Fictitious forces, inertial forces and Mach's principle (II)

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Abstract. According to the principle dynamic equilibrium, we understand the force of inertia is a force that acts on whatever body that accelerated with respect to an inertial frame. It is, therefore, a real force, observed in whatever reference frame. We identify this force with force gravitational induction produced by the whole of the Universe. Therefore, the Universe is an inertial reference frame. In developing the theory, we find that the gravitational induction force produced by the entire Universe is proportional to acceleration and in the opposite sense, as is the force of inertia. In making this identification, we find that the inertial and gravitational mass are proportional, with a coefficient of proportionality depending on the cosmic time.

1 Principle dynamic equilibrium

Usually, the second law of mechanics is expressed by saying that when a force **F** acts on a body, it acquires an acceleration **a** proportional to the force

$$\mathbf{a} = \frac{\mathbf{F}}{m_i}$$

m_i is the inertial mass, a magnitude that characterizes the body. But in this research we make a different formulation: when a force acts on a body it acquires acceleration, and as result of this acceleration, another force called inertia acts on the body $\mathbf{F}_i = -m_i\mathbf{a}$ which opposes the applied force, that is to say

$$\mathbf{F} + \mathbf{F}_i = 0$$

or is the principle of dynamic equilibrium.

Both formulations seem identical, but they are very different. Now the force of inertia is a real force that acts on whatever accelerated body with respect to an inertial reference frame. The force of inertia, as we have defined it, is produced by other bodies. We propose that the force of inertia is produced by the whole Universe, as we will show.

Mach's principle is included in the principle of dynamic equilibrium. Indeed, the inertial mass m_i is the result of the action of the whole Universe. The inertial mass is not an intrinsic property of the body but is the result of the action of the Universe, that is, an acquired property.

In this paper we will demonstrate that the force of inertia is produced when a body is accelerated with respect to the whole Universe. The force of inertia is a gravitational induction force, that is, a gravitational force produced by the movement of the body with respect to the whole Universe.

2 The Universe and the inertial reference frame

Newton assumed the existence of absolute space, which is an inertial reference frame and allows us to define all the remaining inertial reference frames, which are those that move with uniform and rectilinear motion with respect to absolute space. In the Newtonian scheme, there is no way to determine the absolute velocity, although accelerated motion can be detected, that is, the acceleration with respect to absolute space can be measured.

The impossibility of detecting absolute motion was limited to mechanical measurements, but experience showed that the absolute velocity of a body could not be determined from electromagnetic phenomena. The Theory of Special Relativity established, as a hypothesis, that there is no physical phenomenon with which the absolute velocity can be determined. It should be noted that this relativistic statement does not say that this absolute velocity does not exist, but exclusively that it cannot be measured. That is, in the scheme of Special Relativity, the concept of Newtonian absolute space still exists.

In the theory that we are developing, the whole Universe is an inertial reference frame, we say, therefore, that the Universe is the materialization of the Newtonian absolute space. With this vision, the absolute motion is with respect to the whole of the Universe and, therefore, experimentally determinable.

The Universe is not only a referential frame, as the definition of the International Reference Celestial System implies, but it also applies mechanical action on the bodies. There is no possibility of isolating the local phenomena of the Universe, contrary to what Newtonian cosmology affirms based on an infinite and uniform Universe. Local bodies maintain a dynamic relationship with the entire Universe, which is certainly not negligible, as we will see.

We might think that this is not what happens. For example, determine the motion of the Earth in its annual motion, we need to know the action of the Sun, the Moon and the planets, from which we obtain an extremely precise ephemeris, without the need to include in the calculations any information about the whole Universe. The reason for this situation is that the cosmic action is already included in the equation of motion that we use to determine the motion of the Earth.

Gravity is not an action at a distance, but a field that is transmitted by vacuum with the velocity of light. That is, time elapses between the transmission and reception of a gravitational signal. Therefore, we concluded that in addition to the static force of Newtonian gravity, there is a gravitational induction force, that is, a force that depends on the velocity.

The force of induction of gravity is produced by the entire Universe and is exerted on the bodies that move with respect to it. This induction is not a result of the motion in unison of all the bodies of the Universe with respect to a test body, but of the motion of this body with respect to the whole of the Cosmos.

The theory of gravitational induction is, in essence, identical to electromagnetic induction, so that the results obtained in electromagnetism can be extended, although always keeping in mind that it is the body on which the force is acting that is moving, and not sources of the field, that is to say, the bodies that constitute the Universe.

One might think that the motion of the test body with respect to the Universe at rest is equivalent to assuming that it is the Universe that moves with equal velocity but in the opposite sense with respect to the body that would remain at rest. The equivalence between the two situations is only kinematic. However, it is not dynamic since, in general, the bodies that move with acceleration with respect to the Universe emit gravitational radiation, which would not happen if it was the whole of the Universe who moved with acceleration with regarding the test body.

Naturally, the problem of calculating the cosmic induction force is complicated because the gravitational signals that reach the observer were produced at different cosmic time, from the beginning of the Universe to the present day, by bodies that are in different positions and at retarded distances, is say those that had the sources at the time of emitting the signal, which is not the current distance, that is the distance to the sources at the time of arrival of the signal.

The gravitational induction force, like the electromagnetic induction force, depends on the position of the source, its velocity, and acceleration. However, as we will see, when calculating the total induction force it is found that, in the classical approximation, it only depends on the acceleration, being linear with respect to it and in the opposite sense, which confirms its identification with the force of inertia. That is, the total induction force does not depend on velocity; there is no induction force for a body that moves with uniform and rectilinear motion.

When calculating the induction force, we derive the value of the inertial mass, which turns out to be (as we will see) proportional to the gravitational mass, but the constant of proportionality varies with time, even in a static Universe. That is, inertial mass is not a constant characteristic of a body that measures its amount of matter, but is a property that depends on the action of the

Universe, which is different depending on the type of Universe, it depends on cosmic time, and this is the most important thing. Finally, it must be said that the relationship between inertial and gravitational mass is the same for all bodies and depends only on cosmic time.

The consequences of the above considerations, which we will specify later, are far-reaching. There is an equivalence of relativistic origin between the mass of a body and its energy, $E = mc^2$; in this expression, m is the inertial mass, not the gravitational mass. Inertial mass depends on cosmic time; therefore, the energy associated with the matter has equal dependence. This conclusion is significant because energy has an associated gravitational mass that is a source of gravity, but this energy, by the above considerations, depends on cosmic time. For example, the gravitational mass associated with pressure depends on time.

