Review on Electro-gravity via Goemetric Chronon Field

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Abstract: In 1982, Dr. Sam Vaknin pondered the idea of reconstructing physics based on time as a field. His idea appeared in his doctorate dissertation as an amendment to the Dirac spinor equation. Sam saw the Quantum Field Theory particles and momentum and energy as a result of the language of physics and of the way the human mind perceives reality and not as reality. To the author's opinion, it is a revolution of the language itself and is not a new interpretation of the existing language. The Special Theory of Relativity was a revolution and so was the General Theory of Relativity but yet these theories did not challenge the use of momentum and energy but rather gave them new relativistic interpretation. Later on, Quantum Mechanics used Energy and Momentum operators and even Dirac's orthogonal matrices are multiplied by such operators. Quantum Field Theory assumes the existence of particles which are very intuitive and agree with the human visual system. Particles may be merely a human interpretation of events that occur in the human sensory world. This paper elaborates on one specific interpretation of Sam Vaknin's idea that the author has developed from 2003 up to August 2018. It is a major improvement of previously published papers and it summarizes all of them and includes all the appendices along with new ideas.

Keywords: *Time; General Relativity; Foliations; Reeb class; Godbillon-Vey class; Symplectic Geometry; Muon; Higgs; Fine Structure Constant*

Introduction

This paper's approach to the description of matte is geometrical rather than algebraic. In that aspect, it is more loyal to General Relativity than to Super Algebraic approaches [1] that extend the dimensionality of space-time based on algebras such as Grassman's Algebra. In that aspect, it presents a much simpler view of what matter is.

This paper summarizes two previous papers [2], [3] and contains new results. For background, the reader is referred to the work of Zvi Scarr and Yaakov Friedman [4] as a recommended preliminary material, though uniform covariant acceleration is not identical to a model of force fields as curvature of trajectories of different types of clocks and of gravity, as a response of space-time - by curving itself as an error correction mechanism.

On a Big Bang space-time manifold, it is reasonable to define a Morse function [5] which is a submersion [6] from space-time to the real numbers. The idea is that every event can be connected by an integral curve to an S3 manifold of an infinitely small radius on which clocks can be synchronized and then from all such curves, the Morse function is defined as the length of the maximal proper time curve or curves from the sub-manifold to the event. A global Morse function as time may not be measured along a single path and if it represents measurement by material clocks, may not be always geodesic because material clocks interact. This approach, however, in some geometries such as the big bang metric offers global time though no global time coordinate and this idea is not new [7]. Cosmologists will argue that such a definition is problematic because there is a difficulty in defining a limit backwards in time to a sub-manifold on which clocks can be synchronized. To those readers, the author says that also on De-Sitter on open slicing and on Anti De Sitter manifolds, such a Morse function can be defined, although it may not be unique and as we shall see, uniqueness is indeed not required. The definition of a Morse function in space-time, does not even require synchronization of clocks on a sub-manifold. All it requires is to solve some minimum action integral with several symmetries. The longest proper time curve from each event to a three dimensional sub-manifold is valid as long as the resulting submersion function is a Morse function, i.e. its singularities are non - degenerate. Such a sub-manifold can be represented as a leaf of a foliation [8] of space-time where the Morse function is zero. The classification of non degenerate singularities can be found in the Morse lemma [9]. How can we describe time as a Morse function in order to account for matter ? What is matter ? We try to reconstruct physics bottom-up from its very foundations. We borrow from the old language of physics the idea of time and Minkowsky space-time manifold, though it may be arguable that even space can be deduced from time. The 1982 argument of Dr. Sam Vaknin was that as observers, we can imagine being out of space but that even language requires time and therefore, time should account for matter and particles [10] as there is no physical phenomenon out of time.

Matter is characterized by force fields. It is a phenomenological approach. In geometric terminology, forces bend trajectories of clocks because a 4-acceleration of a physical clock is perpendicular to it's 4-velocity. We can say that forces prohibit geodesic motion of material clocks in space-time. Contrary to forces, according to the General Theory of Relativity, motion in a gravitational field is along geodesic curves but in curved space-time. Our objective will be to reach an equation that combines these two types of motion, non-geodetic and geodetic in curved space-time. The non – geodetic motion is not anticipated by the metric alone, which means that we will need an additional structure beside the metric tensor, when we borrow the language of the General Theory of Relativity. This paper is not mathematically difficult to understand but it does offer new perception challenges. Following are important points that the author asks the reader to pay attention to. These are very important points:

1) A foliation which is defined by sub-manifolds perpendicular to a vector field is properly defined as covariant because the property of perpendicular vectors does not depend on choice of coordinates. Foliations can be defined in a non-covariant way but this is not the case in this paper.

2) Time-like curves that describe a physical observable, may not be unique. Such a representation is inherently prone to symmetries.

3) By discussing a "field of time" to account for matter, the idea is to capture non –geodesic motion which is not anticipated by the metric alone and to explain the origin of forces not by the traditional quantum particles exchange or by the classical potentials or vector potentials but by a field that causes non geodesic motion of material clocks that measures time. In another interpretation of such a field, we can say that the extra geometrical information that is needed to represent forces, and thus matter, is stored in the geometry of certain foliations of space time, though due to Lagrangian symmetry, these foliations and the field are not unique.

Throughout this paper x^{μ} denotes the contravariant coordinate system with index μ. The comma denotes ordinary derivative, $\mathrm{P}_{,\mu} \equiv \frac{\mathrm{dP}}{\mathrm{dx}^{\mu}}$, which will often be abbreviated, $\mathrm{P}_{\mu} \equiv \mathrm{P}_{,\mu}$. Likewise, semicolon denotes the ordinary covariant derivative that uses the Christoffel symbols $\mathrm{P}_{\mu;\nu} \equiv \frac{\mathrm{dP}_{\mu}}{\mathrm{dx}^{\nu}} - \mathrm{P}_{\mathrm{k}}\Gamma^{\mathrm{k}}_{\mu\nu}$. $g_{\mu\nu}$ is the metric tensor. g is its determinant and $\sqrt{-g}$ is the volume coefficient in integration and the volume element is $\sqrt{-g}d\Omega$. $d\Omega$ can be also written as $d\Omega = \mathrm{dx}^0\mathrm{dx}^1\mathrm{dx}^2\mathrm{dx}^3$ in Cartesian coordinates.

The Reeb field and its electromagnetic interpretation

Describing a trajectory of a clock, we can write $\frac{dV^{\mu}}{d\tau} = a^{\mu}$ and $a^{\mu}\frac{dV^{\mu}}{d\tau} = 0$. The latter is a result of

$$V^{\mu}V_{\mu} = c^2 \Rightarrow 2V_{\mu}\frac{dV^{\mu}}{d\tau} = 2V_{\mu}a^{\mu} = \frac{dc^2}{d\tau} = 0 \tag{1}$$

where c is the speed of light, V^{μ} is velocity and in the Special Theory of Relativity it is simply

$$\bar{V} = \frac{(c,V_x,V_y,V_z)}{\sqrt{1-V^2/c^2}} \tag{2}$$

In coordinate system (t, x, y, z). τ measures proper time.

We can normalize the velocity 4-vector and get $\frac{\bar{V}}{c} = \frac{(1,\frac{V_x}{c},\frac{V_y}{c},\frac{V_z}{c})}{\sqrt{1-V^2/c^2}}$. A scaled acceleration $\frac{a^{\mu}}{c^2}$ measures the acceleration of this unit vector $\frac{\bar{V}}{c}$ in relation to an arc length $c\tau$ so $dc\tau = cd\tau$ and we can write

$$\frac{d\frac{V^{\mu}}{c}}{cd\tau} = \frac{a^{\mu}}{c^2} \tag{3}$$

Denoting our Morse function P, can we derive a vector which will be equivalent to an acceleration of a clock that moves along the integral curves, generated by $P_{\mu} \equiv \frac{dP}{dx^{\mu}}$? In this case, $P_{\mu}P^{\mu}$ is generally not constant. For example, if it is constant and positive, we may choose a monotonically increasing analytic function of P instead of P such as $\sqrt{P}$ and still have a Morse function. To see a non - vanishing $(P_{\mu}P^{\mu})_{,\nu}$ the reader can refer to Appendix E, see term (80).

We now define the Reeb vector (Reeb, 1948, 1952 [11]) of P_μ, and we will develop the Reeb vector η as it was originally developed in 1948 in the language of De Rham Cohomology, with adjustment to be derived from the normalized vector.

$$\omega = \frac{P_\mu}{\sqrt{Z}} dx^\mu, \quad D\omega = \eta \wedge \omega \tag{4}$$

and for the sake of brevity, we write $Z \equiv P_\lambda P^\lambda$ or if P is a complex scalar field, $Z \equiv \frac{P_\lambda P^{*\lambda} + P^*_\lambda P^\lambda}{2}$ and $Z_\mu \equiv \frac{dZ}{dx^\mu}$.

Note: $\lfloor \eta \wedge D\eta \rfloor$ is the famous Godbillon - Vey cohomology equivalence class [12].

If we limit the discussion to a real scalar function P, The form η can be easily calculated as $\eta = \frac{U_\mu}{2} dx^\mu$ such that

$$U_\mu = \frac{Z_\mu}{Z} - \frac{Z_k P^k}{Z^2} P_\mu \Longrightarrow \tag{5}$$

$$\frac{d}{dx^\nu}\frac{P_\mu}{\sqrt{Z}} - \frac{d}{dx^\mu}\frac{P_\nu}{\sqrt{Z}} =$$

$$\frac{P_{\mu,\nu}}{\sqrt{Z}} - \frac{P_\mu Z_\nu}{2Z^{\frac{3}{2}}} - \frac{P_{\nu,\mu}}{\sqrt{Z}} + \frac{P_\nu Z_\mu}{2Z^{\frac{3}{2}}} =$$

$$\frac{P_\nu Z_\mu}{2Z^{\frac{3}{2}}} - \frac{P_\mu Z_\nu}{2Z^{\frac{3}{2}}} =$$

$$\frac{1}{2}\left(\frac{Z_\mu}{Z}\frac{P_\nu}{\sqrt{Z}} - \frac{Z_k P^k}{Z^2} P_\mu \frac{P_\nu}{\sqrt{Z}}\right) - \frac{1}{2}\left(\frac{Z_\nu}{Z}\frac{P_\mu}{\sqrt{Z}} - \frac{Z_k P^k}{Z^2} P_\nu \frac{P_\mu}{\sqrt{Z}}\right) = \frac{U_\mu}{2}\frac{P_\nu}{\sqrt{Z}} - \frac{U_\nu}{2}\frac{P_\mu}{\sqrt{Z}}$$

But why to use, $\frac{1}{2}U_\mu = \frac{1}{2}\left(\frac{Z_\mu}{Z} - \frac{Z_k P^k}{Z^2}\frac{P_\nu}{\sqrt{Z}}\right)$ and not simply, $\frac{Z_\mu}{Z}$? The reason is that $\frac{U_\mu}{2}\frac{P^\mu}{\sqrt{Z}} = 0$. We can therefore consider $\frac{1}{2}U_\mu$ as a substitute for the 4-acceleration $\frac{a^\mu}{c^2}$ with the very important difference from $V_\mu V^\mu = c^2$ that Z_μ does not vanish because Z is not constant.

We may now write the Lie derivative [13] of $\frac{P_i}{\sqrt{Z}}$ with respect to the vector field $\frac{P^{*m}}{\sqrt{Z}}$,

$$Lie\left(\frac{P^{*m}}{\sqrt{Z}}, \frac{P_i}{\sqrt{Z}}\right) = \frac{P^{*m}}{\sqrt{Z}}\left(\frac{P_i}{\sqrt{Z}}\right)_{,m} + \left(\frac{P^{*m}}{\sqrt{Z}}\right)_{,i}\frac{P_m}{\sqrt{Z}} \tag{6}$$

In which the second term is positive because the differentiated $\frac{P_i}{\sqrt{Z}}$ vector has a low index.
The first term becomes,

$$\frac{P^{*m}}{\sqrt{Z}}\left(\frac{P_i}{\sqrt{Z}}\right)_{,m} = \frac{P^{*m}P_{i,m}}{Z} - \frac{P^{*m}}{\sqrt{Z}}\frac{P_i Z_m}{2Z^{3/2}} = \frac{P^{*m}P_{i,m}}{Z} - \frac{P^{*m}Z_m P_i}{2Z^2} \tag{7}$$

The second term is,

$$\left(\frac{P^{*m}}{\sqrt{Z}}\right)_{,i}\frac{P_m}{\sqrt{Z}} = \frac{P^{*m}{}_{,i}P_m}{Z} - \frac{P^{*m}P_m Z_i}{2Z^2} = \frac{P^{*m}{}_{,i}P_m}{Z} - \frac{Z_i}{2Z} \tag{8}$$

We add (7) and (8) to get (6) and notice that $\frac{P^{*m}P_{i,m}}{Z} + \frac{P^{*m}{}_{,i}P_m}{Z} = \frac{P^{*m}P_{m,i}}{Z} + \frac{P^{*m}{}_{,i}P_m}{Z} = \frac{Z_i}{Z}$ from which (6) becomes

$$Lie\left(\frac{P^{*m}}{\sqrt{Z}}, \frac{P_i}{\sqrt{Z}}\right) = \frac{Z_i}{Z} - \frac{Z_i}{2Z} - \frac{P^{*m}Z_m P_i}{2Z^2} = \frac{Z_i}{2Z} - \frac{P^{*m}Z_m P_i}{2Z^2} = \frac{U_i}{2} \tag{9}$$

(9) is an interesting surprise. In (9), we also saw how we can generalize the definition of $\frac{U_i}{2}$ to the complex case.

We are very close to define a field that describes an acceleration in space-time but we have a problem. The matrix $A_{\mu\nu} = \frac{U_\mu}{2}\frac{P_\nu}{\sqrt{Z}} - \frac{U_\nu}{2}\frac{P_\mu}{\sqrt{Z}}$ is not a regular matrix. It describes rotation and scaling of the vector $\frac{P^{*\nu}}{\sqrt{Z}}$ into $\frac{U_\mu}{2}$ by the following rule,

$$A_{\mu\nu}\frac{P^{*\nu}}{\sqrt{Z}} = \left(\frac{U_\mu}{2}\frac{P_\nu}{\sqrt{Z}} - \frac{U_\nu}{2}\frac{P_\mu}{\sqrt{Z}}\right)\frac{P^{*\nu}}{\sqrt{Z}} = \frac{U_\mu}{2}\frac{P_\nu}{\sqrt{Z}}\frac{P^{*\nu}}{\sqrt{Z}} - \frac{U_\nu}{2}\frac{P^{*\nu}}{\sqrt{Z}}\frac{P_\mu}{\sqrt{Z}} = \frac{U_\mu}{2}\frac{Z}{Z} - 0\frac{P_\mu}{\sqrt{Z}} = \frac{U_\mu}{2} \tag{10}$$

This is where we clearly see why uniqueness of $\frac{P_\mu}{\sqrt{Z}}$ is not required, $A_{\mu\nu}$ describes perpendicular rotation and scaling and there are more than just the two vectors, $\frac{P_\nu}{\sqrt{Z}}$ and $\frac{U_\nu}{2}$ that can represent $A_{\mu\nu}$. The invariance of $\frac{U_\mu}{2}$ if P is replaced by a monotone smooth function of P can be found in Appendix C. Luckily, from the matrix $A_{\mu\nu} = \left(\frac{U_\mu}{2}\frac{P_\nu}{\sqrt{Z}} - \frac{U_\nu}{2}\frac{P_\mu}{\sqrt{Z}}\right)$ we can infer the transformation in the plane which is perpendicular to the vectors $\frac{U_\mu}{2}$ and $\frac{P_\nu}{\sqrt{Z}}$ up to a constant scaling factor. To achieve this goal, we need to use the Levi – Civita alternating tensor $E^{\mu\nu\alpha\beta}$ and not the Levi Civita alternating symbol [14]. If we use the Levi Civita symbol, we will get a tensor density and not a tensor. We then multiply $E^{\mu\nu\alpha\beta}A_{\alpha\beta}$ and get an anti-symmetric matrix $B^{\mu\nu} = \frac{1}{\sqrt{2}}E^{\mu\nu\alpha\beta}A_{\alpha\beta}$. It is easy to see that

$$B^{\mu\nu}B_{\mu\nu} = \frac{1}{\sqrt{2}}E^{\mu\nu\alpha\beta}A_{\alpha\beta}\frac{1}{\sqrt{2}}E_{\mu\nu mn}A^{mn} = -A_{\alpha\beta}A^{\alpha\beta} \tag{11}$$

To remind the reader, the relation between a Levi – Civita symbol $\delta^{\mu\nu\alpha\beta}$ and the Levi – Civita tensor $E^{\mu\nu\alpha\beta}$ is brought here,

$$E^{\mu\nu\alpha\beta} = \frac{\delta^{\mu\nu\alpha\beta}}{\sqrt{-g}} \qquad , E_{\mu\nu\alpha\beta} = \sqrt{-g}\,\delta_{\mu\nu\alpha\beta} \tag{12}$$

where $g = Det(g_{\mu\nu})$. Also please note that $\delta^{\mu\nu\alpha\beta}$ is an alternating symbol and therefore, if it is contacted twice with the same vector, the result is zero, $\delta^{\mu\nu\alpha\beta}V_\alpha V_\beta = 0$. We are finally able to define an accelerating field in a covariant way. Definition, an acceleration field is:

$$F_{\mu\nu} = A_{\mu\nu} + \gamma B_{\mu\nu} \tag{13}$$

Such that $\gamma \in U(1)$ and here $U(1) = \{e^{jx} | x \in \mathbb{R}\}, j = \sqrt{-1}$. The reason for this γ is that

