

On the possible mathematical connections between some equations of certain Dirichlet series, some equations of D-Branes and Rogers-Ramanujan formulas that link π , e and the Golden Ratio. I

Michele Nardelli¹, Antonio Nardelli

Abstract

In this research thesis, we have described some new mathematical connections between some equations of certain Dirichlet series, some equations of D-Branes and Rogers-Ramanujan formulas that link π , e and ϕ .



<https://commons.wikimedia.org/wiki/File:AnatolyA.Karatsuba.jpg>

¹ M.Nardelli have studied by Dipartimento di Scienze della Terra Università degli Studi di Napoli Federico II, Largo S. Marcellino, 10 - 80138 Napoli, Dipartimento di Matematica ed Applicazioni “R. Caccioppoli” - Università degli Studi di Napoli “Federico II” – Polo delle Scienze e delle Tecnologie Monte S. Angelo, Via Cintia (Fuorigrotta), 80126 Napoli, Italy

From:

DOI 10.1070/RM2006v061n03ABEH004328

Behaviour of the argument of the Riemann zeta function on the critical line

A. A. Karatsuba and M. A. Korolev

§ 2. Approximation of the function $S(t)$ by a segment of a Dirichlet series

Let $N(\sigma, T)$ be the number of zeros of the Riemann zeta function $\zeta(s)$ in the domain $\sigma < \operatorname{Re} s \leq 1$, $0 < \operatorname{Im} s \leq T$. If $T^a \leq H \leq T$, where a is a fixed number with $1/2 < a \leq 1$, then the estimate

$$N(\sigma, T + H) - N(\sigma, T) = O\left(H(\log T) \left(\frac{H}{\sqrt{T}}\right)^{-\frac{1}{2}(\sigma - \frac{1}{2})}\right)$$

holds uniformly for $1/2 \leq \sigma \leq 1$.

In §§ 1–3 we use the following notation: ε is an arbitrarily small fixed number such that $0 < \varepsilon < 0.001$, $T \geq T_0(\varepsilon) > 0$, $H = T^{27/82 + \varepsilon}$, $L = \log T$, $k = [L]$, $H_1 = HL^{-1}$, $\Delta = H_1^{-1}L$, $x = T^{0.1\varepsilon}$, and $D = (2H_1)^k$.

For any T and any σ with $0 \leq \sigma \leq 1$

Let $0 \leq \sigma \leq 1$ and let γ range over all rational fractions whose numerator and denominator are positive and do not exceed x . We denote by $r = r(\sigma; T, H, P, x)$ the sum

$$r = \sum_{\gamma} \int_{-2H}^{2H} \int_{-H_1}^{H_1} \cdots \int_{-H_1}^{H_1} |W(\sigma, T_1; \alpha)| dt dt_1 \cdots dt_k,$$

If $h \geq 1$, then $|h + \xi - \eta| \geq 1 - (1 - x^{-1}) = x^{-1} > x^{-2}$.

For $0.5 \leq \sigma \leq 1$ we define numbers $\delta_\nu = \delta_\nu(\sigma)$, $1 \leq \nu \leq x$, by the equalities

Let $s = \sigma + it$, where $0.5 \leq \sigma \leq 1$, $y \geq 1$, $z \geq 1$, and $t = 2\pi yz$.

$$P = \sqrt{T/(2\pi)}.$$

We have that:

$$I = \int_T^{T+H} (\sigma_{z,t} - 0.5)^\nu \xi^{\sigma_{z,t}-0.5} dt$$

satisfies the estimate

$$I < H \frac{(2e)^{8m}}{(\log z)^\nu} \left(1 + \frac{\nu!}{32} \left(\frac{\log z}{\log x} \right)^\nu \frac{\log T}{\log z} \right).$$

For:

$$H = T^{27/82+\varepsilon}; \quad T = 2; \quad z = 3, \nu = 1.0012; \quad m = 1; \quad x = T^{0.1\varepsilon} \quad \varepsilon = 1/24 = 0,04166666;$$

$x = 1.002893; \quad H = 1,256230382233478$, we obtain:

$$1.25623 * (((2*e)^8 / ((\ln 3)^{1.0012}))) * \\ (((1+1.0012!/32*(\ln 3/\ln 1.002893)^{1.0012}*(\ln 2/\ln 3))))$$

Input interpretation:

$$1.25623 \times \frac{(2e)^8}{\log^{1.0012}(3)} \left(1 + \frac{1.0012!}{32} \left(\frac{\log(3)}{\log(1.002893)} \right)^{1.0012} \times \frac{\log(2)}{\log(3)} \right)$$

$\log(x)$ is the natural logarithm

$n!$ is the factorial function

Result:

$$7.46489930912948958098161901454328525321249834928522757... \times 10^6$$

$$7.464899309... * 10^6 = 7464899.309....$$

Alternative representations:

$$\frac{\left(1.25623 \left(1 + \frac{1.0012! \left(\frac{\log(3)}{\log(1.00289)} \right)^{1.0012} \log(2)}{32 \log(3)} \right) \right) (2e)^8}{\log^{1.0012}(3)} = \\ \frac{1.25623 (2e)^8 \left(1 + \frac{0.0012!! \times 1.0012!! \log(2) \left(\frac{\log(3)}{\log(1.00289)} \right)^{1.0012}}{32 \log(3)} \right)}{\log^{1.0012}(3)}$$

•

$$\frac{\left(1.25623 \left(1 + \frac{1.0012! \left(\frac{\log(3)}{\log(1.00289)}\right)^{1.0012} \log(2)}{32 \log(3)}\right)\right) (2e)^8}{\log^{1.0012}(3)} = \frac{1.25623 (2e)^8 \left(1 + \frac{\log_e(2) (1)_{1.0012} \left(\frac{\log_e(3)}{\log_e(1.00289)}\right)^{1.0012}}{32 \log_e(3)}\right)}{\log_e^{1.0012}(3)}$$

$$\frac{\left(1.25623 \left(1 + \frac{1.0012! \left(\frac{\log(3)}{\log(1.00289)}\right)^{1.0012} \log(2)}{32 \log(3)}\right)\right) (2e)^8}{\log^{1.0012}(3)} = \frac{1.25623 (2e)^8 \left(1 + \frac{0.0012!! \times 1.0012!! \log_e(2) \left(\frac{\log_e(3)}{\log_e(1.00289)}\right)^{1.0012}}{32 \log_e(3)}\right)}{\log_e^{1.0012}(3)}$$

$n!!$ is the double factorial function

$\log_b(x)$ is the base- b logarithm

$(a)_n$ is the Pochhammer symbol (rising factorial)

Series representations:

$$\frac{\left(1.25623 \left(1 + \frac{1.0012! \left(\frac{\log(3)}{\log(1.00289)}\right)^{1.0012} \log(2)}{32 \log(3)}\right)\right) (2e)^8}{\log^{1.0012}(3)} = \frac{321.595 e^8}{\log^{1.0012}(3)} + \frac{10.0498 e^8 \log(2) \left(\frac{\log(3)}{\log(1.00289)}\right)^{1.0012} \sum_{k=0}^{\infty} \frac{(1.0012 - n_0)^k \Gamma^{(k)}(1 + n_0)}{k!}}{\log^{2.0012}(3)}$$

for $(n_0 \notin \mathbb{Z} \text{ or } n_0 \geq 0)$ and $n_0 \rightarrow 1.0012$

$$\begin{aligned}
& \left(1.25623 \left(1 + \frac{1.0012! \left(\frac{\log(3)}{\log(1.00289)} \right)^{1.0012} \log(2)}{32 \log(3)} \right) \right) (2e)^8 \\
& \frac{\hspace{10em}}{\log^{1.0012}(3)} = \\
& \left(10.0498 \left(32 e^8 \left(2 i \pi \left[\frac{\arg(3-x)}{2 \pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (3-x)^k x^{-k}}{k} \right)^{2.0012} + 2 e^8 i \pi \right. \right. \\
& \quad \left. \left[\frac{\arg(2-x)}{2 \pi} \right] \left(2 i \pi \left[\frac{\arg(3-x)}{2 \pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (3-x)^k x^{-k}}{k} \right)^{1.0012} \right. \\
& \quad \left. \left(\frac{2 i \pi \left[\frac{\arg(3-x)}{2 \pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (3-x)^k x^{-k}}{k} \right)^{1.0012} \right. \\
& \quad \left. \left(2 i \pi \left[\frac{\arg(1.00289-x)}{2 \pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (1.00289-x)^k x^{-k}}{k} \right) \right. \\
& \quad \left. \sum_{k=0}^{\infty} \frac{(1.0012 - n_0)^k \Gamma^{(k)}(1 + n_0)}{k!} + \right. \\
& \quad e^8 \log(x) \left(2 i \pi \left[\frac{\arg(3-x)}{2 \pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (3-x)^k x^{-k}}{k} \right)^{1.0012} \\
& \quad \left(\frac{2 i \pi \left[\frac{\arg(3-x)}{2 \pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (3-x)^k x^{-k}}{k} \right)^{1.0012} \\
& \quad \left(2 i \pi \left[\frac{\arg(1.00289-x)}{2 \pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (1.00289-x)^k x^{-k}}{k} \right) \\
& \quad \left. \sum_{k=0}^{\infty} \frac{(1.0012 - n_0)^k \Gamma^{(k)}(1 + n_0)}{k!} - \right. \\
& \quad e^8 \left(2 i \pi \left[\frac{\arg(3-x)}{2 \pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (3-x)^k x^{-k}}{k} \right)^{1.0012} \\
& \quad \left(\frac{2 i \pi \left[\frac{\arg(3-x)}{2 \pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (3-x)^k x^{-k}}{k} \right)^{1.0012} \\
& \quad \left(2 i \pi \left[\frac{\arg(1.00289-x)}{2 \pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (1.00289-x)^k x^{-k}}{k} \right) \\
& \quad \left. \sum_{k_1=1}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1} (2-x)^{k_1} x^{-k_1} (1.0012 - n_0)^{k_2} \Gamma^{(k_2)}(1 + n_0)}{k_2! k_1} \right) \Bigg) \Bigg) \Bigg) \\
& \left(2 i \pi \left[\frac{\arg(3-x)}{2 \pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (3-x)^k x^{-k}}{k} \right)^{3.0024}
\end{aligned}$$

for $(n_0 \notin \mathbb{Z} \text{ or } n_0 \geq 0)$ and

$x < 0$

and

$n_0 \rightarrow 1.0012$)

$$\frac{\left(1.25623 \left(1 + \frac{1.0012! \left(\frac{-\log(3)}{\log(1.00289)}\right)^{1.0012} \log(2)}{32 \log(3)}\right)\right) (2e)^8}{\log^{1.0012}(3)} = 10.0498$$

$$\left(32 e^8 \left(2 i \pi \left[\frac{\pi - \arg\left(\frac{3}{z_0}\right) - \arg(z_0)}{2 \pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (3 - z_0)^k z_0^{-k}}{k} \right)^{2.0012} + \right. \\ \left. 2 e^8 i \pi \left[\frac{\pi - \arg\left(\frac{2}{z_0}\right) - \arg(z_0)}{2 \pi} \right] \right. \\ \left. \left(2 i \pi \left[\frac{\pi - \arg\left(\frac{3}{z_0}\right) - \arg(z_0)}{2 \pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (3 - z_0)^k z_0^{-k}}{k} \right)^{1.0012} \right. \\ \left. \left(\frac{2 i \pi \left[\frac{\pi - \arg\left(\frac{3}{z_0}\right) - \arg(z_0)}{2 \pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (3 - z_0)^k z_0^{-k}}{k}}{2 i \pi \left[\frac{\pi - \arg\left(\frac{1.00289}{z_0}\right) - \arg(z_0)}{2 \pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (1.00289 - z_0)^k z_0^{-k}}{k}} \right)^{1.0012} \right. \\ \left. \sum_{k=0}^{\infty} \frac{(1.0012 - n_0)^k \Gamma^{(k)}(1 + n_0)}{k!} + e^8 \log(z_0) \right. \\ \left. \left(2 i \pi \left[\frac{\pi - \arg\left(\frac{3}{z_0}\right) - \arg(z_0)}{2 \pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (3 - z_0)^k z_0^{-k}}{k} \right)^{1.0012} \right. \\ \left. \left(\frac{2 i \pi \left[\frac{\pi - \arg\left(\frac{3}{z_0}\right) - \arg(z_0)}{2 \pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (3 - z_0)^k z_0^{-k}}{k}}{2 i \pi \left[\frac{\pi - \arg\left(\frac{1.00289}{z_0}\right) - \arg(z_0)}{2 \pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (1.00289 - z_0)^k z_0^{-k}}{k}} \right)^{1.0012} \right. \\ \left. \sum_{k=0}^{\infty} \frac{(1.0012 - n_0)^k \Gamma^{(k)}(1 + n_0)}{k!} - \right. \\ \left. e^8 \left(\left(2 i \pi \left[\frac{\pi - \arg\left(\frac{3}{z_0}\right) - \arg(z_0)}{2 \pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (3 - z_0)^k z_0^{-k}}{k} \right)^{1.0012} \right. \right. \\ \left. \left(\left(2 i \pi \left[\frac{\pi - \arg\left(\frac{3}{z_0}\right) - \arg(z_0)}{2 \pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (3 - z_0)^k z_0^{-k}}{k} \right) + \right. \right. \\ \left. \left(2 i \pi \left[\frac{\pi - \arg\left(\frac{1.00289}{z_0}\right) - \arg(z_0)}{2 \pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (1.00289 - z_0)^k z_0^{-k}}{k} \right) \right)^{1.0012} \\ \left. \left. \sum_{k=1}^{\infty} \frac{(-1)^k (1.00289 - z_0)^k z_0^{-k}}{k} \right) \right) \right) \\ \left. \left. \sum_{k_1=1}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1} (1.0012 - n_0)^{k_2} (2 - z_0)^{k_1} z_0^{-k_1} \Gamma^{(k_2)}(1 + n_0)}{k_2! k_1} \right) \right) \Bigg/ \\ \left(2 i \pi \left[\frac{\pi - \arg\left(\frac{3}{z_0}\right) - \arg(z_0)}{2 \pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (3 - z_0)^k z_0^{-k}}{k} \right)^{3.0024}$$

for

$$\begin{aligned} & ((n_0 \notin \mathbb{Z} \text{ or } n_0 \geq 0) \\ & \text{and} \\ & n_0 \rightarrow 1.0012) \end{aligned}$$

$$\begin{aligned}
& \left(1.25623 \left(1 + \frac{1.0012! \left(\frac{\log(3)}{\log(1.00289)} \right)^{1.0012} \log(2)}{32 \log(3)} \right) \right) (2e)^8 \\
& \frac{\log^{1.0012}(3)}{10.0498 \left(32 e^8 \left(\log(z_0) + \left\lfloor \frac{\arg(3-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \right. \right. \\
& \quad \left. \left. \sum_{k=1}^{\infty} \frac{(-1)^k (3-z_0)^k z_0^{-k}}{k} \right)^{2.0012} + \right. \\
& \quad \left. e^8 \left\lfloor \frac{\arg(2-z_0)}{2\pi} \right\rfloor \log\left(\frac{1}{z_0}\right) \left(\log(z_0) + \left\lfloor \frac{\arg(3-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \right. \right. \\
& \quad \left. \left. \sum_{k=1}^{\infty} \frac{(-1)^k (3-z_0)^k z_0^{-k}}{k} \right)^{1.0012} \right. \\
& \quad \left. \left(\left(\log(z_0) + \left\lfloor \frac{\arg(3-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (3-z_0)^k z_0^{-k}}{k} \right) / \right. \right. \\
& \quad \left. \left(\log(z_0) + \left\lfloor \frac{\arg(1.00289-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \right. \right. \\
& \quad \left. \left. \sum_{k=1}^{\infty} \frac{(-1)^k (1.00289-z_0)^k z_0^{-k}}{k} \right)^{1.0012} \right) \\
& \quad \sum_{k=0}^{\infty} \frac{(1.0012-n_0)^k \Gamma^{(k)}(1+n_0)}{k!} + e^8 \log(z_0) \\
& \quad \left(\log(z_0) + \left\lfloor \frac{\arg(3-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \right. \\
& \quad \left. \sum_{k=1}^{\infty} \frac{(-1)^k (3-z_0)^k z_0^{-k}}{k} \right)^{1.0012} \\
& \quad \left(\left(\log(z_0) + \left\lfloor \frac{\arg(3-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (3-z_0)^k z_0^{-k}}{k} \right) / \right. \\
& \quad \left(\log(z_0) + \left\lfloor \frac{\arg(1.00289-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \right. \\
& \quad \left. \sum_{k=1}^{\infty} \frac{(-1)^k (1.00289-z_0)^k z_0^{-k}}{k} \right)^{1.0012} \\
& \quad \sum_{k=0}^{\infty} \frac{(1.0012-n_0)^k \Gamma^{(k)}(1+n_0)}{k!} + e^8 \left\lfloor \frac{\arg(2-z_0)}{2\pi} \right\rfloor \log(z_0) \\
& \quad \left(\log(z_0) + \left\lfloor \frac{\arg(3-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \right. \\
& \quad \left. \sum_{k=1}^{\infty} \frac{(-1)^k (3-z_0)^k z_0^{-k}}{k} \right)^{1.0012} \\
& \quad \left(\left(\log(z_0) + \left\lfloor \frac{\arg(3-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (3-z_0)^k z_0^{-k}}{k} \right) / \right. \\
& \quad \left(\log(z_0) + \left\lfloor \frac{\arg(1.00289-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \right. \\
& \quad \left. \sum_{k=1}^{\infty} \frac{(-1)^k (1.00289-z_0)^k z_0^{-k}}{k} \right)^{1.0012} \\
& \quad \sum_{k=0}^{\infty} \frac{(1.0012-n_0)^k \Gamma^{(k)}(1+n_0)}{k!} - e^8 \left(\left(\log(z_0) + \left\lfloor \frac{\arg(3-z_0)}{2\pi} \right\rfloor \right. \right. \\
& \quad \left. \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (3-z_0)^k z_0^{-k}}{k} \right)^{1.0012} \\
& \quad \left(\left(\log(z_0) + \left\lfloor \frac{\arg(3-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \right. \right. \\
& \quad \left. \left. \sum_{k=1}^{\infty} \frac{(-1)^k (3-z_0)^k z_0^{-k}}{k} \right) / \right. \\
& \quad \left(\log(z_0) + \left\lfloor \frac{\arg(1.00289-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \right. \\
& \quad \left. \sum_{k=1}^{\infty} \frac{(-1)^k (1.00289-z_0)^k z_0^{-k}}{k} \right)^{1.0012} \\
& \quad \left. \left. \sum_{k_1=1}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1} (1.0012-n_0)^{k_2} (2-z_0)^{k_1} z_0^{-k_1} \Gamma^{(k_2)}(1+n_0)}{k_2! k_1} \right) \right) / \\
& \quad \left(\log(z_0) + \left\lfloor \frac{\arg(3-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (3-z_0)^k z_0^{-k}}{k} \right)^{3.0024}
\end{aligned}$$

for

(($n_0 \notin \mathbb{Z}$ or $n_0 \geq 0$) and $n_0 \rightarrow 1.0012$)

$\Gamma(x)$ is the gamma function

\mathbb{Z} is the set of integers

$\text{arg}(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

Integral representations:

$$\frac{\left(1.25623 \left(1 + \frac{1.0012! \left(\frac{\log(3)}{\log(1.00289)}\right)^{1.0012} \log(2)}{32 \log(3)}\right)\right) (2e)^8}{\log^{1.0012}(3)} = \frac{321.595 e^8}{\log^{1.0012}(3)} + \frac{10.0498 e^8 \log(2) \left(\frac{\log(3)}{\log(1.00289)}\right)^{1.0012}}{\log^{2.0012}(3)} \int_0^1 \log^{1.0012}\left(\frac{1}{t}\right) dt$$

•

$$\frac{\left(1.25623 \left(1 + \frac{1.0012! \left(\frac{\log(3)}{\log(1.00289)}\right)^{1.0012} \log(2)}{32 \log(3)}\right)\right) (2e)^8}{\log^{1.0012}(3)} = \frac{10.0498 e^8 \left(32 \left(\int_1^3 \frac{1}{t} dt\right)^{2.0012} + \int_0^1 \int_0^1 \frac{\log^{1.0012}\left(\frac{1}{t_2}\right)}{1+t_1} dt_2 dt_1\right)}{\left(\int_1^3 \frac{1}{t} dt\right)^{3.0024}}$$

•

$$\frac{\left(1.25623 \left(1 + \frac{1.0012! \left(\frac{\log(3)}{\log(1.00289)}\right)^{1.0012} \log(2)}{32 \log(3)}\right)\right) (2e)^8}{\log^{1.0012}(3)} = \frac{321.595 e^8}{\log^{1.0012}(3)} + \frac{10.0498 e^8 \log(2) \left(\frac{\log(3)}{\log(1.00289)}\right)^{1.0012}}{\log^{2.0012}(3)} \int_1^\infty t^{1.0012} \mathcal{A}^{-t} dt + \frac{10.0498 e^8 \log(2) \left(\frac{\log(3)}{\log(1.00289)}\right)^{1.0012} \sum_{k=0}^\infty \frac{(-1)^k}{(2.0012+k)k!}}{\log^{2.0012}(3)}$$

We note that:

$$\left(\left(\left(\left(\left(1.25623 * \left(\left(2e\right)^8 / \left(\left(\ln 3\right)^{1.0012}\right)\right)\right) * \left(\left(1 + 1.0012! / 32 * \left(\ln 3 / \ln 1.002893\right)^{1.0012} * \left(\ln 2 / \ln 3\right)\right)\right)\right)\right)\right)\right)^{1/32}$$

Input interpretation:

$$\sqrt[32]{1.25623 \times \frac{(2e)^8}{\log^{1.0012}(3)} \left(1 + \frac{1.0012!}{32} \left(\frac{\log(3)}{\log(1.002893)}\right)^{1.0012} \times \frac{\log(2)}{\log(3)}\right)}$$

log(x) is the natural logarithm
n! is the factorial function

Result:

1.639766458168004084764132857631915703511047148480676779344...
1.639766458168...

