On the Srinivasa Ramanujan Manuscripts: further and new mathematical developments between various formulas, the Rogers-Ramanujan continued fractions, the mock theta functions and some sectors of Cosmology and Theoretical Physics. III

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Abstract

In this research thesis, concerning the Srinivasa Ramanujan Manuscripts, we have analyzed various formulas, the Rogers-Ramanujan continued fractions, the mock theta functions and some sectors of Cosmology and Theoretical Physics. We have obtained further new possible mathematical connections and developments

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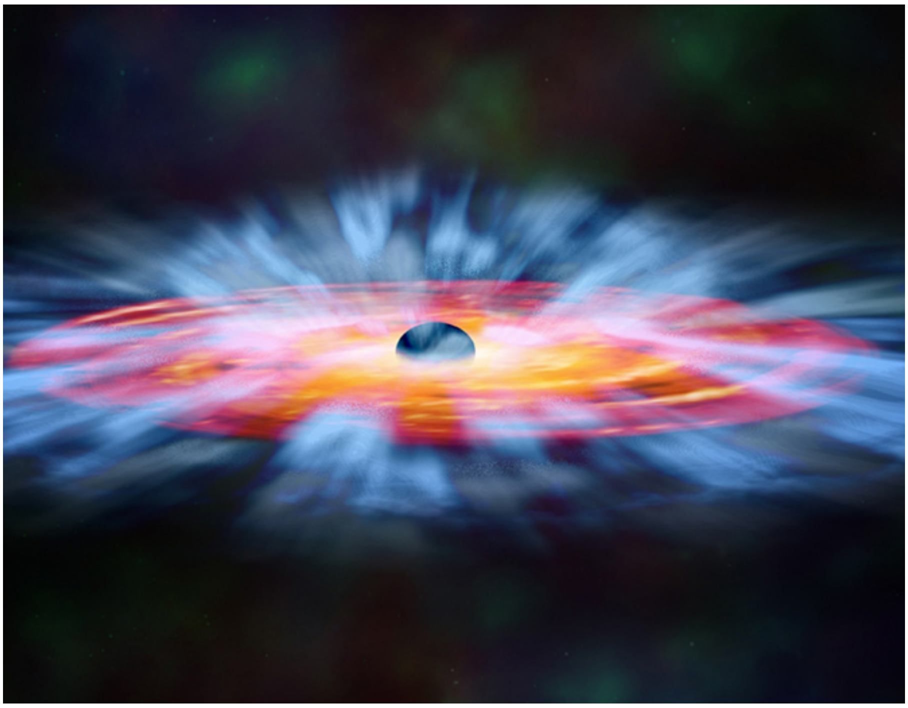

http://esciencecommons.blogspot.com/2012/12/math-formula-gives-new-glimpse-into.html

"...Expansion of modular forms is one of the fundamental tools for computing the entropy of a modular black hole. Some black holes, however, are not modular, but the new formula based on Ramanujan's vision may allow physicists to compute their entropy as though they were....."

https://blogs.royalsociety.org/history-of-science/2014/02/17/movie-maths/

From:

Anomaly Inflow and the η-Invariant
Edward Witten and Kazuya Yonekura - arXiv:1909.08775v2 [hep-th] 7 Oct 2019

The Atiyah-Patodi-Singer η-invariant is a regularized version of $\sum_k \mathrm{sign}(\lambda_k)$. The precise regularization does not matter. We can take, for example,[7]

$$\eta_D = \lim_{\epsilon\to 0^+} \sum_k \exp(-\epsilon|\lambda_k|)\mathrm{sign}(\lambda_k). \tag{2.16}$$

The Pauli-Villars regularization in (2.15) gives a different regularization of $\sum_k \mathrm{sign}(\lambda_k)$ (it gives a regularization since the argument of $\lambda_k/(\lambda_k + \mathrm{i}M)$ vanishes for $|\lambda_k| \to \infty$). The two regularizations are equivalent in the limit $M \to \infty$ or $\epsilon \to 0^+$.

From these expressions, we easily get

$$\langle \mathsf{APS}|\Omega\rangle = \cos\theta_a + \sin\theta_a \to 1 \qquad (\lambda_a \ll |m|), \tag{2.29}$$

and

$$\langle \mathsf{L}|\Omega\rangle = \cos\theta_a \to \begin{cases} 1 & (m>0, \quad \lambda_a \ll |m|) \\ \lambda_a/(2|m|) & (m<0, \quad \lambda_a \ll |m|). \end{cases} \tag{2.30}$$

Essentially, $\cos\theta_a$ for $m<0$ is the eigenvalue λ_a normalized by $2|m|$ as long as $\lambda_a \ll |m|$. But $|m|$ plays the role of a regulator. In the limit $\lambda_a/|m| \to \infty$, we have $\cos\theta_a \to 1/\sqrt{2}$, independent of a or m. Upon taking the ratio between the theories with $m<0$ and $m>0$, the factors of $\cos\theta_a$ associated to eigenvalues with $|\lambda_a| \gg m$ cancel out, and hence the ultraviolet is regularized. Therefore, after taking the ratio, we finally get

$$\frac{\langle \mathsf{L}|\Omega\rangle\langle\Omega|\mathsf{APS}\rangle}{|\langle \mathsf{APS}|\Omega\rangle|^2} = \prod_a \left(\frac{\lambda_a}{2|m|}\right)_{\mathrm{reg}} = |\mathrm{Det}(\mathcal{D}_W^+)| \tag{2.31}$$

Combining this result with eqn. (2.11) and with what we learned in section 2.2, it follows that the total partition function of the bulk massive fermion Ψ with the boundary condition L is given by

$$Z(Y, \mathsf{L}) = |\mathrm{Det}(\mathcal{D}_W^+)| \exp(-\pi \mathrm{i}\eta_D). \tag{2.32}$$

From:

$$\eta_D = \lim_{\epsilon\to 0^+} \sum_k \exp(-\epsilon|\lambda_k|)\mathrm{sign}(\lambda_k). \qquad M > 0 \text{ and } M \gg |\lambda_k|$$

for $\epsilon \to 0^+ = \frac{1}{12}$; $|\lambda_k| = 64$

we obtain:

exp(-1/12*64) sign(64)

Input:

$$\exp\left(-\frac{64}{12}\right)\operatorname{sgn}(64)$$

$\operatorname{sgn}(x)$ is the sign of x

Exact result:

$$\frac{1}{e^{16/3}}$$

Decimal approximation:

0.004827949993831440098727223068817292702877910061330745975...

0.00482794999383144…. = η_D

Property:

$\frac{1}{e^{16/3}}$ is a transcendental number

Alternative representations:

$$\exp\left(\frac{64(-1)}{12}\right)\operatorname{sgn}(64) = \exp\left(-\frac{64}{12}\right)(-\theta(-64)+\theta(64))$$

$$\exp\left(\frac{64(-1)}{12}\right)\operatorname{sgn}(64) = \frac{64\exp\left(-\frac{64}{12}\right)}{|64|}$$

$$\exp\left(\frac{64(-1)}{12}\right)\operatorname{sgn}(64) = \exp\left(-\frac{64}{12}\right)e^{i\arg(64)}$$

Series representations:

$$\exp\left(\frac{64(-1)}{12}\right)\operatorname{sgn}(64) = \frac{1}{\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{16/3}}$$

$$\exp\left(\frac{64(-1)}{12}\right)\operatorname{sgn}(64) = \frac{1}{\left(\sum_{k=0}^{\infty}\frac{(-1+k)^2}{k!}\right)^{16/3}}$$

$$\exp\left(\frac{64(-1)}{12}\right)\operatorname{sgn}(64) = \frac{32\sqrt[3]{2}}{\left(\sum_{k=0}^{\infty}\frac{1+k}{k!}\right)^{16/3}}$$

Integral representation:

$$\exp\left(\frac{64(-1)}{12}\right)\operatorname{sgn}(64) = \frac{\exp\left(-\frac{16}{3}\right)}{i\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{65^{-s}\,\Gamma(-s)}{\Gamma(1-s)}\,ds \quad \text{for } 0<\gamma$$

From:

$$\cos\theta_a \to \lambda_a/(2|m|)$$

$$\prod_a \left(\frac{\lambda_a}{2|m|}\right)_{\text{reg}} = |\text{Det}(\mathcal{D}_W^+)|$$

$$\cos\theta_a \to 1/\sqrt{2},$$

and

$$Z(Y, \text{L}) = |\text{Det}(\mathcal{D}_W^{\dagger})| \exp(-\pi i \eta_D)$$

we obtain:

1/sqrt(2) * exp(-Pi*i*0.004827949)

Input interpretation:

$$\frac{1}{\sqrt{2}} \exp(-\pi (i \times 0.004827949))$$

i is the imaginary unit

Result:

0.707025447... −
0.0107245949... i

Polar coordinates:

$r = 0.707107$ (radius), $\theta = -0.869031°$ (angle)

0.707107

Series representations:

$$\frac{\exp(-\pi (i\, 0.00482795))}{\sqrt{2}} = \frac{\exp(-0.00482795\, i\, \pi)}{\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}}$$

for not $((z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0))$

$$\frac{\exp(-\pi (i\, 0.00482795))}{\sqrt{2}} = \frac{\exp(-0.00482795\, i\, \pi)}{\exp\left(\pi\, \mathcal{A} \left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (2-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}}$$

for $(x \in \mathbb{R}$ and $x < 0)$

$$\frac{\exp(-\pi (i\, 0.00482795))}{\sqrt{2}} = \frac{\exp(-0.00482795\, i\, \pi) \left(\frac{1}{z_0}\right)^{-1/2 \lfloor \arg(2-z_0)/(2\pi) \rfloor} z_0^{-1/2-1/2 \lfloor \arg(2-z_0)/(2\pi) \rfloor}}{\sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}}$$

We note that:

(1/0.707107)

Input interpretation:

$$\frac{1}{0.707107}$$

Result:

1.414213124746325520748627859715714877663493643819110827639...

1.4142131247…

Repeating decimal:

1.414213124746325520748627859715714877663493643819110827639...

(period 3948)

Possible closed forms:

$\sqrt{2} \approx 1.41421356237$

$\frac{140}{99} \approx 1.414141414$

$1 - \frac{3}{\pi} - \sqrt{\pi} + \pi \approx 1.41420914413$

1/4(1/0.707107)^24

Input interpretation:

$$\frac{1}{4} \left(\frac{1}{0.707107}\right)^{24}$$

Result:

1023.992395011969661945937267768583792634522029124223432577...

1023.992395.... result very near to the rest mass of Phi meson 1019.461

(((1/4(1/0.707107)^24)))^1/14

Input interpretation:

$$\sqrt[14]{\frac{1}{4}\left(\frac{1}{0.707107}\right)^{24}}$$

Result:

1.6406698416662140513942794928004902013375580320811 21787188...

1.64066984166621.... ≈ ζ(2) = $\frac{\pi^2}{6}$ = 1.644934 ...

1/10^27 [(29/10^3+2/10^3)+(((1/4(1/0.707107)^24)))^1/14]

Input interpretation:

$$\frac{1}{10^{27}}\left(\left(\frac{29}{10^3}+\frac{2}{10^3}\right)+\sqrt[14]{\frac{1}{4}\left(\frac{1}{0.707107}\right)^{24}}\right)$$

Result:

$1.67167... \times 10^{-27}$

$1.67167...*10^{-27}$

result practically equal to the value of the formula:

$$m_{p'} = 2 \times \frac{\eta}{R} m_P = 1.6714213 \times 10^{-24} \text{ gm}$$

that is the holographic proton mass (N. Haramein)

Furthermore, we obtain:

-1+(((1/sqrt(2) * exp(-Pi*i^2*0.004827949))))^1/64 -i^2

Input interpretation:

$$-1+\sqrt[64]{\frac{1}{\sqrt{2}}\exp(-\pi\,(i^2 \times 0.004827949))} - i^2$$

i is the imaginary unit

Result:

0.99483516292...

0.994835…. result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\dots}}}}\approx 0.9991104684$$

and to the dilaton value **0.989117352243** = $\boldsymbol{\phi}$

Series representations:

$$-1+\sqrt[64]{\frac{\exp(-\pi(i^2\ 0.00482795))}{\sqrt{2}}}-i^2=-1-i^2+\sqrt[64]{\frac{\exp(-0.00482795\ i^2\ \pi)}{\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}}$$

for not $((z_0\in\mathbb{R}$ and $-\infty<z_0\le 0))$

$$-1+\sqrt[64]{\frac{\exp(-\pi(i^2\ 0.00482795))}{\sqrt{2}}}-i^2=$$
$$-1-i^2+\sqrt[64]{\frac{\exp(-0.00482795\ i^2\ \pi)}{\exp\left(\pi\mathcal{A}\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}}\quad\text{for } (x\in\mathbb{R}\text{ and } x<0)$$

$$-1+\sqrt[64]{\frac{\exp(-\pi(i^2\ 0.00482795))}{\sqrt{2}}}-i^2=$$
$$-1-i^2+\sqrt[64]{\frac{\exp(-0.00482795\ i^2\ \pi)\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{-1/2-1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}}{\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}}$$

((1/sqrt(2) * exp(-Pi*i*0.004827949))))^1/64

Input interpretation:

$$\sqrt[64]{\frac{1}{\sqrt{2}}\exp(-\pi\,(i \times 0.004827949))}$$

i is the imaginary unit

Result:

0.99459939555... −
0.00023571149999... i

Polar coordinates:

$r = 0.994599$ (radius), $\theta = -0.0135786°$ (angle)

0.994599..... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1} = 1-\cfrac{e^{-\pi\sqrt{5}}}{1+\cfrac{e^{-2\pi\sqrt{5}}}{1+\cfrac{e^{-3\pi\sqrt{5}}}{1+\cfrac{e^{-4\pi\sqrt{5}}}{1+\dots}}}} \approx 0.9991104684$$

and to the dilaton value $\mathbf{0.989117352243 = \phi}$

Now, we have that:

$$\exp(-\mathrm{i}\pi\eta_{\overline{Y}}/2) = \exp\left(-\mathrm{i}\int_{\overline{Y}}\Phi\right),$$

$$Z_W = |\mathrm{Pf}\,\mathcal{D}_W^+|\exp(-\mathrm{i}\pi\eta_Y/2)\exp\left(\mathrm{i}\int_Y\Phi\right)$$

$$\prod_a\left(\frac{\lambda_a}{2|m|}\right)_{\mathrm{reg}} = |\mathrm{Pf}(\mathcal{D}_W^+)|.$$

Is equal to

$$\prod_a\left(\frac{\lambda_a}{2|m|}\right)_{\mathrm{reg}} = |\mathrm{Det}(\mathcal{D}_W^+)|$$

$\exp(-\mathrm{i}\pi\eta_{\overline{Y}}/2) = \pm 1$ for a closed manifold $\overline{Y}$.

$\exp\left(-\mathrm{i}\pi\eta_{\overline{Y}}/2\right) \neq 1.$

As previously 0.00482794999383144…. = η_Y

1/sqrt(2) * exp(-(-i*Pi*0.00482794999383144)*1/2)

Input interpretation:

$$\frac{1}{\sqrt{2}}\exp\left(-(-i\,(\pi\times 0.00482794999383144))\times\frac{1}{2}\right)$$

i is the imaginary unit

Result:

0.70708644740256586… +
0.0053624527614207895… i

Polar coordinates:

$r = 0.70710678118654752440$ (radius), $\theta = 0.434515499444830°$ (angle)

0.70710678118654752440

Series representations:

$$\frac{\exp\left(-\frac{1}{2}\left(-i\left(\pi\, 0.0048279499938314400000\right)\right)\right)}{\sqrt{2}} = \frac{\exp(0.0024139749969157200000\, i\,\pi)}{\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}}$$

for not $((z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0))$

$$\frac{\exp\left(-\frac{1}{2}\left(-i\left(\pi\, 0.0048279499938314400000\right)\right)\right)}{\sqrt{2}} = \frac{\exp(0.0024139749969157200000\, i\,\pi)}{\exp\left(\pi\,\mathcal{A}\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k (2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}} \quad \text{for } (x \in \mathbb{R} \text{ and } x < 0)$$

$$\frac{\exp\left(-\frac{1}{2}\left(-i\left(\pi\, 0.0048279499938314400000\right)\right)\right)}{\sqrt{2}} = \frac{\exp(0.0024139749969157200000\, i\,\pi)\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{-1/2-1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}}{\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}}$$

Possible closed forms:

$e^{b_4(2)/8} \approx 0.7071067811865475244\,00844$

$\frac{1}{\sqrt{2}} \approx 0.7071067811865475244\,00844$

$$\frac{400914101\pi}{1781214419} \approx 0.70710678118654752422264$$

Now, we have that

$$\exp\left(-\mathrm{i}\pi\eta_{\overline{Y}}/2\right) \neq 1.$$

If the above expression is equal to 64, 4096 or 16777216, we obtain similar results:

1/sqrt(2) * exp(((-64)(-i*Pi*0.00482794999383144)*1/2))

Input interpretation:

$$\frac{1}{\sqrt{2}} \exp\left(-64\,(-i\,(\pi \times 0.00482794999383144)) \times \frac{1}{2}\right)$$

i is the imaginary unit

Result:

0.625441442100499... +
0.329883316497286... i

Polar coordinates:

$r = 0.707106781186548$ (radius), $\theta = 27.8089919644691°$ (angle)

0.707106781186548

Series representations:

$$\frac{\exp\left(-\frac{64}{2}\,(-i\,(\pi\,0.0048279499938314400000))\right)}{\sqrt{2}} = \frac{\exp(0.1544943998026060800\,i\,\pi)}{\sqrt{z_0}\,\sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}}$$

for not $((z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0))$

$$\frac{\exp\left(-\frac{64}{2}\,(-i\,(\pi\,0.0048279499938314400000))\right)}{\sqrt{2}} = \frac{\exp(0.1544943998026060800\,i\,\pi)}{\exp\left(\pi\,\mathcal{A}\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\,\sum_{k=0}^{\infty} \frac{(-1)^k (2-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}}$$

for $(x \in \mathbb{R}$ and $x < 0)$

$$\frac{\exp\left(-\frac{64}{2}\left(-i\left(\pi\,0.00482794999383144000\right)\right)\right)}{\sqrt{2}} = \frac{\exp(0.1544943998026060800\,i\,\pi)\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{-1/2-1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}}{\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}$$

1/sqrt(2) * exp(((-4096)(-i*Pi*0.00482794999383144)*1/2))

Input interpretation:

$$\frac{1}{\sqrt{2}}\exp\left(-4096\left(-i\left(\pi\times 0.00482794999383144\right)\right)\times\frac{1}{2}\right)$$

i is the imaginary unit

Result:

0.66351025357113... –
0.24444660645216... i

Polar coordinates:

$r = 0.7071067811865$ (radius), $\theta = -20.22451427398°$ (angle)

0.7071067811865

Series representations:

$$\frac{\exp\left(-\frac{4096}{2}\left(-i\left(\pi\,0.00482794999383144000\right)\right)\right)}{\sqrt{2}} = \frac{\exp(9.887641587366789120\,i\,\pi)}{\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}$$

for not $((z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0))$

$$\frac{\exp\left(-\frac{4096}{2}\left(-i\left(\pi\,0.00482794999383144000\right)\right)\right)}{\sqrt{2}} = \frac{\exp(9.887641587366789120\,i\,\pi)}{\exp\left(\pi\,\mathcal{A}\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}$$

for $(x \in \mathbb{R}$ and $x < 0)$

$$\frac{\exp\left(-\frac{4096}{2}\left(-i\left(\pi\,0.00482794999383144000\right)\right)\right)}{\sqrt{2}} = \frac{\exp(9.887641587366789120\,i\,\pi)\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{-1/2-1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}}{\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}$$

1/sqrt(2) * exp(((-16777216)(-i*Pi*0.00482794999383144)*1/2))

Input interpretation:

$$\frac{1}{\sqrt{2}} \exp\left(-16\,777\,216\,(-i\,(\pi \times 0.00482794999383144)) \times \frac{1}{2}\right)$$

i is the imaginary unit

Result:

0.5447527957... −
0.4508263431... i

Polar coordinates:

$r = 0.7071067812$ (radius), $\theta = -39.61046621°$ (angle)

0.7071067812

Series representations:

$$\frac{\exp\left(-\frac{16777216}{2}\left(-i\left(\pi\, 0.00482794999383144 0000\right)\right)\right)}{\sqrt{2}} = \frac{\exp(40\,499.77994185436824\, i\,\pi)}{\sqrt{z_0}\,\sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}} \quad \text{for not } ((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$$

$$\frac{\exp\left(-\frac{16777216}{2}\left(-i\left(\pi\, 0.00482794999383144 0000\right)\right)\right)}{\sqrt{2}} = \frac{\exp(40\,499.77994185436824\, i\,\pi)}{\exp\left(\pi\,\mathcal{A}\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\,\sum_{k=0}^{\infty} \frac{(-1)^k (2-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}} \quad \text{for } (x \in \mathbb{R} \text{ and } x < 0)$$

$$\frac{\exp\left(-\frac{16777216}{2}\left(-i\left(\pi\, 0.00482794999383144 0000\right)\right)\right)}{\sqrt{2}} = \frac{\exp(40\,499.77994185436824\, i\,\pi)\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{-1/2-1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}}{\sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}}$$

Possible closed forms:

$$\frac{1}{\sqrt{2}} \approx 0.70710678118654$$

$$\frac{19}{35} - \frac{1}{11e} + \frac{4e}{55} \approx 0.70710678118403$$

$$\frac{4\pi\pi! + 7 - 14\pi + 9\pi^2}{64\pi} \approx 0.707106781273418$$

(0.7071067812)^1/8

Input interpretation:

$$\sqrt[8]{0.7071067812}$$

Result:

0.95760328070...

0.9576032807….. result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}} - \varphi + 1} = 1 - \cfrac{e^{-\pi}}{1 + \cfrac{e^{-2\pi}}{1 + \cfrac{e^{-3\pi}}{1 + \cfrac{e^{-4\pi}}{1 + \dots}}}} \approx 0.9568666373$$

(0.7071067812)^(1/(2e))

Input interpretation:

$$\sqrt[2e]{0.7071067812}$$

Result:

0.9382407973...

0.9382407973…. result very near to the spectral index n_s and to the mesonic Regge slope (see Appendix) and to the inflaton value at the end of the inflation 0.9402

Alternative representation:

$$\sqrt[2e]{0.707107} = \sqrt[2\exp(z)]{0.707107} \quad \text{for } z = 1$$

Series representations:

$$\sqrt[2e]{0.707107} = 0.707107^{1/2\sum_{k=0}^{\infty}(-1)^k/k!}$$

$$\sqrt[2e]{0.707107} = e^{-0.173287/\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)}$$

$$\sqrt[2e]{0.707107} = \sqrt[\sum_{k=0}^{\infty}\frac{1+k}{k!}]{0.707107}$$

Integral representation:

$$(1+z)^a = \frac{\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{\Gamma(s)\,\Gamma(-a-s)}{z^s}\,d\,s}{(2\,\pi\,i)\,\Gamma(-a)} \quad \text{for } (0 < \gamma < -\text{Re}(a) \text{ and } |\arg(z)| < \pi)$$

(0.7071067812)^1/32

Input interpretation:

$$\sqrt[32]{0.7071067812}$$

Result:

0.989228013195...

0.989228013195.... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1} = 1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\dots}}}} \approx 0.9991104684$$

And to the dilaton value $\mathbf{0.989117352243} = \boldsymbol{\phi}$

Now, we have that:

exp(0.7071067812)^32

Input interpretation:

$\exp^{32}(0.7071067812)$

Result:

$6.71370636... \times 10^9$

$6.71370636... * 10^9 \approx 6713706360$

32/ln(6.71370636 × 10^9)

Input interpretation:

$$\frac{32}{\log(6.71370636 \times 10^9)}$$

log(x) is the natural logarithm

Result:

1.414213562...

1.414213562...

From the formula concerning the 5th order' mock theta function psi_1(q). (OEIS – sequence A053261

a(n) ~ sqrt(phi) * exp(Pi*sqrt(n/15)) / (2*5^(1/4)*sqrt(n)).

we obtain, for n = 1106.87772:

Input interpretation:

$$\sqrt{\phi} \times \frac{\exp\left(\pi \sqrt{\frac{1106.87772}{15}}\right)}{2 \sqrt[4]{5} \sqrt{1106.87772}}$$

ϕ is the golden ratio

Result:

$6.7137015... \times 10^9$

$6.7137015… * 10^9 = 6713701500$

Series representations:

$$\frac{\sqrt{\phi}\,\exp\left(\pi\sqrt{\frac{1106.88}{15}}\right)}{2\sqrt[4]{5}\,\sqrt{1106.88}} = \frac{\exp\left(\pi\sqrt{z_0}\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (73.7918-z_0)^k z_0^{-k}}{k!}\right)\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (\phi-z_0)^k z_0^{-k}}{k!}}{2\sqrt[4]{5}\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (1106.88-z_0)^k z_0^{-k}}{k!}}$$

for not $((z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0))$

$$\frac{\sqrt{\phi}\,\exp\left(\pi\sqrt{\frac{1106.88}{15}}\right)}{2\sqrt[4]{5}\,\sqrt{1106.88}} = \left(\exp\left(i\pi\left\lfloor\frac{\arg(\phi-x)}{2\pi}\right\rfloor\right)\exp\left(\pi\exp\left(i\pi\left\lfloor\frac{\arg(73.7918-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k (73.7918-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\sum_{k=0}^{\infty}\frac{(-1)^k(\phi-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right) \Big/ \left(2\sqrt[4]{5}\,\exp\left(i\pi\left\lfloor\frac{\arg(1106.88-x)}{2\pi}\right\rfloor\right)\sum_{k=0}^{\infty}\frac{(-1)^k(1106.88-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)$$

for $(x \in \mathbb{R}$ and $x < 0)$

$$\frac{\sqrt{\phi}\,\exp\left(\pi\sqrt{\frac{1106.88}{15}}\right)}{2\sqrt[4]{5}\,\sqrt{1106.88}} = \left(\exp\left(\pi\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(73.7918-z_0)/(2\pi)\rfloor} z_0^{1/2(1+\lfloor\arg(73.7918-z_0)/(2\pi)\rfloor)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(73.7918-z_0)^k z_0^{-k}}{k!}\right)\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(1106.88-z_0)/(2\pi)\rfloor+1/2\lfloor\arg(\phi-z_0)/(2\pi)\rfloor} z_0^{-1/2\lfloor\arg(1106.88-z_0)/(2\pi)\rfloor+1/2\lfloor\arg(\phi-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(\phi-z_0)^k z_0^{-k}}{k!}\right) \Big/ \left(2\sqrt[4]{5}\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(1106.88-z_0)^k z_0^{-k}}{k!}\right)$$

6713706360 - 6713701500 = 4.860

sqrt(golden ratio) * exp(Pi*sqrt(1106.87772/15)) / (2*5^(1/4)*sqrt(1106.87772)) + 4096 + 64*12 – 24

Input interpretation:

$$\sqrt{\phi}\times\frac{\exp\left(\pi\sqrt{\frac{1106.87772}{15}}\right)}{2\sqrt[4]{5}\sqrt{1106.87772}}+4096+64\times12-24$$

ϕ is the golden ratio

Result:

$6.7137063619365455718345311093822500858909715671691 2471... \times 10^9$

6713706361.936545571834531109382250085890971567 16

6713706361.93654…

Series representations:

$$\frac{\sqrt{\phi}\exp\left(\pi\sqrt{\frac{1106.88}{15}}\right)}{2\sqrt[4]{5}\sqrt{1106.88}}+4096+64\times12-24=$$
$$\left(48400\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(1106.88-z_0)^k z_0^{-k}}{k!}+\right.$$
$$5^{3/4}\exp\left(\pi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(73.7918-z_0)^k z_0^{-k}}{k!}\right)$$
$$\left.\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(\phi-z_0)^k z_0^{-k}}{k!}\right)\Big/$$
$$\left(10\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(1106.88-z_0)^k z_0^{-k}}{k!}\right)\text{ for not }((z_0\in\mathbb{R}\text{ and }-\infty<z_0\le 0))$$

$$\frac{\sqrt{\phi}\,\exp\left(\pi\sqrt{\frac{1106.88}{15}}\right)}{2\sqrt[4]{5}\,\sqrt{1106.88}}+4096+64\times 12-24=$$

$$\left(48\,400\exp\left(i\pi\left\lfloor\frac{\arg(1106.88-x)}{2\pi}\right\rfloor\right)\sum_{k=0}^{\infty}\frac{(-1)^k\,(1106.88-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}+5^{3/4}\exp\left(i\pi\left\lfloor\frac{\arg(\phi-x)}{2\pi}\right\rfloor\right)\exp\left(\pi\exp\left(i\pi\left\lfloor\frac{\arg(73.7918-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k\,(73.7918-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\sum_{k=0}^{\infty}\frac{(-1)^k\,(\phi-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\Bigg/\left(10\exp\left(i\pi\left\lfloor\frac{\arg(1106.88-x)}{2\pi}\right\rfloor\right)\sum_{k=0}^{\infty}\frac{(-1)^k\,(1106.88-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)$$

for ($x\in\mathbb{R}$ and $x<0$)

$$\frac{\sqrt{\phi}\,\exp\left(\pi\sqrt{\frac{1106.88}{15}}\right)}{2\sqrt[4]{5}\,\sqrt{1106.88}}+4096+64\times 12-24=$$

$$\left(\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(1106.88-z_0)/(2\pi)\rfloor}z_0^{-1/2\lfloor\arg(1106.88-z_0)/(2\pi)\rfloor}\left(48\,400\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(1106.88-z_0)/(2\pi)\rfloor}z_0^{1/2\lfloor\arg(1106.88-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(1106.88-z_0)^k\,z_0^{-k}}{k!}+5^{3/4}\exp\left(\pi\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(73.7918-z_0)/(2\pi)\rfloor}z_0^{1/2(1+\lfloor\arg(73.7918-z_0)/(2\pi)\rfloor)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(73.7918-z_0)^k\,z_0^{-k}}{k!}\right)\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(\phi-z_0)/(2\pi)\rfloor}z_0^{1/2\lfloor\arg(\phi-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(\phi-z_0)^k\,z_0^{-k}}{k!}\right)\right)\Bigg/\left(10\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(1106.88-z_0)^k\,z_0^{-k}}{k!}\right)$$

32/ln(6.71370636193654557 × 10^9)

Input interpretation:

$$\frac{32}{\log(6.71370636193654557 \times 10^9)}$$

log(x) is the natural logarithm

Result:

1.414213562286304376...

1.414213562...

32/ln(6.71370635550366316 × 10^9) ≈ 32/ln(6.71370636193654557 × 10^9)

If we approximate both values to 6.71370636 × 10^9, we obtain:

1/((32/ln(6.71370636 × 10^9))) = 1/((32/ln(6.71370636 × 10^9)

Input interpretation:

$$\frac{1}{\frac{32}{\log(6.71370636 \times 10^9)}} = \frac{1}{\frac{32}{\log(6.71370636 \times 10^9)}}$$

log(x) is the natural logarithm

Result:

True

Result:

0.707106781220928905539368942260066879657798748958840643629...

0.7071067812…

Possible closed forms:

$$\frac{1}{\sqrt{2}} \approx 0.70710678118654$$

$$\frac{19}{35} - \frac{1}{11 e} + \frac{4 e}{55} \approx 0.70710678118403$$

$$\frac{4 \pi \pi! + 7 - 14 \pi + 9 \pi^2}{64 \pi} \approx 0.707106781273418$$

We note that the fundamental result in these formulas is ≈ 1/√2 = 0.707106781186

From:

http://www.nat.vu.nl/~wimu/EDUC/QB_Lecture_7b-2014.pdf

Addition of spins in a 2-electron system

$\vec{S} = \vec{s}_1 + \vec{s}_2 \qquad M_S = m_{s_1} + m_{s_2} \quad ;$ S= 0, 1 $\quad$ M_S = -1, 0, 1

$$|S=1, M_S=1\rangle = |\uparrow,\uparrow\rangle$$
$$|S=1, M_S=0\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow,\downarrow\rangle + |\downarrow,\uparrow\rangle\right)$$
$$|S=1, M_S=-1\rangle = |\downarrow,\downarrow\rangle$$

A triplet of symmetric spin wave functions

$$|S=0, M_S=0\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow,\downarrow\rangle - |\downarrow,\uparrow\rangle\right)$$

A singlet of an anti-symmetric spin wave function

From:

Pion family in AdS/QCD: the next generation from configurational entropy
Luiz F. Ferreira and R. da Rocha - arXiv:1902.04534v2 [hep-th] 2 Apr 2019

0.431^2 = 0.185761;

(0.431^2)^1/4096

Input:

$\sqrt[4096]{0.431^2}$

Result:

0.999589124...

0.999589124.... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1} = 1-\cfrac{e^{-\pi\sqrt{5}}}{1+\cfrac{e^{-2\pi\sqrt{5}}}{1+\cfrac{e^{-3\pi\sqrt{5}}}{1+\cfrac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}} \approx 0.9991104684$$

And to the dilaton value $\mathbf{0.989117352243} = \boldsymbol{\phi}$

0.431^2*tanh(0.431^4/0.431^2)

Input:

$$0.431^2 \tanh\left(\frac{0.431^4}{0.431^2}\right)$$

$\tanh(x)$ is the hyperbolic tangent function

Result:

0.034115637763873052948700783822724302904626823348235129924...

