}{-1+\sum_{k=0}^{\infty}\frac{\pi^{2k}}{(2k)!}}}{\sqrt[32]{2}} \quad \text{for } q = e^\pi$$

$$\sqrt[64]{\frac{\coth(\pi)\left(\sin\left(\frac{\pi}{2}\right)\sinh\left(\frac{\pi}{2}\right)\right)}{\cosh\left(\frac{2\pi}{2}\right)+\cos\left(\frac{2\pi}{2}\right)}+\frac{\coth(2\pi)\left(\sin\left(\frac{2\pi}{2}\right)\sinh\left(\frac{2\pi}{2}\right)\right)}{2\left(\cosh\left(\frac{4\pi}{2}\right)+\cos\left(\frac{4\pi}{2}\right)\right)}} =$$

$$\frac{\sqrt[64]{\frac{\left(1+2\sum_{k=1}^{\infty}q^{2k}\right)\sum_{k=0}^{\infty}\frac{\left(\frac{2}{\pi}\right)^{-1-2k}}{(1+2k)!}}{-1+\sqrt{\pi}\sum_{j=0}^{\infty}\operatorname{Res}_{s=-j}\left(\frac{\left(-\frac{1}{4}\right)^{-s}\pi^{-2s}\Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)}\right)}}}{\text{for } q = e^\pi}$$

$$\sqrt[64]{\frac{\coth(\pi)\left(\sin\left(\frac{\pi}{2}\right)\sinh\left(\frac{\pi}{2}\right)\right)}{\cosh\left(\frac{2\pi}{2}\right)+\cos\left(\frac{2\pi}{2}\right)}+\frac{\coth(2\pi)\left(\sin\left(\frac{2\pi}{2}\right)\sinh\left(\frac{2\pi}{2}\right)\right)}{2\left(\cosh\left(\frac{4\pi}{2}\right)+\cos\left(\frac{4\pi}{2}\right)\right)}} =$$

$$\frac{\sqrt[64]{\frac{\sum_{k_1=-\infty}^{\infty}\sum_{k_2=0}^{\infty}\frac{\left(\frac{2}{\pi}\right)^{-1-2k_2}}{(1+2k_2)!(\pi+\pi k_1^2)}}{-1+\sqrt{\pi}\sum_{j=0}^{\infty}\operatorname{Res}_{s=-j}\left(\frac{\left(-\frac{1}{4}\right)^{-s}\pi^{-2s}\Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)}\right)}}}{\sqrt[64]{-1+\sqrt{\pi}\sum_{j=0}^{\infty}\operatorname{Res}_{s=-j}\left(\frac{\left(-\frac{1}{4}\right)^{-s}\pi^{-2s}\Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)}\right)}}$$

$$\sqrt[64]{\frac{\coth(\pi)\left(\sin\left(\frac{\pi}{2}\right)\sinh\left(\frac{\pi}{2}\right)\right)}{\cosh\left(\frac{2\pi}{2}\right)+\cos\left(\frac{2\pi}{2}\right)}+\frac{\coth(2\pi)\left(\sin\left(\frac{2\pi}{2}\right)\sinh\left(\frac{2\pi}{2}\right)\right)}{2\left(\cosh\left(\frac{4\pi}{2}\right)+\cos\left(\frac{4\pi}{2}\right)\right)}} =$$

$$\frac{\pi^{3/128}\sqrt[64]{\frac{i\left(1+2\sum_{k=1}^{\infty}q^{2k}\right)\sum_{j=0}^{\infty}\operatorname{Res}_{s=-j}\left(\frac{\left(-\frac{1}{16}\right)^{-s}\pi^{-2s}\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\right)}{i+\sum_{k=0}^{\infty}\frac{\left(\left(1-\frac{i}{2}\right)\pi\right)^{1+2k}}{(1+2k)!}}}{\sqrt[32]{2}} \quad \text{for } q = e^\pi$$

$$\begin{aligned}
& \sqrt[64]{\frac{\coth(\pi) \left(\sin\left(\frac{\pi}{2}\right) \sinh\left(\frac{\pi}{2}\right) \right)}{\cosh\left(\frac{2\pi}{2}\right) + \cos\left(\frac{2\pi}{2}\right)} + \frac{\coth(2\pi) \left(\sin\left(\frac{2\pi}{2}\right) \sinh\left(\frac{2\pi}{2}\right) \right)}{2 \left(\cosh\left(\frac{4\pi}{2}\right) + \cos\left(\frac{4\pi}{2}\right) \right)}} = \\
& \frac{\pi^{3/128} \sqrt[64]{\frac{\sum_{k_1=-\infty}^{\infty} \sum_{k_2=0}^{\infty} \frac{\operatorname{Res}_{s=-k_2} \left(\left(-\frac{1}{16}\right)^{-s} \pi^{-2s} \Gamma(s) \right)}{\Gamma\left(\frac{3}{2}-s\right)}}{\pi + \pi k_1^2}}}{-1 + \sqrt{\pi} \sum_{j=0}^{\infty} \operatorname{Res}_{s=-j} \frac{\left(-\frac{1}{4}\right)^{-s} \pi^{-2s} \Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)}}} \\
& \frac{\pi^{3/128} \sqrt[64]{\frac{\left(1 + 2 \sum_{k=1}^{\infty} q^{2k}\right) \sum_{j=0}^{\infty} \operatorname{Res}_{s=-j} \frac{\left(-\frac{1}{16}\right)^{-s} \pi^{-2s} \Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}}}{-1 + \sqrt{\pi} \sum_{j=0}^{\infty} \operatorname{Res}_{s=-j} \frac{\left(-\frac{1}{4}\right)^{-s} \pi^{-2s} \Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)}}}}{\sqrt[32]{2}} \quad \text{for } q = e^{\pi}
\end{aligned}$$

Integral representations:

$$\begin{aligned}
& \sqrt[64]{\frac{\coth(\pi) \left(\sin\left(\frac{\pi}{2}\right) \sinh\left(\frac{\pi}{2}\right) \right)}{\cosh\left(\frac{2\pi}{2}\right) + \cos\left(\frac{2\pi}{2}\right)} + \frac{\coth(2\pi) \left(\sin\left(\frac{2\pi}{2}\right) \sinh\left(\frac{2\pi}{2}\right) \right)}{2 \left(\cosh\left(\frac{4\pi}{2}\right) + \cos\left(\frac{4\pi}{2}\right) \right)}} = \\
& \frac{64 \sqrt{-\int_0^1 \int_0^1 \cosh\left(\frac{\pi t_1}{2}\right) \sec^2\left(\left(\frac{1}{2} + i\right) \pi t_2\right) dt_2 dt_1}}{\sqrt[64]{2}} \\
& \sqrt[64]{\frac{\coth(\pi) \left(\sin\left(\frac{\pi}{2}\right) \sinh\left(\frac{\pi}{2}\right) \right)}{\cosh\left(\frac{2\pi}{2}\right) + \cos\left(\frac{2\pi}{2}\right)} + \frac{\coth(2\pi) \left(\sin\left(\frac{2\pi}{2}\right) \sinh\left(\frac{2\pi}{2}\right) \right)}{2 \left(\cosh\left(\frac{4\pi}{2}\right) + \cos\left(\frac{4\pi}{2}\right) \right)}} = \\
& \frac{\pi^{3/128} \sqrt[64]{-\int_0^1 \int_0^1 \cosh\left(\frac{\pi t_1}{2}\right) \sec^2\left(\left(\frac{1}{2} + i\right) \pi t_2\right) dt_2 dt_1}}{\sqrt[64]{2}} \quad \text{for } \gamma > 0 \\
& \sqrt[64]{\frac{\coth(\pi) \left(\sin\left(\frac{\pi}{2}\right) \sinh\left(\frac{\pi}{2}\right) \right)}{\cosh\left(\frac{2\pi}{2}\right) + \cos\left(\frac{2\pi}{2}\right)} + \frac{\coth(2\pi) \left(\sin\left(\frac{2\pi}{2}\right) \sinh\left(\frac{2\pi}{2}\right) \right)}{2 \left(\cosh\left(\frac{4\pi}{2}\right) + \cos\left(\frac{4\pi}{2}\right) \right)}} = \\
& \frac{64 \sqrt{\frac{\pi}{2}} \sqrt[64]{-\int_0^1 \int_0^1 \cosh\left(\frac{\pi t_1}{2}\right) \sec^2\left(\left(\frac{1}{2} + i\right) \pi t_2\right) dt_2 dt_1}}{\sqrt[64]{2}}
\end{aligned}$$

$$\sqrt[64]{\frac{\coth(\pi) \left(\sin\left(\frac{\pi}{2}\right) \sinh\left(\frac{\pi}{2}\right)\right)}{\cosh\left(\frac{2\pi}{2}\right) + \cos\left(\frac{2\pi}{2}\right)} + \frac{\coth(2\pi) \left(\sin\left(\frac{2\pi}{2}\right) \sinh\left(\frac{2\pi}{2}\right)\right)}{2 \left(\cosh\left(\frac{4\pi}{2}\right) + \cos\left(\frac{4\pi}{2}\right)\right)}} =$$

$$\sqrt[64]{\frac{i \left(\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{\pi^2/(16s)+s}}{s^{3/2}} ds\right) \int_{i\pi/2}^{\pi} \operatorname{csch}^2(t) dt}{\int_0^1 \sinh(\pi t) dt}} \quad \text{for } \gamma > 0$$

$$\frac{2^{3/64} 128\sqrt{\pi}}{\sqrt[64]{\pi}}$$

$$\sqrt[64]{\frac{\coth(\pi) \left(\sin\left(\frac{\pi}{2}\right) \sinh\left(\frac{\pi}{2}\right)\right)}{\cosh\left(\frac{2\pi}{2}\right) + \cos\left(\frac{2\pi}{2}\right)} + \frac{\coth(2\pi) \left(\sin\left(\frac{2\pi}{2}\right) \sinh\left(\frac{2\pi}{2}\right)\right)}{2 \left(\cosh\left(\frac{4\pi}{2}\right) + \cos\left(\frac{4\pi}{2}\right)\right)}} =$$

$$\frac{128\sqrt{\pi}}{2^{3/64}} \sqrt[64]{\frac{i \left(\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{\pi^2/(16s)+s}}{s^{3/2}} ds\right) \int_{i\pi/2}^{\pi} \operatorname{csch}^2(t) dt}{-1 + \int_{i\pi/2}^{\pi} \sinh(t) dt}} \quad \text{for } \gamma > 0$$

$$\sqrt[64]{\frac{\coth(\pi) \left(\sin\left(\frac{\pi}{2}\right) \sinh\left(\frac{\pi}{2}\right)\right)}{\cosh\left(\frac{2\pi}{2}\right) + \cos\left(\frac{2\pi}{2}\right)} + \frac{\coth(2\pi) \left(\sin\left(\frac{2\pi}{2}\right) \sinh\left(\frac{2\pi}{2}\right)\right)}{2 \left(\cosh\left(\frac{4\pi}{2}\right) + \cos\left(\frac{4\pi}{2}\right)\right)}} =$$

$$\frac{64\sqrt{\pi}}{32\sqrt{2}} \sqrt[64]{\frac{i \left(\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{\pi^2/(16s)+s}}{s^{3/2}} ds\right) \int_{i\pi/2}^{\pi} \operatorname{csch}^2(t) dt}{2\sqrt{\pi} + i \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{\pi^2/(4s)+s}}{\sqrt{s}} ds}} \quad \text{for } \gamma > 0$$

And:

$$\left(\left(\left(\left(\left(\left(\sin\left(\frac{\pi}{2}\right) \sinh\left(\frac{\pi}{2}\right)\right)\right)\right)\right)\right)\right) / \left(\left(\left(\cosh\left(2 \times \frac{\pi}{2}\right) + \cos\left(2 \times \frac{\pi}{2}\right)\right)\right)\right)\right) \times \coth(\pi) / 1 +$$

$$\left[\left(\left(\left(\sin\left(2 \times \frac{\pi}{2}\right) \sinh\left(2 \times \frac{\pi}{2}\right)\right)\right)\right)\right] / \left(\left(\left(\cosh\left(4 \times \frac{\pi}{2}\right) + \cos\left(4 \times \frac{\pi}{2}\right)\right)\right)\right)\right) \times$$

$$\coth\left(2 \times \frac{\pi}{2}\right)\right)^{1/128}$$

Input:

$$\sqrt[128]{\frac{\sin\left(\frac{\pi}{2}\right) \sinh\left(\frac{\pi}{2}\right)}{\cosh\left(2 \times \frac{\pi}{2}\right) + \cos\left(2 \times \frac{\pi}{2}\right)} \times \frac{\coth(\pi)}{1} + \frac{\sin\left(2 \times \frac{\pi}{2}\right) \sinh\left(2 \times \frac{\pi}{2}\right)}{\cosh\left(4 \times \frac{\pi}{2}\right) + \cos\left(4 \times \frac{\pi}{2}\right)} \left(\frac{1}{2} \coth(2\pi)\right)}$$

Exact result:

$$\sqrt[128]{\frac{\sinh\left(\frac{\pi}{2}\right) \coth(\pi)}{\cosh(\pi) - 1}}$$

Decimal approximation:

0.988172949125626554051191978997501283857357848864886891665...

0.9881729491256265..... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

$$\frac{\sqrt{5}}{1 + \sqrt[5]{\sqrt{\phi^5 4\sqrt{5^3} - 1}}} - \phi + 1$$

and to the dilaton value **0.989117352243 = ϕ**

Property:

$$\sqrt[128]{\frac{\coth(\pi) \sinh\left(\frac{\pi}{2}\right)}{-1 + \cosh(\pi)}} \text{ is a transcendental number}$$

Alternate forms:

$$e^{\pi/256} \sqrt[128]{\frac{\coth(\pi)}{e^\pi - 1}}$$

$$\sqrt[128]{\frac{\cosh(\pi)}{\cosh\left(\frac{3\pi}{2}\right) - \cosh\left(\frac{\pi}{2}\right)}}$$

$$\sqrt[128]{\frac{\cosh(\pi) \sinh\left(\frac{\pi}{2}\right)}{(-1 + \cosh(\pi)) \sinh(\pi)}}$$

All 128th roots of $(\sinh(\pi/2) \coth(\pi))/(\cosh(\pi) - 1)$:

$$e^0 \sqrt[128]{\frac{\sinh\left(\frac{\pi}{2}\right) \coth(\pi)}{\cosh(\pi) - 1}} \approx 0.988173 \text{ (real, principal root)}$$

$$e^{(i\pi)/64} \sqrt[128]{\frac{\sinh\left(\frac{\pi}{2}\right) \coth(\pi)}{\cosh(\pi) - 1}} \approx 0.986983 + 0.048487 i$$

$$e^{(i\pi)/32} \sqrt[128]{\frac{\sinh\left(\frac{\pi}{2}\right) \coth(\pi)}{\cosh(\pi) - 1}} \approx 0.983415 + 0.09686 i$$

$$e^{(3i\pi)/64} \sqrt[128]{\frac{\sinh\left(\frac{\pi}{2}\right) \coth(\pi)}{\cosh(\pi) - 1}} \approx 0.977477 + 0.14500 i$$

$$e^{(i\pi)/16} \sqrt[128]{\frac{\sinh\left(\frac{\pi}{2}\right) \coth(\pi)}{\cosh(\pi) - 1}} \approx 0.96919 + 0.19278 i$$

Alternative representations:

$$\sqrt[128]{\frac{\coth(\pi) \left(\sin\left(\frac{\pi}{2}\right) \sinh\left(\frac{\pi}{2}\right)\right) + \coth(2\pi) \left(\sin\left(\frac{2\pi}{2}\right) \sinh\left(\frac{2\pi}{2}\right)\right)}{\cosh\left(\frac{2\pi}{2}\right) + \cos\left(\frac{2\pi}{2}\right)} + \frac{\coth(2\pi) \left(\sin\left(\frac{2\pi}{2}\right) \sinh\left(\frac{2\pi}{2}\right)\right)}{2 \left(\cosh\left(\frac{4\pi}{2}\right) + \cos\left(\frac{4\pi}{2}\right)\right)} =}$$

$$\sqrt[128]{\frac{\cos(0) \left(-e^{-\pi/2} + e^{\pi/2}\right) \left(1 + \frac{2}{-1+e^{2\pi}}\right) + \cos\left(-\frac{\pi}{2}\right) \left(-e^{-\pi} + e^{\pi}\right) \left(1 + \frac{2}{-1+e^{4\pi}}\right)}{2 \left(\frac{1}{2} \left(e^{-\pi} + e^{\pi}\right) + \frac{1}{2} \left(e^{-i\pi} + e^{i\pi}\right)\right) + 4 \left(\frac{1}{2} \left(e^{-2\pi} + e^{2\pi}\right) + \frac{1}{2} \left(e^{-2i\pi} + e^{2i\pi}\right)\right)}}}$$

$$\sqrt[128]{\frac{\coth(\pi) \left(\sin\left(\frac{\pi}{2}\right) \sinh\left(\frac{\pi}{2}\right)\right) + \coth(2\pi) \left(\sin\left(\frac{2\pi}{2}\right) \sinh\left(\frac{2\pi}{2}\right)\right)}{\cosh\left(\frac{2\pi}{2}\right) + \cos\left(\frac{2\pi}{2}\right)} + \frac{\coth(2\pi) \left(\sin\left(\frac{2\pi}{2}\right) \sinh\left(\frac{2\pi}{2}\right)\right)}{2 \left(\cosh\left(\frac{4\pi}{2}\right) + \cos\left(\frac{4\pi}{2}\right)\right)} =}$$

$$\sqrt[128]{-\frac{i \left(\cos(0) \cos\left(\frac{\pi}{2} - \frac{i\pi}{2}\right) \left(1 + \frac{2}{-1+e^{2\pi}}\right)\right) - i \cos\left(-\frac{\pi}{2}\right) \cos\left(\frac{\pi}{2} - i\pi\right) \left(1 + \frac{2}{-1+e^{4\pi}}\right)}{\frac{1}{2} \left(e^{-\pi} + e^{\pi}\right) + \frac{1}{2} \left(e^{-i\pi} + e^{i\pi}\right)} - \frac{i \cos\left(-\frac{\pi}{2}\right) \cos\left(\frac{\pi}{2} - i\pi\right) \left(1 + \frac{2}{-1+e^{4\pi}}\right)}{2 \left(\frac{1}{2} \left(e^{-2\pi} + e^{2\pi}\right) + \frac{1}{2} \left(e^{-2i\pi} + e^{2i\pi}\right)\right)}}}$$

$$\sqrt[128]{\frac{\coth(\pi) \left(\sin\left(\frac{\pi}{2}\right) \sinh\left(\frac{\pi}{2}\right)\right) + \coth(2\pi) \left(\sin\left(\frac{2\pi}{2}\right) \sinh\left(\frac{2\pi}{2}\right)\right)}{\cosh\left(\frac{2\pi}{2}\right) + \cos\left(\frac{2\pi}{2}\right)} + \frac{\coth(2\pi) \left(\sin\left(\frac{2\pi}{2}\right) \sinh\left(\frac{2\pi}{2}\right)\right)}{2 \left(\cosh\left(\frac{4\pi}{2}\right) + \cos\left(\frac{4\pi}{2}\right)\right)} =}$$

$$\left(\frac{\left(-e^{-\pi/2} + e^{\pi/2}\right) \left(-e^{-i\pi/2} + e^{i\pi/2}\right) \left(1 + \frac{2}{-1+e^{2\pi}}\right)}{2 (2i) \left(\cosh(-i\pi) + \frac{1}{2} \left(e^{-\pi} + e^{\pi}\right)\right)} + \frac{\left(-e^{-\pi} + e^{\pi}\right) \left(-e^{-i\pi} + e^{i\pi}\right) \left(1 + \frac{2}{-1+e^{4\pi}}\right)}{4 (2i) \left(\cosh(-2i\pi) + \frac{1}{2} \left(e^{-2\pi} + e^{2\pi}\right)\right)} \right)^{\wedge (1/128)}$$

Series representations:

$$\sqrt[128]{\frac{\coth(\pi) \left(\sin\left(\frac{\pi}{2}\right) \sinh\left(\frac{\pi}{2}\right)\right) + \coth(2\pi) \left(\sin\left(\frac{2\pi}{2}\right) \sinh\left(\frac{2\pi}{2}\right)\right)}{\cosh\left(\frac{2\pi}{2}\right) + \cos\left(\frac{2\pi}{2}\right)} + \frac{\coth(2\pi) \left(\sin\left(\frac{2\pi}{2}\right) \sinh\left(\frac{2\pi}{2}\right)\right)}{2 \left(\cosh\left(\frac{4\pi}{2}\right) + \cos\left(\frac{4\pi}{2}\right)\right)} =}$$

$$\sqrt[128]{\frac{\left(1 + 2 \sum_{k=1}^{\infty} q^{2k}\right) \sum_{k=0}^{\infty} \frac{\left(\frac{2}{\pi}\right)^{-1-2k}}{(1+2k)!}}{-1 + \sum_{k=0}^{\infty} \frac{\pi^{2k}}{(2k)!}} \quad \text{for } q = e^{\pi}}$$

$$\begin{aligned}
& \sqrt[128]{\frac{\coth(\pi) \left(\sin\left(\frac{\pi}{2}\right) \sinh\left(\frac{\pi}{2}\right) \right) + \coth(2\pi) \left(\sin\left(\frac{2\pi}{2}\right) \sinh\left(\frac{2\pi}{2}\right) \right)}{\cosh\left(\frac{2\pi}{2}\right) + \cos\left(\frac{2\pi}{2}\right)} + \frac{\coth(2\pi) \left(\sin\left(\frac{2\pi}{2}\right) \sinh\left(\frac{2\pi}{2}\right) \right)}{2 \left(\cosh\left(\frac{4\pi}{2}\right) + \cos\left(\frac{4\pi}{2}\right) \right)}} = \\
& \frac{\pi^{3/256} \sqrt[128]{\frac{\sum_{k_1=-\infty}^{\infty} \sum_{k_2=0}^{\infty} \frac{\operatorname{Res}_{s=-k_2} \left(\left(-\frac{1}{16}\right)^{-s} \pi^{-2s} \Gamma(s) \right)}{\Gamma\left(\frac{3}{2}-s\right)}}{\pi + \pi k_1^2}}}{-1 + \sum_{k=0}^{\infty} \frac{\pi^2 k}{(2k)!}}}{\sqrt[64]{2}} \\
& \sqrt[128]{\frac{\coth(\pi) \left(\sin\left(\frac{\pi}{2}\right) \sinh\left(\frac{\pi}{2}\right) \right) + \coth(2\pi) \left(\sin\left(\frac{2\pi}{2}\right) \sinh\left(\frac{2\pi}{2}\right) \right)}{\cosh\left(\frac{2\pi}{2}\right) + \cos\left(\frac{2\pi}{2}\right)} + \frac{\coth(2\pi) \left(\sin\left(\frac{2\pi}{2}\right) \sinh\left(\frac{2\pi}{2}\right) \right)}{2 \left(\cosh\left(\frac{4\pi}{2}\right) + \cos\left(\frac{4\pi}{2}\right) \right)}} = \\
& \frac{\pi^{3/256} \sqrt[128]{\frac{\left(1 + 2 \sum_{k=1}^{\infty} q^{2k}\right) \sum_{j=0}^{\infty} \operatorname{Res}_{s=-j} \left(\left(-\frac{1}{16}\right)^{-s} \pi^{-2s} \Gamma(s) \right)}{\Gamma\left(\frac{3}{2}-s\right)}}}{-1 + \sum_{k=0}^{\infty} \frac{\pi^2 k}{(2k)!}}}{\sqrt[64]{2}} \quad \text{for } q = e^\pi \\
& \sqrt[128]{\frac{\coth(\pi) \left(\sin\left(\frac{\pi}{2}\right) \sinh\left(\frac{\pi}{2}\right) \right) + \coth(2\pi) \left(\sin\left(\frac{2\pi}{2}\right) \sinh\left(\frac{2\pi}{2}\right) \right)}{\cosh\left(\frac{2\pi}{2}\right) + \cos\left(\frac{2\pi}{2}\right)} + \frac{\coth(2\pi) \left(\sin\left(\frac{2\pi}{2}\right) \sinh\left(\frac{2\pi}{2}\right) \right)}{2 \left(\cosh\left(\frac{4\pi}{2}\right) + \cos\left(\frac{4\pi}{2}\right) \right)}} = \\
& \frac{\sqrt[128]{\frac{\left(1 + 2 \sum_{k=1}^{\infty} q^{2k}\right) \sum_{k=0}^{\infty} \frac{\left(\frac{2}{\pi}\right)^{-1-2k}}{(1+2k)!}}}{-1 + \sqrt{\pi} \sum_{j=0}^{\infty} \operatorname{Res}_{s=-j} \left(\left(-\frac{1}{4}\right)^{-s} \pi^{-2s} \Gamma(s) \right)}}}{\text{for } q = e^\pi} \\
& \sqrt[128]{\frac{\coth(\pi) \left(\sin\left(\frac{\pi}{2}\right) \sinh\left(\frac{\pi}{2}\right) \right) + \coth(2\pi) \left(\sin\left(\frac{2\pi}{2}\right) \sinh\left(\frac{2\pi}{2}\right) \right)}{\cosh\left(\frac{2\pi}{2}\right) + \cos\left(\frac{2\pi}{2}\right)} + \frac{\coth(2\pi) \left(\sin\left(\frac{2\pi}{2}\right) \sinh\left(\frac{2\pi}{2}\right) \right)}{2 \left(\cosh\left(\frac{4\pi}{2}\right) + \cos\left(\frac{4\pi}{2}\right) \right)}} = \\
& \frac{\sqrt[128]{\frac{\sum_{k_1=-\infty}^{\infty} \sum_{k_2=0}^{\infty} \frac{\left(\frac{2}{\pi}\right)^{-1-2k_2}}{(1+2k_2)! (\pi + \pi k_1^2)}}}{-1 + \sqrt{\pi} \sum_{j=0}^{\infty} \operatorname{Res}_{s=-j} \left(\left(-\frac{1}{4}\right)^{-s} \pi^{-2s} \Gamma(s) \right)}}}{\Gamma\left(\frac{1}{2}-s\right)} \\
& \sqrt[128]{\frac{\coth(\pi) \left(\sin\left(\frac{\pi}{2}\right) \sinh\left(\frac{\pi}{2}\right) \right) + \coth(2\pi) \left(\sin\left(\frac{2\pi}{2}\right) \sinh\left(\frac{2\pi}{2}\right) \right)}{\cosh\left(\frac{2\pi}{2}\right) + \cos\left(\frac{2\pi}{2}\right)} + \frac{\coth(2\pi) \left(\sin\left(\frac{2\pi}{2}\right) \sinh\left(\frac{2\pi}{2}\right) \right)}{2 \left(\cosh\left(\frac{4\pi}{2}\right) + \cos\left(\frac{4\pi}{2}\right) \right)}} = \\
& \frac{\pi^{3/256} \sqrt[128]{\frac{i \left(1 + 2 \sum_{k=1}^{\infty} q^{2k}\right) \sum_{j=0}^{\infty} \operatorname{Res}_{s=-j} \left(\left(-\frac{1}{16}\right)^{-s} \pi^{-2s} \Gamma(s) \right)}{\Gamma\left(\frac{3}{2}-s\right)}}}{i + \sum_{k=0}^{\infty} \frac{\left(\left(1 - \frac{i}{2}\right)\pi\right)^{1+2k}}{(1+2k)!}}}{\sqrt[64]{2}} \quad \text{for } q = e^\pi
\end{aligned}$$

$$\begin{aligned}
& \sqrt[128]{\frac{\coth(\pi) \left(\sin\left(\frac{\pi}{2}\right) \sinh\left(\frac{\pi}{2}\right) \right) + \coth(2\pi) \left(\sin\left(\frac{2\pi}{2}\right) \sinh\left(\frac{2\pi}{2}\right) \right)}{\cosh\left(\frac{2\pi}{2}\right) + \cos\left(\frac{2\pi}{2}\right)} + \frac{\coth(2\pi) \left(\sin\left(\frac{2\pi}{2}\right) \sinh\left(\frac{2\pi}{2}\right) \right)}{2 \left(\cosh\left(\frac{4\pi}{2}\right) + \cos\left(\frac{4\pi}{2}\right) \right)}} = \\
& \frac{\pi^{3/256} \sqrt[128]{\frac{\sum_{k_1=-\infty}^{\infty} \sum_{k_2=0}^{\infty} \frac{\operatorname{Res}_{s=-k_2} \left(\left(-\frac{1}{16}\right)^{-s} \pi^{-2s} \Gamma(s) \right)}{\Gamma\left(\frac{3}{2}-s\right)}}{\pi + \pi k_1^2}}}{-1 + \sqrt{\pi} \sum_{j=0}^{\infty} \operatorname{Res}_{s=-j} \frac{\left(-\frac{1}{4}\right)^{-s} \pi^{-2s} \Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)}}} \\
& \frac{\pi^{3/256} \sqrt[128]{\frac{\left(1 + 2 \sum_{k=1}^{\infty} q^{2k}\right) \sum_{j=0}^{\infty} \operatorname{Res}_{s=-j} \frac{\left(-\frac{1}{16}\right)^{-s} \pi^{-2s} \Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}}}{-1 + \sqrt{\pi} \sum_{j=0}^{\infty} \operatorname{Res}_{s=-j} \frac{\left(-\frac{1}{4}\right)^{-s} \pi^{-2s} \Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)}}}}{64\sqrt{2}} \quad \text{for } q = e^\pi
\end{aligned}$$