$B_{\mu\nu} = \frac{a_\mu}{c^2}\frac{V_\nu}{\sqrt{\frac{V^{*\alpha}V_\alpha+V^\alpha V^*_\alpha}{2}}} - \frac{a_\nu}{c^2}\frac{V_\mu}{\sqrt{\frac{V^{*\alpha}V_\alpha+V^\alpha V^*_\alpha}{2}}}$ for some acceleration vector $\frac{a_\mu}{c^2}$ and we have a degree of freedom $\frac{a_\mu}{c^2}\frac{a^{*\mu}}{c^2} = \frac{\gamma a_\mu}{c^2}\frac{\gamma^* a^{*\mu}}{c^2}$. We also know that $a_\mu V^{*\mu} = 0$ and $a_\mu U^{*\mu} = 0$ and $U_\mu V^{*\mu} = 0$. The degree of freedom in the representation vectors, a_μ and V_μ together with the degree of freedom of γ is U(1) * SU(2). For a velocity $w^\nu = w^{*\nu}$ and real $F_{\mu\nu}$, we derive an acceleration,

$$F_{\mu\nu}\frac{w^\nu}{c} = \frac{a_{\mu(w)}}{c^2} \tag{14}$$

This rule appears in the Scarr – Friedman interpretation of acceleration [15] and to the author's opinion, it did not receive enough attention from the physics community. Locally, using a real numbers scalar q and $q_\mu = \frac{dq}{dx^\mu}$, $B_{\mu\nu}$ can be represented similar to $A_{\mu\nu}$, i.e. $B_{\mu\nu} = \frac{d}{dx^\nu}\frac{q_\mu}{\sqrt{q_\lambda q^\lambda}} - \frac{d}{dx^\mu}\frac{q_\nu}{\sqrt{q_\lambda q^\lambda}}$ and then $F_{\mu\nu}dx^\mu \wedge dx^\nu = D(\left(\frac{P_\mu}{\sqrt{P_\lambda P^\lambda}} + \frac{q_\mu}{\sqrt{q_\lambda q^\lambda}}\right)dx^\mu)$ and therefore $F_{\mu\nu}dx^\mu \wedge dx^\nu$ becomes a Symplectic form. By Darboux theorem [16], there is a neighborhood around an event e in space time, where $F_{\mu\nu}$ is not degenerated, such that in local coordinates $F_{\mu\nu}$ can be represented as

$$\begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \tag{15}$$

(13) is anti-symmetrical, $F_{\mu\nu} = -F_{\nu\mu}$ and is a regular matrix and a tensor. A short calculation immediately shows that

$$\frac{1}{4}\left(\frac{F_{\mu\nu}F^{*\mu\nu}+F^*_{\mu\nu}F^{\mu\nu}}{2}\right) = \frac{1}{4}\frac{U^*_\mu U^\mu+U_\mu U^{*\mu}}{2} \tag{16}$$

and in the real case

$$\frac{1}{4}F_{\mu\nu}F^{\mu\nu} = \frac{U_\mu U^\mu}{4} \tag{17}$$

(16) has the same format as the classical electromagnetic action in the General Theory of Relativity. The question that arises is, how to relate the older language of energy density to $\frac{U_\mu U^\mu}{4} = \frac{a_\mu a^\mu}{c^4}$, using the real case representation. To answer this question, the reader is referred to Appendix D, that shows that $\frac{U_\mu U^\mu}{4}$ is the squared curvature of the integral curve along $\frac{P_\mu}{\sqrt{Z}}$. That means that as an action, $\frac{U_\mu U^\mu}{4}$ does not need any constants when used in the Einstein – Grossmam action because the Ricci scalar [17] is also a curvature in units $\frac{1}{Length^2}$. Just

before we develop a new version of Einstein – Grossman equations, we can clarify the inevitable result from (17), Appendix D and [17]. In order to interpret the real numbers version $\frac{U_\mu U^\mu}{4}$ as energy density, we need to multiply it by the inverse of Einstein's gravity constant, that is,

$$\frac{c^4}{8\pi K}\frac{U_\mu U^\mu}{4} = \rho = \frac{Energy}{Volume} \tag{18}$$

But then in terms of an acceleration vector a_ν, see (3), $\frac{U_\mu U^\mu}{4} = \frac{a_\lambda a^\lambda}{c^4}$ so

$$\frac{|a|^2}{8\pi K} = \frac{a_\lambda a^\lambda}{8\pi K} = \rho = \frac{Energy}{Volume} \tag{19}$$

$|a|^2$ is the squared norm of acceleration. If we compare that energy density to the classical non-covariant limit of the electrostatic field E then we have,

$$\frac{|a|^2}{8\pi K} \cong \frac{1}{2}\varepsilon_0|E|^2 \Longrightarrow \sqrt{4\pi K\varepsilon_0}|E| \cong |a| \tag{20}$$

where K is Newton's gravity constant and ε_0 is the permittivity of vacuum. The relation between Minkosky norms and the classical non-covariant limit has another inevitable result,

$$\left|\frac{U_\mu}{2}\right| c^2 = |a_\mu| \cong \sqrt{4\pi K\varepsilon_0}|E| \Longrightarrow \sqrt{4\pi K\varepsilon_0}Div(E) \cong \sqrt{4\pi K\varepsilon_0}\frac{\rho}{\varepsilon_0} = \frac{U^\mu{}_{;\mu}}{2}c^2 \tag{21}$$

Where in (21), ρ means charge density. The divergence of the Reeb vector has a classical non-covariant limit which is proportional to the divergence of the electrostatic field and therefore to charge density. (20) means in the old language of physics, that the energy of the electric field is in a very weak acceleration field. We need to understand the interaction between positive and negative charge. By Occam's razor, this can only mean an alignment of the Reeb vector in the classical limit with the classical electric field and that the time-like component of acceleration is very small. Interacting negative and positive charge therefore, must reduce the integration of $\frac{U_\mu U^\mu}{4}$. So how big is this acceleration field and what exactly does it accelerate ? (20) gives us a dauntingly small result, ~8.61cm/sec^2, which is less than 0.01 g, if the electrostatic field is an immense 1,000,000 volts over 1mm. We will see that due to unexpected gravity by electric charge, this acceleration is even smaller, about 4.305 cm/sec^2.

We now need to develop the Euler Lagrange equations of the following action, as the minimum action problem in the General Theory of Relativity language,

$$Z = P_\mu P^\mu, Z_\mu = \frac{dZ}{dx^\mu}, U_\lambda = \frac{Z_\lambda}{Z} - \frac{Z_k P^k P_\lambda}{Z^2}, L = U^k U_k \tag{22}$$

and R is the Ricci curvature [17]. The integral to be minimized over coordinates domain Ω is

$$Action = Min\int_\Omega \left(R - \frac{1}{4}U^k U_k\right)\sqrt{-g}\, d\Omega \tag{23}$$

Locally, this can be written in Regge's Causal Dynamic Triangulation $\sigma(n+1)$ formalism too [18] , where n is the dimension.

$$Action = Min(\textstyle\sum_h \left(2\delta_h - \frac{{a_h}^2}{c^4}\right) V_h) \tag{23.1}$$

Where h is a hinge in a Causal Dynamic Triangulation, the missing angle is $\delta_h = 2\pi - \sum_{h\in\sigma(n)}\theta_h$ and $\theta_{h\in\sigma(n)}$ is the angle between two faces around h that share the hinge h and a_h is the acceleration through the two dimensional hinge h.

From Appendix A there are two minimum action equations, one by the metric $g_{\mu\nu}$ and one by the Morse function, P. The results are,

$$\frac{1}{4}(U_\mu U_\nu - \frac{1}{2}g_{\mu\nu}U_\lambda U^\lambda - 2U^k;_k \frac{P_\mu P_\nu}{Z}) = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} \tag{24}$$

And

$$W^\mu;_\mu = \left(-4U^k;_k \frac{P^\mu}{Z} - 2\frac{Z_\nu P^\nu}{Z^2}U^\mu\right);_\mu = 0 \tag{25}$$

From (25), the following divergence is zero, $\frac{1}{4}\left(U_\mu U_\nu - \frac{1}{2}g_{\mu\nu}U_\lambda U^\lambda - 2U^k;_k \frac{P_\mu P_\nu}{Z}\right);^\nu = 0$. A proof can be found in Appendix B.

Note: By (21), we immediately see a peculiar result, electric charge generates gravity in an unexpected way, by the term $\frac{1}{4}\left(-2U^k;_k \frac{P_\mu P_\nu}{Z}\right) = -\frac{1}{2}U^k;_k \frac{P_\mu P_\nu}{Z}$, which is peculiar because P_μ may not be aligned with the motion of the "source" of the Reeb vector $\frac{U_\mu}{2}$. What is the meaning of "source" ? From the theory of foliations, integration of the reduced Reeb vector [11] in leaves of foliations perpendicular to P_μ is zero along closed curves [11] and the integration of $\frac{U_\mu}{2}$ along leaf-wise curves measures the transverse holonomy expansion. In other words, the field $\frac{U_\mu}{2}$, when reduced to specific three dimensional leaves, behaves exactly as a classical electric field that has a source, as a negative or as a positive charge. (24) can be generalized for a complex P, and P can be described as a sum of a Hilbert orthogonal functions, $\psi(1), \psi(2), \ldots, \psi(n),\ n \to \infty$.

$$P = \lim_{n\to\infty} \psi(1) + \psi(2) + \cdots + \psi(n)$$

$$\int_{\Omega_4} \psi(k)\psi*(k)\sqrt{-g}d\Omega_4 = 1$$

$$0 < j < k \Rightarrow \int_{\Omega_4} \psi(j)\psi*(k)\sqrt{-g}d\Omega_4 = 0$$

$$\frac{1}{8}\left(U_\mu U*_\gamma + U*_\mu U_\gamma - \frac{1}{2}(U*^k U_k + U^k U*_k)g_{\mu\gamma} - 2(U^k;_k + U*^k;_k)\frac{(P_\mu P*_\gamma + P*_\mu P_\gamma)}{2Z}\right) - \frac{1}{2}\lambda PP^* g_{\mu\gamma} = R_{\mu\gamma} - \frac{1}{2}Rg_{\mu\gamma} \tag{26}$$

For some cosmological constant λ whose units are $\frac{1}{Length^2}$. The following constraints can only have one physical meaning, they describe events and not particles.

$$P = \lim_{n\to\infty} \psi(1) + \psi(2) + \cdots + \psi(n)$$
$$\int_{\Omega_4} \psi(k)\psi*(k)\sqrt{-g}d\Omega_4 = 1 \tag{27}$$
$$0 < j < k \Rightarrow \int_{\Omega_4} \psi(j)\psi*(k)\sqrt{-g}d\Omega_4 = 0$$

More profoundly, the meaning of (14) is of an acceleration due to non – geodesic alignment of these events. A reasonable interpretation of (13), (14) is that material clocks or as we see in (17), neutral "electromagnetic energy", even with a total charge 0, cannot move along geodesic curves if the events do not align along geodesic curves. In the model in this paper, charge interacts by increasing or decreasing the energy of the weak acceleration field which results in force due to $\frac{1}{4}\left(U_\mu U_\nu - \frac{1}{2}g_{\mu\nu}U_\lambda U^\lambda - 2U^k;_k \frac{P_\mu P_\nu}{Z}\right);^\nu = 0$. We may have already observed the small acceleration (20) as the Flyby Anomaly [19] above thunderstorms. The Flyby anomaly does not support an energy density $\frac{|a|^2}{4\pi K}$ in which charge induced gravity and charge induced acceleration would cancel out. It does support values such as (19). A field that is a sum of events, quite similar to (27) was already offered in 1982 by Sam Vaknin [10], later, a more set-theory oriented idea was offered by Sorkin in years 1987 and 2000 [20]. Sam Vaknin's work was published in modern variations by other researchers [21] around 2001 – 2002. The approach in this paper does not derive from the Dirac spinor equation [22] as Sam Vaknin's work and as [21] did.

Unexpected gravity and anti-gravity by electric charge

From (21) and (24), $\frac{U_\mu}{2} = \frac{a_\mu}{c^2}$ where $a_\mu$is a 4-acceleration representative of the field.

$$\frac{c^4}{8\pi K}\sqrt{\frac{4\pi K}{\varepsilon_0}}\frac{\rho}{c^2} = \frac{\rho c^2}{\sqrt{16K\varepsilon_0}} = -\frac{c^4}{8\pi K}\frac{U^\mu{}_{;\mu}}{2} \tag{28}$$

which generates gravity equivalently to energy density. That can only mean that

$$\frac{Q}{\sqrt{16\pi K\varepsilon_0}} \tag{29}$$

generates gravity as mass does, however, by the note after (25) the motion of the electric charge need not be aligned with P_μ. (29) yields an assessment

$$\frac{\pm 1 Coulomb}{\sqrt{16\pi K\varepsilon_0}} \cong \pm 5.8023 * 10^9 Kg \tag{30}$$

From (30) and from (20) we can approximate the total acceleration $|\tilde{a}|$ near a charge Q at radius r as,

$$|\tilde{a}| \approx \left|\frac{KQ}{\sqrt{16\pi K\varepsilon_0}r^2} - \frac{Q\sqrt{4\pi K\varepsilon_0}}{4\pi\varepsilon_0 r^2} + Other\ terms\right| \approx \left|\frac{KQ}{\sqrt{16\pi K\varepsilon_0}r^2} - \frac{Q\sqrt{K}}{\sqrt{4\pi\varepsilon_0}r^2}\right|$$

Term (30) requires an experimental evidence which does exist, see Timir Datta's research [23] and a Brazilian experiment [24]. [23] involves high concentration of separated charge close to a tip of a cone. We do not know yet, whether the positive charge or whether the negative charge act as negative energy, however, evidence of free electrons in the galactic center [25] suggests ionization of the galactic center where these electrons move out from the galactic center. We therefore expect the edges of a spiral galaxy to be more negative than its center. Equation (30) by Miguel Alcubierre [26], inevitably generates Warp Drive between the edges of the galaxy and its center, providing that a sufficiently large amounts of electric charge are separated. Since galaxies seem to be stable, the Alcubirre Warp Drive must push the galaxies towards the center. We are inevitably led to the conclusion that it is the negative charge that generates negative gravity in the peripheral mass of the galaxy, where the center of the galaxy is positively charged and generates more gravity than expected. By (30) and [26], static charge separation cannot generate a technologically feasible warp drive without dynamic charge oscillation and/or rotation because an Alcubierre warp drive requires $\pm 10^{27}$Kg $\approx \pm 2 * 10^{17}$Coulombs. The acceleration field around a charge must be opposite in sign to the gravity generated by the positive charge and to the anti-gravity generated by the negative charge. In the language of modern cosmology, electrons generate "negative pressure" and protons generate "pressure" on the neutral clocks they weakly accelerate and cause space-time to respond by gravity. Before we move on, it is worth mentioning that the action $\frac{1}{4}U^k U_k$ can be generalized to more than one Reeb vector. This idea is quite straight forward by roots of determinants of Gram matrices of Reeb vectors and is discussed at the end of Appendix F. Taking the third root of a determinant of the Gram matrix of three complex Reeb vectors, has an $SU(3)$ symmetry. There are other ways to consider $SU(3)$ symmetry, see Appendix F for the conditions of one interesting option. To summarize the idea of an acceleration field that acts on mass in a different way than gravity, if $V^\mu = \frac{v^\mu}{c}$ is a unit velocity of a clock in the hyper-plane spanned by $\frac{U^\mu}{2}$ and $\frac{P^\nu}{\sqrt{Z}}$ then the acceleration $\frac{dV^\mu}{d\tau}$ where τ is proper time, is also in that plane and the clock reference fame is accelerated by the rule,

$$\frac{dV^\mu}{d\tau} = V^\mu{}_{;k}\frac{dx^k}{d\tau} = V^\mu{}_{;k}V^k = \left(\frac{P_k U^\mu}{2\sqrt{z}} - \frac{U_k P^\mu}{2\sqrt{z}}\right)V^k$$

Then multiplying both sides by $\frac{P_\mu}{\sqrt{z}}$ we get $\frac{dV^\mu}{d\tau}\frac{P_\mu}{\sqrt{z}} = \left(\frac{P_k U^\mu}{2\sqrt{z}} - \frac{U_k P^\mu}{2\sqrt{z}}\right) V^k \frac{P_\mu}{\sqrt{z}} = -V^k \frac{U_k}{2}$.

In the same way, assuming $U_\lambda U^\lambda \neq 0$ yields $\frac{dV^\mu}{d\tau}\frac{U_\mu}{\sqrt{|U_\lambda U^\lambda|}} = \left(\frac{P_k U^\mu}{2\sqrt{z}} - \frac{U_k P^\mu}{2\sqrt{z}}\right) V^k \frac{U_\mu}{\sqrt{|U_\lambda U^\lambda|}} = V^k \frac{P_k}{\sqrt{z}}\frac{\sqrt{|U_\lambda U^\lambda|}}{2}$.

So we see that V^k was transformed into a vector with the pseudo-norm $\frac{\sqrt{|U_\lambda U^\lambda|}}{2}$ because,

$$\left\| -V^k \frac{U_k}{2}\frac{P_\mu}{\sqrt{z}} + V^k \frac{P_k}{\sqrt{z}}\frac{\sqrt{|U_\lambda U^\lambda|}}{2}\frac{U_\mu}{\sqrt{|U_\lambda U^\lambda|}} \right\|^2 = \left\| V^k \frac{U_k}{2}\frac{P_\mu}{\sqrt{z}} \right\|^2 + \left\| V^k \frac{P_k}{\sqrt{z}}\frac{U_\mu}{2} \right\|^2 = \frac{|U_\lambda U^\lambda|}{4}$$

Notice that it is possible that $\frac{U_\lambda U^\lambda}{4} < 0$.