For the Standard Model of the Universe or another similar one, the inertial mass of a body increases with cosmic time, because as time goes by there are more and more sources that are acting on this body, increasing its mass. Or, in the past the inertial mass of a body was smaller than it is today. For example, the inertial mass associated with the pressure remains unchanged, but the gravitational mass of that energy decreases with the passage of time.

There are also consequences for nuclear processes, especially those that occurred at the beginning of the Universe. The gravitational masses of the elementary constituents of matter are unalterable, but their inertial masses decrease in the past, and therefore, the associated energy will also decrease. That is, the nuclear bond energy was in the past significantly less than it would have existed if the inertial mass of the elementary particles had remained constant.

Finally, and without wishing to be exhaustive, to indicate that the variation of the inertial mass will result in a variation of the spectral lines, which is superimposed with the variation that causes cosmic expansion.

In summary, the phenomenon of inertia, inertial mass, and force of inertia are the result of the action of the whole of the entire Universe. We conclude, therefore, that the action of the Universe is very prominent in local phenomena. Naturally, the action of the cosmos will depend on its characteristics, the parameters that define it, and the cosmological model. Alternatively, put another way, the local action of the Universe will give us information about its structure and properties. The later means that the theory of Mach's principle that we are developing has important cosmological implications, as we have already pointed out.

3 Inertial mass and gravitational mass

The gravitational mass of a body, which more correctly should be called gravitational charge, is the magnitude that measures the gravitational effects produced by a body. The active and passive gravitational mass must be distinguished. The first is the mass that produces gravity, and the second measures the effect that gravity produces on the body. If the law of action and reaction is valid, or in other words, if the linear momentum is conserved, both types of gravitational masses are equal, and we will suppose so in the future.

Inertial mass is an entirely different concept. It is the mass that measures the inertia of a body, that is, the opposition it offers to the change of motion or the mass that appears in the force of inertia.

By the principle of weak equivalence, the inertial mass is proportional to the gravitational mass. That is, the acceleration that a body acquires due to gravity is independent of mass. However, the equivalence principle does not state that the proportionality between inertial and gravitational mass remains constant over time.

We consider that the gravitational mass is an intrinsic quantity, associated with the amount of matter in the body and therefore invariable. However, we cannot say the same about inertial mass. This mass is the result of the gravitational induction force of the whole Universe, which, with exceptions, varies with cosmic time, which means that the inertial mass also changes.

For the above, we have

$$m_i = \chi(t) m_g$$

m_i is the inertial mass, m_g the gravitational mass and $\chi(t)$ it is a function of cosmic time that we call inertia coefficient and that is a proportionality factor, the same for all the bodies of the Universe.

4 Relative motion

The movement of a body refers to bodies and not to absolute space; in this sense, we consider the motion as «relative», in contrast to Newton's idea that there is an absolute motion or motion with respect to absolute space.

In the development of the «relative» theory of mechanics, not only is absolute space ruled out as unreal, but all reference frames are equivalent, which is not the position that emerges from Mach's principle as we are exposing.

As we have said, there is only the motion of one body with respect to another body, but not all reference frames are equivalent. The reason is the existence of a particular reference frame: the whole of the Universe, which defines the inertial reference frames. The definition of the Universe as an inertial frame breaks the equivalence of all reference frames because each of them has a motion with respect to the Universe.

In our interpretation of Mach's principle, there are still inertial and non-inertial frames, as in Newtonian mechanics, and defined by their motion with respect to the whole Universe. Also, there is no equivalence between all reference frames, since as we have said, the acceleration of a body with respect to the Universe distinguishes it from other frames, in the sense that they are emitters of gravitational radiation.

The Universe is a dynamic system, so what does it mean to say that a body moves with respect to the Universe? We refer to the motion of a body with respect to the current Universe, that is, the observed Universe, and this is a seemingly static. That is, we are referring the observer's motion to the constituents of the Universe in their retarded position

4 Gravitational induction

Induction forces (electromagnetic or gravitational) are those that have their origin in the motion, either of the source, the observer or both. We will call active induction if it is produced by the motion of the source and passive induction if it is produced by the motionof the observer. The gravitational induction forces responsible for generating the force of inertia are those of the second type, that is, forces produced by the motion of a body with respect to the whole Universe.

If the gravitational force were an action at a distance and the propagation were instantaneous, there would be no induction forces. But we admit that gravity is a field, so gravitational signals take time to propagate, because they travel with the velocity of light in vacuum c. And for this reason the forces of gravitational induction arise.

Let us realize that the phenomenon of induction, and therefore the origin of the force of inertia, does not require the Theory of Relativity, but is a classical phenomenon, which only requires that the gravitational signal propagate with a finite velocity.

We will assume a body of mass m, which at a time t'_1 emits a gravitational signal that propagates with the velocity c until it reaches the observer in rest located at point P_0. If the distance between m and P_0 at the time the signal is output is r'_1 then the time t_1 when the signal reaches P_0 is

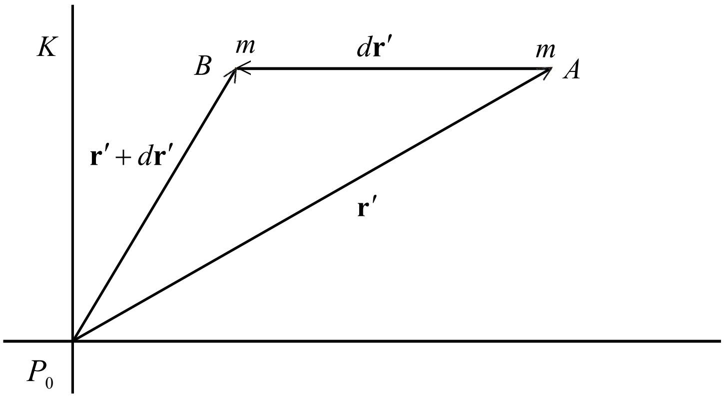
$$r'_1 = c\left(t_1 - t'_1\right) \quad \Rightarrow \quad t_1 = \frac{r'_1}{c} + t'_1,$$

at a later time t'_2 emits m another gravitational signal that will reach P_0 at time t_2

$$r'_2 = c\left(t_2 - t'_2\right) \quad \Rightarrow \quad t_2 = \frac{r'_2}{c} + t'_2$$

if m and P_0 are in relative rest then $r'_1 = r'_2$ and $t_2 - t_1 = t'_2 - t'_1$. But if there is a relative movement between m and P_0, then $r'_1 \neq r'_2$ and therefore $t_2 - t_1 \neq t'_2 - t'_1$. Note that t and t' are not two different time scales, since the time in m and P_0 are synchronized and we do not consider relativistic effects. The measures of t and t' refer to different phenomena: the reception and emission of gravitational signals.