0.034115637…

Alternative representations:

$$0.431^2 \tanh\left(\frac{0.431^4}{0.431^2}\right) = \frac{0.431^2}{\coth\left(\frac{0.431^4}{0.431^2}\right)}$$

$$0.431^2 \tanh\left(\frac{0.431^4}{0.431^2}\right) = 0.431^2\left(-1+\frac{2}{1+e^{-(2\times 0.431^4)/0.431^2}}\right)$$

$$0.431^2 \tanh\left(\frac{0.431^4}{0.431^2}\right) = \coth\left(-\frac{i\pi}{2}+\frac{0.431^4}{0.431^2}\right)0.431^2$$

Series representations:

$$0.431^2 \tanh\left(\frac{0.431^4}{0.431^2}\right) = -0.185761 - 0.371522 \sum_{k=1}^{\infty} (-1)^k q^{2k} \quad \text{for } q = 1.20413$$

$$0.431^2 \tanh\left(\frac{0.431^4}{0.431^2}\right) = 0.276057 \sum_{k=1}^{\infty} \frac{1}{0.138029 + (1-2k)^2 \pi^2}$$

$$0.431^2 \tanh\left(\frac{0.431^4}{0.431^2}\right) = \sum_{k=1}^{\infty} \frac{\left(-1+4^k\right) e^{-1.98029 k} B_{2k}}{(2k)!}$$

Integral representation:

$$0.431^2 \tanh\left(\frac{0.431^4}{0.431^2}\right) = 0.185761 \int_0^{0.185761} \operatorname{sech}^2(t)\, dt$$

2/(((0.431^2*tanh(0.431^4/0.431^2))))^2

Input:

$$\frac{2}{\left(0.431^2 \tanh\left(\frac{0.431^4}{0.431^2}\right)\right)^2}$$

$\tanh(x)$ is the hyperbolic tangent function

Result:

1718.40...

1718.40…

This result is very near to the mass of candidate glueball $f_0(1710)$ meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

Alternative representations:

$$\frac{2}{\left(0.431^2 \tanh\left(\frac{0.431^4}{0.431^2}\right)\right)^2} = \frac{2}{\left(\frac{0.431^2}{\coth\left(\frac{0.431^4}{0.431^2}\right)}\right)^2}$$

$$\frac{2}{\left(0.431^2 \tanh\left(\frac{0.431^4}{0.431^2}\right)\right)^2} = \frac{2}{\left(\coth\left(-\frac{i\pi}{2} + \frac{0.431^4}{0.431^2}\right) 0.431^2\right)^2}$$

$$\frac{2}{\left(0.431^2 \tanh\left(\frac{0.431^4}{0.431^2}\right)\right)^2} = \frac{2}{\left(0.431^2 \left(-1 + \frac{2}{1+e^{-(2\times 0.431^4)/0.431^2}}\right)\right)^2}$$

Integral representation:

$$\frac{2}{\left(0.431^2 \tanh\left(\frac{0.431^4}{0.431^2}\right)\right)^2} = \frac{57.959}{\left(\int_0^{0.185761} \operatorname{sech}^2(t)\, dt\right)^2}$$

(((0.431^2*tanh(0.431^4/0.431^2))))^1/4096

Input:

$$\sqrt[4096]{0.431^2 \tanh\left(\frac{0.431^4}{0.431^2}\right)}$$

tanh(x) is the hyperbolic tangent function

Result:

0.999175633...

0.999175633.... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5 \sqrt[4]{5^3}}-1}} - \varphi + 1} = 1 - \frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\dots}}}} \approx 0.9991104684$$

And to the dilaton value $\mathbf{0.989117352243} = \boldsymbol{\phi}$

(golden ratio+18+76+322)*1/0.185761

Where 18, 76 and 322 are Lucas numbers

Input interpretation:

$$(\phi + 18 + 76 + 322) \times \frac{1}{0.185761}$$

ϕ is the golden ratio

Result:

2248.15...

2248.15…

Alternative representations:

$$\frac{\phi + 18 + 76 + 322}{0.185761} = \frac{416 + 2\sin(54°)}{0.185761}$$

$$\frac{\phi + 18 + 76 + 322}{0.185761} = \frac{416 - 2\cos(216°)}{0.185761}$$

$$\frac{\phi + 18 + 76 + 322}{0.185761} = \frac{416 - 2\sin(666°)}{0.185761}$$

And:

(golden ratio^2+11+47+322)*1/0.185761

Input interpretation:

$$(\phi^2 + 11 + 47 + 322) \times \frac{1}{0.185761}$$

ϕ is the golden ratio

Result:

2059.73...

2059.73…

Alternative representations:

$$\frac{\phi^2+11+47+322}{0.185761}=\frac{380+(2\sin(54\,°))^2}{0.185761}$$

$$\frac{\phi^2+11+47+322}{0.185761}=\frac{380+(-2\cos(216\,°))^2}{0.185761}$$

$$\frac{\phi^2+11+47+322}{0.185761}=\frac{380+(-2\sin(666\,°))^2}{0.185761}$$

And:

(golden ratio-199+29)+76/(((0.431^2*tanh(0.431^4/0.431^2))))

Input:

$$(\phi-199+29)+\frac{76}{0.431^2\tanh\left(\frac{0.431^4}{0.431^2}\right)}$$

$\tanh(x)$ is the hyperbolic tangent function

ϕ is the golden ratio

Result:

2059.34...

2059.34…

Alternative representations:

$$(\phi-199+29)+\frac{76}{0.431^2\tanh\left(\frac{0.431^4}{0.431^2}\right)}=-170+\phi+\frac{76}{\frac{0.431^2}{\coth\left(\frac{0.431^4}{0.431^2}\right)}}$$

$$(\phi-199+29)+\frac{76}{0.431^2\tanh\left(\frac{0.431^4}{0.431^2}\right)}=-170+\phi+\frac{76}{0.431^2\left(-1+\frac{2}{1+e^{-(2\times0.431^4)/0.431^2}}\right)}$$

$$(\phi-199+29)+\frac{76}{0.431^2\tanh\left(\frac{0.431^4}{0.431^2}\right)}=-170+\phi+\frac{76}{\coth\left(-\frac{i\pi}{2}+\frac{0.431^4}{0.431^2}\right)0.431^2}$$

Series representations:

$$(\phi-199+29)+\frac{76}{0.431^2\tanh\left(\frac{0.431^4}{0.431^2}\right)}=-170+\phi-\frac{204.564}{0.5+\sum_{k=1}^{\infty}(-1)^k q^{2k}}$$

for $q = 1.20413$

$$(\phi-199+29)+\frac{76}{0.431^2\tanh\left(\frac{0.431^4}{0.431^2}\right)}=-170+\phi+\frac{275.305}{\sum_{k=1}^{\infty}\frac{1}{0.138029+(1-2k)^2\pi^2}}$$

$$(\phi-199+29)+\frac{76}{0.431^2\tanh\left(\frac{0.431^4}{0.431^2}\right)}=-170+\phi+\frac{76.}{\sum_{k=1}^{\infty}\frac{(-1+4^k)e^{-1.98029k}B_{2k}}{(2k)!}}$$

Integral representation:

$$(\phi-199+29)+\frac{76}{0.431^2\tanh\left(\frac{0.431^4}{0.431^2}\right)}=-170+\phi+\frac{409.128}{\int_0^{0.185761}\operatorname{sech}^2(t)\,dt}$$

(18+2)+76/(((0.431^2*tanh(0.431^4/0.431^2))))

Where 18, 2 and 76 are Lucas numbers

Input:

$$(18+2)+\frac{76}{0.431^2\tanh\left(\frac{0.431^4}{0.431^2}\right)}$$

$\tanh(x)$ is the hyperbolic tangent function

Result:

2247.72...

2247.72…

Alternative representations:

$$(18+2)+\frac{76}{0.431^2\tanh\left(\frac{0.431^4}{0.431^2}\right)}=20+\frac{76}{\frac{0.431^2}{\coth\left(\frac{0.431^4}{0.431^2}\right)}}$$

$$(18+2)+\frac{76}{0.431^2\tanh\left(\frac{0.431^4}{0.431^2}\right)}=20+\frac{76}{0.431^2\left(-1+\frac{2}{1+e^{-(2\times 0.431^4)/0.431^2}}\right)}$$

$$(18+2)+\frac{76}{0.431^2\tanh\left(\frac{0.431^4}{0.431^2}\right)}=20+\frac{76}{\coth\left(-\frac{i\pi}{2}+\frac{0.431^4}{0.431^2}\right)0.431^2}$$

Series representations:

$$(18+2)+\frac{76}{0.431^2\tanh\left(\frac{0.431^4}{0.431^2}\right)}=20-\frac{204.564}{0.5+\sum_{k=1}^{\infty}(-1)^k q^{2k}} \quad \text{for } q=1.20413$$

$$(18+2)+\frac{76}{0.431^2\tanh\left(\frac{0.431^4}{0.431^2}\right)}=20+\frac{275.305}{\sum_{k=1}^{\infty}\frac{1}{0.138029+(1-2k)^2\pi^2}}$$

$$(18+2)+\frac{76}{0.431^2\tanh\left(\frac{0.431^4}{0.431^2}\right)}=20+\frac{76.}{\sum_{k=1}^{\infty}\frac{(-1+4^k)e^{-1.98029k}B_{2k}}{(2k)!}}$$

Integral representation:

$$(18+2)+\frac{76}{0.431^2\tanh\left(\frac{0.431^4}{0.431^2}\right)}=20+\frac{409.128}{\int_0^{0.185761}\operatorname{sech}^2(t)\,dt}$$

We note that, from the two previous equation, we obtain:

((((18+2)+76/(((0.431^2*tanh(0.431^4/0.431^2)))))))^1/16

Input:

$$\sqrt[16]{(18+2)+\frac{76}{0.431^2\tanh\left(\frac{0.431^4}{0.431^2}\right)}}$$

$\tanh(x)$ is the hyperbolic tangent function

Result:

1.619883779830863987026324022904798350478741288124596669262...

1.61988377...

And:

(((1/(((0.431^2*tanh(0.431^4/0.431^2)))))))^1/7

Input:

$$\sqrt[7]{\frac{1}{0.431^2\tanh\left(\frac{0.431^4}{0.431^2}\right)}}$$

$\tanh(x)$ is the hyperbolic tangent function

Result:

1.620235234053204103880884001107038840196492652612601922871...

1.620235234...

The two results 1.61988377... and 1.620235234... are very good approximation to the value of the golden ratio 1,618033988749...

Now, we have that:

Another information provided by the configurational entropic Regge trajectories is the values of the masses of the next generation of the π states. Using the value CE of the n^{th} excitation, Eqs. (22, 23), one can employ Eqs. (22) and (24) to infer the mass spectra of the π_6, π_7 and π_8, as discussed throughtout Sect. III. In the case of the quadratic dilaton the results found are $m_{\pi,6} = 2630 \pm 18$ MeV, $m_{\pi,7} = 2861 \pm 22$ MeV and $m_{\pi,8} = 3074 \pm 25$ MeV. On the other hand, for the deformed dilaton the masses found are $m_{\pi,6} = 2631 \pm 18$ MeV, $m_{\pi,7} = 2801 \pm 22$ MeV and $m_{\pi,8} = 2959 \pm 25$ MeV. It is possible to improve these values of the masses with the eventual detection of the pion excitation states, that shall contribute with more experimental points in Fig. (1).

The values

quadratic dilaton the results found are $m_{\pi,6} = 2630 \pm 18$ MeV, $m_{\pi,7} = 2861 \pm 22$ MeV and $m_{\pi,8} = 3074 \pm 25$ MeV.

And

On the other hand, for the deformed dilaton the masses found are $m_{\pi,6} = 2631 \pm 18$ MeV, $m_{\pi,7} = 2801 \pm 22$ MeV and $m_{\pi,8} = 2959 \pm 25$ MeV. It is possible to improve

Can be related with some Rogers-Ramanujan continued fractions value and Ramanujan mock theta functions. Indeed:

2.630 and 2.631 GeV are connected to the following Ramanujan mock theta functions:

((((1/(1-0.449329) + (0.449329) / ((1-0.449329^2)(1-0.449329^3)))) + ((((0.449329)^2 / ((1-0.449329^3)(1-0.449329^4)(1-0.449329^5))))

Input interpretation:

$$\left(\frac{1}{1-0.449329}+\frac{0.449329}{(1-0.449329^2)(1-0.449329^3)}\right)+\frac{0.449329^2}{(1-0.449329^3)(1-0.449329^4)(1-0.449329^5)}$$

Result:

2.6709253774829457236393175700282750163088358240744567694 61...

$\chi(q)$ **= 2.6709253774829…**

And

2.861 and 2.801 GeV are connected to the following Ramanujan mock theta functions:

2((((((1+(0.449329^2)/(1-0.449329) + (0.449329)^8 / ((1-0.449329)(1-0.449329^3)))))))

Input interpretation:

$$2\left(1+\frac{0.449329^2}{1-0.449329}+\frac{0.449329^8}{(1-0.449329)(1-0.449329^3)}\right)$$

Result:

2.739911418085160509931688101818145762793200697288896419870...

2F(q) = 2.73991141808516...

Thence, from:

$$\left(-\frac{d^2}{dz^2}+V_\pi(z)\right)\pi_n(z)=m_n^2(\pi_n(z)-e^A\xi\varphi_n(z)), \qquad (11)$$

$$\left(-\frac{d^2}{dz^2}+V_\varphi(z)\right)\varphi_n(z)=e^A\xi(\pi_n(z)-e^A\xi\varphi_n(z)), \quad (12)$$

we have the following mathematical connections:

$$\begin{bmatrix} \left(-\frac{d^2}{dz^2}+V_\pi(z)\right)\pi_n(z)=m_n^2(\pi_n(z)-e^A\xi\varphi_n(z)) \\ \left(-\frac{d^2}{dz^2}+V_\varphi(z)\right)\varphi_n(z)=e^A\xi(\pi_n(z)-e^A\xi\varphi_n(z)) \end{bmatrix} = \mathbf{2.630; 2.631} \Rightarrow$$

$$\Rightarrow \left[\left(\frac{1}{1-0.449329}+\frac{0.449329}{(1-0.449329^2)(1-0.449329^3)}\right)+\frac{0.449329^2}{(1-0.449329^3)(1-0.449329^4)(1-0.449329^5)}\right] = \mathbf{2.6709253774829\ldots}$$

And:

$$\left[\begin{array}{c}\left(-\frac{d^2}{dz^2}+V_\pi(z)\right)\pi_n(z)=m_n^2(\pi_n(z)-e^A\xi\varphi_n(z)) \\ \left(-\frac{d^2}{dz^2}+V_\varphi(z)\right)\varphi_n(z)=e^A\xi(\pi_n(z)-e^A\xi\varphi_n(z))\end{array}\right] = \mathbf{2.801; 2.861} \Rightarrow$$

$$\Rightarrow \left[2\left(1+\frac{0.449329^2}{1-0.449329}+\frac{0.449329^8}{(1-0.449329)(1-0.449329^3)}\right) \right] = \mathbf{2.73991141808516...}$$

With regard the Rogers-Ramanujan continued fractions, we have the following connections:

n	Experimental	$\text{mass}_{\phi_1(z)}$	$\text{mass}_{\phi_2(z)}$
4*	2070	2006	2059
5*	2360	2203	2247

the $\phi_2(z) = z^2 \tanh(\mu_{G^2}^4 z^2/\mu_G^2)$ dilaton, for the $\pi(2070)$, $\pi(2360)$ mesons.

The values of dilaton are 2.059 and 2.247 GeV (the mean is 2.153), very near to the following Rogers-Ramanujan continued fraction value:

$$\sqrt{\frac{e\pi}{2}} = \sum_{n=0}^{\infty}\frac{1}{(2n+1)!!}+\cfrac{1}{1+\cfrac{1}{1+\cfrac{2}{1+\cfrac{3}{1+\cfrac{4}{1+\dots}}}}} \approx 2.0663656771$$

Thence, the following mathematical connections:

$$\left[\phi_2(z) = z^2 \tanh\left(\mu_G^4{}_2 z^2/\mu_G^2\right)\right] = 2.059; 2.247 \Rightarrow$$

$$\Rightarrow \left(\sqrt{\frac{e\pi}{2}} = \sum_{n=0}^{\infty} \frac{1}{(2n+1)!!} + \cfrac{1}{1+\cfrac{1}{1+\cfrac{2}{1+\cfrac{3}{1+\cfrac{4}{1+\ldots}}}}} \approx 2.0663656771 \right) = 2.0663656771$$

From:

Vafa-Witten theory and iterated integrals of modular forms
Jan Manschot - https://arxiv.org/abs/1709.10098v2

Using the iterated integral m_2, we can in turn write M_2 as an iterated period integral [29, 24]. One finds for the various domains of u_1 and u_2:

- for $u_1 \neq 0$ and $u_2 - \alpha u_1 \neq 0$:

$$-\frac{u_1 u_2}{2y} q^{\frac{u_1^2}{4y}+\frac{u_2^2}{4y}} \int_{-\bar{\tau}}^{i\infty} dw_2 \int_{w_2}^{i\infty} dw_1 \frac{e^{\frac{\pi i u_1^2 w_1}{2y}+\frac{\pi i u_2^2 w_2}{2y}}}{\sqrt{-(w_1+\tau)(w_2+\tau)}}$$
$$-\frac{(u_1+\alpha u_2)(u_2-\alpha u_1)}{2y(1+\alpha^2)} q^{\frac{u_1^2}{4y}+\frac{u_2^2}{4y}} \int_{-\bar{\tau}}^{i\infty} dw_2 \int_{w_2}^{i\infty} dw_1 \frac{e^{\frac{\pi i (u_2-\alpha u_1)^2 w_1}{2(1+\alpha^2)y}+\frac{\pi i (u_1+\alpha u_2)^2 w_2}{2(1+\alpha^2)y}}}{\sqrt{-(w_1+\tau)(w_2+\tau)}}, \tag{4.27}$$

- for $u_1 = 0$, $u_2 \neq 0$:

$$-\frac{\alpha u_2^2}{2y(1+\alpha^2)} q^{\frac{u_2^2}{4y}} \int_{-\bar{\tau}}^{i\infty} dw_2 \int_{w_2}^{i\infty} dw_1 \frac{e^{\frac{\pi i u_2^2 w_1}{2(1+\alpha^2)y}+\frac{\pi i \alpha^2 u_2^2 w_2}{2(1+\alpha^2)y}}}{\sqrt{-(w_1+\tau)(w_2+\tau)}}, \tag{4.28}$$

- for $u_1 \neq 0$, $u_1 - \alpha u_2 = 0$:

$$-\frac{u_1 u_2}{2y} q^{\frac{u_1^2}{4y}+\frac{u_2^2}{4y}} \int_{-\bar{\tau}}^{i\infty} dw_2 \int_{w_2}^{i\infty} dw_1 \frac{e^{\frac{\pi i u_1^2 w_1}{2y}+\frac{\pi i u_2^2 w_2}{2y}}}{\sqrt{-(w_1+\tau)(w_2+\tau)}}, \tag{4.29}$$

- for $u_1 = u_2 = 0$:

$$\frac{2}{\pi} \arctan \alpha. \tag{4.30}$$

From the eq. (4.30), we obtain:

((2/Pi*arctan(Pi)))^1/248

Input:

$$\sqrt[248]{\frac{2}{\pi} \tan^{-1}(\pi)}$$

$\tan^{-1}(x)$ is the inverse tangent function

Exact Result:

$$\sqrt[248]{\frac{2 \tan^{-1}(\pi)}{\pi}}$$

(result in radians)

Decimal approximation:

0.999119790709955650065346590661826648758006729720900193477...

(result in radians)

0.99911979…. result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}}\approx 0.9991104684$$

and to the dilaton value **0.989117352243** = $\boldsymbol{\phi}$

Alternate form:

$$\sqrt[248]{\frac{i(\log(1-i\pi)-\log(1+i\pi))}{\pi}}$$

All 248th roots of (2 tan^(-1)(π))/π:

$$e^0\sqrt[248]{\frac{2\tan^{-1}(\pi)}{\pi}}\approx 0.9991198 \text{ (real, principal root)}$$

$$e^{(i\pi)/124}\sqrt[248]{\frac{2\tan^{-1}(\pi)}{\pi}}\approx 0.998799+0.025310\,i$$

$$e^{(i\pi)/62}\sqrt[248]{\frac{2\tan^{-1}(\pi)}{\pi}}\approx 0.997837+0.05060\,i$$

$$e^{(3i\pi)/124}\sqrt[248]{\frac{2\tan^{-1}(\pi)}{\pi}}\approx 0.996235+0.07587\,i$$

$$e^{(i\pi)/31}\sqrt[248]{\frac{2\tan^{-1}(\pi)}{\pi}}\approx 0.993994+0.10108\,i$$

Alternative representations:

$$\sqrt[248]{\frac{\tan^{-1}(\pi)\,2}{\pi}} = \sqrt[248]{\frac{2\,\mathrm{sc}^{-1}(\pi \mid 0)}{\pi}}$$

$$\sqrt[248]{\frac{\tan^{-1}(\pi)\,2}{\pi}} = \sqrt[248]{\frac{2\cot^{-1}\left(\frac{1}{\pi}\right)}{\pi}}$$

$$\sqrt[248]{\frac{\tan^{-1}(\pi)\,2}{\pi}} = \sqrt[248]{\frac{2\tan^{-1}(1,\pi)}{\pi}}$$

Series representations:

$$\sqrt[248]{\frac{\tan^{-1}(\pi)\,2}{\pi}} = \sqrt[248]{1 - \frac{2\sum_{k=0}^{\infty}\frac{(-1)^k \pi^{-1-2k}}{1+2k}}{\pi}}$$

$$\sqrt[248]{\frac{\tan^{-1}(\pi)\,2}{\pi}} = \sqrt[248]{\frac{2}{\pi}}\,\sqrt[248]{\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{5}\right)^k (2\pi)^{1+2k}\left(1+\sqrt{1+\frac{4\pi^2}{5}}\right)^{-1-2k} F_{1+2k}}{1+2k}}$$

$$\sqrt[248]{\frac{\tan^{-1}(\pi)\,2}{\pi}} = \frac{\sqrt[248]{2\tan^{-1}(z_0) + i\sum_{k=1}^{\infty}\frac{\left(-(-i-z_0)^{-k}+(i-z_0)^{-k}\right)(\pi-z_0)^k}{k}}}{\sqrt[248]{\pi}}$$

for $(i z_0 \notin \mathbb{R}$ or $($ not $(1 \le i z_0 < \infty)$ and not $(-\infty < i z_0 \le -1)))$

Integral representations:

$$\sqrt[248]{\frac{\tan^{-1}(\pi)\,2}{\pi}} = \sqrt[248]{2}\,\sqrt[248]{\int_0^1 \frac{1}{1+\pi^2 t^2}\,dt}$$

$$\sqrt[248]{\frac{\tan^{-1}(\pi)\,2}{\pi}} = \frac{\sqrt[248]{-i\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\left(1+\pi^2\right)^{-s}\,\Gamma\left(\frac{1}{2}-s\right)\Gamma(1-s)\,\Gamma(s)^2\,ds}}{\sqrt[248]{2}\;\pi^{3/496}} \quad \text{for } 0<\gamma<\frac{1}{2}$$

$$\sqrt[248]{\frac{\tan^{-1}(\pi)\,2}{\pi}} = \frac{\sqrt[248]{-i\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{\pi^{-2s}\,\Gamma\left(\frac{1}{2}-s\right)\Gamma(1-s)\,\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\,ds}}{\sqrt[248]{2\pi}} \quad \text{for } 0<\gamma<\frac{1}{2}$$

Continued fraction representations:

$$\sqrt[248]{\frac{\tan^{-1}(\pi)\,2}{\pi}} = \sqrt[248]{2}\,\sqrt[248]{\frac{1}{1+\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{k^2\pi^2}{1+2k}}} = \sqrt[248]{2}\,\sqrt[248]{\frac{1}{1+\frac{\pi^2}{3+\frac{4\pi^2}{5+\frac{9\pi^2}{7+\frac{16\pi^2}{9+\dots}}}}}}$$

$$\sqrt[248]{\frac{\tan^{-1}(\pi)\,2}{\pi}} = \sqrt[248]{2-\frac{2\pi^2}{3+\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\left(1+(-1)^{1+k}+k\right)^2\pi^2}{3+2k}}} = \sqrt[248]{2-\frac{2\pi^2}{3+\frac{9\pi^2}{5+\frac{4\pi^2}{7+\frac{25\pi^2}{9+\frac{16\pi^2}{11+\dots}}}}}}$$

$$\sqrt[248]{\frac{\tan^{-1}(\pi)\,2}{\pi}} = \sqrt[248]{2}\,\sqrt[248]{\frac{1}{1+\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{(1-2k)^2\pi^2}{1+\pi^2-2k\left(-1+\pi^2\right)}}} = \sqrt[248]{2}$$

$$\sqrt[248]{\frac{1}{1+\frac{\pi^2}{1+\pi^2-2\left(-1+\pi^2\right)+\frac{9\pi^2}{1+\pi^2-4\left(-1+\pi^2\right)+\frac{25\pi^2}{1+\pi^2-6\left(-1+\pi^2\right)+\frac{49\pi^2}{1+\dots+\pi^2-8\left(-1+\pi^2\right)}}}}}}$$

$$\sqrt[248]{\frac{\tan^{-1}(\pi)\,2}{\pi}} = \sqrt[248]{2}\,\sqrt[248]{\frac{1}{1+\pi^2+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{2\pi^2\left(1-2\left\lfloor\frac{1+k}{2}\right\rfloor\right)\left\lfloor\frac{1+k}{2}\right\rfloor}{(1+2k)\left(1+\frac{1}{2}\left(1+(-1)^k\right)\pi^2\right)}}} =$$

$$\sqrt[248]{2}\,\sqrt[248]{\frac{1}{1+\pi^2+-\frac{2\pi^2}{3-\frac{2\pi^2}{5\left(1+\pi^2\right)-\frac{12\pi^2}{7-\frac{12\pi^2}{9\left(1+\pi^2\right)+\ldots}}}}}}$$

Now, we have that:

Next we move on to $N = 3$. Also for this gauge group, there are only two independent 't Hooft fluxes and therefore only two independent partition functions, $f_{3,\mu}$ with $\mu = 0, 1$. The explicit expressions for the refined partition functions are [28]:

$$\begin{aligned} g_{3,0}(\tau,z) =& \frac{1}{b_{3,0}(\tau,2z)} \sum_{k_1,k_2\in\mathbb{Z}} \frac{w^{-2k_1-4k_2} q^{k_1^2+k_2^2+k_1k_2}}{(1-w^4q^{2k_1+k_2})(1-w^4q^{k_2-k_1})} \\ &+ \frac{2i\eta(\tau)^3}{\theta_1(\tau,4z)\,b_{3,0}(\tau,2z)} \sum_{k\in\mathbb{Z}} \frac{w^{-6k}q^{3k^2}}{1-w^6q^{3k}} \\ &- \frac{\eta(\tau)^6\,\theta_1(\tau,2z)}{\theta_1(\tau,4z)^2\,\theta_1(\tau,6z)\,b_{3,0}(\tau,2z)} - g_{2,0}(\tau,z) - \frac{1}{6}, \end{aligned} \tag{6.17}$$

and

$$\begin{aligned} g_{3,1}(\tau,z) =& \frac{1}{b_{3,0}(\tau,2z)} \sum_{k_1,k_2\in\mathbb{Z}} \frac{w^{-2k_1-4k_2+6} q^{k_1^2+k_2^2+k_1k_2-\frac{1}{3}}}{(1-w^4q^{2k_1+k_2-1})(1-w^4q^{k_2-k_1})} \\ &+ \frac{i\eta(\tau)^3}{\theta_1(\tau,4z)\,b_{3,0}(\tau,2z)} \left(\sum_{k\in\mathbb{Z}} \frac{w^{-6k+6}q^{3k^2-\frac{1}{3}}}{1-w^6q^{3k-1}} + \sum_{k\in\mathbb{Z}} \frac{w^{-6k}q^{3k^2+3k+\frac{2}{3}}}{1-w^6q^{3k+1}} \right). \end{aligned} \tag{6.18}$$

$$\begin{aligned} g_{3,0}(\tau,z) =& \frac{1}{4} + \frac{w^4}{b_{3,0}(\tau,2z)} \sum_{k_1,k_2\in\mathbb{Z}} \frac{w^{-2k_1-4k_2} q^{k_1^2+k_2^2+k_1k_2+2k_1+k_2}}{(1-w^4q^{2k_1+k_2})(1-w^4q^{k_2-k_1})} \\ &+ \frac{2i\,\eta(\tau)^3}{\theta_1(\tau,4z)\,b_{3,0}(\tau,2z)} \left(-\tfrac{1}{2}\theta_3(6\tau,6z) + \sum_{k\in\mathbb{Z}} \frac{w^{-6k}q^{3k^2}}{1-w^6q^{3k}} \right) \\ &- \frac{\eta(\tau)^6\,\theta_1(\tau,2z)}{\theta_1(\tau,4z)^2\,\theta_1(\tau,6z)\,b_{3,0}(\tau,2z)} + \frac{1}{12}. \end{aligned} \tag{6.22}$$

From eqs. (6.17) and (6.22), we obtain:

(1/4+1/12-1/6)

Input:

$$\frac{1}{4}+\frac{1}{12}-\frac{1}{6}$$

Exact result:

$$\frac{1}{6}$$

Decimal approximation:

0.1666666666666666666666666666666666666666666666666666666666...

0.16666666666666666.....

From which:

(1/4+1/12-1/6)^1/4096

Input:

$$\sqrt[4096]{\frac{1}{4}+\frac{1}{12}-\frac{1}{6}}$$

Result:

$$\frac{1}{\sqrt[4096]{6}}$$

Decimal approximation:

0.999562654386818699499243199074948827204014747733214939254...

0.99956265438….