Particle mass ratios by added or subtracted area – Muon to electron mass ratio

This section, unlike the previous ones, relies on scaled area ratios. Some of the work can be seen in the remarkable paper of Lee C. Loveridge [27] and is somewhat speculative unless (34.4) is taken into account or claims of a ~40eV neutrino [28] and of a ~0.0002 eV resonance are taken into account and therefore the reader is honestly advised to take it with a grain of salt especially as this model requires a scaling factor $\frac{3}{32}$, that can be shown justifiable due to fractional quantization in a Hole Quantum Wire and Steiner Tree optimization in Causal Dynamic Triangulation that will be discussed, however, the results are very interesting and do have a geometrical basis in [11] that worth further investigation - the Reeb vector on the foliation leaves must have drains and sources. It is therefore the authors opinion that the technique used here will be considered by the reader even if the reader does not fully agree with the idea which is presented in this section. The following development has its roots in the lectures of professor Seth Lloyd of the M.I.T [29] and in a paper by Ted Jacobson [30] combined with equation (24). A method to reduce (24) from 4 dimensional Minkowsky space to Riemannian two dimensions will be discussed along with its possible applications to mass ratios between particles and to the fine structure constant. Einstein tensor means added or subtracted area of the sphere in a ball with an infinitesimally small radius r_0 which is Minkowsky perpendicular to a unit vector, $\frac{P_\mu}{\sqrt{Z}}$, by the equation,

$$-2\left(R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}\right)\frac{P^\mu P^\nu}{Z} = R(3) = 6 \cdot (Dimension = 3) \lim_{r_0 \to 0} \frac{1 - \frac{A}{4\pi r_0{}^2}}{r_0{}^2} = \lim_{r_0 \to 0} 18 \frac{\delta A}{4\pi r_0{}^4} \Rightarrow$$

$$-\left(R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}\right)\frac{P^\mu P^\nu}{Z}\frac{4}{9}\pi r_0{}^4 \longrightarrow \delta A$$

Where $R(3)$ is the scalar curvature in three dimensions.

$\frac{4}{9}\pi\left(R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}\right)\frac{P^\mu P^\nu}{Z} r_0{}^4 = -Sphere\_Area$, so by (24),

$$\left(\frac{4}{9}\pi\right)\frac{1}{4}\left(U_\mu U_\nu - \frac{1}{2}g_{\mu\nu}U_\lambda U^\lambda - 2U^k{}_{;k}\frac{P_\mu P_\nu}{Z}\right)\frac{P^\mu P^\nu}{Z} r_0{}^4 = \frac{4}{9}\pi\left(R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}\right)\frac{P^\mu P^\nu}{Z} r_0{}^4 = -\delta A \quad (31)$$

$U_\mu P^\mu = 0$, $g_{\mu\nu}\frac{P^\mu P^\nu}{Z} = 1$, $\frac{P_\mu P_\nu}{Z}\frac{P^\mu P^\nu}{Z} = 1$ and therefore (31) yields

$$\left(\frac{4}{9}\pi\right)\left(-\frac{1}{8}U_k U^k - \frac{1}{2}U^k{}_{;k}\right) r_0{}^4 = -\delta Sphere\_Area \quad (32)$$

This is not yet the result of contracting the Einstein tensor with a unit vector twice [29], [30] that we want. The factor 2 can also be found in the outstanding work of Lee C. Loveridge [27]. We now make an assumption that in the subatomic scale, there is a relation between the acceleration a_μ and the radius r_0, which is presented by $\sqrt{\frac{|U_\mu U^\mu|}{4}} = \frac{|U_\mu|}{2} = \frac{|a_\mu|}{c^2} = \frac{\zeta}{r_0}$ where ζ depends on the field and c is the speed of light, also the divergence is calculated along the radius r_0 and we also scale r_0 by $\frac{3}{32}$ to get $\frac{3}{32}\frac{4}{9}\pi = \frac{\pi}{24}$. This scaling is not trivial and it leads to some interesting results. In terms of ordinary Riemannian geometry, the curvature of circle of radius r_0 is $\frac{1}{r_0}$. We also know that by Gauss law, if area is added around a charge, the intensity of the electric field is reduced and we expect the same rational reduction in the acceleration field. If the area grows by $x > 1$, then the field is reduced by $\frac{1}{x}$ so, $\frac{\zeta}{r_0}$ becomes $\frac{\zeta}{r_0 x}$. Consider that the divergence is calculated along a distance r_0, which means that there is a minimal distance along which the field can change from $\frac{\zeta}{r_0 x}$ to 0. So the divergence term becomes $\left(\pm\frac{\zeta}{r_0 x} - 0\right)\frac{1}{r_0} = \pm\frac{\zeta}{{r_0}^2 x}$, (32) then yields the following,

$$\left(\frac{4}{9}\pi\right)\left(-\frac{1}{8}U_k U^k - \frac{1}{2}U^k{}_{;k}\right){r_0}^4 =$$

$$\frac{3}{32}\frac{4}{9}\pi\left(-\frac{1}{2}\frac{\zeta^2}{{r_0}^2 x^2} \pm \frac{\zeta}{r_0 x}\frac{1}{r_0}\right){r_0}^4 = \frac{\pi}{24}\left(-\frac{1}{2}\frac{\zeta^2}{x^2} \pm \frac{\zeta}{x}\right){r_0}^2 = \pm\delta A \tag{33}$$

We now divide this area by the area of a sphere $4\pi {r_0}^2$.

$\frac{1}{4\pi {r_0}^2}\frac{\pi}{24}\left(-\frac{1}{2}\frac{\zeta^2}{x^2} \pm \frac{\zeta}{x}\right){r_0}^2 = \frac{1}{96}\left(-\frac{1}{2}\frac{\zeta^2}{x^2} \pm \frac{\zeta}{x}\right) = \frac{\pm Area}{4\pi {r_0}^2}$ and we also know that $x = \frac{4\pi {r_0}^2 \pm Area}{4\pi {r_0}^2}$ from which we infer,

$$1 + \frac{1}{96}\left(-\frac{1}{2}\frac{\zeta^2}{x^2} \pm \frac{\zeta}{x}\right) = x \Longrightarrow x^2 + \frac{1}{96}\left(-\frac{1}{2}\zeta^2 \pm \zeta x\right) = x^3 \tag{34}$$

Note: The real reason for the $\pm\frac{1}{96}\frac{\zeta}{x}$ is because an electromagnetic field of a charged particle can be viewed as comprised of two perpendicular components, see (13). An interaction of such a field can be said to be related to the Fine Structure constant Alpha. So the fine structure constant should be actually multiplied by the square root of two in order to relate to an interaction with two perpendicular components of unit length. The following is the outcome of such a consideration:

$$\frac{1}{Alpha\cdot\sqrt{2}} = Alpha^{-1}\cdot 2^{-\frac{1}{2}} \cong 137.035999149 * \sqrt{0.5} \cong 96.899 \tag{34.1}$$

The nearest integer less than that value is:

$$\left\lfloor \frac{1}{Alpha\cdot\sqrt{2}} \right\rfloor = 96 \tag{34.2}$$

The reader may come to the conclusion that the author first threw a dart and then drew a circle around it and it has to be admitted to be somewhat true. The value $\frac{1}{96}$ was indeed intended due to (34.2) which is also a result of the interpretation of (13) as two perpendicular acceleration fields as the reason for the energy of the electric field.

Note: The easy way to get $\frac{1}{96}$ though not intuitive either, is to divide the area loss from a disk perpendicular to P_μ which is $\frac{\pi}{12}\frac{1}{2}\left(-\frac{1}{8}U_k U^k - \frac{1}{2}U^k{}_{;k}\right){r_0}^4$ by the area of a sphere $4\pi {r_0}^2$ so we have

$$\frac{\frac{\pi}{24}\left(-\frac{1\zeta^2}{2x^2}\pm\frac{\zeta}{x}\right){r_0}^2}{4\pi {r_0}^2} = \frac{1}{96}\left(-\frac{1}{2}\frac{\zeta^2}{x^2} \pm \frac{\zeta}{x}\right) = \frac{\pm\delta Disk\ Area}{Euclidean\ Sphere\ Area} \tag{34.3}$$

Intuitively, we would expect $\frac{\pm\delta Sphere\ Area}{Euclidean\ Sphere\ Area}$ to have a physical meaning when reducing a 4 dimensional Minkowsky geometry to a 2 dimensional Riemannian geometry.

The most surprising argument for 1/96 comes from a simple geometric consideration. Due to (34) consider the following, where $\zeta = \frac{4}{\pi}$ will be discusses.

$${c_1}^2 + \frac{1}{n}\left(-\frac{1}{2}\left(\frac{4}{\pi}\right)^2 + \frac{4}{\pi}c_1\right) = {c_1}^3, {c_2}^2 + \frac{1}{n}\left(-\frac{1}{2}\left(\frac{4}{\pi}\right)^2 - \frac{4}{\pi}c_2\right) = {c_2}^3$$

Now, look for the following minimum for a natural n:

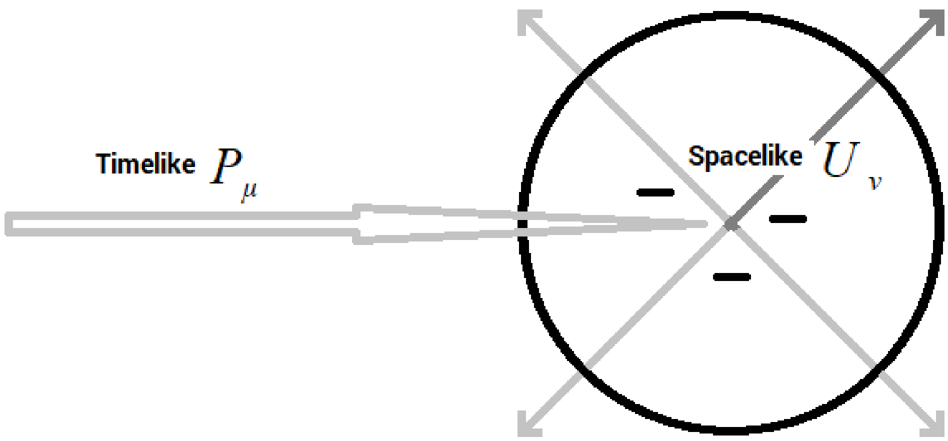
$$\min_n \left|\left((c_1-1)(1-c_2)\right)^{-\frac{1}{2}} - n - \frac{2\pi}{n}\right| \Rightarrow n = 96 \tag{34.4}$$

For n=95 we get ~0.015965593, for n=96, 0.003727368 and for n=97, 0.023393663. In plain English, n=96 minimizes the delta between the residual $\left((c_1-1)(1-c_2)\right)^{-\frac{1}{2}} - n$ and the angle $\frac{2\pi}{n}$. For n = 96 where $\left((c_1 - 1)(1-c_2)\right)^{-\frac{1}{2}} \cong 96.06917721$. These calculations where done, using Excel Datasheet and the reader may prefer Python or C,C++. By professor Ted Jacobson [30], area in the Planck scale is equivalent to energy and therefore if his claims are correct, we should be able to obtain known mass ratios between charged particles, based on area ratios, so $\frac{\pm\delta Area}{4\pi {r_0}^2}$ which is $x-1$ should represent at least one of the known mass ratios in the particles world. The problem here is that we do not have a full solution to either (24) or (26) and we honestly have to make an educated guess about possible values of ζ and get different $x-1$ values as mass ratios. We do that by using the coefficient of a normalized ring potential which is reminded in the book of Ettore Majorana [31] as $V(r) = \frac{4}{\pi}\frac{Q}{r}$. A phenomenological view of the electron as a ring can be seen in the work of O.F. Schilling [32]. Our most obvious first target is the Muon / Electron mass ratio as nominated to have a gravitational reason. Instead of accepting $\zeta = \frac{4}{\pi}$ as factual, we can also say that due to the spin of the electron, the acceleration field around the electron is not evenly distributed, to compensate for that, at any given time, ζ has to be bigger than 1, otherwise Gauss law would be violated. One simple model is of a field which is maximal at an equator of some sphere around the electron and vanishes at its poles. "Equator" means some maximal length circle in which the radius is typically perpendicular to an axis of rotation. Such a model yields the following value,

$$\frac{|a|}{c^2} = Constant \cdot \frac{1}{r_0}\cos(\emptyset) \tag{35}$$

Unlike in (35) a uniform field around a negative charge at a given local time would look as in the followings illustration that represents 3 dimensions, where the dark arrow illustrates an acceleration of a material clock in the electric field. We can also imagine a neutral particle as if having an alternating field pointing in and out of the particle rather than $2(U^k{}_{;k} + U*^k{}_{;k}) = 0$.

(Fig. 1.0) - $2(U^{k}{}_{;k} + U*^{k}{}_{;k}) \neq 0.$



where $\emptyset$ is the angle from the "equator" and $|a| = \sqrt{|a^{\mu}a_{\mu}|}$ is the absolute value of a Minkowsky norm of an acceleration a^{μ}. Field integration on two hemispheres is then,

$$\begin{gathered}2\int_{\emptyset=0}^{\emptyset=\frac{\pi}{2}} \frac{1}{r_0} cos(\emptyset) \cdot 2\pi r_0 \cdot cos(\emptyset) \cdot r_0 \cdot d\emptyset = \\ 4\pi r_0 \int_{\emptyset=0}^{\emptyset=\frac{\pi}{2}} \frac{1+cos^2(\emptyset)}{2} d\emptyset = \\ 4\pi r_0 \cdot \frac{\emptyset + \frac{1}{2}sin(2\emptyset)}{2}\Bigg]_{\emptyset=0}^{\emptyset=\frac{\pi}{2}} = 4\pi r_0 \cdot \frac{\pi}{4}\end{gathered} \tag{36}$$

If the field was uniform then the integration would be

$$2\int_{\emptyset=0}^{\emptyset=\frac{\pi}{2}} \frac{1}{r} 2\pi r_0 \cdot cos(\emptyset) \cdot r_0 \cdot d\emptyset = 4\pi r_0 \tag{37}$$

And the ratio between (37) and (36) is $\frac{4}{\pi}$, which agrees with Ettore Majorana's normalized ring potential coefficient [31] as $V(r) = \frac{4}{\pi}\frac{Q}{r}$, which means that the field $\frac{1}{r_0}$ in (36) has to grow by a factor of $\frac{4}{\pi}$ in order to sum up as in (37). In the charged particle case, we can see U_{λ} as a vector that points toward or outward of an integral curve in space-time but that the Minkowsky norm of the field is always the same, only the probability that this vector points towards a certain direction in space-time changes. This idea leads to the compensating scaling value $\zeta = \frac{4}{\pi}$. So equation (34) around a negative charge becomes,

$$x^2 + \frac{1}{96}\left(-\frac{1}{2}\left(\frac{4}{\pi}\right)^2 + \frac{4}{\pi}x\right) = x^3 \tag{38}$$

The roots of (38) are x1~=1.004836728026, x2~=0.760783659050, x3=0.089280264357. The first root is the only root which is attainable through iterations $x(n+1) = \left(\frac{\left(192x^2(\mathrm{n})+2\frac{4}{\pi}\mathrm{x}(\mathrm{n})-\left(\frac{4}{\pi}\right)^2\right)}{192}\right)^{\frac{1}{3}}$ and it converges starting from 0.1 or from 2 or any other positive number.

But $x = 1 + \frac{Added\ or\ subtracted\ area}{4\pi r^2}$, is either bigger or smaller than 1. The ratio $\frac{4\pi r^2}{Added\ or\ subtracted\ area} = \frac{1}{x-1} \cong \mathbf{206.751340}$, which is very close to the ratio between the mass of the Muon and the mass of the Electron, **206.768277**. We followed the M.I.T professor Seth Lloyd offer that addition or subtraction of quantum area, means addition or subtraction of energy and we have reached 206.751340. For the solution around a positive charge

$$x^2 + \frac{1}{96}\left(-\frac{1}{2}\left(\frac{4}{\pi}\right)^2 - \frac{4}{\pi}x\right) = x^3 \qquad (39)$$

we get about **44.63955017596401 .** These ratios ~1/45 and ~1/207 could mean a decay path for charged leptons where the numerical stability of 1/45 is worse than that of 1/207. A more exact root to (38) yields,

$$\frac{4\pi r^2}{\delta A} = \frac{1}{x-1} \cong 206.75133988502202 \qquad (40)$$

The difference in accuracy in this value by 64 and 128 bits is just the last 2 - 3 digits. If we divide the Muon energy by this value, we get very close to the energy of the electron and the delta in Mega electron volts is: 105.658374524 MeV / 206.75133988502202 – 0.510998946131 (MeV) = 0.00004187509298 MeV, which is **41.875092980 eV** and surprisingly fits the Super Nova 1987a ~40 eV Neutrino claim by Cesare Bacci [28]. That energy is small but beyond the energy of any known Neutrino mass. It is an unknown energy. Should it be a new particle, this particle is beyond the Standard Model. The ratio between the electron's energy and this energy 41.875092980 eV is 0.510998946131 MeV / 0.00004187509298 which is approximately**, 12202.93281199539440,** almost 12203. For Muon energy 105.6583745 MeV and an electron energy 0.5109989461 MeV, the ratio is **12202.95760492718728** .