And:

$$(29+3)/10^3 + ((((((1.25623 * (((2*e)^8 / ((ln3)^1.0012)))) * ((1+1.0012!/32*(ln3/ln1.002893)^1.0012*(ln2/ln3))))))))))^{1/32}$$

where 29 and 3 are Lucas numbers

Input interpretation:

$$\frac{29+3}{10^3} + \sqrt[32]{1.25623 \times \frac{(2e)^8}{\log^{1.0012}(3)} \left(1 + \frac{1.0012!}{32} \left(\frac{\log(3)}{\log(1.002893)}\right)^{1.0012} \times \frac{\log(2)}{\log(3)}\right)}$$

log(x) is the natural logarithm
n! is the factorial function

Result:

1.671766458168004084764132857631915703511047148480676779344...
1.671766458.... result practically equal to the value of the formula:

$$m_{p'} = 2 \times \frac{\eta}{R} m_p = 1.6714213 \times 10^{-24} \text{ gm}$$

that is the holographic proton mass

Alternative representations:

$$\frac{29+3}{10^3} + \sqrt[32]{\frac{\left(1.25623 \left(1 + \frac{1.0012! \left(\frac{\log(3)}{\log(1.002893)}\right)^{1.0012} \log(2)}{32 \log(3)}\right)\right) (2e)^8}{\log^{1.0012}(3)}} = \frac{\frac{32}{10^3} + \sqrt[32]{\frac{1.25623 (2e)^8 \left(1 + \frac{0.0012!! \times 1.0012!! \log(2) \left(\frac{\log(3)}{\log(1.002893)}\right)^{1.0012}}{32 \log(3)}\right)}{\log^{1.0012}(3)}}}$$

$$\frac{29+3}{10^3} + \sqrt[32]{\frac{\left(1.25623 \left(1 + \frac{1.0012! \left(\frac{\log(3)}{\log(1.00289)}\right)^{1.0012} \log(2)\right)}{32 \log(3)}\right) (2e)^8}{\log^{1.0012}(3)}}} =$$

$$\frac{32}{10^3} + \sqrt[32]{\frac{1.25623 (2e)^8 \left(1 + \frac{\log_e(2) (1)_{1.0012} \left(\frac{\log_e(3)}{\log_e(1.00289)}\right)^{1.0012}}{32 \log_e(3)}\right)}{\log_e^{1.0012}(3)}}$$

$$\frac{29+3}{10^3} + \sqrt[32]{\frac{\left(1.25623 \left(1 + \frac{1.0012! \left(\frac{\log(3)}{\log(1.00289)}\right)^{1.0012} \log(2)\right)}{32 \log(3)}\right) (2e)^8}{\log^{1.0012}(3)}}} =$$

$$\frac{32}{10^3} + \sqrt[32]{\frac{1.25623 (2e)^8 \left(1 + \frac{0.0012!! \times 1.0012!! \log_e(2) \left(\frac{\log_e(3)}{\log_e(1.00289)}\right)^{1.0012}}{32 \log_e(3)}\right)}{\log_e^{1.0012}(3)}}$$

Series representations:

$$\frac{29+3}{10^3} + \sqrt[32]{\frac{\left(1.25623 \left(1 + \frac{1.0012! \left(\frac{\log(3)}{\log(1.00289)}\right)^{1.0012} \log(2)\right)}{32 \log(3)}\right) (2e)^8}{\log^{1.0012}(3)}}} = 0.032 +$$

$$1.07477 \sqrt[32]{\frac{e^8 \left(32 \log(3) + \log(2) \left(\frac{\log(3)}{\log(1.00289)}\right)^{1.0012} \sum_{k=0}^{\infty} \frac{(1.0012 - n_0)^k \Gamma^{(k)}(1+n_0)}{k!}\right)}{\log^{2.0012}(3)}}$$

for $(n_0 \notin \mathbb{Z} \text{ or } n_0 \geq 0)$ and $n_0 \rightarrow 1.0012$

$$\begin{aligned}
& \frac{29+3}{10^3} + \sqrt[32]{\frac{\left(1.25623 \left(1 + \frac{1.0012! \left(\frac{-\log(3)}{\log(1.00289)}\right)^{1.0012} \log(2)}{32 \log(3)}\right)\right) (2e)^8}{\log^{1.0012}(3)}}} = \\
& 0.598857 \left(0.0534351 + 1.79471 \left(\left(e^8 \left[64 i \pi \left[\frac{\arg(3-x)}{2\pi} \right] + 32 \log(x) - \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left. \sum_{k=1}^{\infty} \frac{(-1)^k (3-x)^k x^{-k}}{k} + 2 i \pi \left[\frac{\arg(2-x)}{2\pi} \right] \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left. \left(\left(2 i \pi \left[\frac{\arg(3-x)}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (3-x)^k x^{-k}}{k} \right) / \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left. \left(2 i \pi \left[\frac{\arg(1.00289-x)}{2\pi} \right] + \log(x) - \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left. \sum_{k=1}^{\infty} \frac{(-1)^k (1.00289-x)^k x^{-k}}{k} \right) \right) \right) \right) \right) \right)^{1.0012} \\
& \sum_{k=0}^{\infty} \frac{(1.0012 - n_0)^k \Gamma^{(k)}(1 + n_0)}{k!} + \log(x) \\
& \left(\left(2 i \pi \left[\frac{\arg(3-x)}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (3-x)^k x^{-k}}{k} \right) / \right. \\
& \left. \left(2 i \pi \left[\frac{\arg(1.00289-x)}{2\pi} \right] + \log(x) - \right. \right. \\
& \left. \left. \sum_{k=1}^{\infty} \frac{(-1)^k (1.00289-x)^k x^{-k}}{k} \right) \right) \right)^{1.0012} \\
& \sum_{k=0}^{\infty} \frac{(1.0012 - n_0)^k \Gamma^{(k)}(1 + n_0)}{k!} - \\
& \left(\left(2 i \pi \left[\frac{\arg(3-x)}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (3-x)^k x^{-k}}{k} \right) / \right. \\
& \left. \left(2 i \pi \left[\frac{\arg(1.00289-x)}{2\pi} \right] + \log(x) - \right. \right. \\
& \left. \left. \sum_{k=1}^{\infty} \frac{(-1)^k (1.00289-x)^k x^{-k}}{k} \right) \right) \right)^{1.0012} \\
& \sum_{k_1=1}^{\infty} \sum_{k_2=0}^{\infty} \\
& \left. \left. \left. \left. \left. \frac{(-1)^{k_1} (2-x)^{k_1} x^{-k_1} (1.0012 - n_0)^{k_2} \Gamma^{(k_2)}(1 + n_0)}{k_2! k_1} \right) \right) \right) \right) \right) \\
& \left(2 i \pi \left[\frac{\arg(3-x)}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (3-x)^k x^{-k}}{k} \right)^{2.0012} \bigg)^{\wedge (1/32)}
\end{aligned}$$

for $(n_0 \notin \mathbb{Z} \text{ or } n_0 \geq 0)$ and $x < 0$ and $n_0 \rightarrow 1.0012$

$$\begin{aligned}
& \frac{29+3}{10^3} + \sqrt[32]{\frac{\left(1.25623 \left(1 + \frac{1.0012! \left(\frac{\log(3)}{\log(1.00289)}\right)^{1.0012} \log(2)\right)\right)}{\log^{1.0012}(3)}} (2e)^8} = \\
& 0.598857 \left\{ 0.0534351 + 1.79471 \right. \\
& \left. \left(e^8 \left(64 i \pi \left[\frac{\pi - \arg\left(\frac{3}{z_0}\right) - \arg(z_0)}{2\pi} \right] + 32 \log(z_0) - 32 \sum_{k=1}^{\infty} \frac{(-1)^k (3-z_0)^k z_0^{-k}}{k} + \right. \right. \right. \\
& \left. \left. \left. 2 i \pi \left[\frac{\pi - \arg\left(\frac{2}{z_0}\right) - \arg(z_0)}{2\pi} \right] \left(\left(2 i \pi \left[\frac{\pi - \arg\left(\frac{3}{z_0}\right) - \arg(z_0)}{2\pi} \right] + \right. \right. \right. \right. \\
& \left. \left. \left. \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (3-z_0)^k z_0^{-k}}{k} \right) / \right. \right. \right. \\
& \left. \left. \left. \left(2 i \pi \left[\frac{\pi - \arg\left(\frac{1.00289}{z_0}\right) - \arg(z_0)}{2\pi} \right] + \log(z_0) - \right. \right. \right. \\
& \left. \left. \left. \sum_{k=1}^{\infty} \frac{(-1)^k (1.00289 - z_0)^k z_0^{-k}}{k} \right) \right)^{1.0012} \right. \right. \\
& \left. \left. \sum_{k=0}^{\infty} \frac{(1.0012 - n_0)^k \Gamma^{(k)}(1 + n_0)}{k!} + \log(z_0) \right. \right. \\
& \left. \left. \left(\left(2 i \pi \left[\frac{\pi - \arg\left(\frac{3}{z_0}\right) - \arg(z_0)}{2\pi} \right] + \log(z_0) - \right. \right. \right. \right. \\
& \left. \left. \left. \sum_{k=1}^{\infty} \frac{(-1)^k (3-z_0)^k z_0^{-k}}{k} \right) / \right. \right. \right. \\
& \left. \left. \left. \left(2 i \pi \left[\frac{\pi - \arg\left(\frac{1.00289}{z_0}\right) - \arg(z_0)}{2\pi} \right] + \log(z_0) - \right. \right. \right. \\
& \left. \left. \left. \sum_{k=1}^{\infty} \frac{(-1)^k (1.00289 - z_0)^k z_0^{-k}}{k} \right) \right)^{1.0012} \right. \right. \\
& \left. \left. \sum_{k=0}^{\infty} \frac{(1.0012 - n_0)^k \Gamma^{(k)}(1 + n_0)}{k!} - \right. \right. \\
& \left. \left. \left(\left(2 i \pi \left[\frac{\pi - \arg\left(\frac{3}{z_0}\right) - \arg(z_0)}{2\pi} \right] + \log(z_0) - \right. \right. \right. \right. \\
& \left. \left. \left. \sum_{k=1}^{\infty} \frac{(-1)^k (3-z_0)^k z_0^{-k}}{k} \right) / \right. \right. \right. \\
& \left. \left. \left. \left(2 i \pi \left[\frac{\pi - \arg\left(\frac{1.00289}{z_0}\right) - \arg(z_0)}{2\pi} \right] + \log(z_0) - \right. \right. \right. \\
& \left. \left. \left. \sum_{k=1}^{\infty} \frac{(-1)^k (1.00289 - z_0)^k z_0^{-k}}{k} \right) \right)^{1.0012} \right. \right. \\
& \left. \left. \sum_{k_1=1}^{\infty} \sum_{k_2=0}^{\infty} \right. \right. \\
& \left. \left. \frac{(-1)^{k_1} (1.0012 - n_0)^{k_2} (2 - z_0)^{k_1} z_0^{-k_1} \Gamma^{(k_2)}(1 + n_0)}{k_2! k_1} \right) \right) \\
& \left. / \left(2 i \pi \left[\frac{\pi - \arg\left(\frac{3}{z_0}\right) - \arg(z_0)}{2\pi} \right] + \right. \right. \\
& \left. \left. \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (3-z_0)^k z_0^{-k}}{k} \right)^{2.0012} \right) \wedge (1/32) \right)
\end{aligned}$$

for $(n_0 \notin \mathbb{Z} \text{ or } n_0 \geq 0)$ and $n_0 \rightarrow 1.0012$)

$$\begin{aligned}
& \frac{29+3}{10^3} + \sqrt[32]{\frac{\left(1.25623 \left(1 + \frac{1.0012! \left(\frac{\log(3)}{\log(1.00289)}\right)^{1.0012} \log(2)}{32 \log(3)}\right) (2e)^8}{\log^{1.0012}(3)}\right)} = \\
& 0.598857 \left(0.0534351 + 1.79471 \right. \\
& \left. \left(e^8 \left(32 \left[\frac{\arg(3-z_0)}{2\pi} \right] \log\left(\frac{1}{z_0}\right) + 32 \log(z_0) + 32 \left[\frac{\arg(3-z_0)}{2\pi} \right] \log(z_0) - \right. \right. \right. \\
& \quad 32 \sum_{k=1}^{\infty} \frac{(-1)^k (3-z_0)^k z_0^{-k}}{k} + \left[\frac{\arg(2-z_0)}{2\pi} \right] \log\left(\frac{1}{z_0}\right) \\
& \quad \left(\left(\log(z_0) + \left[\frac{\arg(3-z_0)}{2\pi} \right] \right) \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \right. \\
& \quad \quad \left. \left. \sum_{k=1}^{\infty} \frac{(-1)^k (3-z_0)^k z_0^{-k}}{k} \right) / \left(\log(z_0) + \right. \right. \\
& \quad \quad \left. \left. \left[\frac{\arg(1.00289-z_0)}{2\pi} \right] \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \right. \right. \\
& \quad \quad \left. \left. \sum_{k=1}^{\infty} \frac{(-1)^k (1.00289-z_0)^k z_0^{-k}}{k} \right) \right)^{1.0012} \\
& \quad \sum_{k=0}^{\infty} \frac{(1.0012-n_0)^k \Gamma^{(k)}(1+n_0)}{k!} + \log(z_0) \\
& \quad \left(\left(\log(z_0) + \left[\frac{\arg(3-z_0)}{2\pi} \right] \right) \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \right. \\
& \quad \quad \left. \sum_{k=1}^{\infty} \frac{(-1)^k (3-z_0)^k z_0^{-k}}{k} \right) / \left(\log(z_0) + \right. \\
& \quad \quad \left. \left[\frac{\arg(1.00289-z_0)}{2\pi} \right] \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \right. \\
& \quad \quad \left. \sum_{k=1}^{\infty} \frac{(-1)^k (1.00289-z_0)^k z_0^{-k}}{k} \right) \right)^{1.0012} \\
& \quad \sum_{k=0}^{\infty} \frac{(1.0012-n_0)^k \Gamma^{(k)}(1+n_0)}{k!} + \left[\frac{\arg(2-z_0)}{2\pi} \right] \\
& \quad \log(z_0) \left(\left(\log(z_0) + \left[\frac{\arg(3-z_0)}{2\pi} \right] \right) \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \right. \\
& \quad \quad \left. \sum_{k=1}^{\infty} \frac{(-1)^k (3-z_0)^k z_0^{-k}}{k} \right) / \left(\log(z_0) + \right. \\
& \quad \quad \left. \left[\frac{\arg(1.00289-z_0)}{2\pi} \right] \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \right. \\
& \quad \quad \left. \sum_{k=1}^{\infty} \frac{(-1)^k (1.00289-z_0)^k z_0^{-k}}{k} \right) \right)^{1.0012} \\
& \quad \sum_{k=0}^{\infty} \frac{(1.0012-n_0)^k \Gamma^{(k)}(1+n_0)}{k!} - \\
& \quad \left(\left(\log(z_0) + \left[\frac{\arg(3-z_0)}{2\pi} \right] \right) \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \right. \\
& \quad \quad \left. \sum_{k=1}^{\infty} \frac{(-1)^k (3-z_0)^k z_0^{-k}}{k} \right) / \left(\log(z_0) + \right. \\
& \quad \quad \left. \left[\frac{\arg(1.00289-z_0)}{2\pi} \right] \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \right. \\
& \quad \quad \left. \sum_{k=1}^{\infty} \frac{(-1)^k (1.00289-z_0)^k z_0^{-k}}{k} \right) \right)^{1.0012} \\
& \quad \sum_{k_1=1}^{\infty} \sum_{k_2=0}^{\infty} \\
& \quad \quad \frac{(-1)^{k_1} (1.0012-n_0)^{k_2} (2-z_0)^{k_1} z_0^{-k_1} \Gamma^{(k_2)}(1+n_0)}{k_2! k_1} \left. \right) \\
& \left. \left(\left(\left[\frac{\arg(3-z_0)}{2\pi} \right] \log\left(\frac{1}{z_0}\right) + \log(z_0) + \left[\frac{\arg(3-z_0)}{2\pi} \right] \log(z_0) - \right. \right. \right. \\
& \quad \sum_{k=1}^{\infty} \frac{(-1)^k (3-z_0)^k z_0^{-k}}{k} \left(\log(z_0) + \left[\frac{\arg(3-z_0)}{2\pi} \right] \right. \\
& \quad \quad \left. \left. \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (3-z_0)^k z_0^{-k}}{k} \right)^{1.0012} \right) \right) \wedge \\
& (1/32) \left. \right) \text{ for } (n_0 \notin \mathbb{Z} \text{ or } n_0 \geq 0) \text{ and } n_0 \rightarrow 1.0012
\end{aligned}$$

Integral representations:

$$\frac{29+3}{10^3} + \sqrt[32]{\frac{\left(1.25623 \left(1 + \frac{1.0012! \left(\frac{\log(3)}{\log(1.00289)}\right)^{1.0012} \log(2)\right)}{32 \log(3)}\right) (2e)^8}{\log^{1.0012}(3)}} =$$

$$0.032 + 1.07477 \sqrt[32]{\frac{e^8 \left(32 \log(3) + \log(2) \left(\frac{\log(3)}{\log(1.00289)}\right)^{1.0012} \int_0^1 \log^{1.0012}\left(\frac{1}{t}\right) dt\right)}{\log^{2.0012}(3)}}$$

$$\frac{29+3}{10^3} + \sqrt[32]{\frac{\left(1.25623 \left(1 + \frac{1.0012! \left(\frac{\log(3)}{\log(1.00289)}\right)^{1.0012} \log(2)\right)}{32 \log(3)}\right) (2e)^8}{\log^{1.0012}(3)}} =$$

$$1.07477 \left(0.0297737 + \sqrt[32]{\frac{e^8 \left(32 \int_1^3 \frac{1}{t} dt + \int_0^1 \int_0^1 \frac{\log^{1.0012}\left(\frac{1}{t_2}\right)}{1+t_1} dt_2 dt_1\right)}{\left(\int_1^3 \frac{1}{t} dt\right)^{2.0012}}}\right)$$

$$\frac{29+3}{10^3} + \sqrt[32]{\frac{\left(1.25623 \left(1 + \frac{1.0012! \left(\frac{\log(3)}{\log(1.00289)}\right)^{1.0012} \log(2)\right)}{32 \log(3)}\right) (2e)^8}{\log^{1.0012}(3)}} =$$

$$\frac{4}{125} + 1.19771 \sqrt[32]{\frac{e^8 \left(1 + \frac{\log(2) \left(\frac{\log(3)}{\log(1.00289)}\right)^{1.0012} \left(\int_1^\infty t^{1.0012} \mathcal{A}^{-t} dt + \sum_{k=0}^\infty \frac{(-1)^k}{(2.0012+k)k!}\right)}{32 \log(3)}\right)}{\log^{1.0012}(3)}}$$

$$-21/10^3 + ((((((1.25623 * (((2*e)^8 / ((\ln 3)^{1.0012}))) * ((1+1.0012!/32*(\ln 3/\ln 1.00289)^{1.0012}*(\ln 2/\ln 3))))))))))^{1/32}$$

Input interpretation:

$$-\frac{21}{10^3} + \sqrt[32]{1.25623 \times \frac{(2e)^8}{\log^{1.0012}(3)} \left(1 + \frac{1.0012!}{32} \left(\frac{\log(3)}{\log(1.002893)}\right)^{1.0012} \times \frac{\log(2)}{\log(3)}\right)}$$

Result:

1.618766458168004084764132857631915703511047148480676779344...

1.618766458168.... a very good approximation to the golden ratio 1.61803398...

Alternative representations:

$$-\frac{21}{10^3} + \sqrt[32]{\frac{\left(1.25623 \left(1 + \frac{1.0012! \left(\frac{\log(3)}{\log(1.00289)}\right)^{1.0012} \log(2)\right)}{32 \log(3)}\right) (2e)^8}{\log^{1.0012}(3)}}} =$$

$$-\frac{21}{10^3} + \sqrt[32]{\frac{1.25623 (2e)^8 \left(1 + \frac{0.0012!! \times 1.0012!! \log(2) \left(\frac{\log(3)}{\log(1.00289)}\right)^{1.0012}}{32 \log(3)}\right)}{\log^{1.0012}(3)}}$$

•

$$-\frac{21}{10^3} + \sqrt[32]{\frac{\left(1.25623 \left(1 + \frac{1.0012! \left(\frac{\log(3)}{\log(1.00289)}\right)^{1.0012} \log(2)\right) (2e)^8}{\log^{1.0012}(3)}}} =$$

$$-\frac{21}{10^3} + \sqrt[32]{\frac{1.25623 (2e)^8 \left(1 + \frac{\log_e(2) (1)_{1.0012} \left(\frac{\log_e(3)}{\log_e(1.00289)}\right)^{1.0012}}{32 \log_e(3)}\right)}{\log_e^{1.0012}(3)}}$$

•

$$-\frac{21}{10^3} + \sqrt[32]{\frac{\left(1.25623 \left(1 + \frac{1.0012! \left(\frac{\log(3)}{\log(1.00289)}\right)^{1.0012} \log(2)\right) (2e)^8}{\log^{1.0012}(3)}}} =$$

$$-\frac{21}{10^3} + \sqrt[32]{\frac{1.25623 (2e)^8 \left(1 + \frac{0.0012!! \times 1.0012!! \log_e(2) \left(\frac{\log_e(3)}{\log_e(1.00289)}\right)^{1.0012}}{32 \log_e(3)}\right)}{\log_e^{1.0012}(3)}}$$

Series representations:

$$-\frac{21}{10^3} + \sqrt[32]{\frac{\left(1.25623 \left(1 + \frac{1.0012! \left(\frac{\log(3)}{\log(1.00289)}\right)^{1.0012} \log(2)\right) (2e)^8}{\log^{1.0012}(3)}}} = -0.021 +$$

$$1.07477 \sqrt[32]{\frac{e^8 \left(32 \log(3) + \log(2) \left(\frac{\log(3)}{\log(1.00289)}\right)^{1.0012} \sum_{k=0}^{\infty} \frac{(1.0012 - n_0)^k \Gamma^{(k)}(1+n_0)}{k!}\right)}{\log^{2.0012}(3)}}$$

for $(n_0 \notin \mathbb{Z} \text{ or } n_0 \geq 0)$ and $n_0 \rightarrow 1.0012$