Integral representations:

$$\begin{aligned}
& \sqrt[128]{\frac{\coth(\pi) \left(\sin\left(\frac{\pi}{2}\right) \sinh\left(\frac{\pi}{2}\right) \right) + \coth(2\pi) \left(\sin\left(\frac{2\pi}{2}\right) \sinh\left(\frac{2\pi}{2}\right) \right)}{\cosh\left(\frac{2\pi}{2}\right) + \cos\left(\frac{2\pi}{2}\right)} + \frac{\coth(2\pi) \left(\sin\left(\frac{2\pi}{2}\right) \sinh\left(\frac{2\pi}{2}\right) \right)}{2 \left(\cosh\left(\frac{4\pi}{2}\right) + \cos\left(\frac{4\pi}{2}\right) \right)}} = \\
& \frac{128 \sqrt{-\int_0^1 \int_0^1 \cosh\left(\frac{\pi t_1}{2}\right) \sec^2\left(\left(\frac{1}{2} + i\right)\pi t_2\right) dt_2 dt_1}}{128\sqrt{2}} \\
& \sqrt[128]{\frac{\coth(\pi) \left(\sin\left(\frac{\pi}{2}\right) \sinh\left(\frac{\pi}{2}\right) \right) + \coth(2\pi) \left(\sin\left(\frac{2\pi}{2}\right) \sinh\left(\frac{2\pi}{2}\right) \right)}{\cosh\left(\frac{2\pi}{2}\right) + \cos\left(\frac{2\pi}{2}\right)} + \frac{\coth(2\pi) \left(\sin\left(\frac{2\pi}{2}\right) \sinh\left(\frac{2\pi}{2}\right) \right)}{2 \left(\cosh\left(\frac{4\pi}{2}\right) + \cos\left(\frac{4\pi}{2}\right) \right)}} = \\
& \frac{\pi^{3/256} \sqrt[128]{-\int_0^1 \int_0^1 \cosh\left(\frac{\pi t_1}{2}\right) \sec^2\left(\left(\frac{1}{2} + i\right)\pi t_2\right) dt_2 dt_1}}{\sqrt[128]{2}} \quad \text{for } \gamma > 0 \\
& \sqrt[128]{\frac{\coth(\pi) \left(\sin\left(\frac{\pi}{2}\right) \sinh\left(\frac{\pi}{2}\right) \right) + \coth(2\pi) \left(\sin\left(\frac{2\pi}{2}\right) \sinh\left(\frac{2\pi}{2}\right) \right)}{\cosh\left(\frac{2\pi}{2}\right) + \cos\left(\frac{2\pi}{2}\right)} + \frac{\coth(2\pi) \left(\sin\left(\frac{2\pi}{2}\right) \sinh\left(\frac{2\pi}{2}\right) \right)}{2 \left(\cosh\left(\frac{4\pi}{2}\right) + \cos\left(\frac{4\pi}{2}\right) \right)}} = \\
& \frac{128 \sqrt{\frac{\pi}{2}} \sqrt[128]{-\int_0^1 \int_0^1 \cosh\left(\frac{\pi t_1}{2}\right) \sec^2\left(\left(\frac{1}{2} + i\right)\pi t_2\right) dt_2 dt_1}}{\sqrt[128]{2}}
\end{aligned}$$

$$\sqrt[128]{\frac{\coth(\pi)\left(\sin\left(\frac{\pi}{2}\right)\sinh\left(\frac{\pi}{2}\right)\right)}{\cosh\left(\frac{2\pi}{2}\right)+\cos\left(\frac{2\pi}{2}\right)}+\frac{\coth(2\pi)\left(\sin\left(\frac{2\pi}{2}\right)\sinh\left(\frac{2\pi}{2}\right)\right)}{2\left(\cosh\left(\frac{4\pi}{2}\right)+\cos\left(\frac{4\pi}{2}\right)\right)}} =$$

$$\frac{\sqrt[128]{\frac{i\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{\pi^2/(16s)+s}}{s^{3/2}}ds\right)\int_{i\pi/2}^{\pi}\operatorname{csch}^2(t)dt}{\int_0^1\sinh(\pi t)dt}}}{2^{3/128}\sqrt[256]{\pi}} \quad \text{for } \gamma > 0$$

$$\sqrt[128]{\frac{\coth(\pi)\left(\sin\left(\frac{\pi}{2}\right)\sinh\left(\frac{\pi}{2}\right)\right)}{\cosh\left(\frac{2\pi}{2}\right)+\cos\left(\frac{2\pi}{2}\right)}+\frac{\coth(2\pi)\left(\sin\left(\frac{2\pi}{2}\right)\sinh\left(\frac{2\pi}{2}\right)\right)}{2\left(\cosh\left(\frac{4\pi}{2}\right)+\cos\left(\frac{4\pi}{2}\right)\right)}} =$$

$$\frac{\sqrt[256]{\pi}\sqrt[128]{\frac{i\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{\pi^2/(16s)+s}}{s^{3/2}}ds\right)\int_{i\pi/2}^{\pi}\operatorname{csch}^2(t)dt}{-1+\int_{i\pi/2}^{\pi}\sinh(t)dt}}}{2^{3/128}} \quad \text{for } \gamma > 0$$

$$\sqrt[128]{\frac{\coth(\pi)\left(\sin\left(\frac{\pi}{2}\right)\sinh\left(\frac{\pi}{2}\right)\right)}{\cosh\left(\frac{2\pi}{2}\right)+\cos\left(\frac{2\pi}{2}\right)}+\frac{\coth(2\pi)\left(\sin\left(\frac{2\pi}{2}\right)\sinh\left(\frac{2\pi}{2}\right)\right)}{2\left(\cosh\left(\frac{4\pi}{2}\right)+\cos\left(\frac{4\pi}{2}\right)\right)}} =$$

$$\frac{\sqrt[128]{\pi}\sqrt[128]{\frac{i\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{\pi^2/(16s)+s}}{s^{3/2}}ds\right)\int_{i\pi/2}^{\pi}\operatorname{csch}^2(t)dt}{2\sqrt{\pi}+i\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{\pi^2/(4s)+s}}{\sqrt{s}}ds}}}{\sqrt[64]{2}} \quad \text{for } \gamma > 0$$

We have that:

Page 294

For : $\alpha = \pi$; $\beta = \frac{\pi}{2}$, we obtain:

$$\frac{\pi}{2} + \frac{(\pi^2/2)}{(\pi(\pi^2 + (\pi/2)^2)^2)} + \frac{\pi}{2}$$

Input:

$$\frac{\pi}{2} + \frac{\frac{\pi^2}{2}}{\pi \left(\pi^2 + \left(\frac{\pi}{2} \right)^2 \right)^2} + \frac{\pi}{2}$$

Result:

$$\frac{8}{25 \pi^3} + \pi$$

Decimal approximation:

3.151913144608417075001658440139843264546949666942298805530...

3.15191314.... partial result

Property:

$\frac{8}{25 \pi^3} + \pi$ is a transcendental number

Alternate form:

$$\frac{8 + 25 \pi^4}{25 \pi^3}$$

Alternative representations:

$$\frac{\pi}{2} + \frac{\pi^2}{\left(\pi \left(\pi^2 + \left(\frac{\pi}{2} \right)^2 \right)^2 \right)^2} + \frac{\pi}{2} = 2 E(0) + \frac{(2 E(0))^2}{2 (2 E(0) (E(0)^2 + (2 E(0))^2)^2)}$$

$$\frac{\pi}{2} + \frac{\pi^2}{\left(\pi \left(\pi^2 + \left(\frac{\pi}{2} \right)^2 \right)^2 \right)^2} + \frac{\pi}{2} = 2 K(0) + \frac{(2 K(0))^2}{2 (2 K(0) (K(0)^2 + (2 K(0))^2)^2)}$$

$$\frac{\pi}{2} + \frac{\pi^2}{\left(\pi \left(\pi^2 + \left(\frac{\pi}{2} \right)^2 \right)^2 \right)^2} + \frac{\pi}{2} = \cos^{-1}(-1) + \frac{\cos^{-1}(-1)^2}{2 \left(\cos^{-1}(-1) \left(\left(\frac{1}{2} \cos^{-1}(-1) \right)^2 + \cos^{-1}(-1)^2 \right)^2 \right)}$$

Series representations:

$$\frac{\pi}{2} + \frac{\pi^2}{\left(\pi \left(\pi^2 + \left(\frac{\pi}{2} \right)^2 \right)^2 \right)^2} + \frac{\pi}{2} = \frac{1 + 800 \left(\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} \right)^4}{200 \left(\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} \right)^3}$$

$$\frac{\pi}{2} + \frac{\pi^2}{\left(\pi \left(\pi^2 + \left(\frac{\pi}{2} \right)^2 \right)^2 \right)^2} + \frac{\pi}{2} = \frac{8 + 25 \left(\sum_{k=0}^{\infty} \left(-\frac{1}{4} \right)^k \left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k} \right) \right)^4}{25 \left(\sum_{k=0}^{\infty} \left(-\frac{1}{4} \right)^k \left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k} \right) \right)^3}$$

$$\frac{\pi}{2} + \frac{\pi^2}{\left(\pi \left(\pi^2 + \left(\frac{\pi}{2} \right)^2 \right)^2 \right)^2} + \frac{\pi}{2} = \frac{8 + 25 \left(\sum_{k=0}^{\infty} -\frac{4(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k} \right)^4}{25 \left(\sum_{k=0}^{\infty} -\frac{4(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k} \right)^3}$$

Integral representations:

$$\frac{\pi}{2} + \frac{\pi^2}{\left(\pi\left(\pi^2 + \left(\frac{\pi}{2}\right)^2\right)\right)^2} + \frac{\pi}{2} = \frac{1 + 50 \left(\int_0^\infty \frac{1}{1+t^2} dt\right)^4}{25 \left(\int_0^\infty \frac{1}{1+t^2} dt\right)^3}$$

$$\frac{\pi}{2} + \frac{\pi^2}{\left(\pi\left(\pi^2 + \left(\frac{\pi}{2}\right)^2\right)\right)^2} + \frac{\pi}{2} = \frac{1 + 50 \left(\int_0^\infty \frac{\sin(t)}{t} dt\right)^4}{25 \left(\int_0^\infty \frac{\sin(t)}{t} dt\right)^3}$$

$$\frac{\pi}{2} + \frac{\pi^2}{\left(\pi\left(\pi^2 + \left(\frac{\pi}{2}\right)^2\right)\right)^2} + \frac{\pi}{2} = \frac{1 + 50 \left(\int_0^1 \frac{1}{\sqrt{1-t^2}} dt\right)^4}{25 \left(\int_0^1 \frac{1}{\sqrt{1-t^2}} dt\right)^3}$$

3.151913144608417075* (((cosh(2Pi(Pi-Pi/2)))-cos(2Pi(Pi-Pi/2))))/(((cosh(2Pi*Pi)-cos(2Pi*Pi/2)))(((cosh(2Pi*Pi/2)-cos(2Pi*Pi))))))

Input interpretation:

$$3.151913144608417075 \times \frac{\cosh\left(2\pi\left(\pi - \frac{\pi}{2}\right)\right) - \cos\left(2\pi\left(\pi - \frac{\pi}{2}\right)\right)}{\left(\cosh(2\pi\pi) - \cos\left(2\pi \times \frac{\pi}{2}\right)\right)\left(\cosh\left(2\pi \times \frac{\pi}{2}\right) - \cos(2\pi\pi)\right)}$$

cosh(x) is the hyperbolic cosine function

Result:

1.686722419379558613... × 10⁻⁸

1.686722419379... * 10⁻⁸ final result

Alternative representations:

$$\frac{3.1519131446084170750000 \left(\cosh\left(2\pi\left(\pi - \frac{\pi}{2}\right)\right) - \cos\left(2\pi\left(\pi - \frac{\pi}{2}\right)\right)\right)}{\left(\cosh(2\pi\pi) - \cos\left(\frac{2\pi\pi}{2}\right)\right)\left(\cosh\left(\frac{2\pi\pi}{2}\right) - \cos(2\pi\pi)\right)} = \frac{3.1519131446084170750000 \left(-\cosh(-i\pi^2) + \cos(-i\pi^2)\right)}{\left(-\cosh(-2i\pi^2) + \cos(-i\pi^2)\right)\left(-\cosh(-i\pi^2) + \cos(-2i\pi^2)\right)}$$

$$\frac{3.1519131446084170750000 \left(\cosh\left(2\pi\left(\pi - \frac{\pi}{2}\right)\right) - \cos\left(2\pi\left(\pi - \frac{\pi}{2}\right)\right)\right)}{\left(\cosh(2\pi\pi) - \cos\left(\frac{2\pi\pi}{2}\right)\right)\left(\cosh\left(\frac{2\pi\pi}{2}\right) - \cos(2\pi\pi)\right)} = \frac{3.1519131446084170750000 \left(-\cosh(i\pi^2) + \cos(-i\pi^2)\right)}{\left(-\cosh(2i\pi^2) + \cos(-i\pi^2)\right)\left(-\cosh(i\pi^2) + \cos(-2i\pi^2)\right)}$$

$$\frac{3.1519131446084170750000 \left(\cosh\left(2\pi\left(\pi - \frac{\pi}{2}\right)\right) - \cos\left(2\pi\left(\pi - \frac{\pi}{2}\right)\right) \right)}{\left(\cosh(2\pi\pi) - \cos\left(\frac{2\pi\pi}{2}\right) \right) \left(\cosh\left(\frac{2\pi\pi}{2}\right) - \cos(2\pi\pi) \right)} =$$

$$\frac{3.1519131446084170750000 \left(-\cosh(-i\pi^2) + \cos(i\pi^2) \right)}{\left(-\cosh(-2i\pi^2) + \cos(i\pi^2) \right) \left(-\cosh(-i\pi^2) + \cos(2i\pi^2) \right)}$$

Series representations:

$$\frac{3.1519131446084170750000 \left(\cosh\left(2\pi\left(\pi - \frac{\pi}{2}\right)\right) - \cos\left(2\pi\left(\pi - \frac{\pi}{2}\right)\right) \right)}{\left(\cosh(2\pi\pi) - \cos\left(\frac{2\pi\pi}{2}\right) \right) \left(\cosh\left(\frac{2\pi\pi}{2}\right) - \cos(2\pi\pi) \right)} =$$

$$\left(\sum_{k=0}^{\infty} \frac{\left(-3.1519131446084170750 + 3.1519131446084170750 (-1)^k \right) (\pi^2)^{2k}}{(2k)!} \right) /$$

$$\left(\left(\sum_{k=0}^{\infty} \frac{\left(1.00000000000000000000 - 1.00000000000000000000 (-4)^k \right) (\pi^2)^{2k}}{(2k)!} \right) \right.$$

$$\left. \sum_{k=0}^{\infty} \frac{\left((-1)^k - 1.00000000000000000000 \times 4^k \right) (\pi^2)^{2k}}{(2k)!} \right)$$

$$\frac{3.1519131446084170750000 \left(\cosh\left(2\pi\left(\pi - \frac{\pi}{2}\right)\right) - \cos\left(2\pi\left(\pi - \frac{\pi}{2}\right)\right) \right)}{\left(\cosh(2\pi\pi) - \cos\left(\frac{2\pi\pi}{2}\right) \right) \left(\cosh\left(\frac{2\pi\pi}{2}\right) - \cos(2\pi\pi) \right)} =$$

$$\left(\sum_{k=0}^{\infty} \left(-\frac{3.15191314460841707500 (-1)^k (\pi^2)^{2k}}{(2k)!} + \frac{3.1519131446084170750000 i \left(-\frac{i\pi}{2} + \pi^2\right)^{1+2k}}{(1+2k)!} \right) \right) /$$

$$\left(\left(\sum_{k=0}^{\infty} \left(-\frac{1.00000000000000000000 (-4)^k (\pi^2)^{2k}}{(2k)!} + \frac{i \left(-\frac{i\pi}{2} + \pi^2\right)^{1+2k}}{(1+2k)!} \right) \right) \right.$$

$$\left. \sum_{k=0}^{\infty} \left(-\frac{1.00000000000000000000 (-1)^k (\pi^2)^{2k}}{(2k)!} + \frac{i \left(-\frac{i\pi}{2} + 2\pi^2\right)^{1+2k}}{(1+2k)!} \right) \right)$$

$$\begin{aligned}
& \frac{3.1519131446084170750000 \left(\cosh\left(2\pi\left(\pi - \frac{\pi}{2}\right)\right) - \cos\left(2\pi\left(\pi - \frac{\pi}{2}\right)\right) \right)}{\left(\cosh(2\pi\pi) - \cos\left(\frac{2\pi\pi}{2}\right) \right) \left(\cosh\left(\frac{2\pi\pi}{2}\right) - \cos(2\pi\pi) \right)} = \\
& \left(3.15191314460841707500 \right. \\
& \quad \left. \left(1.00000000000000000000000000000000 I_0(\pi^2) + 2.00000000000000000000000000000000 \right. \right. \\
& \quad \quad \left. \left. \sum_{k=1}^{\infty} I_{2k}(\pi^2) - 1.00000000000000000000000000000000 \sum_{k=0}^{\infty} \frac{(-1)^k (\pi^2)^{2k}}{(2k)!} \right) \right) / \\
& \left(\left(1.00000000000000000000000000000000 I_0(\pi^2) + 2.00000000000000000000000000000000 \sum_{k=1}^{\infty} I_{2k}(\pi^2) - \right. \right. \\
& \quad \left. \left. 1.00000000000000000000000000000000 \sum_{k=0}^{\infty} \frac{(-4)^k (\pi^2)^{2k}}{(2k)!} \right) \right) \\
& \left(1.00000000000000000000000000000000 I_0(2\pi^2) + 2.00000000000000000000000000000000 \right. \\
& \quad \left. \sum_{k=1}^{\infty} I_{2k}(2\pi^2) - 1.00000000000000000000000000000000 \sum_{k=0}^{\infty} \frac{(-1)^k (\pi^2)^{2k}}{(2k)!} \right)
\end{aligned}$$

Integral representations:

$$\begin{aligned}
& \frac{3.1519131446084170750000 \left(\cosh\left(2\pi\left(\pi - \frac{\pi}{2}\right)\right) - \cos\left(2\pi\left(\pi - \frac{\pi}{2}\right)\right) \right)}{\left(\cosh(2\pi\pi) - \cos\left(\frac{2\pi\pi}{2}\right) \right) \left(\cosh\left(\frac{2\pi\pi}{2}\right) - \cos(2\pi\pi) \right)} = \\
& \left(3.1519131446084170750000 \int_0^1 (\sinh(\pi^2 t) + \sin(\pi^2 t)) dt \right) / \\
& \left(\pi^2 \left(\int_0^1 (2.00000000000000000000000000000000 \sinh(2\pi^2 t) + \sin(\pi^2 t)) dt \right) \right. \\
& \quad \left. \int_0^1 (\sinh(\pi^2 t) + 2.00000000000000000000000000000000 \sin(2\pi^2 t)) dt \right)
\end{aligned}$$

$$\begin{aligned}
& \frac{3.1519131446084170750000 \left(\cosh\left(2\pi\left(\pi - \frac{\pi}{2}\right)\right) - \cos\left(2\pi\left(\pi - \frac{\pi}{2}\right)\right) \right)}{\left(\cosh(2\pi\pi) - \cos\left(\frac{2\pi\pi}{2}\right) \right) \left(\cosh\left(\frac{2\pi\pi}{2}\right) - \cos(2\pi\pi) \right)} = \\
& \left(3.151913144608417075000 \left(1.00000000000000000000000000000000 \int_{\frac{i\pi}{2}}^{\pi^2} \sinh(t) dt + \right. \right. \\
& \quad \left. \left. 1.00000000000000000000000000000000 \int_{\frac{\pi}{2}}^{\pi^2} \sin(t) dt \right) \right) / \\
& \left(\left(\int_{\frac{i\pi}{2}}^{\pi^2} \left(\sinh(t) + \frac{(1-4\pi) \sin\left(\frac{(-1+2i)\pi^2+t-4\pi t}{i-2\pi}\right)}{i-2\pi} \right) dt \right) \right. \\
& \quad \left. \int_{\frac{i\pi}{2}}^{2\pi^2} \left(\sinh(t) + \frac{(1-2\pi) \sin\left(\frac{(-2+i)\pi^2+t-2\pi t}{i-4\pi}\right)}{i-4\pi} \right) dt \right)
\end{aligned}$$

$$\frac{3.1519131446084170750000 \left(\cosh\left(2\pi\left(\pi - \frac{\pi}{2}\right)\right) - \cos\left(2\pi\left(\pi - \frac{\pi}{2}\right)\right) \right)}{\left(\cosh(2\pi\pi) - \cos\left(\frac{2\pi\pi}{2}\right) \right) \left(\cosh\left(\frac{2\pi\pi}{2}\right) - \cos(2\pi\pi) \right)} =$$

$$\left(3.15191314460841707500 i \left(1.0000000000000000000000000000000000 i \pi^2 \int_{\frac{\pi}{2}}^{\pi^2} \sin(t) dt + \right. \right.$$

$$\left. \left. 0.5000000000000000000000000000000000 \pi \sqrt{\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{\pi^4/(4s)+s}}{\sqrt{s}} ds \right) \right) /$$

$$\left(\left(1.0000000000000000000000000000000000 i \pi \int_{\frac{\pi}{2}}^{2\pi^2} \sin(t) dt + \right. \right.$$

$$\left. \left. 0.5000000000000000000000000000000000 \sqrt{\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{\pi^4/(4s)+s}}{\sqrt{s}} ds \right) \right)$$

$$\left(1.0000000000000000000000000000000000 i \pi \int_{\frac{\pi}{2}}^{\pi^2} \sin(t) dt + \right.$$

$$\left. \left. 0.5000000000000000000000000000000000 \sqrt{\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{\pi^4/s+s}}{\sqrt{s}} ds \right) \right) \text{ for } \gamma > 0$$

$(((((3.151913144608417075 * (((\cosh(2\pi(\pi - \pi/2))) - \cos(2\pi(\pi - \pi/2)))))) / (((\cosh(2\pi\pi) - \cos(2\pi\pi/2))) ((\cosh(2\pi\pi/2) - \cos(2\pi\pi)))))))))^{1/4096}$

Input interpretation:

$$\sqrt[4096]{3.151913144608417075 \times \frac{\cosh\left(2\pi\left(\pi - \frac{\pi}{2}\right)\right) - \cos\left(2\pi\left(\pi - \frac{\pi}{2}\right)\right)}{\left(\cosh(2\pi\pi) - \cos\left(2\pi \times \frac{\pi}{2}\right) \right) \left(\cosh\left(2\pi \times \frac{\pi}{2}\right) - \cos(2\pi\pi) \right)}}$$

$\cosh(x)$ is the hyperbolic cosine function

Result:

0.99563992992080046794012...

0.995639929.... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

and to the dilaton value $0.989117352243 = \phi$

$$64^2 + 76 + 18 + 9 / \left(\left(\left(\left(\left(\left(3.151913144608417075 * \left(\left(\cosh(2\pi(\pi - \pi/2)) \right) - \cos(2\pi(\pi - \pi/2)) \right) \right) \right) \right) \right) / \left(\left(\cosh(2\pi * \pi) - \cos(2\pi * \pi/2) \right) \right) \left(\left(\cosh(2\pi * \pi/2) - \cos(2\pi * \pi) \right) \right) \right) \right) \right) \right) \right)^{1/2}$$

Input interpretation:

$$64^2 + 76 + 18 + \frac{9}{\sqrt{3.151913144608417075 \times \frac{\cosh(2\pi(\pi - \frac{\pi}{2})) - \cos(2\pi(\pi - \frac{\pi}{2}))}{(\cosh(2\pi\pi) - \cos(2\pi \times \frac{\pi}{2}))(\cosh(2\pi \times \frac{\pi}{2}) - \cos(2\pi\pi))}}}$$

cosh(x) is the hyperbolic cosine function

Result:

73488.00000368573772...

73488.0000036...