Alternate form:

$$\frac{6^{4095/4096}}{6}$$

Thence, the new mathematical connection with the following Rogers-Ramanujan continued fraction result:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\cfrac{e^{-\pi\sqrt{5}}}{1+\cfrac{e^{-2\pi\sqrt{5}}}{1+\cfrac{e^{-3\pi\sqrt{5}}}{1+\cfrac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}}\approx 0.9991104684$$

$$\sqrt[4096]{\begin{aligned} g_{3,0}(\tau,z) =& \frac{1}{b_{3,0}(\tau,2z)}\sum_{k_1,k_2\in\mathbb{Z}}\frac{w^{-2k_1-4k_2}q^{k_1^2+k_2^2+k_1k_2}}{(1-w^4q^{2k_1+k_2})(1-w^4q^{k_2-k_1})} \\ &+\frac{2i\eta(\tau)^3}{\theta_1(\tau,4z)\,b_{3,0}(\tau,2z)}\sum_{k\in\mathbb{Z}}\frac{w^{-6k}q^{3k^2}}{1-w^6q^{3k}} \\ &-\frac{\eta(\tau)^6\,\theta_1(\tau,2z)}{\theta_1(\tau,4z)^2\,\theta_1(\tau,6z)\,b_{3,0}(\tau,2z)}-g_{2,0}(\tau,z)-\frac{1}{6}, \\ g_{3,0}(\tau,z) =& \frac{1}{4}+\frac{w^4}{b_{3,0}(\tau,2z)}\sum_{k_1,k_2\in\mathbb{Z}}\frac{w^{-2k_1-4k_2}q^{k_1^2+k_2^2+k_1k_2+2k_1+k_2}}{(1-w^4q^{2k_1+k_2})(1-w^4q^{k_2-k_1})} \\ &+\frac{2i\,\eta(\tau)^3}{\theta_1(\tau,4z)\,b_{3,0}(\tau,2z)}\left(-\tfrac{1}{2}\theta_3(6\tau,6z)+\sum_{k\in\mathbb{Z}}\frac{w^{-6k}q^{3k^2}}{1-w^6q^{3k}}\right) \\ &-\frac{\eta(\tau)^6\,\theta_1(\tau,2z)}{\theta_1(\tau,4z)^2\,\theta_1(\tau,6z)\,b_{3,0}(\tau,2z)}+\frac{1}{12}. \end{aligned}} = 0.99956265438$$

$$\Rightarrow \left[\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\cfrac{e^{-\pi\sqrt{5}}}{1+\cfrac{e^{-2\pi\sqrt{5}}}{1+\cfrac{e^{-3\pi\sqrt{5}}}{1+\cfrac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}}\approx 0.9991104684\right] = 0.9991104684$$

Now, we have that:

$z = 0$, $g_{N,\mu}(\tau, z)$ has a zero of multiplicity $N-1$ at $z=0$. As a result, we can write $f_{N,\mu}$ as the $(N-1)$'th derivative of the refined partition function:

$$f_{N,\mu}(\tau) = \frac{1}{(N-1)!}\left(\tfrac{1}{4\pi i}\partial_z\right)^{N-1} g_{N,\mu}(\tau, z)|_{z=0}. \tag{6.10}$$

The transformation properties of η are given in Equation (3.6), and we are therefore left with determining the modular properties of $f_{N,\mu}$ to verify the modularity of the VW partition function $h_{N,\mu}$. We derive easily from Equation (2.4), that the *expected* transformation properties for the $f_{N,\mu}$ are:

$$\begin{aligned} f_{N,\mu}\left(-\frac{1}{\tau}\right) &= \frac{1}{\sqrt{N}}(-i\tau)^{\frac{3}{2}(N-1)}(-1)^{N-1}\sum_{\nu \mod N} e^{-2\pi i\frac{\mu\nu}{N}} f_{N,\nu}(\tau), \\ f_{N,\mu}(\tau+1) &= (-1)^{\mu} e^{2\pi i\frac{\mu^2}{2N}} f_{N,\mu}(\tau) \end{aligned} \tag{6.11}$$

The q-series $f_{3,\mu}$ is defined in terms of the $g_{3,\mu}(\tau, z)$ by Equation (6.10). Based on the explicit expressions for $g_{3,\mu}(\tau, z)$, (6.18) and (6.22), we can derive explicit q-series for $f_{3,\mu}(\tau)$. To this end, recall the classical Eisenstein series $E_k(\tau)$ of weight $k \in 2\mathbb{N}$, which have the q-expansion

$$E_k(\tau) = 1 - \frac{2k}{B_k}\sum_{n=1}^{\infty}\frac{n^{k-1}q^n}{1-q^n}, \tag{A.1}$$

where $q = e^{2\pi i\tau}$, and B_k are the Bernoulli numbers, $B_2 = \frac{1}{6}$, $B_4 = -\frac{1}{30}$, etc. We define

1-8/(-1/30) sum (2^3*exp(4Pi))/(1-exp(4Pi)), n=1..infinity

(((1-8*1/(-1/30) sum (n^3*exp(2n*Pi))/(1-exp(2n*Pi)), n=1..5)))

Input interpretation:

$$1-\left(8\left(-\frac{1}{\frac{1}{30}}\right)\right)\sum_{n=1}^{5}\frac{n^3\exp(2n\pi)}{1-\exp(2n\pi)}$$

Result:

$$1+240\left(\frac{e^{2\pi}}{1-e^{2\pi}}+\frac{8e^{4\pi}}{1-e^{4\pi}}+\frac{27e^{6\pi}}{1-e^{6\pi}}+\frac{64e^{8\pi}}{1-e^{8\pi}}+\frac{125e^{10\pi}}{1-e^{10\pi}}\right)\approx -53999.5$$

Alternate forms:

$$1-\frac{240e^{2\pi}}{e^{2\pi}-1}-\frac{1920e^{4\pi}}{e^{4\pi}-1}-\frac{6480e^{6\pi}}{e^{6\pi}-1}-\frac{15360e^{8\pi}}{e^{8\pi}-1}-\frac{30000e^{10\pi}}{e^{10\pi}-1}$$

$$1+240\,e^{2\pi}\left(\frac{1}{1-e^{2\pi}}-\frac{8\,e^{2\pi}}{e^{4\pi}-1}-\frac{27\,e^{4\pi}}{e^{6\pi}-1}-\frac{64\,e^{6\pi}}{e^{8\pi}-1}-\frac{125\,e^{8\pi}}{e^{10\pi}-1}\right)$$

$$1+\frac{240\,e^{2\pi}}{1-e^{2\pi}}+\frac{1920\,e^{4\pi}}{1-e^{4\pi}}+\frac{6480\,e^{6\pi}}{1-e^{6\pi}}+\frac{15\,360\,e^{8\pi}}{1-e^{8\pi}}+\frac{30\,000\,e^{10\pi}}{1-e^{10\pi}}$$

We note that:

sqrt[-1/13*(((1-8*1/(-1/30) sum (n^3*exp(2n*Pi))/(1-exp(2n*Pi)), n=1..5)))]

Input interpretation:

$$\sqrt{-\frac{1}{13}\left(1-\left(8\left(-\frac{1}{\frac{1}{30}}\right)\right)\sum_{n=1}^{5}\frac{n^3\exp(2\,n\,\pi)}{1-\exp(2\,n\,\pi)}\right)}$$

Result:

$$\frac{1}{\sqrt{\frac{13}{-1-240\left(\frac{e^{2\pi}}{1-e^{2\pi}}+\frac{8\,e^{4\pi}}{1-e^{4\pi}}+\frac{27\,e^{6\pi}}{1-e^{6\pi}}+\frac{64\,e^{8\pi}}{1-e^{8\pi}}+\frac{125\,e^{10\pi}}{1-e^{10\pi}}\right)}}}\approx 64.45$$

Alternate form:

$$1\Big/\left(\sqrt{\left(\left(13\left(e^{2\pi}-1\right)\left(1+e^{2\pi}\right)\left(1+e^{4\pi}\right)\left(1+e^{2\pi}+e^{4\pi}\right)\left(1+e^{2\pi}+e^{4\pi}+e^{6\pi}+e^{8\pi}\right)\right)\Big/\left(1+242\,e^{2\pi}+2643\,e^{4\pi}+11\,763\,e^{6\pi}+38\,162\,e^{8\pi}+92\,400\,e^{10\pi}+146\,158\,e^{12\pi}+173\,757\,e^{14\pi}+164\,637\,e^{16\pi}+108\,238\,e^{18\pi}+53\,999\,e^{20\pi}\right)\right)}\right)$$

[-1/52*(((1-8*1/(-1/30) sum (n^3*exp(2n*Pi))/(1-exp(2n*Pi)), n=1..5)))]^1/14

Input interpretation:

$$\sqrt[14]{-\frac{1}{52}\left(1-\left(8\left(-\frac{1}{\frac{1}{30}}\right)\right)\sum_{n=1}^{5}\frac{n^3\exp(2\,n\,\pi)}{1-\exp(2\,n\,\pi)}\right)}$$

Result:

$$\frac{1}{\sqrt[7]{2}\,\sqrt[14]{\frac{13}{-1-240\left(\frac{e^{2\pi}}{1-e^{2\pi}}+\frac{8\,e^{4\pi}}{1-e^{4\pi}}+\frac{27\,e^{6\pi}}{1-e^{6\pi}}+\frac{64\,e^{8\pi}}{1-e^{8\pi}}+\frac{125\,e^{10\pi}}{1-e^{10\pi}}\right)}}}\approx 1.64231$$

$1.64231 \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \ldots$

Alternate forms:

$$\frac{1}{\sqrt[7]{2}\ \sqrt[14]{\frac{13}{-1-240\,e^{2\pi}\left(\frac{1}{1-e^{2\pi}}-\frac{8\,e^{2\pi}}{e^{4\pi}-1}-\frac{27\,e^{4\pi}}{e^{6\pi}-1}-\frac{64\,e^{6\pi}}{e^{8\pi}-1}-\frac{125\,e^{8\pi}}{e^{10\pi}-1}\right)}}}$$

$$1\Big/\left(\sqrt[7]{2}\ \left(\left(13\left(e^{2\pi}-1\right)\left(1+e^{2\pi}\right)\left(1+e^{4\pi}\right)\left(1+e^{2\pi}+e^{4\pi}\right)\left(1+e^{2\pi}+e^{4\pi}+e^{6\pi}+e^{8\pi}\right)\right)\Big/\left(1+242\,e^{2\pi}+2643\,e^{4\pi}+11\,763\,e^{6\pi}+38\,162\,e^{8\pi}+92\,400\,e^{10\pi}+146\,158\,e^{12\pi}+173\,757\,e^{14\pi}+164\,637\,e^{16\pi}+108\,238\,e^{18\pi}+53\,999\,e^{20\pi}\right)\right)^{\wedge}(1/14)\right)$$

(((1/[-1/52*(((1-8*1/(-1/30) sum (n^3*exp(2n*Pi))/(1-exp(2n*Pi)), n=1..5)))]^1/14)))^1/512

Input interpretation:

$$\sqrt[512]{\frac{1}{\sqrt[14]{-\frac{1}{52}\left(1-\left(8\left(-\frac{1}{\frac{1}{30}}\right)\right)\sum_{n=1}^{5}\frac{n^3\exp(2\,n\,\pi)}{1-\exp(2\,n\,\pi)}\right)}}}$$

Result:

$$\sqrt[3584]{2}\ \sqrt[7168]{\frac{13}{-1-240\left(\frac{e^{2\pi}}{1-e^{2\pi}}+\frac{8\,e^{4\pi}}{1-e^{4\pi}}+\frac{27\,e^{6\pi}}{1-e^{6\pi}}+\frac{64\,e^{8\pi}}{1-e^{8\pi}}+\frac{125\,e^{10\pi}}{1-e^{10\pi}}\right)}} \approx 0.999032$$

Alternate forms:

$$\sqrt[3584]{2}\ \sqrt[7168]{\frac{13}{-1-240\,e^{2\pi}\left(\frac{1}{1-e^{2\pi}}-\frac{8\,e^{2\pi}}{e^{4\pi}-1}-\frac{27\,e^{4\pi}}{e^{6\pi}-1}-\frac{64\,e^{6\pi}}{e^{8\pi}-1}-\frac{125\,e^{8\pi}}{e^{10\pi}-1}\right)}}$$

$$\sqrt[3584]{2}\ \left(\left(13\left(e^{2\pi}-1\right)\left(1+e^{2\pi}\right)\left(1+e^{4\pi}\right)\left(1+e^{2\pi}+e^{4\pi}\right)\left(1+e^{2\pi}+e^{4\pi}+e^{6\pi}+e^{8\pi}\right)\right)\Big/\left(1+242\,e^{2\pi}+2643\,e^{4\pi}+11\,763\,e^{6\pi}+38\,162\,e^{8\pi}+92\,400\,e^{10\pi}+146\,158\,e^{12\pi}+173\,757\,e^{14\pi}+164\,637\,e^{16\pi}+108\,238\,e^{18\pi}+53\,999\,e^{20\pi}\right)\right)^{\wedge}(1/7168)$$

Thence, we have the following mathematical connection:

$$\left(\sqrt[3584]{2}\ \sqrt[7168]{\frac{13}{-1-240\left(\frac{e^{2\pi}}{1-e^{2\pi}}+\frac{8e^{4\pi}}{1-e^{4\pi}}+\frac{27e^{6\pi}}{1-e^{6\pi}}+\frac{64e^{8\pi}}{1-e^{8\pi}}+\frac{125e^{10\pi}}{1-e^{10\pi}}\right)}} \approx 0.999032 \right) \Rightarrow$$

$$\Rightarrow \left(\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1} = 1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}} \approx 0.9991104684 \right) =$$

$= 0.999032 \approx 0.9991104684$

Furthermore, we have that:

The transformation properties of η are given in Equation (3.6), and we are therefore left with determining the modular properties of $f_{N,\mu}$ to verify the modularity of the VW partition function $h_{N,\mu}$. We derive easily from Equation (2.4), that the *expected* transformation properties for the $f_{N,\mu}$ are:

$$f_{N,\mu}\left(-\frac{1}{\tau}\right) = \frac{1}{\sqrt{N}}(-i\tau)^{\frac{3}{2}(N-1)}(-1)^{N-1}\sum_{\nu \mod N} e^{-2\pi i\frac{\mu\nu}{N}} f_{N,\nu}(\tau),$$
$$f_{N,\mu}(\tau+1) = (-1)^{\mu} e^{2\pi i\frac{\mu^2}{2N}} f_{N,\mu}(\tau) \tag{6.11}$$

(note that (VW) means Vafa-Witten theory)

From the above result -53999.5, for N = 2 and μ = 4, we obtain:

exp(8Pi)i * (-53999.5)

Input interpretation:

$\exp(8\pi)\, i \times (-53\,999.5)$

i is the imaginary unit

Result:

$-4.44018... \times 10^{15}\, i$

Polar coordinates:

$r = 4.44018 \times 10^{15}$ (radius), $\theta = -90°$ (angle)

$4.44018*10^{15}$

And:

1/(((exp(8Pi)i * (-53999.5))))^1/4096

Input interpretation:

$$\frac{1}{\sqrt[4096]{\exp(8\pi)\, i \times (-53\,999.5)}}$$

i is the imaginary unit

Result:

0.991242243... +
0.000380136658... i

Polar coordinates:

$r = 0.991242$ (radius), $\theta = 0.0219727°$ (angle)

0.991242.... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1} = 1-\cfrac{e^{-\pi\sqrt{5}}}{1+\cfrac{e^{-2\pi\sqrt{5}}}{1+\cfrac{e^{-3\pi\sqrt{5}}}{1+\cfrac{e^{-4\pi\sqrt{5}}}{1+\dots}}}} \approx 0.9991104684$$

and to the dilaton value $\mathbf{0.989117352243} = \boldsymbol{\phi}$

Instead, for N = 3 and μ = 4, we obtain:

(((exp(32/6)Pi)i * (-53999.5)))

Input interpretation:

$$\left(\exp\left(\frac{32}{6}\right)\pi\right) i \times (-53\,999.5)$$

i is the imaginary unit

Result:

$-3.51380... \times 10^7 \, i$

Polar coordinates:

$r = 3.5138 \times 10^7$ (radius), $\theta = -90°$ (angle)

$3.5138*10^7$

And:

1/(((exp(32/6)Pi)i * (-53999.5)))^1/4096

Input interpretation:

$$\frac{1}{\sqrt[4096]{\left(\exp\left(\frac{32}{6}\right)\pi\right) i \times (-53\,999.5)}}$$

i is the imaginary unit

Result:

0.995767018... +
0.000381871887... i

Polar coordinates:

$r = 0.995767$ (radius), $\theta = 0.0219727°$ (angle)

0.995767... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1} = 1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}} \approx 0.9991104684$$

and to the dilaton value **0.989117352243** = ϕ

The two results obtained 0.991242 and 0.995767 are very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}}\approx 0.9991104684$$

and to the dilaton value **0.989117352243** = ϕ

From the following ratio between the two previous results $3.5138*10^7$ and $4.44018*10^{15}$, we obtain:

(16+512+1024+4096)i+1/(1e-13)[((((((exp(32/6)Pi)i * (-53999.5)))/((((4.44018*10^15)]

Input interpretation:

$$(16+512+1024+4096)\,i+\frac{1}{1\times 10^{-13}}\times\frac{\left(\exp\left(\frac{32}{6}\right)\pi\right)i\times(-53\,999.5)}{4.44018\times 10^{15}}$$

i is the imaginary unit

Result:

$-73488.4\ldots\,i$

Polar coordinates:

$r = 73\,488.4$ (radius), $\theta = -90°$ (angle)

73488.4

Thence, we have the following mathematical connection:

$$\left[(16+512+1024+4096)i+\frac{1}{1\times 10^{-13}}\times\frac{\left(\exp\left(\frac{32}{6}\right)\pi\right)i\times(-53\,999.5)}{4.44018\times 10^{15}}\right] = 73488.4 \Rightarrow$$

$$\Rightarrow -3927+2\left(\sqrt[13]{\begin{array}{c} N\exp\left[\int d\hat{\sigma}\left(-\frac{1}{4u^2}\mathbf{P}_i D\mathbf{P}_i\right)\right]|Bp\rangle_{\mathrm{NS}} + \\ \int[d\mathbf{X}^\mu]\exp\left\{\int d\hat{\sigma}\left(-\frac{1}{4v^2}D\mathbf{X}^\mu D^2\mathbf{X}^\mu\right)\right\}|\mathbf{X}^\mu,\mathbf{X}^i=0\rangle_{\mathrm{NS}}\end{array}}\right)=$$

$$-3927+2\sqrt[13]{2.2983717437\times 10^{59}+2.0823329825883\times 10^{59}}$$

$= 73490.8437525.... \Rightarrow$

$$\Rightarrow\left(A(r)\times\frac{1}{B(r)}\left(-\frac{1}{\phi(r)}\right)\times\frac{1}{e^{\Lambda(r)}}\right)\Rightarrow$$

$$\Rightarrow\left(-0.000029211892\times\frac{1}{0.0003644621}\left(-\frac{1}{0.0005946833}\right)\times\frac{1}{0.00183393}\right)=$$

$= 73491.78832548118710549159572042220548025195726563413398700...$

$= 73491.7883254... \Rightarrow$

$$\left(\begin{array}{c} I_{21}\ll\int_{-\infty}^{+\infty}\exp\left(-\left(\frac{t}{H}\right)^2\right)\left|\sum_{\lambda\leqslant P^{1-\varepsilon_2}}\frac{a(\lambda)}{\sqrt{\lambda}}B(\lambda)\lambda^{-i(T+t)}\right|^2 dt\ll \\ \ll H\left\{\left(\frac{4}{\varepsilon_2\log T}\right)^{2r}(\log T)(\log X)^{-2\beta}+\left(\varepsilon_2^{-2r}(\log T)^{-2r}+\varepsilon_2^{-r}h_1^r(\log T)^{-r}\right)T^{-\varepsilon_1}\right\}\end{array}\right)/$$

$$/(26\times 4)^2-24 = \left(\frac{7.9313976505275\times 10^8}{(26\times 4)^2-24}\right) = 73493.30662\ldots$$

Mathematical connections with the boundary state corresponding to the NSNS-sector of N Dp-branes in the limit of u → ∞, with the ratio concerning the general asymptotically flat solution of the equations of motion of the p-brane and with the Karatsuba's equation concerning the zeros of a special type of function connected with Dirichlet series.

From:

Interpreting cosmological tensions from the effective field theory of torsional gravity
Sheng-Feng Yan, Pierre Zhang, Jie-Wen Chen, Xin-Zhe Zhang,
Yi-Fu Cai and Emmanuel N. Saridakis - arXiv:1909.06388v1 [astro-ph.CO] 13 Sep 2019

We have that:

$H_0 = 74.03 \pm 1.42\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$

$T = 6H^2$

$$F(T) \approx 375.47\left(\frac{T}{6H_0^2}\right)^{-1.65}, \qquad (10)$$

$$F(T) \approx 375.47\left(\frac{T}{6H_0^2}\right)^{-1.65} + 25T^{1/2}. \qquad (11)$$

From (10), we obtain:

375.47 ((6*74.03^2)/(6*74.03^2))^-1.65

Input interpretation:

$$\frac{375.47}{\left(\frac{6\times 74.03^2}{6\times 74.03^2}\right)^{1.65}}$$

Result:

375.47

375.47

From (11), we obtain:

375.47 ((6*74.03^2)/(6*74.03^2))^-1.65 + 25*(6*74.03^2)^1/2

Input interpretation:

$$\frac{375.47}{\left(\frac{6\times 74.03^2}{6\times 74.03^2}\right)^{1.65}} + 25\sqrt{6\times 74.03^2}$$

Result:

4908.86...

4908.86…

375.47 / ((((375.47 ((6*74.03^2)/(6*74.03^2))^-1.65 + 25*(6*74.03^2)^1/2))))

Input interpretation:

$$\frac{375.47}{\frac{375.47}{\left(\frac{6\times 74.03^2}{6\times 74.03^2}\right)^{1.65}}+25\sqrt{6\times 74.03^2}}$$

Result:

0.0764882...

0.0764882…

(((375.47 / ((((375.47 ((6*74.03^2)/(6*74.03^2))^-1.65 + 25*(6*74.03^2)^1/2)))))))^1/4096

Input interpretation:

$$\sqrt[4096]{\frac{375.47}{\frac{375.47}{\left(\frac{6\times 74.03^2}{6\times 74.03^2}\right)^{1.65}}+25\sqrt{6\times 74.03^2}}}$$

Result:

0.999372604...

0.999372604… result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\cfrac{e^{-\pi\sqrt{5}}}{1+\cfrac{e^{-2\pi\sqrt{5}}}{1+\cfrac{e^{-3\pi\sqrt{5}}}{1+\cfrac{e^{-4\pi\sqrt{5}}}{1+\dots}}}}\approx 0.9991104684$$

and to the dilaton value $\mathbf{0.989117352243} = \boldsymbol{\phi}$

1+ ((((375.47 / ((((375.47 ((6*74.03^2)/(6*74.03^2))^-1.65 + 25*(6*74.03^2)^1/2))))))))^1/6

Input interpretation:

$$1+\sqrt[6]{\frac{375.47}{\frac{375.47}{\left(\frac{6\times 74.03^2}{6\times 74.03^2}\right)^{1.65}}+25\sqrt{6\times 74.03^2}}}$$

Result:

1.651526952513089663269874499030438158331121978838399836340...

1.65152695…. result very near to the 14th root of the following Ramanujan's class invariant $Q=\left(G_{505}/G_{101/5}\right)^3$ = 1164,2696 i.e. 1,65578...

1/10^27*(((((18+2)/10^3+1+ ((((375.47 / ((((375.47 ((6*74.03^2)/(6*74.03^2))^-1.65 + 25*(6*74.03^2)^1/2))))))))^1/6))))

Where 2 and 18 are Lucas numbers

Input interpretation:

$$\frac{1}{10^{27}}\left(\frac{18+2}{10^3}+1+\sqrt[6]{\frac{375.47}{\frac{375.47}{\left(\frac{6\times 74.03^2}{6\times 74.03^2}\right)^{1.65}}+25\sqrt{6\times 74.03^2}}}\right)$$

Result:

$1.67153\ldots\times 10^{-27}$

1.67153… * 10^{-27}

result practically equal to the value of the formula:

$m_{p\prime}=2\times\frac{\eta}{R}m_P=1.6714213\times 10^{-24}$ gm

that is the holographic proton mass (N. Haramein)

-(29+4)/10^3+1+ ((((375.47 / ((((375.47 ((6*74.03^2)/(6*74.03^2))^-1.65 + 25*(6*74.03^2)^1/2))))))))^1/6

Where 29 and 4 are Lucas numbers

Input interpretation:

$$-\frac{29+4}{10^3}+1+\sqrt[6]{\frac{375.47}{\frac{375.47}{\left(\frac{6\times 74.03^2}{6\times 74.03^2}\right)^{1.65}}+25\sqrt{6\times 74.03^2}}}$$

Result:

1.618526952513089663269874499030438158331121978838399836340...

1.61852695...

This result is a very good approximation to the value of the golden ratio 1,618033988749

From:

Post-Newtonian limit of scalar-torsion theories of gravity as analogue to scalar-curvature theories
Elena D. Emtsova∗ Manuel Hohmann† - https://arxiv.org/abs/1909.09355v1

In order to bring the action to the form (5) one has to perform integration by parts. After this step one finds the parameter functions

$$\mathcal{A} = 1 + 2\kappa^2\xi\phi^2\,, \quad \mathcal{B} = -\kappa^2\,, \quad \mathcal{C} = 4\kappa^2\chi\phi\,. \tag{83}$$

Here we restrict ourselves to the massless case $\mathcal{V} = 0$; see [46] for a discussion of the post-Newtonian limit of the theory with a massive scalar field. Note that the parameter functions explicitly depend on κ, so that for the normalization $G = 1$ of the gravitational constant we must insert them into the expression (48). This yields the solution

$$\kappa^2 = \frac{16\pi}{1 - 32\pi(\xi - 6\chi^2)\Phi^2 + \sqrt{(1 - 64\pi\chi^2\Phi^2)(1 - 576\pi\chi^2\Phi^2)}} \tag{84}$$

as the only solution which yields $\kappa^2 \to 8\pi$ in the limit $\Phi \to 0$, as one would expect. Further, observe that $\mathcal{C} \to 0$ in the limit $\chi \to 0$. It is thus helpful to expand the PPN parameters in a Taylor series in χ, since they approach their general relativity values for $\chi \to 0$. This yields the result

$$\gamma = 1 + 128\pi\chi^2\Phi^2 + \mathcal{O}(\chi^4)\,, \quad \beta = 1 + 32\pi\xi\chi\Phi^2 + 32\pi\chi^2\Phi^2 + \mathcal{O}(\chi^3)\,. \tag{85}$$

Comparison of these results with observations of the PPN parameters thus yields bounds on the appearing constants.

Now:

$$\kappa^2 = \frac{16\pi}{1 - 32\pi(\xi - 6\chi^2)\Phi^2 + \sqrt{(1 - 64\pi\chi^2\Phi^2)(1 - 576\pi\chi^2\Phi^2)}}$$

For $\chi = 1/16$; $\Phi = 1/8$ and $\xi = 1/12$ we have:

16Pi / [1-32*Pi(1/12-6*(1/16)^2)*1/64+sqrt(((1-64*Pi*(1/16)^2*1/64)(1-576Pi*(1/16)^2*1/64)))]

Input:

$$16 \times \frac{\pi}{1 - 32\pi\left(\left(\frac{1}{12} - 6\left(\frac{1}{16}\right)^2\right) \times \frac{1}{64}\right) + \sqrt{\left(1 - 64\pi\left(\frac{1}{16}\right)^2 \times \frac{1}{64}\right)\left(1 - 576\pi\left(\frac{1}{16}\right)^2 \times \frac{1}{64}\right)}}$$

Exact result:

$$\frac{16\pi}{1 + \sqrt{\left(1 - \frac{9\pi}{256}\right)\left(1 - \frac{\pi}{256}\right)} - \frac{23\pi}{768}}$$

Decimal approximation:

27.269701502012324026031197257345533980179737891594110717 26...

27.2697015…

Alternate forms:

$$\frac{12\,288\,\pi}{768 - 23\pi + 3\sqrt{65\,536 - 2560\pi + 9\pi^2}}$$

$$\frac{12\,288\,\pi}{768 + 3\sqrt{(256 - 9\pi)(256 - \pi)} - 23\pi}$$

$$-\frac{12\,288}{23} - \frac{36\,864\left(256 + \sqrt{65\,536 - 2560\pi + 9\pi^2}\right)}{23\left(-768 + 23\pi - 3\sqrt{65\,536 - 2560\pi + 9\pi^2}\right)}$$

Series representations:

$$\frac{16\pi}{1-\frac{1}{64}\left(\frac{1}{12}-6\left(\frac{1}{16}\right)^2\right)32\pi+\sqrt{\left(1-\frac{64}{64}\left(\pi\left(\frac{1}{16}\right)^2\right)\right)\left(1-\frac{576}{64}\left(\pi\left(\frac{1}{16}\right)^2\right)\right)}} = \frac{16\pi}{1-\frac{23\pi}{768}+\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{65\,536}\right)^k(\pi(-2560+9\pi))^k\left(-\frac{1}{2}\right)_k}{k!}}$$

$$\frac{16\pi}{1-\frac{1}{64}\left(\frac{1}{12}-6\left(\frac{1}{16}\right)^2\right)32\pi+\sqrt{\left(1-\frac{64}{64}\left(\pi\left(\frac{1}{16}\right)^2\right)\right)\left(1-\frac{576}{64}\left(\pi\left(\frac{1}{16}\right)^2\right)\right)}} = -\frac{12\,288\,\pi\sqrt{\pi}}{(-768+23\pi)\sqrt{\pi}+384\sum_{j=0}^{\infty}\operatorname{Res}_{s=-j}65\,536^s(\pi(-2560+9\pi))^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}$$

$$\frac{16\pi}{1-\frac{1}{64}\left(\frac{1}{12}-6\left(\frac{1}{16}\right)^2\right)32\pi+\sqrt{\left(1-\frac{64}{64}\left(\pi\left(\frac{1}{16}\right)^2\right)\right)\left(1-\frac{576}{64}\left(\pi\left(\frac{1}{16}\right)^2\right)\right)}} = \frac{16\pi}{1-\frac{23\pi}{768}+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(1-\frac{5\pi}{128}+\frac{9\pi^2}{65536}-z_0\right)^k z_0^{-k}}{k!}}$$

$$\text{for not}\left((z_0\in\mathbb{R}\text{ and }-\infty<z_0\le 0)\right)$$

We obtain also:

10^3+(((((16Pi / [1-32*Pi(1/12-6*(1/16)^2)*1/64+sqrt(((1-64*Pi*(1/16)^2*1/64)(1-576Pi*(1/16)^2*1/64)))])))))^2

Input:

$$10^3+\left(16\times\frac{\pi}{1-32\pi\left(\left(\frac{1}{12}-6\left(\frac{1}{16}\right)^2\right)\times\frac{1}{64}\right)+\sqrt{\left(1-64\pi\left(\frac{1}{16}\right)^2\times\frac{1}{64}\right)\left(1-576\pi\left(\frac{1}{16}\right)^2\times\frac{1}{64}\right)}}\right)^2$$

Exact result:

$$1000+\frac{256\pi^2}{\left(1+\sqrt{\left(1-\frac{9\pi}{256}\right)\left(1-\frac{\pi}{256}\right)}-\frac{23\pi}{768}\right)^2}$$

Decimal approximation:

1743.636620008853201026347405776605194652221156597117589024...

1743.63662…. This result is very near to the mass of candidate glueball $f_0(1710)$ meson.

Alternate forms:

$$1000+\frac{150\,994\,944\,\pi^2}{\left(768+3\sqrt{(256-9\pi)(256-\pi)}-23\pi\right)^2}$$

$$-\left(\left(16\left(-73\,728\,000+3\,648\,000\,\pi-9\,475\,309\,\pi^2+375\sqrt{(256-9\pi)(256-\pi)}\,(23\pi-768)\right)\right)\Big/\left(768+3\sqrt{(256-9\pi)(256-\pi)}-23\pi\right)^2\right)$$

$$-\left(\left(16\left(-73\,728\,000+3\,648\,000\,\pi-9\,475\,309\,\pi^2-288\,000\sqrt{65\,536-2560\,\pi+9\pi^2}+8625\,\pi\sqrt{65\,536-2560\,\pi+9\pi^2}\right)\right)\Big/\left(768-23\pi+3\sqrt{65\,536-2560\,\pi+9\pi^2}\right)^2\right)$$

Series representations:

$$10^3+\left(\frac{16\pi}{1-\frac{1}{64}\left(\frac{1}{12}-6\left(\frac{1}{16}\right)^2\right)32\pi+\sqrt{\left(1-\frac{64}{64}\left(\pi\left(\frac{1}{16}\right)^2\right)\right)\left(1-\frac{576}{64}\left(\pi\left(\frac{1}{16}\right)^2\right)\right)}}\right)^2=$$
$$1000+\frac{256\pi^2}{\left(1-\frac{23\pi}{768}+\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{65\,536}\right)^k(\pi(-2560+9\pi))^k\left(-\frac{1}{2}\right)_k}{k!}\right)^2}$$

$$10^3+\left(\frac{16\pi}{1-\frac{1}{64}\left(\frac{1}{12}-6\left(\frac{1}{16}\right)^2\right)32\pi+\sqrt{\left(1-\frac{64}{64}\left(\pi\left(\frac{1}{16}\right)^2\right)\right)\left(1-\frac{576}{64}\left(\pi\left(\frac{1}{16}\right)^2\right)\right)}}\right)^2=$$
$$1000+\frac{256\pi^2}{\left(-1+\frac{23\pi}{768}+\frac{\sum_{j=0}^{\infty}\operatorname{Res}_{s=-j}65\,536^s\,(\pi(-2560+9\pi))^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}\right)^2}$$

$$10^3 + \left(\frac{16\pi}{1 - \frac{1}{64}\left(\frac{1}{12} - 6\left(\frac{1}{16}\right)^2\right) 32\pi + \sqrt{\left(1 - \frac{64}{64}\left(\pi\left(\frac{1}{16}\right)^2\right)\right)\left(1 - \frac{576}{64}\left(\pi\left(\frac{1}{16}\right)^2\right)\right)}} \right)^2 =$$

$$1000 + \frac{256\pi^2}{\left(1 - \frac{23\pi}{768} + \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(1 - \frac{5\pi}{128} + \frac{9\pi^2}{65536} - z_0\right)^k z_0^{-k}}{k!}\right)^2}$$

for not $((z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0))$

And:

[10^3+(((((16Pi / [1-32*Pi(1/12-6*(1/16)^2)*1/64+sqrt(((1-64*Pi*(1/16)^2*1/64)(1-576Pi*(1/16)^2*1/64)))]))))^2]^1/15

Input:

$$\left(10^3 + \left(16 \times \pi \middle/ \left(1 - 32\pi\left(\left(\frac{1}{12} - 6\left(\frac{1}{16}\right)^2\right) \times \frac{1}{64}\right) + \sqrt{\left(1 - 64\pi\left(\frac{1}{16}\right)^2 \times \frac{1}{64}\right)\left(1 - 576\pi\left(\frac{1}{16}\right)^2 \times \frac{1}{64}\right)}\right)\right)^2\right)\text{^}(1/15)$$

Exact result:

$$\sqrt[15]{1000 + \frac{256\pi^2}{\left(1 + \sqrt{\left(1 - \frac{9\pi}{256}\right)\left(1 - \frac{\pi}{256}\right)} - \frac{23\pi}{768}\right)^2}}$$

Decimal approximation:

1.644739283711541327747617596295339675483271825393339291646...

1.6447392837….≈ ζ(2) = $\frac{\pi^2}{6} = 1.644934$ ...

