Refutation of basic logic (bL)

Abstract: Using classical logic we establish the definition of the new connective tensor \( \otimes \) as a theorem. (This is not supposed to be possible from within classical logic as a subset of the conjecture.) Six rules for formation and reflection are not tautologous. This refutes Basic Logic (bL), forming a non tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with tautology as the designated proof value, \( \top \) as contradiction, \( \bot \) as truthity (non-contingency), and \( \bot \) as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)


Abstract  Sambin et al (2000) introduced Basic Logic as an uniform framework for various logics. At the same time, they also introduced the principle of reflection as a criterion for being a connective of Basic Logic. We make explicit a relationship between Hacking’s deducibility of identicals condition (Hacking, 1979) and the principle of reflection by proving the equivalence between them.

Remark 0.0: For Basic Logic (bL) to be a uniform framework for various logics, that is a universal logic for all logics, bL must be an inexact, probabilistic vector space and hence the derivation of a new connective for the principle of reflection. Hacking’s logic is decidedly a two-valued, bivalent logic to enforce the deducibility of identicals (DoI) condition. Hence the conjecture that bL and DoI are equivalent cannot be confirmed on its face. However, we focus here instead on the tensor connective used in bL.

2.2 Principle of reflection … A definitional equation of the tensor connective \([ \otimes ]\) is as follows;

\[ (z@z) \top \text{ as tautology, } \top, \text{ ordinal } 3; \quad (z@z) \bot \text{ as contradiction, } \bot, \text{ Null, } \bot, \text{ zero}; \]

\[ (%z@#z) \top \text{ as non-contingency, } \top, \text{ ordinal } 1; \quad (%z@#z) \bot \text{ as contingency, } \bot, \text{ ordinal } 2; \]

\[ \sim(y < x) \quad (x \leq y), \quad (x \in y), \quad (x \subseteq y); \quad (A=B) \quad (A\sim B). \]

Note for clarity, we usually distribute quantifiers onto each designated variable.

Remark 2.2.1.1: For Eq. 2.2.1.1 to be a theorem, the connectives “@” in the consequent and “\( \otimes \)” in the antecedent must be equivalent, to mean “@” is equivalent to “\( \otimes \)”. However, that flies in the face of defining a new connective, so “@” must mean not equivalent to & or “@&”. Note: this is not the same as the different “\( \otimes & \)” for not &, as “\( \otimes \)”. We build truth table values for “@&” as classical connectives excluding @& and ~@~&.
[p@&q] := (((p+q)&(p-q))&(p<q)&(p<q)) ;  
FFFF FFFF FFFF FFFF  \hspace{1cm} (2.2.1.2.2)

Therefore in Eq. 2.2.1.1 we substitute the truth value of 2.2.1.2.2 for the expression “A⊗B”, such as 2.2.1.2.2 directly:

For all Δ, [Eq. 2.2.1.2.2] Δ if and only if A, B Δ.  \hspace{1cm} (2.2.1.3.1)

LET p, q, r: A, B, Δ.

(#r>(p&q))>(#r>((p+q)&(p-q))(&(p<q)&(p<q))) ;  
TTTT TTTT TTTT TTTT  \hspace{1cm} (2.2.1.3.2)

Remark 2.2.1.3.2: Because Eq. 2.2.1.3.2 is a theorem for the tensor definition in 2.2.1.1.1, the derived formulas in the text invoking ⊗ can be replaced with the truth value of F from 2.2.1.2.2. This value is replaceable by the shortened expression (s@s). We evaluate the formation and reflection rules which follow, ignoring the universal quantifier on Δ, as ignored in the text from 2.2.1.1.1 above.

A, BΔ \hspace{0.5cm} ⊗-formation  \hspace{1cm} (2.2.2.1)

A⊗BΔ

(Δ>(p&q))>(r>(s@s)) ;  
TTTT TTTF TTTT TTTF  \hspace{1cm} (2.2.2.2)

ΓA&B \hspace{0.5cm} implicit &-reflection 1  \hspace{1cm} (2.2.7.1)

ΓA

LET p, q, r, s: A, B, Γ, Γ′.

((p&q)>r)>(r>p) ;  
TFFF TTTT TFFF TTTT  \hspace{1cm} (2.2.7.2)

ΓA&B \hspace{0.5cm} implicit ⊗-reflection 2  \hspace{1cm} (2.2.8.1)

ΓB

((p&q)>r)>(q>r) ;  
TTFT TTTT TTFT TTTT  \hspace{1cm} (2.2.8.2)

AΔ \hspace{0.5cm} explicit &-reflection 1  \hspace{1cm} (2.2.9.1)

A&BΔ

LET p, q, r: A, B, Δ.

(r>p)>(r>(p&q)) ;  
TTTT TFFF TTTT TFFF  \hspace{1cm} (2.2.9.2)

BΔ \hspace{0.5cm} explicit &-reflection 2  \hspace{1cm} (2.2.10.1)

A&BΔ

(r>q)>(r>(p&q)) ;  
TTTT TTFT TTTT TTFT  \hspace{1cm} (2.2.10.2)

Once we establish the definition of the new connective tensor ⊗ as a theorem, which is not supposed to be possible with classical logic as a subset of Basic Logic (bL), six rules for formation and reflection are not tautologous, hence refuting it.