Alternative representations:

$$64^2 + 76 + 18 + \frac{9}{\sqrt{\frac{3.1519131446084170750000 (\cosh(2\pi(\pi - \frac{\pi}{2})) - \cos(2\pi(\pi - \frac{\pi}{2})))}{(\cosh(2\pi\pi) - \cos(\frac{2\pi\pi}{2}))(\cosh(\frac{2\pi\pi}{2}) - \cos(2\pi\pi))}}}} =$$

$$94 + 64^2 + \frac{9}{\sqrt{\frac{3.1519131446084170750000 (-\cosh(-i\pi^2) + \cos(-i\pi^2))}{(-\cosh(-2i\pi^2) + \cos(-i\pi^2))(-\cosh(-i\pi^2) + \cos(-2i\pi^2))}}}}$$

$$64^2 + 76 + 18 + \frac{9}{\sqrt{\frac{3.1519131446084170750000 (\cosh(2\pi(\pi - \frac{\pi}{2})) - \cos(2\pi(\pi - \frac{\pi}{2})))}{(\cosh(2\pi\pi) - \cos(\frac{2\pi\pi}{2}))(\cosh(\frac{2\pi\pi}{2}) - \cos(2\pi\pi))}}}} =$$

$$94 + 64^2 + \frac{9}{\sqrt{\frac{3.1519131446084170750000 (-\cosh(i\pi^2) + \cos(-i\pi^2))}{(-\cosh(2i\pi^2) + \cos(-i\pi^2))(-\cosh(i\pi^2) + \cos(-2i\pi^2))}}}}$$

$$64^2 + 76 + 18 + \frac{9}{\sqrt{\frac{3.1519131446084170750000 (\cosh(2\pi(\pi - \frac{\pi}{2})) - \cos(2\pi(\pi - \frac{\pi}{2})))}{(\cosh(2\pi\pi) - \cos(\frac{2\pi\pi}{2}))(\cosh(\frac{2\pi\pi}{2}) - \cos(2\pi\pi))}}}} =$$

$$94 + 64^2 + \frac{9}{\sqrt{\frac{3.1519131446084170750000 (-\cosh(-i\pi^2) + \cos(i\pi^2))}{(-\cosh(-2i\pi^2) + \cos(i\pi^2))(-\cosh(-i\pi^2) + \cos(2i\pi^2))}}}}$$

Series representations:

$$\begin{aligned}
 & 64^2 + 76 + 18 + \frac{9}{\sqrt{\frac{3.1519131446084170750000 (\cosh(2\pi(\pi-\frac{\pi}{2}))-\cos(2\pi(\pi-\frac{\pi}{2})))}{(\cosh(2\pi\pi)-\cos(\frac{2\pi\pi}{2}))(\cosh(\frac{2\pi\pi}{2})-\cos(2\pi\pi))}}} = \\
 & \frac{1}{\sum_{k=0}^{\infty} \frac{(-1+(-1)^k)(\pi^2)^{2k}}{(2k)!}} \\
 & 4190.0000000000000000000000 \left(1.0000000000000000000000 \sum_{k=0}^{\infty} \frac{(-1+(-1)^k)(\pi^2)^{2k}}{(2k)!} + \right. \\
 & \left. \frac{0.0012098774039385256420704}{\sqrt{\frac{\sum_{k=0}^{\infty} \frac{(-1+(-1)^k)(\pi^2)^{2k}}{(2k)!}}{\sum_{k=0}^{\infty} \frac{(-1+(-4)^k)(\pi^2)^{2k}}{(2k)!} \sum_{k=0}^{\infty} \frac{((-1)^k-4^k)(\pi^2)^{2k}}{(2k)!}}}} \right) \\
 & \left. \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \left(\frac{(\pi^2)^{2k_1}}{(2k_1)!} - \frac{(-4)^{k_1} (\pi^2)^{2k_1}}{(2k_1)!} \right) \left(\frac{(-1)^{k_2} (\pi^2)^{2k_2}}{(2k_2)!} - \frac{4^{k_2} (\pi^2)^{2k_2}}{(2k_2)!} \right) \right)
 \end{aligned}$$

$$64^2 + 76 + 18 + \frac{9}{\sqrt{\frac{3.1519131446084170750000 (\cosh(2\pi(\pi - \frac{\pi}{2})) - \cos(2\pi(\pi - \frac{\pi}{2})))}{(\cosh(2\pi\pi) - \cos(\frac{2\pi\pi}{2}))(\cosh(\frac{2\pi\pi}{2}) - \cos(2\pi\pi))}}} =$$

$$\left(\begin{array}{l} 4190.000000000000000000000000 \\ \sum_{k=0}^{\infty} \frac{(1 + (-1)^{1+k})(\pi^2)^{2k}}{(2k)!} + 0.0012098774039385256420704 \\ \sqrt{\frac{\sum_{k=0}^{\infty} \frac{(1 + (-1)^{1+k})(\pi^2)^{2k}}{(2k)!}}{\sum_{k=0}^{\infty} \frac{(-1 + (-4)^k)(\pi^2)^{2k}}{(2k)!} - \sum_{k=0}^{\infty} \frac{((-1)^{1+k} + 4^k)(\pi^2)^{2k}}{(2k)!}}}} \\ \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \left(\frac{(-1)^{1+k_1} (\pi^2)^{2k_1}}{(2k_1)!} + \frac{2^{2k_1} (\pi^2)^{2k_1}}{(2k_1)!} \right) \\ \left(\frac{(\pi^2)^{2k_2}}{(2k_2)!} + \frac{(-1)^{1+k_2} 2^{2k_2} (\pi^2)^{2k_2}}{(2k_2)!} \right) \Bigg) / \left(\sum_{k=0}^{\infty} \frac{(1 + (-1)^{1+k})(\pi^2)^{2k}}{(2k)!} \right) \end{array} \right)$$

$$64^2 + 76 + 18 + \frac{9}{\sqrt{\frac{3.1519131446084170750000 (\cosh(2\pi(\pi - \frac{\pi}{2})) - \cos(2\pi(\pi - \frac{\pi}{2})))}{(\cosh(2\pi\pi) - \cos(\frac{2\pi\pi}{2}))(\cosh(\frac{2\pi\pi}{2}) - \cos(2\pi\pi))}}} = 4190 +$$

$$\frac{5.0693863225024224402750}{\sqrt{\frac{I_0(\pi^2)+2 \sum_{k=1}^{\infty} I_{2k}(\pi^2) - \sum_{k=0}^{\infty} \frac{(-1)^k (\pi^2)^{2k}}{(2k)!}}{I_0(2\pi^2)+2 \sum_{k=1}^{\infty} I_{2k}(2\pi^2) - \sum_{k=0}^{\infty} \frac{(-1)^k (\pi^2)^{2k}}{(2k)!}}} \left(I_0(\pi^2)+2 \sum_{k=1}^{\infty} I_{2k}(\pi^2) - \sum_{k=0}^{\infty} \frac{(-1)^k 2^{2k} (\pi^2)^{2k}}{(2k)!} \right)$$

Integral representations:

$$64^2 + 76 + 18 + \frac{9}{\sqrt{\frac{3.1519131446084170750000 (\cosh(2\pi(\pi - \frac{\pi}{2})) - \cos(2\pi(\pi - \frac{\pi}{2})))}{(\cosh(2\pi\pi) - \cos(\frac{2\pi\pi}{2}))(\cosh(\frac{2\pi\pi}{2}) - \cos(2\pi\pi))}}} =$$

$$\left(\begin{array}{l} 4190.000000000000000000000000 \\ (1.00000000000000000000000000000000000000 \pi^2 \int_0^1 (\sinh(\pi^2 t) + \sin(\pi^2 t)) dt + \pi^4 \\ \int_0^1 \int_0^1 (2 \sinh(2\pi^2 t_1) + \sin(\pi^2 t_1)) (\sinh(\pi^2 t_2) + 2 \sin(2\pi^2 t_2)) dt_2 dt_1) / \\ (\pi^2 \int_0^1 (\sinh(\pi^2 t) + \sin(\pi^2 t)) dt) \end{array} \right)$$

$$64^2 + 76 + 18 + \frac{9}{\sqrt{\frac{3.1519131446084170750000 (\cosh(2\pi(\pi-\frac{\pi}{2})) - \cos(2\pi(\pi-\frac{\pi}{2})))}{(\cosh(2\pi\pi) - \cos(\frac{2\pi\pi}{2}))(\cosh(\frac{2\pi\pi}{2}) - \cos(2\pi\pi))}}} =$$

$$\left(4190.000000000000000000000000 \right.$$

$$\left. \left(1.0000000000000000000000000000 \int_{\frac{i\pi}{2}}^{\pi^2} \left(\sinh(t) + \frac{(1-2\pi) \sin\left(\frac{(-1+i)\pi^2+t-2\pi t}{i-2\pi}\right)}{i-2\pi} \right) dt + \right.$$

$$\left. \frac{1}{i^2 - 6i\pi + 8\pi^2} \int_0^1 \int_0^1 \left((i-2\pi) \sinh\left(\frac{1}{2}\pi(i-(i-2\pi)t_1)\right) + (1-4\pi) \sin\left(\frac{1}{2}\pi(1+(-1+4\pi)t_1)\right) \right) \left((i-4\pi) \sinh\left(\frac{1}{2}\pi(i-(i-4\pi)t_2)\right) + \right.$$

$$\left. \left. (1-2\pi) \sin\left(\frac{1}{2}\pi(1+(-1+2\pi)t_2)\right) \right) dt_2 dt_1 \right) \right) /$$

$$\left(\int_{\frac{i\pi}{2}}^{\pi^2} \left(\sinh(t) + \frac{(1-2\pi) \sin\left(\frac{(-1+i)\pi^2+t-2\pi t}{i-2\pi}\right)}{i-2\pi} \right) dt \right)$$

$$64^2 + 76 + 18 + \frac{9}{\sqrt{\frac{3.1519131446084170750000 (\cosh(2\pi(\pi-\frac{\pi}{2})) - \cos(2\pi(\pi-\frac{\pi}{2})))}{(\cosh(2\pi\pi) - \cos(\frac{2\pi\pi}{2}))(\cosh(\frac{2\pi\pi}{2}) - \cos(2\pi\pi))}}} =$$

$$4190 + 5.0693863225024224402750 /$$

$$\left(\sqrt{\left(\left(1 + \int_0^1 \left(\pi^2 \sinh(\pi^2 t) + \frac{1}{2}\pi(-1+2\pi) \sin\left(\frac{1}{2}(\pi+\pi(-1+2\pi)t)\right) \right) dt \right) \right) /$$

$$\left(\left(1 + \int_0^1 \frac{1}{2}\pi \left(4\pi \sinh(2\pi^2 t) + (-1+2\pi) \sin\left(\frac{1}{2}(\pi+\pi(-1+2\pi)t)\right) \right) dt \right) \right.$$

$$\left. \left. \left(1 + \int_0^1 \left(\pi^2 \sinh(\pi^2 t) + \frac{1}{2}\pi(-1+4\pi) \sin\left(\frac{1}{2}(\pi+\pi(-1+4\pi)t)\right) \right) dt \right) \right) \right) \right)$$

$\sinh(x)$ is the hyperbolic sine function

$\phi(a, \beta) + \phi(\beta, a) = \frac{\pi}{4} +$
 $\frac{\pi}{4} \cdot \frac{\cosh \pi(a-\beta) - \cos \pi(a-\beta)}{(\cosh \pi a + \cos \pi \beta)(\cosh \pi \beta + \cos \pi a)}$

For : $\alpha = \pi$; $\beta = \frac{\pi}{2}$, we obtain:

$$\frac{\pi}{4} + \frac{\pi}{4} \times \frac{(\cosh(\pi(\pi - \frac{\pi}{2})) - \cos(\pi(\pi - \frac{\pi}{2})))}{((\cosh(\pi \pi) + \cos(\pi \times \frac{\pi}{2}))(\cosh(\pi \times \frac{\pi}{2}) + \cos(\pi \pi)))}$$

Input:

$$\frac{\pi}{4} + \frac{\pi}{4} \times \frac{\cosh\left(\pi\left(\pi - \frac{\pi}{2}\right)\right) - \cos\left(\pi\left(\pi - \frac{\pi}{2}\right)\right)}{\left(\cosh(\pi \pi) + \cos\left(\pi \times \frac{\pi}{2}\right)\right)\left(\cosh\left(\pi \times \frac{\pi}{2}\right) + \cos(\pi \pi)\right)}$$

cosh(x) is the hyperbolic cosine function

Decimal approximation:

0.785480215684966279740922468565947904069734740341881411023...

0.785480215...

Alternate forms:

$$\frac{1}{4} \pi \left(1 + \frac{\cosh\left(\frac{\pi^2}{2}\right) - \cos\left(\frac{\pi^2}{2}\right)}{\left(\cos(\pi^2) + \cosh\left(\frac{\pi^2}{2}\right)\right)\left(\cos\left(\frac{\pi^2}{2}\right) + \cosh(\pi^2)\right)} \right)$$

$$\frac{\pi}{4} - \frac{\pi \cos\left(\frac{\pi^2}{2}\right)}{4\left(\cos(\pi^2) + \cosh\left(\frac{\pi^2}{2}\right)\right)\left(\cos\left(\frac{\pi^2}{2}\right) + \cosh(\pi^2)\right)} +$$

$$\frac{\pi \cosh\left(\frac{\pi^2}{2}\right)}{4\left(\cos(\pi^2) + \cosh\left(\frac{\pi^2}{2}\right)\right)\left(\cos\left(\frac{\pi^2}{2}\right) + \cosh(\pi^2)\right)}$$

$$\left(\pi \left(-\cos\left(\frac{\pi^2}{2}\right) + \cos\left(\frac{\pi^2}{2}\right) \cos(\pi^2) + \cosh\left(\frac{\pi^2}{2}\right) + \right. \right.$$

$$\left. \left. \cosh\left(\frac{\pi^2}{2}\right) \cosh(\pi^2) + \cos\left(\frac{\pi^2}{2}\right) \cosh\left(\frac{\pi^2}{2}\right) + \cos(\pi^2) \cosh(\pi^2) \right) \right) /$$

$$\left(4 \left(\cos(\pi^2) + \cosh\left(\frac{\pi^2}{2}\right) \right) \left(\cos\left(\frac{\pi^2}{2}\right) + \cosh(\pi^2) \right) \right)$$

Alternative representations:

$$\frac{\pi}{4} + \frac{\left(\cosh\left(\pi\left(\pi - \frac{\pi}{2}\right)\right) - \cos\left(\pi\left(\pi - \frac{\pi}{2}\right)\right)\right) \pi}{\left(\left(\cosh(\pi \pi) + \cos\left(\frac{\pi \pi}{2}\right)\right)\left(\cosh\left(\frac{\pi \pi}{2}\right) + \cos(\pi \pi)\right)\right) 4} =$$

$$\frac{\pi}{4} + \frac{\pi \left(-\cosh\left(-\frac{i \pi^2}{2}\right) + \cos\left(-\frac{i \pi^2}{2}\right)\right)}{4 \left(\left(\cosh(-i \pi^2) + \cos\left(-\frac{i \pi^2}{2}\right)\right)\left(\cosh\left(-\frac{i \pi^2}{2}\right) + \cos(-i \pi^2)\right)\right)}$$

$$\frac{\pi}{4} + \frac{(\cosh(\pi(\pi - \frac{\pi}{2})) - \cos(\pi(\pi - \frac{\pi}{2})))\pi}{((\cosh(\pi\pi) + \cos(\frac{\pi\pi}{2}))(\cosh(\frac{\pi\pi}{2}) + \cos(\pi\pi)))4} =$$

$$\frac{\pi}{4} + \frac{\pi(-\cosh(\frac{i\pi^2}{2}) + \cos(-\frac{i\pi^2}{2}))}{4((\cosh(i\pi^2) + \cos(-\frac{i\pi^2}{2}))(\cosh(\frac{i\pi^2}{2}) + \cos(-i\pi^2)))}$$

$$\frac{\pi}{4} + \frac{(\cosh(\pi(\pi - \frac{\pi}{2})) - \cos(\pi(\pi - \frac{\pi}{2})))\pi}{((\cosh(\pi\pi) + \cos(\frac{\pi\pi}{2}))(\cosh(\frac{\pi\pi}{2}) + \cos(\pi\pi)))4} =$$

$$\frac{\pi}{4} + \frac{\pi(-\cosh(-\frac{i\pi^2}{2}) + \cos(\frac{i\pi^2}{2}))}{4((\cosh(-i\pi^2) + \cos(\frac{i\pi^2}{2}))(\cosh(-\frac{i\pi^2}{2}) + \cos(i\pi^2)))}$$

Series representations:

$$\frac{\pi}{4} + \frac{(\cosh(\pi(\pi - \frac{\pi}{2})) - \cos(\pi(\pi - \frac{\pi}{2})))\pi}{((\cosh(\pi\pi) + \cos(\frac{\pi\pi}{2}))(\cosh(\frac{\pi\pi}{2}) + \cos(\pi\pi)))4} =$$

$$\frac{\pi \left(\sum_{k=0}^{\infty} \frac{4^{-k} (1+(-1)^{1+k}) \pi^{4k}}{(2k)!} + \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \left(\frac{\pi^{4k_1}}{(2k_1)!} + \frac{(-\frac{1}{4})^{k_1} \pi^{4k_1}}{(2k_1)!} \right) \left(\frac{(-1)^{k_2} \pi^{4k_2}}{(2k_2)!} + \frac{4^{-k_2} \pi^{4k_2}}{(2k_2)!} \right) \right)}{4 \left(\sum_{k=0}^{\infty} \frac{4^{-k} (1+(-4)^k) \pi^{4k}}{(2k)!} \right) \sum_{k=0}^{\infty} \frac{(1+(-\frac{1}{4})^k) \pi^{4k}}{(2k)!}}$$

$$\frac{\pi}{4} + \frac{(\cosh(\pi(\pi - \frac{\pi}{2})) - \cos(\pi(\pi - \frac{\pi}{2})))\pi}{((\cosh(\pi\pi) + \cos(\frac{\pi\pi}{2}))(\cosh(\frac{\pi\pi}{2}) + \cos(\pi\pi)))4} =$$

$$\left(\pi \left(\sum_{k=0}^{\infty} \left(\frac{2^{-2k} \pi^{4k}}{(2k)!} + \frac{(-1)^k \left(-\frac{\pi}{2} + \frac{\pi^2}{2}\right)^{1+2k}}{(1+2k)!} \right) + \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \left(\frac{\pi^{4k_1}}{(2k_1)!} + \frac{(-1)^{-1+k_1} \left(-\frac{\pi}{2} + \frac{\pi^2}{2}\right)^{1+2k_1}}{(1+2k_1)!} \right) \left(\frac{2^{-2k_2} \pi^{4k_2}}{(2k_2)!} + \frac{(-1)^{-1+k_2} \left(-\frac{\pi}{2} + \pi^2\right)^{1+2k_2}}{(1+2k_2)!} \right) \right) \right) /$$

$$\left(4 \left(\sum_{k=0}^{\infty} \left(\frac{\pi^{4k}}{(2k)!} + \frac{(-1)^{-1+k} \left(-\frac{\pi}{2} + \frac{\pi^2}{2}\right)^{1+2k}}{(1+2k)!} \right) \right) \sum_{k=0}^{\infty} \left(\frac{2^{-2k} \pi^{4k}}{(2k)!} + \frac{(-1)^{-1+k} \left(-\frac{\pi}{2} + \pi^2\right)^{1+2k}}{(1+2k)!} \right) \right)$$

$$\begin{aligned}
& \frac{\pi}{4} + \frac{\left(\cosh\left(\pi\left(\pi - \frac{\pi}{2}\right)\right) - \cos\left(\pi\left(\pi - \frac{\pi}{2}\right)\right)\right)\pi}{\left(\left(\cosh(\pi\pi) + \cos\left(\frac{\pi\pi}{2}\right)\right)\left(\cosh\left(\frac{\pi\pi}{2}\right) + \cos(\pi\pi)\right)\right)4} = \\
& \left(\pi \left(\sum_{k=0}^{\infty} \left(\frac{(-1)^{1+k} 2^{-2k} \pi^{4k}}{(2k)!} + \frac{i\left(-\frac{i\pi}{2} + \frac{\pi^2}{2}\right)^{1+2k}}{(1+2k)!} \right) + \right. \right. \\
& \quad \left. \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \left(\frac{(-1)^{k_1} \pi^{4k_1}}{(2k_1)!} + \frac{i\left(-\frac{i\pi}{2} + \frac{\pi^2}{2}\right)^{1+2k_1}}{(1+2k_1)!} \right) \right. \\
& \quad \left. \left(\frac{(-1)^{k_2} 2^{-2k_2} \pi^{4k_2}}{(2k_2)!} + \frac{i\left(-\frac{i\pi}{2} + \frac{\pi^2}{2}\right)^{1+2k_2}}{(1+2k_2)!} \right) \right) \Bigg) / \\
& \left(4 \left(\sum_{k=0}^{\infty} \left(\frac{(-1)^k \pi^{4k}}{(2k)!} + \frac{i\left(-\frac{i\pi}{2} + \frac{\pi^2}{2}\right)^{1+2k}}{(1+2k)!} \right) \right) \sum_{k=0}^{\infty} \left(\frac{(-1)^k 2^{-2k} \pi^{4k}}{(2k)!} + \frac{i\left(-\frac{i\pi}{2} + \frac{\pi^2}{2}\right)^{1+2k}}{(1+2k)!} \right) \right)
\end{aligned}$$

Integral representations:

$$\begin{aligned}
& \frac{\pi}{4} + \frac{\left(\cosh\left(\pi\left(\pi - \frac{\pi}{2}\right)\right) - \cos\left(\pi\left(\pi - \frac{\pi}{2}\right)\right)\right)\pi}{\left(\left(\cosh(\pi\pi) + \cos\left(\frac{\pi\pi}{2}\right)\right)\left(\cosh\left(\frac{\pi\pi}{2}\right) + \cos(\pi\pi)\right)\right)4} = \\
& - \left(\left(\left(\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-\pi^4/(4s)+s} \left(1 + e^{(5\pi^4)/(16s)}\right)}{\sqrt{s}} ds \right) \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-\pi^4/(16s)+s} \left(1 + e^{(5\pi^4)/(16s)}\right)}{\sqrt{s}} \right. \right. \\
& \quad \left. \left. ds + 4\pi \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{i e^{-\pi^4/(16s)+s} \left(-1 + e^{\pi^4/(8s)}\right)}{2\sqrt{\pi} \sqrt{s}} ds \right) \right) / \\
& \left(4 \left(\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-\pi^4/(4s)+s} \left(1 + e^{(5\pi^4)/(16s)}\right)}{\sqrt{s}} ds \right) \right. \\
& \quad \left. \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-\pi^4/(16s)+s} \left(1 + e^{(5\pi^4)/(16s)}\right)}{\sqrt{s}} ds \right) \Bigg) \text{ for } \gamma > 0
\end{aligned}$$

$$\frac{\pi}{4} + \frac{\left(\cosh\left(\pi\left(\pi - \frac{\pi}{2}\right)\right) - \cos\left(\pi\left(\pi - \frac{\pi}{2}\right)\right)\right)\pi}{\left(\left(\cosh(\pi\pi) + \cos\left(\frac{\pi\pi}{2}\right)\right)\left(\cosh\left(\frac{\pi\pi}{2}\right) + \cos(\pi\pi)\right)\right)4} =$$

$$-\left(\pi\left(4\pi\int_{-i\infty+\gamma}^{i\infty+\gamma}\left(-\frac{i e^{\pi^4/(16s)+s}}{2\sqrt{\pi}\sqrt{s}} + \frac{i 2^{-1+4s}\pi^{-1/2-4s}\Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)}\right)ds - \right.\right.$$

$$\left.\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}\left(\frac{e^{\pi^4/(16s)+s}}{\sqrt{s}} + \frac{4^s\pi^{-4s}\Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)}\right)ds\right)\right.$$

$$\left.\int_{-i\infty+\gamma}^{i\infty+\gamma}\left(\frac{e^{\pi^4/(4s)+s}}{\sqrt{s}} + \frac{16^s\pi^{-4s}\Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)}\right)ds\right)\right)/$$

$$\left(4\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}\left(\frac{e^{\pi^4/(16s)+s}}{\sqrt{s}} + \frac{4^s\pi^{-4s}\Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)}\right)ds\right)\right.$$

$$\left.\int_{-i\infty+\gamma}^{i\infty+\gamma}\left(\frac{e^{\pi^4/(4s)+s}}{\sqrt{s}} + \frac{16^s\pi^{-4s}\Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)}\right)ds\right)\right) \text{ for } 0 < \gamma < \frac{1}{2}$$

$$\frac{\pi}{4} + \frac{\left(\cosh\left(\pi\left(\pi - \frac{\pi}{2}\right)\right) - \cos\left(\pi\left(\pi - \frac{\pi}{2}\right)\right)\right)\pi}{\left(\left(\cosh(\pi\pi) + \cos\left(\frac{\pi\pi}{2}\right)\right)\left(\cosh\left(\frac{\pi\pi}{2}\right) + \cos(\pi\pi)\right)\right)4} =$$

$$\left(\pi\left(-2i\sqrt{\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{\pi^4/(16s)+s}}{\sqrt{s}}ds - \left(\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{\pi^4/(16s)+s}}{\sqrt{s}}ds\right)\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{\pi^4/(4s)+s}}{\sqrt{s}}ds + \right.\right.$$

$$4\pi\int_{\frac{\pi}{2}}^{\frac{\pi^2}{2}}\sin(t)dt + 2i\sqrt{\pi}\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{\pi^4/(16s)+s}}{\sqrt{s}}ds\right)\int_{\frac{\pi}{2}}^{\frac{\pi^2}{2}}\sin(t)dt +$$

$$2i\sqrt{\pi}\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{\pi^4/(4s)+s}}{\sqrt{s}}ds\right)\int_{\frac{\pi}{2}}^{\pi^2}\sin(t)dt +$$

$$\left.\int_0^1\int_0^1\cos\left(\frac{1}{2}(-1+\pi)\pi t_1\right)\cos\left(\frac{1}{2}(1-2\pi)\pi t_2\right)dt_2 dt_1\right)\right)/$$

$$\left(4\left(i\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{\pi^4/(4s)+s}}{\sqrt{s}}ds + 2\sqrt{\pi}\int_{\frac{\pi}{2}}^{\frac{\pi^2}{2}}\sin(t)dt\right)\right.$$

$$\left.\left(i\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{\pi^4/(16s)+s}}{\sqrt{s}}ds + 2\sqrt{\pi}\int_{\frac{\pi}{2}}^{\pi^2}\sin(t)dt\right)\right) \text{ for } \gamma > 0$$

((((Pi/4+Pi/4*(((cosh(Pi(Pi-Pi/2))-cos(Pi(Pi-Pi/2)))))/((((cosh(Pi*Pi)+cos(Pi*Pi/2)))(cosh(Pi*Pi/2)+cos(Pi*Pi))))))))))^(1/32

Input:

$$\sqrt[32]{\frac{\pi}{4} + \frac{\pi}{4} \times \frac{\cosh\left(\pi\left(\pi - \frac{\pi}{2}\right)\right) - \cos\left(\pi\left(\pi - \frac{\pi}{2}\right)\right)}{\left(\cosh(\pi\pi) + \cos\left(\pi \times \frac{\pi}{2}\right)\right)\left(\cosh\left(\pi \times \frac{\pi}{2}\right) + \cos(\pi\pi)\right)}}$$

Decimal approximation:

0.992482771495722580773982919439809428599128982969658968594...