Alternate forms:

$$\left(2^{4/15}\left(73728000 - 3648000\pi + 9475309\pi^2 - 375\sqrt{(256 - 9\pi)(256 - \pi)}\,(23\pi - 768)\right)\text{^}(1/15)\right) \Big/ \left(768 + 3\sqrt{(256 - 9\pi)(256 - \pi)} - 23\pi\right)^{2/15}$$

$$\left(2^{4/15}\left(73\,728\,000-3\,648\,000\,\pi+9\,475\,309\,\pi^2+288\,000\sqrt{65\,536-2560\,\pi+9\,\pi^2}-8625\,\pi\sqrt{65\,536-2560\,\pi+9\,\pi^2}\right)^\wedge(1/15)\right)\Big/\left(768-23\,\pi+3\sqrt{65\,536-2560\,\pi+9\,\pi^2}\right)^{2/15}$$

$$\left(\sqrt[5]{2}\left(73\,728\,000-4\,416\,000\,\pi+18\,940\,493\,\pi^2+576\,000\sqrt{65\,536-2560\,\pi+9\,\pi^2}-17\,250\,\pi\sqrt{65\,536-2560\,\pi+9\,\pi^2}+1125\left(65\,536-2560\,\pi+9\,\pi^2\right)\right)^\wedge(1/15)\right)\Big/\left(768-23\,\pi+3\sqrt{65\,536-2560\,\pi+9\,\pi^2}\right)^{2/15}$$

All 15th roots of 1000 + (256 π^2)/(1 + sqrt((1 - (9 π)/256) (1 - π/256)) - (23 π)/768)^2:

- Polar form

$$\sqrt[15]{1000+\frac{256\,\pi^2}{\left(1+\sqrt{\left(1-\frac{9\pi}{256}\right)\left(1-\frac{\pi}{256}\right)}-\frac{23\,\pi}{768}\right)^2}}\; e^0 \approx 1.64474 \text{ (real, principal root)}$$

$$\sqrt[15]{1000+\frac{256\,\pi^2}{\left(1+\sqrt{\left(1-\frac{9\pi}{256}\right)\left(1-\frac{\pi}{256}\right)}-\frac{23\,\pi}{768}\right)^2}}\; e^{(2\,i\,\pi)/15} \approx 1.50254+0.6690\,i$$

$$\sqrt[15]{1000+\frac{256\,\pi^2}{\left(1+\sqrt{\left(1-\frac{9\pi}{256}\right)\left(1-\frac{\pi}{256}\right)}-\frac{23\,\pi}{768}\right)^2}}\; e^{(4\,i\,\pi)/15} \approx 1.1005+1.2223\,i$$

$$\sqrt[15]{1000+\frac{256\,\pi^2}{\left(1+\sqrt{\left(1-\frac{9\pi}{256}\right)\left(1-\frac{\pi}{256}\right)}-\frac{23\,\pi}{768}\right)^2}}\; e^{(2\,i\,\pi)/5} \approx 0.5083+1.5642\,i$$

$$\sqrt[15]{1000+\frac{256\,\pi^2}{\left(1+\sqrt{\left(1-\frac{9\pi}{256}\right)\left(1-\frac{\pi}{256}\right)}-\frac{23\,\pi}{768}\right)^2}}\; e^{(8\,i\,\pi)/15} \approx -0.17192+1.63573\,i$$

Series representations:

$$\sqrt[15]{10^3+\left(\frac{16\pi}{1-\frac{1}{64}\left(\frac{1}{12}-6\left(\frac{1}{16}\right)^2\right)32\pi+\sqrt{\left(1-\frac{64}{64}\left(\pi\left(\frac{1}{16}\right)^2\right)\right)\left(1-\frac{576}{64}\left(\pi\left(\frac{1}{16}\right)^2\right)\right)}}\right)^2}=$$

$$\sqrt[15]{1000+\frac{256\pi^2}{\left(1-\frac{23\pi}{768}+\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{65536}\right)^k(\pi(-2560+9\pi))^k\left(-\frac{1}{2}\right)_k}{k!}\right)^2}}$$

$$\sqrt[15]{10^3+\left(\frac{16\pi}{1-\frac{1}{64}\left(\frac{1}{12}-6\left(\frac{1}{16}\right)^2\right)32\pi+\sqrt{\left(1-\frac{64}{64}\left(\pi\left(\frac{1}{16}\right)^2\right)\right)\left(1-\frac{576}{64}\left(\pi\left(\frac{1}{16}\right)^2\right)\right)}}\right)^2}=$$

$$\sqrt[15]{1000+\frac{256\pi^2}{\left(1-\frac{23\pi}{768}+\sum_{k=0}^{\infty}\frac{(-1)^k\left(-1+\left(1-\frac{9\pi}{256}\right)\left(1-\frac{\pi}{256}\right)\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^2}}$$

$$\sqrt[15]{10^3+\left(\frac{16\pi}{1-\frac{1}{64}\left(\frac{1}{12}-6\left(\frac{1}{16}\right)^2\right)32\pi+\sqrt{\left(1-\frac{64}{64}\left(\pi\left(\frac{1}{16}\right)^2\right)\right)\left(1-\frac{576}{64}\left(\pi\left(\frac{1}{16}\right)^2\right)\right)}}\right)^2}=$$

$$\sqrt[15]{1000+\frac{256\pi^2}{\left(1-\frac{23\pi}{768}+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\left(1-\frac{9\pi}{256}\right)\left(1-\frac{\pi}{256}\right)-z_0\right)^k z_0^{-k}}{k!}\right)^2}}$$

for not $((z_0\in\mathbb{R}$ and $-\infty<z_0\le 0))$

Integral representation:

$$(1+z)^a=\frac{\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(s)\Gamma(-a-s)}{z^s}\,ds}{(2\pi i)\Gamma(-a)}\quad \text{for } (0<\gamma<-\mathrm{Re}(a) \text{ and } |\arg(z)|<\pi)$$

-(29-3)/(10^3)+[10^3+(((((16Pi / [1-32*Pi(1/12-6*(1/16)^2)*1/64+sqrt(((1-64*Pi*(1/16)^2*1/64)(1-576Pi*(1/16)^2*1/64)))])))))^2]^1/15

Where 3 and 29 are Lucas numbers

Input:

$$-\frac{29-3}{10^3}+\left(10^3+\left(16\times\pi\Big/\left(1-32\pi\left(\left(\frac{1}{12}-6\left(\frac{1}{16}\right)^2\right)\times\frac{1}{64}\right)+\sqrt{\left(1-64\pi\left(\frac{1}{16}\right)^2\times\frac{1}{64}\right)\left(1-576\pi\left(\frac{1}{16}\right)^2\times\frac{1}{64}\right)}\right)\right)^2\right)\text{^}(1/15)$$

Exact result:

$$\sqrt[15]{1000+\frac{256\pi^2}{\left(1+\sqrt{\left(1-\frac{9\pi}{256}\right)\left(1-\frac{\pi}{256}\right)}-\frac{23\pi}{768}\right)^2}}-\frac{13}{500}$$

Decimal approximation:

1.6187392837115413277476175962953396754832718253933392916 46...

1.61873928371154…

This result is a very good approximation to the value of the golden ratio 1,618033988749...

Alternate forms:

$$\left(2^{4/15}\left(73\,728\,000-3\,648\,000\pi+9\,475\,309\pi^2-375\sqrt{(256-9\pi)(256-\pi)}\,(23\pi-768)\right)\text{^}(1/15)\right)\Big/\left(768+3\sqrt{(256-9\pi)(256-\pi)}-23\pi\right)^{2/15}-\frac{13}{500}$$

$$\frac{1}{500}\left(\left(500\times2^{4/15}\left(73\,728\,000-3\,648\,000\pi+9\,475\,309\pi^2-375\sqrt{(256-9\pi)(256-\pi)}\,(23\pi-768)\right)\text{^}(1/15)\right)\Big/\left(768+3\sqrt{(256-9\pi)(256-\pi)}-23\pi\right)^{2/15}-13\right)$$

$$\left(500\times2^{4/15}\left(73\,728\,000-3\,648\,000\pi+9\,475\,309\pi^2+288\,000\sqrt{65\,536-2560\pi+9\pi^2}-8625\pi\sqrt{65\,536-2560\pi+9\pi^2}\right)\text{^}(1/15)-13\left(768-23\pi+3\sqrt{65\,536-2560\pi+9\pi^2}\right)^{2/15}\right)\Big/\left(500\left(768-23\pi+3\sqrt{65\,536-2560\pi+9\pi^2}\right)^{2/15}\right)$$

Series representations:

$$-\frac{29-3}{10^3}+\sqrt[15]{10^3+\left(\frac{16\pi}{1-\frac{1}{64}\left(\frac{1}{12}-6\left(\frac{1}{16}\right)^2\right)32\pi+\sqrt{\left(1-\frac{64}{64}\left(\pi\left(\frac{1}{16}\right)^2\right)\right)\left(1-\frac{576}{64}\left(\pi\left(\frac{1}{16}\right)^2\right)\right)}}\right)^2}=$$

$$-\frac{13}{500}+\sqrt[15]{1000+\frac{256\pi^2}{\left(1-\frac{23\pi}{768}+\sum_{k=0}^{\infty}\frac{(-1)^k\left(-1+\left(1-\frac{9\pi}{256}\right)\left(1-\frac{\pi}{256}\right)\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^2}}$$

$$-\frac{29-3}{10^3}+\sqrt[15]{10^3+\left(\frac{16\pi}{1-\frac{1}{64}\left(\frac{1}{12}-6\left(\frac{1}{16}\right)^2\right)32\pi+\sqrt{\left(1-\frac{64}{64}\left(\pi\left(\frac{1}{16}\right)^2\right)\right)\left(1-\frac{576}{64}\left(\pi\left(\frac{1}{16}\right)^2\right)\right)}}\right)^2}=$$

$$-\frac{13}{500}+\sqrt[15]{1000+\frac{256\pi^2}{\left(1-\frac{23\pi}{768}+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\left(1-\frac{9\pi}{256}\right)\left(1-\frac{\pi}{256}\right)-z_0\right)^k z_0^{-k}}{k!}\right)^2}}$$

for not $((z_0\in\mathbb{R}$ and $-\infty<z_0\le 0))$

$$-\frac{29-3}{10^3}+\sqrt[15]{10^3+\left(\frac{16\pi}{1-\frac{1}{64}\left(\frac{1}{12}-6\left(\frac{1}{16}\right)^2\right)32\pi+\sqrt{\left(1-\frac{64}{64}\left(\pi\left(\frac{1}{16}\right)^2\right)\right)\left(1-\frac{576}{64}\left(\pi\left(\frac{1}{16}\right)^2\right)\right)}}\right)^2}=$$

$$-\frac{13}{500}+\sqrt[15]{1000+\frac{256\pi^2}{\left(1-\frac{23\pi}{768}-\frac{\sum_{j=0}^{\infty}\operatorname{Res}_{s=-j}\left(-1+\left(1-\frac{9\pi}{256}\right)\left(1-\frac{\pi}{256}\right)\right)^{-s}\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}\right)^2}}$$

And:

((((((1/ ((((16Pi / [1-32*Pi(1/12-6*(1/16)^2)*1/64+sqrt(((1-64*Pi*(1/16)^2*1/64)(1-576Pi*(1/16)^2*1/64)))])))))))))))^1/4096

Input:

$$\sqrt[4096]{\frac{1}{16\times\frac{\pi}{1-32\pi\left(\left(\frac{1}{12}-6\left(\frac{1}{16}\right)^2\right)\times\frac{1}{64}\right)+\sqrt{\left(1-64\pi\left(\frac{1}{16}\right)^2\times\frac{1}{64}\right)\left(1-576\pi\left(\frac{1}{16}\right)^2\times\frac{1}{64}\right)}}}}$$

Exact result:

$$\frac{1}{\sqrt[1024]{2}\,\sqrt[4096]{\frac{\pi}{1+\sqrt{\left(1-\frac{9\pi}{256}\right)\left(1-\frac{\pi}{256}\right)}-\frac{23\pi}{768}}}}$$

Decimal approximation:

0.99919325131687239596091640080542698351364206434737735521 1...

0.9991932513168…. result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}}\approx 0.9991104684$$

and to the dilaton value $\mathbf{0.989117352243 = \phi}$

Alternate forms:

$$\frac{\sqrt[4096]{\frac{256+\sqrt{(256-9\pi)(256-\pi)}}{\pi}-\frac{23}{3}}}{2^{3/1024}}$$

$$\frac{\sqrt[4096]{\frac{768-23\pi+3\sqrt{65536-2560\pi+9\pi^2}}{3\pi}}}{2^{3/1024}}$$

$$\frac{\sqrt[4096]{\frac{768+3\sqrt{(256-9\pi)(256-\pi)}-23\pi}{3\pi}}}{2^{3/1024}}$$

All 4096th roots of (1 + sqrt((1 - (9 π)/256) (1 - π/256)) - (23 π)/768)/(16 π):

- Polar form

$$\frac{e^{0}}{\sqrt[1024]{2}\ \sqrt[4096]{\frac{\pi}{1+\sqrt{\left(1-\frac{9\pi}{256}\right)\left(1-\frac{\pi}{256}\right)}-\frac{23\pi}{768}}}} \approx 0.9991933 \text{ (real, principal root)}$$

$$\frac{e^{(i\pi)/2048}}{\sqrt[1024]{2}\ \sqrt[4096]{\frac{\pi}{1+\sqrt{\left(1-\frac{9\pi}{256}\right)\left(1-\frac{\pi}{256}\right)}-\frac{23\pi}{768}}}} \approx 0.9991921+0.0015327\,i$$

$$\frac{e^{(i\pi)/1024}}{\sqrt[1024]{2}\ \sqrt[4096]{\frac{\pi}{1+\sqrt{\left(1-\frac{9\pi}{256}\right)\left(1-\frac{\pi}{256}\right)}-\frac{23\pi}{768}}}} \approx 0.9991885+0.0030655\,i$$

$$\frac{e^{(3i\pi)/2048}}{\sqrt[1024]{2}\ \sqrt[4096]{\frac{\pi}{1+\sqrt{\left(1-\frac{9\pi}{256}\right)\left(1-\frac{\pi}{256}\right)}-\frac{23\pi}{768}}}} \approx 0.9991827+0.0045982\,i$$

$$\frac{e^{(i\pi)/512}}{\sqrt[1024]{2}\ \sqrt[4096]{\frac{\pi}{1+\sqrt{\left(1-\frac{9\pi}{256}\right)\left(1-\frac{\pi}{256}\right)}-\frac{23\pi}{768}}}} \approx 0.9991744+0.006131\,i$$

Series representations:

$$\sqrt[4096]{\frac{1}{\frac{16\pi}{1-\frac{1}{64}\left(\frac{1}{12}-6\left(\frac{1}{16}\right)^2\right)32\pi+\sqrt{\left(1-\frac{64}{64}\left(\pi\left(\frac{1}{16}\right)^2\right)\right)\left(1-\frac{576}{64}\left(\pi\left(\frac{1}{16}\right)^2\right)\right)}}}} = \frac{\sqrt[4096]{\frac{1-\frac{23\pi}{768}+\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{65\,536}\right)^k(\pi(-2560+9\pi))^k\left(-\frac{1}{2}\right)_k}{k!}}{\pi}}}{\sqrt[1024]{2}}$$

$$\sqrt[4096]{\frac{1}{\frac{16\pi}{1-\frac{1}{64}\left(\frac{1}{12}-6\left(\frac{1}{16}\right)^2\right)32\pi+\sqrt{\left(1-\frac{64}{64}\left(\pi\left(\frac{1}{16}\right)^2\right)\right)\left(1-\frac{576}{64}\left(\pi\left(\frac{1}{16}\right)^2\right)\right)}}}} = \frac{\sqrt[4096]{\frac{1-\frac{23\pi}{768}+\sum_{k=0}^{\infty}\frac{(-1)^k\left(-1+\left(1-\frac{9\pi}{256}\right)\left(1-\frac{\pi}{256}\right)\right)^k\left(-\frac{1}{2}\right)_k}{k!}}{\pi}}}{\sqrt[1024]{2}}$$

$$\frac{\sqrt[4096]{\frac{\frac{1}{16\pi}}{1-\frac{1}{64}\left(\frac{1}{12}-6\left(\frac{1}{16}\right)^2\right)32\pi+\sqrt{\left(1-\frac{64}{64}\left(\pi\left(\frac{1}{16}\right)^2\right)\right)\left(1-\frac{576}{64}\left(\pi\left(\frac{1}{16}\right)^2\right)\right)}}} = \sqrt[4096]{\frac{1-\frac{23\pi}{768}-\frac{\sum_{j=0}^{\infty}\operatorname{Res}_{s=-j} 65536^s\,(\pi(-2560+9\pi))^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}}{\pi}}}{\sqrt[1024]{2}}$$

Integral representation:

$$(1+z)^a = \frac{\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma} \frac{\Gamma(s)\,\Gamma(-a-s)}{z^s}\,ds}{(2\pi i)\,\Gamma(-a)} \quad \text{for } (0 < \gamma < -\mathrm{Re}(a) \text{ and } |\arg(z)| < \pi)$$

We have that:

$$\gamma = 1 + 128\pi\chi^2\Phi^2 + \mathcal{O}(\chi^4)\,, \quad \beta = 1 + 32\pi\xi\chi\Phi^2 + 32\pi\chi^2\Phi^2 + \mathcal{O}(\chi^3)\,.$$

For $\chi = 1/16$; $\Phi = 1/8$ and $\xi = 1/12$ we have:

1+128Pi*(1/16)^2*1/64+(1/16)^4

Input:

$$1 + 128\,\pi\left(\frac{1}{16}\right)^2 \times \frac{1}{64} + \left(\frac{1}{16}\right)^4$$

Result:

$$\frac{65537}{65536} + \frac{\pi}{128}$$

Decimal approximation:

1.024558951395232759675489401431871116282790385932618014226...

1.024558951395…

Property:

$\frac{65537}{65536} + \frac{\pi}{128}$ is a transcendental number

Alternate form:

$$\frac{65537 + 512\pi}{65536}$$

Alternative representations:

$$1+\frac{128}{64}\pi\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^4=1+\frac{23040}{64}\circ\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^4$$

$$1+\frac{128}{64}\pi\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^4=1-\frac{128}{64}i\log(-1)\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^4$$

$$1+\frac{128}{64}\pi\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^4=1+\frac{128}{64}\cos^{-1}(-1)\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^4$$

Series representations:

$$1+\frac{128}{64}\pi\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^4=\frac{65537}{65536}+\frac{1}{32}\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}$$

$$1+\frac{128}{64}\pi\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^4=\frac{65537}{65536}+\sum_{k=0}^{\infty}-\frac{(-1)^k\,1195^{-1-2k}\left(5^{1+2k}-4\times239^{1+2k}\right)}{32(1+2k)}$$

$$1+\frac{128}{64}\pi\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^4=\frac{65537}{65536}+\frac{1}{128}\sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)$$

Integral representations:

$$1+\frac{128}{64}\pi\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^4=\frac{65537}{65536}+\frac{1}{32}\int_0^1\sqrt{1-t^2}\,dt$$

$$1+\frac{128}{64}\pi\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^4=\frac{65537}{65536}+\frac{1}{64}\int_0^1\frac{1}{\sqrt{1-t^2}}\,dt$$

$$1+\frac{128}{64}\pi\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^4=\frac{65537}{65536}+\frac{1}{64}\int_0^{\infty}\frac{1}{1+t^2}\,dt$$

And:

1/(((1+128Pi*(1/16)^2*1/64+(1/16)^4)))

Input:

$$\frac{1}{1+128\pi\left(\frac{1}{16}\right)^2\times\frac{1}{64}+\left(\frac{1}{16}\right)^4}$$

Result:

$$\frac{1}{\frac{65537}{65536}+\frac{\pi}{128}}$$

Decimal approximation:

0.9760297332215109162747732825561894816483379520437893939964...

0.9760297332..... result near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}}\approx 0.9991104684$$

and to the dilaton value $\mathbf{0.989117352243=\phi}$

Property:

$\frac{1}{\frac{65537}{65536}+\frac{\pi}{128}}$ is a transcendental number

Alternate form:

$$\frac{65536}{65537+512\pi}$$

Alternative representations:

$$\frac{1}{1+\frac{128}{64}\pi\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^4}=\frac{1}{1+\frac{23040}{64}\circ\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^4}$$

$$\frac{1}{1+\frac{128}{64}\pi\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^4}=\frac{1}{1-\frac{128}{64}i\log(-1)\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^4}$$

$$\frac{1}{1+\frac{128}{64}\pi\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^4}=\frac{1}{1+\frac{128}{64}\cos^{-1}(-1)\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^4}$$

Series representations:

$$\frac{1}{1+\frac{128}{64}\pi\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^4}=\frac{65536}{65537+2048\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}}$$

$$\frac{1}{1+\frac{128}{64}\pi\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^4}=\frac{65536}{65537+\sum_{k=0}^{\infty}-\frac{2048(-1)^k\,1195^{-1-2k}\left(5^{1+2k}-4\times 239^{1+2k}\right)}{1+2k}}$$

$$\frac{1}{1+\frac{128}{64}\pi\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^4}=\frac{65536}{65537+512\sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)}$$

Integral representation:

$$\frac{1}{1+\frac{128}{64}\pi\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^4}=\frac{1}{\frac{65537}{65536}+\frac{1}{48}\int_0^{\infty}\frac{\sin^3(t)}{t^3}\,dt}$$

We obtain also:

1/10^27((((7/10^3+(((1+128Pi*(1/16)^2*1/64+(1/16)^4)))^21))))

Input:

$$\frac{1}{10^{27}}\left(\frac{7}{10^3}+\left(1+128\pi\left(\frac{1}{16}\right)^2\times\frac{1}{64}+\left(\frac{1}{16}\right)^4\right)^{21}\right)$$

Exact result:

$$\frac{\frac{7}{1000}+\left(\frac{65537}{65536}+\frac{\pi}{128}\right)^{21}}{1\,000\,000\,000\,000\,000\,000\,000\,000\,000}$$

Decimal approximation:

$1.671470079165770987661047613863349105583774028183971 4\ldots\times 10^{-27}$

$1.671470079\ldots*10^{-27}$

result practically equal to the value of the formula:

$m_{p\prime}=2\times\frac{\eta}{R}m_P=1.6714213\times 10^{-2}$ gm

that is the holographic proton mass (N. Haramein)

Property:

$$\frac{\frac{7}{1000}+\left(\frac{65537}{65536}+\frac{\pi}{128}\right)^{21}}{1000000000000000000000000000}$$ is a transcendental number

Alternate forms:

$$\frac{7+1000\left(\frac{65537}{65536}+\frac{\pi}{128}\right)^{21}}{1000000000000000000000000000000}$$

Alternative representations:

$$\frac{\frac{7}{10^3}+\left(1+\frac{128}{64}\pi\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^4\right)^{21}}{10^{27}}=\frac{\frac{7}{10^3}+\left(1+\frac{23040}{64}\circ\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^4\right)^{21}}{10^{27}}$$

$$\frac{\frac{7}{10^3}+\left(1+\frac{128}{64}\pi\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^4\right)^{21}}{10^{27}}=\frac{\frac{7}{10^3}+\left(1-\frac{128}{64}i\log(-1)\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^4\right)^{21}}{10^{27}}$$

$$\frac{\frac{7}{10^3}+\left(1+\frac{128}{64}\pi\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^4\right)^{21}}{10^{27}}=\frac{\frac{7}{10^3}+\left(1+\frac{128}{64}\cos^{-1}(-1)\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^4\right)^{21}}{10^{27}}$$

Series representations:

$$\frac{\frac{7}{10^3}+\left(1+\frac{128}{64}\pi\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^4\right)^{21}}{10^{27}}=\frac{\frac{7}{1000}+\left(\frac{65537}{65536}+\frac{1}{32}\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}\right)^{21}}{1000000000000000000000000000}$$

$$\frac{\frac{7}{10^3}+\left(1+\frac{128}{64}\pi\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^4\right)^{21}}{10^{27}}=$$
$$\frac{\frac{7}{1000}+\left(\frac{65537}{65536}+\frac{1}{128}\sum_{k=0}^{\infty}-\frac{4(-1)^k\,1195^{-1-2k}\left(5^{1+2k}-4\times 239^{1+2k}\right)}{1+2k}\right)^{21}}{1000000000000000000000000000}$$

$$\frac{\frac{7}{10^3}+\left(1+\frac{128}{64}\pi\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^4\right)^{21}}{10^{27}}=$$
$$\frac{\frac{7}{1000}+\left(\frac{65537}{65536}+\frac{1}{128}\sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{2}{1+4k}+\frac{2}{2+4k}+\frac{1}{3+4k}\right)\right)^{21}}{1000000000000000000000000000}$$

Integral representations:

$$\frac{\frac{7}{10^3}+\left(1+\frac{128}{64}\pi\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^4\right)^{21}}{10^{27}}=\frac{\frac{7}{1000}+\left(\frac{65537}{65536}+\frac{1}{32}\int_0^1\sqrt{1-t^2}\,dt\right)^{21}}{1000000000000000000000000000}$$

$$\frac{\frac{7}{10^3}+\left(1+\frac{128}{64}\pi\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^4\right)^{21}}{10^{27}}=\frac{\frac{7}{1000}+\left(\frac{65537}{65536}+\frac{1}{64}\int_0^{\infty}\frac{1}{1+t^2}\,dt\right)^{21}}{1000000000000000000000000000}$$

$$\frac{\frac{7}{10^3}+\left(1+\frac{128}{64}\pi\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^4\right)^{21}}{10^{27}} = \frac{\frac{7}{1000}+\left(\frac{65537}{65536}+\frac{1}{64}\int_0^1\frac{1}{\sqrt{1-t^2}}\,dt\right)^{21}}{1\,000\,000\,000\,000\,000\,000\,000\,000\,000}$$

$$\gamma = 1+128\pi\chi^2\Phi^2+\mathcal{O}(\chi^4)\,,\quad \beta = 1+32\pi\xi\chi\Phi^2+32\pi\chi^2\Phi^2+\mathcal{O}(\chi^3)\,.$$

For $\chi = 1/16$; $\Phi = 1/8$ and $\xi = 1/12$ we have

1+32Pi*(1/12)*(1/16)*(1/64)+32Pi*(1/16)^2*(1/64)+(1/16)^3

Input:

$$1+32\,\pi\times\frac{1}{12}\times\frac{1}{16}\times\frac{1}{64}+32\,\pi\left(\frac{1}{16}\right)^2\times\frac{1}{64}+\left(\frac{1}{16}\right)^3$$

Result:

$$\frac{4097}{4096}+\frac{7\,\pi}{1536}$$

Decimal approximation:

1.014561294645265984810702150835258151164961058460693841632...

1.0145612946…

Property:

$\frac{4097}{4096}+\frac{7\,\pi}{1536}$ is a transcendental number

Alternate form:

$$\frac{12\,291+56\,\pi}{12\,288}$$

Alternative representations:

$$1+\frac{32\,\pi}{12\times16\times64}+\frac{32}{64}\,\pi\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^3 = 1+\frac{5760\,^\circ}{12\times16\times64}+\frac{5760}{64}\,^\circ\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^3$$

$$1+\frac{32\,\pi}{12\times16\times64}+\frac{32}{64}\,\pi\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^3 = 1-\frac{32\,i\log(-1)}{12\times16\times64}-\frac{32}{64}\,i\log(-1)\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^3$$

$$1+\frac{32\,\pi}{12\times16\times64}+\frac{32}{64}\,\pi\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^3 = 1+\frac{32\cos^{-1}(-1)}{12\times16\times64}+\frac{32}{64}\cos^{-1}(-1)\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^3$$

Series representations:

$$1+\frac{32\pi}{12\times16\times64}+\frac{32}{64}\pi\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^3=\frac{4097}{4096}+\frac{7}{384}\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}$$

$$1+\frac{32\pi}{12\times16\times64}+\frac{32}{64}\pi\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^3=$$
$$\frac{4097}{4096}+\sum_{k=0}^{\infty}-\frac{7(-1)^k\,1195^{-1-2k}\left(5^{1+2k}-4\times239^{1+2k}\right)}{384(1+2k)}$$

$$1+\frac{32\pi}{12\times16\times64}+\frac{32}{64}\pi\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^3=\frac{4097}{4096}+\frac{7\sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)}{1536}$$

Integral representations:

$$1+\frac{32\pi}{12\times16\times64}+\frac{32}{64}\pi\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^3=\frac{4097}{4096}+\frac{7}{384}\int_0^1\sqrt{1-t^2}\,dt$$

$$1+\frac{32\pi}{12\times16\times64}+\frac{32}{64}\pi\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^3=\frac{4097}{4096}+\frac{7}{768}\int_0^1\frac{1}{\sqrt{1-t^2}}\,dt$$

$$1+\frac{32\pi}{12\times16\times64}+\frac{32}{64}\pi\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^3=\frac{4097}{4096}+\frac{7}{768}\int_0^{\infty}\frac{1}{1+t^2}\,dt$$

And:

1/(((1+32Pi*(1/12)*(1/16)*(1/64)+32Pi*(1/16)^2*(1/64)+(1/16)^3)))

Input:

$$\frac{1}{1+32\pi\times\frac{1}{12}\times\frac{1}{16}\times\frac{1}{64}+32\pi\left(\frac{1}{16}\right)^2\times\frac{1}{64}+\left(\frac{1}{16}\right)^3}$$

Result:

$$\frac{1}{\frac{4097}{4096}+\frac{7\pi}{1536}}$$

Decimal approximation:

0.985647693518253881234253730817470330114796219939391494216...

0.985647693518…. result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{\dfrac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}}}{\dfrac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\cfrac{e^{-\pi\sqrt{5}}}{1+\cfrac{e^{-2\pi\sqrt{5}}}{1+\cfrac{e^{-3\pi\sqrt{5}}}{1+\cfrac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}}\approx 0.9991104684$$

and to the dilaton value **0.989117352243 = $\boldsymbol{\phi}$**

Property:

$\frac{1}{\frac{4097}{4096}+\frac{7\pi}{1536}}$ is a transcendental number

Alternate form:

$$\frac{12288}{12291+56\pi}$$

Alternative representations:

$$\frac{1}{1+\frac{32\pi}{12\times16\times64}+\frac{32}{64}\pi\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^3}=\frac{1}{1+\frac{5760^\circ}{12\times16\times64}+\frac{5760}{64}\circ\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^3}$$

$$\frac{1}{1+\frac{32\pi}{12\times16\times64}+\frac{32}{64}\pi\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^3}=\frac{1}{1-\frac{32\,i\log(-1)}{12\times16\times64}-\frac{32}{64}\,i\log(-1)\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^3}$$

$$\frac{1}{1+\frac{32\pi}{12\times16\times64}+\frac{32}{64}\pi\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^3}=\frac{1}{1+\frac{32\cos^{-1}(-1)}{12\times16\times64}+\frac{32}{64}\cos^{-1}(-1)\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^3}$$

Series representations:

$$\frac{1}{1+\frac{32\pi}{12\times16\times64}+\frac{32}{64}\pi\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^3}=\frac{12288}{12291+224\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}}$$

$$\frac{1}{1+\frac{32\pi}{12\times16\times64}+\frac{32}{64}\pi\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^3}=$$
$$\frac{12288}{12291+\sum_{k=0}^{\infty}-\frac{224(-1)^k\,1195^{-1-2k}\left(5^{1+2k}-4\times239^{1+2k}\right)}{1+2k}}$$

$$\frac{1}{1+\frac{32\pi}{12\times 16\times 64}+\frac{32}{64}\pi\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^3} = \frac{12\,288}{12\,291+56\sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)}$$

Integral representations:

$$\frac{1}{1+\frac{32\pi}{12\times 16\times 64}+\frac{32}{64}\pi\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^3} = \frac{1}{\frac{4097}{4096}+\frac{7}{460}\int_0^{\infty}\frac{\sin^5(t)}{t^5}\,dt}$$

$$\frac{1}{1+\frac{32\pi}{12\times 16\times 64}+\frac{32}{64}\pi\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^3} = \frac{1}{\frac{4097}{4096}+\frac{35}{2112}\int_0^{\infty}\frac{\sin^6(t)}{t^6}\,dt}$$

(((1+32Pi*(1/12)*(1/16)*(1/64)+32Pi*(1/16)^2*(1/64)+(1/16)^3)))^35

Input:

$$\left(1+32\pi\times\frac{1}{12}\times\frac{1}{16}\times\frac{1}{64}+32\pi\left(\frac{1}{16}\right)^2\times\frac{1}{64}+\left(\frac{1}{16}\right)^3\right)^{35}$$

Result:

$$\left(\frac{4097}{4096}+\frac{7\pi}{1536}\right)^{35}$$

Decimal approximation:

1.658594233345089382594153327775853858435230480428186028083...

1.658594233….result very near to the 14th root of the following Ramanujan's class invariant $Q = \left(G_{505}/G_{101/5}\right)^3 = 1164{,}2696$ i.e. 1,65578...

Property:

$\left(\frac{4097}{4096}+\frac{7\pi}{1536}\right)^{35}$ is a transcendental number

Alternative representations:

$$\left(1+\frac{32\pi}{12\times 16\times 64}+\frac{32}{64}\pi\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^3\right)^{35} = \left(1+\frac{5760^\circ}{12\times 16\times 64}+\frac{5760}{64}{}^\circ\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^3\right)^{35}$$

$$\left(1+\frac{32\pi}{12\times16\times64}+\frac{32}{64}\pi\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^3\right)^{35} = \left(1-\frac{32\,i\log(-1)}{12\times16\times64}-\frac{32}{64}\,i\log(-1)\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^3\right)^{35}$$

$$\left(1+\frac{32\pi}{12\times16\times64}+\frac{32}{64}\pi\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^3\right)^{35} = \left(1+\frac{32\cos^{-1}(-1)}{12\times16\times64}+\frac{32}{64}\cos^{-1}(-1)\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^3\right)^{35}$$

Series representations:

$$\left(1+\frac{32\pi}{12\times16\times64}+\frac{32}{64}\pi\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^3\right)^{35} = \left(\frac{4097}{4096}+\frac{7}{384}\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}\right)^{35}$$

$$\left(1+\frac{32\pi}{12\times16\times64}+\frac{32}{64}\pi\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^3\right)^{35} = \left(\frac{4097}{4096}+\frac{7\sum_{k=0}^{\infty}-\frac{4(-1)^k\,1195^{-1-2k}\left(5^{1+2k}-4\times239^{1+2k}\right)}{1+2k}}{1536}\right)^{35}$$

$$\left(1+\frac{32\pi}{12\times16\times64}+\frac{32}{64}\pi\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^3\right)^{35} = \left(\frac{4097}{4096}+\frac{7\sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{2}{1+4k}+\frac{2}{2+4k}+\frac{1}{3+4k}\right)}{1536}\right)^{35}$$

Integral representations:

$$\left(1+\frac{32\pi}{12\times16\times64}+\frac{32}{64}\pi\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^3\right)^{35} = \left(\frac{4097}{4096}+\frac{7}{384}\int_0^1\sqrt{1-t^2}\,dt\right)^{35}$$

$$\left(1+\frac{32\pi}{12\times16\times64}+\frac{32}{64}\pi\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^3\right)^{35} = \left(\frac{4097}{4096}+\frac{7}{768}\int_0^{\infty}\frac{1}{1+t^2}\,dt\right)^{35}$$

$$\left(1+\frac{32\pi}{12\times16\times64}+\frac{32}{64}\pi\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^3\right)^{35} = \left(\frac{4097}{4096}+\frac{7}{768}\int_0^1\frac{1}{\sqrt{1-t^2}}\,dt\right)^{35}$$

In conclusion:

-(29+11)/10^3+(((1+32Pi*(1/12)*(1/16)*(1/64)+32Pi*

(1/16)^2*(1/64)+(1/16)^3)))^35

Input:

$$-\frac{29+11}{10^3}+\left(1+32\,\pi\times\frac{1}{12}\times\frac{1}{16}\times\frac{1}{64}+32\,\pi\left(\frac{1}{16}\right)^2\times\frac{1}{64}+\left(\frac{1}{16}\right)^3\right)^{35}$$

Exact result:

$$\left(\frac{4097}{4096}+\frac{7\,\pi}{1536}\right)^{35}-\frac{1}{25}$$

Decimal approximation:

1.618594233345089382594153327775853858435230480428186028083...

1.6185942333….

This result is a very good approximation to the value of the golden ratio 1,618033988749...