We can get this value if we check the following polynomials for $\zeta = (1 - \frac{1}{96})$, see (34):

$$x^2 + \frac{1}{96}\left(-\frac{1}{2}\left(1-\frac{1}{96}\right)^2 \pm \left(1-\frac{1}{96}\right)x\right) = x^3 \qquad (41)$$

Which is 1 + or 1- the portion of area added around a negative charge or subtracted around a positive charge such that the acceleration field is smaller by a factor of $\zeta = (1 - \frac{1}{96})$. The idea to use a damping of $(1 - \frac{1}{96})$ is because of the factor $\frac{1}{96}$ in (34). This implies that charge quanta can be of the order $\frac{1}{96}$ of the charge of the electron e. The compelling indirect evidence for the $\frac{1}{96}$ in (43) can be seen in [33] as it appears that a fractional quantization in a Hole Quantum Wire yields a lowest fraction $\frac{e}{32}$ and with the direct $\frac{e}{3}$ fraction from Quarks we get $\frac{e}{96} = \frac{e}{32\cdot 3}$ which explains $\frac{1}{96}$ as the coefficient of electric charge. In resemblance to (38), the two polynomials in (41) with the $\pm$ sign have 3 roots each and the big roots are x1=1.00520707510980 for (+Area) and x2=0.98426221868924 for (-Area).

$$\left(\frac{1}{x1-1}\right)\left(\frac{1}{1-x2}\right) = \mathbf{12202.88874066467724} \qquad (42)$$

which with numerical accuracy is even closer to 12202.95760492718728. Other choices except for 96 in $\zeta = (1-\frac{1}{96})$ are further away from 12202.95760492718728 even for small differences after 3 digits after the floating point. The number $1-\frac{1}{96}=\frac{95}{96}=\left(\frac{96}{95}\right)^{-1}$ is the inverse of the Steiner Tree Problem limit. Finding as sub-optimum below $\frac{96}{95}$ of the minimal length of the Steiner Tree [34] that spans a graph with terminal points – in our case on a sphere - is not solvable in polynomial time. This limit strongly suggests a Dynamic Causal Triangulation [18] approach to space-time in (41), i.e. it describes a field truly in the Planck scale. We have to compromise on a graph that its length fits a larger ball by a factor $\frac{96}{95}$, which means that an acceleration field that depends on $\frac{1}{r_0}$ will be smaller, $\frac{95}{96}\frac{1}{r_0}$. Recalling (39), (40), $1+\frac{1}{96}\left(-\frac{1}{2}\left(\frac{4}{\pi}\right)^2 c^{-2}+\frac{4}{\pi}c^{-1}\right)=c$. So the following system of biggest roots as area ratios is

$$1+\frac{1}{96}\left(-\frac{1}{2}(1-\frac{1}{96})^2 a^{-2}+\left(1-\frac{1}{96}\right)a^{-1}\right)=a$$
$$1+\frac{1}{96}\left(-\frac{1}{2}(1-\frac{1}{96})^2 b^{-2}-\left(1-\frac{1}{96}\right)b^{-1}\right)=b$$
$$1+\frac{1}{96}\left(-\frac{1}{2}\left(\frac{4}{\pi}\right)^2 c^{-2}+\frac{4}{\pi}c^{-1}\right)=c$$
$$MuonMass*(c-1)=ElctronMass+ElctronMass*(a-1)(1-b)$$

$$MuonMass\frac{(c-1)}{(1+(a-1)(1-b))}=ElectronMass \qquad (43)$$

which is ~ **0.51099894597978 MeV** instead of **0.510998946131 MeV**. The difference **-0.00015122 eV** may be related to the electron neutrino. The minus sign means this energy is required in addition to the gravitational energy of the Muon in order to create an electron. A Muon mass of 105.65837455 yields an electron mass 0.5109989461 MeV. In Seth Lloyd's and Ted Jacobson's terminology, the physical meaning of this finding could be that the energy of the electron is the gravitational energy of a small surface around the Muon. The code in Python that was used to calculate the result of (43) can be found in Appendix G. To summarize, the factor $\frac{4}{\pi}$ is possibly due to a field distribution, also see [31] and $\frac{95}{96}$ is possibly due to Causal Sets [18] and the Steiner Tree Problem [34].

If we choose the $\frac{1}{1-c}\cong 44.63955018$ solution, we get in (43) a positive charge of 2.366728973 MeV. This is an interesting result about the Up Quark energy as assessed in lattice QCD, 2.36(24) Mev [35] but the charge of an anti-Muon does not match the Up Quark electric charge which could dismiss this option. This fact is used as self criticism on (43). Another research direction is indirect mass ratios, for example, between particle masses and masses from which other particles are derived. For example $MuonMass\frac{(1+(a-1)(1-b))}{(1-c)}\cong\left(1+\frac{1}{12202.88874}\right)*44.63955018*105.6583745\text{ MeV}\cong 4{,}716.92882\text{ MeV}$. This energy, about 4.717 GeV ~ 4.72 GeV [36],[37] appears in QCD as the pole energy of the Bottom Quark and is above the Bottom Quark energy of 4.18 GeV. It is worth mentioning that if we divide the energy of the Muon by 12202.88874066467724, see (42), we get about 8.658472331 KeV which complies with excess of photons with such energy in galactic centers [38]. Another remark is that the energy of the Tauon 1776.82 MeV or 1776.86 MeV divided by 12202.88874066467724 yields about 145.60650 MeV. This value is within one of the spectral lines of the decay of 99Mo and is used among other lines in nuclear medical imaging. Surprisingly, the sharpest image of lymph nodes in the human body can be seen at 145.6 MeV [39], which means that least scattering of photons occurs at that energy. This phenomenon could have a conventional explanation but it could also indicate the conversion of some of these photons into a neutral particle. Another interesting energy is: $ElctronMass * \sqrt{(a-1)(1-b)}$ which is about **4625.8194587 eV.**

Refuting a "speculation" claim attack – mathematical coincidence analysis

Assessment of such an attack must be based on probability as dependent on relative error, $\left(\left(\frac{105.658374524}{0.51099894613} - 206.75133988502202\right) * \left(\frac{0.51099894613}{105.658374524}\right)\right) * \left(\frac{12202.93281199539440-12202.88874066467724}{12202.93281199539440}\right) = \frac{1}{3,379,153,062.118533995049156383 2052} \approx \frac{0.5109989461-0.51099894586371}{0.5109989461} = \frac{1}{2,162,566,044.04535198211670\ldots}$, where a Muon energy of **105.6583745 MeV** and an electron energy of **0.5109989461 MeV** were considered.

This result means less than 5/10,000,000,000 relative error! Before trying $\zeta = \frac{95}{96}$ the author tried $\zeta = 1$. So we can say the probability grows to 1/1,000,000,000 but even 1/10,000,000 is already considered a finding. The significance of $\zeta = \frac{95}{96}$ is of 4 digits and more after the floating point! For example, $\xi = \frac{4}{\pi} - 0.0001$ in the calculation of $c-1$ in (43) yields 0.5110… MeV and $\xi = \frac{4}{\pi} + 0.0001$ yields 0.510969… both further away from the result 0.51099894597978 MeV. 4 digits sensitivity after the point can be seen in the calculation of $(1 + (a-1)(1-b))$ in (43) too, $\zeta = \frac{95}{96} + 0.0001$ yields 0.51099894008... and $\xi = \frac{95}{96} - 0.0001$ yields 0.5109989518… both further away from the result 0.51099894597978 MeV. These numerical sensitivities highly disfavor a "speculation" claim attack. The following shows how significant is the choice $\xi = \frac{95}{96} = 1 - \frac{1}{96}$. We did not refer to annihilation of two Muons. As one quantum system of two entangled particles a Muon and an anti Muon, before annihilation, should be seen as one energy with zero charge. Then (34) turns into

$$x^2 + \frac{1}{96}\left(-\frac{1}{2}\zeta^2 \pm 0 \cdot \zeta x\right) = x^3 \Rightarrow x^2 + \frac{1}{96}\left(-\frac{1}{2}\zeta^2\right) = x^3 \tag{43.1}$$

This equation yields a biggest root smaller than 1. The question we may ask, due to $\zeta = \frac{95}{96}$ in (43), is: Is there a reasonable $\zeta = 1 - \frac{1}{n}$ for which $\frac{1}{1-x} \approx \mathbf{206.768277}$ which is the Muon / Electron mass ratio. The closest fit is

$$\zeta = \frac{23}{24} \Longrightarrow x^2 + \frac{1}{96}\left(-\frac{1}{2}\left(\frac{23}{24}\right)^2\right) = x^3 \Longrightarrow \frac{1}{1-x} \cong 207.0440 \tag{43.2}$$

$1-\frac{1}{n}\approx e^{-\frac{1}{n}}=e^{\frac{-1}{24}}$, e=2.71828… and for $\zeta=e^{\frac{-1}{24}}$, $\frac{1}{1-x}\cong 206.670967462857$. The approximation of $e^{\frac{-1}{24}}$ is $\left(1-\frac{1}{n}\right)^{\frac{n}{24}}$ for which the closest $\frac{1}{1-x}$ to 206.768277 is achieved when $n=96$. This leads to $\zeta=\left(1-\frac{1}{n}\right)^{\frac{n}{24}}=\left(1-\frac{1}{96}\right)^{4}$ and (43.1) becomes

$$x^2+\frac{1}{96}\left(-\frac{1}{2}\left(\left(\frac{95}{96}\right)^4\right)^2\right)=x^3 \Longrightarrow \frac{1}{1-x}\cong 206.762203174798 \qquad (43.3)$$

The Muon mass is divided to yield the delta $2\cdot\left(\frac{105.658374524MeV}{206.7622}\ldots-0.510998946131\right)\cong 2\cdot 15.024999\ldots eV$. Other results are for, $=\frac{23}{24}$, 207.044017583727…, for $\zeta=\left(\frac{47}{48}\right)^2$, 206.854768046788…, for $\zeta=\left(\frac{191}{192}\right)^8$, 206.716421179322… and for e^(-1/24), 206.670967462857…

The relative error $\frac{206.7682826432\ldots-206.762203174798}{206.7682826432\ldots}=34010.914842\ldots^{-1}$ along with the $3{,}379{,}153{,}062.118533995\ldots^{-1}$ error is of extremely low probability of being a result of pure mathematical coincidence. Empirically, it is interesting to know if the energies 41.875092980 eV, **-0.00015122 eV** and 2 * 15 eV have a meaning as neutrino energies. **0.00015122 eV ~ 0.0002 eV** is likely to be the electron neutrino mass**.**

Failed research direction - The Higgs Boson

Consider an electrically neutral particle and Figure 1, where U_v can, at any given time, point inward or outward in equal probability. Then a gravitational source of this particle's energy E can be interpreted from two polynomials

$1+\frac{1}{96}\left(-\frac{1}{2}\zeta^2{c_1}^{-2}+\zeta{c_1}^{-1}\right)=c_1$ and $1+\frac{1}{96}\left(-\frac{1}{2}\zeta^2{c_2}^{-2}-\zeta{c_2}^{-1}\right)=c_2$ and the area addition ratio around some negative charge c_1-1 and area loss ratio around some positive charge $1-c_2$. Then we can calculate two energies, $E_1=-\frac{E}{c_1-1}$ and $E_2=\frac{E}{1-c_2}$ where the negative sign stands for anti-gravity. Recall our choice of $\zeta=\frac{4}{\pi}$,

$$E_2-E_1=\frac{E}{c_1-1}+\frac{E}{1-c_2}\cong E\cdot(206.7513398850\ldots+44.639550\ldots)\cong \qquad (43.4)$$

$$E\cdot 251.39089006098603107 4$$

When the energy of the Higgs Boson, ~125090 MeV is divided by 251.39089006098603107 4 we get,

~497.5916190 MeV which is barely within the assessment **497.611±0.013 MeV** [40] of the three known light neutral Kaons. This result is a total surprise and if true, there must be anomalies related to the neutral Kaons beyond the Standard Model, however, the uncertainty of the energy in [40] is yet too high to draw conclusions. If it was true then the Higgs boson mass would be dictated by the neutral Kaon mass and we know that the opposite is true

The W+ Boson and the Z Boson, the Higgs Boson and the anti-Tau particle

The equation $1+\frac{1}{96}\left(-\frac{1}{2}\left(\frac{4}{\pi}\right)^2 c^{-2} \pm \frac{4}{\pi}c^{-1}\right) = c$ does not take into account the possibility of a null Reeb field $U_k U^k$=0 or in the complex case $U_k U^{*k} + U^*{}_k U^k = 0$. In that case the equation becomes

$$1+\frac{1}{96}\left(\pm\frac{4}{\pi}c^{-1}\right) = 1 \pm \frac{c^{-1}}{24\pi} = c \tag{43.5}$$

And

$$1+\frac{1}{96}\left(\pm\frac{95}{96}b^{-1}\right) = 1 \pm \frac{95b^{-1}}{96^2} = b \tag{43.6}$$

And the biggest roots of these equations are

$$c_1 = \frac{1+\left(1+\frac{1}{6\pi}\right)^{\frac{1}{2}}}{2} \cong 1.0130915\ldots,\ c_2 = \frac{1+\left(1-\frac{1}{6\pi}\right)^{\frac{1}{2}}}{2} \cong 0.986556\ldots \text{ and}$$

$$b_1 = \frac{1+\left(1+\frac{95}{96*24}\right)^{\frac{1}{2}}}{2} \cong 1.010204037\ldots,\ b_2 = \frac{1+\left(1-\frac{95}{96*24}\right)^{\frac{1}{2}}}{2} = \frac{95}{96} \text{ and}$$

$$\frac{1}{c_1-1} \cong 76.38530,\ \frac{1}{1-c_2} \cong 74.3845968,\ \frac{1}{\sqrt{(c_1-1)(1-c_2)}} \cong 75.3783115, \tag{43.7}$$

$$\frac{1}{b_1-1} \cong 98.00042535,\ \frac{1}{1-b_2} = 96,\ \frac{1}{\sqrt{(b_1-1)(1-b_2)}} \cong 96.99505572,$$

$$\frac{\sqrt{(b_1-1)(1-b_2)}}{\sqrt{(c_1-1)(1-c_2)}} \cong 1.134361808^{-2}$$

That number is very close to the mass ratio between the Z boson and the W boson, $\frac{91.1876\ GeV}{80.370\ GeV}$ ≅1.134597487 but the error is too big to rule out mathematical coincidence: $Error^{-1} = \frac{1.134597487}{1.134597487-1.134361808}$ =~4814.16455 and with another W Boson assessment $\frac{91.1876\ GeV}{80.379\ GeV} \cong$ 1.134470446 we get $Error^{-1} = \sim 10442.60078$. It is interesting, but insufficient to rule out mathematical coincidence. Nevertheless, (43.7) means that the coefficient $\frac{95}{96}$ is related to charge-less particles and $\frac{4}{\pi}$ to electrically charged particles. Similar to (43), if we multiply, $1.134361808 \cdot (1+(c_1-1)(1-c_2)) \cong$ 1.134561453 then the relative error for $\frac{91.1876\ GeV}{80.370\ GeV}$ ≅1.134597487 is about 1/31486.95424, i.e. $(1+(c_1-1)(1-c_2))\left(\frac{(c_1-1)(1-c_2)}{(b_1-1)(1-b_2)}\right)^{1/4} \cong 1.134561453$ and for W Boson of 80.3725 GeV the error is about 1/1528961.689, however, in (43) it made sense due to the electron neutrino or another type of neutrino. In this case, this $(1+(c_1-1)(1-c_2))$ calibration requires a more comprehensible theory and is therefore in tension with the assumption of a W boson energy of 80.3725 GeV.

The Higgs Boson

A slightly different Higgs energy of 125.18 GeV yields a very interesting result with $\frac{1}{\sqrt{(b_1-1)(1-b_2)}} \cong 96.99505572$. $125.18\text{ GeV} \cdot \sqrt{(b_1-1)(1-b_2)} \cong 1.289797702$ GeV. Divide this value by the mass of the neutron, $\frac{125.18\text{ GeV}}{0.9395654133\text{ GeV}} \cdot \sqrt{(b_1-1)(1-b_2)} \cong 1.372759878$ and this becomes interesting because the mass ratio between the Higgs Boson and the Z Boson is about, $\frac{125.18\text{ GeV}}{91.1876\text{ GeV}} \cong 1.372774368$. If we check the relative error we get $Error^{-1} \cong \frac{1.372774368}{1.372774368-1.372759878} \cong 94737.81442$. That is a low error. Not enough yet to rule out mathematical coincidence but indeed low. The energy 1.289797702 GeV could be an indication of a new Boson or a vector Boson of about 1.29 GeV but not an isovector resonance [41] and evidence for its existence should be searched for in particle accelerators.

The mass trick

What is it that we can then learn about the Higgs Boson ? If we interpret the Higgs boson as an oscillating charge, then there is a nice mass trick that we can employ. The added area A around a negative charge and the subtracted area around a positive charge should cancel out leaving no arithmetic average gravity effect but we deal with a dynamic field. Writing the geometric average of area ratios we get $\sqrt{(b_1-1)(1-b_2)} < 1$ so the geometic average effect is actually of an area reduction which is equivalent to positive mass. We stated with null Reeb vectors that describe null rest mass and reached a positive mass. If the Higgs boson is polarized and interacts with other diploes, we need to get a head to head dipole interaction that over distances, drops as a 4th power of distance and at close distance, the interaction should depend on the second power of distance. This is also true if we take into account that the positive charge manifests gravity but its acceleration field is outwards and for the electron it is the opposite. Extraordinary claims require extraordinary evidence and whether the idea of the Higgs mass mechanism is indeed dipole based, requires to measure such a dipole in extremely strong electric fields. Another option is that there is no Higgs dipole and that the Higgs Boson behaves as a concentric oscillating charge.