0.9924827714.... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

and to the dilaton value **0.989117352243 = ϕ**

Alternate forms:

$$\sqrt[32]{\frac{\pi \left(-\cos\left(\frac{\pi^2}{2}\right) + \cos\left(\frac{\pi^2}{2}\right) \cos(\pi^2) + \cosh\left(\frac{\pi^2}{2}\right) + \cosh\left(\frac{\pi^2}{2}\right) \cosh(\pi^2) + \cos\left(\frac{\pi^2}{2}\right) \cosh\left(\frac{\pi^2}{2}\right) + \cos(\pi^2) \cosh(\pi^2) \right)}{\left(\cos(\pi^2) + \cosh\left(\frac{\pi^2}{2}\right) \right) \left(\cos\left(\frac{\pi^2}{2}\right) + \cosh(\pi^2) \right)}} \sqrt[16]{2}$$

$$\sqrt[32]{\frac{\pi}{4} + \frac{\left(\frac{1}{2} \left(e^{-\pi^2/2} + e^{\pi^2/2} \right) + \frac{1}{2} \left(-e^{-(i\pi^2)/2} - e^{(i\pi^2)/2} \right) \right) \pi}{4 \left(\frac{1}{2} \left(e^{-\pi^2/2} + e^{\pi^2/2} \right) + \frac{1}{2} \left(e^{-i\pi^2} + e^{i\pi^2} \right) \right) \left(\frac{1}{2} \left(e^{-\pi^2} + e^{\pi^2} \right) + \frac{1}{2} \left(e^{-(i\pi^2)/2} + e^{(i\pi^2)/2} \right) \right)}}$$

$$\frac{1}{\sqrt[16]{2}} \left(\left(\pi \left(e^{-(3\pi^2)/2} + e^{(3\pi^2)/2} + e^{-\pi^2/2} \left(3 + 2 \cos\left(\frac{\pi^2}{2}\right) \right) + e^{\pi^2/2} \left(3 + 2 \cos\left(\frac{\pi^2}{2}\right) \right) + 4 \cos\left(\frac{\pi^2}{2}\right) (\cos(\pi^2) - 1) + 2 e^{-\pi^2} \cos(\pi^2) + 2 e^{\pi^2} \cos(\pi^2) \right) \right) / \left(\left(2 \cos(\pi^2) + 2 \cosh\left(\frac{\pi^2}{2}\right) \right) \left(2 \cos\left(\frac{\pi^2}{2}\right) + 2 \cosh(\pi^2) \right) \right) \right)^{(1/32)}$$

All 32nd roots of $\pi/4 + (\pi (\cosh(\pi^2/2) - \cos(\pi^2/2)))/(4 (\cos(\pi^2) + \cosh(\pi^2/2)) (\cos(\pi^2/2) + \cosh(\pi^2)))$:

$$e^0 \sqrt[32]{\frac{\pi}{4} + \frac{\pi \left(\cosh\left(\frac{\pi^2}{2}\right) - \cos\left(\frac{\pi^2}{2}\right) \right)}{4 \left(\cos(\pi^2) + \cosh\left(\frac{\pi^2}{2}\right) \right) \left(\cos\left(\frac{\pi^2}{2}\right) + \cosh(\pi^2) \right)}} \approx 0.992483 \text{ (real, principal root)}$$

$$e^{(i\pi)/16} \sqrt[32]{\frac{\pi}{4} + \frac{\pi \left(\cosh\left(\frac{\pi^2}{2}\right) - \cos\left(\frac{\pi^2}{2}\right) \right)}{4 \left(\cos(\pi^2) + \cosh\left(\frac{\pi^2}{2}\right) \right) \left(\cos\left(\frac{\pi^2}{2}\right) + \cosh(\pi^2) \right)}} \approx 0.97341 + 0.19362 i$$

$$e^{(i\pi)/8} \sqrt[32]{\frac{\pi}{4} + \frac{\pi \left(\cosh\left(\frac{\pi^2}{2}\right) - \cos\left(\frac{\pi^2}{2}\right) \right)}{4 \left(\cos(\pi^2) + \cosh\left(\frac{\pi^2}{2}\right) \right) \left(\cos\left(\frac{\pi^2}{2}\right) + \cosh(\pi^2) \right)}} \approx 0.91693 + 0.37981 i$$

$$e^{(3i\pi)/16} \sqrt[32]{\frac{\pi}{4} + \frac{\pi \left(\cosh\left(\frac{\pi^2}{2}\right) - \cos\left(\frac{\pi^2}{2}\right) \right)}{4 \left(\cos(\pi^2) + \cosh\left(\frac{\pi^2}{2}\right) \right) \left(\cos\left(\frac{\pi^2}{2}\right) + \cosh(\pi^2) \right)}} \approx 0.82522 + 0.5514 i$$

$$e^{(i\pi)/4} \sqrt[32]{\frac{\pi}{4} + \frac{\pi \left(\cosh\left(\frac{\pi^2}{2}\right) - \cos\left(\frac{\pi^2}{2}\right) \right)}{4 \left(\cos(\pi^2) + \cosh\left(\frac{\pi^2}{2}\right) \right) \left(\cos\left(\frac{\pi^2}{2}\right) + \cosh(\pi^2) \right)}} \approx 0.70179 + 0.70179 i$$

Alternative representations:

$$\sqrt[32]{\frac{\pi}{4} + \frac{\left(\cosh\left(\pi\left(\pi - \frac{\pi}{2}\right)\right) - \cos\left(\pi\left(\pi - \frac{\pi}{2}\right)\right) \right) \pi}{\left(\cosh(\pi\pi) + \cos\left(\frac{\pi\pi}{2}\right) \right) \left(\cosh\left(\frac{\pi\pi}{2}\right) + \cos(\pi\pi) \right) 4}} =$$

$$\sqrt[32]{\frac{\pi}{4} + \frac{\pi \left(-\cosh\left(-\frac{i\pi^2}{2}\right) + \cos\left(-\frac{i\pi^2}{2}\right) \right)}{4 \left(\cosh(-i\pi^2) + \cos\left(-\frac{i\pi^2}{2}\right) \right) \left(\cosh\left(-\frac{i\pi^2}{2}\right) + \cos(-i\pi^2) \right)}}$$

$$\sqrt[32]{\frac{\pi}{4} + \frac{\left(\cosh\left(\pi\left(\pi - \frac{\pi}{2}\right)\right) - \cos\left(\pi\left(\pi - \frac{\pi}{2}\right)\right) \right) \pi}{\left(\cosh(\pi\pi) + \cos\left(\frac{\pi\pi}{2}\right) \right) \left(\cosh\left(\frac{\pi\pi}{2}\right) + \cos(\pi\pi) \right) 4}} =$$

$$\sqrt[32]{\frac{\pi}{4} + \frac{\pi \left(-\cosh\left(\frac{i\pi^2}{2}\right) + \cos\left(-\frac{i\pi^2}{2}\right) \right)}{4 \left(\cosh(i\pi^2) + \cos\left(-\frac{i\pi^2}{2}\right) \right) \left(\cosh\left(\frac{i\pi^2}{2}\right) + \cos(-i\pi^2) \right)}}$$

$$\sqrt[32]{\frac{\pi}{4} + \frac{\left(\cosh\left(\pi\left(\pi - \frac{\pi}{2}\right)\right) - \cos\left(\pi\left(\pi - \frac{\pi}{2}\right)\right) \right) \pi}{\left(\cosh(\pi\pi) + \cos\left(\frac{\pi\pi}{2}\right) \right) \left(\cosh\left(\frac{\pi\pi}{2}\right) + \cos(\pi\pi) \right) 4}} =$$

$$\sqrt[32]{\frac{\pi}{4} + \frac{\pi \left(-\cosh\left(-\frac{i\pi^2}{2}\right) + \cos\left(\frac{i\pi^2}{2}\right) \right)}{4 \left(\cosh(-i\pi^2) + \cos\left(\frac{i\pi^2}{2}\right) \right) \left(\cosh\left(-\frac{i\pi^2}{2}\right) + \cos(i\pi^2) \right)}}$$

Series representations:

$$\begin{aligned}
& \sqrt[32]{\frac{\pi}{4} + \frac{(\cosh(\pi(\pi - \frac{\pi}{2})) - \cos(\pi(\pi - \frac{\pi}{2})))\pi}{((\cosh(\pi\pi) + \cos(\frac{\pi\pi}{2}))(\cosh(\frac{\pi\pi}{2}) + \cos(\pi\pi)))4}} = \\
& \frac{1}{16\sqrt{2}} \sqrt[32]{\pi} \left(\sum_{k=0}^{\infty} \frac{4^{-k} (1 + (-1)^{1+k}) \pi^{4k}}{(2k)!} + \right. \\
& \quad \left. \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \left(\frac{(-1)^{k_1} \pi^{4k_1}}{(2k_1)!} + \frac{2^{-2k_1} \pi^{4k_1}}{(2k_1)!} \right) \left(\frac{\pi^{4k_2}}{(2k_2)!} + \frac{(-1)^{k_2} 2^{-2k_2} \pi^{4k_2}}{(2k_2)!} \right) \right) / \\
& \quad \left(\sum_{k=0}^{\infty} \frac{4^{-k} (1 + (-4)^k) \pi^{4k}}{(2k)!} \right) \sum_{k=0}^{\infty} \frac{4^{-k} ((-1)^k + 4^k) \pi^{4k}}{(2k)!} \Big)^{\wedge (1/32)}
\end{aligned}$$

$$\begin{aligned}
& \sqrt[32]{\frac{\pi}{4} + \frac{(\cosh(\pi(\pi - \frac{\pi}{2})) - \cos(\pi(\pi - \frac{\pi}{2})))\pi}{((\cosh(\pi\pi) + \cos(\frac{\pi\pi}{2}))(\cosh(\frac{\pi\pi}{2}) + \cos(\pi\pi)))4}} = \\
& \frac{1}{16\sqrt{2}} \sqrt[32]{\pi} \left(\sum_{k=0}^{\infty} \left(\frac{2^{-2k} \pi^{4k}}{(2k)!} + \frac{(-1)^k (-\frac{\pi}{2} + \frac{\pi^2}{2})^{1+2k}}{(1+2k)!} \right) + \right. \\
& \quad \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \left(\frac{\pi^{4k_1}}{(2k_1)!} + \frac{(-1)^{-1+k_1} (-\frac{\pi}{2} + \frac{\pi^2}{2})^{1+2k_1}}{(1+2k_1)!} \right) \\
& \quad \left. \left(\frac{2^{-2k_2} \pi^{4k_2}}{(2k_2)!} + \frac{(-1)^{-1+k_2} (-\frac{\pi}{2} + \pi^2)^{1+2k_2}}{(1+2k_2)!} \right) \right) / \\
& \quad \left(\sum_{k=0}^{\infty} \left(\frac{\pi^{4k}}{(2k)!} + \frac{(-1)^{-1+k} (-\frac{\pi}{2} + \frac{\pi^2}{2})^{1+2k}}{(1+2k)!} \right) \right) \\
& \quad \sum_{k=0}^{\infty} \left(\frac{2^{-2k} \pi^{4k}}{(2k)!} + \frac{(-1)^{-1+k} (-\frac{\pi}{2} + \pi^2)^{1+2k}}{(1+2k)!} \right) \Big)^{\wedge (1/32)}
\end{aligned}$$

$$\begin{aligned}
& \sqrt[32]{\frac{\pi}{4} + \frac{(\cosh(\pi(\pi - \frac{\pi}{2})) - \cos(\pi(\pi - \frac{\pi}{2})))\pi}{((\cosh(\pi\pi) + \cos(\frac{\pi\pi}{2}))(\cosh(\frac{\pi\pi}{2}) + \cos(\pi\pi)))4}} = \\
& \frac{1}{\sqrt[16]{2}} \sqrt[32]{\pi} \left(\sum_{k=0}^{\infty} \left(\frac{(-1)^{1+k} 2^{-2k} \pi^{4k}}{(2k)!} + \frac{i(-\frac{i\pi}{2} + \frac{\pi^2}{2})^{1+2k}}{(1+2k)!} \right) + \right. \\
& \quad \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \left(\frac{(-1)^{k_1} \pi^{4k_1}}{(2k_1)!} + \frac{i(-\frac{i\pi}{2} + \frac{\pi^2}{2})^{1+2k_1}}{(1+2k_1)!} \right) \left(\frac{(-1)^{k_2} 2^{-2k_2} \pi^{4k_2}}{(2k_2)!} + \right. \\
& \quad \quad \left. \left. \frac{i(-\frac{i\pi}{2} + \frac{\pi^2}{2})^{1+2k_2}}{(1+2k_2)!} \right) \right) / \left(\sum_{k=0}^{\infty} \left(\frac{(-1)^k \pi^{4k}}{(2k)!} + \frac{i(-\frac{i\pi}{2} + \frac{\pi^2}{2})^{1+2k}}{(1+2k)!} \right) \right) \\
& \quad \left. \sum_{k=0}^{\infty} \left(\frac{(-1)^k 2^{-2k} \pi^{4k}}{(2k)!} + \frac{i(-\frac{i\pi}{2} + \frac{\pi^2}{2})^{1+2k}}{(1+2k)!} \right) \right) \wedge (1/32)
\end{aligned}$$

Integral representations:

$$\begin{aligned}
& \sqrt[32]{\frac{\pi}{4} + \frac{(\cosh(\pi(\pi - \frac{\pi}{2})) - \cos(\pi(\pi - \frac{\pi}{2})))\pi}{((\cosh(\pi\pi) + \cos(\frac{\pi\pi}{2}))(\cosh(\frac{\pi\pi}{2}) + \cos(\pi\pi)))4}} = \frac{1}{\sqrt[16]{2}} \sqrt[32]{\pi} \\
& \left(\left(\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-\pi^4/(4s)+s} (1 + e^{(5\pi^4)/(16s)})}{\sqrt{s}} ds \right) \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-\pi^4/(16s)+s} (1 + e^{(5\pi^4)/(16s)})}{\sqrt{s}} ds - \right. \\
& \quad \left. 4\pi \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{i e^{-\pi^4/(16s)+s} (-1 + e^{\pi^4/(8s)})}{2\sqrt{\pi} \sqrt{s}} ds \right) / \\
& \left(\left(\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-\pi^4/(4s)+s} (1 + e^{(5\pi^4)/(16s)})}{\sqrt{s}} ds \right) \right. \\
& \quad \left. \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-\pi^4/(16s)+s} (1 + e^{(5\pi^4)/(16s)})}{\sqrt{s}} ds \right) \wedge (1/32) \text{ for } \gamma > 0
\end{aligned}$$

$$\sqrt[32]{\frac{\pi}{4} + \frac{(\cosh(\pi(\pi - \frac{\pi}{2})) - \cos(\pi(\pi - \frac{\pi}{2})))\pi}{((\cosh(\pi\pi) + \cos(\frac{\pi\pi}{2}))(\cosh(\frac{\pi\pi}{2}) + \cos(\pi\pi)))4}} =$$

$$\frac{1}{16\sqrt[2]{2}} \sqrt[32]{\pi} \left(\left(-4\pi \int_{-i\infty+\gamma}^{i\infty+\gamma} \left(-\frac{i e^{\pi^4/(16s)+s}}{2\sqrt{\pi}\sqrt{s}} + \frac{i 2^{-1+4s} \pi^{-1/2-4s} \Gamma(s)}{\Gamma(\frac{1}{2}-s)} \right) ds + \right. \right.$$

$$\left. \left(\int_{-i\infty+\gamma}^{i\infty+\gamma} \left(\frac{e^{\pi^4/(16s)+s}}{\sqrt{s}} + \frac{4^s \pi^{-4s} \Gamma(s)}{\Gamma(\frac{1}{2}-s)} \right) ds \right) \right.$$

$$\left. \int_{-i\infty+\gamma}^{i\infty+\gamma} \left(\frac{e^{\pi^4/(4s)+s}}{\sqrt{s}} + \frac{16^s \pi^{-4s} \Gamma(s)}{\Gamma(\frac{1}{2}-s)} \right) ds \right) /$$

$$\left(\left(\int_{-i\infty+\gamma}^{i\infty+\gamma} \left(\frac{e^{\pi^4/(16s)+s}}{\sqrt{s}} + \frac{4^s \pi^{-4s} \Gamma(s)}{\Gamma(\frac{1}{2}-s)} \right) ds \right) \right.$$

$$\left. \int_{-i\infty+\gamma}^{i\infty+\gamma} \left(\frac{e^{\pi^4/(4s)+s}}{\sqrt{s}} + \frac{16^s \pi^{-4s} \Gamma(s)}{\Gamma(\frac{1}{2}-s)} \right) ds \right) \right)^{(1/32)} \text{ for } 0 < \gamma < \frac{1}{2}$$

$$\sqrt[32]{\frac{\pi}{4} + \frac{(\cosh(\pi(\pi - \frac{\pi}{2})) - \cos(\pi(\pi - \frac{\pi}{2})))\pi}{((\cosh(\pi\pi) + \cos(\frac{\pi\pi}{2}))(\cosh(\frac{\pi\pi}{2}) + \cos(\pi\pi)))4}} = \frac{1}{16\sqrt[2]{2}} \sqrt[32]{\pi}$$

$$\left(-\left(\left(-2i\sqrt{\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{\pi^4/(16s)+s}}{\sqrt{s}} ds - \left(\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{\pi^4/(16s)+s}}{\sqrt{s}} ds \right) \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{\pi^4/(4s)+s}}{\sqrt{s}} ds + \right. \right. \right.$$

$$4\pi \int_{\frac{\pi}{2}}^{\frac{\pi^2}{2}} \sin(t) dt + 2i\sqrt{\pi} \left(\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{\pi^4/(16s)+s}}{\sqrt{s}} ds \right) \int_{\frac{\pi}{2}}^{\frac{\pi^2}{2}} \sin(t) dt +$$

$$2i\sqrt{\pi} \left(\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{\pi^4/(4s)+s}}{\sqrt{s}} ds \right) \int_{\frac{\pi}{2}}^{\pi^2} \sin(t) dt +$$

$$\int_0^1 \int_0^1 \cos\left(\frac{1}{2}(-1+\pi)\pi t_1\right) \cos\left(\frac{1}{2}(1-2\pi)\pi t_2\right) dt_2 dt_1 \Big) /$$

$$\left(\left(\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{\pi^4/(4s)+s}}{\sqrt{s}} ds - 2i\sqrt{\pi} \int_{\frac{\pi}{2}}^{\frac{\pi^2}{2}} \sin(t) dt \right) \left(\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{\pi^4/(16s)+s}}{\sqrt{s}} ds - \right. \right.$$

$$\left. \left. 2i\sqrt{\pi} \int_{\frac{\pi}{2}}^{\pi^2} \sin(t) dt \right) \right) \right)^{(1/32)} \text{ for } \gamma > 0$$

1+(((((((Pi/4+Pi/4*(((cosh(Pi(Pi-Pi/2))-cos(Pi(Pi-Pi/2)))))/(((cosh(Pi*Pi)+cos(Pi*Pi/2)))((cosh(Pi*Pi/2)+cos(Pi*Pi)))))))))))))^2

Input:

$$1 + \left(\frac{\pi}{4} + \frac{\pi}{4} \times \frac{\cosh\left(\pi\left(\pi - \frac{\pi}{2}\right)\right) - \cos\left(\pi\left(\pi - \frac{\pi}{2}\right)\right)}{\left(\cosh(\pi\pi) + \cos\left(\pi \times \frac{\pi}{2}\right)\right)\left(\cosh\left(\pi \times \frac{\pi}{2}\right) + \cos(\pi\pi)\right)} \right)^2$$

Decimal approximation:

1.616979169232501146826478654197142656926178946016268751694...
 1.616979169....

This result is a very good approximation to the value of the golden ratio
 1,618033988749...

Alternate forms:

$$\frac{\frac{1}{16} (16 + \pi^2) + \frac{\pi^2 (\cos(\frac{\pi^2}{2}) - \cosh(\frac{\pi^2}{2}))^2}{16 (\cos(\frac{\pi^2}{2}) + \cosh(\frac{\pi^2}{2}))^2 (\cos(\frac{\pi^2}{2}) + \cosh(\pi^2))^2} - \frac{\pi^2 (\cos(\frac{\pi^2}{2}) - \cosh(\frac{\pi^2}{2}))}{8 (\cos(\pi^2) + \cosh(\frac{\pi^2}{2})) (\cos(\frac{\pi^2}{2}) + \cosh(\pi^2))}{1 + \left(\frac{\pi}{4} + \left(\left(\frac{1}{2} (e^{-\pi^2/2} + e^{\pi^2/2}) + \frac{1}{2} (-e^{-(i\pi^2)/2} - e^{(i\pi^2)/2}) \right) \pi \right) / \left(4 \left(\frac{1}{2} (e^{-\pi^2/2} + e^{\pi^2/2}) + \frac{1}{2} (e^{-i\pi^2} + e^{i\pi^2}) \right) \left(\frac{1}{2} (e^{-\pi^2} + e^{\pi^2}) + \frac{1}{2} (e^{-(i\pi^2)/2} + e^{(i\pi^2)/2}) \right) \right) \right)^2}{1 + \left(\pi^2 \left(1 + e^{3\pi^2} + e^{\pi^2} \left(3 + 2 \cos\left(\frac{\pi^2}{2}\right) \right) + e^{2\pi^2} \left(3 + 2 \cos\left(\frac{\pi^2}{2}\right) \right) + 2 e^{\pi^2/2} \cos(\pi^2) + 2 e^{(5\pi^2)/2} \cos(\pi^2) - 2 e^{(3\pi^2)/2} \left(\cos\left(\frac{\pi^2}{2}\right) - \cos\left(\frac{3\pi^2}{2}\right) \right) \right) \right)^2 / \left(16 \left(1 + e^{2\pi^2} + 2 e^{\pi^2} \cos\left(\frac{\pi^2}{2}\right) \right)^2 \left(1 + e^{\pi^2} + 2 e^{\pi^2/2} \cos(\pi^2) \right)^2 \right)}$$

Alternative representations:

$$1 + \left(\frac{\pi}{4} + \frac{(\cosh(\pi(\pi - \frac{\pi}{2})) - \cos(\pi(\pi - \frac{\pi}{2}))) \pi}{((\cosh(\pi\pi) + \cos(\frac{\pi\pi}{2})) (\cosh(\frac{\pi\pi}{2}) + \cos(\pi\pi))) 4} \right)^2 =$$

$$1 + \left(\frac{\pi}{4} + \frac{\pi (-\cosh(-\frac{i\pi^2}{2}) + \cos(-\frac{i\pi^2}{2}))}{4 ((\cosh(-i\pi^2) + \cos(-\frac{i\pi^2}{2})) (\cosh(-\frac{i\pi^2}{2}) + \cos(-i\pi^2)))} \right)^2$$

$$1 + \left(\frac{\pi}{4} + \frac{(\cosh(\pi(\pi - \frac{\pi}{2})) - \cos(\pi(\pi - \frac{\pi}{2}))) \pi}{((\cosh(\pi\pi) + \cos(\frac{\pi\pi}{2})) (\cosh(\frac{\pi\pi}{2}) + \cos(\pi\pi))) 4} \right)^2 =$$

$$1 + \left(\frac{\pi}{4} + \frac{\pi (-\cosh(\frac{i\pi^2}{2}) + \cos(-\frac{i\pi^2}{2}))}{4 ((\cosh(i\pi^2) + \cos(-\frac{i\pi^2}{2})) (\cosh(\frac{i\pi^2}{2}) + \cos(-i\pi^2)))} \right)^2$$

$$1 + \left(\frac{\pi}{4} + \frac{(\cosh(\pi(\pi - \frac{\pi}{2})) - \cos(\pi(\pi - \frac{\pi}{2})))\pi}{((\cosh(\pi\pi) + \cos(\frac{\pi\pi}{2}))(\cosh(\frac{\pi\pi}{2}) + \cos(\pi\pi)))4} \right)^2 =$$

$$1 + \left(\frac{\pi}{4} + \frac{\pi(-\cosh(-\frac{i\pi^2}{2}) + \cos(\frac{i\pi^2}{2}))}{4((\cosh(-i\pi^2) + \cos(\frac{i\pi^2}{2}))(\cosh(-\frac{i\pi^2}{2}) + \cos(i\pi^2)))} \right)^2$$

Series representations:

$$1 + \left(\frac{\pi}{4} + \frac{(\cosh(\pi(\pi - \frac{\pi}{2})) - \cos(\pi(\pi - \frac{\pi}{2})))\pi}{((\cosh(\pi\pi) + \cos(\frac{\pi\pi}{2}))(\cosh(\frac{\pi\pi}{2}) + \cos(\pi\pi)))4} \right)^2 =$$

$$\left(\pi^2 \left(\sum_{k=0}^{\infty} \frac{4^{-k} (1 + (-1)^{1+k}) \pi^{4k}}{(2k)!} \right)^2 + \right.$$

$$16 \left(\sum_{k=0}^{\infty} \frac{4^{-k} (1 + (-4)^k) \pi^{4k}}{(2k)!} \right)^2 \left(\sum_{k=0}^{\infty} \frac{4^{-k} ((-1)^k + 4^k) \pi^{4k}}{(2k)!} \right)^2 +$$

$$\pi^2 \left(\sum_{k=0}^{\infty} \frac{4^{-k} (1 + (-4)^k) \pi^{4k}}{(2k)!} \right)^2 \left(\sum_{k=0}^{\infty} \frac{4^{-k} ((-1)^k + 4^k) \pi^{4k}}{(2k)!} \right)^2 +$$

$$2\pi^2 \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \left(\frac{(-1)^{k_1} \pi^{4k_1}}{(2k_1)!} + \frac{2^{-2k_1} \pi^{4k_1}}{(2k_1)!} \right) \left(\frac{\pi^{4k_2}}{(2k_2)!} + \frac{(-1)^{k_2} 2^{-2k_2} \pi^{4k_2}}{(2k_2)!} \right)$$

$$\left. \left(\frac{2^{-2k_3} \pi^{4k_3}}{(2k_3)!} + \frac{(-1)^{1+k_3} 2^{-2k_3} \pi^{4k_3}}{(2k_3)!} \right) \right) /$$

$$\left(16 \left(\sum_{k=0}^{\infty} \frac{4^{-k} (1 + (-4)^k) \pi^{4k}}{(2k)!} \right)^2 \left(\sum_{k=0}^{\infty} \frac{4^{-k} ((-1)^k + 4^k) \pi^{4k}}{(2k)!} \right)^2 \right)$$

$$1 + \left(\frac{\pi}{4} + \frac{(\cosh(\pi(\pi - \frac{\pi}{2})) - \cos(\pi(\pi - \frac{\pi}{2})))\pi}{((\cosh(\pi\pi) + \cos(\frac{\pi\pi}{2}))(\cosh(\frac{\pi\pi}{2}) + \cos(\pi\pi)))4} \right)^2 =$$

$$1 + \left(\frac{\pi}{4} + \left(\pi \left(\sum_{k=0}^{\infty} \frac{2^{-2k} \pi^{4k}}{(2k)!} - \sqrt{\pi} \sum_{j=0}^{\infty} \text{Res}_{s=-j} \frac{16^s \pi^{-4s} \Gamma(s)}{\Gamma(\frac{1}{2} - s)} \right) \right) /$$

$$\left(4 \left(\sum_{k=0}^{\infty} \frac{2^{-2k} \pi^{4k}}{(2k)!} + \sqrt{\pi} \sum_{j=0}^{\infty} \text{Res}_{s=-j} \frac{4^s \pi^{-4s} \Gamma(s)}{\Gamma(\frac{1}{2} - s)} \right) \right.$$

$$\left. \left(\sum_{k=0}^{\infty} \frac{\pi^{4k}}{(2k)!} + \sqrt{\pi} \sum_{j=0}^{\infty} \text{Res}_{s=-j} \frac{16^s \pi^{-4s} \Gamma(s)}{\Gamma(\frac{1}{2} - s)} \right) \right)^2$$

$$1 + \left(\frac{\pi}{4} + \frac{(\cosh(\pi(\pi - \frac{\pi}{2})) - \cos(\pi(\pi - \frac{\pi}{2})))\pi}{((\cosh(\pi\pi) + \cos(\frac{\pi\pi}{2}))(\cosh(\frac{\pi\pi}{2}) + \cos(\pi\pi)))4} \right)^2 =$$

$$1 + \left(\frac{\pi}{4} + \left(\pi \left(- \sum_{k=0}^{\infty} \frac{(-1)^k 2^{-2k} \pi^{4k}}{(2k)!} + \sqrt{\pi} \sum_{j=0}^{\infty} \text{Res}_{s=-j} \frac{(-\frac{1}{16})^{-s} \pi^{-4s} \Gamma(s)}{\Gamma(\frac{1}{2}-s)} \right) \right) \right)^2 /$$

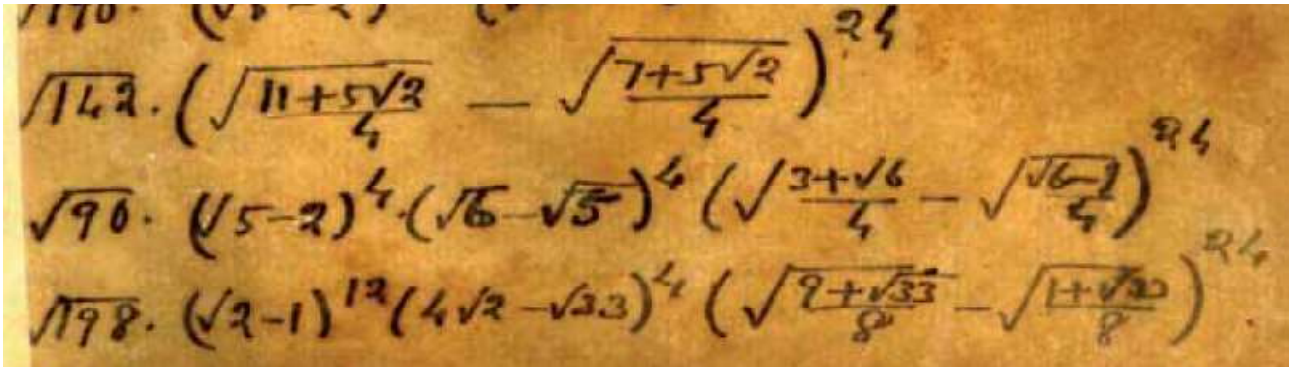
$$\left(4 \left(\sum_{k=0}^{\infty} \frac{(-1)^k 2^{-2k} \pi^{4k}}{(2k)!} + \sqrt{\pi} \sum_{j=0}^{\infty} \text{Res}_{s=-j} \frac{(-\frac{1}{4})^{-s} \pi^{-4s} \Gamma(s)}{\Gamma(\frac{1}{2}-s)} \right) \right)^2$$

$$\left(\sum_{k=0}^{\infty} \frac{(-1)^k \pi^{4k}}{(2k)!} + \sqrt{\pi} \sum_{j=0}^{\infty} \text{Res}_{s=-j} \frac{(-\frac{1}{16})^{-s} \pi^{-4s} \Gamma(s)}{\Gamma(\frac{1}{2}-s)} \right)^2$$

