Proof of the Riemann hypothesis

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Abstract

Up to now, I have tried to expand this equation and prove Riemann hypothesis with the equation of cos, sin, but the proof was impossible.

However, I realized that a simple formula before expansion can prove it.

The real value is zero only when the real part of s is 1/2. Non-trivial zeros must always have a real value of zero.

The real part of s being 1/2 is the minimum requirement for s to be a non-trivial zeros.

key words
Riemann hypothesis, non-trivial zeros, 1/2, minimum requirement

1 introduction

\[
\zeta(s) = 2^n \pi^{s-1} \sin \left( \frac{s\pi}{2} \right) \Gamma(1-s) \zeta(1-s)
\] (1)

{\[2^n \pi^{s-1} \sin \left( \frac{s\pi}{2} \right) \Gamma(1-s)\]}, \{s = 1/2 + i 14.1347\} = -0.950558 - 0.310547i
{\[2^n \pi^{s-1} \sin \left( \frac{s\pi}{2} \right) \Gamma(1-s)\]}, \{s = 1/2 + i 21.022\} = -0.904282 + 0.426936i
{\[2^n \pi^{s-1} \sin \left( \frac{s\pi}{2} \right) \Gamma(1-s)\]}, \{s = 1/2 + i 25.0109\} = -0.784761 - 0.619798i
{\[2^n \pi^{s-1} \sin \left( \frac{s\pi}{2} \right) \Gamma(1-s)\]}, \{s = 1/2 + i 30.4249\} = -0.475849 + 0.879527i
{\[2^n \pi^{s-1} \sin \left( \frac{s\pi}{2} \right) \Gamma(1-s)\]}, \{s = 1/2 + i 32.9351\} = -0.410261 - 0.911968i
{\[2^n \pi^{s-1} \sin \left( \frac{s\pi}{2} \right) \Gamma(1-s)\]}, \{s = 1/2 + i 37.58618\} = -0.832147 + 0.554555i
{\[2^n \pi^{s-1} \sin \left( \frac{s\pi}{2} \right) \Gamma(1-s)\]}, \{s = 1/2 + i 40.91872\} = -0.917431 + 0.397894i
{\[2^n \pi^{s-1} \sin \left( \frac{s\pi}{2} \right) \Gamma(1-s)\]}, \{s = 1/2 + i 43.32707\} = -0.275249 - 0.961373i
{\[2^n \pi^{s-1} \sin \left( \frac{s\pi}{2} \right) \Gamma(1-s)\]}, \{s = 1/2 + i 48.00515\} = 0.130432 + 0.991457i
{\[2^n \pi^{s-1} \sin \left( \frac{s\pi}{2} \right) \Gamma(1-s)\]}, \{s = 1/2 + i 49.77383\} = -0.579292 - 0.81512i

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\{2^{s-1}\pi^{s-1} \sin\left(\frac{\pi s}{2}\right) \Gamma(1-s)\}, \{s = 1/2 + i52.97032\} = -0.867736 - 0.497025i
\{2^{s-1}\pi^{s-1} \sin\left(\frac{\pi s}{2}\right) \Gamma(1-s)\}, \{s = 1/2 + i56.44625\} = -0.752855 + 0.658186i
\{2^{s-1}\pi^{s-1} \sin\left(\frac{\pi s}{2}\right) \Gamma(1-s)\}, \{s = 1/2 + i315.4756\} = -0.286121 - 0.958193i
\{2^{s-1}\pi^{s-1} \sin\left(\frac{\pi s}{2}\right) \Gamma(1-s)\}, \{s = 1/2 + i1393.4334\} = 0.973556 - 0.228449i
\{2^{s-1}\pi^{s-1} \sin\left(\frac{\pi s}{2}\right) \Gamma(1-s)\}, \{s = 1/2 + i74920.8275\} = -0.827399 - 0.561615i

From the above calculation, in Euler’s formula Eq.(1), \(\zeta(s)=0\) (s is non-trivial zeros) is not from \(2^{s-1}\pi^{s-1} \sin\left(\frac{\pi s}{2}\right) \Gamma(1-s)\) but \(\zeta(1-s) = 0\).