The Tau Lepton

What doesn't seem right is the use of null Reeb vectors, which may not be even possible in order to reach a relation between particles that have rest mass, W and Z bosons. In this manner if we multiply the W boson mass by $\sqrt[4]{(c_1-1)(1-c_2)}$ and the Z boson mass by $\sqrt[4]{(b_1-1)(1-b_2)}$ we get almost the same value, about 9.26 GeV. Unfortunately there is no such boson and thus, this is a spurious prediction. The value 1.134361808 gives us some hope to find other particles from the W and Z boson masses from the ratios in (43.7). The geometric average of these values is $\frac{1}{\sqrt{(c_1-1)(1-c_2)}} \cong 75.3783115$ and the ordinary average $\frac{(c_1-1)^{-1}+(1-c_2)^{-1}}{2} \cong 75.384949$. Do these values have a physical meaning ? They could have if the Anti-Tau particle can be derived from the gravitational energy of the W Boson. Now returning to $1+\frac{1}{96}\left(-\frac{1}{2}\left(\frac{4}{\pi}\right)^2 c^{-2} - \frac{4}{\pi}c^{-1}\right) = c$ and to $\frac{1}{1-c} \cong 44.639550$ after (43), this portion from the energy of the W+ Boson [42] yields, $\frac{80370\ MeV}{44.639550\ldots} \cong 1800.4213681\ MeV$. The delta between this value and the Anti-Tau energy

1776.86 MeV is ~23.5613681 MeV. A higher delta is obtained if the Tau energy is taken to be, 1776.82 MeV, ~23.6013681 MeV. We can see that $\frac{1776.86}{75.3783115} \cong 23.5725630\ \mathrm{MeV}$ which means that a good approximation of the mass of the Tau particle can be obtained from

$$\frac{80370(1-c)\ MeV}{1+\sqrt{(c_1-1)(1-c_2)}} \approx 1776.86\ MeV \ \text{ or } \ \frac{80370(1-c)\ MeV}{1+\frac{2}{(c_1-1)^{-1}+(1-c_2)^{-1}}} \approx 1776.86\ MeV \qquad (43.8)$$

And with a more accurate Python code and 80369 MeV W+ Boson we get 1776.82684328632649 MeV.
There are two big problems unfortunately, first, (43.8) cannot be true because a null Reeb vector would imply that the portion of energy 23.5613681 MeV would not have rest mass. If it is a new type of neutrino, then this particle must have rest mass. Second, the averaging in the denominator of (43.7),(43.8) also implies a zero charge oscillating field. The W+ Boson has a rest mass and is not a composite particle and therefore the 23.5613681 MeV cannot be a part of the W+ Boson. Also, the W+ Boson has rest mass which immediately invalidates any claim that it is related to a null Reeb vector. These considerations unfortunately invalidate the feasibility of (43.5), (43.6), (43.7) as a valid theory. Weak evidence for (43.7) can be [43].
Other values for $\zeta = 1 - \frac{1}{96} = \frac{95}{96}$ in the null Reeb vector equations, from which

$$\sqrt{(b_1-1)(1-b_2)} \cong 96.9950557\ldots^{-1} \qquad (43.9)$$

This portion from the Z boson, $\frac{91187.6\ MeV}{96.99505572\ldots} \cong 940.12627\ldots\ MeV$, is close to the neutron energy, 939.5654133 MeV. 939.5654133 MeV * 96.9950557... yields ~91.1331996 GeV which can be viewed as a an intermediate value Z' which is smaller than the energy of the Z boson 91.1876 GeV. (43.9) offers a different process than (43.4), by which mass ratios of neutral particles can be obtained. The Z' intermediate vector boson energy of 91.133 GeV can be found in [44]. (43.9) is problematic because it requires the use of null Reeb vectors which would imply in this case, that a large portion of the Z boson should be mass-less. Obviously this is not true and that is why (43.9) is not considered as a feasible theory. If it was true then the Z' energy and therefore the Z boson energy would be dictated by the neutron's mass. Nevertheless, a researcher's integrity obliges to account for not only successes but also for directions that turned out to be wrong.

Particle mass ratios by added or subtracted area – Approximation of the Fine Structure constant

Like the previous section, this section relies on area ratios to represent mass ratios. Some of the work can be seen in the remarkable work of Lee C. Loveridge [27] and is somewhat speculative unless (34.4) or claims of a ~40eV neutrino [28] and other quantum effects [33] are taken into account and therefore the reader is honestly advised to take it with a grain of salt, however, the results are very interesting and do have geometrical basis in [11] that worth further investigation - the Reeb vector on the foliation leaves must have drains and sources. It is therefore the authors opinion that the technique used here will be considered by the reader even if the reader does not fully agree with the idea which is presented in this section. An approximation to the Fine Structure constant was discovered totally by chance when the author calculated the geometric average of area ratios quite similar to the square root of ones that appear in the denominator of (43). i.e. given two polynomials, $1 + \frac{1}{96}\left(-\frac{1}{2}\zeta^2 a^{-2} + \zeta a^{-1}\right) = a$ and $1 + \frac{1}{96}\left(-\frac{1}{2}\zeta^2 b^{-2} - \zeta b^{-1}\right) = b$, then we are interested in exploring $\sqrt{(a-1)(1-b)}$. The question that was explored was, if ζ is the inverse of the average distance between two points on a sphere S2 then what is $\sqrt{(a-1)(1-b)}$? This question is hard to solve because in (31) and in (32), we were not interested in the geometry within the infinitesimal sphere S2 but only in area deviations from the surface area $4\pi {r_0}^2$, while ignoring the interior of the sphere, which is a three dimensional ball. The greater problem is that in the calculation of the inverse of the average distance, ζ is dependent upon x in the solution of $1+\frac{1}{96}\left(-\frac{1}{2}\zeta^2x^{-2}+\zeta x^{-1}\right)=x$ because if the area $4\pi r^2$ becomes $4\pi r^2 x$ for every radius $r<r_0$ then the average distance between two points on the sphere, depends on rx. We must therefore calculate $\zeta=f(x)$ for some function $f(x)$. In flat geometry, the Riemannian distance between two points on the sphere that have an angle θ with the center is

$$S(\theta)=2r_0 sin\,(\frac{\theta}{2}) \tag{44}$$

Integrating on the sphere and dividing by the area, we get the average distance in flat geometry,

$$D=\frac{1}{4\pi r_0{}^2}\int_0^{\pi} S(\theta)2\pi r_0\, sin(\theta)\, dr_0\theta=\frac{4}{3}r_0 \tag{45}$$

So if the acceleration field depends on the inverse of that distance, it will depend on $\frac{3}{4}\frac{1}{r_0}$ and $\zeta=\frac{3}{4}$. Plugging this value in (34) and calculating the biggest roots a, b of the two resulting polynomials,

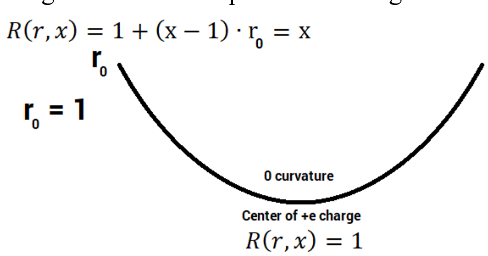
$$\left(\frac{1}{(1-a)}\frac{1}{(b-1)}\right)^{1/2}=\sim 137.2504256 \tag{46}$$

Close to 137.035999173 which is the inverse of the fine structure constant but not good enough.
For the sake of simplicity, we assume $r_0=1$. We need to take into account the effect of dilation and contraction of areas around the electric charge in order to get a closer value. A Riemannian geodesic curve distance $S(\theta)$ in (44) is replaced by

$$S(\theta,x)=Min\int_{\alpha=0}^{\alpha=\theta}\left(\sqrt{\mathrm{R}(\mathrm{r},x)\cdot \mathrm{r}^2+(\frac{\mathrm{dr}}{\mathrm{d\alpha}})^2}\right)\mathrm{d\alpha} \tag{47}$$

With boundary conditions $r(0)=1, r(\theta)=1, \mathrm{r}_0=1$, such that x is the biggest root of one of the two polynomials (34), $x=1+\frac{Added\ or\ subtracted\ disk\ area}{4\pi r^2\ instead\ of\ \pi r^2}$, $x<1$ or $x>1$ and $x>0$. $\mathrm{R}(\mathrm{r},x)\cdot\mathrm{r}^2\mathrm{d\alpha}^2$ is the square length element in a direction perpendicular to the radius r. This is the infinitesimal distance component that is influenced by the ball surface dilation at radius r . dr^2 is the infinitesimal square distance component along the radius and is therefore not influenced by surface dilation or contraction by x. The simplest model in which the area dilation is x at radius r_0 and 1 in the center, which means the curvature is 0 at the center, is $R(r,x)=1+(\mathrm{x}-1)\cdot\mathrm{r}$ so $r=0\Longrightarrow R(r,x)=1,\ \mathrm{r}=1\Longrightarrow R(r,x)=x$. See Fig. 2.0

(Fig. 2.0) – The reduced 3 dimensional curvature, from Minkowsky 4D to Riemann 3D around a hollowed positive charge whose field depends on average distance between two points on the sphere.



(47) becomes,

$$S(\theta,x)=Min\int_{\alpha=0}^{\alpha=\theta}\sqrt{((\mathrm{x}-1)\mathrm{r}+1)\cdot\mathrm{r}^2+(\frac{\mathrm{dr}}{\mathrm{d\alpha}})^2}\mathrm{d\alpha}\ \ \text{and}\ \ \mathrm{r}(0)=\mathrm{r}(\theta)=1 \tag{48}$$

Then (45) becomes,

$$\frac{1}{\zeta} = D(x) = \frac{1}{4\pi x}\int_0^{\pi} xS(\theta,x)2\pi\sin(\theta)\,d\theta = \frac{1}{4\pi}\int_0^{\pi} S(\theta,x)2\pi\sin(\theta)\,d\theta \tag{49}$$

(48) and (49) turned out to be extremely difficult to calculate. The author had to compromise on accuracy, and chose integration along a straight line in the coordinate system as a compromised approximation of (48) but with a systematic error. The term $2\cos\left(\frac{\theta}{2}\right)\int_{\alpha=0}^{\alpha=\frac{\theta}{2}}\frac{\sqrt{\sin^2(\alpha)+R(r,x)\cdot\cos^2(\alpha)}}{\cos^2(\alpha)}d\alpha$ is reduced to $2\sin\left(\frac{\theta}{2}\right)$ as in (44), with $r_0 = 1$ if $R(r,x) = 1$ for all $0 < r \leq 1$. So as an approximation with a systematic error we have,

$$S(\theta,x) \approx$$

$$2\cos\left(\frac{\theta}{2}\right)\int_{\alpha=0}^{\alpha=\frac{\theta}{2}}\frac{\sqrt{1+(R(r,x)-1)\cdot\cos^2(\alpha)}}{\cos^2(\alpha)}d\alpha =$$

$$2\cos\left(\frac{\theta}{2}\right)\int_{\alpha=0}^{\alpha=\frac{\theta}{2}}\frac{\sqrt{1+(x-1)\,(\cos\left(\frac{\theta}{2}\right)/\cos(\alpha))\cdot\cos^2(\alpha)}}{\cos^2(\alpha)}d\alpha \tag{50}$$

and instead of the exact (49) we use,

$$\frac{1}{\zeta} = D(x) \approx \frac{1}{4\pi}\int_{\theta=0}^{\theta=\pi} 2\pi\sin(\theta)\,2\cos\left(\frac{\theta}{2}\right)\left(\int_{\alpha=0}^{\alpha=\frac{\theta}{2}}\frac{\sqrt{1+(x-1)\cos\left(\frac{\theta}{2}\right)\cdot\cos(\alpha)}}{\cos^2(\alpha)}d\alpha\right)d\theta \tag{51}$$

We are now ready to run a computer calculation in C++, with a small systematic error, which is very slow but at least works,

$$a^2 + \frac{1}{96}\left(-\frac{1}{2}D(a)^{-2} + D(a)^{-1}a\right) = a^3 \tag{52}$$

$$b^2 + \frac{1}{96}\left(-\frac{1}{2}D(b)^{-2} - D(b)^{-1}b\right) = b^3 \tag{53}$$

And the approximated $D(x)$ yields a good result despite the systematic error in (50),(51), instead of using the exact terms (48),(49).

$$\left(\frac{1}{(1-a)}\frac{1}{(b-1)}\right)^{1/2} = \sim 137.041002618 \tag{54}$$

The code that was used to calculate (53) was written in C++ and can be found in Appendix H. Integral (50) is further developed in order to avoid the use of time consuming functions.

Conclusion

Using time-like curves which are based on Morse functions, it is possible to describe fields of acceleration that are not predicted by the metric alone. The acceleration fields have a rigid mathematical foundation in the wok of Georges Reeb from 1948 and in the theory of foliations. It is possible to say that space-time codim-1 foliations represent geometric information that is not represented by the metric tensor alone. Although a lot of work has to be done in order to show how to reconstruct the results of Quantum Field Theory, the results of this paper should raise a new interest in this work which cannot progress further as a work of one man. Unexpected gravity by electric charge has an immense importance to the development of the human race and it is especially important in order to understand the Dark Matter effect and in order to develop feasible Alcubierre - White or Alcubierre Froning Warp Drive technology. At least part of the Dark Matter effect may not be due to Dark Particles.

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Appendix A: Euler Lagrange minimum action equations
We assume $\sigma = 8\pi$ (from the previously discussed term, $-a_\mu a^\mu / 8\pi K$ as an energy density).

$$\begin{aligned}
&\mathrm{Z} = N^2 = \mathrm{P}_\mu \mathrm{P}^\mu \text{ and } U_\lambda = \frac{Z_\lambda}{Z} - \frac{Z_k P^k P_\lambda}{Z^2} \text{ and } \mathrm{L} = \frac{1}{4}\mathrm{U}^{\mathrm{k}}\mathrm{U}_{\mathrm{k}} \\
&R = Ricci \text{ curvature.} \\
&Min\ Action = Min\int_\Omega \left(R - \frac{8\pi}{\sigma} L \right)\sqrt{-g}\, d\Omega = \\
&Min\int_\Omega \left(R - \frac{1}{4}\mathrm{U}^{\mathrm{k}}\mathrm{U}_{\mathrm{k}} \right) . \sqrt{-g}\, d\Omega \text{ s.t. } \sigma = 8\pi
\end{aligned} \tag{55}$$

The variation of the Ricci scalar is well known. It uses the Platini identity and Stokes theorem to calculate the variation of the Ricci curvature and reaches the Einstein tensor [45], as follows,

$\delta R = R_{\mu\nu}\delta g^{\mu\nu}$ and $\delta\sqrt{-g} = -\frac{1}{2} g_{\mu\nu}\delta g^{\mu\nu}\sqrt{-g}$ by which we infer $\delta(R\sqrt{-g}) = (R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu})\delta g^{\mu\nu}$ which will be later added to the variation of $\left(\frac{1}{2}R - \frac{8\pi}{\sigma}L\right) . \sqrt{-g}$ by $\delta g^{\mu\nu}$.

The following Euler Lagrange equations have to hold,

$$\left(\frac{\partial}{\partial g^{\mu\nu}} - \frac{d}{dx^m}\frac{\partial}{\partial (g^{\mu\nu}{}_{,m})} + \frac{d^2}{dx^m dx^s}\frac{\partial}{\partial (g^{\mu\nu}{}_{,m,s})}\right)\left((\frac{1}{2}R - \frac{1}{4}\mathrm{U}^{\mathrm{k}}\mathrm{U}_{\mathrm{k}})\sqrt{-g} \right) = 0$$

and

$$\left(\frac{\partial}{\partial p} - \frac{d}{dx^m}\frac{\partial}{\partial (P_m)} + \frac{d^2}{dx^m dx^s}\frac{\partial}{\partial (P_{,m,s})}\right)\left((\frac{1}{2}R - \frac{1}{4}\mathrm{U}^{\mathrm{k}}\mathrm{U}_{\mathrm{k}})\sqrt{-g} \right) = 0$$

$\mathrm{U^k U_k} = \frac{Z_\mu Z^\mu}{Z^2} - \frac{(Z_s P^s)^2}{Z^3}$ which we obtain from the minimum Euler Lagrange equation because $U_\lambda P^\lambda = \frac{Z_\lambda P^\lambda}{Z} - \frac{Z_k P^k P_\lambda P^\lambda}{Z^2} = 0$. In order to calculate the minimum action Euler-Lagrange equations, we will separately treat the Lagrangians, $L = \frac{Z_\mu Z^\mu}{Z^2}$ and $L = \frac{(Z_s P^s)^2}{Z^3}$ to derive the Euler Lagrange equations of the Lagrangian $L = \frac{Z_\mu Z^\mu}{Z^2} - \frac{(Z_s P^s)^2}{Z^3} = U_\mu U^\mu$. The Euler Lagrange operator of the Ricci scalar $(\frac{\partial}{\partial g^{\mu\nu}} - \frac{d}{dx^m}\frac{\partial}{\partial(g^{\mu\nu},_m)} + \frac{d^2}{dx^m dx^s}\frac{\partial}{\partial(g^{\mu\nu},_m,_s)}$.