We consider a reference frame K where the observer P_0 located at its origin of coordinates is fixed. We assume that the particle m moves with respect to K with a velocity $\mathbf{u}$. At time t' the mass m is in the position given by the position vector $\mathbf{r}'$, and emits a signal directed towards the point P_0. As the mass m moves with respect to K, at the time $t' + dt'$ it is in another position $\mathbf{r}' + d\mathbf{r}'$ and there it emits another signal directed towards the point P_0. The distance r' is called



Drawing 1.- *The body of mass m moves relatively to the frame K with a velocity u, which by simplification we assume that it has a horizontal direction and to the left. At point A the mass m emits at time t' a gravitational signal that reaches point P_0 at time t. Over a period of time dt' the body m moves from A to point B and there, at the moment $t'+dt'$, emits another gravitational signal in the direction of point P_0, where it arrives at the time $t+dt$. The velocity of the mass m with respect to the system K is $\mathbf{u}=d\mathbf{r}'/dt'$.*

retarded, that is, the distance between *m* and P_0 at the moment of emitting the signal at time t'. The velocity of *m* with respect to *K* is defined by

$$\mathbf{u}=\frac{d\mathbf{r}'}{dt'}$$

Drawing 1 shows the emission and reception of the two gravitational signals assuming that is the mass *m* that moves with respect to the reference frame *K*. For simplicity, we have assumed that *m* moves with velocity *u* in the horizontal direction and left sense. In drawing 2 the same phenomena are represented but assuming that the frame *K* moves with velocity *u* and that the mass *m* is at rest, the result, as we will see, is the same in both cases.

In drawings 2 and 3 we assume that the frame *K* moves with constant velocity *u* along the horizontal axis and to the right. In drawing a) of 2, the reference frame *K* is represented at the instant t' (that we now call it K_0). At that moment a gravitational signal leaves *m* (that is always in the same position of space), which reach the origin of K at time *t.* At time *t* point P_0 will have moved to point P_1. The distance traveled by the origin of the frame *K,* where the observer is located, is equal to the time it takes the signal to travel r'/c multiplied by the velocity of *K* with respect to *m*, that is

$$P_0P_1=u\frac{r'}{c}$$

r' is the distance traveled by the signal that will reach point P_1 in the instant

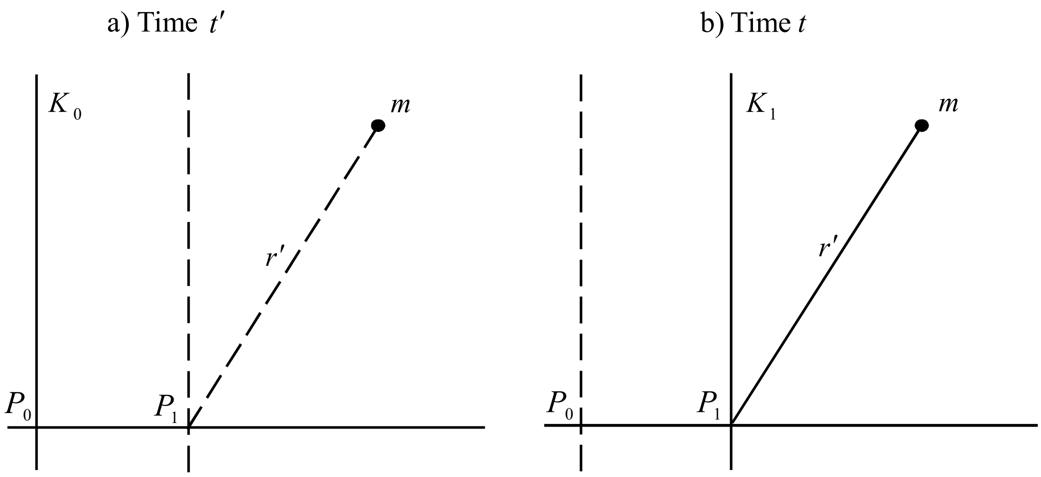
$$t=t'+\frac{r'}{c}.$$

In drawing b) of 2 the time *t* is represented, which is when the signal reaches the origin of the reference frame, which is where we assume the observer is. Drawing c) of 3 shows the situation at time $t'+dt'$. In the time interval dt' which is the one between the emission of the first and the second signal, the reference frame will have moved a distance udt', so that the point P_0 now occupies the position P_2. At this time $t'+dt'$ the mass emits another gravitational signal that will reach the origin of *K* at the time $t+dt$, point that we have represented by P_3.

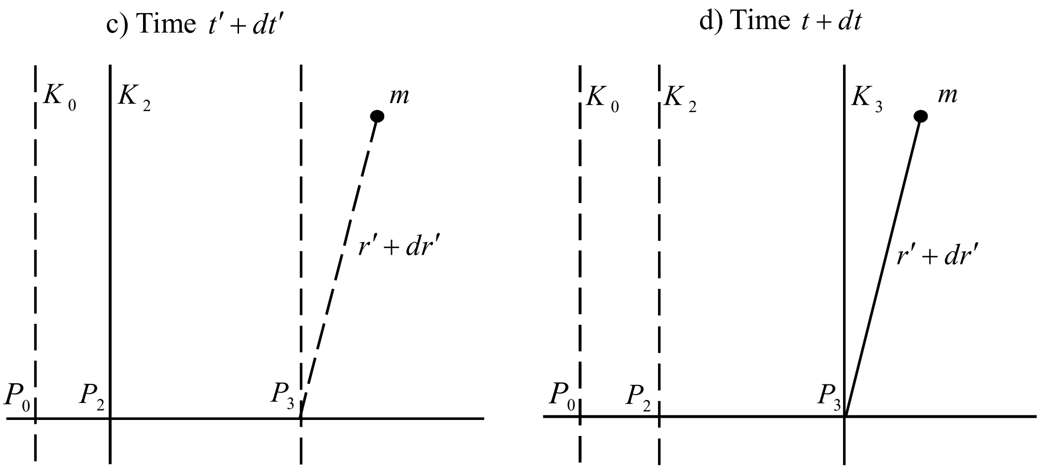
Now the distance traveled by the signal will be $r'+dr'$, taking time $(r'+dr')/c$ and in that time the point P_2 has passed to P_3 carrying a velocity *u*, the distances considered are

$$P_0P_2=udt'; \quad P_2P_3=u\frac{r'+dr'}{c}$$

Drawing 4 shows all the emissions and receptions of the two gravitational signals. At point P_1 the first signal arrived and at point P_3 the second signal, these points are the origin of *K* at time *t* and $t+dt$. So since the frame has a velocity *u*, the distance between P_2 and P_3 must be *udt*. Let's check it out. From drawing 4 we find