\[\zeta(s) = \zeta(1-s) = 0 \quad (2)\]

At the non-trivial zeros, \(\zeta(s) = \zeta(1-s) = 0\) holds. in this case. Eq.(10)=0, Eq(11)=0, and \(\eta(s) = \eta(1-s) = 0\) holds.

\[\eta(1-s) = (1 - \frac{2}{21-s})\zeta(1-s) = \frac{2^{1-s} - 2}{21-s} \zeta(1-s) = \frac{2 - 2^{s+1}}{2} \zeta(1-s) = (1 - 2^s)\zeta(1-s) \quad (3)\]

\[\eta(s) + \frac{2}{2^s} \zeta(s) = \eta(1-s) + \frac{2}{21-s} \zeta(1-s) \quad (4)\]

\[\eta(s) = \frac{2^s - 2}{2^s} \frac{2^{1-s}}{21-s} \eta(1-s) = \frac{2^s - 2}{2^s} \zeta(1-s) = (1 - \frac{2}{21-s})\zeta(1-s) = 0 \quad (5)\]

\{\frac{2^{s-2}}{2^s} \zeta(1-s)\}, \{s = 1/2 + i25.0109\} = 0.0000600703 - 0.0000774542i
\{\frac{2^{s-2}}{2^s} \zeta(1-s)\}, \{s = 1/2 + i30.4249\} = 2.30973 \times 10^{-7} - 0.0000678699i
\{\frac{2^{s-2}}{2^s} \zeta(1-s)\}, \{s = 1/2 + i32.9351\} = -9.25931 \times 10^{-6} - 0.000117068i
\{\frac{2^{s-2}}{2^s} \zeta(1-s)\}, \{s = 1/2 + i37.5862\} = -0.0000437932 - 0.0000195875i
\{\frac{2^{s-2}}{2^s} \zeta(1-s)\}, \{s = 1/2 + i40.9187\} = -0.0000173311 + 0.000066198i
\{\frac{2^{s-2}}{2^s} \zeta(1-s)\}, \{s = 1/2 + i74919.0752\} = -0.0000827382 - 0.0000177009i
\{\frac{2^{s-2}}{2^s} \zeta(1-s)\}, \{s = 1/2 + i74920.8275\} = -0.0000166426 + 8.31396 \times 10^{-6}i

From the above calculation, \(\eta(s)=0\) (s is non-trivial zeros).

\[\eta(1-s) = \frac{2^{1-s} - 2}{2^{1-s} - 2} \eta(s) = \frac{2^{1-s} - 2}{2^{1-s} - 2} \zeta(s) = (1 - \frac{2}{21-s})\zeta(1-s) = 0 \quad (6)\]

\[\zeta(s) = \frac{2^s}{2^s - 2} \eta(s) = \left(\frac{2^s - 2 + 2}{2^s - 2}\right) \eta(s) = \left(\frac{2^s}{2^s - 2}\right) \eta(s) \quad (7)\]

\[= \left(2^s - 2 + 2\right) \eta(s) = \eta(s) + \frac{2}{2^s} \zeta(s) = \eta(s) + \frac{2}{2^s} \left[\eta(s) + \frac{2}{2^s} \zeta(s)\right] \quad (8)\]

\[= \eta(s) + \frac{2}{2^s} \left[\eta(s) + \frac{2}{2^s} \eta(s) + \frac{2}{2^s} \zeta(s)\right] = \eta(s) + \frac{2}{2^s} \eta(s) + \left(\frac{2}{2^s}\right)^2 \eta(s) + \left(\frac{2}{2^s}\right)^3 \zeta(s) \quad (9)\]
\[ \eta(s)[1 + \frac{2}{2^s} + \left(\frac{2}{2^s}\right)^2] + \left(\frac{2}{2^s}\right)^3 \zeta(s) \neq \eta(s)\left[\frac{1 - \left(\frac{2}{2^s}\right)^k}{1 - \frac{2}{2^s}}\right] + \left(\frac{2}{2^s}\right)^{k+1} \zeta(s) \] (10)

when \( k=2 \) (If the formula is a geometric sequence and is the same up to the \( k \)-th term, \( = \) holds.)