Property:

$$-\frac{1}{25}+\left(\frac{4097}{4096}+\frac{7\,\pi}{1536}\right)^{35}\text{ is a transcendental number}$$

Alternative representations:

$$-\frac{29+11}{10^3}+\left(1+\frac{32\,\pi}{12\times16\times64}+\frac{32}{64}\,\pi\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^3\right)^{35}=-\frac{40}{10^3}+\left(1+\frac{5760\,^\circ}{12\times16\times64}+\frac{5760}{64}\,^\circ\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^3\right)^{35}$$

$$-\frac{29+11}{10^3}+\left(1+\frac{32\,\pi}{12\times16\times64}+\frac{32}{64}\,\pi\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^3\right)^{35}=-\frac{40}{10^3}+\left(1-\frac{32\,i\log(-1)}{12\times16\times64}-\frac{32}{64}\,i\log(-1)\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^3\right)^{35}$$

$$-\frac{29+11}{10^3}+\left(1+\frac{32\,\pi}{12\times16\times64}+\frac{32}{64}\,\pi\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^3\right)^{35}=-\frac{40}{10^3}+\left(1+\frac{32\cos^{-1}(-1)}{12\times16\times64}+\frac{32}{64}\cos^{-1}(-1)\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^3\right)^{35}$$

Series representations:

$$-\frac{29+11}{10^3}+\left(1+\frac{32\,\pi}{12\times16\times64}+\frac{32}{64}\,\pi\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^3\right)^{35}=-\frac{1}{25}+\left(\frac{4097}{4096}+\frac{7}{384}\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2\,k}\right)^{35}$$

$$-\frac{29+11}{10^3}+\left(1+\frac{32\pi}{12\times16\times64}+\frac{32}{64}\pi\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^3\right)^{35}=$$
$$-\frac{1}{25}+\left(\frac{4097}{4096}+\frac{7\sum_{k=0}^{\infty}-\frac{4(-1)^k\,1195^{-1-2k}\left(5^{1+2k}-4\times239^{1+2k}\right)}{1+2k}}{1536}\right)^{35}$$

$$-\frac{29+11}{10^3}+\left(1+\frac{32\pi}{12\times16\times64}+\frac{32}{64}\pi\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^3\right)^{35}=$$
$$-\frac{1}{25}+\left(\frac{4097}{4096}+\frac{7\sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{2}{1+4k}+\frac{2}{2+4k}+\frac{1}{3+4k}\right)}{1536}\right)^{35}$$

Integral representations:

$$-\frac{29+11}{10^3}+\left(1+\frac{32\pi}{12\times16\times64}+\frac{32}{64}\pi\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^3\right)^{35}=$$
$$-\frac{1}{25}+\left(\frac{4097}{4096}+\frac{7}{384}\int_0^1\sqrt{1-t^2}\,dt\right)^{35}$$

$$-\frac{29+11}{10^3}+\left(1+\frac{32\pi}{12\times16\times64}+\frac{32}{64}\pi\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^3\right)^{35}=$$
$$-\frac{1}{25}+\left(\frac{4097}{4096}+\frac{7}{768}\int_0^{\infty}\frac{1}{1+t^2}\,dt\right)^{35}$$

$$-\frac{29+11}{10^3}+\left(1+\frac{32\pi}{12\times16\times64}+\frac{32}{64}\pi\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^3\right)^{35}=$$
$$-\frac{1}{25}+\left(\frac{4097}{4096}+\frac{7}{768}\int_0^1\frac{1}{\sqrt{1-t^2}}\,dt\right)^{35}$$

Now, we have that:

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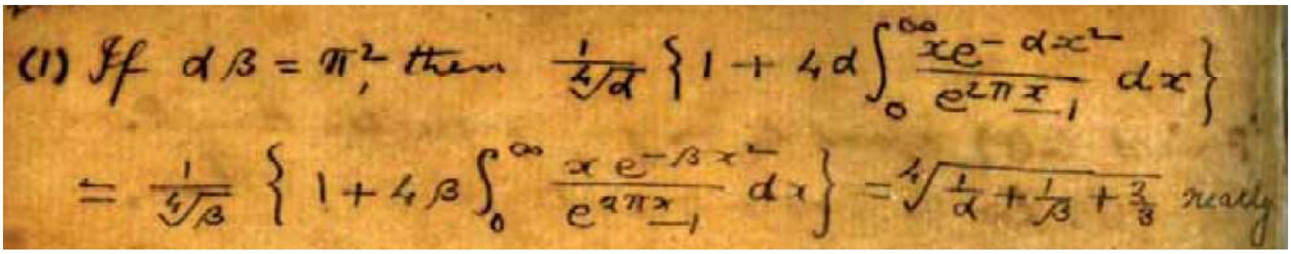
(1) If $\alpha\beta=\pi^2$, then $\frac{1}{\sqrt[4]{\alpha}}\left\{1+4\alpha\int_0^\infty\frac{xe^{-\alpha x^2}}{e^{2\pi x}-1}dx\right\}$
$=\frac{1}{\sqrt[4]{\beta}}\left\{1+4\beta\int_0^\infty\frac{xe^{-\beta x^2}}{e^{2\pi x}-1}dx\right\}=\sqrt[4]{\frac{1}{\alpha}+\frac{1}{\beta}+\frac{2}{3}}$ nearly

(1/Pi + 1/Pi + 2/3)^1/4

Input:

$$\sqrt[4]{\frac{1}{\pi}+\frac{1}{\pi}+\frac{2}{3}}$$

Exact result:

$$\sqrt[4]{\frac{2}{3}+\frac{2}{\pi}}$$

Decimal approximation:

1.0684641848256444258975743779642393458802855347366759251 61...

1.0684641848….

Property:

$\sqrt[4]{\frac{2}{3}+\frac{2}{\pi}}$ is a transcendental number

Alternate form:

$$\sqrt[4]{\frac{2(3+\pi)}{3\pi}}$$

Alternative representations:

$$\sqrt[4]{\frac{1}{\pi}+\frac{1}{\pi}+\frac{2}{3}}=\sqrt[4]{\frac{2}{3}+\frac{2}{180\,°}}$$

$$\sqrt[4]{\frac{1}{\pi}+\frac{1}{\pi}+\frac{2}{3}}=\sqrt[4]{\frac{2}{3}+-\frac{2}{i\log(-1)}}$$

$$\sqrt[4]{\frac{1}{\pi}+\frac{1}{\pi}+\frac{2}{3}}=\sqrt[4]{\frac{2}{3}+\frac{2}{\cos^{-1}(-1)}}$$

Series representations:

$$\sqrt[4]{\frac{1}{\pi}+\frac{1}{\pi}+\frac{2}{3}}=\sqrt[4]{\frac{2}{3}+\frac{1}{2\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}}}$$

$$\sqrt[4]{\frac{1}{\pi}+\frac{1}{\pi}+\frac{2}{3}}=\sqrt[4]{\frac{2}{3}+\frac{1}{\sum_{k=0}^{\infty}-\frac{2(-1)^k\,1195^{-1-2k}\left(5^{1+2k}-4\times 239^{1+2k}\right)}{1+2k}}}$$

$$\sqrt[4]{\frac{1}{\pi}+\frac{1}{\pi}+\frac{2}{3}} = \sqrt[4]{\frac{2}{3}+\frac{2}{\sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)}}$$

Integral representations:

$$\sqrt[4]{\frac{1}{\pi}+\frac{1}{\pi}+\frac{2}{3}} = \sqrt[4]{\frac{2}{3}+\frac{1}{\int_0^{\infty}\frac{1}{1+t^2}\,dt}}$$

$$\sqrt[4]{\frac{1}{\pi}+\frac{1}{\pi}+\frac{2}{3}} = \sqrt[4]{\frac{2}{3}+\frac{1}{\int_0^{1}\frac{1}{\sqrt{1-t^2}}\,dt}}$$

$$\sqrt[4]{\frac{1}{\pi}+\frac{1}{\pi}+\frac{2}{3}} = \sqrt[4]{\frac{2}{3}+\frac{1}{\int_0^{\infty}\frac{\sin(t)}{t}\,dt}}$$

And:

1/ ((((1/Pi + 1/Pi + 2/3)^1/4)))

Input:

$$\frac{1}{\sqrt[4]{\frac{1}{\pi}+\frac{1}{\pi}+\frac{2}{3}}}$$

Exact result:

$$\frac{1}{\sqrt[4]{\frac{2}{3}+\frac{2}{\pi}}}$$

Decimal approximation:

0.935922807897565006810718841026160004421420806231548562723...

0.9359228078…. result very near to the spectral index n_s and to the mesonic Regge slope (see Appendix) and to the inflaton value at the end of the inflation 0.9402

Property:

$$\frac{1}{\sqrt[4]{\frac{2}{3}+\frac{2}{\pi}}}$$ is a transcendental number

Alternate form:

$$\sqrt[4]{\frac{3\pi}{2(3+\pi)}}$$

Alternative representations:

$$\frac{1}{\sqrt[4]{\frac{1}{\pi}+\frac{1}{\pi}+\frac{2}{3}}}=\frac{1}{\sqrt[4]{\frac{2}{3}+\frac{2}{180\,^{\circ}}}}$$

$$\frac{1}{\sqrt[4]{\frac{1}{\pi}+\frac{1}{\pi}+\frac{2}{3}}}=\frac{1}{\sqrt[4]{\frac{2}{3}+\frac{2}{\cos^{-1}(-1)}}}$$

$$\frac{1}{\sqrt[4]{\frac{1}{\pi}+\frac{1}{\pi}+\frac{2}{3}}}=\frac{1}{\sqrt[4]{\frac{2}{3}+-\frac{2}{i\log(-1)}}}$$

Series representations:

$$\frac{1}{\sqrt[4]{\frac{1}{\pi}+\frac{1}{\pi}+\frac{2}{3}}}=\frac{1}{\sqrt[4]{\frac{2}{3}+\frac{1}{2\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}}}}$$

$$\frac{1}{\sqrt[4]{\frac{1}{\pi}+\frac{1}{\pi}+\frac{2}{3}}}=\frac{1}{\sqrt[4]{\frac{2}{3}+\frac{1}{\sum_{k=0}^{\infty}-\frac{2(-1)^k\,1195^{-1-2k}\left(5^{1+2k}-4\times 239^{1+2k}\right)}{1+2k}}}}$$

$$\frac{1}{\sqrt[4]{\frac{1}{\pi}+\frac{1}{\pi}+\frac{2}{3}}}=\frac{1}{\sqrt[4]{\frac{2}{3}+\frac{2}{\sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)}}}$$

Integral representations:

$$\frac{1}{\sqrt[4]{\frac{1}{\pi}+\frac{1}{\pi}+\frac{2}{3}}} = \frac{1}{\sqrt[4]{\frac{2}{3}+\frac{1}{\int_0^\infty \frac{1}{1+t^2}\,dt}}}$$

$$\frac{1}{\sqrt[4]{\frac{1}{\pi}+\frac{1}{\pi}+\frac{2}{3}}} = \frac{1}{\sqrt[4]{\frac{2}{3}+\frac{1}{\int_0^1 \frac{1}{\sqrt{1-t^2}}\,dt}}}$$

$$\frac{1}{\sqrt[4]{\frac{1}{\pi}+\frac{1}{\pi}+\frac{2}{3}}} = \frac{1}{\sqrt[4]{\frac{2}{3}+\frac{1}{\int_0^\infty \frac{\sin(t)}{t}\,dt}}}$$

Now:

If n is a positive integer

If $\alpha\beta = 4\pi^2$, then

$$(4)\quad \int_0^\infty \frac{x \sin 2nx}{e^{x^2}+e^{-x^2}}\,dx = \frac{n\sqrt{\pi}}{2}\left(e^{-n^2} - \frac{e^{-\frac{n^2}{3}}}{3\sqrt{3}} + \frac{e^{-\frac{n^2}{5}}}{5\sqrt{5}} - \&c\right)$$

$$= \frac{\pi}{2}\left(e^{-n\sqrt{\pi}} \sin n\sqrt{\pi} - e^{-n\sqrt{3\pi}} \sin n\sqrt{3\pi} + \&c\right).$$

Pi/2((((((e^(-4sqrt(Pi))*((sin(4sqrt(pi)))-e^(-4sqrt(3Pi))*sin((4sqrt(3Pi)))))))))

Input:

$$\frac{\pi}{2}\left(e^{-4\sqrt{\pi}}\left(\sin\left(4\sqrt{\pi}\right) - e^{-4\sqrt{3\pi}} \sin\left(4\sqrt{3\pi}\right)\right)\right)$$

Exact result:

$$\frac{1}{2} e^{-4\sqrt{\pi}} \pi \left(\sin\left(4\sqrt{\pi}\right) - e^{-4\sqrt{3\pi}} \sin\left(4\sqrt{3\pi}\right)\right)$$

Decimal approximation:

0.000945291620585502568170161455260046037670259110665998863...

0.0009452916205855….

Alternate forms:

$$-\frac{1}{2} e^{-4\sqrt{\pi}} \pi \left(e^{-4\sqrt{3\pi}} \sin\left(4\sqrt{3\pi}\right) - \sin\left(4\sqrt{\pi}\right)\right)$$

$$\frac{1}{2} e^{-4\sqrt{\pi}} \pi \sin\left(4\sqrt{\pi}\right) - \frac{1}{2} e^{-4\sqrt{\pi}-4\sqrt{3\pi}} \pi \sin\left(4\sqrt{3\pi}\right)$$

$$\frac{1}{2} e^{-4\sqrt{\pi}-4\sqrt{3\pi}} \pi \left(e^{4\sqrt{3\pi}} \sin\left(4\sqrt{\pi}\right) - \sin\left(4\sqrt{3\pi}\right)\right)$$

Alternative representations:

$$\frac{1}{2}\left(e^{-4\sqrt{\pi}}\left(\sin\left(4\sqrt{\pi}\right) - e^{-4\sqrt{3\pi}} \sin\left(4\sqrt{3\pi}\right)\right)\right)\pi = \frac{1}{2}\pi e^{-4\sqrt{\pi}}\left(\cos\left(\frac{\pi}{2} - 4\sqrt{\pi}\right) - \cos\left(\frac{\pi}{2} - 4\sqrt{3\pi}\right) e^{-4\sqrt{3\pi}}\right)$$

$$\frac{1}{2}\left(e^{-4\sqrt{\pi}}\left(\sin\left(4\sqrt{\pi}\right) - e^{-4\sqrt{3\pi}} \sin\left(4\sqrt{3\pi}\right)\right)\right)\pi = \frac{1}{2}\pi e^{-4\sqrt{\pi}}\left(\frac{1}{\csc\left(4\sqrt{\pi}\right)} - \frac{e^{-4\sqrt{3\pi}}}{\csc\left(4\sqrt{3\pi}\right)}\right)$$

$$\frac{1}{2}\left(e^{-4\sqrt{\pi}}\left(\sin\left(4\sqrt{\pi}\right) - e^{-4\sqrt{3\pi}} \sin\left(4\sqrt{3\pi}\right)\right)\right)\pi = \frac{1}{2}\pi e^{-4\sqrt{\pi}}\left(-\cos\left(\frac{\pi}{2} + 4\sqrt{\pi}\right) + \cos\left(\frac{\pi}{2} + 4\sqrt{3\pi}\right) e^{-4\sqrt{3\pi}}\right)$$

Series representations:

$$\frac{1}{2}\left(e^{-4\sqrt{\pi}}\left(\sin\left(4\sqrt{\pi}\right) - e^{-4\sqrt{3\pi}} \sin\left(4\sqrt{3\pi}\right)\right)\right)\pi = \sum_{k=0}^{\infty} \frac{(-1)^{1+k}\, 2^{1+4k}\, e^{-4\left(1+\sqrt{3}\right)\sqrt{\pi}} \left(3^{1/2+k} - e^{4\sqrt{3\pi}}\right) \pi^{3/2+k}}{(1+2k)!}$$

$$\frac{1}{2}\left(e^{-4\sqrt{\pi}}\left(\sin\left(4\sqrt{\pi}\right) - e^{-4\sqrt{3\pi}} \sin\left(4\sqrt{3\pi}\right)\right)\right)\pi = \sum_{k=0}^{\infty}\left(\frac{(-1)^k\, 2^{1+4k}\, e^{-4\sqrt{\pi}}\, \pi^{1+1/2(1+2k)}}{(1+2k)!} + \frac{(-1)^{1+k}\, 2^{1+4k} \times 3^{1/2(1+2k)}\, e^{-4\sqrt{\pi}-4\sqrt{3\pi}}\, \pi^{1+1/2(1+2k)}}{(1+2k)!}\right)$$

$$\frac{1}{2}\left(e^{-4\sqrt{\pi}}\left(\sin\left(4\sqrt{\pi}\right)-e^{-4\sqrt{3\pi}}\sin\left(4\sqrt{3\pi}\right)\right)\right)\pi = \sum_{k=0}^{\infty}\frac{(-1)^k e^{-4\left(1+\sqrt{3}\right)\sqrt{\pi}}\pi\left(e^{4\sqrt{3\pi}}\left(4\sqrt{\pi}-\frac{\pi}{2}\right)^{2k}-\left(-\frac{\pi}{2}+4\sqrt{3\pi}\right)^{2k}\right)}{2(2k)!}$$

Integral representations:

$$\frac{1}{2}\left(e^{-4\sqrt{\pi}}\left(\sin\left(4\sqrt{\pi}\right)-e^{-4\sqrt{3\pi}}\sin\left(4\sqrt{3\pi}\right)\right)\right)\pi = \int_0^1\left(2e^{-4\sqrt{\pi}}\pi^{3/2}\cos\left(4\sqrt{\pi}\,t\right)-2\sqrt{3}\,e^{-4\sqrt{\pi}-4\sqrt{3\pi}}\pi^{3/2}\cos\left(4\sqrt{3\pi}\,t\right)\right)dt$$

$$\frac{1}{2}\left(e^{-4\sqrt{\pi}}\left(\sin\left(4\sqrt{\pi}\right)-e^{-4\sqrt{3\pi}}\sin\left(4\sqrt{3\pi}\right)\right)\right)\pi = \int_{-i\infty+\gamma}^{i\infty+\gamma}\left(\frac{i\sqrt{3}\,e^{-4\sqrt{\pi}-4\sqrt{3\pi}-(12\pi)/s+s}\pi}{2s^{3/2}}-\frac{i\,e^{-4\sqrt{\pi}-(4\pi)/s+s}\pi}{2s^{3/2}}\right)ds \quad \text{for } \gamma > 0$$

$$\frac{1}{2}\left(e^{-4\sqrt{\pi}}\left(\sin\left(4\sqrt{\pi}\right)-e^{-4\sqrt{3\pi}}\sin\left(4\sqrt{3\pi}\right)\right)\right)\pi = \int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{i\,2^{-1-2s}\times 3^{-s}e^{-4\left(1+\sqrt{3}\right)\sqrt{\pi}}\left(\sqrt{3}-3^s e^{4\sqrt{3\pi}}\right)\pi^{1-s}\,\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}ds \quad \text{for } 0 < \gamma < 1$$

Multiple-argument formulas:

$$\frac{1}{2}\left(e^{-4\sqrt{\pi}}\left(\sin\left(4\sqrt{\pi}\right)-e^{-4\sqrt{3\pi}}\sin\left(4\sqrt{3\pi}\right)\right)\right)\pi = \frac{1}{2}e^{-4\sqrt{\pi}}\pi\left(2\cos\left(2\sqrt{\pi}\right)\sin\left(2\sqrt{\pi}\right)-2e^{-4\sqrt{3\pi}}\cos\left(2\sqrt{3\pi}\right)\sin\left(2\sqrt{3\pi}\right)\right)$$

$$\frac{1}{2}\left(e^{-4\sqrt{\pi}}\left(\sin\left(4\sqrt{\pi}\right)-e^{-4\sqrt{3\pi}}\sin\left(4\sqrt{3\pi}\right)\right)\right)\pi = \prod_{k=0}^{3}-4e^{-4\left(1+\sqrt{3}\right)\sqrt{\pi}}\pi\left(-e^{4\sqrt{3\pi}}\sin\left(\sqrt{\pi}+\frac{k\pi}{4}\right)+\sin\left(\frac{k\pi}{4}+\sqrt{3\pi}\right)\right)$$

$$\frac{1}{2}\left(e^{-4\sqrt{\pi}}\left(\sin\left(4\sqrt{\pi}\right)-e^{-4\sqrt{3\pi}}\sin\left(4\sqrt{3\pi}\right)\right)\right)\pi = \prod_{k=0}^{3}\left(4e^{-4\sqrt{\pi}}\pi\sin\left(\sqrt{\pi}+\frac{k\pi}{4}\right)-4e^{-4\sqrt{\pi}-4\sqrt{3\pi}}\pi\sin\left(\frac{k\pi}{4}+\sqrt{3\pi}\right)\right)$$

1+((((Pi/2((((((e^(-4sqrt(Pi))*((sin(4sqrt(Pi)))-e^(-4sqrt(3Pi))*sin((4sqrt(3Pi))))))))))))))^1/16

Input:

$$1+\sqrt[16]{\frac{\pi}{2}\left(e^{-4\sqrt{\pi}}\left(\sin\left(4\sqrt{\pi}\right)-e^{-4\sqrt{3\pi}}\sin\left(4\sqrt{3\pi}\right)\right)\right)}$$

Exact result:

$$1+e^{-\sqrt{\pi}/4}\sqrt[16]{\frac{1}{2}\pi\left(\sin\left(4\sqrt{\pi}\right)-e^{-4\sqrt{3\pi}}\sin\left(4\sqrt{3\pi}\right)\right)}$$

Decimal approximation:

1.6471021801579573713354391625345586846951860482181653040 90...

$1.64710218\ldots \approx \zeta(2)=\frac{\pi^2}{6}=1.644934\ldots$

Alternate forms:

$$1+e^{-\sqrt{\pi}/4}\sqrt[16]{\frac{1}{2}\left(\frac{1}{2}i\left(e^{-4i\sqrt{\pi}}-e^{4i\sqrt{\pi}}\right)-\frac{1}{2}i\,e^{-4\sqrt{3\pi}}\left(e^{-4i\sqrt{3\pi}}-e^{4i\sqrt{3\pi}}\right)\right)\pi}$$

$$\frac{1}{2}e^{-\sqrt{\pi}/4-\sqrt{3\pi}/4}\left(2e^{\sqrt{\pi}/4+\sqrt{3\pi}/4}+2^{15/16}\sqrt[16]{\pi\left(e^{4\sqrt{3\pi}}\sin\left(4\sqrt{\pi}\right)-\sin\left(4\sqrt{3\pi}\right)\right)}\right)$$

Alternative representations:

$$1+\sqrt[16]{\frac{1}{2}\left(e^{-4\sqrt{\pi}}\left(\sin\left(4\sqrt{\pi}\right)-e^{-4\sqrt{3\pi}}\sin\left(4\sqrt{3\pi}\right)\right)\right)\pi}=1+\sqrt[16]{\frac{1}{2}\pi e^{-4\sqrt{\pi}}\left(\frac{1}{\csc\left(4\sqrt{\pi}\right)}-\frac{e^{-4\sqrt{3\pi}}}{\csc\left(4\sqrt{3\pi}\right)}\right)}$$

$$1+\sqrt[16]{\frac{1}{2}\left(e^{-4\sqrt{\pi}}\left(\sin\left(4\sqrt{\pi}\right)-e^{-4\sqrt{3\pi}}\sin\left(4\sqrt{3\pi}\right)\right)\right)\pi}=1+\sqrt[16]{\frac{1}{2}\pi e^{-4\sqrt{\pi}}\left(\cos\left(\frac{\pi}{2}-4\sqrt{\pi}\right)-\cos\left(\frac{\pi}{2}-4\sqrt{3\pi}\right)e^{-4\sqrt{3\pi}}\right)}$$

$$1+\sqrt[16]{\frac{1}{2}\left(e^{-4\sqrt{\pi}}\left(\sin\left(4\sqrt{\pi}\right)-e^{-4\sqrt{3\pi}}\sin\left(4\sqrt{3\pi}\right)\right)\right)\pi}=$$
$$1+\sqrt[16]{\frac{1}{2}\pi e^{-4\sqrt{\pi}}\left(-\cos\left(\frac{\pi}{2}+4\sqrt{\pi}\right)+\cos\left(\frac{\pi}{2}+4\sqrt{3\pi}\right)e^{-4\sqrt{3\pi}}\right)}$$

Series representations:

$$1+\sqrt[16]{\frac{1}{2}\left(e^{-4\sqrt{\pi}}\left(\sin\left(4\sqrt{\pi}\right)-e^{-4\sqrt{3\pi}}\sin\left(4\sqrt{3\pi}\right)\right)\right)\pi}=$$
$$1+e^{-\sqrt{\pi}/4}\sqrt[16]{\frac{\pi}{2}}\sqrt[16]{\sum_{k=0}^{\infty}\frac{(-1)^k 4^{1+2k} e^{-4\sqrt{3\pi}}\left(-3^{1/2+k}+e^{4\sqrt{3\pi}}\right)\pi^{1/2+k}}{(1+2k)!}}$$

$$1+\sqrt[16]{\frac{1}{2}\left(e^{-4\sqrt{\pi}}\left(\sin\left(4\sqrt{\pi}\right)-e^{-4\sqrt{3\pi}}\sin\left(4\sqrt{3\pi}\right)\right)\right)\pi}=$$
$$1+e^{-\sqrt{\pi}/4}\sqrt[16]{\frac{\pi}{2}}\sqrt[16]{\sum_{k=0}^{\infty}\frac{(-1)^k\left(\left(4\sqrt{\pi}-\frac{\pi}{2}\right)^{2k}-e^{-4\sqrt{3\pi}}\left(-\frac{\pi}{2}+4\sqrt{3\pi}\right)^{2k}\right)}{(2k)!}}$$

$$1+\sqrt[16]{\frac{1}{2}\left(e^{-4\sqrt{\pi}}\left(\sin\left(4\sqrt{\pi}\right)-e^{-4\sqrt{3\pi}}\sin\left(4\sqrt{3\pi}\right)\right)\right)\pi}=$$
$$1+e^{-\sqrt{\pi}/4}\sqrt[16]{\frac{\pi}{2}}\sqrt[16]{\sum_{k=0}^{\infty}\left(\frac{(-1)^k\left(4\sqrt{\pi}-\frac{\pi}{2}\right)^{2k}}{(2k)!}+\frac{(-1)^{1+k}e^{-4\sqrt{3\pi}}\left(-\frac{\pi}{2}+4\sqrt{3\pi}\right)^{2k}}{(2k)!}\right)}$$

Integral representations:

$$1+\sqrt[16]{\frac{1}{2}\left(e^{-4\sqrt{\pi}}\left(\sin\left(4\sqrt{\pi}\right)-e^{-4\sqrt{3\pi}}\sin\left(4\sqrt{3\pi}\right)\right)\right)\pi}=$$
$$1+e^{-\sqrt{\pi}/4}\sqrt[16]{\frac{\pi}{2}}\sqrt[16]{\int_0^1\left(4\sqrt{\pi}\cos\left(4\sqrt{\pi}\,t\right)-4e^{-4\sqrt{3\pi}}\sqrt{3\pi}\cos\left(4\sqrt{3\pi}\,t\right)\right)dt}$$

$$1+\sqrt[16]{\frac{1}{2}\left(e^{-4\sqrt{\pi}}\left(\sin\left(4\sqrt{\pi}\right)-e^{-4\sqrt{3\pi}}\sin\left(4\sqrt{3\pi}\right)\right)\right)\pi}=$$
$$1+e^{-\sqrt{\pi}/4}\sqrt[16]{\frac{\pi}{2}}\sqrt[16]{\int_{-i\infty+\gamma}^{i\infty+\gamma}\left(\frac{i\sqrt{3}\,e^{-4\sqrt{3\pi}-(12\pi)/s+s}}{s^{3/2}}-\frac{i\,e^{-(4\pi)/s+s}}{s^{3/2}}\right)ds}\quad\text{for }\gamma>0$$

$$1+\sqrt[16]{\frac{1}{2}\left(e^{-4\sqrt{\pi}}\left(\sin\left(4\sqrt{\pi}\right)-e^{-4\sqrt{3\pi}}\sin\left(4\sqrt{3\pi}\right)\right)\right)\pi}=1+e^{-\sqrt{\pi}/4}\sqrt[16]{\frac{\pi}{2}}\sqrt[16]{\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{i\,e^{-4\sqrt{3\pi}}\left(\sqrt{3}-3^{s}e^{4\sqrt{3\pi}}\right)(12\pi)^{-s}\,\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\,ds}\quad\text{for } 0<\gamma<1$$

1/10^27 * [24/10^3+1+((((Pi/2((((((e^(-4sqrt(Pi))*((sin(4sqrt(Pi)))-e^(-4sqrt(3Pi))*sin((4sqrt(3Pi)))))))))))))))^1/16]

Input:

$$\frac{1}{10^{27}}\left(\frac{24}{10^{3}}+1+\sqrt[16]{\frac{\pi}{2}\left(e^{-4\sqrt{\pi}}\left(\sin\left(4\sqrt{\pi}\right)-e^{-4\sqrt{3\pi}}\sin\left(4\sqrt{3\pi}\right)\right)\right)}\right)$$

Exact result:

$$\frac{\frac{128}{125}+e^{-\sqrt{\pi}/4}\sqrt[16]{\frac{1}{2}\pi\left(\sin(4\sqrt{\pi})-e^{-4\sqrt{3\pi}}\sin(4\sqrt{3\pi})\right)}}{1\,000\,000\,000\,000\,000\,000\,000\,000\,000}$$

Decimal approximation:

$1.671102180157957371335439162534558684695186048218165 3\ldots\times 10^{-27}$

$1.67110218\ldots * 10^{-27}$

result practically equal to the value of the formula:

$$m_{p\prime}=2\times\frac{\eta}{R}m_{P}=1.6714213\times 10^{-24}\text{ gm}$$

that is the holographic proton mass (N. Haramein)

Alternate forms:

$$\frac{128+125\,e^{-\sqrt{\pi}/4}\sqrt[16]{\frac{1}{2}\pi\left(\sin(4\sqrt{\pi})-e^{-4\sqrt{3\pi}}\sin(4\sqrt{3\pi})\right)}}{125\,000\,000\,000\,000\,000\,000\,000\,000\,000}$$

$$\frac{1}{976\,562\,500\,000\,000\,000\,000\,000\,000}+\frac{e^{-\sqrt{\pi}/4}\sqrt[16]{\frac{1}{2}\pi\left(\sin(4\sqrt{\pi})-e^{-4\sqrt{3\pi}}\sin(4\sqrt{3\pi})\right)}}{1\,000\,000\,000\,000\,000\,000\,000\,000\,000}$$

$$\left(e^{-\sqrt{\pi}/4-\sqrt{3\pi}/4}\left(256\, e^{\sqrt{\pi}/4+\sqrt{3\pi}/4} + 125 \times 2^{15/16}\sqrt[16]{\pi\left(e^{4\sqrt{3\pi}}\sin\left(4\sqrt{\pi}\right)-\sin\left(4\sqrt{3\pi}\right)\right)}\right)\right)/$$
$$250\,000\,000\,000\,000\,000\,000\,000\,000\,000$$

Alternative representations:

$$\frac{\frac{24}{10^3}+1+\sqrt[16]{\frac{1}{2}\pi\left(e^{-4\sqrt{\pi}}\left(\sin\left(4\sqrt{\pi}\right)-e^{-4\sqrt{3\pi}}\sin\left(4\sqrt{3\pi}\right)\right)\right)}}{10^{27}} = \frac{1+\frac{24}{10^3}+\sqrt[16]{\frac{1}{2}\pi e^{-4\sqrt{\pi}}\left(\frac{1}{\csc\left(4\sqrt{\pi}\right)}-\frac{e^{-4\sqrt{3\pi}}}{\csc\left(4\sqrt{3\pi}\right)}\right)}}{10^{27}}$$

$$\frac{\frac{24}{10^3}+1+\sqrt[16]{\frac{1}{2}\pi\left(e^{-4\sqrt{\pi}}\left(\sin\left(4\sqrt{\pi}\right)-e^{-4\sqrt{3\pi}}\sin\left(4\sqrt{3\pi}\right)\right)\right)}}{10^{27}} = \frac{1+\frac{24}{10^3}+\sqrt[16]{\frac{1}{2}\pi e^{-4\sqrt{\pi}}\left(\cos\left(\frac{\pi}{2}-4\sqrt{\pi}\right)-\cos\left(\frac{\pi}{2}-4\sqrt{3\pi}\right)e^{-4\sqrt{3\pi}}\right)}}{10^{27}}$$

$$\frac{\frac{24}{10^3}+1+\sqrt[16]{\frac{1}{2}\pi\left(e^{-4\sqrt{\pi}}\left(\sin\left(4\sqrt{\pi}\right)-e^{-4\sqrt{3\pi}}\sin\left(4\sqrt{3\pi}\right)\right)\right)}}{10^{27}} = \frac{1+\frac{24}{10^3}+\sqrt[16]{\frac{1}{2}\pi e^{-4\sqrt{\pi}}\left(-\cos\left(\frac{\pi}{2}+4\sqrt{\pi}\right)+\cos\left(\frac{\pi}{2}+4\sqrt{3\pi}\right)e^{-4\sqrt{3\pi}}\right)}}{10^{27}}$$

Series representations:

$$\frac{\frac{24}{10^3}+1+\sqrt[16]{\frac{1}{2}\pi\left(e^{-4\sqrt{\pi}}\left(\sin\left(4\sqrt{\pi}\right)-e^{-4\sqrt{3\pi}}\sin\left(4\sqrt{3\pi}\right)\right)\right)}}{10^{27}} = \frac{1}{976\,562\,500\,000\,000\,000\,000\,000\,000} + \frac{e^{-\sqrt{\pi}/4}\sqrt[16]{\frac{\pi}{2}}\sqrt[16]{\sum_{k=0}^{\infty}\frac{(-1)^k 4^{1+2k} e^{-4\sqrt{3\pi}}\left(-3^{1/2+k}+e^{4\sqrt{3\pi}}\right)\pi^{1/2+k}}{(1+2k)!}}}{1\,000\,000\,000\,000\,000\,000\,000\,000\,000}$$

$$\frac{\frac{24}{10^3}+1+\sqrt[16]{\frac{1}{2}\pi\left(e^{-4\sqrt{\pi}}\left(\sin(4\sqrt{\pi})-e^{-4\sqrt{3\pi}}\sin(4\sqrt{3\pi})\right)\right)}}{10^{27}} = \frac{1}{976562500000000000000000000} + \frac{e^{-\sqrt{\pi}/4}\sqrt[16]{\frac{\pi}{2}}\sqrt[16]{\sum_{k=0}^{\infty}\frac{(-1)^k\left(\left(4\sqrt{\pi}-\frac{\pi}{2}\right)^{2k}-e^{-4\sqrt{3\pi}}\left(-\frac{\pi}{2}+4\sqrt{3\pi}\right)^{2k}\right)}{(2k)!}}}{1000000000000000000000000000}$$

$$\frac{\frac{24}{10^3}+1+\sqrt[16]{\frac{1}{2}\pi\left(e^{-4\sqrt{\pi}}\left(\sin(4\sqrt{\pi})-e^{-4\sqrt{3\pi}}\sin(4\sqrt{3\pi})\right)\right)}}{10^{27}} = \frac{1}{976562500000000000000000000} + \frac{e^{-\sqrt{\pi}/4}\sqrt[16]{\frac{\pi}{2}}\sqrt[16]{\sum_{k=0}^{\infty}\left(\frac{(-1)^k\left(4\sqrt{\pi}-\frac{\pi}{2}\right)^{2k}}{(2k)!}+\frac{(-1)^{1+k}e^{-4\sqrt{3\pi}}\left(-\frac{\pi}{2}+4\sqrt{3\pi}\right)^{2k}}{(2k)!}\right)}}{1000000000000000000000000000}$$

Integral representations:

$$\frac{\frac{24}{10^3}+1+\sqrt[16]{\frac{1}{2}\pi\left(e^{-4\sqrt{\pi}}\left(\sin(4\sqrt{\pi})-e^{-4\sqrt{3\pi}}\sin(4\sqrt{3\pi})\right)\right)}}{10^{27}} = \frac{1}{976562500000000000000000000} + \frac{e^{-\sqrt{\pi}/4}\sqrt[16]{\frac{\pi}{2}}\sqrt[16]{\int_0^1\left(4\sqrt{\pi}\cos(4\sqrt{\pi}\,t)-4e^{-4\sqrt{3\pi}}\sqrt{3\pi}\cos(4\sqrt{3\pi}\,t)\right)dt}}{1000000000000000000000000000}$$

$$\frac{\frac{24}{10^3}+1+\sqrt[16]{\frac{1}{2}\pi\left(e^{-4\sqrt{\pi}}\left(\sin(4\sqrt{\pi})-e^{-4\sqrt{3\pi}}\sin(4\sqrt{3\pi})\right)\right)}}{10^{27}} = \frac{1}{976562500000000000000000000} + \frac{e^{-\sqrt{\pi}/4}\sqrt[16]{\frac{\pi}{2}}\sqrt[16]{\int_{-i\infty+\gamma}^{i\infty+\gamma}\left(\frac{i\sqrt{3}\,e^{-4\sqrt{3\pi}-(12\pi)/s+s}}{s^{3/2}}-\frac{i\,e^{-(4\pi)/s+s}}{s^{3/2}}\right)ds}}{1000000000000000000000000000} \quad \text{for } \gamma > 0$$

$$\frac{\frac{24}{10^3}+1+\sqrt[16]{\frac{1}{2}\pi\left(e^{-4\sqrt{\pi}}\left(\sin(4\sqrt{\pi})-e^{-4\sqrt{3\pi}}\sin(4\sqrt{3\pi})\right)\right)}}{10^{27}} =$$

$$\frac{1}{976562500000000000000000000} + \frac{e^{-\sqrt{\pi}/4}\sqrt[16]{\frac{\pi}{2}}\sqrt[16]{\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{i e^{-4\sqrt{3\pi}}\left(\sqrt{3}-3^s e^{4\sqrt{3\pi}}\right)(12\pi)^{-s}\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}ds}}{1000000000000000000000000000} \quad \text{for } 0<\gamma<1$$

$\Gamma(x)$ is the gamma function

-29/10^3+1+((((Pi/2((((((e^(-4sqrt(Pi))*((sin(4sqrt(Pi)))-e^(-4sqrt(3Pi))*sin((4sqrt(3Pi)))))))))))))^1/16

Input:

$$-\frac{29}{10^3}+1+\sqrt[16]{\frac{\pi}{2}\left(e^{-4\sqrt{\pi}}\left(\sin\left(4\sqrt{\pi}\right)-e^{-4\sqrt{3\pi}}\sin\left(4\sqrt{3\pi}\right)\right)\right)}$$

Exact result:

$$\frac{971}{1000}+e^{-\sqrt{\pi}/4}\sqrt[16]{\frac{1}{2}\pi\left(\sin\left(4\sqrt{\pi}\right)-e^{-4\sqrt{3\pi}}\sin\left(4\sqrt{3\pi}\right)\right)}$$

Decimal approximation:

1.618102180157957371335439162534558684695186048218165304090...