Drawing 2.- *On the left, the reference frame is represented in continuous strokes at time t', which we now call K_0, which for simplicity we have assumed moves in the horizontal direction and in the sense to the right. The origin, point P_0, is where the observer is. In the time t' the mass m emits a gravitational signal that will reach the point P_0 at the time t. The point represented by P_1 is where P_0 will be when the gravitational signal arrives, since K is in motion with a constant velocity u to the right. In the drawing on the right, the situation is represented at time t. The mass m is in the same position, since we assume that it is the frame K that moves, and the frame K, now called K_1, has shifted to the right. The distance traveled by the gravitational signal is r'.*



Drawing 3.- *At the time $t' + dt'$ the frame K, which we now call K_2, has shifted to the right, passing point P_0 to P_2 during the interval dt'. At the time $t' + dt'$ the mass emits another gravitational signal that reach the point P_0 at the time t + dt. At that time the point P_0 will be in position P_3, since K moves to the right with a velocity u. The moment t + dt is shown in the drawing on the right. The signal has reached the point P_0, which at the time t + dt coincides with P_3. The distance traveled by the signal is now $r' + dr'$.*

$$P_1P_3 = P_2P_3 - P_2P_1 ; \quad P_2P_1 = P_0P_1 - P_0P_2$$

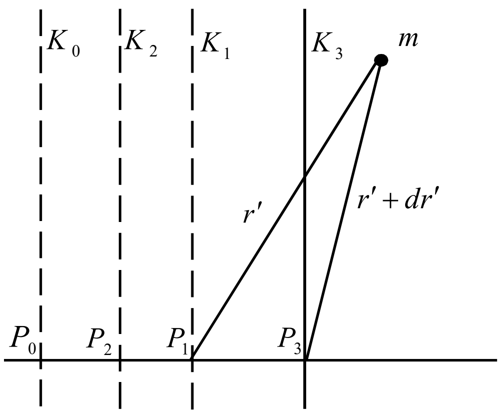
therefore

$$P_1P_3 = udt$$

as we had assumed; which tells us that it is equivalent to assume that the mass *m* moves with velocity *u* or or that it is the frame *K* that moves with equal velocity but in the opposite sense.

It is easy to generalize the reasoning for any velocity, so the quantities that interest us are (drawing 4)

$$\overrightarrow{P_0P_1} = \int_{t'}^{t} \mathbf{u}(t')dt' = \int_{t'}^{t'+r'/c} \mathbf{u}(t')dt' = \int_{t'}^{t'+dt'} \mathbf{u}(t')dt' + \int_{t'+dt'}^{t'+r'/c} \mathbf{u}(t')dt' = \mathbf{u}(t')dt' + \int_{t'+dt'}^{t'+r'/c} \mathbf{u}(t')dt'$$



Drawing 4.- *We assume that the frame K moves to the right in a horizontal direction with a velocity u, while the mass m is at rest. The observer is at the origin of K, which is in various positions according to the moment:* P_0 *is the origin at time* t' *;* P_2 *in* $t'+dt'$ *;* P_1 *in t and* P_3 *in t + dt.* P_1 *and* P_3 *are the positions in which the origin of K is when the signals arrive, which starting from m, were emitted at times* t' *and* $t'+dt'$*. The distance between* P_1 *and* P_3 *is that traveled by K in the time interval dt.*

$$\overrightarrow{P_0P_2} = \mathbf{u}(t')dt'$$

$$\overrightarrow{P_2P_3} = \int_{t'+dt'}^{t+dt} \mathbf{u}(t')dt' = \int_{t'+dt'}^{t} \mathbf{u}(t')dt' + \int_{t}^{t+dt} \mathbf{u}(t')dt' = \int_{t'+dt'}^{t'+r'/c} \mathbf{u}(t')dt' + \mathbf{u}(t)dt$$

and so

$$\overrightarrow{P_1P_3} = \overrightarrow{P_2P_3} - \overrightarrow{P_0P_1} + \overrightarrow{P_0P_2} = \mathbf{u}(t)dt$$

As we wanted to check. Naturally all this reasoning extends when both *K* and *m* move, once again **u** will be the relative velocity between them.

5 Relationship between the times *t* and *t'*

We have said that *t* and t' are not two different time scales. We are in classical mechanics and only consider absolute time. The clocks linked to *m* and *K* are synchronized with each other and therefore give the same time for any event. However, *t* and t' are different because they refer to different phenomena, t' measures the emission of a signal and *t* its reception. Moreover, as a result of the relative motion between the source *m* and the frame *K*, the interval dt' is different from the associated interval *dt*. Next we will calculate the relationship between these intervals.

As the gravitational signal propagates at uniform velocity *c* and travels a distance r' so

$$r' = c(t - t')$$

making the derivation

$$dr' = c(dt - dt') \;\Rightarrow\; \frac{dr'}{dt} = c\left(1 - \frac{dt'}{dt}\right),$$

we deduce

$$\frac{dr'}{dt} = \frac{dr'}{dt'}\frac{dt'}{dt} = c\left(1 - \frac{dt'}{dt}\right) \;\Rightarrow\; \frac{dt'}{dt} = \frac{1}{1 + \frac{1}{c}\frac{dr'}{dt'}}.$$

On the other hand we have

$$r'^2 = \mathbf{r}'^2 \;\Rightarrow\; r'\frac{dr'}{dt'} = \mathbf{r}'\frac{d\mathbf{r}'}{dt'} \tag{1}$$

$\mathbf{r}'$ is the vector that joins the source point and the origin of *K* at the retarded time, then

$$\frac{dr'}{dt'} = -\frac{\mathbf{r}'\cdot\mathbf{u}}{r'} \;\Rightarrow\; \frac{dt'}{dt} = \frac{1}{1 - \frac{\mathbf{r}'\cdot\mathbf{u}}{cr'}}, \tag{2}$$

u is the velocity of the source with respect to the frame *K*. (2) is the relationship we were looking for and that we will use later.

6 Force derived from the potentials

In field theories we have two groups of equations: the first is the field equation which is a second-order differential equation of the potentials and the second is the equation of motion that gives us the expression with which the force is calculated of the potentials. In a nonlinear theory, that is whose field equation is nonlinear, such as General Relativity, the equation of motion is derived from field equations, something that does not occur in linear field theory.

Potentials are functions of space and time without direct physical significance, but field strengths are calculated from differential operators, that is, from partial derivatives of the spatio-temporal coordinates. And from these field strength the force is calculated.