•

$$\begin{aligned}
& -\frac{21}{10^3} + \sqrt[32]{\frac{\left(1.25623 \left(1 + \frac{1.0012! \left(\frac{\log(3)}{\log(1.00289)}\right)^{1.0012} \log(2)}{32 \log(3)}\right)\right) (2e)^8}{\log^{1.0012}(3)}}} = \\
& 0.598857 \left(-0.0350668 + 1.79471 \left(e^8 \left[64 i \pi \left[\frac{\arg(3-x)}{2\pi} \right] + 32 \log(x) - \right. \right. \right. \\
& \quad \left. \left. \left. 32 \sum_{k=1}^{\infty} \frac{(-1)^k (3-x)^k x^{-k}}{k} + 2 i \pi \left[\frac{\arg(2-x)}{2\pi} \right] \right. \right. \right. \\
& \quad \left. \left. \left. \left(\left[2 i \pi \left[\frac{\arg(3-x)}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (3-x)^k x^{-k}}{k} \right] / \right. \right. \right. \\
& \quad \left. \left. \left. \left(\left[2 i \pi \left[\frac{\arg(1.00289-x)}{2\pi} \right] + \log(x) - \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \sum_{k=1}^{\infty} \frac{(-1)^k (1.00289-x)^k x^{-k}}{k} \right) \right] \right)^{1.0012} \right. \right. \\
& \quad \left. \left. \sum_{k=0}^{\infty} \frac{(1.0012 - n_0)^k \Gamma^{(k)}(1 + n_0)}{k!} + \log(x) \right. \right. \\
& \quad \left. \left. \left(\left[2 i \pi \left[\frac{\arg(3-x)}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (3-x)^k x^{-k}}{k} \right] / \right. \right. \right. \\
& \quad \left. \left. \left. \left(\left[2 i \pi \left[\frac{\arg(1.00289-x)}{2\pi} \right] + \log(x) - \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \sum_{k=1}^{\infty} \frac{(-1)^k (1.00289-x)^k x^{-k}}{k} \right) \right] \right)^{1.0012} \right. \right. \\
& \quad \left. \left. \sum_{k=0}^{\infty} \frac{(1.0012 - n_0)^k \Gamma^{(k)}(1 + n_0)}{k!} - \right. \right. \\
& \quad \left. \left. \left(\left[2 i \pi \left[\frac{\arg(3-x)}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (3-x)^k x^{-k}}{k} \right] / \right. \right. \right. \\
& \quad \left. \left. \left. \left(\left[2 i \pi \left[\frac{\arg(1.00289-x)}{2\pi} \right] + \log(x) - \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \sum_{k=1}^{\infty} \frac{(-1)^k (1.00289-x)^k x^{-k}}{k} \right) \right] \right)^{1.0012} \right. \right. \\
& \quad \left. \left. \sum_{k_1=1}^{\infty} \sum_{k_2=0}^{\infty} \right. \right. \\
& \quad \left. \left. \left. \frac{(-1)^{k_1} (2-x)^{k_1} x^{-k_1} (1.0012 - n_0)^{k_2} \Gamma^{(k_2)}(1 + n_0)}{k_2! k_1} \right) \right) / \right. \\
& \quad \left. \left(\left[2 i \pi \left[\frac{\arg(3-x)}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (3-x)^k x^{-k}}{k} \right]^{2.0012} \right)^{\wedge (1/32)} \right)
\end{aligned}$$

for $(n_0 \notin \mathbb{Z} \text{ or } n_0 \geq 0)$ and $x < 0$ and $n_0 \rightarrow 1.0012$

$$\begin{aligned}
& -\frac{21}{10^3} + 32 \sqrt[3]{\frac{\left(1.25623 \left(1 + \frac{1.0012! \left(\frac{\log(3)}{\log(1.00289)}\right)^{1.0012} \log(2)}{32 \log(3)}\right)\right) (2e)^8}{\log^{1.0012}(3)}}} = \\
& 0.598857 \left\{ -0.0350668 + 1.79471 \right. \\
& \left. \left(\left(e^8 \left[64 i \pi \left| \frac{\pi - \arg\left(\frac{3}{z_0}\right) - \arg(z_0)}{2\pi} \right| + 32 \log(z_0) - 32 \sum_{k=1}^{\infty} \frac{(-1)^k (3-z_0)^k z_0^{-k}}{k} + \right. \right. \right. \\
& \left. \left. \left. 2 i \pi \left| \frac{\pi - \arg\left(\frac{2}{z_0}\right) - \arg(z_0)}{2\pi} \right| \right) \left(\left(2 i \pi \left| \frac{\pi - \arg\left(\frac{3}{z_0}\right) - \arg(z_0)}{2\pi} \right| + \right. \right. \right. \\
& \left. \left. \left. \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (3-z_0)^k z_0^{-k}}{k} \right) / \right. \right. \\
& \left. \left. \left(2 i \pi \left| \frac{\pi - \arg\left(\frac{1.00289}{z_0}\right) - \arg(z_0)}{2\pi} \right| + \log(z_0) - \right. \right. \\
& \left. \left. \left. \sum_{k=1}^{\infty} \frac{(-1)^k (1.00289 - z_0)^k z_0^{-k}}{k} \right) \right)^{1.0012} \right. \\
& \left. \sum_{k=0}^{\infty} \frac{(1.0012 - n_0)^k \Gamma^{(k)}(1 + n_0)}{k!} + \log(z_0) \right. \\
& \left. \left(\left(2 i \pi \left| \frac{\pi - \arg\left(\frac{3}{z_0}\right) - \arg(z_0)}{2\pi} \right| + \log(z_0) - \right. \right. \right. \\
& \left. \left. \left. \sum_{k=1}^{\infty} \frac{(-1)^k (3-z_0)^k z_0^{-k}}{k} \right) / \right. \right. \\
& \left. \left. \left(2 i \pi \left| \frac{\pi - \arg\left(\frac{1.00289}{z_0}\right) - \arg(z_0)}{2\pi} \right| + \log(z_0) - \right. \right. \\
& \left. \left. \left. \sum_{k=1}^{\infty} \frac{(-1)^k (1.00289 - z_0)^k z_0^{-k}}{k} \right) \right)^{1.0012} \right. \\
& \left. \sum_{k=0}^{\infty} \frac{(1.0012 - n_0)^k \Gamma^{(k)}(1 + n_0)}{k!} - \right. \\
& \left. \left(\left(2 i \pi \left| \frac{\pi - \arg\left(\frac{3}{z_0}\right) - \arg(z_0)}{2\pi} \right| + \log(z_0) - \right. \right. \right. \\
& \left. \left. \left. \sum_{k=1}^{\infty} \frac{(-1)^k (3-z_0)^k z_0^{-k}}{k} \right) / \right. \right. \\
& \left. \left. \left(2 i \pi \left| \frac{\pi - \arg\left(\frac{1.00289}{z_0}\right) - \arg(z_0)}{2\pi} \right| + \log(z_0) - \right. \right. \\
& \left. \left. \left. \sum_{k=1}^{\infty} \frac{(-1)^k (1.00289 - z_0)^k z_0^{-k}}{k} \right) \right)^{1.0012} \right. \\
& \left. \sum_{k_1=1}^{\infty} \sum_{k_2=0}^{\infty} \right. \\
& \left. \left. \frac{(-1)^{k_1} (1.0012 - n_0)^{k_2} (2 - z_0)^{k_1} z_0^{-k_1} \Gamma^{(k_2)}(1 + n_0)}{k_2! k_1} \right) \right) \\
& \left. / \left(2 i \pi \left| \frac{\pi - \arg\left(\frac{3}{z_0}\right) - \arg(z_0)}{2\pi} \right| + \right. \right. \\
& \left. \left. \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (3-z_0)^k z_0^{-k}}{k} \right)^{2.0012} \right) \wedge (1/32) \Big)
\end{aligned}$$

for $(n_0 \notin \mathbb{Z} \text{ or } n_0 \geq 0)$ and $n_0 \rightarrow 1.0012$

$$\begin{aligned}
& -\frac{21}{10^3} + 32 \sqrt[3]{\frac{\left(1.25623 \left(1 + \frac{1.0012! \left(\frac{\log(3)}{\log(1.00289)}\right)^{1.0012} \log(2)\right)}{32 \log(3)}\right) (2e)^8}{\log^{1.0012}(3)}}} = \\
& 0.598857 \left(-0.0350668 + 1.79471 \right. \\
& \left. \left(\left(e^8 \left(32 \left[\frac{\arg(3-z_0)}{2\pi} \right] \log\left(\frac{1}{z_0}\right) + 32 \log(z_0) + 32 \left[\frac{\arg(3-z_0)}{2\pi} \right] \log(z_0) - \right. \right. \right. \right. \\
& \left. \left. \left. 32 \sum_{k=1}^{\infty} \frac{(-1)^k (3-z_0)^k z_0^{-k}}{k} + \left[\frac{\arg(2-z_0)}{2\pi} \right] \log\left(\frac{1}{z_0}\right) \right. \right. \right. \\
& \left. \left. \left. \left(\left(\log(z_0) + \left[\frac{\arg(3-z_0)}{2\pi} \right] \right) \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \right. \right. \right. \right. \\
& \left. \left. \left. \sum_{k=1}^{\infty} \frac{(-1)^k (3-z_0)^k z_0^{-k}}{k} \right) / \left(\log(z_0) + \right. \right. \right. \\
& \left. \left. \left. \left[\frac{\arg(1.00289-z_0)}{2\pi} \right] \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \right. \right. \right. \\
& \left. \left. \left. \left. \sum_{k=1}^{\infty} \frac{(-1)^k (1.00289-z_0)^k z_0^{-k}}{k} \right) \right) \right)^{1.0012} \right) \\
& \sum_{k=0}^{\infty} \frac{(1.0012-n_0)^k \Gamma^{(k)}(1+n_0)}{k!} + \log(z_0) \\
& \left(\left(\log(z_0) + \left[\frac{\arg(3-z_0)}{2\pi} \right] \right) \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \right. \\
& \left. \sum_{k=1}^{\infty} \frac{(-1)^k (3-z_0)^k z_0^{-k}}{k} \right) / \left(\log(z_0) + \right. \\
& \left. \left[\frac{\arg(1.00289-z_0)}{2\pi} \right] \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \right. \\
& \left. \left. \sum_{k=1}^{\infty} \frac{(-1)^k (1.00289-z_0)^k z_0^{-k}}{k} \right) \right)^{1.0012} \\
& \sum_{k=0}^{\infty} \frac{(1.0012-n_0)^k \Gamma^{(k)}(1+n_0)}{k!} + \left[\frac{\arg(2-z_0)}{2\pi} \right] \\
& \log(z_0) \left(\left(\log(z_0) + \left[\frac{\arg(3-z_0)}{2\pi} \right] \right) \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \right. \\
& \left. \sum_{k=1}^{\infty} \frac{(-1)^k (3-z_0)^k z_0^{-k}}{k} \right) / \left(\log(z_0) + \right. \\
& \left. \left[\frac{\arg(1.00289-z_0)}{2\pi} \right] \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \right. \\
& \left. \left. \sum_{k=1}^{\infty} \frac{(-1)^k (1.00289-z_0)^k z_0^{-k}}{k} \right) \right)^{1.0012} \\
& \sum_{k=0}^{\infty} \frac{(1.0012-n_0)^k \Gamma^{(k)}(1+n_0)}{k!} - \\
& \left(\left(\log(z_0) + \left[\frac{\arg(3-z_0)}{2\pi} \right] \right) \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \right. \\
& \left. \sum_{k=1}^{\infty} \frac{(-1)^k (3-z_0)^k z_0^{-k}}{k} \right) / \left(\log(z_0) + \right. \\
& \left. \left[\frac{\arg(1.00289-z_0)}{2\pi} \right] \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \right. \\
& \left. \left. \sum_{k=1}^{\infty} \frac{(-1)^k (1.00289-z_0)^k z_0^{-k}}{k} \right) \right)^{1.0012} \\
& \sum_{k_1=1}^{\infty} \sum_{k_2=0}^{\infty} \\
& \left. \frac{(-1)^{k_1} (1.0012-n_0)^{k_2} (2-z_0)^{k_1} z_0^{-k_1} \Gamma^{(k_2)}(1+n_0)}{k_2! k_1} \right) \\
& \left(\left(\left[\frac{\arg(3-z_0)}{2\pi} \right] \log\left(\frac{1}{z_0}\right) + \log(z_0) + \left[\frac{\arg(3-z_0)}{2\pi} \right] \log(z_0) - \right. \right. \\
& \left. \sum_{k=1}^{\infty} \frac{(-1)^k (3-z_0)^k z_0^{-k}}{k} \right) \left(\log(z_0) + \left[\frac{\arg(3-z_0)}{2\pi} \right] \right. \\
& \left. \left. \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (3-z_0)^k z_0^{-k}}{k} \right) \right)^{1.0012} \right) \wedge \\
& (1/32) \left. \right) \text{ for } ((n_0 \notin \mathbb{Z} \text{ or } n_0 \geq 0) \text{ and } n_0 \rightarrow 1.0012)
\end{aligned}$$

$\Gamma(x)$ is the gamma function

\mathbb{Z} is the set of integers

$\text{arg}(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

Integral representations:

$$-\frac{21}{10^3} + \sqrt[32]{\frac{\left(1.25623 \left(1 + \frac{1.0012! \left(\frac{\log(3)}{\log(1.00289)}\right)^{1.0012} \log(2)\right)}{32 \log(3)}\right) (2e)^8}{\log^{1.0012}(3)}}} =$$

$$-0.021 + 1.07477 \sqrt[32]{\frac{e^8 \left(32 \log(3) + \log(2) \left(\frac{\log(3)}{\log(1.00289)}\right)^{1.0012} \int_0^1 \log^{1.0012}\left(\frac{1}{t}\right) dt\right)}{\log^{2.0012}(3)}}$$

$$-\frac{21}{10^3} + \sqrt[32]{\frac{\left(1.25623 \left(1 + \frac{1.0012! \left(\frac{\log(3)}{\log(1.00289)}\right)^{1.0012} \log(2)\right)}{32 \log(3)}\right) (2e)^8}{\log^{1.0012}(3)}} =$$

$$1.07477 \left(-0.019539 + \sqrt[32]{\frac{e^8 \left(32 \int_1^3 \frac{1}{t} dt + \int_0^1 \int_0^1 \frac{\log^{1.0012}\left(\frac{1}{t_2}\right)}{1+t_1} dt_2 dt_1\right)}{\left(\int_1^3 \frac{1}{t} dt\right)^{2.0012}}}\right)$$

$$-\frac{21}{10^3} + \sqrt[32]{\frac{\left(1.25623 \left(1 + \frac{1.0012! \left(\frac{\log(3)}{\log(1.00289)}\right)^{1.0012} \log(2)\right)}{32 \log(3)}\right) (2e)^8}{\log^{1.0012}(3)}} =$$

$$-\frac{21}{1000} + 1.19771 \sqrt[32]{\frac{e^8 \left(1 + \frac{\log(2) \left(\frac{\log(3)}{\log(1.00289)}\right)^{1.0012} \left(\int_1^\infty t^{1.0012} \mathcal{A}^{-t} dt + \sum_{k=0}^\infty \frac{(-1)^k}{(2.0012+k)k!}\right)}{32 \log(3)}\right)}{\log^{1.0012}(3)}}$$

Now, we have that:

Lemma 7. Let m be an integer with $1 \leq m < (\log x)/192$ and let $x^{1/(4m)} < y \leq x^{1/m}$. Then the following estimate holds:

$$\int_T^{T+H} \left(S(t) + \frac{1}{\pi} \sum_{p < y} \frac{\sin(t \log p)}{\sqrt{p}} \right)^{2m} dt < (e^{37} \pi^{-2} \varepsilon^{-3} m^2)^m H.$$

$H = T^{27/82+\varepsilon}$; $T = 2$; $z = 3$, $v = 1.0012$; $m = 1$ or 0.5 ; $x = T^{0.1\varepsilon} \varepsilon = 1/24 = 0,04166666$;
 $x = 1.002893$; $H = 1,256230382233478$, we obtain:

$$(((\exp(37) * \text{Pi}^{(-2)} * (1/24)^{(-3)} * (0.5)^2)))^{0.5} * 1.256230382233478$$

Input interpretation:

$$\sqrt{\frac{\exp(37) \times 0.5^2}{\pi^2 \left(\frac{1}{24}\right)^3}} \times 1.256230382233478$$

Result:

$$2.54480319953865374485272324217174470817053823953696203... \times 10^9$$

$$2.5448031995... * 10^9$$

From the ratio with the result of previous expression:

$$7.46489930912948958098161901454328525321249834928522757... \times 10^6$$

We obtain:

$$2.5448031995 \times 10^9 / (((((1.25623 * (((2*e)^8 / ((\ln 3)^{1.0012}))) * ((1+1.0012!/32*(\ln 3/\ln 1.002893)^{1.0012}*(\ln 2/\ln 3))))))))))$$

Input interpretation:

$$\frac{2.5448031995 \times 10^9}{1.25623 \times \frac{(2e)^8}{\log^{1.0012}(3)} \left(1 + \frac{1.0012!}{32} \left(\frac{\log(3)}{\log(1.002893)} \right)^{1.0012} \times \frac{\log(2)}{\log(3)} \right)}$$

$\log(x)$ is the natural logarithm

$n!$ is the factorial function

Result:

$$340.9025485967015970940602940419611062704272803061341727583...$$

$$340.9025485967...$$

We note that 340 is the sum of two Lucas numbers: 18 + 322. Furthermore, we obtain:

$$\left(\left(\left(\left(\left(\left(2.5448031995 \times 10^9 / \left(\left(\left(\left(1.25623 * \left(\left(2 * e \right)^8 / \left(\ln 3 \right)^{1.0012} \right) \right) * \left(\left(1 + 1.0012! / 32 * \left(\ln 3 / \ln 1.002893 \right)^{1.0012} * \left(\ln 2 / \ln 3 \right) \right) \right) \right) \right) \right) \right) \right) \right)^{1/12}$$

Input interpretation:

$$\sqrt[12]{\frac{2.5448031995 \times 10^9}{1.25623 \times \frac{(2e)^8}{\log^{1.0012}(3)} \left(1 + \frac{1.0012!}{32} \left(\frac{\log(3)}{\log(1.002893)} \right)^{1.0012} \times \frac{\log(2)}{\log(3)} \right)}}$$

log(x) is the natural logarithm
n! is the factorial function

Result:

1.625745350631718082229037481258608740550471968797917955126...
1.6257453506....

$$47/10^3 + \left(\left(\left(\left(\left(\left(2.5448031995 \times 10^9 / \left(\left(\left(\left(1.25623 * \left(\left(2 * e \right)^8 / \left(\ln 3 \right)^{1.0012} \right) \right) * \left(\left(1 + 1.0012! / 32 * \left(\ln 3 / \ln 1.002893 \right)^{1.0012} * \left(\ln 2 / \ln 3 \right) \right) \right) \right) \right) \right) \right) \right) \right)^{1/12}$$

Where 47 is a Lucas number

Input interpretation:

$$\frac{47}{10^3} + \sqrt[12]{\frac{2.5448031995 \times 10^9}{1.25623 \times \frac{(2e)^8}{\log^{1.0012}(3)} \left(1 + \frac{1.0012!}{32} \left(\frac{\log(3)}{\log(1.002893)} \right)^{1.0012} \times \frac{\log(2)}{\log(3)} \right)}}$$

log(x) is the natural logarithm
n! is the factorial function

Result:

1.672745350631718082229037481258608740550471968797917955126...
1.67274535.... result practically equal to the value of proton mass

$$18/10^3 + \left(\left(\left(\left(\left(\left(2.5448031995 \times 10^9 / \left(\left(\left(\left(1.25623 * \left(\left(2 * e \right)^8 / \left(\ln 3 \right)^{1.0012} \right) \right) * \left(\left(1 + 1.0012! / 32 * \left(\ln 3 / \ln 1.002893 \right)^{1.0012} * \left(\ln 2 / \ln 3 \right) \right) \right) \right) \right) \right) \right) \right) \right)^{1/12}$$

Where 18 is a Lucas number

Input interpretation:

$$\frac{18}{10^3} + \sqrt[12]{\frac{2.5448031995 \times 10^9}{1.25623 \times \frac{(2e)^8}{\log^{1.0012}(3)} \left(1 + \frac{1.0012!}{32} \left(\frac{\log(3)}{\log(1.002893)} \right)^{1.0012} \times \frac{\log(2)}{\log(3)} \right)}}$$

log(x) is the natural logarithm

$n!$ is the factorial function

Result:

1.643745350631718082229037481258608740550471968797917955126...

$$1.64374535... \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 ...$$

$$-7/10^3 + ((((((2.5448031995 \times 10^9 / ((((((1.25623 * ((2*e)^8 / ((\ln 3)^{1.0012}))) * ((1+1.0012!/32*(\ln 3/\ln 1.002893))^{1.0012}*(\ln 2/\ln 3)))))))))))))^{1/12}$$

Where 7 is a Lucas number

Input interpretation:

$$-\frac{7}{10^3} + \sqrt[12]{\frac{2.5448031995 \times 10^9}{1.25623 \times \frac{(2e)^8}{\log^{1.0012}(3)} \left(1 + \frac{1.0012!}{32} \left(\frac{\log(3)}{\log(1.002893)}\right)^{1.0012} \times \frac{\log(2)}{\log(3)}\right)}}$$

$\log(x)$ is the natural logarithm

$n!$ is the factorial function

Result:

1.618745350631718082229037481258608740550471968797917955126...

1.61874535.... This result is a very good approximation to the value of the golden ratio 1,618033988749...

We observe that, from the previous expression, we obtain also:

$$((((((((((\exp(37) * \pi^{-2}) * (1/24)^{-3}) * (0.5)^2))))))^{0.5} * 1.256230382233478))))^{1/45}$$

Where 45 = 47 – 2 with 47 and 2 that are Lucas numbers.

Input interpretation:

$$\sqrt[45]{\sqrt{\frac{\exp(37) \times 0.5^2}{\pi^2 \left(\frac{1}{24}\right)^3} \times 1.256230382233478}}$$

Result:

1.618134203098387316230469884880777746323758159943377881228...

1.6181342.... This result is a very good approximation to the value of the golden ratio 1,618033988749...