$n!$ is the factorial function

$\Gamma(x)$ is the gamma function
 $\text{Res } f$ is a complex residue
 $\Rightarrow 0$

Page 302



$$\text{sqrt}(142) * (((((\text{sqrt}((11+5*\text{sqrt}(2))/4)) - (((\text{sqrt}((7+5*\text{sqrt}(2))/4))))))))^24$$

Input:

$$\sqrt{142} \left(\sqrt{\frac{1}{4}(11+5\sqrt{2})} - \sqrt{\frac{1}{4}(7+5\sqrt{2})} \right)^{24}$$

Result:

$$\sqrt{142} \left(\frac{1}{2} \sqrt{11+5\sqrt{2}} - \frac{1}{2} \sqrt{7+5\sqrt{2}} \right)^{24}$$

Decimal approximation:

$$4.2063313414753316252174540855706339431072025172919219... \times 10^{-14}$$

$$4.2063313414... * 10^{-14}$$

Alternate forms:

$$\frac{\sqrt{142} \left(\sqrt{11+5\sqrt{2}} - \sqrt{7+5\sqrt{2}} \right)^{24}}{16777216}$$

$$\sqrt{\left(142 \left(20064178973593989354119025601 + \right.\right. \\ \left.14187517011168852729798806400\sqrt{2} - 2520 \sqrt{\left(142 \right.\right. \\ \left.\left.\left(892859977067899354121985954835246279027023601409 \right.\right.\right. \\ \left.\left.\left.\left.+ \right.\right.\right.\right. \\ \left.631347344434776949083787862200150313637006 \cdot \right.\right. \\ \left.\left.916146\sqrt{2}\right)\right)\right)\right)}$$

$$\frac{\sqrt{\frac{71}{2}} \left(\sqrt{7+5\sqrt{2}} - \sqrt{11+5\sqrt{2}} \right)^{24}}{8388608}$$

Minimal polynomial:

$$x^8 - 11396453657001385953139606541368x^6 + \\ 3207593437472821049302768754348184x^4 - \\ 229798091539775946359107026300144352x^2 + 406586896$$

$$\sqrt{90} \left((\sqrt{5}-2)^4 \right) (\sqrt{6}-\sqrt{5})^4 \left(\left(\left(\left(\left(\left(\sqrt{\frac{3+\sqrt{6}}{4}} \right) \right) \right) \right) \right) \right) - \\ \left(\left(\left(\left(\left(\sqrt{\frac{\sqrt{6}-3}{4}} \right) \right) \right) \right) \right) \right)^{24}$$

Input:

$$\sqrt{90} (\sqrt{5}-2)^4 (\sqrt{6}-\sqrt{5})^4 \left(\sqrt{\frac{1}{4}(3+\sqrt{6})} - \sqrt{\frac{1}{4}(\sqrt{6}-3)} \right)^{24}$$

Result:

$$3\sqrt{10} (\sqrt{5}-2)^4 (\sqrt{6}-\sqrt{5})^4 \left(\frac{\sqrt{3+\sqrt{6}}}{2} - \frac{1}{2}i\sqrt{3-\sqrt{6}} \right)^{24}$$

Decimal approximation:

$$0.00357921577730507229864923703881610138254467705255447778... - \\ 0.00707725449138392906070823928879439801666534761562474954... i$$

Polar coordinates:

$$r \approx 0.00793086 \text{ (radius)}, \quad \theta \approx -63.1727^\circ \text{ (angle)}$$

$$0.00793086$$

Alternate forms:

$$\frac{3(161 - 72\sqrt{5})(241\sqrt{10} - 440\sqrt{3})\left(\sqrt{3+\sqrt{6}} - i\sqrt{3-\sqrt{6}}\right)^{24}}{16\,777\,216}$$

$$\frac{2187(329 - 460i\sqrt{2})(\sqrt{5} - 2)^4(241\sqrt{10} - 440\sqrt{3})}{4096}$$

$$\frac{3\sqrt{\frac{5}{2}}(\sqrt{5} - 2)^4(\sqrt{5} - \sqrt{6})^4\left(\sqrt{3-\sqrt{6}} + i\sqrt{3+\sqrt{6}}\right)^{24}}{8\,388\,608}$$

$$\sqrt[24]{198 \left((\sqrt{2}-1)^{12} \right) (4\sqrt{2}-\sqrt{33})^4 \left(\left(\sqrt{\frac{9+\sqrt{33}}{8}} \right) - \left(\sqrt{\frac{1+\sqrt{33}}{8}} \right) \right)^2}$$

Input:

$$\sqrt{198} (\sqrt{2} - 1)^{12} (4\sqrt{2} - \sqrt{33})^4 \left(\sqrt{\frac{1}{8}(9 + \sqrt{33})} - \sqrt{\frac{1}{8}(1 + \sqrt{33})} \right)^{24}$$

Result:

$$3\sqrt{22} (\sqrt{2} - 1)^{12} (4\sqrt{2} - \sqrt{33})^4 \left(\frac{1}{2} \sqrt{\frac{1}{2}(9 + \sqrt{33})} - \frac{1}{2} \sqrt{\frac{1}{2}(1 + \sqrt{33})} \right)^{24}$$

Decimal approximation:

$$5.70210896104951444448980153678853736270841816487651930... \times 10^{-17}$$

$$5.702108961... * 10^{-17}$$

Alternate forms:

$$\frac{3(19\,601 - 13\,860\sqrt{2})(8\,449\sqrt{22} - 22\,880\sqrt{3})\left(\sqrt{2(9+\sqrt{33})} - \sqrt{2(1+\sqrt{33})}\right)^{24}}{281\,474\,976\,710\,656}$$

$$3\sqrt{22} (\sqrt{2} - 1)^{12} (8\,449 - 10\,400\sqrt{66})$$

$\text{root of } x^8 - 5x^6 - 5x^2 + 1 \text{ near } x = 0.439409$

24

$$\frac{3\sqrt{\frac{11}{2}} (\sqrt{2} - 1)^{12} (4\sqrt{2} - \sqrt{33})^4 \left(\sqrt{1+\sqrt{33}} - \sqrt{9+\sqrt{33}}\right)^{24}}{34\,359\,738\,368}$$

Minimal polynomial:

$$\begin{aligned}
& x^{16} - 12\,057\,557\,915\,772\,134\,092\,386\,196\,727\,564\,622\,384\,x^{14} + \\
& 2533592\,302\,767\,616\,739\,823\,926\,559\,444\,298\,928\,112\,x^{12} - \\
& 10\,296\,208\,352\,103\,720\,215\,855\,121\,114\,781\,503\,307\,781\,952\,x^{10} + \\
& 9\,050\,372\,678\,682\,251\,025\,552\,579\,128\,979\,586\,235\,405\,505\,120\,x^8 - \\
& 403\,652\,552\,235\,874\,247\,342\,384\,168\,183\,894\,055\,678\,283\,646\,208\,x^6 + \\
& 3894013\,851\,208\,455\,355\,974\,515\,128\,856\,354\,188\,146\,662\,452\,992\,x^4 - \\
& 726524891\,388\,950\,974\,692\,631\,170\,026\,093\,950\,526\,110\,002\,686\,976\,x^2 + \\
& 2362226\,417735475456
\end{aligned}$$

We take the three results, and obtain:

$$(76+18)\pi / (5.702108961049514444898 \times 10^{-17} * 1 / 4.206331341475331625217454 \times 10^{-14} * 1 / 0.00793086)$$

Input interpretation:

$$(76 + 18) \times \frac{\pi}{5.702108961049514444898 \times 10^{-17} \times \frac{1}{4.206331341475331625217454 \times 10^{-14}} \times \frac{1}{0.00793086}}$$

Result:

1727.69...

1727.69...

This result is very near to the mass of candidate glueball $f_0(1710)$ meson. Furthermore, 1728 occurs in the algebraic formula for the j -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

$$(5.702108961049514444898 \times 10^{-17} * 1 / 4.206331341475331625217454 \times 10^{-14} * 1 / 0.00793086)^{1/128}$$

Input interpretation:

$$\left(5.702108961049514444898 \times 10^{-17} \times \frac{1}{4.206331341475331625217454 \times 10^{-14}} \times \frac{1}{0.00793086} \right)^{1/128}$$

Result:

0.98629389...

0.98629389.... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

$$\frac{1 + \sqrt[5]{\sqrt{\phi^5 4\sqrt{5^3} - 1}}}{\sqrt{5}} - \phi + 1$$

and to the dilaton value **0.989117352243 = φ**

$$24 * 1 / (5.7021089610495144444898 \times 10^{-17} * 1 / 4.206331341475331625217454 \times 10^{-14} * 1 / 0.00793086) - (1 / \text{golden ratio})$$

Input interpretation:

$$24 \times \frac{1}{5.7021089610495144444898 \times 10^{-17} \times \frac{1}{4.206331341475331625217454 \times 10^{-14}} \times \frac{1}{0.00793086}} - \frac{1}{\phi}$$

φ is the golden ratio

Result:

139.7924360772430828108811259289202010716867955221403936046...

139.792436.... result very near to the rest mass of the Pion meson 139.57

$$24 * 1 / (5.7021089610495144444898 \times 10^{-17} * 1 / 4.206331341475331625217454 \times 10^{-14} * 1 / 0.00793086) - (1 / \text{golden ratio}) - 5$$

Input interpretation:

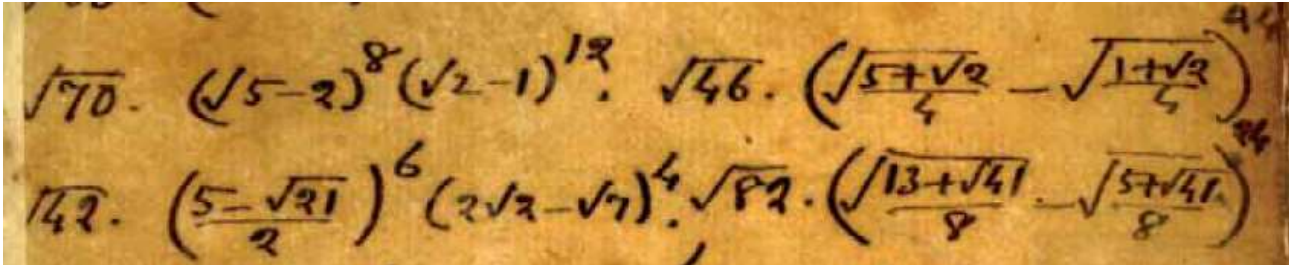
$$24 \times \frac{1}{5.7021089610495144444898 \times 10^{-17} \times \frac{1}{4.206331341475331625217454 \times 10^{-14}} \times \frac{1}{0.00793086}} - \frac{1}{\phi} - 5$$

φ is the golden ratio

Result:

134.7924360772430828108811259289202010716867955221403936046...

134.792436... result very near to the rest mass of the Pion meson 134.9766



$$\sqrt{70} \left((\sqrt{5}-2)^8 \right) (\sqrt{2}-1)^{12} * \sqrt{46} \left(\left(\left(\left(\left(\sqrt{\frac{5+\sqrt{2}}{4}} \right) - \sqrt{\frac{1+\sqrt{2}}{4}} \right) \right) \right) \right) - \left(\left(\left(\left(\sqrt{\frac{1+\sqrt{2}}{4}} \right) \right) \right) \right) \right)^{24}$$

Input:

$$\sqrt{70} (\sqrt{5} - 2)^8 (\sqrt{2} - 1)^{12} \sqrt{46} \left(\sqrt{\frac{1}{4}(5 + \sqrt{2})} - \sqrt{\frac{1}{4}(1 + \sqrt{2})} \right)^{24}$$

Exact result:

$$2\sqrt{805} (\sqrt{2} - 1)^{12} (\sqrt{5} - 2)^8 \left(\frac{\sqrt{5 + \sqrt{2}}}{2} - \frac{1}{2} \sqrt{1 + \sqrt{2}} \right)^{24}$$

Decimal approximation:

4.9824812456009075790315405129245724670950311590232375... × 10⁻¹⁶

4.9824812456... * 10⁻¹⁶

Alternate forms:

$$\sqrt{\text{root of } x^8 + 106\,356\,847\,048\,126\,276\,950\,880x^7 + 2\,827\,934\,228\,787\,970\,847\,137\,211\,739\,659\,343\,955\,353\,918\,400x^6 - 50\,043\,261\,381\,920\,343\,079\,823\,090\,739\,476\,807\,551\,519\,667\,838\,662\,326 \cdot 256\,847\,296\,000x^5 - 46\,057\,206\,754\,622\,431\,631\,070\,519\,228\,569\,947\,303\,761\,500\,086\,868\,056 \cdot 623\,656\,498\,240\,000x^4 - 10\,597\,123\,166\,687\,536\,241\,219\,503\,815\,551\,247\,417\,085\,484\,241\,223\,920 \cdot 871\,900\,603\,366\,400\,000x^3 + 1\,622\,747\,015\,272\,977\,243\,631\,067\,873\,374\,314\,041\,427\,496\,220\,995\,076\,410 \cdot 522\,879\,744\,000\,000x^2 - 46\,553\,852\,714\,046\,249\,068\,603\,854\,999\,208\,620\,425\,208\,502\,341\,806\,080 \cdot 000\,000x + 11\,557\,049\,504\,227\,688\,473\,600\,000\,000}$$

near $x = 2.48251 \times 10^{-31}$

$$\sqrt{42} \left(\frac{(5-\sqrt{21})}{2} \right)^6 (2\sqrt{2}-\sqrt{7})^4 \sqrt{82} \left(\sqrt{\frac{1}{8}(13+\sqrt{41})} - \sqrt{\frac{1}{8}(5+\sqrt{41})} \right)^{24}$$

Input:

$$\sqrt{42} \left(\frac{1}{2} (5 - \sqrt{21}) \right)^6 (2\sqrt{2} - \sqrt{7})^4 \sqrt{82} \left(\sqrt{\frac{1}{8} (13 + \sqrt{41})} - \sqrt{\frac{1}{8} (5 + \sqrt{41})} \right)^{24}$$

Result:

$$\frac{1}{32} \sqrt{861} (2\sqrt{2} - \sqrt{7})^4 (5 - \sqrt{21})^6 \left(\frac{1}{2} \sqrt{\frac{1}{2} (13 + \sqrt{41})} - \frac{1}{2} \sqrt{\frac{1}{2} (5 + \sqrt{41})} \right)^{24}$$

Decimal approximation:

$$1.5267531076537129490789921102790012152788703609432591... \times 10^{-16}$$

$$1.5267531076... * 10^{-16}$$

Alternate forms:

$$\frac{1}{281474976710656} (-12446280\sqrt{2} - 10162320\sqrt{3} + 6652800\sqrt{7} + 2716001\sqrt{42}) \sqrt{82} \left(\sqrt{2(13+\sqrt{41})} - \sqrt{2(5+\sqrt{41})} \right)^{24}$$

$$\frac{\sqrt{861} (449 - 120\sqrt{14}) (\sqrt{21} - 5)^6 \left(\sqrt{2(5+\sqrt{41})} - \sqrt{2(13+\sqrt{41})} \right)^{24}}{9007199254740992}$$

$$\frac{\sqrt{861} (2\sqrt{2} - \sqrt{7})^4 (\sqrt{21} - 5)^6 \left(\sqrt{5+\sqrt{41}} - \sqrt{13+\sqrt{41}} \right)^{24}}{2199023255552}$$

$$(4.982481245600907579 \times 10^{-16} / 1.526753107653712949 \times 10^{-16})/2$$

Input interpretation:

$$\frac{1}{2} \times \frac{4.982481245600907579 \times 10^{-16}}{1.526753107653712949 \times 10^{-16}}$$

Result:

$$1.631724612389326076661517292922069983139334658735759338683...$$

1.631724612... result that is a golden number, belonging to the range of the golden ratio

$$\sqrt{14} \left(\left(\left(\left(\left(\sqrt{\frac{3+\sqrt{2}}{4}} - \sqrt{\frac{\sqrt{2}-1}{4}} \right) \right) \right) \right) \right) \right)^{24}$$

Input:

$$\sqrt{14} \left(\sqrt{\frac{1}{4}(3+\sqrt{2})} - \sqrt{\frac{1}{4}(\sqrt{2}-1)} \right)^{24}$$

Result:

$$\sqrt{14} \left(\frac{\sqrt{3+\sqrt{2}}}{2} - \frac{1}{2} \sqrt{\sqrt{2}-1} \right)^{24}$$

Decimal approximation:

0.001880602329157560948116868181688668604992011199064131109...
0.0018806023...

Alternate forms:

$$\frac{\sqrt{14} \left(\sqrt{3+\sqrt{2}} - \sqrt{\sqrt{2}-1} \right)^{24}}{16777216}$$

$$\sqrt{14 \left(989633 + 699776\sqrt{2} - 8 \sqrt{14 \left(2186100865 + 1545806746\sqrt{2} \right)} \right)}$$

$$\frac{\sqrt{\frac{7}{2}} \left(\sqrt{\sqrt{2}-1} - \sqrt{3+\sqrt{2}} \right)^{24}}{8388608}$$

Minimal polynomial:

$$x^8 - 55419448x^6 + 450280600x^4 - 10862211808x^2 + 38416$$

$$\sqrt{34} * \left(\left(\left(\left(\left(\sqrt{\frac{7+\sqrt{17}}{8}} - \sqrt{\frac{\sqrt{17}-1}{8}} \right) \right) \right) \right) \right) \right)^{24}$$

Input:

$$\sqrt{34} \left(\sqrt{\frac{1}{8}(7+\sqrt{17})} - \sqrt{\frac{1}{8}(\sqrt{17}-1)} \right)^{24}$$

Result:

$$\sqrt{34} \left(\frac{1}{2} \sqrt{\frac{1}{2}(7+\sqrt{17})} - \frac{1}{2} \sqrt{\frac{1}{2}(\sqrt{17}-1)} \right)^{24}$$

Decimal approximation:

$$4.1333926439269591981213714369492937167052179586385156... \times 10^{-6}$$

$$4.133392643926... * 10^{-6}$$

Alternate forms:

$$\frac{\sqrt{34} \left(\sqrt{2(7+\sqrt{17})} - \sqrt{2(\sqrt{17}-1)} \right)^{24}}{281474976710656}$$

$$\sqrt{\left(34 \left(497514337921 + 120664950912 \sqrt{17} - 144 \sqrt{23873506600747119133 + 5790175845221223489 \sqrt{17}} \right) \right)}$$

$$\sqrt{34} \left[\text{root of } x^8 - 3x^6 - 3x^2 + 1 \text{ near } x = 0.554337 \right]^{24}$$

Minimal polynomial:

$$x^8 - 67661949957256 x^6 + 3928989044632344 x^4 - 78217214150587936 x^2 + 1336336$$

From the six results, we obtain:

$$\frac{1}{0.0018806023} * \frac{1}{4.206331341475331625217454 \times 10^{-14}} * \frac{1}{0.00793086} * \frac{1}{5.702108961049514444898 \times 10^{-17}} * \frac{1}{4.982481245600907579 \times 10^{-16}} * \frac{1}{1.526753107653712949 \times 10^{-16}}$$

Input interpretation:

$$\frac{1}{0.0018806023} \times \frac{1}{4.206331341475331625217454 \times 10^{-14}} \times \frac{1}{0.00793086} \times \frac{1}{5.702108961049514444898 \times 10^{-17}} \times \frac{1}{4.982481245600907579 \times 10^{-16}} \times \frac{1}{1.526753107653712949 \times 10^{-16}}$$

Result:

3.6747625037020751609144248856119887699144170173889139... × 10⁶⁵
3.6747625037... * 10⁶⁵

$$-16 + \text{colog}\left(\left(\frac{1}{0.0018806023} \times \frac{1}{4.206331341475331625217454 \times 10^{-14}} \times \frac{1}{0.00793086} \times \frac{1}{5.702108961049514444898 \times 10^{-17}} \times \frac{1}{4.982481245600907579 \times 10^{-16}} \times \frac{1}{1.526753107653712949 \times 10^{-16}}\right)\right)$$

Input interpretation:

$$-16 - \log\left(1 / \left(\frac{1}{0.0018806023} \times \frac{1}{4.206331341475331625217454 \times 10^{-14}} \times \frac{1}{0.00793086} \times \frac{1}{5.702108961049514444898 \times 10^{-17}} \times \frac{1}{4.982481245600907579 \times 10^{-16}} \times \frac{1}{1.526753107653712949 \times 10^{-16}}\right)\right)$$

log(x) is the natural logarithm

Result:

134.96952...
134.96952.... result very near to the rest mass of Pion meson 134.9766

$$-24 + 13\left(\left(-16 + \text{colog}\left(\left(\frac{1}{0.0018806023} \times \frac{1}{4.206331341475 \times 10^{-14}} \times \frac{1}{0.00793086} \times \frac{1}{5.70210896104 \times 10^{-17}} \times \frac{1}{4.9824812456 \times 10^{-16}} \times \frac{1}{1.52675310765 \times 10^{-16}}\right)\right)\right)\right)$$

Input interpretation:

$$-24 + 13\left(-16 - \log\left(1 / \left(\frac{1}{0.0018806023} \times \frac{1}{4.206331341475 \times 10^{-14}} \times \frac{1}{0.00793086} \times \frac{1}{5.70210896104 \times 10^{-17}} \times \frac{1}{4.9824812456 \times 10^{-16}} \times \frac{1}{1.52675310765 \times 10^{-16}}\right)\right)\right)$$

log(x) is the natural logarithm

Result:

1730.6038...
1730.6038...

This result is very near to the mass of candidate glueball f₀(1710) meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–

Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

$$1/(1/0.0018806023 * 1/4.206331341475 \times 10^{-14} * 1 / 0.00793086 * 1/ 5.70210896104 \times 10^{-17} * 1/ 4.9824812456 \times 10^{-16} * 1 / 1.52675310765 \times 10^{-16})^{1/4096}$$

Input interpretation:

$$1/\left(\left(\frac{1}{0.0018806023} \times \frac{1}{4.206331341475 \times 10^{-14}} \times \frac{0.00793086}{5.70210896104 \times 10^{-17}} \times \frac{4.9824812456 \times 10^{-16}}{1.52675310765 \times 10^{-16}}\right)^{(1/4096)}\right)$$

Result:

0.9638131867...

0.9638131867.... result very near to the spectral index n_s and to the mesonic Regge slope (see Appendix) and to the inflaton value at the end of the inflation 0.9402 and to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}} - \varphi + 1} = 1 - \frac{e^{-\pi}}{1 + \frac{e^{-2\pi}}{1 + \frac{e^{-3\pi}}{1 + \frac{e^{-4\pi}}{1 + \dots}}}}} \approx 0.9568666373$$

$$1/(1/0.0018806023 * 1/4.206331341475 \times 10^{-14} * 1 / 0.00793086 * 1/ 5.70210896104 \times 10^{-17} * 1/ 4.9824812456 \times 10^{-16} * 1 / 1.52675310765 \times 10^{-16})^{1/2}$$

Input interpretation:

$$1 / \left(\sqrt{ \left(\frac{1}{0.0018806023} \times \frac{1}{4.206331341475 \times 10^{-14}} \times \frac{1}{0.00793086} \times \frac{1}{5.70210896104 \times 10^{-17}} \times \frac{1}{4.9824812456 \times 10^{-16}} \times \frac{1}{1.52675310765 \times 10^{-16}} \right) } \right)$$

Result:

1.64963... × 10⁻³³
 1.64963... * 10⁻³³

We note that:

Input interpretation:

1.64963 × 10⁻³³ cm (centimeters)

Unit conversions:

1.6496 × 10⁻³⁵ meters
 6.4946 × 10⁻³⁴ inches

Comparisons as length:

≈ (0.2 ≈ 1/6) × length of a putative string in M-theory (≈ 1 × 10⁻³⁴ m)
 ≈ Planck length (≈ 1.6 × 10⁻³⁵ m)

Interpretations:

length

Corresponding quantities:

Wavelength λ from λ = 2πλ̄:

1 × 10⁻³⁴ meters

Spectroscopic wavenumber ν̄ from ν̄ = 2π/λ̄:

3.8088 × 10³⁵ m⁻¹ (reciprocal meters)

Wavenumber k from k = 1/λ̄:

6.062 × 10³⁴ m⁻¹ (reciprocal meters)

Angular wavelength λ̄ from λ̄ = λ/(2π):

2.6 × 10⁻³⁶ meters

Spectroscopic wavenumber ν̄ from ν̄ = 1/λ:

6.062 × 10³⁴ m⁻¹ (reciprocal meters)

Wavenumber k from k = 2π/λ:

3.8088 × 10³⁵ m⁻¹ (reciprocal meters)

Now, we have that:

$$\frac{1}{(0.0018806023 * 4.2063313414753316 * 10^{-14} * 0.00793086 * 5.7021089610495 * 10^{-17} * 4.98248124560 * 10^{-16} * 4.133392643926 * 10^{-6} * 1.5267531076537 * 10^{-16})}$$

Input interpretation:

$$\frac{1}{(0.0018806023 \times 4.2063313414753316 \times 0.00793086 \times 5.7021089610495 \times 4.98248124560 \times 4.133392643926 \times 1.5267531076537) / (10^{14} \times 10^{17} \times 10^{16} \times 10^6 \times 10^{16})}$$

Result:

$$8.8904268727115250336413927240225004866967797646399225... \times 10^{70}$$

$$8.8904268... * 10^{70}$$

$$-24 + \ln\left(\left(\frac{1}{(0.0018806023 * 4.2063313414753316 * 10^{-14} * 0.00793086 * 5.7021089610495 * 10^{-17} * 4.98248124560 * 10^{-16} * 4.133392643926 * 10^{-6} * 1.5267531076537 * 10^{-16})}\right)\right)$$

Input interpretation:

$$-24 + \log\left(\frac{1}{(0.0018806023 \times 4.2063313414753316 \times 0.00793086 \times 5.7021089610495 \times 4.98248124560 \times 4.133392643926 \times 1.5267531076537) / (10^{14} \times 10^{17} \times 10^{16} \times 10^6 \times 10^{16})}\right)$$

log(x) is the natural logarithm

Result:

$$139.36593...$$

$$139.36593... \text{ result very near to the rest mass of Pion meson } 139.57$$

Alternative representations:

$$-24 + \log\left(\frac{1}{(0.0018806 \times 4.20633134147533160000 \times 0.00793086 \times 5.70210896104950000 \times 4.982481245600000 \times 4.1333926439260000 \times 1.52675310765370000) / (10^{14} \times 10^{17} \times 10^{16} \times 10^6 \times 10^{16})}\right) = -24 + \log_e \left(\frac{1}{\frac{0.0112481 \left(\frac{1}{10^{16}}\right)^2}{10^{17} \times 10^{14} \times 10^6}} \right)$$

$$-24 + \log\left(1 / (0.0018806 \times 4.20633134147533160000 \times 0.00793086 \times 5.70210896104950000 \times 4.982481245600000 \times 4.1333926439260000 \times 1.52675310765370000) / (10^{14} \times 10^{17} \times 10^{16} \times 10^6 \times 10^{16})\right) =$$

$$-24 + \log(a) \log_a \left(\frac{1}{\frac{0.0112481 \left(\frac{1}{10^{16}}\right)^2}{10^{17} \times 10^{14} \times 10^6}} \right)$$

$$-24 + \log\left(1 / (0.0018806 \times 4.20633134147533160000 \times 0.00793086 \times 5.70210896104950000 \times 4.982481245600000 \times 4.1333926439260000 \times 1.52675310765370000) / (10^{14} \times 10^{17} \times 10^{16} \times 10^6 \times 10^{16})\right) = -24 - \text{Li}_1 \left(1 - \frac{1}{\frac{0.0112481 \left(\frac{1}{10^{16}}\right)^2}{10^{17} \times 10^{14} \times 10^6}} \right)$$

$$\left(10^{14} \times 10^{17} \times 10^{16} \times 10^6 \times 10^{16} \right) = -24 - \text{Li}_1 \left(1 - \frac{1}{\frac{0.0112481 \left(\frac{1}{10^{16}}\right)^2}{10^{17} \times 10^{14} \times 10^6}} \right)$$