If with $\phi = \phi(x, y, z, t)$ we represent the potential generically, where *x, y, z* are the coordinates of the observer, then to find the field strength, we have to calculate their difference at the same point at two different times, that is

$$\left(\frac{\partial \phi}{\partial t}\right)_{x,y,z}$$

the time used in the previous derivative is the reception time of the gravitational signal since the derivation is made at the point of observation, of coordinates *x, y, z*; but as we will see later the potential ϕ depends on the relative positions and velocities, so from the previous partial derivative we would find expressions such as

$$\frac{d\mathbf{r}'}{dt}; \quad \frac{d\mathbf{u}}{dt}$$

that do not have a definite significance, but their derivatives with respect to dt' have meaning

$$\frac{d\mathbf{r}'}{dt'} = -\mathbf{u}; \quad \frac{d\mathbf{u}}{dt'} = -\mathbf{a}$$

a is the relative acceleration of the source.

Therefore the calculation we have to do is

$$\left(\frac{\partial \phi}{\partial t}\right)_{x,y,z} = \left(\frac{\partial \phi}{\partial t'}\right)_{x,y,z} \frac{dt'}{dt}$$

and then use (2)

$$\left(\frac{\partial \phi}{\partial t}\right)_{x,y,z} = \frac{1}{1 - \dfrac{\mathbf{r}' \cdot \mathbf{u}}{c\, r'}} \left(\frac{\partial \phi}{\partial t'}\right)_{x,y,z} = \frac{r'}{s}\left(\frac{\partial \phi}{\partial t'}\right)_{x,y,z}. \tag{3}$$

we have defined

$$s = r' - \frac{\mathbf{r}' \cdot \mathbf{u}}{c}.$$

Something similar occurs with spatial derivatives such as

$$\left(\frac{\partial \phi}{\partial x}\right)_{y,z,t}$$

which is the difference between the potential at two different points at the same time *t*. When developing the previous expression we will find expressions of the type

$$\left(\frac{\partial \mathbf{r}'}{\partial x}\right)_{y,z,t}$$

but the expressions that have physical meaning are

$$\left(\frac{\partial \mathbf{r}'}{\partial x}\right)_{y,z,t'}$$

Now the derivation is made keeping t' constant, that is to say, that we compare two vectors r' that, starting from different point, emit signals at the same time retrased, that is, the two signals left the source at the same time t', then the vector r'

$$\mathbf{r}' = (x' - x)\mathbf{i} + (y' - y)\mathbf{j} + (z' - z)\mathbf{k}$$

in the derivatives of *x, y, z* remain constant x', y', z'.

$\phi\left(x^{\alpha},t'\right)$ is the potential, so using Cartesian coordinates we have

$$d\phi = \frac{\partial\phi}{\partial x}dx + \frac{\partial\phi}{\partial y}dy + \frac{\partial\phi}{\partial z}dz + \frac{\partial\phi}{\partial t'}dt'$$

$$\frac{d\phi}{dx} = \frac{\partial\phi}{\partial x} + \frac{\partial\phi}{\partial y}\frac{dy}{dx} + \frac{\partial\phi}{\partial z}\frac{dz}{dx} + \frac{\partial\phi}{\partial t'}\frac{dt'}{dx}$$

$$\left(\frac{d\phi}{dx}\right)_{y,z,t} = \left(\frac{\partial\phi}{\partial x}\right)_{y,z,t'} + \left(\frac{\partial\phi}{\partial t'}\right)_{x,y,z}\left(\frac{dt'}{dx}\right)_{y,z,t}$$

and similar formulas for the remaining coordinates. So

$$\nabla = \nabla' + \nabla t'\frac{\partial}{\partial t'}. \tag{4}$$

where ∇ is calculated with the coordinates x^{α} taking t as a constant, while ∇' is also calculated with respect to the x^{α} coordinates but taking t' constant.

Now we need to calculate $\nabla t'$. For this we apply (3) to the function $r'\left(x^{\alpha},t'\right)$

$$\nabla r' = \nabla' r' + \nabla t'\frac{\partial r'}{\partial t'} = \frac{\mathbf{r}'}{r'} + \nabla t'\frac{\partial r'}{\partial t'} \tag{5}$$

of $r' = c\left(t - t'\right)$

$$\nabla r' = -c\nabla t' \tag{6}$$

of (1), (5) y (6)

$$\nabla t' = -\frac{\mathbf{r}'}{cs}$$

equation that we include in (5), obtaining

$$\nabla = \nabla' - \frac{\mathbf{r}'}{cs}\frac{\partial}{\partial t'}, \tag{7}$$

result we were looking for.

7 Vector gravitational theory

To simplify the calculations, we will consider a vector gravitational field compatible with the Special Relativity. To get this theory we have to adapt the electromagnetic theory to gravity. The gavitational potential is a tetravector that we define by

$$\phi^{k} = \left(\phi, c\mathbf{A}\right)$$

ϕ is the scalar potential and **A** the vector potential. The source of the field is the current tetradensity

$$j^{k} = \rho u^{k} = \left(j^{o}, \mathbf{j}\right)$$

ρ is the proper density of gravitational mass and u^{k} is the tetravelocity of the source defined by

$$u^{k} = \frac{dx^{k}}{d\tau}$$

$d\tau$ is the proper time of the source particle.

The field equation is

$$\nabla^{2}\phi^{k} - \frac{1}{c^{2}}\frac{\partial^{2}\phi^{k}}{\partial t^{2}} = \frac{4\pi G}{c}j^{k}.$$

This equation is valid in inertial reference frames and in Cartesian coordinates, although it is generalizable to non-inertial frames and to other coordinates, but then the partial derivatives must be substituted by covariant derivatives.

Following the similarity of the electromagnetic theory with vector gravitational theory we postulate that the force of gravity that acts on a body of gravitational mass m_{g} and has velocity **w** is an expression similar to the Lorentz force

$$\mathbf{F} = m_{g}\mathbf{E} + m_{g}\mathbf{w}\wedge\mathbf{B} \tag{8}$$

where **E** and **B** are the gravitoelectric and gravitomagnetic fields defined by

$$\mathbf{E} = -\nabla\phi - \frac{\partial \mathbf{A}}{\partial t}; \quad \mathbf{B} = \nabla \wedge \mathbf{A}. \tag{9}$$

8 Potentials of Liénard-Wiechert

The phenomena of gravitational induction is produced by the movement of the source or by the movement of the observer. In the first case we speak of active induction and in the second of passive induction.

In our problem the observer moves with respect to the Universe, which is at rest in an inertial reference frame K_0.

The observer P_0 is at rest at the origin of the frame *K*, which has a velocity $-\mathbf{u}$ with respect to K_0. This reference frame, in general, will not be inertial (that is, it will have acceleration with respect to K_0).