The reader may skip the following equations up to equation (61). Equations (61), (62) and (63) are however crucial.

$$L = \frac{(P_\lambda Z^\lambda)^2}{Z^3}\ s.t.\ Z = P_\mu P^\mu\ and\ Z_s \equiv Z,_s = \frac{dZ}{dx^s}$$

$$\frac{\partial(L\sqrt{-g}}{\partial g^{\mu\nu}} - \frac{d}{dx^m}\frac{\partial(L\sqrt{-g}\,)}{\partial g^{\mu\nu},_m}$$
$$= \Bigg(-2\left(\frac{Z,_s P^s}{Z^3}P_\mu P_\nu P^m\right);_m + 2\left(\frac{Z,_s P^s}{Z^3}\right)\left(\Gamma^{\mathrm{i}}_{\mathrm{\mu m}}P_i P_\nu P^m + \Gamma^{\mathrm{i}}_{\mathrm{\nu m}}P_\mu P_i P^m\right)$$
$$+ 2\left(\frac{Z,_s P^s}{Z^3}\right)(P_\mu P_\nu);_m P^m - 2\left(\frac{Z,_s P^s}{Z^3}\right)\left(\Gamma^{\mathrm{i}}_{\mathrm{\mu m}}P_i P_\nu P^m + \Gamma^{\mathrm{i}}_{\mathrm{\nu m}}P_\mu P_i P^m\right)$$
$$+ 2\left(\frac{Z,_s P^s}{Z^3}\right)Z_\mu P_\nu - 3\frac{(Z,_s P^s)^2}{Z^4}P_\mu P_\nu - \frac{1}{2}\frac{(Z,_s P^s)^2}{Z^3}g_{\mu\nu}\Bigg)\sqrt{-g} =$$

$$\left(-2\left(\frac{Z,_s P^S}{Z^3}P^k\right);_k P_\mu P_\nu - 2\frac{(Z,_s P^S)^2}{Z^3}\frac{P_\mu P_\nu}{Z} - \frac{(Z,_s P^S)^2}{Z^3}\frac{P_\mu P_\nu}{Z} + 2\left(\frac{Z,_s P^S}{Z^3}\right)Z_\mu P_\nu - \frac{1}{2}\frac{(Z,_s P^S)^2}{Z^3}g_{\mu\nu}\right)\sqrt{-g} \quad (56)$$

$$L = \frac{Z^\lambda Z_\lambda}{Z^2}\ s.t.\ Z = P_\mu P^\mu, s.t.\ Z = P_\mu P^\mu\ and\ Z_s \equiv Z,_s = \frac{dZ}{dx^s}$$

$$\frac{\partial(L\sqrt{-g}}{\partial g^{\mu\nu}} - \frac{d}{dx^m}\frac{\partial(L\sqrt{-g}\,)}{\partial g^{\mu\nu},_m} = \Bigg(-2\left(\frac{Z^m P_\mu P_\nu}{Z^2}\right);_m + 2\frac{(\Gamma^{\mathrm{i}}_{\mathrm{\mu m}}\mathrm{P_i P_\nu Z^m} + \Gamma^{\mathrm{i}}_{\mathrm{\nu m}}P_i P_\mu Z^m)}{Z^2} + 2\frac{(P_\mu P_\nu);_m Z^m}{Z^2} -$$
$$2\frac{(\Gamma^{\mathrm{i}}_{\mathrm{\mu m}}\mathrm{P_i P_\nu Z^m} + \Gamma^{\mathrm{i}}_{\mathrm{\nu m}}P_i P_\mu Z^m)}{Z^2} + \frac{Z_\mu Z_\nu}{Z^2} - 2\frac{Z_s Z^s}{Z^3}P_\mu P_\nu - \frac{1}{2}\frac{Z_m Z^m}{Z^2}g_{\mu\nu}\Bigg)\sqrt{-g} = \left(-2\left(\frac{Z^m}{Z^2}\right);_m P_\mu P_\nu -\right.$$
$$\left.2\frac{Z_s Z^s}{Z^3}P_\mu P_\nu - \frac{1}{2}\frac{Z_m Z^m}{Z^2}g_{\mu\nu} + \frac{Z_\mu Z_\nu}{Z^2}\right)\sqrt{-g} \quad (57)$$

We subtract (56) from (57)

$$Z = P_\mu P^\mu, s.t.\ Z = P_\mu P^\mu\ and\ Z_s \equiv Z,_s = \frac{dZ}{dx^s}, U_\lambda = \frac{Z_\lambda}{Z} - \frac{Z_k P^k P_\lambda}{Z^2}, L = U^k U_k = \frac{Z_\lambda Z^\lambda}{Z^2} - \frac{(Z_k P^k)^2}{Z^3}$$

$$\frac{\partial(L\sqrt{-g}}{\partial g^{\mu\nu}} - \frac{d}{dx^m}\frac{\partial(L\sqrt{-g}\,)}{\partial g^{\mu\nu},_m} = \Big(+2\left(\frac{Z_m P^m}{Z^3}P^k\right);_k P_\mu P_\nu + 2\frac{(Z_m P^m)^2}{Z^3}\frac{P_\mu P_\nu}{Z} - 2\frac{Z_m P^m}{Z^3}Z_\mu P_\nu + \frac{1}{2}\frac{(Z_m P^m)^2}{Z^3}g_{\mu\nu} + \frac{(Z_m P^m)^2}{Z^3}\frac{P_\mu P_\nu}{Z} +$$
$$(-2\left(\frac{Z^m}{Z^2}\right);_m P_\mu P_\nu - 2\frac{Z_\lambda Z^\lambda}{Z^2}\frac{P_\mu P_\nu}{Z} - \frac{1}{2}\frac{Z_\lambda Z^\lambda}{Z^2}g_{\mu\nu} + \frac{Z_\mu Z_\nu}{Z^2})\Big)\sqrt{-g} = \Bigg(\left(+2\left(\frac{Z_m P^m}{Z^3}P^k\right);_k - 2\left(\frac{Z^m}{Z^2}\right);_m\right)P_\mu P_\nu +$$

$$2\frac{\left(P^{\lambda}Z_{\lambda}\right)^{2}}{Z^{3}}\frac{P_{\mu}P_{\nu}}{z}-2\frac{Z^{\lambda}Z_{\lambda}}{Z^{2}}\frac{P_{\mu}P_{\nu}}{z}+\frac{1}{2}\frac{\left(P^{\lambda}Z_{\lambda}\right)^{2}}{Z^{3}}g_{\mu\nu}-\frac{1}{2}\frac{Z_{k}Z^{k}}{Z^{2}}g_{\mu\nu}+\frac{Z_{\mu}Z_{\nu}}{Z^{2}}-2\left(\frac{Z_{S}P^{S}}{Z^{3}}\right)Z_{\mu}P_{\nu}+\frac{\left(P^{\lambda}Z_{\lambda}\right)^{2}}{Z^{3}}\frac{P_{\mu}P_{\nu}}{z}\Bigg)\sqrt{-g}=$$

$$\left(\left(+2\left(\frac{Z_{m}P^{m}}{Z^{3}}P^{k}\right);_{k}-2\left(\frac{Z^{m}}{Z^{2}}\right);_{m}\right)P_{\mu}P_{\nu}+2\frac{\left(P^{\lambda}Z_{\lambda}\right)^{2}}{Z^{3}}\frac{P_{\mu}P_{\nu}}{z}-2\frac{Z^{\lambda}Z_{\lambda}}{Z^{2}}\frac{P_{\mu}P_{\nu}}{z}+U_{\mu}U_{\nu}-\frac{1}{2}U^{\lambda}U_{\lambda}g_{\mu\nu}\right)\sqrt{-g}=$$

$$\left(U_{\mu}U_{\nu}-\frac{1}{2}U^{\lambda}U_{\lambda}g_{\mu\nu}-2U^{k};_{k}\frac{P_{\mu}P_{\nu}}{z}\right)\sqrt{-g} \tag{58}$$

$$L=\frac{(Z^{s}P_{s})^{2}}{Z^{3}}\quad s.t.\ Z=P^{\lambda}P_{\lambda}\ and\ Z_{m}=(P^{\lambda}P_{\lambda}),_{m}$$

$$\frac{\partial(L\sqrt{-g})}{\partial P_{\mu}}-\frac{d}{dx^{\nu}}\frac{\partial(L\sqrt{-g})}{\partial P_{\mu},_{\nu}}=$$

$$\begin{pmatrix}-4(\frac{(Z_{s}P^{s})}{Z^{3}}P^{\mu}P^{\nu});_{\nu}+4\frac{(Z_{s}P^{s})}{Z^{3}}\Gamma_{i}{}^{\mu}{}_{\nu}P^{i}P^{\nu}+\\+4\frac{(Z_{s}P^{s})}{Z^{3}}P^{\mu};_{\nu}P^{\nu}-4\frac{(Z_{s}P^{s})}{Z^{3}}\Gamma_{i}{}^{\mu}{}_{k}P^{i}P^{k}+\\+2\frac{Z_{m}P^{m}Z^{\mu}}{Z^{3}}-6\frac{(Z_{m}P^{m})^{2}}{Z^{4}}P^{\mu}\end{pmatrix}\sqrt{-g}=$$

$$(-4(\frac{(Z_{s}P^{s})P^{\nu}}{Z^{3}});_{\nu}P^{\mu}+2\frac{Z_{m}P^{m}Z^{\mu}}{Z^{3}}-6\frac{(Z_{m}P^{m})^{2}}{Z^{4}}P^{\mu})\sqrt{-g} \tag{59}$$

$$L=\frac{Z^{s}Z_{s}}{Z^{2}}\quad s.t.\ Z=P^{\lambda}P_{\lambda}\ and\ Z_{m}=(P^{\lambda}P_{\lambda}),_{m}$$

$$\frac{\partial(L\sqrt{-g})}{\partial P_{\mu}}-\frac{d}{dx^{\nu}}\frac{\partial(L\sqrt{-g})}{\partial P_{\mu},_{\nu}}=$$

$$\begin{pmatrix}-4(\frac{P^{\mu}Z^{\nu}}{Z^{2}});_{\nu}+\frac{4}{Z^{2}}\Gamma_{i}{}^{\mu}{}_{k}P^{i}Z^{k}+\\+\frac{4}{Z^{2}}P^{\mu};_{\nu}Z^{\nu}-\frac{4}{Z^{2}}\Gamma_{i}{}^{\mu}{}_{k}P^{i}Z^{k}+\\-4\frac{Z_{m}Z^{m}}{Z^{3}}P^{\mu}\sqrt{-g}\end{pmatrix}\sqrt{-g}=$$

$$(-4(\frac{Z^{\nu}}{Z^{2}});_{\nu}-4\frac{Z_{m}Z^{m}}{Z^{3}})P^{\mu}\sqrt{-g} \tag{60}$$

We subtracted the Euler Lagrange operators of $\frac{(Z^{s}P_{s})^{2}}{Z^{3}}\sqrt{-g}$ in (56) from the Euler Lagrange operators of $\frac{Z^{\lambda}Z_{\lambda}}{Z^{2}}\sqrt{-g}$ in (57) and got (58) and we will subtract (59) from (60) to get two tensor equations of gravity, these will be (61), and (63). Assuming $\sigma=8\pi$, where the metric variation equations (55), (56), (57) and (58) yield

$$Z = N^2 = P_\mu P^\mu \ , \ U_\lambda = \frac{Z_\lambda}{Z} - \frac{Z_k P^k P_\lambda}{Z^2} \ , L = \frac{1}{4} U_i U^i \ \text{ and } Z = P^k P_k$$

$$\frac{8\pi}{\sigma}\frac{1}{4}\begin{pmatrix} +2\left(\left(\frac{(P^\lambda P_\lambda),_m P^m}{Z^3} P^k\right);_k - 2\left(\frac{Z^m}{Z^2}\right);_m\right) P_\mu P_\nu + \\ +2\frac{(P^\lambda Z_\lambda)^2}{Z^3}\frac{P_\mu P_\nu}{Z} - 2\frac{Z^\lambda Z_\lambda}{Z^2}\frac{P_\mu P_\nu}{Z} + \\ +U_\mu U_\nu - \frac{1}{2} U_k U^k g_{\mu\nu} \end{pmatrix} =$$

$$\frac{8\pi}{\sigma}\frac{1}{4}\left(U_\mu U_\nu - \frac{1}{2}U_k U^k g_{\mu\nu} - 2U^k;_k \frac{P_\mu P_\nu}{Z}\right) = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$$

$$\text{s.t. } R = R_{\mu\upsilon} g^{\mu\upsilon}$$

$$\text{s.t. } R_{kj} = (\Gamma_{jk}{}^P),_p - (\Gamma_{pk}{}^P),_j + \Gamma_{p\mu}{}^P \Gamma_{jk}{}^\mu - \Gamma_{pj}{}^\mu \Gamma_{k\mu}{}^p \tag{61}$$

$R_{\mu\nu}$ is the Ricci tensor and $R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$ is the Einstein tensor [45]. In general, by (28) and $\sigma = 8\pi$, (61) can be written as

$$\frac{1}{4}\left(U_\mu U_\nu - \frac{1}{2}U_k U^k g_{\mu\nu} - 2U^k;_k \frac{P_\mu P_\nu}{Z}\right) = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \tag{62}$$

Charge-less field: The term $-2U^k;_k \frac{P_\mu P_\nu}{Z}$ in (62) can be generalized to:

$-2((U^k;_k + U^{*k};_k)/2)\frac{(P_\mu P^*_\nu + P^*_\mu P_\nu)/2}{Z}$ and can be zero under the following condition:

$$4\left(A_{\mu\nu};^\mu \frac{P^{*\nu}}{\sqrt{Z}} + A^*_{\mu\nu};^\mu \frac{P^\nu}{\sqrt{Z}}\right) = U_\mu U^{*\mu} + U^*_\mu U^\mu \Rightarrow U^k;_k + U^{*k};_k = 0$$

Note: The complimentary matrix $B_{\mu\nu} = \frac{1}{\sqrt{2}} E^{\mu\nu\alpha\beta} A_{\alpha\beta}$, see (11), can be transformed to a real matrix due to the SU(2) x U(1) degrees of freedom and also be imaginary.

From (59), (60) we have, $\frac{d}{dx^\mu}\left(\frac{\partial}{\partial P_\mu} - \frac{d}{dx^\nu}\frac{\partial}{\partial P_{\mu},_\nu}\right)(U_k U^k \sqrt{-g}) = W^\mu;_\mu \sqrt{-g} = 0$

We recall, $W^\mu = \left(\frac{\partial}{\partial P_\mu} - \frac{d}{dx^\nu}\frac{\partial}{\partial P_{\mu},_\nu}\right)(U_k U^k \sqrt{-g})$

$W^{\mu} =$

$(-4(\frac{Z^{\nu}}{Z^2});_{\nu} -4\frac{Z_{\mathrm{m}}Z^{m}}{Z^3})\mathrm{P}^{\mu} + 4(\frac{(Z_s P^s)P^{\nu}}{Z^3});_{\nu} P^{\mu} - 2\frac{Z_{\mathrm{m}}P^{m}Z^{\mu}}{Z^3} + 6\frac{(Z_m P^m)^2}{Z^4}P^{\mu} =$

$-4(\frac{Z^{\nu}}{Z^2});_{\nu} \mathrm{P}^{\mu} - 4\frac{Z_{\mathrm{m}}Z^{m}}{Z^3}\mathrm{P}^{\mu} +$

$+4(\frac{(Z_s P^s)P^{\nu}}{Z^3});_{\nu} P^{\mu} + 4\frac{(Z_m P^m)^2}{Z^4}P^{\mu}$

$-2\frac{Z_{\mathrm{m}}P^{m}}{Z^2}(\frac{Z^{\mu}}{Z} - \frac{Z_m P^m P^{\mu}}{Z^2}) =$

$-4((\frac{U^k}{Z});_k + \frac{U^k U_k}{Z})P^{\mu} - 2\frac{Z_{\mathrm{m}}P^{m}}{Z^2}U^{\mu} = 0$

$$W^{\mu};_{\mu} = \left(-4U^{\nu};_{\nu}\frac{P^{\mu}}{Z} - 2\frac{(Z_m P^m)}{Z^2}U^{\mu}\right);_{\mu} = 0 \tag{63}$$

Appendix B: Proof of conservation

Theorem: Conservation law of the real Reeb vector.

From the vanishing of the divergence of Einstein tensor and (62) in the paper, we have to prove the following:

$$\frac{1}{4}\left(U_{\mu}U_{\nu} - \frac{1}{2}U_{\mathrm{k}}U^{k}g_{\mu\nu} - 2U^{k};_{k}\frac{P_{\mu}P_{\nu}}{Z}\right);^{\mu} = G_{\mu\nu};^{\mu} = (R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu});^{\mu} = 0 \tag{64}$$

Proof:

From the zero variation by the scalar time field (63)

$$W^{\mu};_{\mu} = \left(-4U^{\nu};_{\nu}\frac{P^{\mu}}{Z} - 2\frac{(Z_m P^m)}{Z^2}U^{\mu}\right);_{\mu} = 0 \tag{65}$$

$$-\left(2U^{\nu};_{\nu}\frac{P^{\mu}}{Z}\right);_{\mu} = \left(\frac{(Z_m P^m)}{Z^2}U^{\mu}\right);_{\mu} \tag{66}$$

$$\left(-2U^{k};_{k}\frac{P^{\mu}P^{\nu}}{Z}\right);_{\mu} = \left(\frac{(Z_m P^m)}{Z^2}U^{\mu}\right);_{\mu}P^{\nu} - \left(2U^{k};_{k}\frac{P^{\mu}}{Z}\right)P^{\nu};_{\mu} =$$
$$\left(\frac{(Z_m P^m)}{Z^2}U^{\mu}\right);_{\mu}P^{\nu} - U^{k};_{k}\frac{Z^{\nu}}{Z} \tag{67}$$

Now let $t \equiv Z_m P^m$

$$\left(\frac{t}{Z^2}U^{\mu}\right);_{\mu}P^{\nu} - U^{k};_{k}\frac{Z^{\nu}}{Z} = (\frac{t}{Z^2});_{\mu}U^{\mu}P^{\nu} + \frac{t}{Z^2}U^{\mu};_{\mu}P^{\nu} - U^{k};_{k}\frac{Z^{\nu}}{Z} =$$
$$-U^{\mu};_{\mu}U^{\nu} + (\frac{t}{Z^2});_{\mu}U^{\mu}P^{\nu}$$

This is because $-\mathrm{U}^{\nu} = -\frac{\mathrm{Z}^{\nu}}{\mathrm{Z}} + \frac{\mathrm{t}}{\mathrm{Z}^2}\mathrm{P}^{\nu} \Rightarrow -\mathrm{U}^{\mu};_{\mu}\frac{\mathrm{Z}^{\nu}}{\mathrm{Z}} + \frac{\mathrm{t}}{\mathrm{Z}^2}\mathrm{U}^{\mu};_{\mu}\mathrm{P}^{\nu} = -\mathrm{U}^{\mu};_{\mu}\mathrm{U}^{\nu}$. Note that $-\mathrm{U}^{\nu}$ is minus twice the real numbered Reeb vector. So,

$$(-2U^k;_k \frac{P^\mu P^\nu}{Z});_\mu = -U^\mu;_\mu U^\nu + (\frac{t}{Z^2});_\mu U^\mu P^\nu \tag{68}$$

Returning to the theorem we have to prove and using equation (68), we have to show,

$$\begin{aligned}
&\left(U^\mu U^\nu - \frac{1}{2}U_k U^k g^{\mu\nu} - 2U^k;_k \frac{P^\mu P^\nu}{Z}\right);^\mu = \\
&U^\mu;_\mu U^\nu + U^\mu U^\nu;_\mu - \frac{1}{2}(U_k;_\mu U_s + U_k U_s;_\mu)g^{ks}g^{\mu\nu} - \\
&U^\mu;_\mu U^\nu + (\frac{t}{Z^2});_\mu U^\mu P^\nu = \\
&U^\mu U^\nu;_\mu - \frac{1}{2}(U^s U_s);^\nu + (\frac{t}{Z^2});_\mu U^\mu P^\nu = 0
\end{aligned} \tag{69}$$

Notice that

$$\begin{aligned}
&U^\mu U^\nu;_\mu - \frac{1}{2}U^s U_s;^\nu = \\
&U^\mu\left((\frac{Z_k}{Z});_\mu - (\frac{t}{Z^2});_\mu P_k - (\frac{t}{Z^2})P_k;_\mu\right)g^{k\nu} - \\
&U^s\left((\frac{Z_s}{Z});_k - (\frac{t}{Z^2});_k P_s - (\frac{t}{Z^2})P_s;_k\right)g^{k\nu} = \\
&-U^\mu(\frac{t}{Z^2});_\mu P^\nu
\end{aligned} \tag{70}$$

Since $-(\frac{t}{Z^2});_k P_s U^s = 0$ because the Reeb vector is perpendicular to the foliation kernel $\frac{P_\lambda}{\sqrt{Z}}, \frac{P^k}{\sqrt{Z}}\frac{U_k}{2} = 0$.