1.61810218…

This result is a very good approximation to the value of the golden ratio 1,618033988749...

Alternate forms:

$$\frac{971}{1000}+e^{-\sqrt{\pi}/4}\sqrt[16]{\frac{1}{2}\left(\frac{1}{2}i\left(e^{-4i\sqrt{\pi}}-e^{4i\sqrt{\pi}}\right)-\frac{1}{2}i e^{-4\sqrt{3\pi}}\left(e^{-4i\sqrt{3\pi}}-e^{4i\sqrt{3\pi}}\right)\right)\pi}$$

$$\frac{1}{1000}e^{-\sqrt{\pi}/4-\sqrt{3\pi}/4}\left(971 e^{\sqrt{\pi}/4+\sqrt{3\pi}/4}+500\times 2^{15/16}\sqrt[16]{\pi\left(e^{4\sqrt{3\pi}}\sin\left(4\sqrt{\pi}\right)-\sin\left(4\sqrt{3\pi}\right)\right)}\right)$$

Alternative representations:

$$-\frac{29}{10^3}+1+\sqrt[16]{\frac{1}{2}\left(e^{-4\sqrt{\pi}}\left(\sin\left(4\sqrt{\pi}\right)-e^{-4\sqrt{3\pi}}\sin\left(4\sqrt{3\pi}\right)\right)\right)\pi} = 1-\frac{29}{10^3}+\sqrt[16]{\frac{1}{2}\pi e^{-4\sqrt{\pi}}\left(\frac{1}{\csc(4\sqrt{\pi})}-\frac{e^{-4\sqrt{3\pi}}}{\csc(4\sqrt{3\pi})}\right)}$$

$$-\frac{29}{10^3}+1+\sqrt[16]{\frac{1}{2}\left(e^{-4\sqrt{\pi}}\left(\sin\left(4\sqrt{\pi}\right)-e^{-4\sqrt{3\pi}}\sin\left(4\sqrt{3\pi}\right)\right)\right)\pi} = 1-\frac{29}{10^3}+\sqrt[16]{\frac{1}{2}\pi e^{-4\sqrt{\pi}}\left(\cos\left(\frac{\pi}{2}-4\sqrt{\pi}\right)-\cos\left(\frac{\pi}{2}-4\sqrt{3\pi}\right)e^{-4\sqrt{3\pi}}\right)}$$

$$-\frac{29}{10^3}+1+\sqrt[16]{\frac{1}{2}\left(e^{-4\sqrt{\pi}}\left(\sin\left(4\sqrt{\pi}\right)-e^{-4\sqrt{3\pi}}\sin\left(4\sqrt{3\pi}\right)\right)\right)\pi} = 1-\frac{29}{10^3}+\sqrt[16]{\frac{1}{2}\pi e^{-4\sqrt{\pi}}\left(-\cos\left(\frac{\pi}{2}+4\sqrt{\pi}\right)+\cos\left(\frac{\pi}{2}+4\sqrt{3\pi}\right)e^{-4\sqrt{3\pi}}\right)}$$

Series representations:

$$-\frac{29}{10^3}+1+\sqrt[16]{\frac{1}{2}\left(e^{-4\sqrt{\pi}}\left(\sin\left(4\sqrt{\pi}\right)-e^{-4\sqrt{3\pi}}\sin\left(4\sqrt{3\pi}\right)\right)\right)\pi} = \frac{971}{1000}+e^{-\sqrt{\pi}/4}\sqrt[16]{\frac{\pi}{2}}\sqrt[16]{\sum_{k=0}^{\infty}\frac{(-1)^k\,4^{1+2k}\,e^{-4\sqrt{3\pi}}\left(-3^{1/2+k}+e^{4\sqrt{3\pi}}\right)\pi^{1/2+k}}{(1+2k)!}}$$

$$-\frac{29}{10^3}+1+\sqrt[16]{\frac{1}{2}\left(e^{-4\sqrt{\pi}}\left(\sin\left(4\sqrt{\pi}\right)-e^{-4\sqrt{3\pi}}\sin\left(4\sqrt{3\pi}\right)\right)\right)\pi} = \frac{971}{1000}+e^{-\sqrt{\pi}/4}\sqrt[16]{\frac{\pi}{2}}\sqrt[16]{\sum_{k=0}^{\infty}\frac{(-1)^k\left(\left(4\sqrt{\pi}-\frac{\pi}{2}\right)^{2k}-e^{-4\sqrt{3\pi}}\left(-\frac{\pi}{2}+4\sqrt{3\pi}\right)^{2k}\right)}{(2k)!}}$$

$$-\frac{29}{10^3}+1+\sqrt[16]{\frac{1}{2}\left(e^{-4\sqrt{\pi}}\left(\sin\left(4\sqrt{\pi}\right)-e^{-4\sqrt{3\pi}}\sin\left(4\sqrt{3\pi}\right)\right)\right)\pi} = \frac{971}{1000}+e^{-\sqrt{\pi}/4}\sqrt[16]{\frac{\pi}{2}}\sqrt[16]{\sum_{k=0}^{\infty}\left(\frac{(-1)^k\left(4\sqrt{\pi}-\frac{\pi}{2}\right)^{2k}}{(2k)!}+\frac{(-1)^{1+k}e^{-4\sqrt{3\pi}}\left(-\frac{\pi}{2}+4\sqrt{3\pi}\right)^{2k}}{(2k)!}\right)}$$

Integral representations:

$$-\frac{29}{10^3}+1+\sqrt[16]{\frac{1}{2}\left(e^{-4\sqrt{\pi}}\left(\sin\left(4\sqrt{\pi}\right)-e^{-4\sqrt{3\pi}}\sin\left(4\sqrt{3\pi}\right)\right)\right)\pi} = \frac{971}{1000}+e^{-\sqrt{\pi}/4}\sqrt[16]{\frac{\pi}{2}}\sqrt[16]{\int_0^1\left(4\sqrt{\pi}\cos\left(4\sqrt{\pi}\,t\right)-4e^{-4\sqrt{3\pi}}\sqrt{3\pi}\cos\left(4\sqrt{3\pi}\,t\right)\right)dt}$$

$$-\frac{29}{10^3}+1+\sqrt[16]{\frac{1}{2}\left(e^{-4\sqrt{\pi}}\left(\sin\left(4\sqrt{\pi}\right)-e^{-4\sqrt{3\pi}}\sin\left(4\sqrt{3\pi}\right)\right)\right)\pi} =$$

$$\frac{971}{1000}+e^{-\sqrt{\pi}/4}\sqrt[16]{\frac{\pi}{2}}\sqrt[16]{\int_{-i\infty+\gamma}^{i\infty+\gamma}\left(\frac{i\sqrt{3}\,e^{-4\sqrt{3\pi}-(12\pi)/s+s}}{s^{3/2}}-\frac{i\,e^{-(4\pi)/s+s}}{s^{3/2}}\right)ds} \quad \text{for } \gamma>0$$

$$-\frac{29}{10^3}+1+\sqrt[16]{\frac{1}{2}\left(e^{-4\sqrt{\pi}}\left(\sin\left(4\sqrt{\pi}\right)-e^{-4\sqrt{3\pi}}\sin\left(4\sqrt{3\pi}\right)\right)\right)\pi} = \frac{971}{1000}+$$

$$e^{-\sqrt{\pi}/4}\sqrt[16]{\frac{\pi}{2}}\sqrt[16]{\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{i\,e^{-4\sqrt{3\pi}}\left(\sqrt{3}-3^s\,e^{4\sqrt{3\pi}}\right)(12\pi)^{-s}\,\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}ds} \quad \text{for } 0<\gamma<1$$

(((((Pi/2((((((e^(-4sqrt(Pi))*((sin(4sqrt(pi)))-e^(-4sqrt(3Pi))*sin((4sqrt(3Pi)))))))))))))))^1/1024

Input:

$$\sqrt[1024]{\frac{\pi}{2}\left(e^{-4\sqrt{\pi}}\left(\sin\left(4\sqrt{\pi}\right)-e^{-4\sqrt{3\pi}}\sin\left(4\sqrt{3\pi}\right)\right)\right)}$$

Exact result:

$$e^{-\sqrt{\pi}/256}\sqrt[1024]{\frac{1}{2}\pi\left(\sin\left(4\sqrt{\pi}\right)-e^{-4\sqrt{3\pi}}\sin\left(4\sqrt{3\pi}\right)\right)}$$

Decimal approximation:

0.993222275157061609755056215479801504881257255924359742559...

0.9932222…. result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\dots}}}}\approx 0.9991104684$$

and to the dilaton value $\mathbf{0.989117352243=\phi}$

Alternate forms:

$$e^{-\sqrt{\pi}/256-\sqrt{3\pi}/256}\sqrt[1024]{\frac{1}{2}\pi\left(e^{4\sqrt{3\pi}}\sin\left(4\sqrt{\pi}\right)-\sin\left(4\sqrt{3\pi}\right)\right)}$$

$$\frac{e^{-\sqrt{\pi}/256-\sqrt{3\pi}/256}}{\sqrt[1024]{\frac{2}{e^{4\sqrt{3\pi}}\pi\sin\left(4\sqrt{\pi}\right)-\pi\sin\left(4\sqrt{3\pi}\right)}}}$$

$$e^{-\sqrt{\pi}/256}\sqrt[1024]{\frac{1}{2}\left(\frac{1}{2}i\left(e^{-4i\sqrt{\pi}}-e^{4i\sqrt{\pi}}\right)-\frac{1}{2}ie^{-4\sqrt{3\pi}}\left(e^{-4i\sqrt{3\pi}}-e^{4i\sqrt{3\pi}}\right)\right)\pi}$$

All 1024th roots of 1/2 e^(-4 sqrt(π)) π (sin(4 sqrt(π)) - e^(-4 sqrt(3 π)) sin(4 sqrt(3 π))):

$$e^{-\sqrt{\pi}/256}e^{0}\sqrt[1024]{\frac{1}{2}\pi\left(\sin\left(4\sqrt{\pi}\right)-e^{-4\sqrt{3\pi}}\sin\left(4\sqrt{3\pi}\right)\right)}\approx 0.993222 \text{ (real, principal root)}$$

$$e^{-\sqrt{\pi}/256}e^{(i\pi)/512}\sqrt[1024]{\frac{1}{2}\pi\left(\sin\left(4\sqrt{\pi}\right)-e^{-4\sqrt{3\pi}}\sin\left(4\sqrt{3\pi}\right)\right)}\approx 0.993204+0.006094i$$

$$e^{-\sqrt{\pi}/256}e^{(i\pi)/256}\sqrt[1024]{\frac{1}{2}\pi\left(\sin\left(4\sqrt{\pi}\right)-e^{-4\sqrt{3\pi}}\sin\left(4\sqrt{3\pi}\right)\right)}\approx 0.993147+0.012188i$$

$$e^{-\sqrt{\pi}/256}e^{(3i\pi)/512}\sqrt[1024]{\frac{1}{2}\pi\left(\sin\left(4\sqrt{\pi}\right)-e^{-4\sqrt{3\pi}}\sin\left(4\sqrt{3\pi}\right)\right)}\approx 0.993054+0.018282i$$

$$e^{-\sqrt{\pi}/256}e^{(i\pi)/128}\sqrt[1024]{\frac{1}{2}\pi\left(\sin\left(4\sqrt{\pi}\right)-e^{-4\sqrt{3\pi}}\sin\left(4\sqrt{3\pi}\right)\right)}\approx 0.992923+0.024375i$$

Alternative representations:

$$\sqrt[1024]{\frac{1}{2}\left(e^{-4\sqrt{\pi}}\left(\sin\left(4\sqrt{\pi}\right)-e^{-4\sqrt{3\pi}}\sin\left(4\sqrt{3\pi}\right)\right)\right)\pi}=\sqrt[1024]{\frac{1}{2}\pi e^{-4\sqrt{\pi}}\left(\frac{1}{\csc\left(4\sqrt{\pi}\right)}-\frac{e^{-4\sqrt{3\pi}}}{\csc\left(4\sqrt{3\pi}\right)}\right)}$$

$$\sqrt[1024]{\frac{1}{2}\left(e^{-4\sqrt{\pi}}\left(\sin\left(4\sqrt{\pi}\right)-e^{-4\sqrt{3\pi}}\sin\left(4\sqrt{3\pi}\right)\right)\right)\pi}=\sqrt[1024]{\frac{1}{2}\pi e^{-4\sqrt{\pi}}\left(\cos\left(\frac{\pi}{2}-4\sqrt{\pi}\right)-\cos\left(\frac{\pi}{2}-4\sqrt{3\pi}\right)e^{-4\sqrt{3\pi}}\right)}$$

$$\sqrt[1024]{\frac{1}{2}\left(e^{-4\sqrt{\pi}}\left(\sin\left(4\sqrt{\pi}\right)-e^{-4\sqrt{3\pi}}\sin\left(4\sqrt{3\pi}\right)\right)\right)\pi} = \sqrt[1024]{\frac{1}{2}\pi e^{-4\sqrt{\pi}}\left(-\cos\left(\frac{\pi}{2}+4\sqrt{\pi}\right)+\cos\left(\frac{\pi}{2}+4\sqrt{3\pi}\right)e^{-4\sqrt{3\pi}}\right)}$$

Series representations:

$$\sqrt[1024]{\frac{1}{2}\left(e^{-4\sqrt{\pi}}\left(\sin\left(4\sqrt{\pi}\right)-e^{-4\sqrt{3\pi}}\sin\left(4\sqrt{3\pi}\right)\right)\right)\pi} = e^{-1/256\left(1+\sqrt{3}\right)\sqrt{\pi}}\sqrt[1024]{\frac{\pi}{2}}\sqrt[1024]{\sum_{k=0}^{\infty}\frac{(-1)^{1+k}4^{1+2k}\left(3^{1/2+k}-e^{4\sqrt{3\pi}}\right)\pi^{1/2+k}}{(1+2k)!}}$$

$$\sqrt[1024]{\frac{1}{2}\left(e^{-4\sqrt{\pi}}\left(\sin\left(4\sqrt{\pi}\right)-e^{-4\sqrt{3\pi}}\sin\left(4\sqrt{3\pi}\right)\right)\right)\pi} = e^{-\sqrt{\pi}/256}\sqrt[1024]{\frac{\pi}{2}}\sqrt[1024]{\sum_{k=0}^{\infty}\frac{(-1)^{k}4^{1+2k}e^{-4\sqrt{3\pi}}\left(-3^{1/2+k}+e^{4\sqrt{3\pi}}\right)\pi^{1/2+k}}{(1+2k)!}}$$

$$\sqrt[1024]{\frac{1}{2}\left(e^{-4\sqrt{\pi}}\left(\sin\left(4\sqrt{\pi}\right)-e^{-4\sqrt{3\pi}}\sin\left(4\sqrt{3\pi}\right)\right)\right)\pi} = e^{-\sqrt{\pi}/256}\sqrt[1024]{\frac{\pi}{2}}\sqrt[1024]{\sum_{k=0}^{\infty}\frac{(-1)^{k}\left(\left(4\sqrt{\pi}-\frac{\pi}{2}\right)^{2k}-e^{-4\sqrt{3\pi}}\left(-\frac{\pi}{2}+4\sqrt{3\pi}\right)^{2k}\right)}{(2k)!}}$$

Integral representations:

$$\sqrt[1024]{\frac{1}{2}\left(e^{-4\sqrt{\pi}}\left(\sin\left(4\sqrt{\pi}\right)-e^{-4\sqrt{3\pi}}\sin\left(4\sqrt{3\pi}\right)\right)\right)\pi} = e^{-\sqrt{\pi}/256}\sqrt[1024]{\frac{\pi}{2}}\sqrt[1024]{\int_0^1\left(4\sqrt{\pi}\cos\left(4\sqrt{\pi}\,t\right)-4e^{-4\sqrt{3\pi}}\sqrt{3\pi}\cos\left(4\sqrt{3\pi}\,t\right)\right)dt}$$

$$\sqrt[1024]{\frac{1}{2}\left(e^{-4\sqrt{\pi}}\left(\sin\left(4\sqrt{\pi}\right)-e^{-4\sqrt{3\pi}}\sin\left(4\sqrt{3\pi}\right)\right)\right)\pi} = e^{-\sqrt{\pi}/256}\sqrt[1024]{\frac{\pi}{2}}\sqrt[1024]{\int_{-i\infty+\gamma}^{i\infty+\gamma}\left(\frac{i\sqrt{3}\,e^{-4\sqrt{3\pi}-(12\pi)/s+s}}{s^{3/2}}-\frac{i\,e^{-(4\pi)/s+s}}{s^{3/2}}\right)ds} \quad \text{for } \gamma > 0$$

$$\sqrt[1024]{\frac{1}{2}\left(e^{-4\sqrt{\pi}}\left(\sin\left(4\sqrt{\pi}\right)-e^{-4\sqrt{3\pi}}\sin\left(4\sqrt{3\pi}\right)\right)\right)\pi} = e^{-\sqrt{\pi}/256}\sqrt[1024]{\frac{\pi}{2}}\sqrt[1024]{\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{i\,e^{-4\sqrt{3\pi}}\left(\sqrt{3}-3^{s}e^{4\sqrt{3\pi}}\right)(12\pi)^{-s}\,\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}ds} \quad \text{for } 0 < \gamma < 1$$

Multiple-argument formulas:

$$\sqrt[1024]{\frac{1}{2}\left(e^{-4\sqrt{\pi}}\left(\sin\left(4\sqrt{\pi}\right)-e^{-4\sqrt{3\pi}}\sin\left(4\sqrt{3\pi}\right)\right)\right)\pi} = e^{-\sqrt{\pi}/256}\sqrt[1024]{\frac{1}{2}\pi\left(2\cos\left(2\sqrt{\pi}\right)\sin\left(2\sqrt{\pi}\right)-2e^{-4\sqrt{3\pi}}\cos\left(2\sqrt{3\pi}\right)\sin\left(2\sqrt{3\pi}\right)\right)}$$

$$\sqrt[1024]{\frac{1}{2}\left(e^{-4\sqrt{\pi}}\left(\sin\left(4\sqrt{\pi}\right)-e^{-4\sqrt{3\pi}}\sin\left(4\sqrt{3\pi}\right)\right)\right)\pi} = e^{-\sqrt{\pi}/256}\sqrt[1024]{\frac{\pi}{2}}\sqrt[1024]{\prod_{k=0}^{3}\left(8\sin\left(\sqrt{\pi}+\frac{k\pi}{4}\right)-8e^{-4\sqrt{3\pi}}\sin\left(\frac{k\pi}{4}+\sqrt{3\pi}\right)\right)}$$

$$\sqrt[1024]{\frac{1}{2}\left(e^{-4\sqrt{\pi}}\left(\sin\left(4\sqrt{\pi}\right)-e^{-4\sqrt{3\pi}}\sin\left(4\sqrt{3\pi}\right)\right)\right)\pi} = e^{-\sqrt{\pi}/256}\sqrt[1024]{\frac{1}{2}\pi\left(-e^{-4\sqrt{3\pi}}\left(3\sin\left(4\sqrt{\frac{\pi}{3}}\right)-4\sin^3\left(4\sqrt{\frac{\pi}{3}}\right)\right)+3\sin\left(\frac{4\sqrt{\pi}}{3}\right)-4\sin^3\left(\frac{4\sqrt{\pi}}{3}\right)\right)}$$

Now,

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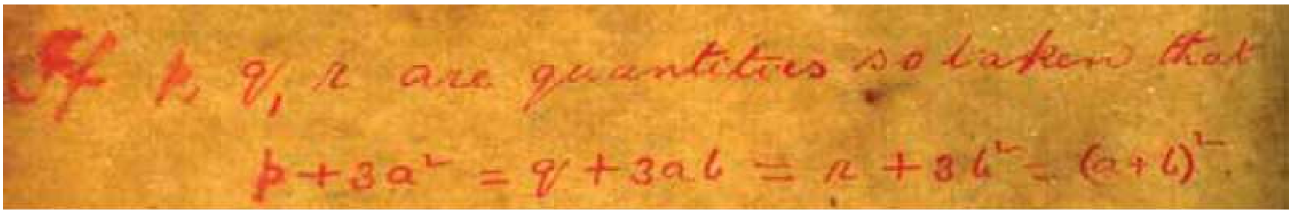

If p, q, r are quantities so taken that

$p+3a^2 = q+3ab = r+3b^2 = (a+b)^2.$

If:

$6+3\times1^2 = 3+3\times1\times2 = -3+3\times2^2 = (1+2)^2$

True

and

$6+3\times1^2$

9

and:

$$-3+3\times 2^2 = 9$$
$$(1+2)^2 = 9$$

Thence: a = 1, b = 2, p = 6, q = 3 and r = – 3

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$$P_{12} = \frac{691}{2730} + \frac{691p}{210} + \frac{616p^2}{45} + \frac{451p^3}{18} + \frac{385p^4}{18} + \frac{385p^5}{54}.$$

We obtain:

691/2730 + (691*6)/210 + (616*6^2)/45 + (451*6^3)/18 + (385*6^4)/18 + (385*6^5)/54

Input:

$$\frac{691}{2730}+\frac{691\times 6}{210}+\frac{1}{45}\left(616\times 6^2\right)+\frac{1}{18}\left(451\times 6^3\right)+\frac{1}{18}\left(385\times 6^4\right)+\frac{1}{54}\left(385\times 6^5\right)$$

Exact result:

$$\frac{243201493}{2730}$$

Decimal approximation:

89084.795970695970695970695970695970695970695970695970695970695970695 97...

Repeating decimal:

$89084.7\overline{959706}$ (period 6)

89084.7959706….

Further, we have:

$$P_{10} = \frac{5}{66} + \frac{5p}{6} + \frac{17p^2}{6} + \frac{35p^3}{9} + \frac{35p^4}{18}$$

5/66+(5*6)/6+(17*6^2)/6+(35*6^3)/6+(35*6^4)/18

Input:

$$\frac{5}{66} + \frac{5 \times 6}{6} + \frac{1}{6}(17 \times 6^2) + \frac{1}{6}(35 \times 6^3) + \frac{1}{18}(35 \times 6^4)$$

Exact result:

$$\frac{256547}{66}$$

Decimal approximation:

3887.0757575757575757575757575757575757575757575757575757...

3887.0757575…..

$$P_8 = \frac{1}{30} + \frac{3p}{10} + \frac{7p^2}{9} + \frac{35p^3}{54}.$$

1/30 + (3*6)/10 + (7*6^2)/9 + (35*6^3)/54

Input:

$$\frac{1}{30} + \frac{3 \times 6}{10} + \frac{1}{9}(7 \times 6^2) + \frac{1}{54}(35 \times 6^3)$$

Exact result:

$$\frac{1019}{6}$$

Decimal approximation:

169.83333333333333333333333333333333333333333333333333333...

169.833333……

$$P_6 = \frac{1}{42} + \frac{p}{6} + \frac{5p^2}{18}$$

1/42 + 6/6 + (5*6^2)/18

Input:

$$\frac{1}{42} + \frac{6}{6} + \frac{1}{18}(5 \times 6^2)$$

Exact result:

$$\frac{463}{42}$$

Decimal approximation:

11.02380952380952380952380952380952380952380952380952...

Repeating decimal:

$11.0\overline{238095}$ (period 6)

11.0238095

$$P_4 = \frac{1}{30} + \frac{p}{6}$$

1/30 + 6/6

Input:

$$\frac{1}{30} + 1$$

Exact result:

$$\frac{31}{30}$$

Decimal approximation:

1.033333333333333333333333333333333333333333333333333...

1.0333333333…………..

$$P_2 = \frac{1}{6}$$

Exact result:

$$\frac{1}{6}$$ (irreducible)

Decimal approximation:

0.1666666666666666666666666666666666666666666666666666...

0.1666666666666……

$$P_{14} = \frac{7}{6} + \frac{35p}{2} + \frac{7709p^2}{90} + \frac{26026p^3}{185} + \frac{8002p^4}{9} + \frac{7007p^5}{54} + \frac{5005p^6}{162}$$

&c &c &c.

7/6 + (35*6)/2 + (7709*6^2)/90 + (26026*6^3)/185 + (2002*6^4)/9 + (7007*6^5)/54 + (5005*6^6)/162

Input:

$$\frac{7}{6}+\frac{35\times 6}{2}+\frac{1}{90}\left(7709\times 6^2\right)+\frac{1}{185}\left(26\,026\times 6^3\right)+\frac{1}{9}\left(2002\times 6^4\right)+\frac{1}{54}\left(7007\times 6^5\right)+\frac{1}{162}\left(5005\times 6^6\right)$$

Exact result:

$$\frac{3\,077\,267\,297}{1110}$$

Decimal approximation:

$$2.77231288018018018018018018018018018018018018018018018\ldots\times 10^6$$

Decimal form:

2772312.880180180180180180180180180180180180180180180180180180

2772312.88018….

From the six results, we obtain:

2772312.88018; 89084.7959; 3887.07575; 11.0238095; 1.03333333: 0.1666666666

1/(((144*(2772312.88018 * 1/89084.7959 * 1/ 3887.07575 *1/11.0238095 *1/1.03333333 *1/ 0.1666666666))))

Input interpretation:

$$1\Big/\left(144\left(2.77231288018\times 10^6\times\frac{1}{89\,084.7959}\times\frac{1}{3887.07575}\times\frac{1}{11.0238095}\times\frac{1}{1.03333333}\times\frac{1}{0.1666666666}\right)\right)$$

Result:

1.646807155866853506901034660392575791791730501344558698817…

1.64680715586…..≈ ζ(2) = $\frac{\pi^2}{6} = 1.644934$ ...

(((144*(2772312.88018 * 1/89084.7959 * 1/ 3887.07575 *1/11.0238095 *1/1.03333333 *1/ 0.1666666666))))

Input interpretation:

$$144\left(2.77231288018\times10^{6}\times\frac{1}{89\,084.7959}\times\frac{1}{3887.07575}\times\frac{1}{11.0238095}\times\frac{1}{1.03333333}\times\frac{1}{0.1666666666}\right)$$

Result:

0.607235641670269312439820551044844826679424587857889310637...

0.60723564167….

(((144*(2772312.88018 * 1/89084.7959 * 1/ 3887.07575 *1/11.0238095 *1/1.03333333 *1/ 0.1666666666)))^1/64

Input interpretation:

$$\left(144\left(2.77231288018\times10^{6}\times\frac{1}{89\,084.7959}\times\frac{1}{3887.07575}\times\frac{1}{11.0238095}\times\frac{1}{1.03333333}\times\frac{1}{0.1666666666}\right)\right)^{\wedge}(1/64)$$

Result:

0.9922359479...

0.9922359479…. result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\dfrac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\cfrac{e^{-\pi\sqrt{5}}}{1+\cfrac{e^{-2\pi\sqrt{5}}}{1+\cfrac{e^{-3\pi\sqrt{5}}}{1+\cfrac{e^{-4\pi\sqrt{5}}}{1+\dots}}}}\approx 0.9991104684$$

and to the dilaton value $\mathbf{0.989117352243} = \boldsymbol{\phi}$

From:

(2772312.88018 + 89084.7959 + 3887.07575 +11.0238095 +1.03333333 + 0.1666666666)

Input interpretation:

$$2.77231288018\times10^6 + 89\,084.7959 + 3887.07575 + 11.0238095 + 1.03333333 + 0.1666666666$$

Result:

$$2.8652969756394966 \times 10^6$$

2.8652969756….*10^6

21+1/(13*3)(2.8652969756394966 × 10^6)

Input interpretation:

$$21 + \frac{1}{13\times3}\times 2.8652969756394966\times 10^6$$

Result:

73490.15322152555384615384615384615384615384615384615384615...

73490.1532215…

We have the following mathematical connection:

$$\left(21 + \frac{1}{13\times3}\times 2.8652969756394966\times 10^6\right) = 73490.1532215 \ldots \Rightarrow$$

$$\Rightarrow -3927 + 2\left(\sqrt[13]{\begin{array}{c} N\exp\left[\int d\hat{\sigma}\left(-\frac{1}{4u^2}\mathbf{P}_i D\mathbf{P}_i\right)\right]|Bp\rangle_{\rm NS} + \\ \int[d\mathbf{X}^\mu]\exp\left\{\int d\hat{\sigma}\left(-\frac{1}{4v^2}D\mathbf{X}^\mu D^2\mathbf{X}^\mu\right)\right\}|\mathbf{X}^\mu,\mathbf{X}^i=0\rangle_{\rm NS}\end{array}}\right) =$$

$$-3927 + 2\sqrt[13]{2.2983717437\times10^{59} + 2.0823329825883\times10^{59}}$$

$$= 73490.8437525 \ldots. \Rightarrow$$

$$\Rightarrow\left(A(r)\times\frac{1}{B(r)}\left(-\frac{1}{\phi(r)}\right)\times\frac{1}{e^{\Lambda(r)}}\right)\Rightarrow$$

$$\Rightarrow \left(-0.000029211892 \times \frac{1}{0.0003644621} \left(-\frac{1}{0.0005946833} \right) \times \frac{1}{0.00183393} \right) =$$

$$= 73491.788325481187105491595720422205480251957265634133987 00\ldots$$

$$= 73491.7883254\ldots \Rightarrow$$

$$\left(\begin{array}{c} I_{21} \ll \int_{-\infty}^{+\infty} \exp\left(-\left(\frac{t}{H}\right)^2\right) \left| \sum_{\lambda \leq P^{1-\varepsilon_1}} \frac{a(\lambda)}{\sqrt{\lambda}} B(\lambda)\, \lambda^{-i(T+t)} \right|^2 dt \ll \\ \ll H \left\{ \left(\frac{4}{\varepsilon_2 \log T} \right)^{2r} (\log T)\, (\log X)^{-2\beta} + \left(\varepsilon_2^{-2r} (\log T)^{-2r} + \varepsilon_2^{-r} h_1^r (\log T)^{-r} \right) T^{-\varepsilon_1} \right\} \end{array} \right) /$$

$$/(26 \times 4)^2 - 24 = \left(\frac{7.9313976505275 \times 10^8}{(26 \times 4)^2 - 24} \right) = 73493.30662\ldots$$

Mathematical connections with the boundary state corresponding to the NSNS-sector of N Dp-branes in the limit of $u \to \infty$, with the ratio concerning the general asymptotically flat solution of the equations of motion of the p-brane and with the Karatsuba's equation concerning the zeros of a special type of function connected with Dirichlet series.