\[ \neq \eta(s)\left[\frac{1 - \left(\frac{2}{2^s}\right)^2}{1 - \frac{2}{2^s}}\right] + \left(\frac{2}{2^s}\right)^3 \zeta(s) = (1 - \frac{2}{2^s}) \zeta(s)\left[\frac{1 - \left(\frac{2}{2^s}\right)^2}{1 - \frac{2}{2^s}}\right] + \left(\frac{2}{2^s}\right)^3 \zeta(s) \] (11)

\[ \zeta(s)[1 - \left(\frac{2}{2^s}\right)^2 + \left(\frac{2}{2^s}\right)^3] \] (12)

\[ \zeta(1-s) = \frac{2^{1-s}}{2^{1-s} - 2} \eta(1-s) = (\frac{2^{1-s} - 2 + 2}{2^{1-s} - 2}) \eta(1-s) = (1 + \frac{2}{2^{1-s} - 2}) \eta(1-s) \] (13)

\[ = (1 + \frac{2}{2^{1-s}} \frac{2^{1-s}}{2^{1-s} - 2}) \eta(1-s) = \eta(1-s) + \frac{2}{2^{1-s}} \zeta(1-s) \] (14)

\[ = \eta(1-s) + \frac{2}{2^{1-s}} [\eta(1-s) + \frac{2}{2^{1-s}} \zeta(1-s)] = \eta(1-s) + \frac{2}{2^{1-s}} \eta(1-s) + \left(\frac{2}{2^{1-s}}\right)^2 \zeta(1-s) \] (15)

\[ = \eta(1-s) + \frac{2}{2^{1-s}} \eta(1-s) + \left(\frac{2}{2^{1-s}}\right)^2 \eta(1-s) + \left(\frac{2}{2^{1-s}}\right)^3 \zeta(1-s) \] (16)

\[ = \eta(1-s) + \frac{2}{2^{1-s}} \eta(1-s) + \left(\frac{2}{2^{1-s}}\right)^2 \eta(1-s) + \left(\frac{2}{2^{1-s}}\right)^3 \zeta(1-s) \] (17)

when \( \frac{2}{2^{1-s}} = 2^s \)

\[ = \eta(1-s) + 2^s \eta(1-s) + (2^s)^2 \eta(1-s) + (2^s)^3 \zeta(1-s) \] (18)

\[ = \eta(1-s)\left[1 + 2^s + (2^s)^2\right] + (2^s)^3 \zeta(1-s) \] (19)

\[ \neq \eta(1-s)\left[\frac{1 - (2^s)^k}{1 - 2^s}\right] + (2^s)^{k+1} \zeta(1-s) \] (20)

when \( k=2 \)

\[ \neq \eta(1-s)\left[\frac{1 - 2^{2s}}{1 - 2^s}\right] + 2^{3s} \zeta(1-s) \] (21)
\[
= \zeta(1-s)(1-2^s)[1-\frac{2^{2s}}{1-2^s}]+2^{3s}\zeta(1-s) 
\] (22)

\[
= \zeta(1-s)[1-2^{2s}]+2^{3s}\zeta(1-s) 
\] (23)

\[
= \zeta(1-s)[1-2^{2s}+2^{3s}] 
\] (24)

from Eq.(12) and Eq.(24)

\[
\zeta(s)[1-(\frac{2}{2s})^2+(\frac{2}{2s})^3]=\zeta(1-s)[1-2^{2s}+2^{3s}] 
\] (25)

2 Discussion

Define \(0 < \Re(s) < 1\)

from Eq.(25)

\[
\{\zeta(s)[1-\frac{2}{2s}+(\frac{2}{2s})^3]-\zeta(1-s)[1-2^s+2^{2s}]\}, \{s = 1/2 + i14.1347\} = -0.000160889i
\]

\[
\{\zeta(s)[1-\frac{2}{2s}+(\frac{2}{2s})^3]-\zeta(1-s)[1-2^s+2^{2s}]\}, \{s = 1/2 + i15.1347\} = -0.280343i
\]

\[
\{\zeta(s)[1-\frac{2}{2s}+(\frac{2}{2s})^3]-\zeta(1-s)[1-2^s+2^{2s}]\}, \{s = 1/2 + i16.1347\} = -4.17572i
\]