We consider a small mass m_g at rest with respect to K_0. For an observer at rest at K_0, in the classical approach, there is only the scalar potential, which will correspond to the Newtonian potential

$$\phi = -G\frac{m_g}{r_0} \tag{10}$$

where r_0 is the distance from the mass m_g to the observer. Now we calculate the tetrapotential of the mass m_g for the observer *C* that is at rest in *K*, for which we express (10) in tetravectorial notation

$$\phi^k = \left(-G\frac{m_g}{r_0}, 0\right).$$

Now we make use of the relative movement. Instead of considering the movement of *K* with respect to K_0, we now consider the movement of K_0 with respect to *K*. Then, the mass m_g moves with velocity **u** with respect to P_0. For the observer *K* this is a retarded velocity, that is to say, it is the velocity that the mass m_g had when it emitted the gravitational action at the moment t' (retarded time), which reaches observer P_0 at the current time *t*. r' is the distance at which the mass m_g was at time t', that is to say, the distance at which P_0 observes the mass m_g at the time *t*: $r' = c(t - t')$ and $\mathbf{r}'$ is the position of P_0 with respect to the position retarded of m_g and therefore $-\mathbf{r}'$ is the vector of position of m_g with respect to P_0. Therefore, the coordinates of the emission and reception of the gravitational signal with respect to *K* are

$$x^k(m_g) = (ct', -\mathbf{r}'); \quad x^k(C) = (ct, 0),$$

the tetravelocity of m_g with respect to *K* is

$$u^k = \frac{dx^k(m_g)}{d\tau'} = \frac{dx^k(m_g)}{dt'\sqrt{1-u^2/c^2}} = \left(\frac{c}{\sqrt{1-u^2/c^2}}, \frac{\mathbf{u}}{\sqrt{1-u^2/c^2}}\right)$$

$d\tau'$ is the proper time of the mass m_g at the time retarded and **u** is the velocity with respect to the observer at rest in *K*, that is

$$\mathbf{u} = -\frac{d\mathbf{r}'}{dt'}.$$

We define the tetravector

$$R^k = x^k(m_g) - x^k(C) = \left[c(t'-t), -\mathbf{r}'\right] = (-r', -\mathbf{r}')$$

then the potential (10) in tetravectorial form is

$$\phi^k = Gm_g\frac{u^k}{u^iR_i} \tag{11}$$

since it is a covariant expression it is the same for all reference frames (inertial or non-inertial), therefore it is the tetrapotential in the frame *K*. From the above definitions we find

$$u^iR_i = \frac{\mathbf{r}'\cdot\mathbf{u}}{\sqrt{1-u^2/c^2}} - \frac{r'c}{\sqrt{1-u^2/c^2}} = -\frac{cs}{\sqrt{1-u^2/c^2}}$$

therefore the tetrapotential (11) is

$$\phi^k = -Gm_g\left(\frac{1}{s},\frac{\mathbf{u}}{cs}\right) \tag{12}$$

which corresponds to the following scalar and vector potentials

$$\mathbf{A} = -\frac{G}{c^2}m_g\frac{\mathbf{u}}{s};\quad \phi = -G\frac{m_g}{s}, \tag{13}$$

(13) are the potentials of Liénard-Wiechert. The equation (11) is valid for any observer at rest with respect to K. In the equations (13) $\mathbf{u}$ is the velocity with which the particle m_g moves with respect to the frame K, in which the observer is at rest. However, m is at rest with respect to the system K_0, which is an inertial reference system, which means that mass m_g does not emit gravitational radiation.

For the observer P_0 the mass m_g, that has a velocity $\mathbf{u}$, emitted a gravitational signal at the previous moment t'. Therefore, $\mathbf{r}'$ is the position vector of P_0 with respect to the retarded position of m_g.

P_0 would observe all the masses of the Universe move with the same velocity $\mathbf{u}$, since this movement is the reflection of the movement of P_0 with respect to K_0 with velocity $-\mathbf{u}$.

9 Calculation of differential operators

For finding the divergence of the potential ϕ of (13), we have to calculate the divergence of $1/s$ for what we have to use (7)

$$\nabla\frac{1}{s} = -\frac{1}{s^2}\nabla s = -\frac{1}{s^2}\left(\nabla' s - \frac{\mathbf{r}'}{cs}\frac{\partial s}{\partial t'}\right) \tag{14}$$

doing the derivatives by parts, we found

$$\begin{aligned} \nabla' s &= \nabla'\left(r' - \frac{\mathbf{u}\cdot\mathbf{r}'}{c}\right) = \frac{\mathbf{r}'}{r'} - \frac{\mathbf{u}}{c} \\ \frac{\partial s}{\partial t'} &= \frac{\partial r'}{\partial t'} - \frac{\dot{\mathbf{u}}\cdot\mathbf{r}'}{c} - \frac{\mathbf{u}}{c}\cdot\frac{\partial \mathbf{r}'}{\partial t'} = -\frac{\mathbf{u}\cdot\mathbf{r}'}{r'} - \frac{\dot{\mathbf{u}}\cdot\mathbf{r}'}{c} + \frac{u^2}{c} \end{aligned} \tag{15}$$

by replacing it in (14), we get

$$\nabla\left(\frac{1}{s}\right) = -\frac{\mathbf{r}'}{s^2 r'} + \frac{\mathbf{u}}{cs^2} - \frac{\mathbf{r}'(\mathbf{r}'\cdot\mathbf{u})}{cs^3 r'} - \frac{\mathbf{r}'(\mathbf{r}'\cdot\dot{\mathbf{u}})}{c^2 s^3} + \frac{u^2\mathbf{r}'}{c^2 s^3}, \tag{16}$$

Now we calculate the temporary derivative that appears in (13)

$$\frac{\partial}{\partial t}\left(\frac{\mathbf{u}}{s}\right),$$

for its calculation we use (3)

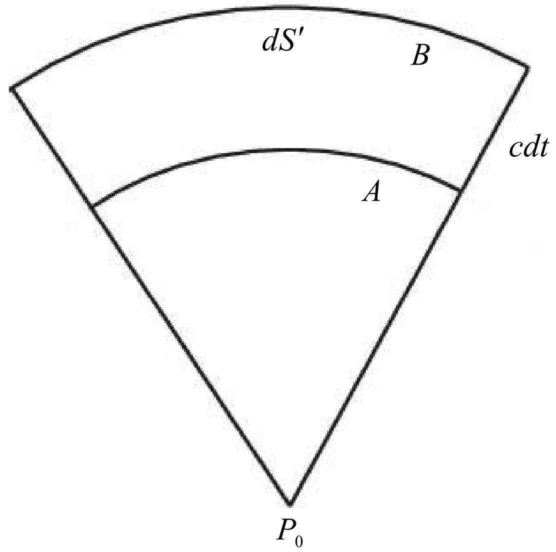
$$\frac{\partial}{\partial t}\left(\frac{\mathbf{u}}{s}\right) = \frac{r'}{s}\frac{\partial}{\partial t'}\left(\frac{\mathbf{u}}{s}\right) = \frac{r'}{s}\left(\frac{\dot{\mathbf{u}}}{s} - \frac{\mathbf{u}}{s^2}\frac{\partial s}{\partial t'}\right)$$

and by (15)

$$\frac{\partial}{\partial t}\left(\frac{\mathbf{u}}{s}\right) = \frac{r'}{s}\left[\frac{\mathbf{a}}{s} + \frac{\mathbf{u}(\mathbf{r}'\cdot\mathbf{u})}{s^2 r'} + \frac{\mathbf{u}(\mathbf{r}'\cdot\mathbf{a})}{cs^2} - \frac{\mathbf{u}u^2}{cs^2}\right]. \tag{17}$$