We obtain also:

((((137401)/((636390*Pi)))) (2.8652969756394966 × 10^6))-34

Input interpretation:

$$\frac{137401}{636390\,\pi} \times 2.8652969756394966 \times 10^6 - 34$$

Result:

196884.40776762271...

196884.40776…

196884 is a fundamental number of the following j-invariant

$$j(\tau) = q^{-1} + 744 + 196884q + 21493760q^2 + 864299970q^3 + 20245856256q^4 + \cdots$$

(In mathematics, Felix Klein's *j*-invariant or *j* function, regarded as a function of a complex variable τ, is a modular function of weight zero for SL(2, Z) defined on the upper half plane of complex numbers. Several remarkable properties of *j* have to do with its q expansion (Fourier series expansion), written as a Laurent series in terms of $q = e^{2\pi i\tau}$ (the square of the nome), which begins:

$$j(\tau) = q^{-1} + 744 + 196884q + 21493760q^2 + 864299970q^3 + 20245856256q^4 + \cdots$$

Note that *j* has a simple pole at the cusp, so its q-expansion has no terms below q^{-1}.

All the Fourier coefficients are integers, which results in several almost integers, notably Ramanujan's constant:

$$e^{\pi\sqrt{163}} \approx 640320^3 + 744.$$

The asymptotic formula for the coefficient of q^n is given by

$$\frac{e^{4\pi\sqrt{n}}}{\sqrt{2}\,n^{3/4}},$$

as can be proved by the Hardy–Littlewood circle method)

Now, we have that:

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(3) when x is small, $\frac{1}{1+}\frac{1}{1+}\frac{2}{1+}\frac{3}{1+}\frac{4}{1+}$ &c =

$x e^{\frac{1}{2}} \left\{ e^{-\frac{(1+x)^2}{2}} + e^{-\frac{(1+3x)^2}{2}} + e^{-\frac{(1+3x)^2}{2}} + \&c \right\} +$

$\frac{x}{2} - \frac{x^2}{12} - \frac{x^4}{360} - \frac{x^6}{5040} - \frac{x^8}{60480} - \frac{x^{10}}{1710720}$ nearly

For x = 2, we obtain:

2*e^0.5*(((((((e^(((-(1+2)^2))/2))))+(e^(((-(1+4)^2))/2))+(e^(((-(1+6)^2))/2))))))+1+4/12+16/360+64/5040+256/60480-1024/1710720

Input:

$$2\sqrt{e}\left(e^{-1/2(1+2)^2}+e^{-1/2(1+4)^2}+e^{-1/2(1+6)^2}\right)+1+\frac{4}{12}+\frac{16}{360}+\frac{64}{5040}+\frac{256}{60480}-\frac{1024}{1710720}$$

Exact result:

$$\frac{130426}{93555}+2\left(\frac{1}{e^{49/2}}+\frac{1}{e^{25/2}}+\frac{1}{e^{9/2}}\right)\sqrt{e}$$

Decimal approximation:

1.430753982610316262753312445195044707966358468334924983890...

1.43075398261….

Property:

$\frac{130426}{93555}+2\left(\frac{1}{e^{49/2}}+\frac{1}{e^{25/2}}+\frac{1}{e^{9/2}}\right)\sqrt{e}$ is a transcendental number

Alternate forms:

$$\frac{130426}{93555}+\frac{2\left(1+e^{12}+e^{20}\right)}{e^{24}}$$

$$\frac{130426}{93555}+\frac{2}{e^{24}}+\frac{2}{e^{12}}+\frac{2}{e^{4}}$$

$$\frac{2\left(93555+93555\,e^{12}+93555\,e^{20}+65213\,e^{24}\right)}{93555\,e^{24}}$$

And:

1/(((((2*e^0.5*(((((((e^(((-(1+2)^2))/2))))+(e^(((-(1+4)^2))/2))+(e^(((-(1+6)^2))/2))))))+1+4/12+16/360+64/5040+256/60480-1024/1710720)))))^1/8

Input:

$$\frac{1}{\sqrt[8]{2\sqrt{e}\left(e^{-1/2(1+2)^2}+e^{-1/2(1+4)^2}+e^{-1/2(1+6)^2}\right)+1+\frac{4}{12}+\frac{16}{360}+\frac{64}{5040}+\frac{256}{60480}-\frac{1024}{1710720}}}$$

Exact result:

$$\frac{1}{\sqrt[8]{\frac{130426}{93555}+2\left(\frac{1}{e^{49/2}}+\frac{1}{e^{25/2}}+\frac{1}{e^{9/2}}\right)\sqrt{e}}}$$

Decimal approximation:

0.9562124183021051216146332612459873092823767906108704617 82...

0.9562124183021… result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}}-\varphi+1}=1-\cfrac{e^{-\pi}}{1+\cfrac{e^{-2\pi}}{1+\cfrac{e^{-3\pi}}{1+\cfrac{e^{-4\pi}}{1+\ldots}}}}\approx 0.9568666373$$

and also very near to the spectral index n_s and to the mesonic Regge slope (see Appendix) and to the inflaton value at the end of the inflation 0.9402

Property:

$$\frac{1}{\sqrt[8]{\frac{130426}{93555}+2\left(\frac{1}{e^{49/2}}+\frac{1}{e^{25/2}}+\frac{1}{e^{9/2}}\right)\sqrt{e}}}$$ is a transcendental number

Alternate forms:

$$\frac{1}{\sqrt[8]{\frac{130426}{93555}+\frac{2\left(1+e^{12}+e^{20}\right)}{e^{24}}}}$$

$$\frac{1}{\sqrt[8]{\frac{130426}{93555}+\frac{2}{e^{24}}+\frac{2}{e^{12}}+\frac{2}{e^{4}}}}$$

$$3^{5/8}\,e^{3}\,\sqrt[8]{\frac{385}{2\left(93555+93555\,e^{12}+93555\,e^{20}+65213\,e^{24}\right)}}$$

Now, we have that:

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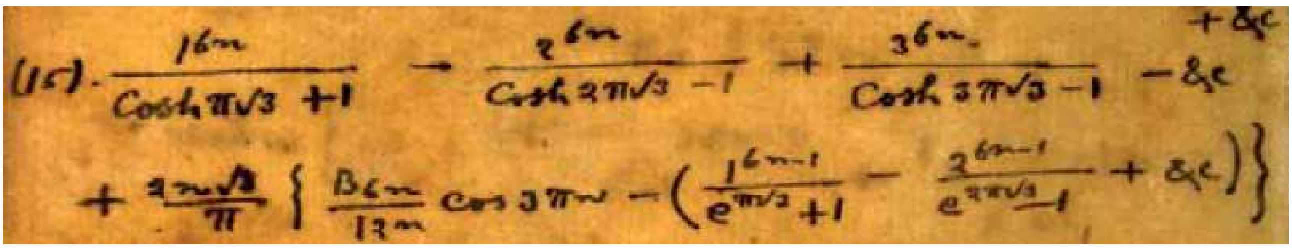

$$(15).\ \frac{1^{6m}}{\cosh\pi\sqrt{3}+1}-\frac{2^{6m}}{\cosh 2\pi\sqrt{3}-1}+\frac{3^{6m}}{\cosh 3\pi\sqrt{3}-1}-\&c+\&c$$
$$+\frac{2m\sqrt{3}}{\pi}\left\{\frac{B_{6m}}{12m}\cos 3\pi w-\left(\frac{1^{6m-1}}{e^{\pi\sqrt{3}}+1}-\frac{2^{6m-1}}{e^{2\pi\sqrt{3}}-1}+\&c\right)\right\}$$

For n = 2, from the first expression, we obtain:

(1^12)/(cosh(Pi*sqrt(3)+1)) – (2^12)/(cosh(2Pi*sqrt(3)-1))+
(3^12)/(cosh(3Pi*sqrt(3)-1))

Input:

$$\frac{1^{12}}{\cosh(\pi\sqrt{3}+1)} - \frac{2^{12}}{\cosh(2\pi\sqrt{3}-1)} + \frac{3^{12}}{\cosh(3\pi\sqrt{3}-1)}$$

$\cosh(x)$ is the hyperbolic cosine function

Exact result:

$$531441\operatorname{sech}\left(1-3\sqrt{3}\,\pi\right) - 4096\operatorname{sech}\left(1-2\sqrt{3}\,\pi\right) + \operatorname{sech}\left(1+\sqrt{3}\,\pi\right)$$

$\operatorname{sech}(x)$ is the hyperbolic secant function

Decimal approximation:

−0.179864620990768805093060712197253438797057360058550520 57...

-0.17986462099….

Alternate forms:

$$\frac{2}{e^{-1-\sqrt{3}\,\pi}+e^{1+\sqrt{3}\,\pi}} - \frac{8192}{e^{1-2\sqrt{3}\,\pi}+e^{2\sqrt{3}\,\pi-1}} + \frac{1062882}{e^{1-3\sqrt{3}\,\pi}+e^{3\sqrt{3}\,\pi-1}}$$

$$\frac{1062882\cosh(1-3\sqrt{3}\,\pi)}{1+\cosh(2(1-3\sqrt{3}\,\pi))} - \frac{8192\cosh(1-2\sqrt{3}\,\pi)}{1+\cosh(2(1-2\sqrt{3}\,\pi))} + \frac{2\cosh(1+\sqrt{3}\,\pi)}{1+\cosh(2(1+\sqrt{3}\,\pi))}$$

$$\left(2e^{\sqrt{3}\,\pi}\left(e^5-4096e^{3+\sqrt{3}\,\pi}+531441e^{3+2\sqrt{3}\,\pi}-4096e^{5+3\sqrt{3}\,\pi}+e^{3+4\sqrt{3}\,\pi}+531441e^{5+4\sqrt{3}\,\pi}+531441e^{1+6\sqrt{3}\,\pi}+e^{3+6\sqrt{3}\,\pi}-4096e^{1+7\sqrt{3}\,\pi}+531441e^{3+8\sqrt{3}\,\pi}-4096e^{3+9\sqrt{3}\,\pi}+e^{1+10\sqrt{3}\,\pi}\right)\right)\Big/\left(\left(e^2+e^{4\sqrt{3}\,\pi}\right)\left(e^2+e^{6\sqrt{3}\,\pi}\right)\left(1+e^{2+2\sqrt{3}\,\pi}\right)\right)$$

Alternative representations:

$$\frac{1^{12}}{\cosh(\pi\sqrt{3}+1)} - \frac{2^{12}}{\cosh(2\pi\sqrt{3}-1)} + \frac{3^{12}}{\cosh(3\pi\sqrt{3}-1)} = \frac{1^{12}}{\cos(i(1+\pi\sqrt{3}))} - \frac{2^{12}}{\cos(i(-1+2\pi\sqrt{3}))} + \frac{3^{12}}{\cos(i(-1+3\pi\sqrt{3}))}$$

$$\frac{1^{12}}{\cosh(\pi\sqrt{3}+1)} - \frac{2^{12}}{\cosh(2\pi\sqrt{3}-1)} + \frac{3^{12}}{\cosh(3\pi\sqrt{3}-1)} = \frac{1^{12}}{\cos(-i(1+\pi\sqrt{3}))} - \frac{2^{12}}{\cos(-i(-1+2\pi\sqrt{3}))} + \frac{3^{12}}{\cos(-i(-1+3\pi\sqrt{3}))}$$

$$\frac{1^{12}}{\cosh(\pi\sqrt{3}+1)} - \frac{2^{12}}{\cosh(2\pi\sqrt{3}-1)} + \frac{3^{12}}{\cosh(3\pi\sqrt{3}-1)} = \frac{1^{12}}{\frac{1}{\sec(i(1+\pi\sqrt{3}))}} - \frac{2^{12}}{\frac{1}{\sec(i(-1+2\pi\sqrt{3}))}} + \frac{3^{12}}{\frac{1}{\sec(i(-1+3\pi\sqrt{3}))}}$$

Series representations:

$$\frac{1^{12}}{\cosh(\pi\sqrt{3}+1)} - \frac{2^{12}}{\cosh(2\pi\sqrt{3}-1)} + \frac{3^{12}}{\cosh(3\pi\sqrt{3}-1)} = -1054692\sum_{k=1}^{\infty}(-1)^k q^{-1+2k}$$
$$\text{for } \left(e^{3\sqrt{3}\pi} q = e \text{ and } e^{2\sqrt{3}\pi} q = e \text{ and } q = e^{1+\sqrt{3}\pi}\right)$$

$$\frac{1^{12}}{\cosh(\pi\sqrt{3}+1)} - \frac{2^{12}}{\cosh(2\pi\sqrt{3}-1)} + \frac{3^{12}}{\cosh(3\pi\sqrt{3}-1)} = \sum_{k=0}^{\infty}(-1)^k (1+2k)\pi\left(\frac{1}{1+2\sqrt{3}\pi+\left(\frac{13}{4}+k+k^2\right)\pi^2} - \frac{4096}{1-4\sqrt{3}\pi+\left(\frac{49}{4}+k+k^2\right)\pi^2} + \frac{531441}{1-6\sqrt{3}\pi+\left(\frac{109}{4}+k+k^2\right)\pi^2}\right)$$

$$\frac{1^{12}}{\cosh(\pi\sqrt{3}+1)} - \frac{2^{12}}{\cosh(2\pi\sqrt{3}-1)} + \frac{3^{12}}{\cosh(3\pi\sqrt{3}-1)} = \sum_{k=0}^{\infty}\frac{1}{k!} i\left(\mathrm{Li}_{-k}(-i e^{z_0}) - \mathrm{Li}_{-k}(i e^{z_0})\right)\left(531441\left(1-3\sqrt{3}\pi-z_0\right)^k - 4096\left(1-2\sqrt{3}\pi-z_0\right)^k + \left(1+\sqrt{3}\pi-z_0\right)^k\right) \text{ for } \frac{1}{2}+\frac{i z_0}{\pi}\notin\mathbb{Z}$$

Integral representation:

$$\frac{1^{12}}{\cosh(\pi\sqrt{3}+1)} - \frac{2^{12}}{\cosh(2\pi\sqrt{3}-1)} + \frac{3^{12}}{\cosh(3\pi\sqrt{3}-1)} = \int_0^{\infty}\frac{2t^{-6i\sqrt{3}+(2i)/\pi}\left(531441-4096t^{2i\sqrt{3}}+t^{8i\sqrt{3}}\right)}{\pi(1+t^2)}\,dt$$

And:

Input:

$$\sqrt[64]{\frac{1^{12}}{\cosh(\pi\sqrt{3}+1)}-\frac{2^{12}}{\cosh(2\pi\sqrt{3}-1)}+\frac{3^{12}}{\cosh(3\pi\sqrt{3}-1)}}$$

$\cosh(x)$ is the hyperbolic cosine function

Exact result:

$$\sqrt[64]{531441\,\mathrm{sech}\left(1-3\sqrt{3}\,\pi\right)-4096\,\mathrm{sech}\left(1-2\sqrt{3}\,\pi\right)+\mathrm{sech}\left(1+\sqrt{3}\,\pi\right)}$$

$\mathrm{sech}(x)$ is the hyperbolic secant function

Decimal approximation:

0.9723779123685147459450070781060790176773744324571860280... +
0.0477698636200762184612009214706264690327763370345457 1063... i

Polar coordinates:

$r \approx 0.973551$ (radius), $\theta \approx 2.8125°$ (angle)

0.973551.... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}}\approx 0.9991104684$$

and to the dilaton value $\mathbf{0.989117352243 = \phi}$

Alternate forms:

$$\sqrt[64]{\frac{2}{e^{-1-\sqrt{3}\,\pi}+e^{1+\sqrt{3}\,\pi}}-\frac{8192}{e^{1-2\sqrt{3}\,\pi}+e^{2\sqrt{3}\,\pi-1}}+\frac{1062882}{e^{1-3\sqrt{3}\,\pi}+e^{3\sqrt{3}\,\pi-1}}}$$

$$\left(\frac{1}{-\frac{\sinh(\sqrt{3}\,\pi)}{2e}+\frac{1}{2}e\sinh(\sqrt{3}\,\pi)+\frac{\cosh(\sqrt{3}\,\pi)}{2e}+\frac{1}{2}e\cosh(\sqrt{3}\,\pi)}-\frac{4096}{\frac{\sinh(2\sqrt{3}\,\pi)}{2e}-\frac{1}{2}e\sinh(2\sqrt{3}\,\pi)+\frac{\cosh(2\sqrt{3}\,\pi)}{2e}+\frac{1}{2}e\cosh(2\sqrt{3}\,\pi)}+\frac{531441}{\frac{\sinh(3\sqrt{3}\,\pi)}{2e}-\frac{1}{2}e\sinh(3\sqrt{3}\,\pi)+\frac{\cosh(3\sqrt{3}\,\pi)}{2e}+\frac{1}{2}e\cosh(3\sqrt{3}\,\pi)}\right)\hat{}\,(1/64)$$

$$e^{(\sqrt{3}\,\pi)/64}\left(\left(2\left(e^5-4096\,e^{3+\sqrt{3}\,\pi}+531441\,e^{3+2\sqrt{3}\,\pi}-4096\,e^{5+3\sqrt{3}\,\pi}+e^{3+4\sqrt{3}\,\pi}+531441\,e^{5+4\sqrt{3}\,\pi}+531441\,e^{1+6\sqrt{3}\,\pi}+e^{3+6\sqrt{3}\,\pi}-4096\,e^{1+7\sqrt{3}\,\pi}+531441\,e^{3+8\sqrt{3}\,\pi}-4096\,e^{3+9\sqrt{3}\,\pi}+e^{1+10\sqrt{3}\,\pi}\right)\right)\Big/\left(\left(e^2+e^{4\sqrt{3}\,\pi}\right)\left(e^2+e^{6\sqrt{3}\,\pi}\right)\left(1+e^{2+2\sqrt{3}\,\pi}\right)\right)\right)\hat{}\,(1/64)$$

sinh(x) is the hyperbolic sine function

All 64th roots of 531441 sech(1 - 3 sqrt(3) π) - 4096 sech(1 - 2 sqrt(3) π) + sech(1 + sqrt(3) π):

- Polar form

$$e^{(i\pi)/64}\sqrt[64]{-531441\,\mathrm{sech}\left(1-3\sqrt{3}\,\pi\right)+4096\,\mathrm{sech}\left(1-2\sqrt{3}\,\pi\right)-\mathrm{sech}\left(1+\sqrt{3}\,\pi\right)}$$
$\approx 0.9724+0.04777\,i$ (principal root)

$$e^{(3i\pi)/64}\sqrt[64]{-531441\,\mathrm{sech}\left(1-3\sqrt{3}\,\pi\right)+4096\,\mathrm{sech}\left(1-2\sqrt{3}\,\pi\right)-\mathrm{sech}\left(1+\sqrt{3}\,\pi\right)}$$
$\approx 0.9630+0.14285\,i$

$$e^{(5i\pi)/64}\sqrt[64]{-531441\,\mathrm{sech}\left(1-3\sqrt{3}\,\pi\right)+4096\,\mathrm{sech}\left(1-2\sqrt{3}\,\pi\right)-\mathrm{sech}\left(1+\sqrt{3}\,\pi\right)}$$
$\approx 0.9444+0.23655\,i$

$$e^{(7i\pi)/64}\sqrt[64]{-531441\,\mathrm{sech}\left(1-3\sqrt{3}\,\pi\right)+4096\,\mathrm{sech}\left(1-2\sqrt{3}\,\pi\right)-\mathrm{sech}\left(1+\sqrt{3}\,\pi\right)}$$
$\approx 0.9166+0.3280\,i$

$$e^{(9i\pi)/64}\sqrt[64]{-531441\,\mathrm{sech}\left(1-3\sqrt{3}\,\pi\right)+4096\,\mathrm{sech}\left(1-2\sqrt{3}\,\pi\right)-\mathrm{sech}\left(1+\sqrt{3}\,\pi\right)}$$
$\approx 0.8801+0.4162\,i$

Alternative representations:

$$\sqrt[64]{\frac{1^{12}}{\cosh(\pi\sqrt{3}+1)}-\frac{2^{12}}{\cosh(2\pi\sqrt{3}-1)}+\frac{3^{12}}{\cosh(3\pi\sqrt{3}-1)}} = \sqrt[64]{\frac{1^{12}}{\cos(i(1+\pi\sqrt{3}))}-\frac{2^{12}}{\cos(i(-1+2\pi\sqrt{3}))}+\frac{3^{12}}{\cos(i(-1+3\pi\sqrt{3}))}}$$

$$\sqrt[64]{\frac{1^{12}}{\cosh(\pi\sqrt{3}+1)}-\frac{2^{12}}{\cosh(2\pi\sqrt{3}-1)}+\frac{3^{12}}{\cosh(3\pi\sqrt{3}-1)}} = \sqrt[64]{\frac{1^{12}}{\cos(-i(1+\pi\sqrt{3}))}-\frac{2^{12}}{\cos(-i(-1+2\pi\sqrt{3}))}+\frac{3^{12}}{\cos(-i(-1+3\pi\sqrt{3}))}}$$

$$\sqrt[64]{\frac{1^{12}}{\cosh(\pi\sqrt{3}+1)}-\frac{2^{12}}{\cosh(2\pi\sqrt{3}-1)}+\frac{3^{12}}{\cosh(3\pi\sqrt{3}-1)}} = \sqrt[64]{\frac{1^{12}}{\frac{1}{\sec(i(1+\pi\sqrt{3}))}}-\frac{2^{12}}{\frac{1}{\sec(i(-1+2\pi\sqrt{3}))}}+\frac{3^{12}}{\frac{1}{\sec(i(-1+3\pi\sqrt{3}))}}}$$

Series representations:

$$\sqrt[64]{\frac{1^{12}}{\cosh(\pi\sqrt{3}+1)}-\frac{2^{12}}{\cosh(2\pi\sqrt{3}-1)}+\frac{3^{12}}{\cosh(3\pi\sqrt{3}-1)}} = \sqrt[32]{6}\ \sqrt[64]{29\,297}\ \sqrt[64]{-\sum_{k=1}^{\infty}(-1)^k q^{-1+2k}}$$

$$\text{for } \left(e^{3\sqrt{3}\pi} q = e \text{ and } e^{2\sqrt{3}\pi} q = e \text{ and } q = e^{1+\sqrt{3}\pi}\right)$$

$$\sqrt[64]{\frac{1^{12}}{\cosh(\pi\sqrt{3}+1)}-\frac{2^{12}}{\cosh(2\pi\sqrt{3}-1)}+\frac{3^{12}}{\cosh(3\pi\sqrt{3}-1)}} = \left(\sum_{k=0}^{\infty}(-1)^k (1+2k)\pi\left(\frac{1}{1+2\sqrt{3}\pi+\left(\frac{13}{4}+k+k^2\right)\pi^2} - \frac{4096}{1-4\sqrt{3}\pi+\left(\frac{49}{4}+k+k^2\right)\pi^2}+\frac{531\,441}{1-6\sqrt{3}\pi+\left(\frac{109}{4}+k+k^2\right)\pi^2}\right)\right)^\wedge(1/64)$$

$$\sqrt[64]{\frac{1^{12}}{\cosh(\pi\sqrt{3}+1)} - \frac{2^{12}}{\cosh(2\pi\sqrt{3}-1)} + \frac{3^{12}}{\cosh(3\pi\sqrt{3}-1)}} =$$
$$\left(\sum_{k=0}^{\infty} \frac{1}{k!}\, i\left(\mathrm{Li}_{-k}(-i e^{z_0}) - \mathrm{Li}_{-k}(i e^{z_0})\right)\left(531441\left(1-3\sqrt{3}\,\pi-z_0\right)^k - 4096\left(1-2\sqrt{3}\,\pi-z_0\right)^k + \left(1+\sqrt{3}\,\pi-z_0\right)^k\right)\right)\text{^}(1/64) \quad \text{for } \frac{1}{2}+\frac{i z_0}{\pi} \notin \mathbb{Z}$$

Integral representation:

$$\sqrt[64]{\frac{1^{12}}{\cosh(\pi\sqrt{3}+1)} - \frac{2^{12}}{\cosh(2\pi\sqrt{3}-1)} + \frac{3^{12}}{\cosh(3\pi\sqrt{3}-1)}} = \sqrt[64]{\int_0^{\infty} \frac{2\, t^{-6 i\sqrt{3}+(2 i)/\pi}\left(531441 - 4096\, t^{2 i\sqrt{3}} + t^{8 i\sqrt{3}}\right)}{\pi\left(1+t^2\right)}\, dt}$$

And:

-3^2(((((1^12)/(cosh(Pi*sqrt(3)+1)) – (2^12)/(cosh(2Pi*sqrt(3)-1))+ (3^12)/(cosh(3Pi*sqrt(3)-1))))))

Input:

$$-3^2\left(\frac{1^{12}}{\cosh(\pi\sqrt{3}+1)} - \frac{2^{12}}{\cosh(2\pi\sqrt{3}-1)} + \frac{3^{12}}{\cosh(3\pi\sqrt{3}-1)}\right)$$

$\cosh(x)$ is the hyperbolic cosine function

Exact result:

$$-9\left(531441\,\mathrm{sech}\left(1-3\sqrt{3}\,\pi\right) - 4096\,\mathrm{sech}\left(1-2\sqrt{3}\,\pi\right) + \mathrm{sech}\left(1+\sqrt{3}\,\pi\right)\right)$$

$\mathrm{sech}(x)$ is the hyperbolic secant function

Decimal approximation:

1.618781588916919245837546409775280949173516240526954685134...

1.6187815889169…..

This result is a very good approximation to the value of the golden ratio 1,618033988749...

Alternate forms:

$$-4\,782\,969\,\text{sech}\left(1-3\sqrt{3}\,\pi\right)+36\,864\,\text{sech}\left(1-2\sqrt{3}\,\pi\right)-9\,\text{sech}\left(1+\sqrt{3}\,\pi\right)$$

$$-9\left(531\,441\,\text{sech}\left(1-3\sqrt{3}\,\pi\right)-4096\,\text{sech}\left(1-2\sqrt{3}\,\pi\right)\right)-9\,\text{sech}\left(1+\sqrt{3}\,\pi\right)$$

$$-\frac{18}{e^{-1-\sqrt{3}\,\pi}+e^{1+\sqrt{3}\,\pi}}+\frac{73\,728}{e^{1-2\sqrt{3}\,\pi}+e^{2\sqrt{3}\,\pi-1}}-\frac{9\,565\,938}{e^{1-3\sqrt{3}\,\pi}+e^{3\sqrt{3}\,\pi-1}}$$

Alternative representations:

$$-3^2\left(\frac{1^{12}}{\cosh(\pi\sqrt{3}+1)}-\frac{2^{12}}{\cosh(2\pi\sqrt{3}-1)}+\frac{3^{12}}{\cosh(3\pi\sqrt{3}-1)}\right)=$$
$$-9\left(\frac{1^{12}}{\cos(i(1+\pi\sqrt{3}))}-\frac{2^{12}}{\cos(i(-1+2\pi\sqrt{3}))}+\frac{3^{12}}{\cos(i(-1+3\pi\sqrt{3}))}\right)$$

$$-3^2\left(\frac{1^{12}}{\cosh(\pi\sqrt{3}+1)}-\frac{2^{12}}{\cosh(2\pi\sqrt{3}-1)}+\frac{3^{12}}{\cosh(3\pi\sqrt{3}-1)}\right)=$$
$$-9\left(\frac{1^{12}}{\cos(-i(1+\pi\sqrt{3}))}-\frac{2^{12}}{\cos(-i(-1+2\pi\sqrt{3}))}+\frac{3^{12}}{\cos(-i(-1+3\pi\sqrt{3}))}\right)$$

$$-3^2\left(\frac{1^{12}}{\cosh(\pi\sqrt{3}+1)}-\frac{2^{12}}{\cosh(2\pi\sqrt{3}-1)}+\frac{3^{12}}{\cosh(3\pi\sqrt{3}-1)}\right)=$$
$$-9\left(\frac{1^{12}}{\frac{1}{\sec\left(i\left(1+\pi\sqrt{3}\right)\right)}}-\frac{2^{12}}{\frac{1}{\sec\left(i\left(-1+2\pi\sqrt{3}\right)\right)}}+\frac{3^{12}}{\frac{1}{\sec\left(i\left(-1+3\pi\sqrt{3}\right)\right)}}\right)$$

Series representations:

$$-3^2\left(\frac{1^{12}}{\cosh(\pi\sqrt{3}+1)}-\frac{2^{12}}{\cosh(2\pi\sqrt{3}-1)}+\frac{3^{12}}{\cosh(3\pi\sqrt{3}-1)}\right)=$$
$$9\,492\,228\sum_{k=1}^{\infty}(-1)^k q^{-1+2k}\quad\text{for}\left(e^{3\sqrt{3}\,\pi}q=e\text{ and }e^{2\sqrt{3}\,\pi}q=e\text{ and }q=e^{1+\sqrt{3}\,\pi}\right)$$

$$-3^2\left(\frac{1^{12}}{\cosh(\pi\sqrt{3}+1)}-\frac{2^{12}}{\cosh(2\pi\sqrt{3}-1)}+\frac{3^{12}}{\cosh(3\pi\sqrt{3}-1)}\right)=$$
$$\sum_{k=0}^{\infty}9(-1)^k(1+2k)\pi\left(-\frac{1}{1+2\sqrt{3}\,\pi+\left(\frac{13}{4}+k+k^2\right)\pi^2}+\frac{4096}{1-4\sqrt{3}\,\pi+\left(\frac{49}{4}+k+k^2\right)\pi^2}-\frac{531441}{1-6\sqrt{3}\,\pi+\left(\frac{109}{4}+k+k^2\right)\pi^2}\right)$$

$$-3^2\left(\frac{1^{12}}{\cosh(\pi\sqrt{3}+1)}-\frac{2^{12}}{\cosh(2\pi\sqrt{3}-1)}+\frac{3^{12}}{\cosh(3\pi\sqrt{3}-1)}\right)=$$
$$\sum_{k=0}^{\infty}-\frac{1}{k!}9i\left(\mathrm{Li}_{-k}(-ie^{z_0})-\mathrm{Li}_{-k}(ie^{z_0})\right)\left(531441\left(1-3\sqrt{3}\,\pi-z_0\right)^k-4096\left(1-2\sqrt{3}\,\pi-z_0\right)^k+\left(1+\sqrt{3}\,\pi-z_0\right)^k\right)\quad\text{for }\frac{1}{2}+\frac{iz_0}{\pi}\notin\mathbb{Z}$$

Integral representation:

$$-3^2\left(\frac{1^{12}}{\cosh(\pi\sqrt{3}+1)}-\frac{2^{12}}{\cosh(2\pi\sqrt{3}-1)}+\frac{3^{12}}{\cosh(3\pi\sqrt{3}-1)}\right)=$$
$$\int_0^{\infty}-\frac{18\,t^{-6i\sqrt{3}+(2i)/\pi}\left(531441-4096\,t^{2i\sqrt{3}}+t^{8i\sqrt{3}}\right)}{\pi(1+t^2)}\,dt$$

Now, we have that:

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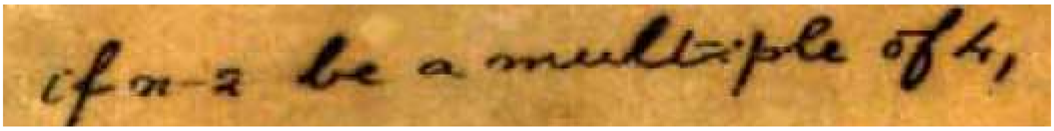
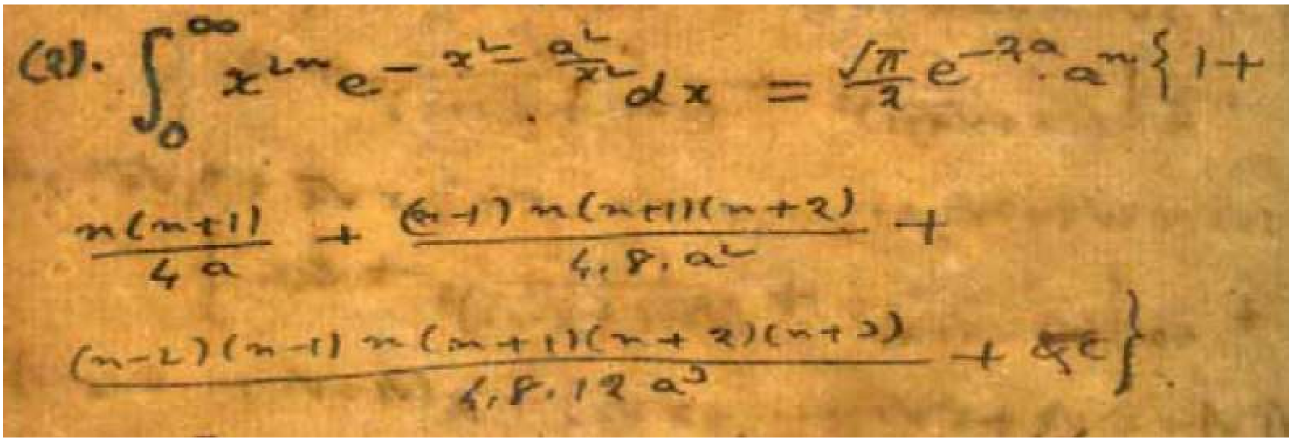

if n-2 be a multiple of 4,

We take n – 2 = 24; n = 26



$$(2).\ \int_0^\infty x^{2n} e^{-x^2-\frac{a^2}{x^2}}\,dx = \frac{\sqrt{\pi}}{2} e^{-2a} a^n \Big\{ 1 + \frac{n(n+1)}{4a} + \frac{(n-1)n(n+1)(n+2)}{4.8.a^2} + \frac{(n-2)(n-1)n(n+1)(n+2)(n+3)}{4.8.12\,a^3} + \&c \Big\}.$$

Thence: a = 1, b = 2, p = 6, q = 3 and r = – 3 ; n = 26; x = 2

(((1/2* sqrt(Pi) 1/e^(2)))) *
((((26*27)/4+(25*26(27*28))/32+(24*25*26(27*28*29))/(32*12))))

Input:

$$\left(\frac{1}{2}\sqrt{\pi}\times\frac{1}{e^2}\right)\left(\frac{26\times 27}{4}+\frac{1}{32}(25\times 26\,(27\times 28))+\frac{24\times 25\times 26\,(27\times 28\times 29)}{32\times 12}\right)$$

Exact result:

$$\frac{3\,624\,777\sqrt{\pi}}{8\,e^2}$$

Decimal approximation:

108686.9193152684672515923115733778728353477376809511182260...