Returning to the estimate for $I(n)$ and using the Cauchy inequality, we see that

$$\begin{aligned}
 I(n) &\leq (15\pi)^{2m} \int_T^{T+H} (\sigma_{z,t} - 0.5)^{2(1+n)m} z^{2m(\sigma_{z,t}-0.5)} \\
 &\quad \times \left(\int_0^{+\infty} z^{-v} \left| \sum_{p < z^3} \frac{\Lambda_z(p)(\log p)^n \log(zp)}{p^{0.5+v}} p^{it} \right| dv \right)^{2m} dt \\
 &\leq (15\pi)^{2m} \sqrt{j_1} \sqrt{j_2},
 \end{aligned}$$

$$j_1 \leq H \frac{(2e)^{8m}}{(\log z)^\nu} \left(1 + 60m^{1.5} \varepsilon^{-1} \left(\frac{2}{3e} \right)^{4m} \right) < (4\varepsilon)^{-1} \frac{(2e)^{8m}}{(\log z)^\nu} H.$$

j_2 does not exceed the product

$$(\log z)^\nu \cdot 13(72^2 m)^{2m} H.$$

Thence, for $H = T^{27/82+\varepsilon}$; $T = 2$; $z = 3$, $\nu = 1.0012$; $m = 1$ or 0.5 ; $x = T^{0.1\varepsilon}$ $\varepsilon = 1/24 = 0,04166666$; $x = 1.002893$; $H = 1,256230382233478$, we obtain:

$$4 * (1/24)^{-1} * (((2*e)^8 / (\ln 3)^{1.0012})) * 1.256230382233478$$

Input interpretation:

$$\frac{4 \times \frac{(2e)^8}{\log^{1.0012}(3)} \times 1.256230382233478}{\frac{1}{24}}$$

$\log(x)$ is the natural logarithm

Result:

$$8.37611980263202966496082874946434404730001186125892694... \times 10^7$$

$$8.3761198026320... * 10^7$$

•

Alternative representations:

$$\frac{4(2e)^8 1.2562303822334780000}{\frac{\log^{1.0012}(3)}{24}} = \frac{5.0249215289339120000 (2e)^8}{\frac{1}{24} \log_e^{1.0012}(3)}$$

•

$$\frac{4(2e)^8 1.2562303822334780000}{\frac{\log^{1.0012}(3)}{24}} = \frac{5.0249215289339120000 (2e)^8}{\frac{1}{24} (2 \coth^{-1}(2))^{1.0012}}$$

$$\frac{4(2e)^8 1.2562303822334780000}{\frac{\log^{1.0012}(3)}{24}} = \frac{5.0249215289339120000 (2e)^8}{\frac{1}{24} (\log(a) \log_a(3))^{1.0012}}$$

$\log_b(x)$ is the base- b logarithm

$\coth^{-1}(x)$ is the inverse hyperbolic cotangent function

Series representations:

$$\frac{4(2e)^8 1.2562303822334780000}{\frac{\log^{1.0012}(3)}{24}} = \frac{30\,873.117873769955328 e^8}{\left(\log(2) - \sum_{k=1}^{\infty} \frac{(-1)^k}{k}\right)^{1.0012}}$$

$$\frac{4(2e)^8 1.2562303822334780000}{\frac{\log^{1.0012}(3)}{24}} = \frac{30\,873.117873769955328 e^8}{\left(2i\pi \left\lfloor \frac{\text{arg}(3-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (3-x)^k x^{-k}}{k}\right)^{1.0012}} \text{ for } x < 0$$

$$\frac{4(2e)^8 1.2562303822334780000}{\frac{\log^{1.0012}(3)}{24}} = \frac{30\,873.117873769955328 e^8}{\left(\log(z_0) + \left\lfloor \frac{\text{arg}(3-z_0)}{2\pi} \right\rfloor \right) \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (3-z_0)^k z_0^{-k}}{k}\right)^{1.0012}}$$

$\text{arg}(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

Integral representations:

$$\frac{4(2e)^8 \cdot 1.2562303822334780000}{\frac{\log^{1.0012}(3)}{24}} = \frac{30873.117873769955328 e^8}{\left(\int_1^3 \frac{1}{t} dt\right)^{1.0012}}$$

$$\frac{4(2e)^8 \cdot 1.2562303822334780000}{\frac{\log^{1.0012}(3)}{24}} = \frac{61797.6 e^8}{\left(\frac{1}{i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{2^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds\right)^{1.0012}} \text{ for } -1 < \gamma < 0$$

$\Gamma(x)$ is the gamma function

$$(\log z)^\nu \cdot 13(72^2 m)^{2m} H.$$

$$(((\ln 3)^{1.0012}) * (13(72^2))^2) * 1.256230382233478$$

Input interpretation:

$$(\log^{1.0012}(3)(13 \times 72^2)^2) \times 1.256230382233478$$

$\log(x)$ is the natural logarithm

Result:

$$6.26873... \times 10^9$$

i.e.

$$6.26872832830030487776064395727841779947810160638925081... \times 10^9$$

$$6.2687283283003... * 10^9$$

Alternative representations:

$$1.2562303822334780000 \log^{1.0012}(3)(13 \times 72^2)^2 = 1.2562303822334780000 \log_e^{1.0012}(3)(13 \times 72^2)^2$$

$$1.2562303822334780000 \log^{1.0012}(3)(13 \times 72^2)^2 = 1.2562303822334780000 (\log(a) \log_a(3))^{1.0012} (13 \times 72^2)^2$$

$$1.2562303822334780000 \log^{1.0012}(3)(13 \times 72^2)^2 = 1.2562303822334780000 (2 \coth^{-1}(2))^{1.0012} (13 \times 72^2)^2$$

$\log_b(x)$ is the base- b logarithm

$\coth^{-1}(x)$ is the inverse hyperbolic cotangent function

Series representations:

$$1.2562303822334780000 \log^{1.0012}(3) (13 \times 72^2)^2 = 5.7053984927494983995 \times 10^9 \left(\log(2) - \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{2}\right)^k}{k} \right)^{1.0012}$$

- $$1.2562303822334780000 \log^{1.0012}(3) (13 \times 72^2)^2 = 5.7053984927494983995 \times 10^9 \left(2i\pi \left\lfloor \frac{\arg(3-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (3-x)^k x^{-k}}{k} \right)^{1.0012} \text{ for } x < 0$$

- $$1.2562303822334780000 \log^{1.0012}(3) (13 \times 72^2)^2 = 5.7053984927494983995 \times 10^9 \left(\log(z_0) + \left\lfloor \frac{\arg(3-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (3-z_0)^k z_0^{-k}}{k} \right)^{1.0012}$$

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

- **Integral representations:**

$$1.2562303822334780000 \log^{1.0012}(3) (13 \times 72^2)^2 = 5.7053984927494983995 \times 10^9 \left(\int_1^3 \frac{1}{t} dt \right)^{1.0012}$$

$$1.2562303822334780000 \log^{1.0012}(3) (13 \times 72^2)^2 = 2.85033 \times 10^9 \left(\frac{1}{i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{2^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right)^{1.0012} \text{ for } -1 < \gamma < 0$$

$\Gamma(x)$ is the gamma function

$$\sqrt{\left(\left(\left(\left(4 \times \left(\frac{1}{24} \right)^{-1} \times \left(\left(\left(2 \times e \right)^8 / \left(\ln 3 \right)^{1.0012} \right) \right) \times 1.256230382233478 \right) \right) \right) \right) \times \left(\left(\left(\left(\ln 3 \right)^{1.0012} \right) \times \left(13 \left(72^2 \right)^2 \right) \times 1.256230382233478 \right) \right) \right)$$

Input interpretation:

$$\sqrt{\frac{4 \times \frac{(2e)^8}{\log^{1.0012}(3)} \times 1.256230382233478}{\frac{1}{24}}}$$

$$\sqrt{(\log^{1.0012}(3)(13 \times 72^2)^2) \times 1.256230382233478}$$

$\log(x)$ is the natural logarithm

Result:

- $7.24621414864317703420849413412197114326569681730709267... \times 10^8$

$7.246214148643... * 10^8$

Alternative representations:

- $$\sqrt{\frac{4 (2e)^8 1.2562303822334780000}{\frac{\log^{1.0012}(3)}{24}}}$$

$$\sqrt{1.2562303822334780000 \log^{1.0012}(3) (13 \times 72^2)^2} =$$

$$\sqrt{1.2562303822334780000 \log_e^{1.0012}(3) (13 \times 72^2)^2}$$

$$\sqrt{\frac{5.0249215289339120000 (2e)^8}{\frac{1}{24} \log_e^{1.0012}(3)}}$$

- $$\sqrt{\frac{4 (2e)^8 1.2562303822334780000}{\frac{\log^{1.0012}(3)}{24}}}$$

$$\sqrt{1.2562303822334780000 \log^{1.0012}(3) (13 \times 72^2)^2} =$$

$$\sqrt{1.2562303822334780000 (\log(a) \log_a(3))^{1.0012} (13 \times 72^2)^2}$$

$$\sqrt{\frac{5.0249215289339120000 (2e)^8}{\frac{1}{24} (\log(a) \log_a(3))^{1.0012}}}$$

$$\sqrt{\frac{4(2e)^8 1.2562303822334780000}{\frac{\log^{1.0012}(3)}{24}}}$$

$$\sqrt{1.2562303822334780000 \log^{1.0012}(3) (13 \times 72^2)^2} =$$

$$\sqrt{1.2562303822334780000 (2 \coth^{-1}(2))^{1.0012} (13 \times 72^2)^2}$$

$$\sqrt{\frac{5.0249215289339120000 (2e)^8}{\frac{1}{24} (2 \coth^{-1}(2))^{1.0012}}}$$

$\log_b(x)$ is the base- b logarithm

$\coth^{-1}(x)$ is the inverse hyperbolic cotangent function

Series representations:

$$\sqrt{\frac{4(2e)^8 1.2562303822334780000}{\frac{\log^{1.0012}(3)}{24}}}$$

$$\sqrt{1.2562303822334780000 \log^{1.0012}(3) (13 \times 72^2)^2} =$$

$$\sqrt{\frac{30873.117873769955328 e^8}{\left(\log(2) - \sum_{k=1}^{\infty} \frac{(-\frac{1}{2})^k}{k}\right)^{1.0012}}}$$

$$\sqrt{5.7053984927494983995 \times 10^9 \left(\log(2) - \sum_{k=1}^{\infty} \frac{(-\frac{1}{2})^k}{k}\right)^{1.0012}}$$

•

$$\sqrt{\frac{4(2e)^8 1.2562303822334780000}{\frac{\log^{1.0012}(3)}{24}}}$$

$$\sqrt{1.2562303822334780000 \log^{1.0012}(3) (13 \times 72^2)^2} =$$

$$\sqrt{-1 + \frac{30873.117873769955328 e^8}{\log^{1.0012}(3)}}$$

$$\sqrt{-1.00000000000000000000 + 5.7053984927494983995 \times 10^9 \log^{1.0012}(3)}$$

$$\sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \binom{\frac{1}{2}}{k_1} \binom{\frac{1}{2}}{k_2} \left(-1 + \frac{30873.117873769955328 e^8}{\log^{1.0012}(3)}\right)^{-k_1}$$

$$(-1.00000000000000000000 +$$

$$5.7053984927494983995 \times 10^9 \log^{1.0012}(3))^{-k_2}$$

$$\sqrt{\frac{4(2e)^8 1.2562303822334780000}{\frac{\log^{1.0012}(3)}{24}}}$$

$$\sqrt{1.2562303822334780000 \log^{1.0012}(3) (13 \times 72^2)^2} =$$

$$\sqrt{-1 + \frac{30873.117873769955328 e^8}{\log^{1.0012}(3)}}$$

$$\sqrt{-1.00000000000000000000 + 5.7053984927494983995 \times 10^9 \log^{1.0012}(3)}$$

$$\sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{1}{k_1! k_2!} (-1)^{k_1+k_2} \left(-1 + \frac{30873.117873769955328 e^8}{\log^{1.0012}(3)}\right)^{-k_1}$$

$$(-1.00000000000000000000 +$$

$$5.7053984927494983995 \times 10^9 \log^{1.0012}(3))^{-k_2} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2}$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$$(15\pi)^2 * \text{sqrt}((((4*(1/24)^{-1} * (((2*e)^8 / (\ln 3)^{1.0012}))) * 1.256230382233478)))) * \text{sqrt}((((((\ln 3)^{1.0012}) * (13(72^2))^2) * 1.256230382233478))))$$

Input interpretation:

$$(15 \pi)^2 \sqrt{\frac{4 \times \frac{(2 e)^8}{\log^{1.0012}(3)} \times 1.256230382233478}{\frac{1}{24}}}$$

$$\sqrt{(\log^{1.0012}(3)(13 \times 72^2)^2) \times 1.256230382233478}$$

$\log(x)$ is the natural logarithm

Result:

$$1.60914... \times 10^{12}$$

$$1.6091385086854052982798296090138338157308526310267718... \times 10^{12}$$

$$1.6091385086... * 10^{12}$$

Alternative representations:

$$(15 \pi)^2 \sqrt{\frac{4 (2 e)^8 1.2562303822334780000}{\frac{\log^{1.0012}(3)}{24}}}$$

$$\sqrt{1.2562303822334780000 \log^{1.0012}(3) (13 \times 72^2)^2} =$$

$$(15 \pi)^2 \sqrt{1.2562303822334780000 \log_e^{1.0012}(3) (13 \times 72^2)^2}$$

$$\sqrt{\frac{5.0249215289339120000 (2 e)^8}{\frac{1}{24} \log_e^{1.0012}(3)}}$$

$$(15 \pi)^2 \sqrt{\frac{4 (2 e)^8 1.2562303822334780000}{\frac{\log^{1.0012}(3)}{24}}}$$

$$\sqrt{1.2562303822334780000 \log^{1.0012}(3) (13 \times 72^2)^2} =$$

$$(15 \pi)^2 \sqrt{1.2562303822334780000 (\log(a) \log_a(3))^{1.0012} (13 \times 72^2)^2}$$

$$\sqrt{\frac{5.0249215289339120000 (2 e)^8}{\frac{1}{24} (\log(a) \log_a(3))^{1.0012}}}$$

$$\begin{aligned}
& (15 \pi)^2 \sqrt{\frac{4 (2 e)^8 1.2562303822334780000}{\frac{\log^{1.0012}(3)}{24}}} \\
& \sqrt{1.2562303822334780000 \log^{1.0012}(3) (13 \times 72^2)^2} = \\
& (15 \pi)^2 \sqrt{1.2562303822334780000 (2 \coth^{-1}(2))^{1.0012} (13 \times 72^2)^2} \\
& \sqrt{\frac{5.0249215289339120000 (2 e)^8}{\frac{1}{24} (2 \coth^{-1}(2))^{1.0012}}}
\end{aligned}$$

$\log_b(x)$ is the base- b logarithm

$\coth^{-1}(x)$ is the inverse hyperbolic cotangent function

Series representations:

$$\begin{aligned}
& (15 \pi)^2 \sqrt{\frac{4 (2 e)^8 1.2562303822334780000}{\frac{\log^{1.0012}(3)}{24}}} \\
& \sqrt{1.2562303822334780000 \log^{1.0012}(3) (13 \times 72^2)^2} = \\
& 225 \pi^2 \sqrt{\frac{30\,873.117873769955328 e^8}{\left(\log(2) - \sum_{k=1}^{\infty} \frac{(-\frac{1}{2})^k}{k}\right)^{1.0012}}} \\
& \sqrt{5.7053984927494983995 \times 10^9 \left(\log(2) - \sum_{k=1}^{\infty} \frac{(-\frac{1}{2})^k}{k}\right)^{1.0012}}
\end{aligned}$$

•

$$\begin{aligned}
& (15 \pi)^2 \sqrt{\frac{4 (2 e)^8 1.2562303822334780000}{\frac{\log^{1.0012}(3)}{24}}} \\
& \sqrt{1.2562303822334780000 \log^{1.0012}(3) (13 \times 72^2)^2} = \\
& 225 \pi^2 \sqrt{-1 + \frac{30873.117873769955328 e^8}{\log^{1.0012}(3)}} \\
& \sqrt{-1.00000000000000000000 + 5.7053984927494983995 \times 10^9 \log^{1.0012}(3)} \\
& \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \binom{\frac{1}{2}}{k_1} \binom{\frac{1}{2}}{k_2} \left(-1 + \frac{30873.117873769955328 e^8}{\log^{1.0012}(3)}\right)^{-k_1} \\
& \quad (-1.00000000000000000000 + \\
& \quad 5.7053984927494983995 \times 10^9 \log^{1.0012}(3))^{-k_2}
\end{aligned}$$

$$\begin{aligned}
& (15 \pi)^2 \sqrt{\frac{4 (2 e)^8 1.2562303822334780000}{\frac{\log^{1.0012}(3)}{24}}} \\
& \sqrt{1.2562303822334780000 \log^{1.0012}(3) (13 \times 72^2)^2} = \\
& 225 \pi^2 \sqrt{-1 + \frac{30873.117873769955328 e^8}{\log^{1.0012}(3)}} \\
& \sqrt{-1.00000000000000000000 + 5.7053984927494983995 \times 10^9 \log^{1.0012}(3)} \\
& \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{1}{k_1! k_2!} (-1)^{k_1+k_2} \left(-1 + \frac{30873.117873769955328 e^8}{\log^{1.0012}(3)}\right)^{-k_1} \\
& \quad (-1.00000000000000000000 + \\
& \quad 5.7053984927494983995 \times 10^9 \log^{1.0012}(3))^{-k_2} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2}
\end{aligned}$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

The reciprocal of the result, is:

$$\begin{aligned}
& 1 / [(15\text{Pi})^2 * \text{sqrt}(\frac{4 * (2 * e)^8 / (\ln 3)^{1.0012}}{\ln 3}) * \\
& 1.256230382233478) * \text{sqrt}(\frac{(\ln 3)^{1.0012} * (13 * 72^2)^2}{1.256230382233478})]
\end{aligned}$$

Input interpretation:

$$1 / \left((15 \pi)^2 \sqrt{\frac{4 \times \frac{(2e)^8}{\log^{1.0012}(3)} \times 1.256230382233478}{\frac{1}{24}}}} \right. \\ \left. \sqrt{(\log^{1.0012}(3)(13 \times 72^2)^2) \times 1.256230382233478} \right)$$

$\log(x)$ is the natural logarithm

Result:

- More digits
6.21451... $\times 10^{-13}$

Alternative representations:

$$1 / \left((15 \pi)^2 \sqrt{\frac{4 (2e)^8 1.2562303822334780000}{\frac{\log^{1.0012}(3)}{24}}}} \right. \\ \left. \sqrt{1.2562303822334780000 \log^{1.0012}(3)(13 \times 72^2)^2} \right) = \\ 1 / \left((15 \pi)^2 \sqrt{1.2562303822334780000 \log_e^{1.0012}(3)(13 \times 72^2)^2} \right. \\ \left. \sqrt{\frac{5.0249215289339120000 (2e)^8}{\frac{1}{24} \log_e^{1.0012}(3)}} \right)$$

$$1 / \left((15 \pi)^2 \sqrt{\frac{4 (2e)^8 1.2562303822334780000}{\frac{\log^{1.0012}(3)}{24}}}} \right. \\ \left. \sqrt{1.2562303822334780000 \log^{1.0012}(3)(13 \times 72^2)^2} \right) = \\ 1 / \left((15 \pi)^2 \sqrt{1.2562303822334780000 (2 \coth^{-1}(2))^{1.0012} (13 \times 72^2)^2} \right. \\ \left. \sqrt{\frac{5.0249215289339120000 (2e)^8}{\frac{1}{24} (2 \coth^{-1}(2))^{1.0012}}} \right)$$

$$\begin{aligned}
& 1 / \left((15 \pi)^2 \sqrt{\frac{4 (2 e)^8 1.2562303822334780000}{\frac{\log^{1.0012}(3)}{24}}} \right. \\
& \quad \left. \sqrt{1.2562303822334780000 \log^{1.0012}(3) (13 \times 72^2)^2} \right) = \\
& 1 / \left((15 \pi)^2 \sqrt{1.2562303822334780000 (\log(a) \log_a(3))^{1.0012} (13 \times 72^2)^2} \right. \\
& \quad \left. \sqrt{\frac{5.0249215289339120000 (2 e)^8}{\frac{1}{24} (\log(a) \log_a(3))^{1.0012}}} \right)
\end{aligned}$$

$\log_b(x)$ is the base- b logarithm

$\coth^{-1}(x)$ is the inverse hyperbolic cotangent function

Series representations:

$$\begin{aligned}
& 1 / \left((15 \pi)^2 \sqrt{\frac{4 (2 e)^8 1.2562303822334780000}{\frac{\log^{1.0012}(3)}{24}}} \right. \\
& \quad \left. \sqrt{1.2562303822334780000 \log^{1.0012}(3) (13 \times 72^2)^2} \right) = \\
& 1 / \left(225 \pi^2 \sqrt{\frac{30\,873.117873769955328 e^8}{\left(\log(2) - \sum_{k=1}^{\infty} \frac{(-\frac{1}{2})^k}{k}\right)^{1.0012}}} \right. \\
& \quad \left. \sqrt{5.7053984927494983995 \times 10^9 \left(\log(2) - \sum_{k=1}^{\infty} \frac{(-\frac{1}{2})^k}{k}\right)^{1.0012}} \right)
\end{aligned}$$

•

$$\begin{aligned}
& 1 / \left((15 \pi)^2 \sqrt{\frac{4 (2 e)^8 1.2562303822334780000}{\frac{\log^{1.0012}(3)}{24}}} \right. \\
& \quad \left. \sqrt{1.2562303822334780000 \log^{1.0012}(3) (13 \times 72^2)^2} \right) = \\
& 1 / \left(225 \pi^2 \sqrt{-1 + \frac{30873.117873769955328 e^8}{\log^{1.0012}(3)}} \right. \\
& \quad \sqrt{-1.00000000000000000000 + 5.7053984927494983995 \times 10^9 \log^{1.0012}(3)} \\
& \quad \left(\sum_{k=0}^{\infty} \binom{\frac{1}{2}}{k} \left(-1 + \frac{30873.117873769955328 e^8}{\log^{1.0012}(3)} \right)^{-k} \right) \\
& \quad \left. \sum_{k=0}^{\infty} \binom{\frac{1}{2}}{k} (-1.00000000000000000000 + \right. \\
& \quad \quad \left. 5.7053984927494983995 \times 10^9 \log^{1.0012}(3))^{-k} \right)
\end{aligned}$$

$$\begin{aligned}
& 1 / \left((15 \pi)^2 \sqrt{\frac{4 (2 e)^8 1.2562303822334780000}{\frac{\log^{1.0012}(3)}{24}}} \right. \\
& \quad \left. \sqrt{1.2562303822334780000 \log^{1.0012}(3) (13 \times 72^2)^2} \right) = \\
& 1 / \left(225 \pi^2 \sqrt{-1 + \frac{30873.117873769955328 e^8}{\log^{1.0012}(3)}} \right. \\
& \quad \sqrt{-1.00000000000000000000 + 5.7053984927494983995 \times 10^9 \log^{1.0012}(3)} \\
& \quad \left(\sum_{k=0}^{\infty} \frac{(-1)^k \left(-1 + \frac{30873.117873769955328 e^8}{\log^{1.0012}(3)} \right)^{-k} \left(-\frac{1}{2} \right)_k}{k!} \right) \\
& \quad \left. \sum_{k=0}^{\infty} \frac{1}{k!} (-1)^k (-1.00000000000000000000 + \right. \\
& \quad \quad \left. 5.7053984927494983995 \times 10^9 \log^{1.0012}(3))^{-k} \left(-\frac{1}{2} \right)_k \right)
\end{aligned}$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(\alpha)_n$ is the Pochhammer symbol (rising factorial)

We have that:

Thence, for $H = T^{27/82+\varepsilon}$; $T = 2$; $z = 3$, $v = 1.0012$; $m = 1$ or 0.5 ; $x = T^{0.1\varepsilon}$ $\varepsilon = 1/24 = 0,04166666$; $x = 1.002893$; $H = 1,256230382233478$, we obtain:

$$I_5 + I_6 < \frac{2H}{\sqrt{\varepsilon}} (e^{24} m)^m$$

$$(e^{24}) * (((2*1.256230382233478))) / (((\text{sqrt}(1/24))))$$

Input interpretation:

$$e^{24} \times \frac{2 \times 1.256230382233478}{\sqrt{\frac{1}{24}}}$$

Result:

$$3.260411940037072... \times 10^{11}$$

$$3.2604119400... * 10^{11}$$

Series representations:

$$\frac{e^{24} (2 \times 1.2562303822334780000)}{\sqrt{\frac{1}{24}}} = \frac{2.5124607644669560000 e^{24}}{\sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{23}{24}\right)^k \left(-\frac{1}{2}\right)_k}{k!}}$$

$$\frac{e^{24} (2 \times 1.2562303822334780000)}{\sqrt{\frac{1}{24}}} = - \frac{5.0249215289339120000 e^{24} \sqrt{\pi}}{\sum_{j=0}^{\infty} \text{Res}_{s=-j} \left(-\frac{23}{24}\right)^{-s} \Gamma\left(-\frac{1}{2} - s\right) \Gamma(s)}$$

$$\frac{e^{24} (2 \times 1.2562303822334780000)}{\sqrt{\frac{1}{24}}} = \frac{2.5124607644669560000 e^{24}}{\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{1}{24} - z_0\right)^k z_0^{-k}}{k!}}$$

for not $((z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0))$

$n!$ is the factorial function

$(\alpha)_n$ is the Pochhammer symbol (rising factorial)

$\Gamma(x)$ is the gamma function

$\text{Res}_{z=z_0} f$ is a complex residue

\mathbb{R} is the set of real numbers

Now, we have that:

$$\begin{aligned}
 I_7 &\leq (15\pi)^{2m} \int_T^{T+H} (\sigma_{z,t} - 0.5)^{2m} \left| \sum_{p < z^3} \frac{\Lambda_z(p)}{\sqrt{p}} p^{it} \right|^{2m} dt \leq (15\pi)^{2m} \sqrt{j_3} \sqrt{j_4}, \\
 j_3 &= \int_T^{T+H} (\sigma_{z,t} - 0.5)^{4m} dt, \\
 j_4 &= \int_T^{T+H} \left| \sum_{p < z^3} \frac{\Lambda_z(p)}{\sqrt{p}} p^{it} \right|^{4m} dt \\
 &= (\log z)^{4m} \int_0^H \left| \sum_{p < z^3} \frac{a(p)}{\sqrt{p}} p^{it} \right|^{4m} dt, \quad a(p) = \frac{\Lambda_z(p)}{\log z} p^{iT}.
 \end{aligned}$$

Setting $\xi = 1$ and $\nu = 4m$ in Lemma 6, we obtain

$$j_3 \leq H \frac{(2e)^{8m}}{(\log z)^{4m}} (1 + 45m^{1.5} \varepsilon^{-1} (3e)^{-4m}) < (80\varepsilon)^{-1} H \frac{(2e)^{8m}}{(\log z)^{4m}}.$$

Since the absolute value of $a(p)$ does not exceed the fraction

$$\frac{3 \log p}{\log(z^3)}$$

for $p \leq z^3$, it follows from Lemma 3 applied to j_4 with $\varkappa = H$, $\xi = z^3$, and $B = 3$ that

$$j_4 \leq 13(2 \cdot 3^2 \cdot 2m)^{2m} H (\log z)^{4m}.$$

Passing to an estimate for I_7 , we have

$$I_7 \leq (15\pi)^{2m} \frac{H}{\sqrt{\varepsilon}} (2e)^{4m} (36m)^m < \frac{H}{\sqrt{\varepsilon}} (e^{19} m)^m.$$

Thence, for $H = T^{27/82+\varepsilon}$; $T = 2$; $z = 3$, $\nu = 1.0012$; $m = 1$ or 0.5 ; $x = T^{0.1\varepsilon}$ $\varepsilon = 1/24 = 0,04166666$; $x = 1.002893$; $H = 1,256230382233478$, we obtain:

$$(15\pi)^2 * (((((\sqrt{((80 * 1/24)^{-1} * 1.25623038 * (((2 * e)^8 / (\ln 3)^4)))))))) * (((((\sqrt{((13 * 3^2 * 2)^2 * 1.25623038 (\ln 3)^4))))))))$$

Input interpretation:

$$(15 \pi)^2 \sqrt{\frac{1.25623038 \times \frac{(2 e)^8}{\log^4(3)}}{80 \times \frac{1}{24}}} \sqrt{13 (2 \times 3^2 \times 2)^2 \times 1.25623038 \log^4(3)}$$

log(x) is the natural logarithm

Result:

$$1.73254399... \times 10^8$$

$$1.73254398860430049678048005183511593009420209130208425... \times 10^8$$

$$1.7325439886... * 10^8$$

Alternative representations:

$$(15 \pi)^2 \sqrt{\frac{1.25623 (2 e)^8}{\frac{80 \log^4(3)}{24}}} \sqrt{13 (2 \times 3^2 \times 2)^2 1.25623 \log^4(3)} =$$

$$(15 \pi)^2 \sqrt{16.331 \times 36^2 \log_e^4(3)} \sqrt{\frac{1.25623 (2 e)^8}{\frac{80}{24} \log_e^4(3)}}$$

$$(15 \pi)^2 \sqrt{\frac{1.25623 (2 e)^8}{\frac{80 \log^4(3)}{24}}} \sqrt{13 (2 \times 3^2 \times 2)^2 1.25623 \log^4(3)} =$$

$$(15 \pi)^2 \sqrt{16.331 \times 36^2 (\log(a) \log_a(3))^4} \sqrt{\frac{1.25623 (2 e)^8}{\frac{80}{24} (\log(a) \log_a(3))^4}}$$

$$(15 \pi)^2 \sqrt{\frac{1.25623 (2 e)^8}{\frac{80 \log^4(3)}{24}}} \sqrt{13 (2 \times 3^2 \times 2)^2 1.25623 \log^4(3)} =$$

$$(15 \pi)^2 \sqrt{16.331 \times 36^2 (2 \coth^{-1}(2))^4} \sqrt{\frac{1.25623 (2 e)^8}{\frac{80}{24} (2 \coth^{-1}(2))^4}}$$

$\log_b(x)$ is the base- b logarithm

$\coth^{-1}(x)$ is the inverse hyperbolic cotangent function

• **Series representations:**

$$(15 \pi)^2 \sqrt{\frac{1.25623 (2 e)^8}{80 \log^4(3)}} \sqrt{13 (2 \times 3^2 \times 2)^2 1.25623 \log^4(3)} =$$

$$225 \pi^2 \sqrt{-1 + \frac{96.4785 e^8}{\log^4(3)}} \sqrt{-1 + 21 165. \log^4(3)}$$

$$\sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \binom{\frac{1}{2}}{k_1} \binom{\frac{1}{2}}{k_2} \left(-1 + \frac{96.4785 e^8}{\log^4(3)}\right)^{-k_1} (-1 + 21 165. \log^4(3))^{-k_2}$$

$$(15 \pi)^2 \sqrt{\frac{1.25623 (2 e)^8}{80 \log^4(3)}} \sqrt{13 (2 \times 3^2 \times 2)^2 1.25623 \log^4(3)} =$$

$$225 \pi^2 \sqrt{\frac{96.4785 e^8}{\left(\log(2) - \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{2}\right)^k}{k}\right)^4}} \sqrt{21 165. \left(\log(2) - \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{2}\right)^k}{k}\right)^4}$$

$$(15 \pi)^2 \sqrt{\frac{1.25623 (2 e)^8}{80 \log^4(3)}} \sqrt{13 (2 \times 3^2 \times 2)^2 1.25623 \log^4(3)} =$$

$$225 \pi^2 \sqrt{-1 + \frac{96.4785 e^8}{\log^4(3)}} \sqrt{-1 + 21 165. \log^4(3)}$$

$$\sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1+k_2} \left(-1 + \frac{96.4785 e^8}{\log^4(3)}\right)^{-k_1} (-1 + 21 165. \log^4(3))^{-k_2} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2}}{k_1! k_2!}$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(\alpha)_n$ is the Pochhammer symbol (rising factorial)

We have also:

$$I_8 = \int_0^H \left| \sum_{p < z^{1.5}} \frac{a(p)}{p} p^{2it} \right|^{2m} dt \leq m^m H.$$

$$I_9 = \int_0^H \left| \sum_{p < z^3} \frac{a(p)}{\sqrt{p}} p^{it} \right|^{2m} dt \leq 13(2 \cdot 3^2 m)^m H < (e^{5.5} m)^m H.$$

$$\begin{aligned} I_{10} &\leq \int_T^{T+H} \left| \sum_{z^3 < p < y} \frac{p^{it}}{\sqrt{p}} \right|^{2m} dt \\ &= \int_0^H \left| \sum_{p < y} \frac{a(p)}{\sqrt{p}} p^{it} \right|^{2m} dt \leq 13(2 \cdot 4^2 m)^m H < (e^{6.1} m)^m H. \end{aligned}$$

Thence, for $H = T^{27/82+\varepsilon}$; $T = 2$; $z = 3$, $v = 1.0012$; $m = 1$ or 0.5 ; $x = T^{0.1\varepsilon}$ $\varepsilon = 1/24 = 0,04166666$; $x = 1.002893$; $H = 1,256230382233478$, we obtain:

1.256230382233478;

$(e^{5.5}) \cdot 1.256230382233478$

Input interpretation:

$e^{5.5} \times 1.256230382233478$

Result:

307.3894395977298858040716453770991459586906306760301266548...

307.389439597...

$(e^{6.1}) \cdot 1.256230382233478$

Input interpretation:

$e^{6.1} \times 1.256230382233478$

Result:

560.1000769325263971902753089815851263031169201505484678901...
560.1000769325...

From the sum of the results:

$1.6091385086... \times 10^{12}$ $3.2604119400... \times 10^{11}$ $1.7325439886... \times 10^8$
 1.256230382233478 ; $307.389439597...$ $560.1000769325...$

We obtain:

$(1.6091385086 \times 10^{12} + 3.2604119400 \times 10^{11} + 1.7325439886 \times 10^8$
 $+ 1.256230382233478 + 307.389439597 + 560.1000769325)$

Input interpretation:

$1.6091385086 \times 10^{12} + 3.2604119400 \times 10^{11} + 1.7325439886 \times 10^8 +$
 $1.256230382233478 + 307.389439597 + 560.1000769325$

Result:

$1.935352957867605746911733478 \times 10^{12}$

• **Repeating decimal:**

$1.935352957867605746911733478 \times 10^{12}$
 $1.9353529578676... \times 10^{12}$

And:

$(1.6091385086 \times 10^{12} + 3.2604119400 \times 10^{11} + 1.7325439886 \times 10^8$
 $+ 1.256230382233478 + 307.389439597 + 560.1000769325)^{1/57}$

Input interpretation:

$(1.6091385086 \times 10^{12} + 3.2604119400 \times 10^{11} + 1.7325439886 \times 10^8 +$
 $1.256230382233478 + 307.389439597 + 560.1000769325)^{(1/57)}$

Result:

1.64269598685...

$$1.64269598\dots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$$

$$29/10^3 + (1.6091385086 \times 10^{12} + 3.2604119400 \times 10^{11} + 1.7325439886 \times 10^8 + 1.256230382233478 + 307.389439597 + 560.1000769325)^{1/57}$$

Where 29 is a Lucas number

Input interpretation:

$$\frac{29}{10^3} + (1.6091385086 \times 10^{12} + 3.2604119400 \times 10^{11} + 1.7325439886 \times 10^8 + 1.256230382233478 + 307.389439597 + 560.1000769325)^{(1/57)}$$

Result:

1.67169598685...

1.67169598.... result practically equal to the value of the formula:

$$m_{p'} = 2 \times \frac{\eta}{R} m_P = 1.6714213 \times 10^{-24} \text{ gm}$$

that is the holographic proton mass

$$-24/10^3 + (1.6091385086 \times 10^{12} + 3.2604119400 \times 10^{11} + 1.7325439886 \times 10^8 + 1.256230382233478 + 307.389439597 + 560.1000769325)^{1/57}$$

Input interpretation:

$$-\frac{24}{10^3} + (1.6091385086 \times 10^{12} + 3.2604119400 \times 10^{11} + 1.7325439886 \times 10^8 + 1.256230382233478 + 307.389439597 + 560.1000769325)^{(1/57)}$$

Result:

1.61869598685...

1.618695986.... result that is a very good approximation to the value of the golden ratio 1,618033988749...

In conclusion, we have:

$$\begin{aligned} \pi^{2m} I &< 7^{2m-1} \left((e^{16.2} m)^{2m} \varepsilon^{-(2m+1)} + \frac{2}{\sqrt{\varepsilon}} (e^{24} m)^m \right. \\ &\quad \left. + \frac{1}{\sqrt{\varepsilon}} (e^{19} m)^m + m^m H + (e^{5.5} m)^m + (e^{6.1} m)^m \right) H \\ &< (7e^{16.2} m)^{2m} \varepsilon^{-(2m+1)} H < (e^{37} \varepsilon^{-3} m^2)^m H. \end{aligned}$$

$$(((\exp(37) * (1/24)^{-3}))) * 1.256230382233478$$

Input interpretation:

$$\frac{\exp(37)}{\left(\frac{1}{24}\right)^3} \times 1.256230382233478$$

Result:

$$2.035161359184528... \times 10^{20}$$

$$2.035161359... * 10^{20}$$

$$1.9353529578676... * 10^{12}$$

We note that, from the ratio of the two results, we obtain also:

$$\begin{aligned} & [((((((((\exp(37) * (1/24)^{-3}))) * \\ & 1.256230382233478)))))) / 1.9353529578676 * 10^{12}]^{1/((1/0.6290748)^6)} \end{aligned}$$

Input interpretation:

$$\left(\frac{1}{0.6290748}\right)^6 \sqrt{\frac{\frac{\exp(37)}{\left(\frac{1}{24}\right)^3} \times 1.256230382233478}{1.9353529578676 \times 10^{12}}}$$

Result:

$$3.141595303845995591603733311601075135919913410677183998480...$$

$$3.1415953038459... \cong \pi$$

From which we can to obtain $C = 2\pi r = 6,2831906076919$ that is a circle with radius equal to 1,000000843602750128912

$$2[\(((((((((\exp(37) * (1/24)^{-3}))) * 1.256230382233478)))))))/1.9353529578676 * 10^{12}]^{1/((1/0.6290748)^6)}$$

Input interpretation:

$$2 \left(\frac{1}{0.6290748} \right)^6 \sqrt[6]{\frac{\frac{\exp(37)}{(1/24)^3} \times 1.256230382233478}{1.9353529578676 \times 10^{12}}}$$

Result:

6.283190607691991183207466623202150271839826821354367996961...

6.28319060769...

Input interpretation:

$$\frac{1}{2\pi} \times 2 \left(\frac{1}{0.6290748} \right)^6 \sqrt[6]{\frac{\frac{\exp(37)}{(1/24)^3} \times 1.256230382233478}{1.9353529578676 \times 10^{12}}}$$

Result:

1.000000843602750128913639254617090890505252772361143816622...

1.000000843602750...

We note that:

Input:

$\zeta(20.18)$

$\zeta(s)$ is the Riemann zeta function

Result:

1.000000842047231832874744577386700923673234902125175719030...

1.00000084204723... a result vary near to the value of the circle radius above

And

$$1/6[\((((((((([(((((((((\exp(37) * (1/24)^{-3}))) * 1.256230382233478)))))))/1.9353529578676 * 10^{12}]^{1/((1/0.6290748)^6)})))))))]^2$$

Input interpretation:

$$\frac{1}{6} \left(\frac{1}{0.6290748} \right)^6 \sqrt[6]{\frac{\frac{\exp(37)}{(1/24)^3} \times 1.256230382233478}{1.9353529578676 \times 10^{12}}}$$

Result:

1.644936842191202227266283026499016139064224075588841405359...

$$1.644936842\dots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$$

$$(29-2)/10^3 + 1/6((((([((((((\exp(37) * (1/24)^{-3}))) * 1.256230382233478)))))))/1.9353529578676 * 10^{12}]^{1/((1/0.6290748)^6)}))^{2}$$

Input interpretation:

$$\frac{29 - 2}{10^3} + \frac{1}{6} \left(\frac{1}{0.6290748} \right)^6 \sqrt[6]{\frac{\frac{\exp(37)}{\left(\frac{1}{24}\right)^3} \times 1.256230382233478}{1.9353529578676 \times 10^{12}}}$$

Result:

1.671936842191202227266283026499016139064224075588841405359...

1.6719368.... a result practically equal to the value of the formula:

$$m_{p'} = 2 \times \frac{\eta}{R} m_p = 1.6714213 \times 10^{-24} \text{ gm}$$

that is the holographic proton mass

$$-(29-2)/10^3 + 1/6((((([((((((\exp(37) * (1/24)^{-3}))) * 1.256230382233478)))))))/1.9353529578676 * 10^{12}]^{1/((1/0.6290748)^6)}))^{2}$$

Input interpretation:

$$-\frac{29 - 2}{10^3} + \frac{1}{6} \left(\frac{1}{0.6290748} \right)^6 \sqrt[6]{\frac{\frac{\exp(37)}{\left(\frac{1}{24}\right)^3} \times 1.256230382233478}{1.9353529578676 \times 10^{12}}}$$

Result:

1.617936842191202227266283026499016139064224075588841405359...

1.61793684219.... This result is a very good approximation to the value of the golden ratio 1,618033988749...

We have also that:

$$\left(\frac{\left(\frac{\exp(37) \cdot (1/24)^{-3}}{(1/24)^3} \cdot 1.256230382233478\right)}{1.9353529578676 \cdot 10^{12}}\right)^{1/(1.589506383^6)}$$

Where $1.589506383 = 1,075226 + 2,103786766 = 3,179012766$;

$3,179012766 \div 2 = 1,589506383$ where **2.103786766...** and 1.075226 are two results of Ramanujan mock theta functions!

Input interpretation:

$$1.589506383^6 \sqrt{\frac{\frac{\exp(37)}{(1/24)^3} \times 1.256230382233478}{1.9353529578676 \times 10^{12}}}$$

Result:

3.14335682...

$3.14335682 \dots \cong \pi$

$$2 \cdot \left(\frac{\left(\frac{\exp(37) \cdot (1/24)^{-3}}{(1/24)^3} \cdot 1.256230382233478\right)}{1.9353529578676 \cdot 10^{12}}\right)^{1/(1.589506383^6)}$$

Input interpretation:

$$2 \cdot 1.589506383^6 \sqrt{\frac{\frac{\exp(37)}{(1/24)^3} \times 1.256230382233478}{1.9353529578676 \times 10^{12}}}$$

Result:

6.286713647892617703097355636434740273200272077817291197942...

$6.2867136478926 \dots = 2\pi r$ with $r = 1.000561552865 \dots$

Indeed:

Input interpretation:

$$\frac{1}{2\pi} \times 2 \cdot 1.589506383^6 \sqrt{\frac{\frac{\exp(37)}{(1/24)^3} \times 1.256230382233478}{1.9353529578676 \times 10^{12}}}$$

Result:

1.000561552865391302213166756457585888541263003621725199866...

1.00056155286539...

We note that:

Input:

$\zeta(10.82)$

$\zeta(s)$ is the Riemann zeta function

Result:

1.000560383437973775703943629544312740741189100259521364546...

1.000560383437.... result very near to the value of the radius of above circle

From the above radius, computing the mass and the temperature, considering the brane a black hole, with the Ramanujan-Nardelli mock formula, we can to obtain the golden ratio:

$\sqrt{\left[\frac{1}{\left(\frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \right) \times \frac{1}{6.738462 \times 10^{26}} \right]} \times \sqrt{\frac{0.0001821192 \times 4 \pi \times 1.000562^3 - 1.000562^2}{6.67 \times 10^{-11}}}$

Input interpretation:

$$\sqrt{\frac{1}{\frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{6.738462 \times 10^{26}}}} \times \sqrt{\frac{0.0001821192 \times 4 \pi \times 1.000562^3 - 1.000562^2}{6.67 \times 10^{-11}}}$$

Result:

1.618248882916651279772588461162778420462637620122863547662...

1.6182488829...

And the coniugate:

$\frac{1}{\sqrt{\left[\frac{1}{\left(\frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \right) \times \frac{1}{6.738462 \times 10^{26}} \right]} \times \sqrt{\frac{0.0001821192 \times 4 \pi \times 1.000562^3 - 1.000562^2}{6.67 \times 10^{-11}}}}$

Input interpretation:

$$\frac{1}{\sqrt{\frac{1}{\frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{6.738462 \times 10^{26}}}} \times \sqrt{\frac{0.0001821192 \times 4 \pi \times 1.000562^3 - 1.000562^2}{6.67 \times 10^{-11}}}}$$

Result:

0.617952...

0.617952...

We know that:

dual coordinate.¹⁻⁸ This is the Dirichlet condition: the X^{25} coordinate of the endpoint is fixed, so the endpoint is constrained to lie on a hyperplane.

In fact, all endpoints are constrained to lie on the same hyperplane. To see this, integrate

$$\begin{aligned}
X'^{25}(\pi) - X'^{25}(0) &= \int_0^\pi d\sigma \partial_\sigma X'^{25} = i \int_0^\pi d\sigma \partial_\tau X^{25} \\
&= 2\pi\alpha' p^{25} = \frac{2\pi\alpha' n}{R} = 2\pi n R'. \quad (31)
\end{aligned}$$

Thence, we have the following mathematical connections with the Dirichlet condition concerning the D-branes:

$$\int_0^\pi d\sigma \partial_\sigma X'^{25} = i \int_0^\pi d\sigma \partial_\tau X^{25} = 2\pi\alpha' p^{25} = \frac{2\pi\alpha' n}{R} = 2\pi n R'.$$

$$2 \left(\frac{1}{0.6290748} \right)^6 \sqrt{\frac{\frac{\exp(37)}{\left(\frac{1}{24}\right)^3} \times 1.256230382233478}{1.9353529578676 \times 10^{12}}}$$

6.283190607691991183207466623202150271839826821354367996961...

r = 1.000000843602750128913639254617090890505252772361143816622...

Or:

$$\int_0^\pi d\sigma \partial_\sigma X'^{25} = i \int_0^\pi d\sigma \partial_\tau X^{25} = 2\pi\alpha' p^{25} = \frac{2\pi\alpha' n}{R} = 2\pi n R'.$$

$$2^{1.5895063836} \sqrt{\frac{\frac{\exp(37)}{\left(\frac{1}{24}\right)^3} \times 1.256230382233478}{1.9353529578676 \times 10^{12}}}$$

6.286713647892617703097355636434740273200272077817291197942...

$$r = 1.000561552865391302213166756457585888541263003621725199866...$$

From:

Mathematical Notes, vol. 73, no. 2, 2003, pp. 212–217.
 Translated from *Matematicheskie Zametki*, vol. 73, no. 2, 2003, pp. 228–233.
 Original Russian Text Copyright c_2003 by A. A. Karatsuba.