10 Relationship between mass and density

We will assume an elementary portion of a spherical shell (drawing 5), which has a surface area dS', a thickness cdt (where dt is an arbitrary elementary time interval), therefore it has the volume $dV' = dS'cdt$; this spherical shell belongs to a sphere whose center is at the point where the observer is P. In that elementary volume dV' there is a mass distribution that we assume is uniform. At a given moment t' all the matter contained in that volume element emits a gravitational signal in the direction of the observer. The signal of the part of the volume element dV' closest to the observer (surface A in drawing 5) will arrive at time t, while the signals leaving the furthest part



Drawing 5.- *The portion of a spherical shell where is the mass that emits at the moment t' a gravitational signal in the direction of the observer P0.*

of the volume element dV' (surface *B* of the drawing 5) will reach the observer in $t+dt$.

If the mass contained in that volume dV' were at rest, the observed mass would be

$$dm_g = [\rho]dV'$$

where $[\rho]$ is the density of gravitational mass at the retarded time t'. The situation changes if the matter that creates the field is in motion relative to the observer. Let us assume that all matter moves in the direction of the center of the sphere. Again at the instant t', a signal is emitted in the direction of the observer. The gravitational signal of the matter of dV' closest to the observer (surface *A*), will arrive at time *t*, as before. But during the time interval *dt* a portion of mass that was initially farther away than the volume element dV' will have shifted and entered that volume, and therefore, the gravitational signal that this matter that at the time t' was external to dV' and that emitted the signal at time t' reach the observer within the range between *t* and $t+dt$. Then the observer feels the gravitational effect of a mass higher than m_g, which is containing dV' initially.

The mass that has entered in dV' by the movement of matter towards the center of the sphere occupied the volume $dS'udt$ and therefore will have a mass $[\rho]dS'udt$. Then the observer will feel a mass higher than if there was no movement. If $[\rho]$ is the mass density observed, then

$$[\rho]dV' = dm_g + [\rho]dSudt \quad \Rightarrow \quad dm_g = [\rho]\left(1-\frac{u}{c}\right)$$

that is the mass that was at the beginning dm_g, plus the mass that due to the movement has entered the volume dV'. If the matter that creates the field moved in the opposite direction, that is, moving away from the center of the sphere, we would find the result

$$dm_g = [\rho]\left(1+\frac{u}{c}\right).$$

In general, if the velocity of the source **u** had whatever direction and sense, then

$$dm_g = \left(1-\frac{\mathbf{r}'\cdot\mathbf{u}}{r'c}\right)[\rho]dV' \tag{18}$$

r 'is the delayed position of the material contained in dV', where

$$\frac{\mathbf{r}'}{r'}\cdot\mathbf{u}$$

is the component of the velocity in the direction of the observer.

11 Induction force on an accelerated body

We suppose a body *C* of gravitational mass m_g that has velocity $-\mathbf{u}$ and acceleration $-\mathbf{a}$

with respect to the Universe, that is to say, with respect to the inertial reference frame K_0. The Liénard-Wiechert potentials (13) act on the body *C*, from which the gravitoelectromagnetic field strengths are calculated using the relations (9). Finally by (8) we calculate the force of induction that acts on the body *C*.

In order to see the basic concepts without technical difficulties, we consider a simplified cosmic model. We assume that the Universe is large enough, of finite age, static and with a constant and uniform density.

To make the integration of the induction force acting on *C* we divide the Universe into spherical layers of negligible thickness dr', of center in the body *C* and of radius r'. Let dm_g be the gravitational mass of a portion of a spherical shell, its position vector with respect to the frame *K* in which is at rest *C* in spherical coordinates is

$$-\mathbf{r}' = -r'\sin\theta\cos\varphi\mathbf{i} - r'\sin\theta\sin\varphi\mathbf{j} - r'\cos\theta\mathbf{k}.$$

The divergence of the potential $d\phi$ produced by dm_g is

$$\nabla d\phi = -Gdm_g\nabla\left(\frac{1}{s}\right)$$

and the time derivative of the potential vector $d\mathbf{A}$ is

$$\frac{\partial d\mathbf{A}}{\partial t} = -\frac{G}{c^2}dm_g\frac{\partial}{\partial t}\left(\frac{\mathbf{u}}{s}\right).$$

Now integrate over the whole mass of the spherical shell considered

$$\frac{\partial\delta\mathbf{A}}{\partial t} = -\frac{G}{c^2}\int\frac{\partial}{\partial t}\left(\frac{\mathbf{u}}{s}\right)dm_g; \quad \nabla\delta\phi = -G\int\nabla\left(\frac{1}{s}\right)dm_g, \tag{19}$$

the relation between mass and density is given in (18), $[\rho]$ is the density in the retarded moment, which in our cosmological model always has the same value ρ and we identify dV' with the proper volume. Of (19) we obtain in spherical coordinates

$$\begin{aligned}\frac{\partial\delta\mathbf{A}}{\partial t} &= -\frac{G}{c^2}\iint\left(1-\frac{\mathbf{r}'\cdot\mathbf{u}}{r'c}\right)\frac{\partial}{\partial t}\left(\frac{\mathbf{u}}{s}\right)\rho r'^2\sin\theta dr'd\theta d\varphi \\ \nabla\delta\phi &= -G\iint\left(1-\frac{\mathbf{r}'\cdot\mathbf{u}}{r'c}\right)\nabla\left(\frac{1}{s}\right)r'^2\sin\theta dr'd\theta d\varphi,\end{aligned} \tag{20}$$

then the integration is made on all the spherical shells and the force induced is calculated by (8).