Series representations:

$$-24 + \log\left(1 / (0.0018806 \times 4.20633134147533160000 \times 0.00793086 \times 5.70210896104950000 \times 4.982481245600000 \times 4.1333926439260000 \times 1.52675310765370000) / (10^{14} \times 10^{17} \times 10^{16} \times 10^6 \times 10^{16})\right) =$$

$$-24 + \log(8.89043 \times 10^{70}) - \sum_{k=1}^{\infty} \frac{(-1)^k e^{-163.366k}}{k}$$

$$-24 + \log\left(1 / (0.0018806 \times 4.20633134147533160000 \times 0.00793086 \times 5.70210896104950000 \times 4.982481245600000 \times 4.1333926439260000 \times 1.52675310765370000) / (10^{14} \times 10^{17} \times 10^{16} \times 10^6 \times 10^{16})\right) =$$

$$-24 + 2i\pi \left[\frac{\arg(8.89043 \times 10^{70} - x)}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (8.89043 \times 10^{70} - x)^k x^{-k}}{k} \quad \text{for } x < 0$$

$$\begin{aligned}
& -24 + \log\left(1 / (0.0018806 \times 4.20633134147533160000 \times \right. \\
& \quad \left. 0.00793086 \times 5.70210896104950000 \times 4.982481245600000 \times \right. \\
& \quad \left. 4.1333926439260000 \times 1.52675310765370000) / \right. \\
& \quad \left. (10^{14} \times 10^{17} \times 10^{16} \times 10^6 \times 10^{16})\right) = \\
& -24 + \left[\frac{\arg(8.89043 \times 10^{70} - z_0)}{2\pi} \right] \log\left(\frac{1}{z_0}\right) + \\
& \quad \log(z_0) + \\
& \quad \left[\frac{\arg(8.89043 \times 10^{70} - z_0)}{2\pi} \right] \log(z_0) - \\
& \quad \sum_{k=1}^{\infty} \frac{(-1)^k (8.89043 \times 10^{70} - z_0)^k z_0^{-k}}{k}
\end{aligned}$$

Integral representations:

$$\begin{aligned}
& -24 + \log\left(1 / (0.0018806 \times 4.20633134147533160000 \times \right. \\
& \quad \left. 0.00793086 \times 5.70210896104950000 \times 4.982481245600000 \times \right. \\
& \quad \left. 4.1333926439260000 \times 1.52675310765370000) / \right. \\
& \quad \left. (10^{14} \times 10^{17} \times 10^{16} \times 10^6 \times 10^{16})\right) = -24 + \int_1^{8.89043 \times 10^{70}} \frac{1}{t} dt
\end{aligned}$$

$$\begin{aligned}
& -24 + \log\left(1 / (0.0018806 \times 4.20633134147533160000 \times 0.00793086 \times \right. \\
& \quad \left. 5.70210896104950000 \times 4.982481245600000 \times \right. \\
& \quad \left. 4.1333926439260000 \times 1.52675310765370000) / \right. \\
& \quad \left. (10^{14} \times 10^{17} \times 10^{16} \times 10^6 \times 10^{16})\right) = \\
& -24 + \frac{1}{2i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-163.366s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds
\end{aligned}$$

for
 $-1 < \gamma < 0$

$$-4-64+11\ln\left(\left(\frac{1}{(0.0018806023 * 4.2063313414753316 * 10^{-14} * 0.00793086 * 5.7021089610495 * 10^{-17} * 4.98248124560 * 10^{-16} * 4.133392643926 * 10^{-6} * 1.5267531076537 * 10^{-16})}\right)\right)$$

Input interpretation:

$$\begin{aligned}
& -4 - 64 + 11 \log\left(\right. \\
& \quad \left. 1 / (0.0018806023 \times 4.2063313414753316 \times 0.00793086 \times 5.7021089610495 \times \right. \\
& \quad \left. 4.98248124560 \times 4.133392643926 \times 1.5267531076537) / \right. \\
& \quad \left. (10^{14} \times 10^{17} \times 10^{16} \times 10^6 \times 10^{16})\right)
\end{aligned}$$

$\log(x)$ is the natural logarithm

Result:

1729.0252...

1729.0252...

This result is very near to the mass of candidate glueball $f_0(1710)$ meson. Furthermore, 1728 occurs in the algebraic formula for the j -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

Alternative representations:

$$-4 - 64 + 11 \log\left(\frac{1}{(0.0018806 \times 4.20633134147533160000 \times 0.00793086 \times 5.70210896104950000 \times 4.982481245600000 \times 4.1333926439260000 \times 1.52675310765370000)}\right)$$

$$\left(10^{14} \times 10^{17} \times 10^{16} \times 10^6 \times 10^{16}\right) = -68 + 11 \log_e \left(\frac{1}{\frac{0.0112481 \left(\frac{1}{10^{16}}\right)^2}{10^{17} \times 10^{14} \times 10^6}} \right)$$

$$-4 - 64 + 11 \log\left(\frac{1}{(0.0018806 \times 4.20633134147533160000 \times 0.00793086 \times 5.70210896104950000 \times 4.982481245600000 \times 4.1333926439260000 \times 1.52675310765370000)}\right)$$

$$\left(10^{14} \times 10^{17} \times 10^{16} \times 10^6 \times 10^{16}\right) =$$

$$-68 + 11 \log(a) \log_a \left(\frac{1}{\frac{0.0112481 \left(\frac{1}{10^{16}}\right)^2}{10^{17} \times 10^{14} \times 10^6}} \right)$$

$$-4 - 64 + 11 \log\left(\frac{1}{(0.0018806 \times 4.20633134147533160000 \times 0.00793086 \times 5.70210896104950000 \times 4.982481245600000 \times 4.1333926439260000 \times 1.52675310765370000)}\right)$$

$$\left(10^{14} \times 10^{17} \times 10^{16} \times 10^6 \times 10^{16}\right) =$$

$$-68 - 11 \operatorname{Li}_1 \left(1 - \frac{1}{\frac{0.0112481 \left(\frac{1}{10^{16}}\right)^2}{10^{17} \times 10^{14} \times 10^6}} \right)$$

Series representations:

$$\begin{aligned}
& -4 - 64 + 11 \log\left(1 / (0.0018806 \times 4.20633134147533160000 \times \right. \\
& \quad \left. 0.00793086 \times 5.70210896104950000 \times 4.982481245600000 \times \right. \\
& \quad \left. 4.1333926439260000 \times 1.52675310765370000) / \right. \\
& \quad \left. (10^{14} \times 10^{17} \times 10^{16} \times 10^6 \times 10^{16})\right) = \\
& -68 + 11 \log(8.89043 \times 10^{70}) - 11 \sum_{k=1}^{\infty} \frac{(-1)^k e^{-163.366k}}{k}
\end{aligned}$$

$$\begin{aligned}
& -4 - 64 + 11 \log\left(1 / (0.0018806 \times 4.20633134147533160000 \times 0.00793086 \times \right. \\
& \quad \left. 5.70210896104950000 \times 4.982481245600000 \times \right. \\
& \quad \left. 4.1333926439260000 \times 1.52675310765370000) / \right. \\
& \quad \left. (10^{14} \times 10^{17} \times 10^{16} \times 10^6 \times 10^{16})\right) = \\
& -68 + 22 i \pi \left[\frac{\arg(8.89043 \times 10^{70} - x)}{2 \pi} \right] + \\
& 11 \log(x) - \\
& 11 \sum_{k=1}^{\infty} \frac{(-1)^k (8.89043 \times 10^{70} - x)^k x^{-k}}{k} \text{ for } x < \\
& 0
\end{aligned}$$

$$\begin{aligned}
& -4 - 64 + 11 \log\left(1 / (0.0018806 \times 4.20633134147533160000 \times \right. \\
& \quad \left. 0.00793086 \times 5.70210896104950000 \times 4.982481245600000 \times \right. \\
& \quad \left. 4.1333926439260000 \times 1.52675310765370000) / \right. \\
& \quad \left. (10^{14} \times 10^{17} \times 10^{16} \times 10^6 \times 10^{16})\right) = \\
& -68 + 11 \left[\frac{\arg(8.89043 \times 10^{70} - z_0)}{2 \pi} \right] \log\left(\frac{1}{z_0}\right) + \\
& 11 \\
& \log(z_0) + \\
& 11 \left[\frac{\arg(8.89043 \times 10^{70} - z_0)}{2 \pi} \right] \log(z_0) - \\
& 11 \sum_{k=1}^{\infty} \frac{(-1)^k (8.89043 \times 10^{70} - z_0)^k z_0^{-k}}{k}
\end{aligned}$$

Integral representations:

$$\begin{aligned}
& -4 - 64 + 11 \log\left(1 / (0.0018806 \times 4.20633134147533160000 \times \right. \\
& \quad \left. 0.00793086 \times 5.70210896104950000 \times 4.982481245600000 \times \right. \\
& \quad \left. 4.1333926439260000 \times 1.52675310765370000) / \right. \\
& \quad \left. (10^{14} \times 10^{17} \times 10^{16} \times 10^6 \times 10^{16})\right) = -68 + 11 \int_1^{8.89043 \times 10^{70}} \frac{1}{t} dt
\end{aligned}$$

$$\begin{aligned}
& -4 - 64 + 11 \log\left(1 / (0.0018806 \times 4.20633134147533160000 \times 0.00793086 \times \right. \\
& \quad \left. 5.70210896104950000 \times 4.982481245600000 \times \right. \\
& \quad \left. 4.1333926439260000 \times 1.52675310765370000) / \right. \\
& \quad \left. (10^{14} \times 10^{17} \times 10^{16} \times 10^6 \times 10^{16})\right) = \\
& -68 + \frac{11}{2i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-163.366s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \\
& \text{for} \\
& -1 < \\
& \quad \gamma < \\
& \quad 0
\end{aligned}$$

$$\begin{aligned}
& (((0.0018806023 * 4.2063313414753316 * 10^{-14} * 0.00793086 * \\
& 5.7021089610495 * 10^{-17} * 4.98248124560 * 10^{-16} * 4.133392643926 * 10^{-6} * \\
& 1.5267531076537 * 10^{-16})))^{(1/(4096*3))}
\end{aligned}$$

Input interpretation:

$$\begin{aligned}
& ((0.0018806023 \times 4.2063313414753316 \times 0.00793086 \times 5.7021089610495 \times \\
& \quad 4.98248124560 \times 4.133392643926 \times 1.5267531076537) / \\
& \quad (10^{14} \times 10^{17} \times 10^{16} \times 10^6 \times 10^{16}))^{(1 / (4096 \times 3))}
\end{aligned}$$

Result:

0.9867932313...

0.9867932313.... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}}}{1 + \sqrt[5]{\sqrt{\varphi^5 \sqrt[4]{5^3}} - 1}} - \varphi + 1 = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

and to the dilaton value **0.989117352243 = ϕ**

$$\left((0.0018806023 * 4.2063313414753316 * 10^{-14} * 0.00793086 * 5.7021089610495 * 10^{-17} * 4.98248124560 * 10^{-16} * 4.133392643926 * 10^{-6} * 1.5267531076537 * 10^{-16}) \right)^{(1/4096)}$$

Input interpretation:

$$\left((0.0018806023 \times 4.2063313414753316 \times 0.00793086 \times 5.7021089610495 \times 4.98248124560 \times 4.133392643926 \times 1.5267531076537) / (10^{14} \times 10^{17} \times 10^{16} \times 10^6 \times 10^{16}) \right)^{(1/4096)}$$

Result:

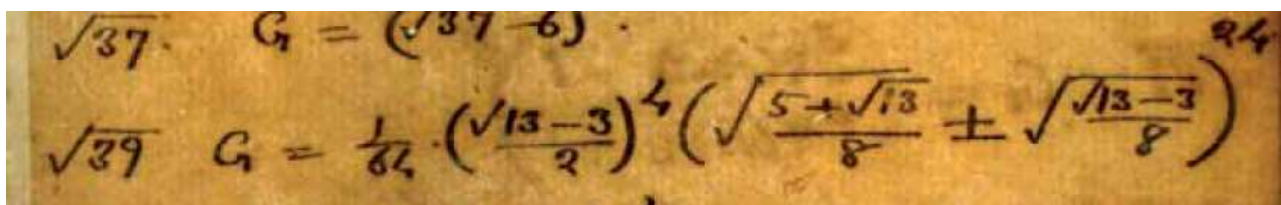
0.9609006467...

0.9609006467.... result very near to the spectral index n_s and to the mesonic Regge slope (see Appendix) and to the inflaton value at the end of the inflation 0.9402 and to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}} - \varphi + 1} = 1 - \frac{e^{-\pi}}{1 + \frac{e^{-2\pi}}{1 + \frac{e^{-3\pi}}{1 + \frac{e^{-4\pi}}{1 + \dots}}}} \approx 0.9568666373$$

Now, we have that:

Page 298-301



$$1/64 * (((((sqrt(13)-3)/2)))^4 (((((((((sqrt(((5+sqrt(13))/8)))))))))) + (((sqrt(((sqrt(13)-3)/8))))))^24$$

Input:

$$\frac{1}{64} \left(\frac{1}{2} (\sqrt{13} - 3) \right)^4 \left(\sqrt{\frac{1}{8} (5 + \sqrt{13})} + \sqrt{\frac{1}{8} (\sqrt{13} - 3)} \right)^{24}$$

Exact result:

$$\frac{(\sqrt{13} - 3)^4 \left(\frac{1}{2} \sqrt{\frac{1}{2}(\sqrt{13} - 3)} + \frac{1}{2} \sqrt{\frac{1}{2}(5 + \sqrt{13})} \right)^{24}}{1024}$$

Decimal approximation:

0.089321319219383532717820532693319698548662535994269836508...

0.08932131921...

Alternate forms:

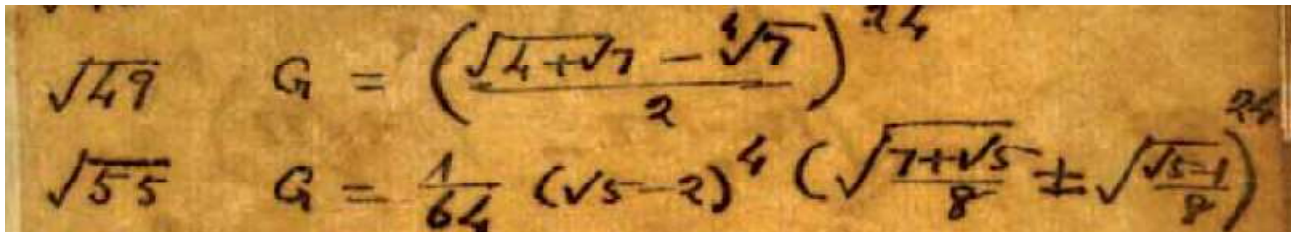
$$\frac{1}{128} \left(-70 + 21 \sqrt{13} + 3 \sqrt{546 \sqrt{13} - 1965} \right)$$

$$\frac{1}{128} \left(-70 + 21 \sqrt{13} + 64 \sqrt{\frac{2457 \sqrt{13}}{2048} - \frac{17685}{4096}} \right)$$

$$\frac{(\sqrt{13} - 3)^4 \left(\sqrt{\sqrt{13} - 3} + \sqrt{5 + \sqrt{13}} \right)^{24}}{70368744177664}$$

Minimal polynomial:

$$16777216x^4 + 36700160x^3 + 54583296x^2 - 5180224x + 1$$



$$\left(\left(\frac{1}{2} \left(\sqrt{4+\sqrt{7}} - \sqrt[4]{7} \right) \right) \right)^{24}$$

Input:

$$\left(\frac{1}{2} \left(\sqrt{4+\sqrt{7}} - \sqrt[4]{7} \right) \right)^{24}$$

Result:

$$\frac{\left(\sqrt{4+\sqrt{7}} - \sqrt[4]{7} \right)^{24}}{16777216}$$

Decimal approximation:

1.8011318006254695078679221289122711103977839590548607... $\times 10^{-8}$

1.801131800625... $\times 10^{-8}$

Alternate forms:

$$\frac{(\sqrt{2} - 2\sqrt[4]{7} + \sqrt{14})^{24}}{281474976710656}$$

$$13880161 + 5246208\sqrt{7} - 72\sqrt{74328271227 + 28093445864\sqrt{7}}$$

$$\frac{(\sqrt[4]{7} - \sqrt{4 + \sqrt{7}})^{24}}{16777216}$$

Minimal polynomial:

$$x^4 - 55520644x^3 - 77075706x^2 - 55520644x + 1$$

$$1/64 * (((\sqrt{5}-2)))^4 (((\sqrt{((7+\sqrt{5}))/8}))) + ((\sqrt{((\sqrt{5}-1))/8})))^24$$

Input:

$$\frac{1}{64} (\sqrt{5} - 2)^4 \left(\sqrt{\frac{1}{8}(7 + \sqrt{5})} + \sqrt{\frac{1}{8}(\sqrt{5} - 1)} \right)^{24}$$

Exact result:

$$\frac{1}{64} (\sqrt{5} - 2)^4 \left(\frac{1}{2} \sqrt{\frac{1}{2}(\sqrt{5} - 1)} + \frac{1}{2} \sqrt{\frac{1}{2}(7 + \sqrt{5})} \right)^{24}$$

Decimal approximation:

0.483328677491088752226784108880734992447114767859336959699...

0.483328677491...

Alternate forms:

$$\frac{1}{256} \left(-401 + 207\sqrt{5} + 9\sqrt{110(23\sqrt{5} - 51)} \right)$$

$$\frac{1}{256} \left(-401 + 207\sqrt{5} + 128\sqrt{\frac{102465\sqrt{5}}{8192} - \frac{227205}{8192}} \right)$$

$$\frac{(\sqrt{5} - 2)^4 \left(\sqrt{\sqrt{5} - 1} + \sqrt{7 + \sqrt{5}} \right)^{24}}{4398046511104}$$

Minimal polynomial:

$$16777216x^4 + 105119744x^3 + 369954816x^2 - 205260736x + 1$$

Handwritten expression: $G = \frac{1}{64} \cdot \left(\frac{5-\sqrt{21}}{2}\right)^4 \left(\sqrt{\frac{5+\sqrt{21}}{8}} - \sqrt{\frac{\sqrt{21}-3}{8}}\right)^{24}$

$$\frac{1}{64} \cdot \left(\frac{5-\sqrt{21}}{2}\right)^4 \left(\sqrt{\sqrt{\frac{5+\sqrt{21}}{8}}}\right) - \left(\sqrt{\sqrt{\frac{\sqrt{21}-3}{8}}}\right)^{24}$$

Input:

$$\frac{1}{64} \left(\frac{1}{2}(5-\sqrt{21})\right)^4 \left(\sqrt{\frac{1}{8}(5+\sqrt{21})} - \sqrt{\frac{1}{8}(\sqrt{21}-3)}\right)^{24}$$

Exact result:

$$\frac{(5-\sqrt{21})^4 \left(\frac{1}{2}\sqrt{\frac{1}{2}(5+\sqrt{21})} - \frac{1}{2}\sqrt{\frac{1}{2}(\sqrt{21}-3)}\right)^{24}}{1024}$$

Decimal approximation:

$$9.4793463810790645294767007300671522140861764864488681... \times 10^{-10}$$

$$9.479346381... \cdot 10^{-10}$$

Alternate forms:

$$\frac{1}{256} \left(829 - 155\sqrt{21} - 5\sqrt{42(217\sqrt{21} - 981)}\right)$$

$$\frac{1}{256} \left(829 - 155\sqrt{21} - 128\sqrt{\frac{113925\sqrt{21}}{8192} - \frac{515025}{8192}}\right)$$

$$\frac{(\sqrt{21}-5)^4 \left(\sqrt{\sqrt{21}-3} - \sqrt{5+\sqrt{21}}\right)^{24}}{70368744177664}$$

Minimal polynomial:

$$16777216x^4 - 217317376x^3 + 1324670976x^2 - 1054925056x + 1$$

Handwritten expression: $G = \left(\sqrt{\frac{9+\sqrt{73}}{8}} - \sqrt{\frac{1+\sqrt{73}}{8}}\right)^{24}$

$$\left(\sqrt{\sqrt{\sqrt{\frac{9+\sqrt{73}}{8}}}}\right) - \left(\sqrt{\sqrt{\frac{1+\sqrt{73}}{8}}}\right)^{24}$$

Input:

$$\left(\sqrt{\frac{1}{8}(9 + \sqrt{73})} - \sqrt{\frac{1}{8}(1 + \sqrt{73})} \right)^{24}$$

Result:

$$\left(\frac{1}{2} \sqrt{\frac{1}{2}(9 + \sqrt{73})} - \frac{1}{2} \sqrt{\frac{1}{2}(1 + \sqrt{73})} \right)^{24}$$

Decimal approximation:

$$1.4091048078505405721949267887471891427399742657427102... \times 10^{-10}$$

$$1.40910480785... * 10^{-10}$$

Alternate forms:

$$\frac{\left(\sqrt{2(9 + \sqrt{73})} - \sqrt{2(1 + \sqrt{73})} \right)^{24}}{281474976710656}$$

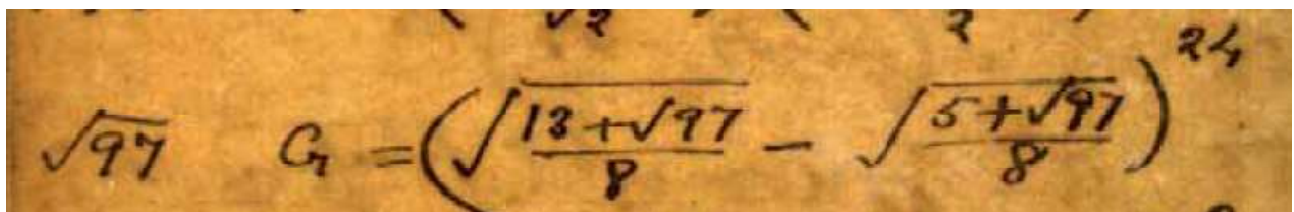
$$1774176049 + 207651600\sqrt{73} - 180$$

$$\sqrt{2(97151254669323 + 11370694298041\sqrt{73})}$$

$$\frac{\left(\sqrt{1 + \sqrt{73}} - \sqrt{9 + \sqrt{73}} \right)^{24}}{68719476736}$$

Minimal polynomial:

$$x^4 - 7096704196x^3 + 12473481606x^2 - 7096704196x + 1$$



$$\left(\left(\left(\left(\sqrt{\frac{13 + \sqrt{97}}{8}} \right) \right) - \left(\sqrt{\frac{5 + \sqrt{97}}{8}} \right) \right) \right)^{24}$$

Input:

$$\left(\sqrt{\frac{1}{8}(13 + \sqrt{97})} - \sqrt{\frac{1}{8}(5 + \sqrt{97})} \right)^{24}$$

Result:

$$\left(\frac{1}{2} \sqrt{\frac{1}{2} (13 + \sqrt{97})} - \frac{1}{2} \sqrt{\frac{1}{2} (5 + \sqrt{97})} \right)^{24}$$

Decimal approximation:

$$2.3368523701975162017491646602966621622386382697868389... \times 10^{-12}$$

$$2.33685237... \times 10^{-12}$$

Alternate forms:

$$\frac{\left(\sqrt{2(13 + \sqrt{97})} - \sqrt{2(5 + \sqrt{97})} \right)^{24}}{281474976710656}$$

$$106981512049 + 10862326800\sqrt{97} - 180$$

$$\sqrt{2(353242096308027183 + 35866300785011593\sqrt{97})}$$

$$\frac{\left(\sqrt{5 + \sqrt{97}} - \sqrt{13 + \sqrt{97}} \right)^{24}}{68719476736}$$

Minimal polynomial:

$$x^4 - 427926048196x^3 - 717988406394x^2 - 427926048196x + 1$$

From these six results, we obtain the following interesting expression:

$$(-8/10^3 * 1/10^{35}) + 1/(1 + \sqrt{5}) [\sqrt{39} * 0.08932131921 * \sqrt{49} * 1.801131800625e-8 * \sqrt{55} * 0.483328677491 * \sqrt{63} * 9.479346381e-10 * \sqrt{73} * 1.40910480785e-10 * \sqrt{97} * 2.33685237e-12]$$

Input interpretation:

$$-\frac{8}{10^3} \times \frac{1}{10^{35}} + \frac{1}{1 + \sqrt{5}} \left(\sqrt{39} \times 0.08932131921 \sqrt{49} \times 1.801131800625 \times 10^{-8} \sqrt{55} \times 0.483328677491 \sqrt{63} \times 9.479346381 \times 10^{-10} \sqrt{73} \times 1.40910480785 \times 10^{-10} \sqrt{97} \times 2.33685237 \times 10^{-12} \right)$$

Result:

$$1.61608134... \times 10^{-35}$$

1.61608134... * 10⁻³⁵ result practically equal to the value of the Planck length

$$1.616255 * 10^{-35}$$

Or:

$$6.67430 \times 10^{-11} \times 10^{16} \times [0.08932131921 \times 1.801131800625 \times 10^{-8} \times 0.483328677491 \times 9.479346381 \times 10^{-10} \times 1.40910480785 \times 10^{-10} \times 2.33685237 \times 10^{-12}]$$

Input interpretation:

$$6.67430 \times 10^{-11} \times 10^{16} (0.08932131921 \times 1.801131800625 \times 10^{-8} \times 0.483328677491 \times 9.479346381 \times 10^{-10} \times 1.40910480785 \times 10^{-10} \times 2.33685237 \times 10^{-12})$$

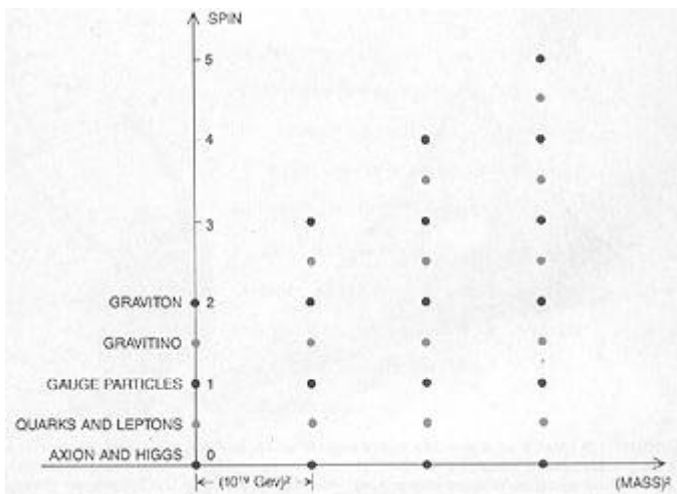
Result:

$$1.6199509951699523203430449485657397899083641659698295... \times 10^{-34}$$

$$1.619950995... \times 10^{-34}$$

From:

<http://www.damtp.cam.ac.uk/user/mbg15/superstrings/superstrings.html>



SPECTRUM OF STRING STATES is plotted for the heterotic string theory in which the extra six dimensions of spacetime have been curled up. Each black dot represents a set of bosons and each colored dot represents a set of fermions. All string states that correspond to known particles are massless states; the states with nonzero mass form an infinite series whose masses are a whole number times the square of the Planck mass, which is 10^{11} GeV. For each mass the number of fermion states is equal to the number of boson states. If each possible spin direction is counted as a different state, there are 8,064 massless states, and 18,883,584 states at the first mass level; the number increases exponentially thereafter.

