Refutation of Frege’s inference rule of generalization

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Abstract: We evaluate Frege’s inference rule of generalization which is not tautologous. This result forms a non tautologous fragment of the universal logic $VŁ_4$.

We assume the method and apparatus of Meth8/$VŁ_4$ with $\top$ as tautology (non-contradiction), $\bot$ as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET  $\sim$ Not, $\neg$;  $+$ Or, $\lor$, $\cup$;  $-$ Not Or;  $\&$ And, $\land$, $\cap$, $\cdot$;  $\Theta$;  \ Not And;
\> Imply, greater than, $\rightarrow$, $\Rightarrow$, $\supset$, \ $\supseteq$;  \< Not Imply, less than, $\in$, $\subset$, $\subseteq$, $\neq$, $\approx$;  \@ Not Equivalent, $\neq$, $\oplus$;
\% possibility, for one or some, $\exists$, $\diamond$, $M$;  \# necessity, for every or all, $\forall$, $\square$, $L$;
\(z=z\) $\top$ as tautology, $\top$, ordinal 3;  \(z@z\) $\bot$ as contradiction, $\emptyset$, Null, $\perp$, zero;
\(\%z>#z\) $\neg$ as non-contingency, $\Delta$, ordinal 1;  \(\%z<#z\) $\neg\bot$ as contingency, $\nabla$, ordinal 2;
\(~(y<x)\) (\(x \leq y\)), (\(x \leq y\)), (\(x \subseteq y\));  \(A=B\) (\(A\sim B\)).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: en.wikipedia.org/wiki/Begriffsschrift

The rule of generalization allows us to infer [ if $x$ does not occur in $P$, ]

\[ \vdash P \rightarrow \forall x A(x) \text{ from } \vdash P \rightarrow A(x) \]  \hspace{1cm} (2.1)

LET  $p$, $q$, $r$, $s$:  $P$, $x$, $A$.

\(~(r<p)>(p>(q&r)))>(p>(q\&r))\);

\[ \begin{array}{cccccc}
   & T & T & T & T & T \\
\vdash (r<p) & T & T & T & T & T \\
\vdash (p>(q\&r)) & T & T & T & T & T \\
\vdash (p>(q\&r)) & T & T & T & T & T \\
\end{array} \]  \hspace{1cm} (2.2)

Remark 2.2: Eq. 2.2 as rendered is not tautologous. This refutes Frege’s inference rule of generalization, and any conjecture making use of it.