Equation (70) is also a result of $\ln(Z)_{,k};_\mu U^\mu g^{k\nu} = \ln(Z)_{,s};_k U^s g^{k\nu}$ and of $P_k;_\mu U^\mu g^{k\nu} = P_s;_k U^s g^{k\nu}$.

$$U^\mu U^\nu;_\mu - \frac{1}{2}(U^s U_s);^\nu + (\frac{t}{Z^2});_\mu U^\mu P^\nu = -U^\mu(\frac{t}{Z^2});_\mu P^\nu + (\frac{t}{Z^2});_\mu U^\mu P^\nu = 0 \tag{71}$$

and we are done.

Appendix C: Invariance of the Reeb vector under different functions of P

Here we wish to explore another degree of freedom in the action operator of the "acceleration field" which results from the Reeb vector, as shown by a representative vector field $\frac{dP}{dx^i}$ which is tangent to a non-geodesic integral curve. We wish to show that P can be replaced with a smooth function $f(P)$ and that U_m is invariant under such a transformation. We revisit our acceleration field and write $U_m = \frac{N^2{}_{,m}}{N^2} - \frac{N^2{}_{,\mu}P^{*\mu}}{N^4}P_m$ s.t. $Z = N^2 = \frac{P^{*k}P_k + P^k P_k^*}{2}$.

We can omit the comma for the sake of brevity the same way we write P_i instead of $P_{,i}$ for $\frac{dP}{dx^i}$ and write $U_m = \frac{N^2{}_m}{N^2} - \frac{N^2{}_\mu P^{*\mu}}{N^4}P_m$. We will prove the invariance of U_m where P is real, however, a similar proof is also valid where P is complex and where P is replaced with a smooth function of P.

Suppose that we replace P by $f(P)$ such that f is positive and increasing or decreasing, then $f(P)_i \equiv \frac{df(P)}{dx^i} = \frac{df(P)}{dP}\frac{dP}{dx^i} = f_p(P)P_i$. Let $N^2 \equiv P^\lambda P_\lambda$ then $\hat{N}^2 \equiv f(P)_\lambda f(P)^\lambda = N^2 f_p(P)^2$
and $\frac{\hat{N}^2{}_k}{\hat{N}^2} = \frac{N^2{}_k}{N^2} + \frac{2f_{pp}(P)}{f_p(P)}P_k$ but also

$$\hat{U}_k = \frac{\hat{N}^2{}_k}{\hat{N}^2} - \frac{\hat{N}^2{}_s f_p(P)P^s f_p(P)P_k}{\hat{N}^2} =$$
$$\frac{N^2{}_k}{N^2} + \frac{2f_{pp}(P)}{f_p(P)}P_k - \left(\frac{N^2{}_s}{N^2} + \frac{2f_{pp}(P)}{f_p(P)}P_s\right)\frac{f_p(P)P^s f_p(P)P_k}{N^2 f_p(P)^2} =$$
$$\frac{N^2{}_k}{N^2} - \frac{N^2{}_\mu P^\mu}{N^4}P_k = U_k \tag{72}$$

Appendix D: The curvature of the gradient of P

The second power of the curvature of the integral curve by $P_\mu = \frac{dP}{dx^\mu}$ where x^μ denote the coordinates is expressible by

$$Curv^2 = \frac{d}{dt}\frac{P_\lambda}{\sqrt{P_k P^k}}\frac{d}{dt}\frac{P_\mu}{\sqrt{P_k P^k}}g^{\lambda\mu} \tag{73}$$

such that $g^{\lambda\mu}$ is the metric tensor. (73) is an excellent candidate for an action operator. For convenience, we will write $Norm \equiv \sqrt{P^k P_k}$ and $\dot{P}_\lambda \equiv \frac{d}{dt}P_\lambda$. For the arc length parameter t. Here is the main trick, as was mentioned about $Z = Norm^2$, $Norm$ may not be constant because P_λ is not the 4-velocity of any particle, (to see an example of a variable $Norm$, see Appendix E – The time field in the Schwarzschild solution), An arc length parameterization along these curves is equivalent to proper time measured by a particle that moves along the curves, and in the real numbers case, P can be indeed time. Unlike velocity's squared norms, Z is not constant.

Let W_λ denote: $W_\lambda = \frac{d}{dt}\left(\frac{P_\lambda}{\sqrt{P_k P_k}}\right) = \frac{\dot{P}_\lambda}{Norm} - \frac{P_\lambda}{Norm^3}P_k\dot{P}_\nu g^{k\nu}$

Obviously

$$W_\lambda P_k g^{\lambda k} = \frac{\dot{P}_\lambda P_k g^{\lambda k}}{Norm} - \frac{P_\lambda P_s g^{\lambda s}}{Norm^3}P_k\dot{P}_\nu g^{k\nu} = \frac{\dot{P}_\lambda P_k g^{\lambda k}}{Norm} - \frac{P_k\dot{P}_\nu g^{k\nu}}{Norm} = 0$$

Thus

$$Curv^2 = W_\lambda W^\lambda = \frac{\dot{P}_\lambda\dot{P}_\nu g^{\lambda\nu}}{Norm^2} - \frac{P_\lambda\dot{P}_s g^{\lambda s}}{Norm^4}P_k\dot{P}_\nu g^{k\nu} = \frac{\dot{P}_\lambda\dot{P}^\lambda}{Norm^2} - \left(\frac{P_\lambda\dot{P}^\lambda}{Norm^2}\right)^2$$

Following the curves formed by $P_\lambda = P,_\lambda = \frac{dP}{dx^\lambda}$, the term $\frac{dx^r}{dt} = \frac{P_\lambda}{Norm}$ is the derivative of the normalized curve or normalized "velocity", using the upper Christoffel symbols, $P_\lambda;_r \equiv \frac{d}{dx^r}P_\lambda - P_s\Gamma^s_{\lambda r}$.

$\frac{d}{dt}P_\lambda = (\frac{d}{dx^r}P_\lambda - P_s\Gamma^s_{\lambda r})\frac{dx^r}{dt} = (P_\lambda;_r)\frac{P^r}{Norm}$ such that x^r denotes the local coordinates. If P_λ is a conserving field, then $P_\lambda;_r = P_r;_\lambda$ and thus $P_\lambda,_r P^r = \frac{1}{2}Norm^2,_\lambda$ and

$$Curv^2 = \frac{\dot{P}_\lambda \dot{P}^\lambda}{Norm^2} - (\frac{P_\lambda \dot{P}^\lambda}{Norm^2})^2 = \frac{1}{4}(\frac{Norm^2,_\lambda Norm^2,_k g^{\lambda k}}{Norm^4} - (\frac{Norm^2,_s P_r g^{sr}}{Norm^3})^2)$$

In the real case, we have achieved the Reeb vector,

$$U_m = \frac{(\mathrm{P}^\lambda \mathrm{P}_\lambda),_\mathrm{m}}{\mathrm{P}^\mathrm{i}\mathrm{P}_\mathrm{i}} - \frac{(\mathrm{P}^\lambda \mathrm{P}_\lambda),_\mu \mathrm{P}^\mu}{(\mathrm{P}^\mathrm{i}\mathrm{P}_\mathrm{i})^2} P_m = \frac{Z_\mathrm{m}}{Z} - \frac{Z_\mu \mathrm{P}^\mu}{Z^2} P_m \tag{74}$$

and our candidate for a trajectory curvature action is

$Action = \frac{1}{4}U_m U^m$ where in the complex case we saw $\mathrm{U}_\mu = \frac{\mathrm{Z}_\mu}{\mathrm{Z}} - \frac{\mathrm{Z}_\mathrm{k}\mathrm{P}^{*\mathrm{k}}}{\mathrm{Z}^2}\mathrm{P}_\mu$

$$Action = \frac{1}{8}(U^*_m U^m + U_m U^{*m}) \tag{75}$$

Non-geodesic motion, as a result of interaction with a field, is not a geodesic motion in a gravitational field, i.e. it is not free fall. Moreover, material fields by this interpretation prohibit geodesic motion curves of particles moving at speeds less than the speed of light and by this, reduce the measurement of proper time.

Appendix E – Time-like field from geodesic curves in the Schwarzschild solution

Motivation: To show a non vanishing $(\mathrm{P}_\mu \mathrm{P}^\mu),_\nu$, to make the reader familiar with the idea of maximal proper time from a sub-manifold and to calculate the background scalar time field of the Schwarzschild solution from that sub-manifold. We choose as a sub-manifold, a small 3 dimensional 3-spehere around the "Big Bang" singularity or a synchronized big sphere around the gravity source and far from the source and therefore this example is either limited to a "Big Bang" manifold or to a big sphere. So, we want to connect each event in a Schwarzschild solution to a primordial sub-manifold a fraction of second after the presumed "Big Bang" or to a synchronized big sphere around the gravity source, with the longest possible curve under the assumption that no closed time-like curves occur.

In this limited case, the scalar field is uninteresting as it does not represent interactions with any charged particle or with other force fields and therefore, the Reeb vector should be zero.

We would like to calculate $\frac{U_s U^s}{4} = \frac{1}{4}(\frac{(P_\lambda P^\lambda),_m (P_\lambda P^\lambda),_k g^{mk}}{(P_i P^i)^2} - \frac{((P_\lambda P^\lambda),_m P^m)^2}{(P_i P^i)^3})$ in Schwarzschild coordinates for a freely falling particle. This theory predicts that where there is no matter, the result must be zero. The speed U of a falling particle from very far away, as measured by an observer in the gravitational field is

$$V^2 = \frac{U^2}{c^2} = \frac{R}{r} = \frac{2GM}{rc^2} \tag{76}$$

Where R is the Schwarzschild radius. If speed V is normalized in relation to the speed of light then $V = \frac{U}{c}$. For a far observer, the deltas are denoted by dt', dr' and,

$$\dot{r}^2 = (\frac{dr}{dt})^2 = V^2(1-\frac{R}{r}) \tag{77}$$

because $dr = dr' / \sqrt{1-R/r}$ and $dt = dt'\sqrt{1-R/r}$.

$$P = \int_0^t \left((1-\frac{R}{r}) - \frac{\dot{r}^2}{(1-\frac{R}{r})} \right)^{\frac{1}{2}} dt = \int_0^t \left((1-\frac{R}{r}) - \frac{\frac{R}{r}(1-\frac{R}{r})^2}{(1-\frac{R}{r})} \right)^{\frac{1}{2}} dt =$$

$$\int_0^t \left((1-\frac{R}{r})^2 \right)^{\frac{1}{2}} dt = \int_0^t (1-\frac{R}{r}) dt$$

which results in,

$$P_t = \frac{dP}{dt} = (1-\frac{R}{r}) \tag{78}$$

Here t is not a tensor index and it denotes derivative by t !

On the other hand

$$P = \int_0^r \left((1-\frac{R}{r})\frac{1}{\dot{r}^2} - \frac{1}{(1-\frac{R}{r})} \right)^{\frac{1}{2}} dr = \int_0^r \left(\frac{(1-\frac{R}{r})\frac{r}{R}}{(1-\frac{R}{r})^2} - \frac{1}{(1-\frac{R}{r})} \right)^{\frac{1}{2}} dr = \int_0^r \left(\frac{\frac{r-R}{R}}{\frac{r-R}{r}} \right)^{\frac{1}{2}} dr =$$

$$\int_0^r \sqrt{\frac{r}{R}} dr$$

Which results in

$$P_r = \frac{dP}{dr} = \sqrt{\frac{r}{R}} \tag{79}$$

Here, r is not a tensor index and it denotes derivative by r !

For the square norms of gradients, we use the inverse of the metric tensor,

So, we have $(1-\frac{R}{r}) \to (1-\frac{R}{r})^{-1}$ and $(1-\frac{R}{r})^{-1} \to (1-\frac{R}{r})$

So, we can write

$$N^2 = P_\lambda P^\lambda = (1-\frac{R}{r})P_r^2 - (1-\frac{R}{r})^{-1}P_t^2 = (1-\frac{R}{r})(\frac{r}{R}-1) = \frac{r}{R} + \frac{R}{r} - 2$$

$$N^2 = \frac{r}{R} + \frac{R}{r} - 2 \tag{80}$$

$N^2{}_{\lambda} = \frac{dN^2}{dx^{\lambda}}$ And we can calculate

$$\frac{N^2{}_{\lambda} N^{2\lambda}}{(N^2)^2} = \frac{(1-\frac{R}{r})^2(\frac{1}{R}-\frac{R}{r^2})^2}{(\frac{r}{R}+\frac{R}{r}-2)^2} \tag{81}$$

We continue to calculate

$$N^2{}_t P_t = (1-\frac{R}{r})^2(\frac{1}{R}-\frac{R}{r^2})\sqrt{\frac{R}{r}} \text{ and } \frac{N^2{}_t P_t}{(1-\frac{R}{r})} = (1-\frac{R}{r})(\frac{1}{R}-\frac{R}{r^2})\sqrt{\frac{R}{r}} \tag{82}$$

Note that here t is not a tensor index and it denotes derivative by t !

$$(1-\frac{R}{r})N^2{}_r P_r = (1-\frac{R}{r})(\frac{1}{R}-\frac{R}{r^2})\sqrt{\frac{r}{R}} \tag{83}$$

Please note, here r is not a tensor index and it denotes derivative by r !

$N^2{}_{\lambda} P^{\lambda} = (1-\frac{R}{r})(\frac{1}{R}-\frac{R}{r^2})(\sqrt{\frac{r}{R}}-\sqrt{\frac{R}{r}})$ and

$$(N^2{}_{\lambda} P^{\lambda})^2 = (1-\frac{R}{r})^2(\frac{1}{R}-\frac{R}{r^2})^2(\frac{r}{R}+\frac{R}{r}-2) \tag{84}$$

So

$$\frac{(N^2{}_{\lambda} P^{\lambda})^2}{(N^2)^3} = \frac{(1-\frac{R}{r})^2(\frac{1}{R}-\frac{R}{r^2})^2}{(\frac{r}{R}+\frac{R}{r}-2)^2} \tag{85}$$

And finally, from (81) and (85) we have,

$$\frac{(P^{\lambda}P_{\lambda}),_m (P^s P_s),_k g^{mk}}{(P_i P^i)^2} - \frac{(P^{\lambda}P_{\lambda}),_m P^m}{(P_i P^i)^3} =$$

$$\frac{N^2{}_{\lambda} N^{2\lambda}}{(N^2)^2} - \frac{N^2{}_{\lambda} P^{\lambda}}{(N^2)^3} = \frac{(1-\frac{R}{r})^2(\frac{1}{R}-\frac{R}{r^2})^2}{(\frac{r}{R}+\frac{R}{r}-2)^2} - \frac{(1-\frac{R}{r})^2(\frac{1}{R}-\frac{R}{r^2})^2}{(\frac{r}{R}+\frac{R}{r}-2)^2} = 0 \tag{86}$$

which shows that indeed the gradient of time measured, by a falling particle until it hits an event in the gravitational field, has zero curvature as expected.

Appendix F: Conditions for SU(3) symmetry by three complex scalars in the kernel of P_{μ}

We may want to express the acceleration matrix $A_{\alpha\beta}$ by three scalar fields that are defined in the foliation F that is perpendicular to $\frac{P_i}{\sqrt{Z}}$. This is because P_i is a geometric object that defines foliations of space-time and can be conversely defined by the foliations. Another motivation is to show that $SU(3)$ that is seen in Quantum Chromo-Dynamics, may originate from geometry. By a theorem of Frobenius, necessary conditions for 3 vectors $h(j=1,2,3)$ to span the foliation F is that the vectors $h(s)$ are Holonomic if their Lie brackets depend on them $[h(i),h(k)]=\sum_{j=1}^{3}c_j h(j)$ for some coefficients c_j. The Lie brackets of each two vectors must depend on the vectors that span $T(F)$. We may write our 3 scalars a,b,c (here c is not the speed of light and not the previous coefficients but a scalar field) and their gradients that span the foliation's tangent space $T(F)$ as follows, $h_k(1)=\frac{d}{dx^k}a$, $h_k(2)=\frac{d}{dx^k}b$, $h_k(3)=\frac{d}{dx^k}c$.

