Refutation of multi-valued concurrent dynamic logic

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Abstract: We evaluate two operators for parallel and sequential composition which are not tautologous. The conjecture of multi-valued, concurrent, propositional, dynamic logics fails. These results form a non tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, F as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩, . , ⊓ ; \ Not And; > Imply, greater than, →, ⇒, ↦, ⊢ ; ≤ Not Imply, less than, ∈, ⊂, ⊆, ≠, ≅, ≤ ; = Equivalent, ≡, ≅, :=, ↔, ≡, ≅, ≈, ♦, ⊤ ; @ Not Equivalent, ≠, ⊭; % possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L; (z=z) T as tautology, ⊤, ordinal 3; (z@z) F as contradiction, Ø, Null, ⊥, zero; (%z>#z) N as non-contingency, Δ, ordinal 1; (%z=#z) C as contingency, ∇, ordinal 2; ~(y<x) (x≤y), (x∈y), (x∈y); (A=B) (A~B); pipe symbol | means “such that”.

Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Gomes, L. (2019). On the construction of multi-valued concurrent dynamic logic. arxiv.org/pdf/1911.00462.pdf leandro.r.gomes@inesctec.pt

Abstract. Dynamic logic is a powerful framework for reasoning about imperative programs. An extension with a concurrent operator .. was introduced to formalise programs running in parallel. In other direction, other authors proposed a systematic method for generating multi-valued propositional dynamic logics to reason about weighted programs .. . This paper presents the first step of combining these two frameworks to introduce uncertainty in concurrent computations. In the developed framework, a weight is assigned to each branch of the parallel execution, resulting in a (possible) asymmetric parallelism, inherent to fuzzy programming paradigm .. . By adopting such an approach, a family of logics is obtained, called multi-valued concurrent propositional dynamic logics (CGDL(A)), parametric on an action lattice A specifying a notion of “weight” assigned to program execution. Additionally, the validity of some axioms of CPDL is discussed in the new family of generated logics.

2 Preliminaries. 2.1 Semantics for concurrency. Definition 1 (Binary multirelation .. ). Given a set X, a binary multirelation is a subset of the Cartesian product X × P(X), i.e. a set of ordered pairs (a, A), where a ∈ X and A ⊆ X. The following operations over multirelations are defined: … the parallel composition

\[ R \cap S = \{(a, A \cup B) \mid (a, A) \in R \land (a, B) \in S\}. \]  

(2.1.3.1)

\[ \text{LET } p, q, r, s, t, u, v, w: A, B, R, S, a, b, n, f. \]
\[ (s\&r)=(((t\&p)<r)\&(t\&q)<s)>((t\&p)+q)); \]
\[ \text{FFFF FFFF FFFF TTTT} \]  

(2.1.3.2)

4 Conclusion. Although we base our definition of sequential composition for fuzzy multirelations in that of Peleg, there are other versions of the operator worth to be analysed. One corresponds to the definition introduced for giving semantics to Parikh’s game logic ..
\[ R \cdot S = \{ n (a, A) \mid \exists B.(a, B) \in R \land \exists f.(\forall b \in B.(b, A) \in S) \} \quad (4.1.1) \]

\[ (r\&s)=(((t\&%q)<r)\&(w\&((u<(q&(u&p))<s)))>(v&(t&p))) ; \]

\[
\begin{array}{cccc}
\text{FFFF} & \text{FFFF} & \text{FFFF} & \text{TTTT} (11) \\
\text{FNF} & \text{FFFF} & \text{FNN} & \text{TTTT} \\
\text{FFFF} & \text{FFFF} & \text{FFFF} & \text{TTTT (3)} \\
\text{FNF} & \text{FFFF} & \text{FNF} & \text{TTTT} \end{array}
\]

It is clearly stronger than Peleg’s, since it requires that every intermediate state \( b \) must be related with the arriving set of states \( A \).

Eqs. 2.1.3.2 and 4.1.2 as rendered are not tautologous. This refutes the parallel composition operator and a stronger version than Peleg’s for the Parikh fuzzy, game logic operator of sequential composition. What follows is that the conjecture of multi-valued, concurrent, propositional, dynamic logics fails.