Omega Theorems for Zeta Sums

A. A. Karatsuba

Received April 8, 2002

Averaging (over t) the square of the modulus of S shows that the “bulk” of $|S|$ can be estimated from above slightly less accurately than the quadratic root of the number of summands. Indeed, if $T \geq T_1 > 0$, $2 \leq N \leq \sqrt{T}$ and the set $E \subset (T, 2T)$ is the set of the numbers t for which the inequality

$$\left| \sum_{n \leq N} n^{it} \right| \geq \sqrt{N} h(T), \quad h(T) > 0,$$

holds, then we can easily find that

$$Nh^2(T)\mu(E) \leq \int_E \left| \sum_{n \leq N} n^{it} \right|^2 dt \leq \int_T^{2T} \left| \sum_{n \leq N} n^{it} \right|^2 dt \leq TN + 16N \log N \leq 17TN.$$

$$\operatorname{Re} s = \sigma \geq 1 - A \frac{(\log \log t)^\beta}{(\log t)^{1-\beta}}, \quad t \geq 14,$$

where $A > 0$ is an absolute constant.

We can seek omega estimates of $|S(N)|$ of the form other than (7). Suppose that for $1 \leq N \leq t$ we have the estimate

$$|S(N)| \ll N \exp\left(-c \frac{\log^a N}{\log^b t}\right),$$

where $c > 0$ is a constant, $a > b > 0$, and $t \geq t_1 > 0$. For $a = 3$, $b = 2$ such an estimate was obtained by Vinogradov (see [8, 1]). Using the omega estimate of $|\zeta(1 + it)|$ or the Dirichlet theorem, from the theory of Diophantine approximations we can easily prove that

$$\frac{a}{b} \ll \frac{\log \log t}{\log \log \log t}; \quad t \rightarrow +\infty.$$

For $N = 11$ and $T = 144$, (11 is a Lucas number and 144 is a Fibonacci's number), from:

$$Nh^2(T)\mu(F) \leq \int_E \left| \sum_{n < N} n^{it} \right|^2 dt \leq \int_T^{2T} \left| \sum_{n < N} n^{it} \right|^2 dt \leq TN + 16N \log N \leq 17TN.$$

We obtain:

$$144 \cdot 11 + 16 \cdot 11 \ln(11) \leq 17 \cdot 144 \cdot 11$$

Input:

$$144 \times 11 + 16 \times 11 \log(11) \leq 17 \times 144 \times 11$$

$\log(x)$ is the natural logarithm

Difference:

$$176 \log(11) - 25\,344$$

$$-25\,344 + 176 \log(11)$$

Input:

$$-25\,344 + 176 \log(11)$$

$\log(x)$ is the natural logarithm

Decimal approximation:

$$-24921.9704319874867842450979302781372432313795937070145771\dots$$

$$-24921.97043198\dots$$

•

Property:

$$-25\,344 + 176 \log(11) \text{ is a transcendental number}$$

•

Alternate form:

$$176 (\log(11) - 144)$$

•

Alternative representations:

• More

$$-25\,344 + 176 \log(11) = -25\,344 + 176 \log_e(11)$$

•

$$-25\,344 + 176 \log(11) = -25\,344 + 176 \log(a) \log_a(11)$$

•

$$-25\,344 + 176 \log(11) = -25\,344 - 176 \operatorname{Li}_1(-10)$$

$\log_b(x)$ is the base- b logarithm

$\operatorname{Li}_n(x)$ is the polylogarithm function

•

Series representations:

$$-25\,344 + 176 \log(11) = -25\,344 + 176 \log(10) - 176 \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{10}\right)^k}{k}$$

•

$$-25\,344 + 176 \log(11) = -25\,344 + 352 i \pi \left\lfloor \frac{\arg(11-x)}{2\pi} \right\rfloor + 176 \log(x) - 176 \sum_{k=1}^{\infty} \frac{(-1)^k (11-x)^k x^{-k}}{k} \quad \text{for } x < 0$$

•

$$-25\,344 + 176 \log(11) = -25\,344 + 176 \left\lfloor \frac{\arg(11-z_0)}{2\pi} \right\rfloor \log\left(\frac{1}{z_0}\right) + 176 \log(z_0) + 176 \left\lfloor \frac{\arg(11-z_0)}{2\pi} \right\rfloor \log(z_0) - 176 \sum_{k=1}^{\infty} \frac{(-1)^k (11-z_0)^k z_0^{-k}}{k}$$

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

•

Integral representations:

$$-25\,344 + 176 \log(11) = -25\,344 + 176 \int_1^{11} \frac{1}{t} dt$$

•

$$-25\,344 + 176 \log(11) = -25\,344 - \frac{88 i}{\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{10^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \quad \text{for } -1 < \gamma < 0$$

$\Gamma(x)$ is the gamma function

$$-\left(\left(\left(-25344 + 176 \log(11)\right)\right)\right) / 15127$$

Where 15127 is a Lucas number

Input:

$$\frac{-25\,344 + 176 \log(11)}{15\,127}$$

$\log(x)$ is the natural logarithm

Exact result:

$$\frac{25\,344 - 176 \log(11)}{15\,127}$$

Decimal approximation:

1.647515728960632431033588810093087673909656878013288462825...

$$1.64751572896\dots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$$

Property:

$$\frac{25\,344 - 176 \log(11)}{15\,127} \text{ is a transcendental number}$$

•

Alternate forms:

$$\frac{176 (\log(11) - 144)}{15\,127}$$

•

$$\frac{25\,344}{15\,127} - \frac{176 \log(11)}{15\,127}$$

•

Alternative representations:

$$-\frac{-25\,344 + 176 \log(11)}{15\,127} = \frac{25\,344 - 176 \log_e(11)}{15\,127}$$

•

$$-\frac{-25\,344 + 176 \log(11)}{15\,127} = \frac{25\,344 - 176 \log(a) \log_a(11)}{15\,127}$$

•

$$-\frac{-25\,344 + 176 \log(11)}{15\,127} = \frac{25\,344 + 176 \operatorname{Li}_1(-10)}{15\,127}$$

$\log_b(x)$ is the base- b logarithm

$\operatorname{Li}_n(x)$ is the polylogarithm function

•

Series representations:

$$-\frac{-25\,344 + 176 \log(11)}{15\,127} = \frac{25\,344}{15\,127} - \frac{176 \log(10)}{15\,127} + \frac{176 \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{10}\right)^k}{k}}{15\,127}$$

•

$$-\frac{-25\,344 + 176 \log(11)}{15\,127} = \frac{25\,344}{15\,127} - \frac{352 i \pi \left\lfloor \frac{\arg(11-x)}{2\pi} \right\rfloor}{15\,127} - \frac{176 \log(x)}{15\,127} + \frac{176 \sum_{k=1}^{\infty} \frac{(-1)^k (11-x)^k x^{-k}}{k}}{15\,127} \quad \text{for } x < 0$$

•

$$-\frac{-25\,344 + 176 \log(11)}{15\,127} = \frac{25\,344}{15\,127} - \frac{176 \left\lfloor \frac{\arg(11-z_0)}{2\pi} \right\rfloor \log\left(\frac{1}{z_0}\right)}{15\,127} - \frac{176 \log(z_0)}{15\,127} - \frac{176 \left\lfloor \frac{\arg(11-z_0)}{2\pi} \right\rfloor \log(z_0)}{15\,127} + \frac{176 \sum_{k=1}^{\infty} \frac{(-1)^k (11-z_0)^k z_0^{-k}}{k}}{15\,127}$$

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

•

Integral representations:

$$-\frac{-25\,344 + 176 \log(11)}{15\,127} = \frac{25\,344}{15\,127} - \frac{176}{15\,127} \int_1^{11} \frac{1}{t} dt$$

•

$$-\frac{-25\,344 + 176 \log(11)}{15\,127} = \frac{25\,344}{15\,127} + \frac{88 i}{15\,127 \pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{10^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds$$

for $-1 < \gamma < 0$

$\Gamma(x)$ is the gamma function

$$1/6 \left(\left(\left(\left(\left(\left(-25344 + 176 \log(11) \right) \right)^{1/4} \right) \right)^2 \right) \right)^{1/4}$$

Input:

$$\frac{1}{6} \left(\frac{1}{4} \sqrt[4]{-(-25\,344 + 176 \log(11))} \right)^2$$

$\log(x)$ is the natural logarithm

Exact result:

$$\frac{1}{96} \sqrt{25\,344 - 176 \log(11)}$$

Decimal approximation:

1.644447281340983986578801891727760223918055024106365671192...

$$1.64444728134\dots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$$

Property:

$$\frac{1}{96} \sqrt{25\,344 - 176 \log(11)} \text{ is a transcendental number}$$

•

Alternate forms:

$$\frac{1}{24} \sqrt{1584 - 11 \log(11)}$$

•

$$\frac{1}{24} \sqrt{11 (144 - \log(11))}$$

•

Alternative representations:

$$\frac{1}{6} \left(\frac{1}{4} \sqrt[4]{-(-25\,344 + 176 \log(11))} \right)^2 = \frac{1}{6} \left(\frac{1}{4} \sqrt[4]{25\,344 - 176 \log_e(11)} \right)^2$$

$$\frac{1}{6} \left(\frac{1}{4} \sqrt[4]{-(-25\,344 + 176 \log(11))} \right)^2 = \frac{1}{6} \left(\frac{1}{4} \sqrt[4]{25\,344 - 176 \log(a) \log_a(11)} \right)^2$$

$$\frac{1}{6} \left(\frac{1}{4} \sqrt[4]{-(-25\,344 + 176 \log(11))} \right)^2 = \frac{1}{6} \left(\frac{1}{4} \sqrt[4]{25\,344 + 176 \operatorname{Li}_1(-10)} \right)^2$$

$\log_b(x)$ is the base- b logarithm

$\operatorname{Li}_n(x)$ is the polylogarithm function

Series representations:

$$\frac{1}{6} \left(\frac{1}{4} \sqrt[4]{-(-25\,344 + 176 \log(11))} \right)^2 = \frac{1}{96} \sqrt{25\,344 - 176 \left(\log(10) - \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{10}\right)^k}{k} \right)}$$

$$\frac{1}{6} \left(\frac{1}{4} \sqrt[4]{-(-25\,344 + 176 \log(11))} \right)^2 = \frac{1}{96} \sqrt{25\,344 - 176 \left(2i\pi \left\lfloor \frac{\arg(11-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (11-x)^k x^{-k}}{k} \right)} \quad \text{for } x < 0$$

$$\frac{1}{6} \left(\frac{1}{4} \sqrt[4]{-(-25\,344 + 176 \log(11))} \right)^2 = \frac{1}{96} \sqrt{\left(25\,344 - 176 \left(\log(z_0) + \left\lfloor \frac{\arg(11-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (11-z_0)^k z_0^{-k}}{k} \right) \right)}$$

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

Integral representations:

$$\frac{1}{6} \left(\frac{1}{4} \sqrt[4]{-(-25344 + 176 \log(11))} \right)^2 = \frac{1}{96} \sqrt{25344 - 176 \int_1^{11} \frac{1}{t} dt}$$

$$\frac{1}{6} \left(\frac{1}{4} \sqrt[4]{-(-25344 + 176 \log(11))} \right)^2 = \frac{1}{96} \sqrt{25344 + \frac{88i}{\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{10^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds} \quad \text{for } -1 < \gamma < 0$$

$$(29-2)/10^3 + 1/6 \left(\left(\frac{1}{4} \sqrt[4]{-(-25344 + 176 \log(11))} \right) \right)^2$$

Input:

$$\frac{29-2}{10^3} + \frac{1}{6} \left(\frac{1}{4} \sqrt[4]{-(-25344 + 176 \log(11))} \right)^2$$

log(x) is the natural logarithm

Exact result:

$$\frac{27}{1000} + \frac{1}{96} \sqrt{25344 - 176 \log(11)}$$

Decimal approximation:

1.671447281340983986578801891727760223918055024106365671192...

1.67144728.... result practically equal to the value of the formula:

$$m_{p'} = 2 \times \frac{\eta}{R} m_P = 1.6714213 \times 10^{-24} \text{ gm}$$

that is the holographic proton mass

Property:

$$\frac{27}{1000} + \frac{1}{96} \sqrt{25344 - 176 \log(11)} \text{ is a transcendental number}$$

Alternate forms:

$$\frac{27}{1000} + \frac{1}{24} \sqrt{1584 - 11 \log(11)}$$

- $$\frac{27}{1000} + \frac{1}{24} \sqrt{11(144 - \log(11))}$$

- $$\frac{81 + 125 \sqrt{11(144 - \log(11))}}{3000}$$

- **Alternative representations:**

$$\frac{29-2}{10^3} + \frac{1}{6} \left(\frac{1}{4} \sqrt[4]{-(-25\,344 + 176 \log(11))} \right)^2 = \frac{27}{10^3} + \frac{1}{6} \left(\frac{1}{4} \sqrt[4]{25\,344 - 176 \log_e(11)} \right)^2$$

- $$\frac{29-2}{10^3} + \frac{1}{6} \left(\frac{1}{4} \sqrt[4]{-(-25\,344 + 176 \log(11))} \right)^2 =$$

$$\frac{27}{10^3} + \frac{1}{6} \left(\frac{1}{4} \sqrt[4]{25\,344 - 176 \log(a) \log_a(11)} \right)^2$$

- $$\frac{29-2}{10^3} + \frac{1}{6} \left(\frac{1}{4} \sqrt[4]{-(-25\,344 + 176 \log(11))} \right)^2 = \frac{27}{10^3} + \frac{1}{6} \left(\frac{1}{4} \sqrt[4]{25\,344 + 176 \operatorname{Li}_1(-10)} \right)^2$$

$\log_b(x)$ is the base- b logarithm

$\operatorname{Li}_n(x)$ is the polylogarithm function

- **Series representations:**

$$\frac{29-2}{10^3} + \frac{1}{6} \left(\frac{1}{4} \sqrt[4]{-(-25\,344 + 176 \log(11))} \right)^2 =$$

$$\frac{27}{1000} + \frac{1}{96} \sqrt{25\,344 - 176 \left(\log(10) - \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{10}\right)^k}{k} \right)}$$

$$\frac{29-2}{10^3} + \frac{1}{6} \left(\frac{1}{4} \sqrt[4]{-(-25\,344 + 176 \log(11))} \right)^2 =$$

$$\frac{27}{1000} + \frac{1}{96} \sqrt{25\,344 - 176 \left(2i\pi \left\lfloor \frac{\arg(11-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (11-x)^k x^{-k}}{k} \right)}$$

for $x < 0$

$$\frac{29-2}{10^3} + \frac{1}{6} \left(\frac{1}{4} \sqrt[4]{-(-25\,344 + 176 \log(11))} \right)^2 = \frac{27}{1000} + \frac{1}{96} \sqrt{\left(25\,344 - \right.$$

$$\left. 176 \left(\log(z_0) + \left\lfloor \frac{\arg(11-z_0)}{2\pi} \right\rfloor \right) \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (11-z_0)^k z_0^{-k}}{k} \right)}$$

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

Integral representations:

$$\frac{29-2}{10^3} + \frac{1}{6} \left(\frac{1}{4} \sqrt[4]{-(-25\,344 + 176 \log(11))} \right)^2 = \frac{27}{1000} + \frac{1}{96} \sqrt{25\,344 - 176 \int_1^{11} \frac{1}{t} dt}$$

$$\frac{29-2}{10^3} + \frac{1}{6} \left(\frac{1}{4} \sqrt[4]{-(-25\,344 + 176 \log(11))} \right)^2 =$$

$$\frac{27}{1000} + \frac{1}{96} \sqrt{25\,344 + \frac{88i}{\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{10^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \quad \text{for } -1 < \gamma < 0$$

For $N = 11$, $c = 5$, $a = 3$, $b = 2$ and $t = 14$, we obtain from:

$$|S(N)| \ll N \exp\left(-c \frac{\log^a N}{\log^b t}\right),$$

we obtain:

$$11 * \exp(-5 * (((\ln^3(11)) / ((\ln^2(14))))))$$

Input:

$$11 \exp\left(-5 \times \frac{\log^3(11)}{\log^2(14)}\right)$$

$\log(x)$ is the natural logarithm

Exact result:

$$11 e^{-(5 \log^3(11))/\log^2(14)}$$

Decimal approximation:

0.000552830615211073109887405550616657560035243405929988337...

0.000552830615211...

Property:

$11 e^{-(5 \log^3(11))/\log^2(14)}$ is a transcendental number

•

Alternate form:

$$11 e^{-(5 \log^3(11))/(\log(2)+\log(7))^2}$$

•

Alternative representations:

$$11 \exp\left(-\frac{5 \log^3(11)}{\log^2(14)}\right) = 11 \exp\left(-\frac{5 \log_e^3(11)}{\log_e^2(14)}\right)$$

•

$$11 \exp\left(-\frac{5 \log^3(11)}{\log^2(14)}\right) = 11 \exp\left(-\frac{5 (\log(a) \log_a(11))^3}{(\log(a) \log_a(14))^2}\right)$$

$\log_b(x)$ is the base- b logarithm

•

Series representations:

$$11 \exp\left(-\frac{5 \log^3(11)}{\log^2(14)}\right) = 11 \exp\left(-\frac{5 \left(\log(10) - \sum_{k=1}^{\infty} \frac{(-\frac{1}{10})^k}{k}\right)^3}{\left(\log(13) - \sum_{k=1}^{\infty} \frac{(-\frac{1}{13})^k}{k}\right)^2}\right)$$

- $$11 \exp\left(-\frac{5 \log^3(11)}{\log^2(14)}\right) = 11 \exp\left(-\frac{5 \left(2 i \pi \left\lfloor \frac{\text{arg}(11-x)}{2 \pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (11-x)^k x^{-k}}{k} \right)^3}{\left(2 i \pi \left\lfloor \frac{\text{arg}(14-x)}{2 \pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (14-x)^k x^{-k}}{k} \right)^2}\right)$$

for $x < 0$

- $$11 \exp\left(-\frac{5 \log^3(11)}{\log^2(14)}\right) = 11 \exp\left(-\frac{5 \left(\log(z_0) + \left\lfloor \frac{\text{arg}(11-z_0)}{2 \pi} \right\rfloor \right) \left(\log\left(\frac{1}{z_0}\right) + \log(z_0)\right) - \sum_{k=1}^{\infty} \frac{(-1)^k (11-z_0)^k z_0^{-k}}{k} \right)^3}{\left(\log(z_0) + \left\lfloor \frac{\text{arg}(14-z_0)}{2 \pi} \right\rfloor \right) \left(\log\left(\frac{1}{z_0}\right) + \log(z_0)\right) - \sum_{k=1}^{\infty} \frac{(-1)^k (14-z_0)^k z_0^{-k}}{k} \right)^2}\right)$$

$\text{arg}(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

$$-76 + 1 / ((((((11 * \exp(-5 * (((\ln^3(11)) / ((\ln^2(14))))))))))))))$$

Where 76 is a Lucas number

Input:

$$-76 + \frac{1}{11 \exp\left(-5 \times \frac{\log^3(11)}{\log^2(14)}\right)}$$

$\log(x)$ is the natural logarithm

Exact result:

$$\frac{1}{11} e^{(5 \log^3(11)) / \log^2(14)} - 76$$

Decimal approximation:

1732.872324515160386107279602340921200572579471630789252801...

1732.8723245...

Alternate forms:

$$\frac{1}{11} \left(e^{(5 \log^3(11)) / \log^2(14)} - 836 \right)$$

$$\frac{1}{11} \left(e^{(5 \log^3(11)) / (\log(2) + \log(7))^2} - 836 \right)$$

$$\frac{1}{11} e^{(5 \log^3(11)) / (\log(2) + \log(7))^2} - 76$$

Alternative representations:

$$-76 + \frac{1}{11 \exp\left(-\frac{5 \log^3(11)}{\log^2(14)}\right)} = -76 + \frac{1}{11 \exp\left(-\frac{5 \log_e^3(11)}{\log_e^2(14)}\right)}$$

$$-76 + \frac{1}{11 \exp\left(-\frac{5 \log^3(11)}{\log^2(14)}\right)} = -76 + \frac{1}{11 \exp\left(-\frac{5 (\log(a) \log_a(11))^3}{(\log(a) \log_a(14))^2}\right)}$$

$\log_b(x)$ is the base- b logarithm

Series representations:

$$-76 + \frac{1}{11 \exp\left(-\frac{5 \log^3(11)}{\log^2(14)}\right)} = \frac{1}{11} \left(-836 + \exp \left(\frac{5 \left(\log(10) - \sum_{k=1}^{\infty} \frac{\left(\frac{-1}{10}\right)^k}{k} \right)^3}{\left(\log(13) - \sum_{k=1}^{\infty} \frac{\left(\frac{-1}{13}\right)^k}{k} \right)^2} \right) \right)$$

$$-76 + \frac{1}{11 \exp\left(-\frac{5 \log^3(11)}{\log^2(14)}\right)} = \frac{1}{11} \left(-836 + \exp \left(\frac{5 \left(2i\pi \left\lfloor \frac{\arg(11-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (11-x)^k x^{-k}}{k} \right)^3}{\left(2i\pi \left\lfloor \frac{\arg(14-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (14-x)^k x^{-k}}{k} \right)^2} \right) \right) \text{ for } x < 0$$

$$-76 + \frac{1}{11 \exp\left(-\frac{5 \log^3(11)}{\log^2(14)}\right)} = \frac{1}{11} \left(-836 + \exp \left(\frac{5 \left(\log(z_0) + \left\lfloor \frac{\arg(11-z_0)}{2\pi} \right\rfloor \right) \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (11-z_0)^k z_0^{-k}}{k} \right)^3}{\left(\log(z_0) + \left\lfloor \frac{\arg(14-z_0)}{2\pi} \right\rfloor \right) \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (14-z_0)^k z_0^{-k}}{k} \right)^2} \right) \right)$$

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

$$2\text{sqrt}[6((((((-76+1/ (((((11*\exp(-5*(((\ln^3(11))/((\ln^2(14))))))))))))))))))^{1/15}]$$

Input:

$$2 \sqrt{6} \sqrt[15]{\sqrt{-76 + \frac{1}{11 \exp\left(-5 \times \frac{\log^2(11)}{\log^2(14)}\right)}}$$

$\log(x)$ is the natural logarithm

Exact result:

$$2 \sqrt{6} \sqrt[30]{\frac{1}{11} e^{(5 \log^3(11))/\log^2(14)} - 76}$$

Decimal approximation:

6.281516521465835536716642965319628146647221544302618224701...

$$6.281516521465... = C = 2\pi r$$

Alternate forms:

$$2 \sqrt{6} \sqrt[30]{\frac{1}{11} \left(e^{(5 \log^3(11))/\log^2(14)} - 836 \right)}$$

•

$$2 \sqrt{6} \sqrt[30]{\frac{1}{11} e^{(5 \log^3(11))/(\log(2)+\log(7))^2} - 76}$$