We now express $A_{\alpha\beta}$ by $h_k(1)$, $h_k(2)$, $h_k(3)$ in a covariant formalism but we need some constraint on P_μ.

Condition: $P_\mu;^\lambda P*^\mu = P*_\mu;^\lambda P^\mu$

This condition is not trivial and in general, $P_\mu;^\lambda P*^\mu \neq P*_\mu;^\lambda P^\mu$.

Consider the following matrix:

$$D_{ij}=g_{ij}-a*_i a_j - b*_i b_j - c*_i c_j \tag{87}$$

and $a_i=\frac{\alpha_i}{\sqrt{(\alpha^\lambda\alpha*_\lambda+\alpha*^\lambda\alpha_\lambda)/2}}$ for some scalar function α whose gradient α_i is in the foliation perpendicular to P_μ etc. and in the same manner replace β_i by a normalized unit vector b_i and γ_i by a normalized $c_i=\frac{\gamma_i}{\sqrt{\gamma_\mu\gamma*^\mu+\gamma*_\mu\gamma^\mu}}$ vector. Also, we demand orthogonality, $a_i b*^i = a_i c*^i = b_i c*^i = 0$. Obviously $D_{ij}a*^j = D_{ij}b*^j = D_{ij}c*^j = 0$ and $D_{ij}\frac{P*^j}{\sqrt{Z}}=\frac{P*^i}{\sqrt{Z}}$ which then shows that $D_{ij}=\frac{P*_i P_j}{Z}$, $Z=(P_\lambda P*^\lambda + P*_\lambda P^\lambda)/2$. That is very interesting, because as was already said, we can represent $P*_i P_j$ by orthogonal fields that span the tangent space of the perpendicular foliation to P_j, namely T(F). Consider the following:

$$\begin{aligned}D_{ij};_k - D_{ik};_j &= \left(\frac{P*_i;_k P_j + P*_i P_j;_k}{Z} - \frac{P*_i P_j Z_k}{Z^2}\right) - \left(\frac{P*_i;_j P_k + P*_i P_k;_j}{Z} - \frac{P*_i P_k Z_j}{Z^2}\right) = \\ &\left(\frac{P*_i P_k Z_j}{Z^2} - \frac{P*_i P_j Z_k}{Z^2}\right) + \left(\frac{P*_i;_k P_j}{Z} - \frac{P*_i;_j P_k}{Z}\right)\end{aligned} \tag{88}$$

Now comes a little trick:

$$(D_{ij};_k - D_{ik};_j)\frac{P^i P^{*j}}{Z} =$$
$$\left(\frac{P^*_i P_k Z_j}{Z^2} - \frac{P^*_i P_j Z_k}{Z^2}\right)\frac{P^i P^{*j}}{Z} + \left(\frac{P^*_i;_k P_j}{Z} - \frac{P^*_i;_j P_k}{Z}\right)\frac{P^i P^{*j}}{Z} =$$
$$\left(\frac{Z_j P^{*j} P_k}{Z^2} - \frac{Z_j}{Z}\right) + \left(\frac{P^*_i;_k P^i}{Z} - \frac{P^i P^{*j} P^*_i;_j P_k}{Z^2}\right)$$

By (10), it is obvious that the first two terms constitute minus twice the Reeb vector,

$\frac{Z_j P^{*j} P_k}{Z^2} - \frac{Z_j}{Z} = -U_k = -2(\frac{U_k}{2})$. For the last two terms, we need a special condition $P_\mu;^\lambda P^{*\mu} = P^*_\mu;^\lambda P^\mu$ although usually $P_\mu;^\lambda P^{*\mu} \neq P^*_\mu;^\lambda P^\mu$. Then by this condition,

$\frac{P^*_i;_k P^i}{Z} - \frac{P^i P^{*j} P^*_i;_j P_k}{Z^2} = \frac{U_k}{2}$ and therefore

$$(D_{ij};_k - D_{ik};_j)D^{*ij} = (D_{ij};_k - D_{ik};_j)\frac{P^i P^{*j}}{Z} = -\frac{U_k}{2} \tag{89}$$

Consider our assumption, $D_{ij} = g_{ij} - a^*_i a_j - b^*_i b_j - c^*_i c_j$ and we have obtained an expression of the Reeb vector by the orthonormal vectors that represent the foliation. The additives of (87) are tensors. This leads us to an open question as follows: Is the condition $P_\mu;^\lambda P^{*\mu} = P^*_\mu;^\lambda P^\mu$, the minimal condition which is needed for a representation of the Reeb vector by a_j, b_j, c_j as the sum of tensor terms ? In other words, is the condition $P_\mu;^\lambda P^{*\mu} = P^*_\mu;^\lambda P^\mu$ a necessity for the tensor representation of the acceleration matrix by the foliation scalars, a, b, c?

There are other ways to achieve a Lagrangian with higher symmetries than U(1) that are shortly discussed.
The following action can be extended to U(1) x SU(2) and to SU(3) symmetries by considering more than one Reeb vector.

$$\mathrm{Z} = N^2 = (\mathrm{P}_\mu \mathrm{P}^{*\mu} + \mathrm{P}^*_\mu \mathrm{P}^\mu)/2 \text{ and } U_\lambda = \frac{Z_\lambda}{Z} - \frac{Z_k P^{*k} P_\lambda}{Z^2} \text{ and}$$
$$\mathrm{L} = \frac{1}{4}\left(\frac{\mathrm{U}^k \mathrm{U}^*_k + \mathrm{U}^{*k} \mathrm{U}_k}{2}\right) \tag{90}$$

Since the matrix of a Symplectic form can be described as two rotation and scaling hyper-planes, there is a possibility to locally add another scalar $P(2)$ and the Reeb vector of its gradient $P(2)_\mu$

$$\mathrm{Z}(2) = N^2(2) = (\mathrm{P}(2)_\mu \mathrm{P}(2)^{*\mu} + \mathrm{P}(2)^*_\mu \mathrm{P}(2)^\mu)/2$$

And $Z(2)_\lambda = \frac{dZ(2)}{dx^\lambda}$

$$\hat{U}(2)_\lambda = \frac{Z(2)_\lambda}{Z(2)} - \frac{Z(2)_k P(2)^{*k} P_\lambda}{Z(2)^2} \text{ and } U(2)_\lambda = \hat{U}(2)_\lambda - \hat{U}(2)_\nu \frac{P^{*\nu}}{Z} P_\lambda$$

And such that

$$U(2)_\lambda U^{*\lambda} = P(2)_\lambda P^{*\lambda} = P(2)_\lambda U^{*\lambda} = P_\lambda U(2)^{*\lambda} = 0 \tag{91}$$

and obviously $P_\lambda U^{*\lambda} = 0$ and $P(2)_\lambda U(2)^{*\lambda} = 0$ from the definition of a Reeb vector.
The action is then dictated by the root of the Gram determinant and is added to the previous action,

$$L = \frac{1}{4}\left(\frac{U^k U_k^* + U^{*k} U_k}{2}\right) + \frac{1}{4}\left(\frac{U(2)^k U(2)_k^* + U(2)^{*k} U(2)_k}{2}\right) \tag{92}$$

The physical meaning of $U(2)_\mu$ is another acceleration field in another plane. We will consider (91) as the "Electro-Weak Geometric Chronon Action".

In the three dimensional space, Minkowsky perpendicular to P_λ we can view three holonomic vectors fields that span the foliation tangent space as required by the Froben ius theorem . These can be locally described by three gradients,
$P(3)_\lambda, P(4)_\lambda, P(5)_\lambda$ and accordingly we can discuss their Reeb vectors, $\hat{S}_\mu, \hat{W}_\mu, \hat{T}_\mu$ and their projection on the foliation perpendicular to $\frac{P_\lambda}{\sqrt{Z}}$.,

$$S_\lambda = \hat{S}_\lambda - \hat{S}_\nu \frac{P^{*\nu}}{Z} P_\lambda, W_\lambda = \hat{W}_\lambda - \hat{W}_\nu \frac{P^{*\nu}}{Z} P_\lambda, T_\lambda = \hat{T}_\lambda - T_\nu \frac{P^{*\nu}}{Z} P_\lambda$$

This time we can't require the orthogonality condition which is described in (91) because there are no three Minkowsky – perpendicular hyper planes in space-time.
Now we need the third root of the determinant of the Gram matrix of these new three Reeb vectors and the action becomes,

$$L = \frac{1}{4}\left(\frac{U^k U_k^* + U^{*k} U_k}{2}\right) + \begin{vmatrix} \frac{U^k U_k^* + U^{*k} U_k}{8} & \frac{U(2)^k U_k^* + U(2)^{*k} U_k}{8} \\ \frac{U(2)^k U_k^* + U(2)^{*k} U_k}{8} & \frac{U(2)^k U(2)_k^* + U(2)^{*k} U(2)_k}{8} \end{vmatrix}^{\frac{1}{2}} +$$

$$\begin{vmatrix} \frac{S_\mu S^{*\mu} + S_\mu^* S^\mu}{8} & \frac{S_\mu W^{*\mu} + S_\mu^* W^\mu}{8} & \frac{S_\mu T^{*\mu} + S_\mu^* T^\mu}{8} \\ \frac{W_\mu S^{*\mu} + W_\mu^* S^\mu}{8} & \frac{W_\mu W^{*\mu} + W_\mu^* W^\mu}{8} & \frac{W_\mu T^{*\mu} + W_\mu^* T^\mu}{8} \\ \frac{T_\mu S^{*\mu} + T_\mu^* S^\mu}{8} & \frac{T_\mu W^{*\mu} + T_\mu^* W^\mu}{8} & \frac{T_\mu T^{*\mu} + T_\mu^* T^\mu}{8} \end{vmatrix}^{\frac{1}{3}} \tag{93}$$

Appendix G: The Python code that was used for the mass ratio calculations out of area ratios

```
import numpy as NP

class ELECTROGRAVITY_CLASS:

    def function_cubic_viete(self, a, b, c, d): # If all roots are real.

        # Viete's algorithm when all roots are real.

        b2 = NP.longdouble(b * b)
        b3 = NP.longdouble(b2 * b)
        a2 = NP.longdouble(a * a)
        a3 = a2 * a

        p = (3 * a * c - b2) / (3 * a2)

        q = (2 * b3 - 9 * a * b * c + 27 * a2 * d) / (27 * a3)

        offset = b / (3 * a)

        t1 = 2 * NP.sqrt(-p / 3) * NP.cos(NP.arccos(NP.sqrt(-3 / p) * (3 * q) / (2 * p)) / 3)
        t2 = 2 * NP.sqrt(-p / 3) * NP.cos(NP.arccos(NP.sqrt(-3 / p) * (3 * q) / (2 * p)) / 3 -
NP.pi / 3)
        t3 = 2 * NP.sqrt(-p / 3) * NP.cos(NP.arccos(NP.sqrt(-3 / p) * (3 * q) / (2 * p)) / 3 - 2
* NP.pi / 3)

        x1 = t1 - offset
        x2 = t2 - offset
        x3 = t3 - offset

        return (x1, x2, x3)

MAIN_electrogravity_class = ELECTROGRAVITY_CLASS()

f = 1 - 1/96

x1, x2, x3 = \
MAIN_electrogravity_class.function_cubic_viete(1, -1, -f / 96, (f * f) / 192)

x4, x5, x6 = \
MAIN_electrogravity_class.function_cubic_viete(1, -1, f / 96, (f * f) / 192)

f = 4 / NP.pi

x7, x8, x9 = \
MAIN_electrogravity_class.function_cubic_viete(1, -1, -f / 96, (f * f) / 192)

print("Anti-gravity: X1,X2,X3 = (%.14lf, %.14lf, %.14lf)" %(x1, x2, x3))
print("Gravity: X4,X5,X6 = (%.14lf, %.14lf, %.14lf)" %(x4, x5, x6))
print("Anti-gravity: X7,X8,X9 = (%.14lf, %.14lf, %.14lf)" %(x7, x8, x9))

print("Muon mass in MeV/C^2 105.658374524")
print("Predicted electron mass im MeV/C^2 %.14lf" % ((105.658374524 * (x7 - 1)) / (1 + (x1-1)*(1-
x4))))
0
x4, x5, x6 = \
MAIN_electrogravity_class.function_cubic_viete(1, -1, f / 96, f*f / 192)

x8 = (1 + NP.sqrt(1 - 1/(NP.pi * 6)))/2
```

```
print("Gravity: X4,X5,X6 = (%.14lf, %.14lf, %.14lf)" %(x4, x5, x6))
print("Gravity: X8 = %.14lf" % x8)

print("Predicted Tau particle out of the W Boson 80385 MeV/C^2 = %.14lf " % (80385*(1-x4)/(1+1-
x8)))

f = (1 - 1/96) * (4 / NP.pi)

x7, x8, x9 = \
MAIN_electrogravity_class.function_cubic_viete(1, -1, -f / 96, (f * f) / 192)

x10, x11, x12 = \
MAIN_electrogravity_class.function_cubic_viete(1, -1, f / 96, (f * f) / 192)

print("Anti-gravity: X7,X8,X9 = (%.14lf, %.14lf, %.14lf)" %(x7, x8, x9))
print("Gravity: X10,X11,X12 = (%.14lf, %.14lf, %.14lf)" %(x10, x11, x12))
print("Average 1/(1 - (1/(X7-1) + 1/(1-x10))/2) = %.14lf" %(1/((x7 - x10)/2)))

print("Better prediction: Tau out of the W Boson 80385 MeV/C^2 = %.14lf " \
          % (80385*(1-x4)/(1+0.5*(x7-x10))))

x7 = (1 + NP.sqrt(1 + 1/(NP.pi * 6)))/2
x10 = (1 + NP.sqrt(1 - 1/(NP.pi * 6)))/2

print("Another prediction: Tau out of the W Boson 80369 MeV/C^2 = %.14lf " \
         % (80369 *(1-x4)/(1+0.5*(x7-x10))))

input("Press Enter to exit> ")
```

Appendix H: The C++ code that was used to approximate the Fine Structure Constant

```
#include "stdafx.h"

#include <math.h>

#include <conio.h>

// #define FINE_STUCTURE_STEP 0.0001

#define FINE_STRUCTURE_LOOP 100000 // Must be 1 / step.

#define FINE_STUCTURE_STEP ((long double)1.0/FINE_STRUCTURE_LOOP)

#define FINE_STRUCTURE_PI 3.14159265358979323846264338327950

#define FINE_STRUCTURE_ITERATIONS 32

// 0.5 of the distance.

long double FUNCTION_distance(long double d_angle,long double d_area_ratio)

{

  long double d_square_height,d_r,d_scaled,d_l,d_step,d_sum,

              d_l_square,d_delta_ratio;

  int i;

  d_step = sin(d_angle) / FINE_STRUCTURE_LOOP;
```

```
  d_square_height = cos(d_angle);
  d_square_height *= d_square_height;
  d_delta_ratio = d_area_ratio - 1;
  // d_scaled = d_square_height * d_area_ratio;
  d_l = 0;
  d_sum = 0;
  for (i = 0; i < FINE_STRUCTURE_LOOP; i++,d_l+=d_step)
  {
    d_l_square = d_l * d_l;
    d_r = sqrt(d_l_square + d_square_height);
    d_scaled = d_square_height * (1 + d_r * d_delta_ratio);
    d_sum += sqrt(d_l_square + d_scaled) / d_r;
  }
  return d_sum * d_step;
}

// Calculate 0.5 of the distance.
long double FUNCTION_average_distance(long double ad_area_ratio)
{
  long double ad_x,ad_step,ad_sum,ad_distance;
  int i;

  ad_sum = 0;
  ad_step = (long double)FINE_STRUCTURE_PI * FINE_STUCTURE_STEP;

  for (i = 0, ad_x = 0; i < FINE_STRUCTURE_LOOP; i++,ad_x+=ad_step)
  {
    ad_distance = FUNCTION_distance(ad_x * 0.5, ad_area_ratio);
    // 2 * 2Pi / 4Pi = 1.
    // Also area that grows by b and is divided by an area that grows by b is 1.
    ad_sum += sin(ad_x) * ad_distance;
  }
```

```
  return ad_sum * ad_step;
}
void FUNCTION_roots(void)
{
  long double r_root1,r_root2,r_f1, r_f2;
  long double r_result;
  int i;
  // Start close to already known attractors & save time.
  r_root1 = 1.0048700774565755;
  r_root2 = 0.98905515302403790;
  r_f1 = 1.3354957147970252;
  r_f2 = 1.3284582439903136;
  for (i = 0; i < FINE_STRUCTURE_ITERATIONS; i++)
  {
    r_f1 = FUNCTION_average_distance(r_root1);
    r_root1 = (192 * r_root1 * r_root1 +
               2 * r_root1 / r_f1 -
               1.0/(r_f1 * r_f1))/192;
    r_root1 = cbrt(r_root1);
    r_f2 = FUNCTION_average_distance(r_root2);
    r_root2 = (192 * r_root2 * r_root2 -
               2 * r_root2 / r_f2 -
               1.0 / (r_f2 * r_f2)) / 192;
    r_root2 = cbrt(r_root2);
    r_result = sqrt(1 / ((r_root1 - 1)*(1 - r_root2)));
    printf("%.9lf, x1=%.9lf, x2=%.9lf2, 1/f1=%.9lf, 1/f2=%.9lf2\n",
           r_result,r_root1,r_root2,1/r_f1,1/r_f2);
  }
}
int main()
{
  while (_kbhit()) _getch(); // Clear keyboard input.
```

```
  FUNCTION_roots();

  while (_kbhit()) _getch(); // Clear keyboard input.

  puts("Press Enter to exit the console.");

  getchar();

  return 0;

}
```

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