We limit ourselves to non-relativistic velocities, because we intend is to identify the inertial force of classical mechanics with the force of gravitational induction (8). So $u \ll c$, therefore $s \approx r'$

$$\begin{aligned}\frac{\partial\delta\mathbf{A}}{\partial t} &= -\frac{G}{c^2}\iint\left(1-\frac{\mathbf{r}'\cdot\mathbf{u}}{r'c}\right)\left[\frac{\mathbf{a}}{r'}+\frac{\mathbf{u}(\mathbf{r}'\cdot\mathbf{u})}{r'^3}+\frac{\mathbf{u}(\mathbf{r}'\cdot\mathbf{a})}{cr'^2}-\frac{\mathbf{u}u^2}{cr'^2}\right]\rho r'^2\sin\theta dr'd\theta d\varphi \\ \nabla\delta\phi &= -G\iint\left(1-\frac{\mathbf{r}'\cdot\mathbf{u}}{r'c}\right)\left[-\frac{\mathbf{r}'}{r'^3}+\frac{\mathbf{u}}{cr'^2}-\frac{\mathbf{r}'(\mathbf{r}'\cdot\mathbf{u})}{cr'^4}-\frac{\mathbf{r}'(\mathbf{r}'\cdot\mathbf{a})}{c^2r'^3}+\frac{u^2\mathbf{r}'}{c^2r'^3}\right]r'^2\sin\theta dr'd\theta d\varphi,\end{aligned} \tag{21}$$

since r' has cosmic dimensions, we neglect in equation (21) the terms that depend on $1/r'^2$ versus the terms that depend on $1/r'$. With this simplification the only terms we consider they are the first and third terms of the first equation (22) and the first and fourth terms of the second equation (21)

$$\frac{\partial\delta\mathbf{A}}{\partial t} \approx -\frac{G}{c^2}\iint\left[\frac{\mathbf{a}}{r'}-\frac{\mathbf{a}(\mathbf{r}'\cdot\mathbf{u})}{cr'^2}+\frac{\mathbf{u}(\mathbf{r}'\cdot\mathbf{a})}{cr'^2}\right]\rho r'^2\sin\theta dr'd\theta d\varphi$$

$$\nabla\delta\phi \approx -G\iint\left[-\frac{\mathbf{r}'(\mathbf{r}'\cdot\mathbf{a})}{c^2r'^3}\right]r'^2\sin\theta dr'd\theta d\varphi,$$

the direct calculation gives us

$$\frac{\partial\delta\mathbf{A}}{\partial t} = -4\pi\frac{G}{c^2}\rho\mathbf{a}r'dr'; \quad \nabla\delta\phi = \frac{4\pi}{3}\frac{G}{c^2}\rho\mathbf{a}r'dr',$$

now we integrate for all the spherical shells, from the radius 0 to the radius r' that is at the distance *ct*, where *t* is the age of the Universe and finally we apply (8), resulting

$$\mathbf{F} = \frac{4\pi}{3} G\rho t^2 m_g \mathbf{a} \tag{22}$$

which is the force of induction that the whole of the Universe exerts on a particle of gravitational mass m_g that has an acceleration $-\mathbf{a}$ with respect to the whole of the Universe. The body *C* is at rest relative to the frame *K*, then the second term of (8) is zero.

We identify the force (22) with the force of inertia $-m_i(-\mathbf{a})$ so we find the relation between the inertial mass and the gravitational mass

$$m_i = \frac{4\pi}{3} G\rho t^2 m_g , \tag{23}$$

to the coefficient of proportionality between the inertial mass and the gravitational mass $\xi(t) = 4\pi/3G\rho t^2$ we call it the coefficient of inertia and it is a magnitude that depends on the cosmic model and that in general depends on the age of the Universe.

The cosmic data are

$$\rho = 9{,}9 \cdot 10^{-27}\, kg/m^3 ; \quad t_0 = 4{,}35 \cdot 10^{17}\, s$$

then the inertia coefficient for the current moment t_0 is

$$\xi(t_0) = 0{,}52 \tag{24}$$

At present the inertial mass is identical to the gravitational mass, that is $\xi(t_0) = 1$. Considering the simplified cosmological theory we have adopted and that cosmic values depend of the Universe model that is considered, the result (24) is quite encouraging and speaks very favorably of the idea that we have presented, which is identifying the force of inertia with the gravitational induction force produced by the action of all the Universe over everybody that is accelerated.

11 Proportionality of the inertial and gravitational mass

According to equation (22) when a body moves with acceleration with respect to the whole of the Universe, gravitational induction force acts on it, proportional to the acceleration of the body and in the opposite sense, exactly like the force of inertia. Therefore we must identify both forces, concluding that the force of inertia acting on a body is the inductive force of the Universe.

(22) also shows us that the force of inertia does not depend on the velocity, at least at the classical level, and is proportional to the acceleration. Note that **a** is any type of acceleration, therefore (22) also explains the centrifugal force.

By the equation (23) we find that the inertial mass is proportional to the gravitational mass, although both magnitudes are conceptually diferent. The universal gravitation constant is chosen so that the inertial and gravitational mass are equal in the current epoch, that is to say

$$G = \frac{3}{4\pi} \frac{1}{\rho t_0^2}$$

t_0 is the current age of the Universe.

The inertial mass varies as the square of the age of the Universe

$$m_i(t) = \left(\frac{t}{t_0}\right)^2 m_i(t_0). \tag{25}$$

In our simplified model this increase in inertial mass is explained because as time passes there are more masses of the Universe that are causally connected to the body that undergoes the force of induction.

Of (23) we deduce that gravity has to be an exclusively attractive force. In fact, if gravity were repulsive the scalar potential of equation (10) would be positive and the same will happen with the vector and scalar potentials of (13). From these new equations we find again (23) but with a negative sign. But then the inertial mass would be negative, contrary to the observation that inertia is opposed to the change of movement of a body.

The previous reasoning is valid even in the case in which gravitational mass was negative. By equation (22) the gravitational mass can only have one sign, either positive or negative. If the gravitational mass had two signs (like the electric charge), there would be bodies with a negative inertial mass, which is absurd. Indeed, if the active gravitational mass (which produces the force) had a sign and the passive gravitational mass (on which the force acts) had the opposite sign, then its inertial mass would be negative, which is not observed in nature. Or in other words, if there were gravitational masses of the two signs, there would be bodies with a negative inertial mass.

The relation (25) produces effects that could be detectable. Among others, we point out that the variation of the inertial mass with time will affect the emission frequency of the spectral lines, producing a shift of these lines, an effect that would overlap the shift caused by the cosmic expansion. The variation of the inertial mass will affect the orbital movements, so the rotation of distant galaxies will have a different law from the rotation of nearby galaxies. The equivalence between mass and the energy would also be affected by the variation of the inertial mass. In fact, in the equation $E = mc^2$, m is the inertial mass; therefore its variation will affect the nuclear processes, which would have an impact on stellar evolution.