Shorter refutation of Craig’s interpolation and derived conjectures

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Abstract: We evaluate the formula for Craig’s interpolation as not tautologous. This means Craig’s interpolation is refuted, along with these eight conjectures in the following cascade: proving Beth definability; verifying the division of larger problems into smaller problems; representing knowledge by suppression of information; proving intuitionistic and bi-intuitionistic logics, converse modality, tense logic, and extensions with path axioms; and; and an orthogonality condition for defining duality between interpolates. These results form a non tautologous fragment of the universal logic $V_L^4$.

We assume the method and apparatus of Meth8/$V_L^4$ with $T$ autology as the designated proof value, $F$ as contradiction, $N$ as truthity (non-contingency), and $C$ as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET $\sim$ Not, $\neg$; $\lor$ Or, $\vee$, $\cup$; $\neg\lor$ Not Or; $\land$ And, $\land$, $\cap$, $\cap$; $\neg\land$ Not And;
$\rightarrow$ Imply, greater than, $\Rightarrow$, $\Rightarrow$, $\Rightarrow$, $\Rightarrow$; $\leftarrow$ Not Imply, less than, $\in$, $\in$, $\subseteq$, $\subseteq$, $\subseteq$;
$=$ Equivalent, $\equiv$, $\equiv$, $\equiv$, $\equiv$, $\equiv$; $\neg$ Not Equivalent, $\neq$, $\neq$;
$\%$ possibility, for one or some, $\exists$, $\diamond$, $M$; $\#$ necessity, for every or all, $\forall$, $\square$, $L$;
$\left(z=z\right)$ $T$ as tautology, $\top$, ordinal 3; $\left(z@z\right)$ $F$ as contradiction, $\emptyset$, Null, $\bot$, zero;
$\left(\%z>\%z\right)$ $N$ as non-contingency, $\Delta$, ordinal 1; $\left(\%z<\%z\right)$ $C$ as contingency, $\nabla$, ordinal 2;
$\left(\neg y<x\right)$ $\left(x \leq y\right)$, $\left(x \leq y\right)$, $\left(x \leq y\right)$; $\left(A=B\right)$ $\left(A-B\right)$.
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Lyone, T.; et al. (2019). Syntactic interpolation for tense logics and bi-intuitionistic logic via nested sequents. arxiv.org/pdf/1910.05215.pdf lyon@logic.at

Abstract. We provide a direct method for proving Craig interpolation for a range of modal and intuitionistic logics, including those containing a “converse” modality. We demonstrate this method for classical tense logic, its extensions with path axioms, and for bi-intuitionistic logic. ... The proof of the interpolation theorem uses these calculi and is purely syntactic, without resorting to embeddings, semantic arguments, or interpreted connectives external to the underlying logical language. A novel feature of our proof includes an orthogonality condition for defining duality between interpolates.

Introduction. The Craig interpolation property for a logic $L$ states that

if $A \Rightarrow B \in L$, then there exists a formula $C$ in the language of $L$ such that $A \Rightarrow C \in L$ and $C \Rightarrow B \in L$, and every propositional variable appearing in $C$ appears in $A$ and $B$. \hspace{1em} (1.1)

Remark 1.0: We interpret the compound sentence of Eq. 1.0 to read with the antecedent preceding and the consequent following the last conjunction of “and”. We take the consequent to mean “the necessity of (variables in) $C$ implies the necessity of (variables in) $A$ and $B$. In other words, $C$ is quantified separately twice: in the antecedent $C$ is quantified existentially; and in the consequent $C$ is quantified universally. (The difficulty of putting Craig’s interpolation into words is not unique to the text evaluated.)

LET $p$, $q$, $r$, $s$: $A$, $B$, $C$, $L$. 

\hspace{1em}
\[(p>(q<s))>((((p>(r<s))&(r>(q<s))))>(r<s)))&(r<#(p&q)) \]

\[(1.2)\]

[The text continues as] This property has many useful applications: it can be used to prove Beth definability ..; in computer-aided verification it can be used to split a large problem involving \(A \Rightarrow B\) into smaller problems involving \(A \Rightarrow C\) and \(C \Rightarrow B\) ..; and in knowledge representation (uniform) interpolation can be used to conceal or forget irrelevant or confidential information in ontology querying .. . Therefore, demonstrating that a logic possesses the Craig interpolation property is of practical value.

Eq. 1.2 as rendered is not tautologous. This means Craig’s interpolation is refuted, along with the nine conjectures in the following cascade: proving Beth definability; verifying the division of larger problems into smaller problems; representing knowledge by suppression of information; proving intuitionistic and bi-intuitionistic logics, converse modality, tense logic, and extensions with path axioms; and an orthogonality condition for defining duality between interpolates.