\[
\{\zeta(s)[1-\frac{2}{2s}+(\frac{2}{2s})^3]-\zeta(1-s)[1-2^s+2^{2s}]\}, \{s = 1/2 - i16.1347\} = 4.17572i
\]

\[
\{\zeta(s)[1-\frac{2}{2s}+(\frac{2}{2s})^3]-\zeta(1-s)[1-2^s+2^{2s}]\}, \{s = 1/2 + i17.1347\} = 4.82094i
\]

\[
\{\zeta(s)[1-\frac{2}{2s}+(\frac{2}{2s})^3]-\zeta(1-s)[1-2^s+2^{2s}]\}, \{s = 1/2 - i17.1347\} = -4.82094i
\]

\[
\{\zeta(s)[1-\frac{2}{2s}+(\frac{2}{2s})^3]-\zeta(1-s)[1-2^s+2^{2s}]\}, \{s = 1/2 + i21.022\} = -0.0000820167i
\]

\[
\{\zeta(s)[1-\frac{2}{2s}+(\frac{2}{2s})^3]-\zeta(1-s)[1-2^s+2^{2s}]\}, \{s = 1/2 - i21.022\} = 0.0000820167i
\]

\[
\{\zeta(s)[1-\frac{2}{2s}+(\frac{2}{2s})^3]-\zeta(1-s)[1-2^s+2^{2s}]\}, \{s = 1/2 - i74879.422\} = 0.00056128i
\]

\[
\{\zeta(s)[1-\frac{2}{2s}+(\frac{2}{2s})^3]-\zeta(1-s)[1-2^s+2^{2s}]\}, \{s = 1/2 - i74879.8804\} = -0.00111728i
\]

\[
\{\zeta(s)[1-\frac{2}{2s}+(\frac{2}{2s})^3]-\zeta(1-s)[1-2^s+2^{2s}]\}, \{s = 1/2 - i74891.93\} = 0.000554776i
\]

\[
\{\zeta(s)[1-\frac{2}{2s}+(\frac{2}{2s})^3]-\zeta(1-s)[1-2^s+2^{2s}]\}, \{s = 1/2 + i74892.5452\} = -0.000641199i
\]

\[
\{\zeta(s)[1-\frac{2}{2s}+(\frac{2}{2s})^3]-\zeta(1-s)[1-2^s+2^{2s}]\}, \{s = 1/2 + i74895.7013\} = 0.00117245i
\]

\[
\{\zeta(s)[1-\frac{2}{2s}+(\frac{2}{2s})^3]-\zeta(1-s)[1-2^s+2^{2s}]\}, \{s = 1/2 + i74896.2133\} = 0.000808722i
\]

\[
\{\zeta(s)[1-\frac{2}{2s}+(\frac{2}{2s})^3]-\zeta(1-s)[1-2^s+2^{2s}]\}, \{s = 1/2 + i74896.6987\} = -0.00106666i
\]

\[
\{\zeta(s)[1-\frac{2}{2s}+(\frac{2}{2s})^3]-\zeta(1-s)[1-2^s+2^{2s}]\}, \{s = 1/2 + i74897.0517\} = 0.000224195i
\]

\[
\{\zeta(s)[1-\frac{2}{2s}+(\frac{2}{2s})^3]-\zeta(1-s)[1-2^s+2^{2s}]\}, \{s = 1/2 + i74898.1134\} = -0.000935263i
\]

\[
\{\zeta(s)[1-\frac{2}{2s}+(\frac{2}{2s})^3]-\zeta(1-s)[1-2^s+2^{2s}]\}, \{s = 1/2 + i74898.9041\} = 0.000102353i
\]

\[
\{\zeta(s)[1-\frac{2}{2s}+(\frac{2}{2s})^3]-\zeta(1-s)[1-2^s+2^{2s}]\}, \{s = 0.49999+i74911.8951\} = 0.000232008+0.000914211i
\]

\[
\{\zeta(s)[1-\frac{2}{2s}+(\frac{2}{2s})^3]-\zeta(1-s)[1-2^s+2^{2s}]\}, \{s = 1/2 + i74911.9\} = 2.7105110^{-20}+0.000914218i
\]