We note that

$$76 + (4096 + 1.0061571663 + 0.50970737445 + 248)^2$$

Where 76 is a Lucas number, while 0.50970737445 and 1.0061571663 are Ramanujan mock theta functions

Input interpretation:

$$76 + (4096 + 1.0061571663 + 0.50970737445 + 248)^2$$

Result:

$$1.88835841289753419032084105625 \times 10^7$$

Decimal form:

$$18883584.1289753419032084105625$$

$$18883584.1289\dots \approx 18883584$$

From the previous result,

$$\left((0.0018806023 \times 4.2063313414753316 \times 0.00793086 \times 5.7021089610495 \times 4.98248124560 \times 4.133392643926 \times 1.5267531076537) / (10^{14} \times 10^{17} \times 10^{16} \times 10^6 \times 10^{16}) \right)^{1/4096}$$

$$= 0.9609006467\dots$$

we obtain the following interesting formula:

$$76 + ((\log_{0.9609006467}(((0.0018806023 * 4.2063313 * 10^{-14} * 0.00793086 * 5.7021089 * 10^{-17} * 4.9824812 * 10^{-16} * 4.1333926 * 10^{-6} * 1.5267531 * 10^{-16})))) + 1.0061571663 + 0.50970737445 + 248)^2$$

Input interpretation:

$$76 + \left(\log_{0.9609006467} \left(\frac{(0.0018806023 \times 4.2063313 \times 0.00793086 \times 5.7021089 \times 4.9824812 \times 4.1333926 \times 10^{-6} \times 1.5267531)}{(10^{14} \times 10^{17} \times 10^{16} \times 10^{16})} \right) + 1.0061571663 + 0.50970737445 + 248 \right)^2$$

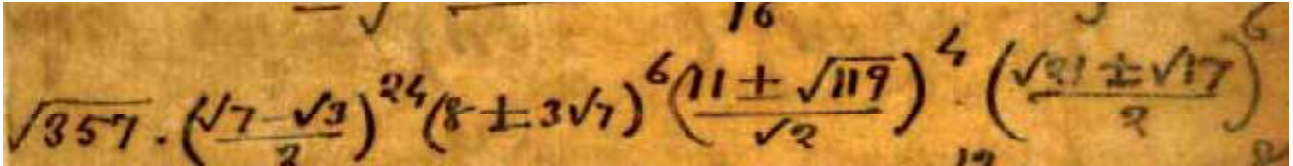
$\log_b(x)$ is the base- b logarithm

Result:

$$1.8883584\dots \times 10^7$$

$$18883584$$

These results equals to 18883584 corresponding to the number of the string states at the first mass level



$$\sqrt{357} \left(\frac{\sqrt{7}-\sqrt{3}}{2}\right)^{24} (8+3\sqrt{7})^6 \left(\frac{11+\sqrt{119}}{\sqrt{2}}\right)^4 \left(\frac{\sqrt{21}+\sqrt{17}}{2}\right)^6$$

Input:

$$\sqrt{357} \left(\frac{1}{2}(\sqrt{7}-\sqrt{3})\right)^{24} (8+3\sqrt{7})^6 \left(\frac{11+\sqrt{119}}{\sqrt{2}}\right)^4 \left(\frac{1}{2}(\sqrt{21}+\sqrt{17})\right)^6$$

Result:

$$\frac{\sqrt{357} (\sqrt{7}-\sqrt{3})^{24} (8+3\sqrt{7})^6 (\sqrt{17}+\sqrt{21})^6 (11+\sqrt{119})^4}{4294967296}$$

Decimal approximation:

$$8.28766007584362840898141524504466606118817701083371669... \times 10^8$$

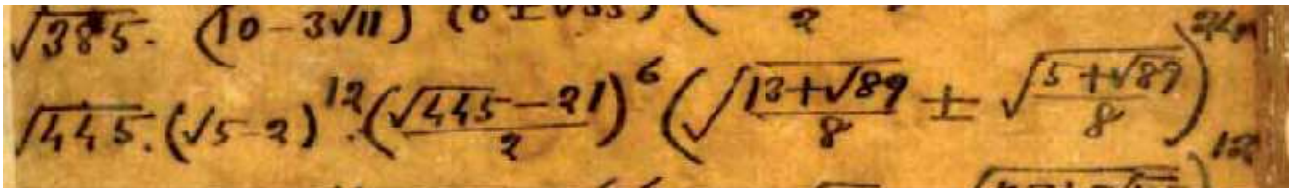
$$8.28766007584362... \times 10^8$$

Alternate forms:

$$\frac{\begin{aligned} &(73\,180\,801 - 15\,969\,360\sqrt{21})(8\,193\,151 + 3\,096\,720\sqrt{7}) \\ &(28\,799 + 2\,640\sqrt{119})(64\,260 + 3\,401\sqrt{357}) \\ &-47\,295\,902\,119\,915\,260 + 27\,306\,311\,483\,534\,160\sqrt{3} - \\ &17\,876\,176\,967\,880\,000\sqrt{7} - 11\,795\,424\,479\,047\,440\sqrt{17} + \\ &10\,320\,812\,022\,628\,800\sqrt{21} + 6\,810\,089\,212\,578\,960\sqrt{51} - \\ &4\,458\,249\,881\,870\,400\sqrt{119} + 2\,573\,972\,655\,221\,449\sqrt{357} \\ &\sqrt{357}(\sqrt{3}-\sqrt{7})^{24}(8+3\sqrt{7})^6(11+\sqrt{119})^4(19+\sqrt{357})^3 \end{aligned}}{536\,870\,912}$$

Minimal polynomial:

$$\begin{aligned} &x^8 + 378\,367\,216\,959\,322\,080x^7 - 2\,053\,476\,757\,124\,410\,105\,890\,999\,890\,295\,828x^6 + \\ &137\,473\,397\,591\,521\,948\,593\,825\,150\,186\,230\,516\,945\,188\,320x^5 - \\ &112\,522\,842\,219\,551\,674\,720\,780\,327\,332\,251\,385\,491\,997\,241\,482\,953\,706x^4 - \\ &49\,078\,002\,940\,173\,335\,647\,995\,578\,616\,484\,294\,549\,432\,230\,240x^3 - \\ &261\,713\,559\,218\,748\,943\,585\,702\,045\,018\,312\,982\,772x^2 - \\ &17\,215\,440\,866\,026\,764\,399\,289\,440x + 16\,243\,247\,601 \end{aligned}$$



$$\sqrt{445} \left((\sqrt{5}-2) \right)^{12} \left((\sqrt{445}-21) \right)^6 \left(\left(\left(\left(\left(\left(\left(\sqrt{\frac{13+\sqrt{89}}{8}} + \sqrt{\frac{5+\sqrt{89}}{8}} \right) \right) \right) \right) \right) \right) \right)^{24}$$

Input:

$$\sqrt{445} \left(\sqrt{5} - 2 \right)^{12} \left(\frac{1}{2} \left(\sqrt{445} - 21 \right) \right)^6 \left(\sqrt{\frac{1}{8} \left(13 + \sqrt{89} \right)} + \sqrt{\frac{1}{8} \left(5 + \sqrt{89} \right)} \right)^{24}$$

Result:

$$\frac{1}{64} \sqrt{445} \left(\sqrt{5} - 2 \right)^{12} \left(\sqrt{445} - 21 \right)^6 \left(\frac{1}{2} \sqrt{\frac{1}{2} \left(5 + \sqrt{89} \right)} + \frac{1}{2} \sqrt{\frac{1}{2} \left(13 + \sqrt{89} \right)} \right)^{24}$$

Decimal approximation:

0.002366091682043984276364364455153178399824146953710234692...
0.002366091682....

Alternate forms:

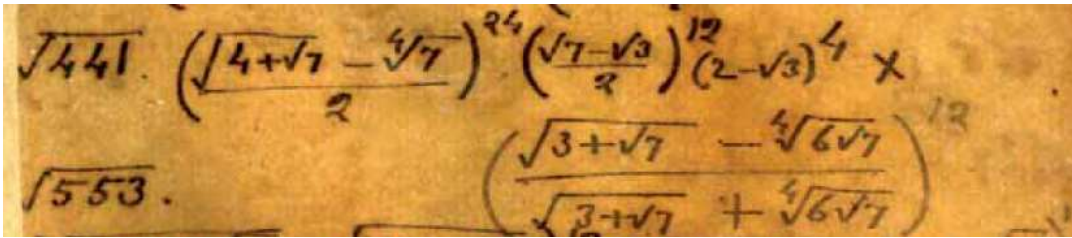
$$\frac{1}{281474976710656} \left(16692641 - 7465176\sqrt{5} \right) \left(43468489\sqrt{445} - 916968780 \right) \left(\sqrt{2(5+\sqrt{89})} + \sqrt{2(13+\sqrt{89})} \right)^{24}$$

$$\frac{\sqrt{445} \left(\sqrt{5} - 2 \right)^{12} \left(\sqrt{445} - 21 \right)^6 \left(\sqrt{5+\sqrt{89}} + \sqrt{13+\sqrt{89}} \right)^{24}}{4398046511104}$$

$$\frac{1}{64} \sqrt{445} \left(\sqrt{5} - 2 \right)^{12} \left(\sqrt{445} - 21 \right)^6 \left(\text{root of } x^8 - 9x^6 - 9x^2 + 1 \text{ near } x = 3.01781 \right)^{24}$$

Minimal polynomial:

$$x^8 + 19931993926651868607104761440x^7 + 3749178296437495879657758726727820x^6 + 92445655024451323900781466465333600x^5 + 11095468429541633056004222130641604150x^4 - 41138316485880839135847752577073452000x^3 + 742431032152035121569227671860276555500x^2 - 1756429728309730145010254570949420000x + 39213900625$$



$((\sqrt{441})) \left(\left(\left(\left(\left(\sqrt{((4)+\sqrt{7})} \right) - 7^{(1/4)} \right) / 2 \right) \right) \right)^{24} \left(\left(\left(\left(\sqrt{7} - \sqrt{3} \right) / 2 \right) \right) \right)^{12} (2 - \sqrt{3})^4$

Input:

$$\sqrt{441} \left(\frac{1}{2} \left(\sqrt{4+\sqrt{7}} - \sqrt[4]{7} \right) \right)^{24} \left(\frac{1}{2} (\sqrt{7} - \sqrt{3}) \right)^{12} (2 - \sqrt{3})^4$$

Result:

$$\frac{21(2 - \sqrt{3})^4 (\sqrt{7} - \sqrt{3})^{12} \left(\sqrt{4+\sqrt{7}} - \sqrt[4]{7} \right)^{24}}{68719476736}$$

Decimal approximation:

1.6116139633724039328007999712947585885627476577576094... $\times 10^{-13}$

1.6116139633... $\times 10^{-13}$ partial result

Alternate forms:

$$\frac{21 \left(\sqrt{2} - 2\sqrt[4]{7} + \sqrt{14} \right)^{24} (6049 - 1320\sqrt{21})(97 - 56\sqrt{3})}{281474976710656}$$

$$\frac{21(56\sqrt{3} - 97)(1320\sqrt{21} - 6049) \left(\sqrt[4]{7} - \sqrt{4+\sqrt{7}} \right)^{24}}{16777216}$$

$$\frac{21(\sqrt{3} - 2)^4 (\sqrt{3} - \sqrt{7})^{12} \left(\sqrt[4]{7} - \sqrt{4+\sqrt{7}} \right)^{24}}{68719476736}$$

Minimal polynomial:

$$x^8 - 2736387311245224x^7 - 3614336567069941188x^6 - 2220155221033548790488x^5 + 2332421523793822092744294x^4 - 979088452475795016605208x^3 - 702919789900329232183428x^2 - 234689325239122542256104x + 37822859361$$

1.6116139633724039328007999712947585885627476577576094 $\times 10^{-13}$ *

$\left(\left(\left(\left(\left(\sqrt{((3)+\sqrt{7})} \right) - ((6\sqrt{7})^{1/4}) \right) \right) \right) \right) / \left(\left(\left(\left(\sqrt{((3)+\sqrt{7})} \right) + ((6\sqrt{7})^{1/4}) \right) \right) \right) \right)^{12}$

Input interpretation:

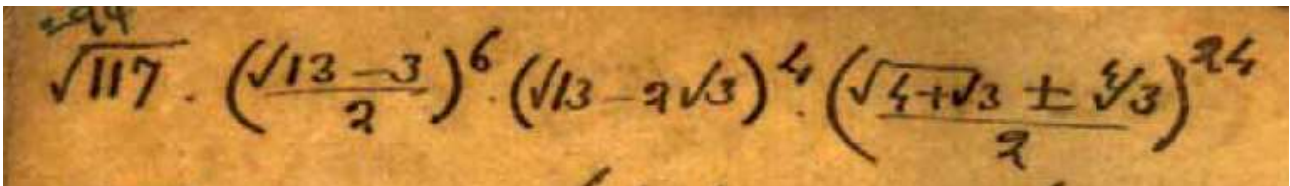
$$1.6116139633724039328007999712947585885627476577576094 \times 10^{-13}$$

$$\left(\frac{\sqrt{3+\sqrt{7}} - \sqrt[4]{6\sqrt{7}}}{\sqrt{3+\sqrt{7}} + \sqrt[4]{6\sqrt{7}}} \right)^{12}$$

Result:

$$2.9956828838554071283013240868820414082519145382139993... \times 10^{-26}$$

2.9956828838554... * 10⁻²⁶ final result



$$\sqrt{117} \left(\frac{\sqrt{13}-3}{2} \right)^6 \left(\sqrt{13}-2\sqrt{3} \right)^4 \left(\frac{\sqrt{4+\sqrt{3}} + \sqrt[4]{3}}{2} \right)^{24}$$

Input:

$$\sqrt{117} \left(\frac{1}{2} (\sqrt{13} - 3) \right)^6 (\sqrt{13} - 2\sqrt{3})^4 \left(\frac{1}{2} \left(\sqrt{4+\sqrt{3}} + \sqrt[4]{3} \right) \right)^{24}$$

Result:

$$\frac{3\sqrt{13} (\sqrt{13} - 3)^6 (\sqrt{13} - 2\sqrt{3})^4 \left(\sqrt[4]{3} + \sqrt{4+\sqrt{3}} \right)^{24}}{1073741824}$$

Decimal approximation:

9.208016037890824458855570254304688845619458953437052978389...
 9.20801603789.....

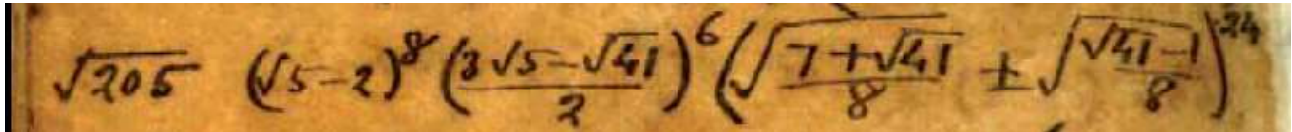
Alternate forms:

$$\frac{3(649\sqrt{13} - 2340)(1249 - 200\sqrt{39}) \left(\sqrt{4+\sqrt{3}} + \sqrt[4]{3} \right)^{24}}{16777216}$$

root of $x^8 + 96805418168160x^7 + 461446278882732x^6 + 34031477760639840x^5 + 128439125155493334x^4 - 3981682897994861280x^3 + 6316738111625718348x^2 - 155044816208561242080x + 187388721$ near $x = 9.20802$

Minimal polynomial:

$$x^8 + 96\,805\,418\,168\,160 x^7 + 461\,446\,278\,882\,732 x^6 + 34\,031\,477\,760\,639\,840 x^5 + 128\,439\,125\,155\,493\,334 x^4 - 3\,981\,682\,897\,994\,861\,280 x^3 + 6\,316\,738\,111\,625\,718\,348 x^2 - 155\,044\,816\,208\,561\,242\,080 x + 187\,388\,721$$



$$\sqrt{205} ((\sqrt{5}-2))^8 ((3\sqrt{5}-\sqrt{41})/2)^6$$

$$((((\sqrt{((7+\sqrt{41})/8)}+\sqrt{((\sqrt{41}-1)/8)}))))^24$$

Input:

$$\sqrt{205} (\sqrt{5} - 2)^8 \left(\left(\frac{1}{2} (3\sqrt{5} - \sqrt{41}) \right)^6 \left(\sqrt{\frac{1}{8} (7 + \sqrt{41})} + \sqrt{\frac{1}{8} (\sqrt{41} - 1)} \right) \right)^{24}$$

Result:

$$\frac{\sqrt{205} (\sqrt{5} - 2)^8 (3\sqrt{5} - \sqrt{41})^{144} \left(\frac{1}{2} \sqrt{\frac{1}{2} (\sqrt{41} - 1)} + \frac{1}{2} \sqrt{\frac{1}{2} (7 + \sqrt{41})} \right)^{24}}{22\,300\,745\,198\,530\,623\,141\,535\,718\,272\,648\,361\,505\,980\,416}$$

Decimal approximation:

$$2.294472816480531091837526681167287239636295312746917... \times 10^{-114}$$

$$2.29447281648... * 10^{-114}$$

Alternate forms:

$$\frac{1}{281\,474\,976\,710\,656} (51\,841 - 23\,184\sqrt{5})$$

$$\left(1957824\,041\,217\,313\,769\,464\,018\,127\,701\,593\,290\,409\,190\,610\,257\,259\,371\,123 \right.$$

$$048\,697\,209\,633\,031\,486\,243\,334\,963\,133\,277\,424\,779\,647\,745\,485\,062\,319\,618 \left. \right.$$

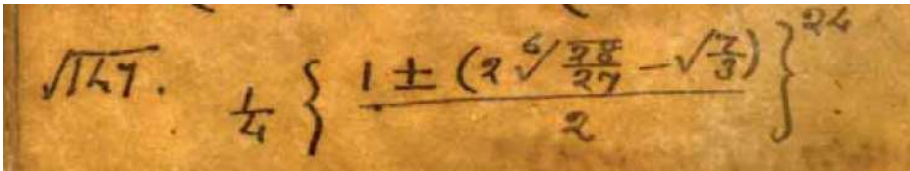
$$561\sqrt{205} -$$

$$28\,031\,774\,295\,530\,086\,098\,339\,937\,487\,685\,293\,327\,720\,357\,150\,478\,811\,074 \left. \right.$$

$$035\,436\,416\,879\,923\,466\,021\,793\,094\,435\,855\,490\,000\,889\,898\,750\,729\,955 \left. \right.$$

$$316\,881\,760) \left(\sqrt{2(\sqrt{41} - 1)} + \sqrt{2(7 + \sqrt{41})} \right)^{24}$$

$$\frac{(\sqrt{205} (\sqrt{5} - 2)^8 (3\sqrt{5} - \sqrt{41})^{144} \text{root of } x^8 - 3x^6 - 6x^4 - 3x^2 + 1 \text{ near } x = 2.11619^{24})}{22\,300\,745\,198\,530\,623\,141\,535\,718\,272\,648\,361\,505\,980\,416}$$



$$\text{sqrt}(147) \cdot \frac{1}{4} \left(\left(\left(\left(1 + \left(2 \sqrt{\frac{28}{27}} - \sqrt{\frac{7}{3}} \right) \right) \right) \right) \right)^{24}$$

Input:

$$\sqrt{147} \times \frac{1}{4} \left(\frac{1}{2} \left(1 + \left(2 \sqrt{\frac{28}{27}} - \sqrt{\frac{7}{3}} \right) \right) \right)^{24}$$

Result:

$$\frac{7\sqrt{3} \left(1 - \sqrt{\frac{7}{3}} + \frac{2\sqrt[3]{2}\sqrt[6]{7}}{\sqrt{3}} \right)^{24}}{67\,108\,864}$$

Decimal approximation:

0.002375446998015593652699176828666312896463588049572173350...

0.002375446998....

Alternate forms:

$$\frac{7\sqrt{3} \left(3 + 2\sqrt[3]{2}\sqrt{3}\sqrt[6]{7} - \sqrt{21} \right)^{24}}{18\,953\,525\,353\,286\,467\,584}$$

$$\frac{7\sqrt{3} \left(-1 + \sqrt{\frac{7}{3}} - \frac{2\sqrt[3]{2}\sqrt[6]{7}}{\sqrt{3}} \right)^{24}}{67\,108\,864}$$

$$\frac{7 \left(\sqrt{3} + 2\sqrt[3]{2}\sqrt[6]{7} - \sqrt{7} \right)^{24}}{11\,888\,133\,931\,008\sqrt{3}}$$

Minimal polynomial:

$$16\,777\,216\,x^{12} + 41\,965\,406\,750\,929\,707\,859\,968\,x^{10} +$$

$$26\,242\,365\,893\,781\,074\,497\,078\,567\,129\,850\,511\,360\,x^8 +$$

$$613\,126\,613\,976\,783\,772\,404\,361\,354\,938\,184\,718\,499\,840\,x^6 +$$

$$3\,606\,652\,938\,269\,232\,022\,348\,555\,370\,061\,318\,335\,121\,719\,040\,x^4 -$$

$$20\,351\,435\,261\,937\,652\,270\,756\,815\,897\,837\,426\,960\,672\,x^2 + 10\,090\,298\,369\,529$$

Now, from this results

$$8.28766007584362... \cdot 10^8; \quad 0.002366091682; \quad 2.9956828838554... \cdot 10^{-26}$$

$$9.20801603789; \quad 2.29447281648... \cdot 10^{-114}; \quad 0.002375446998$$

we obtain:

$$-2 \cdot 4096 \cdot \left(\left(\left(2.29447281648 \cdot 10^{-114} \cdot \frac{1}{(1/8.28766007584362 \cdot 10^8 \cdot 2.9956828838554 \cdot 10^{-26})} \right) \cdot \frac{1}{0.002366091682} \right) \cdot \frac{1}{9.20801603789} \right) \cdot 0.002375446998$$

Input interpretation:

$$-2 \times 4096 \left(\left(\frac{2.29447281648}{10^{114}} \times \frac{1}{\frac{1}{8.28766007584362 \times 10^8} \times 2.9956828838554 \times 10^{-26}} \right) \times \frac{1}{0.002366091682 \times \frac{1}{9.20801603789} \times 0.002375446998} \right)$$

Result:

$$-8.519186108004345273146039066277327207193732306577354... \times 10^{-70}$$

$$-8.5191861... \cdot 10^{-70}$$

From the previous six results, we obtain also:

$$2 \cdot 6.67430 \cdot 10^{-11} \cdot 10^{17} \cdot [0.08932131921 \cdot 1.801131800625 \cdot 10^{-8} \cdot 0.483328677491$$

$$\cdot 9.479346381 \cdot 10^{-10} \cdot 1.40910480785 \cdot 10^{-10} \cdot 2.33685237 \cdot 10^{-12}]$$

Input interpretation:

$$2 \times 6.67430 \times 10^{-11} \times 10^{17}$$

$$(0.08932131921 \times 1.801131800625 \times 10^{-8} \times 0.483328677491 \times$$

$$9.479346381 \times 10^{-10} \times 1.40910480785 \times 10^{-10} \times 2.33685237 \times 10^{-12})$$

Result:

$$3.2399019903399046406860898971314795798167283319396590... \times 10^{-33}$$

$$3.23990199... \cdot 10^{-33}$$

We have also:

$$2.103786766 * 4096 * 10^6 \left(\left(\left(\left(\frac{2.29447281648 * 10^{-114} * 1/}{(1/8.28766007584362 * 10^8 * 2.9956828838554 * 10^{-26})} * 1/ \right. \right. \right. \right. \\ \left. \left. \left. \left. \frac{0.002366091682 * 1/9.20801603789 * 0.002375446998 \right) \right) \right) \right)$$

Where 2.103786766... is the following Ramanujan mock theta function:

$$\left(\frac{1}{1-0.449329} + \frac{(0.449329)^2}{((1-0.449329^2)(1-0.449329^3))} \right) + \\ \left(\frac{(0.449329)^6}{((1-0.449329^3)(1-0.449329^4)(1-0.449329^5))} \right)$$

$$\left(\frac{1}{1-0.449329} + \frac{0.449329^2}{(1-0.449329^2)(1-0.449329^3)} \right) + \\ \frac{0.449329^6}{(1-0.449329^3)(1-0.449329^4)(1-0.449329^5)}$$

2.103786766736423428513160268563462427442372252987548543251...

2.103786766...

Input interpretation:

$$2.103786766 \times 4096 \times 10^6 \\ \left(\left(\frac{2.29447281648}{10^{114}} \times \frac{1}{8.28766007584362 \times 10^8 \times 2.9956828838554 \times 10^{-26}} \right) \times \right. \\ \left. \frac{1}{0.002366091682 \times \frac{1}{9.20801603789} \times 0.002375446998} \right)$$

Result:

8.9612754955552941280696460864766189321729570123620470... × 10⁻⁶⁴

8.961275495... * 10⁻⁶⁴

We note also that:

$$\left(\left(\left(\left(\left(\left(2 * 4096 * \left(\left(\left(\left(\left(\frac{2.29447281648 * 10^{-114} * 1/}{(1/8.28766007584362 * 10^8 * 2.9956828838554 * 10^{-26})} * 1/ \right. \right. \right. \right. \right. \right. \right. \right. \\ \left. \left. \left. \left. \frac{0.002366091682 * 1/9.20801603789 * 0.002375446998 \right) \right) \right) \right) \right) \right) \right) \right) \right)^{1/4096}$$

Input interpretation:

$$\left(2 \times 4096 \left(\left(\frac{2.29447281648}{10^{114}} \times \frac{1}{8.28766007584362 \times 10^8} \times 2.9956828838554 \times 10^{-26} \right) \times \frac{1}{0.002366091682 \times \frac{1}{9.20801603789} \times 0.002375446998} \right) \right)^{(1/4096)}$$

Result:

0.9619163446518...

0.9619163446518.... result very near to the spectral index n_s and to the mesonic Regge slope (see Appendix) and to the inflaton value at the end of the inflation 0.9402 and to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}} - \varphi + 1} = 1 - \frac{e^{-\pi}}{1 + \frac{e^{-2\pi}}{1 + \frac{e^{-3\pi}}{1 + \frac{e^{-4\pi}}{1 + \dots}}}}} \approx 0.9568666373$$

And:

$$\left(\left(\left(\left(2 \times 6.67430 \times 10^{-11} \times 10^{17} \times [0.08932131921 \times 1.801131800625 \times 10^{-8} \times 0.483328677491 \times 9.479346381 \times 10^{-10} \times 1.40910480785 \times 10^{-10} \times 2.33685237 \times 10^{-12}] \right) \right) \right) \right)^{1/4096}$$

Input interpretation:

$$\left(2 \times 6.67430 \times 10^{-11} \times 10^{17} \times (0.08932131921 \times 1.801131800625 \times 10^{-8} \times 0.483328677491 \times 9.479346381 \times 10^{-10} \times 1.40910480785 \times 10^{-10} \times 2.33685237 \times 10^{-12}) \right)^{(1/4096)}$$

Result:

0.9819016751...