•

Alternative representations:

$$2 \sqrt{6} \sqrt[15]{\sqrt{-76 + \frac{1}{11 \exp\left(-\frac{5 \log^3(11)}{\log^2(14)}\right)}} = 2 \sqrt{6} \sqrt[15]{\sqrt{-76 + \frac{1}{11 \exp\left(-\frac{5 \log_e^3(11)}{\log_e^2(14)}\right)}}}$$

•

$$2 \sqrt{6} \sqrt[15]{\sqrt{-76 + \frac{1}{11 \exp\left(-\frac{5 \log^3(11)}{\log^2(14)}\right)}} = 2 \sqrt{6} \sqrt[15]{\sqrt{-76 + \frac{1}{11 \exp\left(-\frac{5 (\log(\alpha) \log_{\alpha}(11))^3}{(\log(\alpha) \log_{\alpha}(14))^2}\right)}}}$$

$\log_b(x)$ is the base- b logarithm

Series representations:

$$2 \sqrt{6} \sqrt[15]{-76 + \frac{1}{11 \exp\left(-\frac{5 \log^3(11)}{\log^2(14)}\right)}} = \frac{2 \sqrt{6} \sqrt[30]{-836 + \exp\left(\frac{5 \left(\log(10) - \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{10}\right)^k}{k}\right)^3}{\left(\log(13) - \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{13}\right)^k}{k}\right)^2}\right)}}{\sqrt[30]{11}}$$

$$2 \sqrt{6} \sqrt[15]{-76 + \frac{1}{11 \exp\left(-\frac{5 \log^3(11)}{\log^2(14)}\right)}} = \frac{2 \sqrt{6} \sqrt[30]{-836 + \exp\left(\frac{5 \left(2i\pi \left\lfloor \frac{\arg(11-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (11-x)^k x^{-k}}{k} \right)^3}{\left(2i\pi \left\lfloor \frac{\arg(14-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (14-x)^k x^{-k}}{k} \right)^2}\right)}}{\sqrt[30]{11}} \quad \text{for } x < 0$$

$$2 \sqrt{6} \sqrt[15]{-76 + \frac{1}{11 \exp\left(-\frac{5 \log^3(11)}{\log^2(14)}\right)}} = \frac{2 \sqrt{6} \sqrt[30]{-836 + \exp\left(\frac{5 \left(\log(z_0) + \left\lfloor \frac{\arg(11-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0)\right) - \sum_{k=1}^{\infty} \frac{(-1)^k (11-z_0)^k z_0^{-k}}{k} \right)^3}{\left(\log(z_0) + \left\lfloor \frac{\arg(14-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0)\right) - \sum_{k=1}^{\infty} \frac{(-1)^k (14-z_0)^k z_0^{-k}}{k} \right)^2}\right)}}{\sqrt[30]{11}}$$

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

$$(2\pi) * 1 / (((2\sqrt{6} * (((-76 + 1 / (11 * \exp(-5 * ((\ln^3(11)) / (\ln^2(14))))))))))^{1/15}))))$$

Input:

$$(2\pi) \times \frac{1}{2 \sqrt{6 \sqrt[15]{-76 + \frac{1}{11 \exp\left(-5 \times \frac{\log^3(11)}{\log^2(14)}\right)}}}}$$

$\log(x)$ is the natural logarithm

Exact result:

$$\frac{\pi}{\sqrt{6} \sqrt[30]{\frac{1}{11} e^{(5 \log^3(11))/\log^2(14)} - 76}}$$

Decimal approximation:

1.000265666055012065376898475018531988133572536841624152905...

1.000265666055..... = r = radius of circumference

Alternate forms:

$$\frac{\pi \sqrt[30]{\frac{11}{e^{(5 \log^3(11))/\log^2(14)} - 836}}}{\sqrt{6}}$$

•

$$\frac{\pi}{\sqrt{6} \sqrt[30]{\frac{1}{11} e^{(5 \log^3(11))/(\log(2)+\log(7))^2} - 76}}$$

•

Alternative representations:

$$\frac{2\pi}{2 \sqrt{6 \sqrt[15]{-76 + \frac{1}{11 \exp\left(-\frac{5 \log^3(11)}{\log^2(14)}\right)}}}} = \frac{2\pi}{2 \sqrt{6 \sqrt[15]{-76 + \frac{1}{11 \exp\left(-\frac{5 \log_e^3(11)}{\log_e^2(14)}\right)}}}}$$

•

$$\frac{2\pi}{2 \sqrt{6 \sqrt[15]{-76 + \frac{1}{11 \exp\left(-\frac{5 \log^3(11)}{\log^2(14)}\right)}}}} = \frac{2\pi}{2 \sqrt{6 \sqrt[15]{-76 + \frac{1}{11 \exp\left(-\frac{5 (\log(a) \log_a(11))^3}{(\log(a) \log_a(14))^2}\right)}}}}$$

$\log_b(x)$ is the base- b logarithm

Series representations:

$$\frac{2\pi}{2\sqrt{6}\sqrt[15]{-76 + \frac{1}{11\exp\left(-\frac{5\log^3(11)}{\log^2(14)}\right)}}} = \frac{\pi}{\sqrt{6}\sqrt[30]{-76 + \frac{1}{11}\exp\left(\frac{5\left(\log(10) - \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{10}\right)^k}{k}\right)^3}{\left(\log(13) - \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{13}\right)^k}{k}\right)^2}\right)}}$$

- $$\frac{2\pi}{2\sqrt{6}\sqrt[15]{-76 + \frac{1}{11\exp\left(-\frac{5\log^3(11)}{\log^2(14)}\right)}}} = \frac{\pi}{\sqrt{6}\sqrt[30]{-76 + \frac{1}{11}\exp\left(\frac{5\left(2i\pi\left\lfloor\frac{\arg(11-x)}{2\pi}\right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k(11-x)^k x^{-k}}{k}\right)^3}{\left(2i\pi\left\lfloor\frac{\arg(14-x)}{2\pi}\right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k(14-x)^k x^{-k}}{k}\right)^2}\right)}}$$
 for $x < 0$

- $$\frac{2\pi}{2\sqrt{6}\sqrt[15]{-76 + \frac{1}{11\exp\left(-\frac{5\log^3(11)}{\log^2(14)}\right)}}} = \frac{\pi}{\sqrt{6}\sqrt[30]{-76 + \frac{1}{11}\exp\left(\frac{5\left(\log(z_0) + \left\lfloor\frac{\arg(11-z_0)}{2\pi}\right\rfloor\right)\left(\log\left(\frac{1}{z_0}\right) + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k(11-z_0)^k z_0^{-k}}{k}\right)^3}{\left(\log(z_0) + \left\lfloor\frac{\arg(14-z_0)}{2\pi}\right\rfloor\right)\left(\log\left(\frac{1}{z_0}\right) + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k(14-z_0)^k z_0^{-k}}{k}\right)^2}\right)}}$$

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

We note that **1.000265666055.....** is very near to the value of $\zeta(12)$. Indeed:

zeta (12)

Input:

$\zeta(12)$

$\zeta(s)$ is the Riemann zeta function

Exact result:

$$\frac{691 \pi^{12}}{638512875}$$

Decimal approximation:

1.000246086553308048298637998047739670960416088458003404533...

1.0002460865533...

Property:

$\frac{691 \pi^{12}}{638512875}$ is a transcendental number

•

Alternative representations:

$$\zeta(12) = \zeta(12, 1)$$

•

$$\zeta(12) = S_{11,1}(1)$$

•

$$\zeta(12) = \frac{\psi^{(11)}(1)(-1)^{12}}{11!}$$

$\zeta(s, a)$ is the generalized Riemann zeta function

$S_{n,p}(x)$ is the Nielsen generalized polylogarithm function

$\psi^{(n)}(x)$ is the n^{th} derivative of the digamma function

$n!$ is the factorial function

•

Integral representations:

$$\zeta(12) = \frac{2830336 \left(\int_0^\infty \frac{1}{1+t^2} dt \right)^{12}}{638512875}$$

•

$$\zeta(12) = \frac{2830336 \left(\int_0^1 \frac{1}{\sqrt{1-t^2}} dt \right)^{12}}{638512875}$$

$$\zeta(12) = \frac{11593056256 \left(\int_0^1 \sqrt{1-t^2} dt \right)^{12}}{638512875}$$

Thence, we have the following mathematical connection with the Dirichlet condition concerning the D-branes:

$$\int_0^\pi d\sigma \partial_\sigma X^{r25} = i \int_0^\pi d\sigma \partial_\tau X^{25} = 2\pi\alpha' p^{25} = \frac{2\pi\alpha' n}{R} = 2\pi n R'$$

$$2 \sqrt{6} \sqrt[15]{-76 + \frac{1}{11 \exp\left(-5 \times \frac{\log^3(11)}{\log^2(14)}\right)}} \quad 2 \sqrt{6} \sqrt[30]{\frac{1}{11} e^{(5 \log^3(11))/\log^2(14)} - 76}$$

$$6.281516521465835536716642965319628146647221544302618224701... = 2\pi r$$

$$\frac{\pi}{\sqrt{6} \sqrt[30]{\frac{1}{11} e^{(5 \log^3(11))/\log^2(14)} - 76}}$$

$$1.000265666055012065376898475018531988133572536841624152905... = r$$

Now, we insert this radius in the Hawking Radiation calculator and obtain temperature and mass of this quantum black hole (a black brane).

$$\text{Mass} = 6.736469e+26$$

$$\text{Radius} = 1.000266$$

$$\text{Temperature} = 0.0001821731$$

From the Ramanujan-Nardelli mock formula, we obtain:

$$\sqrt{\frac{1}{\frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{6.736469 \times 10^{26}} \sqrt{\frac{0.0001821731 \times 4 \pi \times 1.000266^3 - 1.000266^2}{6.67 \times 10^{-11}}}}}$$

Input interpretation:

$$\sqrt{\frac{1}{\frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{6.736469 \times 10^{26}} \sqrt{\frac{0.0001821731 \times 4 \pi \times 1.000266^3 - 1.000266^2}{6.67 \times 10^{-11}}}}}$$

Result:

1.618248938735178025577670247874304978307846795256872628616...

1.618248938...

And the conjugate

$$1 / \sqrt{\frac{1}{\frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{6.736469 \times 10^{26}} \sqrt{\frac{0.0001821731 \times 4 \pi \times 1.000266^3 - 1.000266^2}{6.67 \times 10^{-11}}}}}$$

Input interpretation:

$$\sqrt{\frac{1}{\frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{6.736469 \times 10^{26}} \sqrt{\frac{0.0001821731 \times 4 \pi \times 1.000266^3 - 1.000266^2}{6.67 \times 10^{-11}}}}}$$

Result:

0.617951896067115073952485206270777030337420984711450298039...

0.617951896...

Now, we have that:

was obtained by Vinogradov (see [8, 1]). Using the omega estimate of $|\zeta(1 + it)|$ or the Dirichlet theorem, from the theory of Diophantine approximations we can easily prove that

$$\frac{a}{b} \ll \frac{\log \log t}{\log \log \log t}, \quad t \rightarrow +\infty.$$

We obtain, for $t = 14$:

$$\ln(\ln 14) / \ln(\ln(\ln(14)))$$

Input:

$$\frac{\log(\log(14))}{\log(\log(\log(14)))}$$

$\log(x)$ is the natural logarithm

Decimal approximation:

-32.3210231338405437985817608079562125106041355188346290265...

-32.32102313384...

Alternate form:

$$\frac{\log(\log(2) + \log(7))}{\log(\log(\log(2) + \log(7)))}$$

Alternative representations:

$$\frac{\log(\log(14))}{\log(\log(\log(14)))} = \frac{\log_e(\log(14))}{\log_e(\log(\log(14)))}$$

- $$\frac{\log(\log(14))}{\log(\log(\log(14)))} = \frac{\log(a) \log_a(\log(14))}{\log(a) \log_a(\log(\log(14)))}$$
- $$\frac{\log(\log(14))}{\log(\log(\log(14)))} = \frac{-\text{Li}_1(1 - \log(14))}{-\text{Li}_1(1 - \log(\log(14)))}$$

$\log_b(x)$ is the base- b logarithm

$\text{Li}_n(x)$ is the polylogarithm function

Series representations:

$$\frac{\log(\log(14))}{\log(\log(\log(14)))} = \frac{2\pi \left[\frac{\text{arg}(-x + \log(14))}{2\pi} \right] - i \log(x) + i \sum_{k=1}^{\infty} \frac{(-1)^k x^{-k} (-x + \log(14))^k}{k}}{2\pi \left[\frac{\text{arg}(-x + \log(\log(14)))}{2\pi} \right] - i \log(x) + i \sum_{k=1}^{\infty} \frac{(-1)^k x^{-k} (-x + \log(\log(14)))^k}{k}}$$

for $x < 0$

$$\frac{\log(\log(14))}{\log(\log(\log(14)))} = \frac{2\pi \left\lfloor \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right\rfloor - i \log(z_0) + i \sum_{k=1}^{\infty} \frac{(-1)^k (\log(14) - z_0)^k z_0^{-k}}{k}}{2\pi \left\lfloor \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right\rfloor - i \log(z_0) + i \sum_{k=1}^{\infty} \frac{(-1)^k (\log(\log(14)) - z_0)^k z_0^{-k}}{k}}$$

$$\frac{\log(\log(14))}{\log(\log(\log(14)))} = \left(\left\lfloor \frac{\arg(\log(14) - z_0)}{2\pi} \right\rfloor \log\left(\frac{1}{z_0}\right) + \log(z_0) + \left\lfloor \frac{\arg(\log(14) - z_0)}{2\pi} \right\rfloor \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (\log(14) - z_0)^k z_0^{-k}}{k} \right) / \left(\left\lfloor \frac{\arg(\log(\log(14)) - z_0)}{2\pi} \right\rfloor \log\left(\frac{1}{z_0}\right) + \log(z_0) + \left\lfloor \frac{\arg(\log(\log(14)) - z_0)}{2\pi} \right\rfloor \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (\log(\log(14)) - z_0)^k z_0^{-k}}{k} \right)$$

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

Integral representation:

$$\frac{\log(\log(14))}{\log(\log(\log(14)))} = \frac{\int_1^{\log(14)} \frac{1}{t} dt}{\int_1^{\log(\log(14))} \frac{1}{t} dt}$$

And for $t = 18$, where 18 is a Lucas number, we obtain:

$$\ln(\ln 18) / \ln(\ln(\ln(18)))$$

Input:

$$\frac{\log(\log(18))}{\log(\log(\log(18)))}$$

$\log(x)$ is the natural logarithm

Decimal approximation:

17.81601385916840590783241471903272500240518751073911212258...

17.81601385916....

Alternate form:

$$\frac{\log(\log(2) + 2 \log(3))}{\log(\log(\log(2) + 2 \log(3)))}$$

• **Alternative representations:**

$$\frac{\log(\log(18))}{\log(\log(\log(18)))} = \frac{\log_e(\log(18))}{\log_e(\log(\log(18)))}$$

•
$$\frac{\log(\log(18))}{\log(\log(\log(18)))} = \frac{\log(a) \log_a(\log(18))}{\log(a) \log_a(\log(\log(18)))}$$

•
$$\frac{\log(\log(18))}{\log(\log(\log(18)))} = \frac{-\text{Li}_1(1 - \log(18))}{-\text{Li}_1(1 - \log(\log(18)))}$$

$\log_b(x)$ is the base- b logarithm

$\text{Li}_n(x)$ is the polylogarithm function

• **Series representations:**

$$\frac{\log(\log(18))}{\log(\log(\log(18)))} = \frac{2 \pi \left[\frac{\text{arg}(-x+\log(18))}{2 \pi} \right] - i \log(x) + i \sum_{k=1}^{\infty} \frac{(-1)^k x^{-k} (-x+\log(18))^k}{k}}{2 \pi \left[\frac{\text{arg}(-x+\log(\log(18)))}{2 \pi} \right] - i \log(x) + i \sum_{k=1}^{\infty} \frac{(-1)^k x^{-k} (-x+\log(\log(18)))^k}{k}}$$

for $x < 0$

•
$$\frac{\log(\log(18))}{\log(\log(\log(18)))} = \frac{2 \pi \left[\frac{\pi - \text{arg}\left(\frac{1}{z_0}\right) - \text{arg}(z_0)}{2 \pi} \right] - i \log(z_0) + i \sum_{k=1}^{\infty} \frac{(-1)^k (\log(18) - z_0)^k z_0^{-k}}{k}}{2 \pi \left[\frac{\pi - \text{arg}\left(\frac{1}{z_0}\right) - \text{arg}(z_0)}{2 \pi} \right] - i \log(z_0) + i \sum_{k=1}^{\infty} \frac{(-1)^k (\log(\log(18)) - z_0)^k z_0^{-k}}{k}}$$

$$\frac{\log(\log(18))}{\log(\log(\log(18)))} = \left(\left[\frac{\arg(\log(18) - z_0)}{2\pi} \right] \log\left(\frac{1}{z_0}\right) + \log(z_0) + \left[\frac{\arg(\log(18) - z_0)}{2\pi} \right] \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (\log(18) - z_0)^k z_0^{-k}}{k} \right) / \left(\left[\frac{\arg(\log(\log(18)) - z_0)}{2\pi} \right] \log\left(\frac{1}{z_0}\right) + \log(z_0) + \left[\frac{\arg(\log(\log(18)) - z_0)}{2\pi} \right] \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (\log(\log(18)) - z_0)^k z_0^{-k}}{k} \right)$$

$\arg(z)$ is the complex argument

$[x]$ is the floor function

Integral representations:

$$\frac{\log(\log(18))}{\log(\log(\log(18)))} = \frac{\int_1^{\log(18)} \frac{1}{t} dt}{\int_1^{\log(\log(18))} \frac{1}{t} dt}$$

$$\frac{\log(\log(18))}{\log(\log(\log(18)))} = \frac{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2 \Gamma(1+s) (-1+\log(18))^{-s}}{\Gamma(1-s)} ds}{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2 \Gamma(1+s) (-1+\log(\log(18)))^{-s}}{\Gamma(1-s)} ds} \quad \text{for } -1 < \gamma < 0$$

$\Gamma(x)$ is the gamma function

We have that:

$$-3 / [(((\ln(\ln 14) / \ln(\ln(\ln(14)))))) * 1 / (((\ln(\ln 18) / \ln(\ln(\ln(18)))))))]$$

Input:

$$3 - \frac{\frac{\log(\log(14))}{\log(\log(\log(14)))}}{\frac{\log(\log(18))}{\log(\log(\log(18)))}}$$

$\log(x)$ is the natural logarithm

Exact result:

$$\frac{3 \log(\log(18)) \log(\log(\log(14)))}{\log(\log(14)) \log(\log(\log(18)))}$$

Decimal approximation:

1.653661808791702600993459739806921200486284037140320196941...

1.65366180879.... is very near to the 14th root of the following Ramanujan's class invariant $Q = (G_{505}/G_{101/5})^3 = 1164,2696$ i.e. 1,65578...

Alternate forms:

$$\frac{3 \log(\log(2) + 2 \log(3)) \log(\log(\log(14)))}{\log(\log(14)) \log(\log(\log(2) + 2 \log(3)))}$$

$$\frac{3 \log(\log(2) + 2 \log(3)) \log(\log(\log(2) + \log(7)))}{\log(\log(2) + \log(7)) \log(\log(\log(2) + 2 \log(3)))}$$

Alternative representations:

$$-\frac{3}{\frac{\log(\log(14))}{\frac{\log(\log(18)) \log(\log(\log(14)))}{\log(\log(\log(18)))}}} = -\frac{3}{\frac{\log_e(\log(14))}{\frac{\log_e(\log(\log(14))) \log_e(\log(18))}{\log_e(\log(\log(18)))}}}$$

$$-\frac{3}{\frac{\log(\log(14))}{\frac{\log(\log(18)) \log(\log(\log(14)))}{\log(\log(\log(18)))}}} = -\frac{3}{\frac{\log(a) \log_a(\log(14))}{\frac{(\log(a) \log_a(\log(\log(14)))) (\log(a) \log_a(\log(18)))}{\log(a) \log_a(\log(\log(18)))}}}$$

$$-\frac{3}{\frac{\log(\log(14))}{\frac{\log(\log(18)) \log(\log(\log(14)))}{\log(\log(\log(18)))}}} = \frac{-3}{\frac{-\text{Li}_1(1-\log(14))}{\frac{-\text{Li}_1(1-\log(\log(14))) (-\text{Li}_1(1-\log(18)))}{-\text{Li}_1(1-\log(\log(18)))}}}$$