\[
\zeta(s)[1-\frac{2}{2s}+(\frac{2}{2s})^3]-\zeta(1-s)[1-2^s+2^{2s}], \{s = 0.51 + i74911.8951\} = -0.232499 - 0.00585949i
\]

\[
\{\zeta(s)[1-\frac{2}{2s}+(\frac{2}{2s})^3]-\zeta(1-s)[1-2^s+2^{2s}], \{s = 1/2 - i74912.4918\} = -0.0000277175i
\]

\[
\{\zeta(s)[1-\frac{2}{2s}+(\frac{2}{2s})^3]-\zeta(1-s)[1-2^s+2^{2s}], \{s = 1/2 + i74916.2765\} = 0.000952275i
\]
\{\zeta(s)[1 - \frac{1}{2^s} + (\frac{1}{2^s})^2] - \zeta(1 - s)[1 - 2^s + 2^{2s}]), \{s = 1/2 + i74918.7} = 0.000152616i \\
\{\zeta(s)[1 - \frac{1}{2^s} + (\frac{1}{2^s})^2] - \zeta(1 - s)[1 - 2^s + 2^{2s}]), \{s = 1/2 + i74919.1} = -8.4703310^{-22} + 0.000171143i \\
\{\zeta(s)[1 - \frac{1}{2^s} + (\frac{1}{2^s})^2] - \zeta(1 - s)[1 - 2^s + 2^{2s}]), \{s = 1/2 + i74920.2598} = 0.00360484i \\
\{\zeta(s)[1 - \frac{1}{2^s} + (\frac{1}{2^s})^2] - \zeta(1 - s)[1 - 2^s + 2^{2s}]), \{s = 1/2 + i74920.8275} = 0.0000329616i \\
\{\zeta(s)[1 - \frac{1}{2^s} + (\frac{1}{2^s})^2] - \zeta(1 - s)[1 - 2^s + 2^{2s}]), \{s = 1/2 + i99999.422} = -9.48598i \\
\{\zeta(s)[1 - \frac{1}{2^s} + (\frac{1}{2^s})^2] - \zeta(1 - s)[1 - 2^s + 2^{2s}]), \{s = 1/2 + i999999.422} = -0.270142i \\
\{\zeta(s)[1 - \frac{1}{2^s} + (\frac{1}{2^s})^2] - \zeta(1 - s)[1 - 2^s + 2^{2s}]), \{s = 1/2 + i9999999.422} = -6.86408i \\
\{\zeta(s)[1 - \frac{1}{2^s} + (\frac{1}{2^s})^2] - \zeta(1 - s)[1 - 2^s + 2^{2s}]), \{s = 1/2 + i99999999.422} = 0.0172762i \\
\{\zeta(s)[1 - \frac{1}{2^s} + (\frac{1}{2^s})^2] - \zeta(1 - s)[1 - 2^s + 2^{2s}]), \{s = 1/2 + i999999999.422} = -0.0048036i \\
\{\zeta(s)[1 - \frac{1}{2^s} + (\frac{1}{2^s})^2] - \zeta(1 - s)[1 - 2^s + 2^{2s}]), \{s = 1/2 + i9999999999.422} = no result \\

As in these examples, when the real part of s is 1/2, the real value is 0 or almost 0, but the imaginary value remains. Even if s is a non-trivial zero, the imaginary value is close to 0 but not 0.

However, even if the real part of s is 1/2, a real value close to 0 but not 0 is frequently generated.

If the real value of s is 1/2, the output real value is 0 or a value very close to 0 even if the imaginary value is other than the non-trivial zero value (However, in this case, the output imaginary value is far from 0).

That is, the minimum requirement for the non-trivial zeros is that the real part of s is 1/2.

That is, the lowest condition in which a non-trivial zero exists is a real part value of 1/2.

That is, a non-trivial zero can have a real part only 1/2.

\[ \Re(s) = \frac{1}{2} \quad (26) \]

Proof complete.

3 Postscript

These calculations were performed with WolframAlpha.

References

I was finally crazy because of the curse of Riemann.

Please raise the prize money to my little son and daughter who are still young.