108686.9193…

Series representations:

$$\frac{\left(\frac{26\times 27}{4}+\frac{25}{32}\times 26\,(27\times 28)+\frac{24\times 27\,(28\times 29)\,25\times 26}{32\times 12}\right)\sqrt{\pi}}{2\,e^2} = \frac{3\,624\,777\sqrt{-1+\pi}\,\sum_{k=0}^{\infty}(-1+\pi)^{-k}\binom{\frac{1}{2}}{k}}{8\,e^2}$$

$$\frac{\left(\frac{26\times 27}{4}+\frac{25}{32}\times 26\,(27\times 28)+\frac{24\times 27\,(28\times 29)\,25\times 26}{32\times 12}\right)\sqrt{\pi}}{2\,e^2}=$$

$$\frac{3\,624\,777\,\sqrt{-1+\pi}\,\sum_{k=0}^{\infty}\frac{(-1)^k\,(-1+\pi)^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{8\,e^2}$$

$$\frac{\left(\frac{26\times 27}{4}+\frac{25}{32}\times 26\,(27\times 28)+\frac{24\times 27\,(28\times 29)\,25\times 26}{32\times 12}\right)\sqrt{\pi}}{2\,e^2}=$$

$$\frac{3\,624\,777\,\sqrt{z_0}\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\,(\pi-z_0)^k\,z_0^{-k}}{k!}}{8\,e^2}\quad\text{for not }((z_0\in\mathbb{R}\text{ and }-\infty<z_0\le 0))$$

And, we obtain also:

(29+2)+1/64((((((((1/2* sqrt(Pi) 1/e^(2)))) * ((((26*27)/4+(25*26(27*28))/32+(24*25*26(27*28*29))/(32*12))))))))

Input:

$$(29+2)+\frac{1}{64}\left(\left(\frac{1}{2}\sqrt{\pi}\times\frac{1}{e^2}\right)\left(\frac{26\times 27}{4}+\frac{1}{32}\,(25\times 26\,(27\times 28))+\frac{24\times 25\times 26\,(27\times 28\times 29)}{32\times 12}\right)\right)$$

Exact result:

$$31+\frac{3\,624\,777\,\sqrt{\pi}}{512\,e^2}$$

Decimal approximation:

1729.233114301069800806129868334029263052308401264861222281...

1729.2331143…

This result is very near to the mass of candidate glueball $f_0(1710)$ meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

Alternate form:

$$\frac{15\,872\,e^2+3\,624\,777\,\sqrt{\pi}}{512\,e^2}$$

Series representations:

$$(29+2)+\frac{\sqrt{\pi}\left(\frac{26\times 27}{4}+\frac{25}{32}\times 26\,(27\times 28)+\frac{24\times 27\,(28\times 29)\,25\times 26}{32\times 12}\right)}{(2\,e^2)\,64}=31+\frac{3\,624\,777\,\sqrt{-1+\pi}\,\sum_{k=0}^{\infty}(-1+\pi)^{-k}\binom{\frac{1}{2}}{k}}{512\,e^2}$$

$$(29+2)+\frac{\sqrt{\pi}\left(\frac{26\times 27}{4}+\frac{25}{32}\times 26\,(27\times 28)+\frac{24\times 27\,(28\times 29)\,25\times 26}{32\times 12}\right)}{(2\,e^2)\,64}=31+\frac{3\,624\,777\,\sqrt{-1+\pi}\,\sum_{k=0}^{\infty}\frac{(-1)^k(-1+\pi)^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{512\,e^2}$$

$$(29+2)+\frac{\sqrt{\pi}\left(\frac{26\times 27}{4}+\frac{25}{32}\times 26\,(27\times 28)+\frac{24\times 27\,(28\times 29)\,25\times 26}{32\times 12}\right)}{(2\,e^2)\,64}=31+\frac{3\,624\,777\,\sqrt{z_0}\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(\pi-z_0)^k z_0^{-k}}{k!}}{512\,e^2}\quad\text{for not }((z_0\in\mathbb{R}\text{ and }-\infty<z_0\leq 0))$$

We obtain also:

((((((((1/2* sqrt(Pi) 1/e^(2)))) * ((((26*27)/4+(25*26(27*28))/32+(24*25*26(27*28*29))/(32*12))))))))+(4096*21+ 2048+128+4)

Input:

$$\left(\frac{1}{2}\sqrt{\pi}\times\frac{1}{e^2}\right)\left(\frac{26\times 27}{4}+\frac{1}{32}\,(25\times 26\,(27\times 28))+\frac{24\times 25\times 26\,(27\times 28\times 29)}{32\times 12}\right)+(4096\times 21+2048+128+4)$$

Exact result:

$$88\,196+\frac{3\,624\,777\,\sqrt{\pi}}{8\,e^2}$$

Decimal approximation:

196882.91931526846725159231157337787283534773768095111822 60...

Alternate form:

$$\frac{705568\,e^2 + 3624777\sqrt{\pi}}{8\,e^2}$$

Series representations:

$$\frac{\left(\frac{26\times 27}{4} + \frac{25}{32}\times 26\,(27\times 28) + \frac{24\times 27\,(28\times 29)\,25\times 26}{32\times 12}\right)\sqrt{\pi}}{2\,e^2} + (4096\times 21 + 2048 + 128 + 4) = 88196 + \frac{3624777\sqrt{-1+\pi}\,\sum_{k=0}^{\infty}(-1+\pi)^{-k}\binom{\frac{1}{2}}{k}}{8\,e^2}$$

$$\frac{\left(\frac{26\times 27}{4} + \frac{25}{32}\times 26\,(27\times 28) + \frac{24\times 27\,(28\times 29)\,25\times 26}{32\times 12}\right)\sqrt{\pi}}{2\,e^2} + (4096\times 21 + 2048 + 128 + 4) = 88196 + \frac{3624777\sqrt{-1+\pi}\,\sum_{k=0}^{\infty}\frac{(-1)^k(-1+\pi)^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{8\,e^2}$$

$$\frac{\left(\frac{26\times 27}{4} + \frac{25}{32}\times 26\,(27\times 28) + \frac{24\times 27\,(28\times 29)\,25\times 26}{32\times 12}\right)\sqrt{\pi}}{2\,e^2} + (4096\times 21 + 2048 + 128 + 4) = 88196 + \frac{3624777\sqrt{z_0}\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(\pi-z_0)^k z_0^{-k}}{k!}}{8\,e^2}$$

for not $((z_0\in\mathbb{R}$ and $-\infty < z_0 \le 0))$

196882.919 ≈ 196883 result very near to 196884

196884 is a fundamental number of the following j-invariant

$$j(\tau) = q^{-1} + 744 + 196884q + 21493760q^2 + 864299970q^3 + 20245856256q^4 + \cdots$$

(In mathematics, Felix Klein's j-invariant or j function, regarded as a function of a complex variable τ, is a modular function of weight zero for SL(2, Z) defined on the upper half plane of complex numbers. Several remarkable properties of j have to do with its q expansion (Fourier series expansion), written as a Laurent series in terms of $q = e^{2\pi i\tau}$ (the square of the nome), which begins:

$$j(\tau) = q^{-1} + 744 + 196884q + 21493760q^2 + 864299970q^3 + 20245856256q^4 + \cdots$$

Note that j has a simple pole at the cusp, so its q-expansion has no terms below q^{-1}.

All the Fourier coefficients are integers, which results in several almost integers, notably Ramanujan's constant:

$$e^{\pi\sqrt{163}} \approx 640320^3 + 744.$$

The asymptotic formula for the coefficient of q^n is given by

$$\frac{e^{4\pi\sqrt{n}}}{\sqrt{2}\,n^{3/4}},$$

as can be proved by the Hardy–Littlewood circle method)

Appendix

Scen.	λ_1	ℓ^{-1}/M_P	$m_{\rm rad}/m_G$	ρ_1/TeV	$m_{\rm rad}$/TeV	$\langle\mu\rangle$/TeV	$\mu_0/\langle\mu\rangle$	$T_c/\langle\mu\rangle$	$T_n/\langle\mu\rangle$
A_1	−1.250	0.501	0.0645	0.758	0.1998	0.750	—	0.305	—
B_1	−3.000	0.554	0.1969	1.085	1.018	0.828	0.9995	0.903	0.609
B_2	−2.583	0.554	0.1905	1.007	0.915	0.767	0.989	0.825	0.428
B_3	−2.500	0.554	0.1888	0.989	0.890	0.752	0.974	0.806	0.367
B_4	2.438	0.554	0.1874	0.973	0.870	0.741	0.937	0.790	0.297
B_5	−2.375	0.554	0.1859	0.957	0.849	0.728	0.982	0.774	0.193
B_6	−2.292	0.554	0.1836	0.934	0.818	0.710	0.971	0.750	0.149
B_7	−2.208	0.554	0.1809	0.908	0.784	0.690	0.949	0.724	0.0990
B_8	−2.125	0.554	0.1776	0.879	0.745	0.667	0.890	0.694	0.0388
B_9	−2.096	0.554	0.1763	0.8675	0.7303	0.6585	0.827	0.682	0.0122
B_{10}	−2.092	0.554	0.1761	0.8658	0.7281	0.6572	0.808	0.680	0.0073
B_{11}	−2.090	0.554	0.1760	0.8650	0.7270	0.6565	0.793	0.679	0.0039
C_1	−3.125	0.377	0.289	0.554	0.890	0.378	0.989	1.123	0.601
C_2	−2.604	0.377	0.271	0.496	0.751	0.336	0.937	0.976	0.098
D_1	−3.462	1.49	0.106	0.468	0.477	0.250	0.9996	1.007	0.445
E_1	−2.429	0.554	0.155	0.877	0.643	0.667	0.895	0.694	0.142

Table 1. List of benchmark scenarios defined by the classes in eqs. (4.12)–(4.16) and the input values of λ_1 (second column). The outputs obtained in each scenario are presented from the third column on. The foreground red [blue] color on the value of λ_1 indicates that the corresponding phase transition is driven by $O(3)$ [$O(4)$] symmetric bounce solutions. In scenario A_1 there is no phase transition.

Scen.	$T_i/\langle\mu\rangle$	N_e	$T_R/\langle\mu\rangle$	T_R/GeV	α	$\log_{10}(\beta/H_\star)$
B_1	0.663	0.09	1.272	1053	1.60	2.36
B_2	0.605	0.35	1.071	821.8	4.61	1.99
B_3	0.591	0.48	1.024	770.4	7.86	1.79
B_4	0.580	0.67	0.986	730.6	17.1	1.48
B_5	0.568	1.08	0.953	694.0	90.1	1.97
B_6	0.551	1.31	0.921	654.2	228	1.86
B_7	0.531	1.68	0.887	612.0	1047	1.67
B_8	0.509	2.57	0.849	566.4	$4.0\cdot10^4$	1.23
B_9	0.5004	3.71	0.834	549.3	$4.1\cdot10^6$	0.64
B_{10}	0.4991	4.22	0.832	546.8	$3.3\cdot10^7$	0.34
B_{11}	0.4985	4.86	0.831	545.6	$4.5\cdot10^8$	-0.32
C_1	0.828	0.32	1.531	578.4	4.3	2.03
C_2	0.718	1.99	1.239	416.2	$5.0\cdot10^3$	1.45
D_1	–	–	0.535	133.7	5.0	1.05
E_1	0.509	1.28	0.850	567.2	203	1.89

Table 2. Some physical parameters for the cases B_i, C_i, D and E considered in the text.

Table of connection between the physical and mathematical constants and the very closed approximations to the dilaton value.

Table 1

Elementary charge = 1.602176	**$1 / (1,602176)^{1/64} = 0,992662013$**
Golden ratio = 1.61803398	**$1 / (1,61803398)^{1/64} = 0,992509261$**
ζ(2) = 1.644934	**$1 / (1,644934)^{1/64} = 0,992253592$**
$\sqrt[14]{Q = (G_{505}/G_{101/5})^3} = 1.65578$	**$1 / (1,65578)^{1/64} = 0,992151706$**
Proton mass = 1.672621	**$1 / (1,672621)^{1/64} = 0,991994840$**
Neutron mass = 1.674927	**$1 / (1,674927)^{1/64} = 0,991973486$**

From:

Rotating strings confronting PDG mesons

Jacob Sonnenschein and Dorin Weissman - arXiv:1402.5603v1 [hep-ph] 23 Feb 2014

$c\bar{c}$. **The Ψ trajectory:** The left side of figure (15) depicts the Ψ trajectory. Here we use the states $J/\Psi(1S)(3097)1^{--}, \chi_{c1}(1P)(3510)1^{++}$, and $\Psi(3770)1^{--}$. Since no $J = 3$ state has been observed, we use three states with $J = 1$, but with increasing orbital angular momentum ($L = 0, 1, 2$) and do the fit to L instead of J. To give an idea of the shifts in mass involved, the $J^{PC} = 2^{++}$ state χ_{c2} has a mass of 3556 MeV, and the $J^{PC} = 3^{--}$ state is expected to lie 30 − 60 MeV above the $\Psi(3770)$[23].

The best linear fit is

$$\alpha' = 0.418, a = -4.04$$

with $\chi_l^2 = 3.41 \times 10^{-4}$, but the optimal fit is far from the linear, with endpoint masses in the range of the constituent c quark mass:

$$m_c = 1500, \alpha' = 0.979, a = -0.09$$

with $\chi_m^2 = 5 \times 10^{-7}$ ($\chi_m^2/\chi_l^2 = 0.002$). Aside from the improvement in χ^2, by adding the mass we also get a value for the slope (and to a lesser extent, the intercept) that is much closer to that obtained in fits for the light meson trajectories.

where α' is the Regge slope (string tension)

We know also that:

ω	6	$m_{u/d} = 0 - 60$	0.910 − 0.918
ω/ω_3	5 + 3	$m_{u/d} = 255 - 390$	0.988 − 1.18
ω/ω_3	5 + 3	$m_{u/d} = 240 - 345$	0.937 − 1.000

The average of the various Regge slope of Omega mesons are:

1/7 * (0.979 + 0.910 + 0.918 + 0.988 + 0.937 + 1.18 +1) = 0.987428571

result very near to the value of dilaton and to the solution 0.987516007… of the above expression.

From:

Astronomy & Astrophysics manuscript no. ms c ESO 2019 - September 24, 2019
Planck 2018 results. VI. Cosmological parameters

The primordial fluctuations are consistent with Gaussian purely adiabatic scalar perturbations characterized by a power spectrum with a spectral index $n_s = 0.965 \pm 0.004$, consistent with the predictions of slow-roll, single-field, inflation.

from:

Modular equations and approximations to π - *Srinivasa Ramanujan*
Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

We have that:

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \cdots, \\ 64g_{22}^{-24} &= \qquad\qquad 4096e^{-\pi\sqrt{22}} + \cdots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\ldots.$$

Again

$$G_{37} = (6+\sqrt{37})^{\frac{1}{4}},$$

$$\begin{aligned} 64G_{37}^{24} &= e^{\pi\sqrt{37}} + 24 + 276e^{-\pi\sqrt{37}} + \cdots, \\ 64G_{37}^{-24} &= \qquad\qquad 4096e^{-\pi\sqrt{37}} - \cdots, \end{aligned}$$

so that

$$64(G_{37}^{24} + G_{37}^{-24}) = e^{\pi\sqrt{37}} + 24 + 4372e^{-\pi\sqrt{37}} - \cdots = 64\{(6+\sqrt{37})^6 + (6-\sqrt{37})^6\}.$$

Hence

$$e^{\pi\sqrt{37}} = 199148647.999978\ldots.$$

Similarly, from

$$g_{58} = \sqrt{\left(\frac{5+\sqrt{29}}{2}\right)},$$

we obtain

$$64(g_{58}^{24} + g_{58}^{-24}) = e^{\pi\sqrt{58}} - 24 + 4372e^{-\pi\sqrt{58}} + \cdots = 64\left\{\left(\frac{5+\sqrt{29}}{2}\right)^{12} + \left(\frac{5-\sqrt{29}}{2}\right)^{12}\right\}.$$

Hence

$$e^{\pi\sqrt{58}} = 24591257751.99999982\ldots.$$

From:

An Update on Brane Supersymmetry Breaking
J. Mourad and A. Sagnotti - arXiv:1711.11494v1 [hep-th] 30 Nov 2017

From the following vacuum equations:

$$T\, e^{\gamma_E \phi} \;=\; -\,\frac{\beta_E^{(p)}\, h^2}{\gamma_E}\; e^{-2\,(8-p)\,C\,+\,2\,\beta_E^{(p)}\,\phi}$$

$$16\,k'\,e^{-2C} \;=\; \frac{h^2\left(p\,+\,1\,-\,\frac{2\,\beta_E^{(p)}}{\gamma_E}\right)e^{-2\,(8-p)\,C\,+\,2\,\beta_E^{(p)}\,\phi}}{(7\,-\,p)}$$

$$(A')^2 \;=\; k\,e^{-2A} \,+\, \frac{h^2}{16(p\,+\,1)}\,\left(7\,-\,p\,+\,\frac{2\,\beta_E^{(p)}}{\gamma_E}\right)\, e^{-2\,(8-p)\,C\,+\,2\,\beta_E^{(p)}\,\phi}$$

we have obtained, from the results almost equals of the equations, putting

$4096\, e^{-\pi\sqrt{18}}$ instead of

$$e^{-2\,(8-p)\,C\,+\,2\,\beta_E^{(p)}\,\phi}$$

a new possible mathematical connection between the two exponentials. Thence, also the values concerning p, C, β_E and ϕ correspond to the exponents of *e* (i.e. of exp). Thence we obtain for p = 5 and β_E = 1/2:

$$e^{-6C+\phi} = 4096e^{-\pi\sqrt{18}}$$

Therefore, with respect to the exponentials of the vacuum equations, the Ramanujan's exponential has a coefficient of 4096 which is equal to 64^2, while -6C+ϕ is equal to -$\pi\sqrt{18}$. From this it follows that it is possible to establish mathematically, the dilaton value.

For

exp((-Pi*sqrt(18)) we obtain:

Input:

$\exp\left(-\pi\sqrt{18}\right)$

Exact result:

$e^{-3\sqrt{2}\,\pi}$

Decimal approximation:

$1.627201622607250929294215673911797954183858113695401\ldots \times 10^{-6}$

1.6272016… * 10^{-6}

Property:

$e^{-3\sqrt{2}\,\pi}$ is a transcendental number

Series representations:

$$e^{-\pi\sqrt{18}} = e^{-\pi\sqrt{17}\sum_{k=0}^{\infty} 17^{-k}\binom{1/2}{k}}$$

$$e^{-\pi\sqrt{18}} = \exp\left(-\pi\sqrt{17}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{17}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$e^{-\pi\sqrt{18}} = \exp\left(-\frac{\pi\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j} 17^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}\right)$$

Now, we have the following calculations:

$$e^{-6C+\phi} = 4096e^{-\pi\sqrt{18}}$$

$$e^{-\pi\sqrt{18}} = 1.6272016\ldots * 10^{-6}$$

from which:

$$\frac{1}{4096}e^{-6C+\phi} = 1.6272016\ldots * 10^{-6}$$

$$0.000244140625\ e^{-6C+\phi} = e^{-\pi\sqrt{18}} = 1.6272016\ldots * 10^{-6}$$

Now:

$$\ln\left(e^{-\pi\sqrt{18}}\right) = -13.328648814475 = -\pi\sqrt{18}$$

And:

(1.6272016* 10^-6) *1/ (0.000244140625)

Input interpretation:

$$\frac{1.6272016}{10^6} \times \frac{1}{0.000244140625}$$

Result:

0.0066650177536

0.006665017...

Thence:

$$0.000244140625\ e^{-6C+\phi} = e^{-\pi\sqrt{18}}$$

Dividing both sides by 0.000244140625, we obtain:

$$\frac{0.000244140625}{0.000244140625} e^{-6C+\phi} = \frac{1}{0.000244140625} e^{-\pi\sqrt{18}}$$

$$e^{-6C+\phi} = 0.0066650177536$$

((((exp((-Pi*sqrt(18)))))))*1/0.000244140625

Input interpretation:

$$\exp\left(-\pi\sqrt{18}\right)\times\frac{1}{0.000244140625}$$

Result:

0.00666501785...

0.00666501785…

Series representations:

$$\frac{\exp(-\pi\sqrt{18})}{0.000244141}=4096\exp\left(-\pi\sqrt{17}\sum_{k=0}^{\infty}17^{-k}\binom{\frac{1}{2}}{k}\right)$$

$$\frac{\exp(-\pi\sqrt{18})}{0.000244141}=4096\exp\left(-\pi\sqrt{17}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{17}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$\frac{\exp(-\pi\sqrt{18})}{0.000244141}=4096\exp\left(-\frac{\pi\sum_{j=0}^{\infty}\mathrm{Res}_{s=-\frac{1}{2}+j}\,17^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}\right)$$

Now:

$$e^{-6C+\phi}=0.0066650177536$$

$$\exp\left(-\pi\sqrt{18}\right)\times\frac{1}{0.000244140625}=$$

$$e^{-\pi\sqrt{18}}\times\frac{1}{0.000244140625}$$

$$=0.00666501785\ldots$$

From:

ln(0.00666501784619)

Input interpretation:

$\log(0.00666501784619)$

Result:

$-5.010882647757\ldots$

-5.010882647757…

Alternative representations:

$$\log(0.006665017846190000) = \log_e(0.006665017846190000)$$

$$\log(0.006665017846190000) = \log(a)\log_a(0.006665017846190000)$$

$$\log(0.006665017846190000) = -\text{Li}_1(0.993334982153810000)$$

Series representations:

$$\log(0.006665017846190000) = -\sum_{k=1}^{\infty} \frac{(-1)^k (-0.993334982153810000)^k}{k}$$

$$\log(0.006665017846190000) = 2 i \pi \left\lfloor \frac{\arg(0.006665017846190000 - x)}{2 \pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (0.006665017846190000 - x)^k x^{-k}}{k} \quad \text{for } x < 0$$

$$\log(0.00666501784619000) = \left\lfloor \frac{\arg(0.00666501784619000 - z_0)}{2\pi} \right\rfloor \log\left(\frac{1}{z_0}\right) + \log(z_0) + \left\lfloor \frac{\arg(0.00666501784619000 - z_0)}{2\pi} \right\rfloor \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (0.00666501784619000 - z_0)^k z_0^{-k}}{k}$$

Integral representation:

$$\log(0.00666501784619000) = \int_1^{0.00666501784619000} \frac{1}{t}\, dt$$

In conclusion:

$$-6C + \phi = -5.010882647757 \ldots$$

and for C = 1, we obtain:

$$\phi = -5.010882647757 + 6 = \mathbf{0.989117352243} = \boldsymbol{\phi}$$

Note that the values of n_s (spectral index) 0.965, of the average of the Omega mesons Regge slope 0.987428571 and of the dilaton 0.989117352243, are also connected to the following two Rogers-Ramanujan continued fractions:

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}} - \varphi + 1} = 1 - \cfrac{e^{-\pi}}{1 + \cfrac{e^{-2\pi}}{1 + \cfrac{e^{-3\pi}}{1 + \cfrac{e^{-4\pi}}{1 + \ldots}}}} \approx 0.9568666373$$

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\dfrac{\sqrt{5}}{1 + \sqrt[5]{\sqrt{\varphi^5 \sqrt[4]{5^3}} - 1}} - \varphi + 1} = 1 - \cfrac{e^{-\pi\sqrt{5}}}{1 + \cfrac{e^{-2\pi\sqrt{5}}}{1 + \cfrac{e^{-3\pi\sqrt{5}}}{1 + \cfrac{e^{-4\pi\sqrt{5}}}{1 + \ldots}}}} \approx 0.9991104684$$

(http://www.bitman.name/math/article/102/109/)

Also performing the 512th root of the inverse value of the Pion meson rest mass 139.57, we obtain:

((1/(139.57)))^1/512

Input interpretation:

$$\sqrt[512]{\frac{1}{139.57}}$$

Result:

0.990400732708644027550973755713301415460732796178555551684...

0.99040073.... result very near to the dilaton value $\mathbf{0.989117352243 = \phi}$ and to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\dots}}}}\approx 0.9991104684$$

From:

Eur. Phys. J. C (2019) 79:713 - https://doi.org/10.1140/epjc/s10052-019-7225-2-Regular Article - Theoretical Physics

Generalized dilaton–axion models of inflation, de Sitter vacua and spontaneous SUSY breaking in supergravity

Yermek Aldabergenov, Auttakit Chatrabhuti, Sergei V. Ketov

Table 1 The predictions for the inflationary parameters (n_s, r), and the values of φ at the horizon crossing (φ_i) and at the end of inflation (φ_f), in the case $3 \le \alpha \le \alpha_*$ with both signs of ω_1. The α parameter is taken to be integer, except of the upper limit $\alpha_* \equiv (7+\sqrt{33})/2$

α	3	4		5	6		α_*
sgn(ω_1)	$-$	$+$	$-$	$+/-$	$+$	$-$	$-$
n_s	0.9650	0.9649	0.9640	0.9639	0.9634	0.9637	0.9632
r	0.0035	0.0010	0.0013	0.0007	0.0005	0.0004	0.0003
$-\kappa\varphi_i$	5.3529	3.5542	3.9899	3.2657	3.0215	2.7427	2.5674
$-\kappa\varphi_f$	0.9402	0.7426	0.8067	0.7163	0.6935	0.6488	0.6276

From:

Pion family in AdS/QCD: the next generation from configurational entropy
Luiz F. Ferreira and R. da Rocha - arXiv:1902.04534v2 [hep-th] 2 Apr 2019

The AdS/QCD setup can be then employed to derive configurational entropic Regge trajectories for the pion family. Based upon a two flavour soft wall model, with gluon and chiral condensates, coupled to gravity with a dilaton [33], informational Regge trajectories were studied to the a_1, f_0 and ρ meson families in such a setup [17]. The following dilatons were introduced in Refs. [17, 35] to model mesons and glueballs,

$$\phi_1(z) = \mu_G^2 z^2, \tag{15}$$
$$\phi_2(z) = \mu_G^2 z^2 \tanh\left(\mu_{G^2}^4 z^2/\mu_G^2\right). \tag{16}$$

and shall be employed, respectively as the prototypical dilaton in the soft wall AdS/QCD [31], Eq. (15), and its deformation, Eq. (16). The deformed dilaton in the UV limit yields the quadratic dilaton. The holographic gluon condensate is dual to the quadratic dilaton (15) and has μ_G energy scale when it corresponds to a dimension-2 system, whereas it has μ_G^2 energy scale when describing a dimension-4 dual system [38, 52]. A graviton-gluon-dilaton action in AdS can be given by [35],

$$S=\kappa_5^2\int\sqrt{-g}e^{-2\phi}\Big\{\big[R+4\partial^M\phi\partial_M\phi-4V_g(\phi)\\ -16\lambda e^{-\phi}\big(\partial^M\xi\partial_M\xi+V(\phi,\xi)\big)\big]\Big\}d^5x, \quad (17)$$

where λ denotes a general coupling, and V_g denotes the gluon system potential. Ref. [35] studied a heavy quark potential in the background given by Eq. (17), deriving the physical effective potential $V(\phi,\xi)\approx\xi^2\phi^2$. For both the $\phi_1(z)$ and $\phi_2(z)$, respectively in Eqs. (15) and (16), the parameters $\mu_{G^2}=\mu_G\approxeq 0.431$ were adopted in Refs. [17, 35], in full compliance to data from experiments in PDG. Numerical analysis of the EOMs derived from (17), in Ref. [35], yields the solutions for $\xi(z)$, for both the dilatonic backgrounds. The first column in Table I replicates the mass spectra in the PDG 2018 for $\pi_1=\{\pi_\pm,\pi_0\}$, $\pi_2=\pi(1300)$, $\pi_3=\pi(1800)$, as well as for $\pi_4=\pi(2070)$, $\pi_5=\pi(2360)$ that are still left out the summary table in PDG (few events registered [50]).

——— pseudoscalar pion mass spectra (MeV) ———

n	Experimental	$\text{mass}_{\phi_1(z)}$	$\text{mass}_{\phi_2(z)}$
1	139.57018 ± 0.00035	139.3	139.6
1	134.9766 ± 0.0006	139.3	139.6
2	1300 ± 100	1343	1505
3	1816 ± 14	1755	1832
4*	2070	2006	2059
5*	2360	2203	2247

TABLE I: Mass spectra for the pseudoscalar pion family, in the $\phi_2(z) = z^2 \tanh\left(\mu_{G^2}^4 z^2/\mu_G^2\right)$ dilaton, for the π_0, $\pi(1300)$, $\pi(1800)$, $\pi(2070)$, $\pi(2360)$ mesons. The modes indicated with asterisk are not established particles and therefore are omitted from the summary table in PDG.

Table I shows the pseudoscalar pion family, identifying the π_n eigenfunctions in Eq. (12), as $\pi_1 = \{\pi_\pm, \pi_0\}$, $\pi_2 = \pi(1300)$, $\pi_3 = \pi(1800)$, $\pi_4 = \pi(2070)$, $\pi_5 = \pi(2360)$. whereas the other ones have not been experimentally confirmed states yet [50]. Besides, the pseudoscalar sector can be implemented by considering the following action, pion and ϕ meson wavefunctions:

$$S_\pi^{(2)} = -\frac{1}{3L^3}\int d^5x e^{-\phi}\sqrt{g}(\xi^2\partial^z\pi\partial_z\pi + \xi^2\partial^\mu(\varphi-\pi)\partial_\mu(\varphi-\pi) + L^2\partial^z\partial^\mu\varphi\partial_z\partial_\mu\varphi). \quad (18)$$

It is observed that in the graviton-dilaton-scalar system, the lowest pseudoscalar state has a mass around 140MeV, Another information provided by the configurational entropic Regge trajectories is the values of the masses of the next generation of the π states. Using the value CE of the n^{th} excitation, Eqs. (22, 23), one can employ Eqs. (22) and (24) to infer the mass spectra of the π_6, π_7 and π_8, as discussed throughtout Sect. III. In the case of the quadratic dilaton the results found are $m_{\pi,6} = 2630 \pm 18$ MeV, $m_{\pi,7} = 2861 \pm 22$ MeV and $m_{\pi,8} = 3074 \pm 25$ MeV. On the other hand, for the deformed dilaton the masses found are $m_{\pi,6} = 2631 \pm 18$ MeV, $m_{\pi,7} = 2801 \pm 22$ MeV and $m_{\pi,8} = 2959 \pm 25$ MeV. It is possible to improve these values of the masses with the eventual detection of the pion excitation states, that shall contribute with more experimental points in Fig. (1).

From:

Citation: M. Tanabashi et al. (Particle Data Group), Phys. Rev. D 98, 030001 (2018) and 2019 update

Further States

OMITTED FROM SUMMARY TABLE

This section contains states observed by a single group or states poorly established that thus need confirmation.

QUANTUM NUMBERS, MASSES, WIDTHS, AND BRANCHING RATIOS

$\pi(2070)$ $I^G(J^{PC}) = 1^-(0^{-+})$

MASS (MeV)	*WIDTH* (MeV)	*DOCUMENT ID*		*TECN*	*COMMENT*
2070 ± 35	310^{+100}_{-50}	ANISOVICH	01F	SPEC	2.0 $\bar{p}p \to 3\pi^0, \pi^0\eta, \pi^0\eta'$

$\pi(2360)$ $I^G(J^{PC}) = 1^-(0^{-+})$

MASS (MeV)	*WIDTH* (MeV)	*DOCUMENT ID*		*TECN*	*COMMENT*
2360 ± 25	300^{+100}_{-50}	ANISOVICH	01F	SPEC	2.0 $\bar{p}p \to 3\pi^0, \pi^0\eta, \pi^0\eta'$

From:

Generalized dilaton–axion models of inflation, de Sitter vacua and spontaneous SUSY breaking in supergravity
Yermek Aldabergenov, Auttakit Chatrabhuti, Sergei V. Ketov
Received: 5 August 2019 / Accepted: 13 August 2019 / Published online: 24 August 2019

Table 2 The masses of inflaton, axion and gravitino, and the VEVs of F- and D-fields derived from our models by fixing the amplitude A_s according to PLANCK data – see Eq. (57). The value of $\langle F_T \rangle$ for a positive ω_1 is not fixed by A_s

α	3	4		5		6		7	
$\mathrm{sgn}(\omega_1)$	$-$	$+$	$-$	$+$	$-$	$+$	$-$	$-$	
m_φ	2.83	2.95	2.73	2.71	2.71	2.53	2.58	1.86	$\times 10^{13}$ GeV
$m_{t'}$	0	0.93	1.73	2.02	2.02	4.97	2.01	1.56	$\times 10^{13}$ GeV
$m_{3/2}$	≥ 1.41	2.80	0.86	2.56	0.64	3.91	0.49	0.29	$\times 10^{13}$ GeV
$\langle F_T \rangle$	any	$\neq 0$	0	$\neq 0$	0	$\neq 0$	0	0	$\times 10^{31}$ GeV^2
$\langle D \rangle$	8.31	4.48	5.08	3.76	3.76	3.25	2.87	1.73	$\times 10^{31}$ GeV^2

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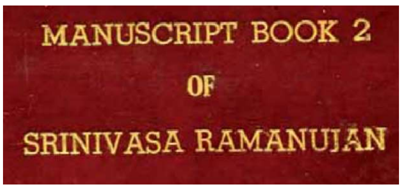

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