0.9819016751.... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

$$\frac{1 + \sqrt[5]{\sqrt{\sqrt{\varphi^5 4\sqrt{5^3}} - 1}}}{\sqrt{5}} - \varphi + 1$$

and to the dilaton value **0.989117352243 = ϕ**

From:

Physics Letters B - Vol. 694, No. 3 (2010) pp. 181–185 - c Elsevier B. V.
COSMOLOGY WITH TORSION: AN ALTERNATIVE TO COSMIC INFLATION - *Nikodem J. Poplawski*

IV. DENSITY PARAMETERS

Seven-year Wilkinson Microwave Anisotropy Probe (WMAP) observations show that our Universe may be indeed closed, with $\Omega = 1.002$ [29]. The WMAP data give also $H_0^{-1} = 4.4 \times 10^{17}$ s and $\Omega_R = 8.8 \times 10^{-5}$. Thus $a_0 = 2.9 \times 10^{27}$ m. To estimate Ω_S , we use the relic background neutrinos which are the most abundant fermions in the Universe, with $n = 5.6 \times 10^7 \text{ m}^{-3}$ for each type (out of 6) [28]. Equations (13) and (15) then give

$$\Omega_S = -8.6 \times 10^{-70}. \quad (23)$$

While in general relativity the torsion density parameter Ω_S vanishes, the ECKS theory of gravity gives Ω_S a nonzero, though extremely small, negative value.

V. FLATNESS PROBLEM

Gravitational repulsion induced by torsion, which becomes significant at extremely high densities, prevents the cosmological singularity. Equation (21) shows that the expansion of the Universe started when $H = 0$, at which $\hat{a} = \hat{a}_m$, where

$$\hat{a}_m = \sqrt{\frac{\Omega_S}{\Omega_R}} = 3.1 \times 10^{-33}, \quad (24)$$

In the ECKS gravity, where $\Omega_S < 0$, $\Omega(\hat{a})$ is infinite at $\hat{a} = \hat{a}_m$. The function (22) has a local minimum at $\hat{a} = \sqrt{2}\hat{a}_m$, where it is equal to

$$\Omega(\sqrt{2}\hat{a}_m) = 1 - \frac{4\Omega_S(\Omega - 1)}{\Omega_R^2} = 1 + 8.9 \times 10^{-64}. \quad (26)$$

With regard the following results that we have obtained, it is possible to have some new mathematical connections with three important equations of the above cosmology.

Our results:

$$-2 \times 4096 \left(\left(\frac{2.29447281648}{10^{114}} \times \frac{1}{8.28766007584362 \times 10^8 \times 2.9956828838554 \times 10^{-26}} \right) \times \frac{1}{0.002366091682 \times \frac{1}{9.20801603789} \times 0.002375446998} \right) \times$$

$$-8.5191861... \times 10^{-70}$$

$$2 \times 6.67430 \times 10^{-11} \times 10^{17}$$

$$(0.08932131921 \times 1.801131800625 \times 10^{-8} \times 0.483328677491 \times 9.479346381 \times 10^{-10} \times 1.40910480785 \times 10^{-10} \times 2.33685237 \times 10^{-12})$$

$$3.23990199... \times 10^{-33}$$

$$2.103786766 \times 4096 \times 10^6$$

$$\left(\left(\frac{2.29447281648}{10^{114}} \times \frac{1}{8.28766007584362 \times 10^8 \times 2.9956828838554 \times 10^{-26}} \right) \times \frac{1}{0.002366091682 \times \frac{1}{9.20801603789} \times 0.002375446998} \right) \times$$

$$8.961275495... \times 10^{-64}$$

Poplawski formulas:

$$\Omega_S = -8.6 \times 10^{-70}.$$

$$\hat{a}_m = \sqrt{-\frac{\Omega_S}{\Omega_R}} = 3.1 \times 10^{-33},$$

$$\Omega(\hat{a}) = 1 + \frac{(\Omega - 1)\hat{a}^4}{\Omega_R \hat{a}^2 + \Omega_S}, \quad \Omega(\sqrt{2\hat{a}_m}) = 1 - \frac{4\Omega_S(\Omega - 1)}{\Omega_R^2} = 1 + 8.9 \times 10^{-64}.$$

Now, we have that:

Page 305

$$8(1+\sqrt{5}) \cdot \frac{1}{\pi}$$

Input:

$$8(1+\sqrt{5}) \times \frac{1}{\pi}$$

Result:

$$\frac{8(1+\sqrt{5})}{\pi}$$

Decimal approximation:

8.240579436807741840116241625190654312835794522646634343106...

8.2405794368....

Property:

$\frac{8(1+\sqrt{5})}{\pi}$ is a transcendental number

Alternate forms:

$$\frac{8}{\pi} + \frac{8\sqrt{5}}{\pi}$$

$$\frac{8+8\sqrt{5}}{\pi}$$

Series representations:

$$\frac{8(1+\sqrt{5})}{\pi} = \frac{8}{\pi} + \frac{8\sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k}}{\pi}$$

$$\frac{8(1+\sqrt{5})}{\pi} = \frac{8}{\pi} + \frac{8\sqrt{4} \sum_{k=0}^{\infty} \frac{(-\frac{1}{4})^k (-\frac{1}{2})_k}{k!}}{\pi}$$

$$\frac{8(1+\sqrt{5})}{\pi} = \frac{8}{\pi} + \frac{4 \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 4^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{\pi \sqrt{\pi}}$$

$$(6+\sqrt{5}) + (66+19\sqrt{5}) \cdot \left(\frac{1}{2}\right)^3 \cdot \frac{1}{64} \left(\frac{(\sqrt{5}-1)}{2}\right)^8$$

Input:

$$(6+\sqrt{5}) + (66+19\sqrt{5}) \left(\frac{1}{2}\right)^3 \times \frac{1}{64} \left(\frac{(\sqrt{5}-1)}{2}\right)^8$$

Result:

$$6+\sqrt{5} + \frac{(\sqrt{5}-1)^8 (66+19\sqrt{5})}{131072}$$

Decimal approximation:

8.240578218801160477337178923922175469745086278275117343347...

8.2405782188....

Alternate forms:

$$\frac{3(2417+177\sqrt{5})}{1024}$$

$$\frac{7251}{1024} + \frac{531\sqrt{5}}{1024}$$

$$\frac{7251+531\sqrt{5}}{1024}$$

Minimal polynomial:

$$262144x^2 - 3712512x + 12791799$$

$$\frac{1}{5} \left((8(1+\sqrt{5})) \cdot \frac{1}{\pi} \right)$$

Input:

$$\frac{1}{5} \left(8(1+\sqrt{5}) \times \frac{1}{\pi} \right)$$

Result:

$$\frac{8(1+\sqrt{5})}{5\pi}$$

Decimal approximation:

1.648115887361548368023248325038130862567158904529326868621...

$$1.64811588736\dots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934\dots$$

Property:

$\frac{8(1+\sqrt{5})}{5\pi}$ is a transcendental number

Alternate forms:

$$\frac{8\sqrt{5}}{\pi} \frac{1}{5} + \frac{8}{5\pi}$$

$$\frac{\frac{8}{5} + \frac{8}{\sqrt{5}}}{\pi}$$

$$\frac{8 + 8\sqrt{5}}{5\pi}$$

Series representations:

$$\frac{8(1+\sqrt{5})}{\pi 5} = \frac{8}{5\pi} + \frac{8\sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k}}{5\pi}$$

$$\frac{8(1+\sqrt{5})}{\pi 5} = \frac{8}{5\pi} + \frac{8\sqrt{4} \sum_{k=0}^{\infty} \frac{(-\frac{1}{4})^k \binom{-\frac{1}{2}}{k}}{k!}}{5\pi}$$

$$\frac{8(1+\sqrt{5})}{\pi 5} = \frac{8}{5\pi} + \frac{4 \sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 4^{-s} \Gamma(-\frac{1}{2}-s) \Gamma(s)}{5\pi\sqrt{\pi}}$$

$$\left(\left(\left(\left(\left(8(1+\sqrt{5})\right) \times \frac{1}{\pi}\right)\right)\right)\right)^{1/256}$$

Input:

$$\sqrt[256]{\frac{1}{8(1+\sqrt{5}) \times \frac{1}{\pi}}}$$

Exact result:

$$\frac{\sqrt[256]{\frac{\pi}{1+\sqrt{5}}}}{2^{3/256}}$$

Decimal approximation:

0.991795286636236647635601182981934557973120711593215999556...

0.991795286636... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} - \phi + 1 \approx 0.9991104684$$

and to the dilaton value **0.989117352243 = ϕ**

Property:

$$\frac{\sqrt[256]{\frac{\pi}{1+\sqrt{5}}}}{2^{3/256}} \text{ is a transcendental number}$$

Alternate form:

$$\frac{\sqrt[256]{(\sqrt{5} - 1)\pi}}{2^{5/256}}$$

All 256th roots of $\pi/(8(1 + \text{sqrt}(5)))$:

$$\frac{\sqrt[256]{\frac{\pi}{1+\sqrt{5}}} e^0}{2^{3/256}} \approx 0.991795 \text{ (real, principal root)}$$

$$\frac{\sqrt[256]{\frac{\pi}{1+\sqrt{5}}} e^{(i\pi)/128}}{2^{3/256}} \approx 0.991497 + 0.024340 i$$

$$\frac{256 \sqrt{\frac{\pi}{1+\sqrt{5}}} e^{(i\pi)/64}}{2^{3/256}} \approx 0.990601 + 0.048665 i$$

$$\frac{256 \sqrt{\frac{\pi}{1+\sqrt{5}}} e^{(3i\pi)/128}}{2^{3/256}} \approx 0.989108 + 0.07296 i$$

$$\frac{256 \sqrt{\frac{\pi}{1+\sqrt{5}}} e^{(i\pi)/32}}{2^{3/256}} \approx 0.987020 + 0.09721 i$$

Series representations:

$$\sqrt[256]{\frac{1}{8(1+\sqrt{5})\pi}} = \frac{256 \sqrt{\frac{\pi}{1+\sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{1/2}{k}}}}{2^{3/256}}$$

$$\sqrt[256]{\frac{1}{8(1+\sqrt{5})\pi}} = \frac{256 \sqrt{\frac{\pi}{1+\sqrt{4} \sum_{k=0}^{\infty} \frac{(-1)^k (-1/2)_k}{k!}}}}{2^{3/256}}$$

$$\sqrt[256]{\frac{1}{8(1+\sqrt{5})\pi}} = \frac{256 \sqrt{\frac{\pi \sqrt{\pi}}{2\sqrt{\pi} + \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 4^{-s} \Gamma(-\frac{1}{2}-s) \Gamma(s)}}}{128\sqrt{2}}$$

Integral representation:

$$(1+z)^a = \frac{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(s)\Gamma(-a-s)}{z^s} ds}{(2\pi i)\Gamma(-a)} \quad \text{for } (0 < \gamma < -\operatorname{Re}(a) \text{ and } |\arg(z)| < \pi)$$

And:

$$\pi + 16 \left[(6 + \sqrt{5}) + (66 + 19\sqrt{5}) \left(\frac{1}{2} \right)^3 \times \frac{1}{64} \left(\left(\frac{\sqrt{5} - 1}{2} \right) \right)^8 \right]$$

Input:

$$\pi + 16 \left((6 + \sqrt{5}) + (66 + 19\sqrt{5}) \left(\frac{1}{2} \right)^3 \times \frac{1}{64} \left(\frac{1}{2} (\sqrt{5} - 1) \right)^8 \right)$$

Result:

$$16 \left(6 + \sqrt{5} + \frac{(\sqrt{5} - 1)^8 (66 + 19\sqrt{5})}{131072} \right) + \pi$$

Decimal approximation:

134.9908441544083608758575061660343104001185498517769833145...

134.9908441544... result very near to the rest mass of Pion meson 134.9766

Property:

$$16 \left(6 + \sqrt{5} + \frac{(-1 + \sqrt{5})^8 (66 + 19\sqrt{5})}{131072} \right) + \pi \text{ is a transcendental number}$$

Alternate forms:

$$\frac{1}{64} (64\pi + 7251 + 531\sqrt{5})$$

$$\frac{7251}{64} + \frac{531\sqrt{5}}{64} + \pi$$

$$\frac{3}{64} (2417 + 177\sqrt{5}) + \pi$$

Series representations:

$$\begin{aligned} \pi + 16 \left((6 + \sqrt{5}) + \frac{1}{64} (66 + 19\sqrt{5}) \left(\frac{1}{2} \right)^3 \left(\frac{1}{2} (\sqrt{5} - 1) \right)^8 \right) = \\ 96 + \pi + 16\sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k} + \frac{\left(-1 + \sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k} \right)^8 \left(66 + 19\sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k} \right)}{8192} \end{aligned}$$

$$\begin{aligned} \pi + 16 \left((6 + \sqrt{5}) + \frac{1}{64} (66 + 19\sqrt{5}) \left(\frac{1}{2} \right)^3 \left(\frac{1}{2} (\sqrt{5} - 1) \right)^8 \right) = 96 + \pi + \\ 16\sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4} \right)^k \left(-\frac{1}{2} \right)_k}{k!} + \frac{\left(-1 + \sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4} \right)^k \left(-\frac{1}{2} \right)_k}{k!} \right)^8 \left(66 + 19\sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4} \right)^k \left(-\frac{1}{2} \right)_k}{k!} \right)}{8192} \end{aligned}$$

$$\pi + 16 \left((6 + \sqrt{5}) + \frac{1}{64} (66 + 19\sqrt{5}) \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}(\sqrt{5} - 1)\right)^8 \right) =$$

$$96 + \pi + 16 \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5 - z_0)^k z_0^{-k}}{k!} +$$

$$\frac{\left(-1 + \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5 - z_0)^k z_0^{-k}}{k!} \right)^8 \left(66 + 19 \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5 - z_0)^k z_0^{-k}}{k!} \right)}{8192}$$

for not $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

Appendix

From:

Rotating strings confronting PDG mesons

Jacob Sonnenschein and Dorin Weissman - arXiv:1402.5603v1 [hep-ph] 23 Feb 2014

$c\bar{c}$. **The Ψ trajectory:** The left side of figure (15) depicts the Ψ trajectory. Here we use the states $J/\Psi(1S)(3097)1^{--}$, $\chi_{c1}(1P)(3510)1^{++}$, and $\Psi(3770)1^{--}$. Since no $J = 3$ state has been observed, we use three states with $J = 1$, but with increasing orbital angular momentum ($L = 0, 1, 2$) and do the fit to L instead of J . To give an idea of the shifts in mass involved, the $J^{PC} = 2^{++}$ state χ_{c2} has a mass of 3556 MeV, and the $J^{PC} = 3^{--}$ state is expected to lie 30 – 60 MeV above the $\Psi(3770)$ [23].

The best linear fit is

$$\alpha' = 0.418, a = -4.04$$

with $\chi_l^2 = 3.41 \times 10^{-4}$, but the optimal fit is far from the linear, with endpoint masses in the range of the constituent c quark mass:

$$m_c = 1500, \alpha' = 0.979, a = -0.09$$

with $\chi_m^2 = 5 \times 10^{-7}$ ($\chi_m^2/\chi_l^2 = 0.002$). Aside from the improvement in χ^2 , by adding the mass we also get a value for the slope (and to a lesser extent, the intercept) that is much closer to that obtained in fits for the light meson trajectories.

where α' is the Regge slope (string tension)

We know also that:

$$\omega \quad | \quad 6 \quad | \quad m_{u/d} = 0 - 60 \quad | \quad 0.910 - 0.918$$

$$\omega/\omega_3 \quad | \quad 5 + 3 \quad | \quad m_{u/d} = 255 - 390 \quad | \quad 0.988 - 1.18$$

$$\omega/\omega_3 \quad | \quad 5 + 3 \quad | \quad m_{u/d} = 240 - 345 \quad | \quad 0.937 - 1.000$$

The average of the various Regge slope of Omega mesons are:

$$1/7 * (0.979 + 0.910 + 0.918 + 0.988 + 0.937 + 1.18 + 1) = \mathbf{0.987428571}$$

result very near to the value of dilaton and to the solution 0.987516007... of the above expression.

From:

Astronomy & Astrophysics manuscript no. ms c ESO 2019 - September 24, 2019
Planck 2018 results. VI. Cosmological parameters

The primordial fluctuations are consistent with Gaussian purely adiabatic scalar perturbations characterized by a power spectrum with a [spectral index \$n_s = 0.965 \pm 0.004\$](#) , consistent with the predictions of slow-roll, single-field, inflation.

from:

Modular equations and approximations to π - *Srinivasa Ramanujan*
Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

We have that:

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} (24 + 276e^{-\pi\sqrt{22}} + \dots), \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \dots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} (24 + 4372e^{-\pi\sqrt{22}} + \dots) = 64\{(1 + \sqrt{2})^{12} + (1 - \sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\dots$$

Again

$$G_{37} = (6 + \sqrt{37})^{\frac{1}{4}},$$

$$\begin{aligned} 64G_{37}^{24} &= e^{\pi\sqrt{37}} (24 + 276e^{-\pi\sqrt{37}} + \dots), \\ 64G_{37}^{-24} &= 4096e^{-\pi\sqrt{37}} + \dots, \end{aligned}$$

so that

$$64(G_{37}^{24} + G_{37}^{-24}) = e^{\pi\sqrt{37}} (24 + 4372e^{-\pi\sqrt{37}} + \dots) = 64\{(6 + \sqrt{37})^6 + (6 - \sqrt{37})^6\}.$$

Hence

$$e^{\pi\sqrt{37}} = 199148647.999978\dots$$

Similarly, from

$$g_{58} = \sqrt{\left(\frac{5 + \sqrt{29}}{2}\right)},$$

we obtain

$$64(g_{58}^{24} + g_{58}^{-24}) = e^{\pi\sqrt{58}} (24 + 4372e^{-\pi\sqrt{58}} + \dots) = 64 \left\{ \left(\frac{5 + \sqrt{29}}{2}\right)^{12} + \left(\frac{5 - \sqrt{29}}{2}\right)^{12} \right\}.$$

Hence

$$e^{\pi\sqrt{58}} = 24501257751.99999982\dots$$

From:

An Update on Brane Supersymmetry Breaking

J. Mourad and A. Sagnotti - arXiv:1711.11494v1 [hep-th] 30 Nov 2017

From the following vacuum equations:

$$T e^{\gamma_E \phi} = - \frac{\beta_E^{(p)} h^2}{\gamma_E} e^{-2(8-p)C + 2\beta_E^{(p)} \phi}$$

$$16 k' e^{2C} = \frac{h^2 \left(p + 1 - \frac{2\beta_E^{(p)}}{\gamma_E} \right) e^{-2(8-p)C + 2\beta_E^{(p)} \phi}}{(7-p)}$$

$$(A')^2 = k e^{-2A} + \frac{h^2}{16(p+1)} \left(7 - p + \frac{2\beta_E^{(p)}}{\gamma_E} \right) e^{-2(8-p)C + 2\beta_E^{(p)} \phi}$$

we have obtained, from the results almost equals of the equations, putting

$4096 e^{-\pi\sqrt{18}}$ instead of

$$e^{-2(8-p)C + 2\beta_E^{(p)} \phi}$$

a new possible mathematical connection between the two exponentials. Thence, also the values concerning p , C , β_E and ϕ correspond to the exponents of e (i.e. of exp). Thence we obtain for $p = 5$ and $\beta_E = 1/2$:

$$e^{-6C + \phi} = 4096 e^{-\pi\sqrt{18}}$$

Therefore, with respect to the exponentials of the vacuum equations, the Ramanujan's exponential has a coefficient of 4096 which is equal to 64^2 , while $-6C + \phi$ is equal to $-\pi\sqrt{18}$. From this it follows that it is possible to establish mathematically, the dilaton value.

For

$\exp(-\pi\sqrt{18})$ we obtain:

Input:

$$\exp(-\pi\sqrt{18})$$

Exact result:

$$e^{-3\sqrt{2}\pi}$$

Decimal approximation:

$$1.6272016226072509292942156739117979541838581136954016... \times 10^{-6}$$

$$1.6272016... * 10^{-6}$$

Property:

$e^{-3\sqrt{2}\pi}$ is a transcendental number

Series representations:

$$e^{-\pi\sqrt{18}} = e^{-\pi\sqrt{17} \sum_{k=0}^{\infty} 17^{-k} \binom{1/2}{k}}$$

$$e^{-\pi\sqrt{18}} = \exp\left(-\pi\sqrt{17} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{17}\right)^k \binom{-\frac{1}{2}}{k}}{k!}\right)$$

$$e^{-\pi\sqrt{18}} = \exp\left(-\frac{\pi \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 17^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{2\sqrt{\pi}}\right)$$

Now, we have the following calculations:

$$e^{-6C+\phi} = 4096e^{-\pi\sqrt{18}}$$

$$e^{-\pi\sqrt{18}} = 1.6272016... * 10^{-6}$$

from which:

$$\frac{1}{4096}e^{-6C+\phi} = 1.6272016... * 10^{-6}$$

$$0.000244140625 e^{-6C+\phi} = e^{-\pi\sqrt{18}} = 1.6272016... * 10^{-6}$$

Now:

$$\ln\left(e^{-\pi\sqrt{18}}\right) = -13.328648814475 = -\pi\sqrt{18}$$

And:

$$(1.6272016 * 10^{-6}) * 1 / (0.000244140625)$$

Input interpretation:

$$\frac{1.6272016}{10^6} \times \frac{1}{0.000244140625}$$

Result:

0.0066650177536

0.006665017...

Thence:

$$0.000244140625 e^{-6C+\phi} = e^{-\pi\sqrt{18}}$$

Dividing both sides by 0.000244140625, we obtain:

$$\frac{0.000244140625}{0.000244140625} e^{-6C+\phi} = \frac{1}{0.000244140625} e^{-\pi\sqrt{18}}$$

$$e^{-6C+\phi} = 0.0066650177536$$

$$(((\exp((-Pi*\sqrt{18})))))) * 1 / 0.000244140625$$

Input interpretation:

$$\exp(-\pi\sqrt{18}) \times \frac{1}{0.000244140625}$$

Result:

0.00666501785...

0.00666501785...

Series representations:

$$\frac{\exp(-\pi \sqrt{18})}{0.000244141} = 4096 \exp\left(-\pi \sqrt{17} \sum_{k=0}^{\infty} 17^{-k} \binom{\frac{1}{2}}{k}\right)$$

$$\frac{\exp(-\pi \sqrt{18})}{0.000244141} = 4096 \exp\left(-\pi \sqrt{17} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{17}\right)^k \left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$\frac{\exp(-\pi \sqrt{18})}{0.000244141} = 4096 \exp\left(-\frac{\pi \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 17^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{2\sqrt{\pi}}\right)$$

Now:

$$e^{-6C+\phi} = 0.0066650177536$$

$$\exp(-\pi \sqrt{18}) \times \frac{1}{0.000244140625} =$$

$$e^{-\pi \sqrt{18}} \times \frac{1}{0.000244140625}$$

$$= 0.00666501785\dots$$

From:

$$\ln(0.00666501784619)$$

Input interpretation:

$$\log(0.00666501784619)$$

Result:

$$-5.010882647757\dots$$

$$-5.010882647757\dots$$

Alternative representations:

$$\log(0.006665017846190000) = \log_e(0.006665017846190000)$$

$$\log(0.006665017846190000) = \log(a) \log_a(0.006665017846190000)$$

$$\log(0.006665017846190000) = -\text{Li}_1(0.993334982153810000)$$

Series representations:

$$\log(0.006665017846190000) = -\sum_{k=1}^{\infty} \frac{(-1)^k (-0.993334982153810000)^k}{k}$$

$$\log(0.006665017846190000) = 2i\pi \left[\frac{\arg(0.006665017846190000 - x)}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (0.006665017846190000 - x)^k x^{-k}}{k} \quad \text{for } x < 0$$

$$\log(0.006665017846190000) = \left[\frac{\arg(0.006665017846190000 - z_0)}{2\pi} \right] \log\left(\frac{1}{z_0}\right) + \log(z_0) + \left[\frac{\arg(0.006665017846190000 - z_0)}{2\pi} \right] \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (0.006665017846190000 - z_0)^k z_0^{-k}}{k}$$

Integral representation:

$$\log(0.006665017846190000) = \int_1^{0.006665017846190000} \frac{1}{t} dt$$

In conclusion:

$$-6C + \phi = -5.010882647757 \dots$$

and for $C = 1$, we obtain:

$$\phi = -5.010882647757 + 6 = 0.989117352243 = \phi$$

Note that the values of n_s (spectral index) 0.965, of the average of the Omega mesons Regge slope 0.987428571 and of the dilaton 0.989117352243, are also connected to the following two Rogers-Ramanujan continued fractions:

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\phi-1)\sqrt{5}} - \phi + 1} = 1 - \frac{e^{-\pi}}{1 + \frac{e^{-2\pi}}{1 + \frac{e^{-3\pi}}{1 + \frac{e^{-4\pi}}{1 + \dots}}}} \approx 0.9568666373$$

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

(<http://www.bitman.name/math/article/102/109/>)

Also performing the 512th root of the inverse value of the Pion meson rest mass 139.57, we obtain:

$$((1/(139.57)))^{1/512}$$

Input interpretation:

$$\sqrt[512]{\frac{1}{139.57}}$$

Result:

0.990400732708644027550973755713301415460732796178555551684...

0.99040073.... result very near to the dilaton value **0.989117352243 = ϕ** and to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

$$\frac{\sqrt{5}}{1 + \sqrt[5]{\sqrt{\phi^5 \sqrt[4]{5^3}} - 1}} - \phi + 1$$

From:

Eur. Phys. J. C (2019) 79:713 - <https://doi.org/10.1140/epjc/s10052-019-7225-2> - Regular Article - Theoretical Physics
Generalized dilaton–axion models of inflation, de Sitter vacua and spontaneous SUSY breaking in supergravity
Yermek Aldabergenov, Auttakit Chatrabhuti, Sergei V. Ketov

Table 1 The predictions for the inflationary parameters (n_s, r), and the values of φ at the horizon crossing (φ_i) and at the end of inflation (φ_f), in the case $3 \leq \alpha \leq \alpha_*$ with both signs of ω_1 . The α parameter is taken to be integer, except of the upper limit $\alpha_* \equiv (7 + \sqrt{33})/2$

α	3	4	5	6	α_*	
$\text{sgn}(\omega_1)$	–	+	–	+/–	+	–
n_s	0.9650	0.9649	0.9640	0.9639	0.9634	0.9637
r	0.0035	0.0010	0.0013	0.0007	0.0005	0.0004
$-\kappa\varphi_i$	5.3529	3.5542	3.9899	3.2657	3.0215	2.7427
$-\kappa\varphi_f$	0.9402	0.7426	0.8067	0.7163	0.6935	0.6488

Acknowledgments

I would like to thank Prof. **George E. Andrews** Evan Pugh Professor of Mathematics at Pennsylvania State University for his availability and kindness towards me

References

Ramanujan's Notebooks, Part I, by Bruce C. Berndt (Springer, 1985, [ISBN 0-387-96110-0](#))

Ramanujan's Notebooks, Part II, by Bruce C. Berndt (Springer, 1999, [ISBN 0-387-96794-X](#))

Ramanujan's Notebooks, Part III, by Bruce C. Berndt (Springer, 2004, [ISBN 0-387-97503-9](#))

Ramanujan's Notebooks, Part IV, by Bruce C. Berndt (Springer, 1993, [ISBN 0-387-94109-6](#))

Ramanujan's Notebooks, Part V, by Bruce C. Berndt (Springer, 2005, [ISBN 0-387-94941-0](#))

Ramanujan's Lost Notebook, Part I, by [George Andrews](#) and Bruce C. Berndt (Springer, 2005, [ISBN 0-387-25529-X](#))

Ramanujan's Lost Notebook, Part II, George E. Andrews, Bruce C. Berndt (Springer, 2008, [ISBN 978-0-387-77765-8](#))

Ramanujan's Lost Notebook: Part III, George E. Andrews, Bruce C. Berndt (Springer, 2012, [ISBN 978-1-4614-3809-0](#))

Ramanujan's Lost Notebook: Part IV, George E. Andrews, Bruce C. Berndt (Springer, 2013, [ISBN 978-1-4614-4080-2](#))

Manuscript Book 2 - *Srinivasa Ramanujan*