$\log_b(x)$ is the base- b logarithm

$\text{Li}_n(x)$ is the polylogarithm function

Series representations:

$$\begin{aligned}
& - \frac{3}{\frac{\log(\log(14))}{\frac{\log(\log(18)) \log(\log(\log(14)))}{\log(\log(\log(18)))}}} = \\
& - \left(\left(3 \left(2\pi \left[\frac{\arg(-x + \log(18))}{2\pi} \right] - i \log(x) + i \sum_{k=1}^{\infty} \frac{(-1)^k x^{-k} (-x + \log(18))^k}{k} \right) \right. \right. \\
& \quad \left. \left(2\pi \left[\frac{\arg(-x + \log(\log(14)))}{2\pi} \right] - i \log(x) + \right. \right. \\
& \quad \quad \left. \left. i \sum_{k=1}^{\infty} \frac{(-1)^k x^{-k} (-x + \log(\log(14)))^k}{k} \right) \right) / \\
& \left(\left(2\pi \left[\frac{\arg(-x + \log(14))}{2\pi} \right] - i \log(x) + i \sum_{k=1}^{\infty} \frac{(-1)^k x^{-k} (-x + \log(14))^k}{k} \right) \right. \\
& \quad \left(2\pi \left[\frac{\arg(-x + \log(\log(18)))}{2\pi} \right] - i \log(x) + \right. \\
& \quad \quad \left. \left. i \sum_{k=1}^{\infty} \frac{(-1)^k x^{-k} (-x + \log(\log(18)))^k}{k} \right) \right) \right) \text{ for } x < 0
\end{aligned}$$

$$\begin{aligned}
& - \frac{3}{\frac{\log(\log(14))}{\frac{\log(\log(18)) \log(\log(\log(14)))}{\log(\log(\log(18)))}}} = \\
& - \left(\left(3 \left(2\pi \left[\frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] - i \log(z_0) + i \sum_{k=1}^{\infty} \frac{(-1)^k (\log(18) - z_0)^k z_0^{-k}}{k} \right) \right. \right. \\
& \quad \left(2\pi \left[\frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] - i \log(z_0) + \right. \\
& \quad \quad \left. \left. i \sum_{k=1}^{\infty} \frac{(-1)^k (\log(\log(14)) - z_0)^k z_0^{-k}}{k} \right) \right) / \\
& \left(\left(2\pi \left[\frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] - i \log(z_0) + i \sum_{k=1}^{\infty} \frac{(-1)^k (\log(14) - z_0)^k z_0^{-k}}{k} \right) \right. \\
& \quad \left(2\pi \left[\frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] - i \log(z_0) + i \sum_{k=1}^{\infty} \frac{(-1)^k (\log(\log(18)) - z_0)^k z_0^{-k}}{k} \right) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& -\frac{3}{\frac{\log(\log(14))}{\frac{\log(\log(18))\log(\log(\log(14)))}{\log(\log(\log(18)))}}} = \\
& -\left(3\left(\left\lfloor\frac{\arg(\log(18)-z_0)}{2\pi}\right\rfloor\log\left(\frac{1}{z_0}\right)+\log(z_0)+\left\lfloor\frac{\arg(\log(18)-z_0)}{2\pi}\right\rfloor\log(z_0)-\right.\right. \\
& \quad \left.\left.\sum_{k=1}^{\infty}\frac{(-1)^k(\log(18)-z_0)^k z_0^{-k}}{k}\right)\right. \\
& \quad \left(\left\lfloor\frac{\arg(\log(\log(14))-z_0)}{2\pi}\right\rfloor\log\left(\frac{1}{z_0}\right)+\log(z_0)+\right. \\
& \quad \left.\left\lfloor\frac{\arg(\log(\log(14))-z_0)}{2\pi}\right\rfloor\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k(\log(\log(14))-z_0)^k z_0^{-k}}{k}\right)\right) \\
& \left(\left(\left\lfloor\frac{\arg(\log(14)-z_0)}{2\pi}\right\rfloor\log\left(\frac{1}{z_0}\right)+\log(z_0)+\left\lfloor\frac{\arg(\log(14)-z_0)}{2\pi}\right\rfloor\log(z_0)-\right.\right. \\
& \quad \left.\left.\sum_{k=1}^{\infty}\frac{(-1)^k(\log(14)-z_0)^k z_0^{-k}}{k}\right)\right) \\
& \quad \left(\left\lfloor\frac{\arg(\log(\log(18))-z_0)}{2\pi}\right\rfloor\log\left(\frac{1}{z_0}\right)+\log(z_0)+\right. \\
& \quad \left.\left\lfloor\frac{\arg(\log(\log(18))-z_0)}{2\pi}\right\rfloor\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k(\log(\log(18))-z_0)^k z_0^{-k}}{k}\right)\right)
\end{aligned}$$

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

Integral representation:

$$\begin{aligned}
& -\frac{3}{\frac{\log(\log(14))}{\frac{\log(\log(18))\log(\log(\log(14)))}{\log(\log(\log(18)))}}} = \\
& \int_0^1 \int_0^1 \frac{1}{(1+(-1+\log(18))t_1)(1+(-1+\log(\log(14)))t_2)} dt_2 dt_1
\end{aligned}$$

$$-8/10^3 + 3 / [(((\ln(\ln 14) / \ln(\ln(\ln(14)))))) * 1 / (((\ln(\ln 18) / \ln(\ln(\ln(18))))))]$$

Input:

$$-\frac{8}{10^3} - \frac{3}{\frac{\log(\log(14))}{\log(\log(\log(14)))}} \times \frac{1}{\frac{\log(\log(18))}{\log(\log(\log(18)))}}$$

$\log(x)$ is the natural logarithm

Exact result:

$$\frac{1}{125} - \frac{3 \log(\log(18)) \log(\log(\log(14)))}{\log(\log(14)) \log(\log(\log(18)))}$$

Decimal approximation:

1.645661808791702600993459739806921200486284037140320196941...

$$1.64566180879\dots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934\dots$$

Alternate forms:

$$\frac{1}{125} - \frac{3 \log(\log(2) + 2 \log(3)) \log(\log(\log(14)))}{\log(\log(14)) \log(\log(\log(2) + 2 \log(3)))}$$

$$\frac{375 \log(\log(18)) \log(\log(\log(14))) + \log(\log(14)) \log(\log(\log(18)))}{125 \log(\log(14)) \log(\log(\log(18)))}$$

- $$-\frac{((\log(\log(2) + \log(7)) \log(\log(\log(2) + 2 \log(3))) + 375 \log(\log(2) + 2 \log(3)) \log(\log(\log(2) + \log(7))))}{(125 \log(\log(2) + \log(7)) \log(\log(\log(2) + 2 \log(3))))}$$

Alternative representations:

- $$-\frac{8}{10^3} - \frac{3 \frac{\log(\log(14))}{\frac{\log(\log(18)) \log(\log(\log(14)))}{\log(\log(\log(18)))}}}{10^3} = -\frac{8}{10^3} - \frac{3 \frac{\log_e(\log(14))}{\frac{\log_e(\log(\log(14))) \log_e(\log(18))}{\log_e(\log(\log(18)))}}}{10^3}$$

- $$-\frac{8}{10^3} - \frac{3 \frac{\log(\log(14))}{\frac{\log(\log(18)) \log(\log(\log(14)))}{\log(\log(\log(18)))}}}{10^3} = -\frac{8}{10^3} - \frac{3 \frac{\log(a) \log_a(\log(14))}{\frac{(\log(a) \log_a(\log(\log(14)))) (\log(a) \log_a(\log(18)))}{\log(a) \log_a(\log(\log(18)))}}}{10^3}$$

- $$-\frac{8}{10^3} - \frac{3 \frac{\log(\log(14))}{\frac{\log(\log(18)) \log(\log(\log(14)))}{\log(\log(\log(18)))}}}{10^3} = -\frac{8}{10^3} - \frac{3 \frac{\text{Li}_1(1-\log(14))}{\frac{\text{Li}_1(1-\log(\log(14))) (-\text{Li}_1(1-\log(18)))}{\text{Li}_1(1-\log(\log(18)))}}}{10^3}$$

$\log_b(x)$ is the base- b logarithm

Series representations:

$$\begin{aligned}
 & -\frac{8}{10^3} - \frac{3}{\frac{\log(\log(14))}{\frac{\log(\log(18))\log(\log(\log(14)))}{\log(\log(\log(18)))}}} = \\
 & -\frac{1}{125} - \left(3 \left(2i\pi \left[\frac{\arg(-x + \log(18))}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k x^{-k} (-x + \log(18))^k}{k} \right) \right. \\
 & \quad \left(2i\pi \left[\frac{\arg(-x + \log(\log(14)))}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k x^{-k} (-x + \log(\log(14)))^k}{k} \right) \Bigg) / \\
 & \quad \left(\left(2i\pi \left[\frac{\arg(-x + \log(14))}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k x^{-k} (-x + \log(14))^k}{k} \right) \right. \\
 & \quad \left. \left(2i\pi \left[\frac{\arg(-x + \log(\log(18)))}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k x^{-k} (-x + \log(\log(18)))^k}{k} \right) \right) \Bigg) \\
 & \text{for } x < 0
 \end{aligned}$$

$$\begin{aligned}
 & -\frac{8}{10^3} - \frac{3}{\frac{\log(\log(14))}{\frac{\log(\log(18))\log(\log(\log(14)))}{\log(\log(\log(18)))}}} = -\frac{1}{125} - \\
 & \left(3 \left(\log(z_0) + \left[\frac{\arg(\log(18) - z_0)}{2\pi} \right] \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (\log(18) - z_0)^k z_0^{-k}}{k} \right) \right. \\
 & \quad \left(\log(z_0) + \left[\frac{\arg(\log(\log(14)) - z_0)}{2\pi} \right] \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (\log(\log(14)) - z_0)^k z_0^{-k}}{k} \right) \Bigg) / \\
 & \quad \left(\left(\log(z_0) + \left[\frac{\arg(\log(14) - z_0)}{2\pi} \right] \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (\log(14) - z_0)^k z_0^{-k}}{k} \right) \right. \\
 & \quad \left. \left(\log(z_0) + \left[\frac{\arg(\log(\log(18)) - z_0)}{2\pi} \right] \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (\log(\log(18)) - z_0)^k z_0^{-k}}{k} \right) \right) \Bigg)
 \end{aligned}$$

$$\begin{aligned}
& -\frac{8}{10^3} - \frac{3}{\frac{\log(\log(14))}{\frac{\log(\log(18))\log(\log(\log(14)))}{\log(\log(\log(18)))}}} = \\
& -\left(\left(1504 \pi^2 \left[\frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2 \pi} \right]^2 - 1504 i \pi \left[\frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2 \pi} \right] \log(z_0) - \right. \right. \\
& \quad 376 \log^2(z_0) + 2 i \pi \left[\frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2 \pi} \right] \sum_{k=1}^{\infty} \frac{(-1)^k (\log(14) - z_0)^k z_0^{-k}}{k} + \\
& \quad \log(z_0) \sum_{k=1}^{\infty} \frac{(-1)^k (\log(14) - z_0)^k z_0^{-k}}{k} + 750 i \pi \left[\frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2 \pi} \right] \\
& \quad \sum_{k=1}^{\infty} \frac{(-1)^k (\log(18) - z_0)^k z_0^{-k}}{k} + 375 \log(z_0) \sum_{k=1}^{\infty} \frac{(-1)^k (\log(18) - z_0)^k z_0^{-k}}{k} + \\
& \quad 750 i \pi \left[\frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2 \pi} \right] \sum_{k=1}^{\infty} \frac{(-1)^k (\log(\log(14)) - z_0)^k z_0^{-k}}{k} + \\
& \quad 375 \log(z_0) \sum_{k=1}^{\infty} \frac{(-1)^k (\log(\log(14)) - z_0)^k z_0^{-k}}{k} + \\
& \quad 2 i \pi \left[\frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2 \pi} \right] \sum_{k=1}^{\infty} \frac{(-1)^k (\log(\log(18)) - z_0)^k z_0^{-k}}{k} + \\
& \quad \log(z_0) \sum_{k=1}^{\infty} \frac{(-1)^k (\log(\log(18)) - z_0)^k z_0^{-k}}{k} - \\
& \quad 375 \sum_{k_1=1}^{\infty} \sum_{k_2=1}^{\infty} \frac{(-1)^{k_1+k_2} (\log(18) - z_0)^{k_1} (\log(\log(14)) - z_0)^{k_2} z_0^{-k_1-k_2}}{k_1 k_2} - \\
& \quad \left. \sum_{k_1=1}^{\infty} \sum_{k_2=1}^{\infty} \frac{(-1)^{k_1+k_2} (\log(14) - z_0)^{k_1} (\log(\log(18)) - z_0)^{k_2} z_0^{-k_1-k_2}}{k_1 k_2} \right) / \\
& \quad \left(125 \left(2 \pi \left[\frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2 \pi} \right] - i \log(z_0) + i \sum_{k=1}^{\infty} \frac{(-1)^k (\log(14) - z_0)^k z_0^{-k}}{k} \right) \right. \\
& \quad \left. \left(2 \pi \left[\frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2 \pi} \right] - i \log(z_0) + i \sum_{k=1}^{\infty} \frac{(-1)^k (\log(\log(18)) - z_0)^k z_0^{-k}}{k} \right) \right) \right)
\end{aligned}$$

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

Integral representation:

$$\frac{8}{10^3} - \frac{3}{\frac{\log(\log(14))}{\frac{\log(\log(18)) \log(\log(\log(14)))}{\log(\log(\log(18)))}} =$$

$$-\left(\int_0^1 \int_0^1 \frac{1}{(1 + (-1 + \log(18)) t_1) (1 + (-1 + \log(\log(14))) t_2)} dt_2 dt_1 + \right.$$

$$\left. \int_0^1 \int_0^1 \frac{1}{(1 + (-1 + \log(14)) t_1) (1 + (-1 + \log(\log(18))) t_2)} dt_2 dt_1 \right) /$$

$$\left(125 \left(\int_1^{\log(14)} \frac{1}{t} dt \right) \int_1^{\log(\log(18))} \frac{1}{t} dt \right)$$

$$18/10^3 + -3/ [(((\ln(\ln 14) / \ln(\ln(\ln(14)))))) * 1/(((\ln(\ln 18) / \ln(\ln(\ln(18))))))]]$$

Where 18 is a Lucas number

Input:

$$\frac{18}{10^3} - \frac{3}{\frac{\log(\log(14))}{\frac{\log(\log(\log(14)))}{\log(\log(\log(18)))}} \times \frac{1}{\frac{\log(\log(18))}{\log(\log(\log(18)))}}$$

log(x) is the natural logarithm

Exact result:

$$\frac{9}{500} - \frac{3 \log(\log(18)) \log(\log(\log(14)))}{\log(\log(14)) \log(\log(\log(18)))}$$

Decimal approximation:

1.671661808791702600993459739806921200486284037140320196941...

1.6716618087917.... a result practically equal to the value of the formula:

$$m_{p'} = 2 \times \frac{\eta}{R} m_p = 1.6714213 \times 10^{-24} \text{ gm}$$

that is the holographic proton mass

Alternate forms:

- $$\frac{9}{500} - \frac{3 \log(\log(2) + 2 \log(3)) \log(\log(\log(14)))}{\log(\log(14)) \log(\log(\log(2) + 2 \log(3)))}$$

$$\frac{3(3 \log(\log(14)) \log(\log(\log(18))) - 500 \log(\log(18)) \log(\log(\log(14))))}{500 \log(\log(14)) \log(\log(\log(18)))}$$

$$\frac{-((3(500 \log(\log(2)) + 2 \log(3)) \log(\log(\log(2)) + \log(7))) - 3 \log(\log(2) + \log(7)) \log(\log(\log(2) + 2 \log(3))))}{(500 \log(\log(2) + \log(7)) \log(\log(\log(2) + 2 \log(3))))}$$

Alternative representations:

$$\frac{18}{10^3} - \frac{3}{\frac{\log(\log(14))}{\frac{\log(\log(18)) \log(\log(\log(14)))}{\log(\log(\log(18)))}}} = \frac{18}{10^3} - \frac{3}{\frac{\log_e(\log(14))}{\frac{\log_e(\log(\log(14))) \log_e(\log(18))}{\log_e(\log(\log(18)))}}}$$

$$\frac{18}{10^3} - \frac{3}{\frac{\log(\log(14))}{\frac{\log(\log(18)) \log(\log(\log(14)))}{\log(\log(\log(18)))}}} = \frac{18}{10^3} - \frac{3}{\frac{\log(a) \log_a(\log(14))}{\frac{(\log(a) \log_a(\log(\log(14)))) (\log(a) \log_a(\log(18)))}{\log(a) \log_a(\log(\log(18)))}}}$$

$$\frac{18}{10^3} - \frac{3}{\frac{\log(\log(14))}{\frac{\log(\log(18)) \log(\log(\log(14)))}{\log(\log(\log(18)))}}} = \frac{18}{10^3} - \frac{3}{\frac{\text{Li}_1(1-\log(14))}{\frac{-\text{Li}_1(1-\log(\log(14))) (-\text{Li}_1(1-\log(18)))}{\text{Li}_1(1-\log(\log(18)))}}}$$

$\log_b(x)$ is the base- b logarithm

$\text{Li}_n(x)$ is the polylogarithm function

Series representations:

$$\frac{18}{10^3} - \frac{3}{\frac{\log(\log(14))}{\frac{\log(\log(18)) \log(\log(\log(14)))}{\log(\log(\log(18)))}}} =$$

$$\frac{9}{500} - \left(3 \left(2 i \pi \left[\frac{\arg(-x + \log(18))}{2 \pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k x^{-k} (-x + \log(18))^k}{k} \right) \right.$$

$$\left. \left(2 i \pi \left[\frac{\arg(-x + \log(\log(14)))}{2 \pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k x^{-k} (-x + \log(\log(14)))^k}{k} \right) \right) /$$

$$\left(\left(2 i \pi \left[\frac{\arg(-x + \log(14))}{2 \pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k x^{-k} (-x + \log(14))^k}{k} \right) \right.$$

$$\left. \left(2 i \pi \left[\frac{\arg(-x + \log(\log(18)))}{2 \pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k x^{-k} (-x + \log(\log(18)))^k}{k} \right) \right)$$

for $x < 0$

$$\frac{18}{10^3} - \frac{3}{\frac{\log(\log(14))}{\frac{\log(\log(18)) \log(\log(\log(14)))}{\log(\log(\log(18)))}}} = \frac{9}{500} -$$

$$\left(3 \left(\log(z_0) + \left[\frac{\arg(\log(18) - z_0)}{2 \pi} \right] \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (\log(18) - z_0)^k z_0^{-k}}{k} \right) \right.$$

$$\left(\log(z_0) + \left[\frac{\arg(\log(\log(14)) - z_0)}{2 \pi} \right] \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (\log(\log(14)) - z_0)^k z_0^{-k}}{k} \right) \right) /$$

$$\left(\left(\log(z_0) + \left[\frac{\arg(\log(14) - z_0)}{2 \pi} \right] \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (\log(14) - z_0)^k z_0^{-k}}{k} \right) \right.$$

$$\left. \left(\log(z_0) + \left[\frac{\arg(\log(\log(18)) - z_0)}{2 \pi} \right] \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (\log(\log(18)) - z_0)^k z_0^{-k}}{k} \right) \right)$$

$$\begin{aligned}
& \frac{18}{10^3} - \frac{3}{\frac{\log(\log(14))}{\frac{\log(\log(18)) \log(\log(\log(14)))}{\log(\log(\log(18)))}}} = \\
& - \left(\left(3 \left(1988 \pi^2 \left[\frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2 \pi} \right]^2 - 1988 i \pi \left[\frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2 \pi} \right] \log(z_0) - \right. \right. \\
& \quad 497 \log^2(z_0) - 6 i \pi \left[\frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2 \pi} \right] \\
& \quad \left. \sum_{k=1}^{\infty} \frac{(-1)^k (\log(14) - z_0)^k z_0^{-k}}{k} - 3 \log(z_0) \sum_{k=1}^{\infty} \frac{(-1)^k (\log(14) - z_0)^k z_0^{-k}}{k} + \right. \\
& \quad 1000 i \pi \left[\frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2 \pi} \right] \sum_{k=1}^{\infty} \frac{(-1)^k (\log(18) - z_0)^k z_0^{-k}}{k} + \\
& \quad 500 \log(z_0) \sum_{k=1}^{\infty} \frac{(-1)^k (\log(18) - z_0)^k z_0^{-k}}{k} + \\
& \quad 1000 i \pi \left[\frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2 \pi} \right] \sum_{k=1}^{\infty} \frac{(-1)^k (\log(\log(14)) - z_0)^k z_0^{-k}}{k} + \\
& \quad 500 \log(z_0) \sum_{k=1}^{\infty} \frac{(-1)^k (\log(\log(14)) - z_0)^k z_0^{-k}}{k} - \\
& \quad 6 i \pi \left[\frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2 \pi} \right] \sum_{k=1}^{\infty} \frac{(-1)^k (\log(\log(18)) - z_0)^k z_0^{-k}}{k} - \\
& \quad 3 \log(z_0) \sum_{k=1}^{\infty} \frac{(-1)^k (\log(\log(18)) - z_0)^k z_0^{-k}}{k} - \\
& \quad 500 \sum_{k_1=1}^{\infty} \sum_{k_2=1}^{\infty} \frac{(-1)^{k_1+k_2} (\log(18) - z_0)^{k_1} (\log(\log(14)) - z_0)^{k_2} z_0^{-k_1-k_2}}{k_1 k_2} + \\
& \quad \left. \left. 3 \sum_{k_1=1}^{\infty} \sum_{k_2=1}^{\infty} \frac{(-1)^{k_1+k_2} (\log(14) - z_0)^{k_1} (\log(\log(18)) - z_0)^{k_2} z_0^{-k_1-k_2}}{k_1 k_2} \right) \right) / \\
& \left(500 \left(2 \pi \left[\frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2 \pi} \right] - i \log(z_0) + i \sum_{k=1}^{\infty} \frac{(-1)^k (\log(14) - z_0)^k z_0^{-k}}{k} \right) \right. \\
& \quad \left. \left(2 \pi \left[\frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2 \pi} \right] - i \log(z_0) + i \sum_{k=1}^{\infty} \frac{(-1)^k (\log(\log(18)) - z_0)^k z_0^{-k}}{k} \right) \right) \right)
\end{aligned}$$

$\arg(z)$ is the complex argument

$[x]$ is the floor function

Integral representation:

$$\frac{18}{10^3} - \frac{3}{\frac{\log(\log(14))}{\frac{\log(\log(18)) \log(\log(\log(14)))}{\log(\log(\log(18)))}}} =$$

$$\left(3 \left(\int_0^1 \int_0^1 \frac{1}{(1 + (-1 + \log(18)) t_1) (1 + (-1 + \log(\log(14))) t_2)} dt_2 dt_1 + \int_0^1 \int_0^1 \frac{1}{(1 + (-1 + \log(14)) t_1) (1 + (-1 + \log(\log(18))) t_2)} dt_2 dt_1 \right) \right) /$$

$$\left(500 \left(\int_1^{\log(14)} \frac{1}{t} dt \right) \int_1^{\log(\log(18))} \frac{1}{t} dt \right)$$

$$-(34+1)/10^3 - 3 / [(((\ln(\ln 14) / \ln(\ln(\ln(14)))))) * 1 / (((\ln(\ln 18) / \ln(\ln(\ln(18)))))))]$$

Input:

$$-\frac{34+1}{10^3} - \frac{3}{\frac{\log(\log(14))}{\log(\log(\log(14)))} \times \frac{1}{\frac{\log(\log(18))}{\log(\log(\log(18)))}}}$$

log(x) is the natural logarithm

Exact result:

$$-\frac{7}{200} - \frac{3 \log(\log(18)) \log(\log(\log(14)))}{\log(\log(14)) \log(\log(\log(18)))}$$

Decimal approximation:

1.618661808791702600993459739806921200486284037140320196941...

1.61866180879.... result that is a very good approximation to the value of the golden ratio 1,618033988749...

Alternate forms:

$$-\frac{7}{200} - \frac{3 \log(\log(2) + 2 \log(3)) \log(\log(\log(14)))}{\log(\log(14)) \log(\log(\log(2) + 2 \log(3)))}$$

$$-\frac{600 \log(\log(18)) \log(\log(\log(14))) + 7 \log(\log(14)) \log(\log(\log(18)))}{200 \log(\log(14)) \log(\log(\log(18)))}$$

- $$-\frac{((7 \log(\log(2) + \log(7)) \log(\log(\log(2) + 2 \log(3))) + 600 \log(\log(2) + 2 \log(3)) \log(\log(\log(2) + \log(7))))}{(200 \log(\log(2) + \log(7)) \log(\log(\log(2) + 2 \log(3))))}$$

Alternative representations:

$$-\frac{34+1}{10^3} - \frac{3}{\frac{\log(\log(14))}{\frac{\log(\log(\log(14))) \log(\log(18))}{\log(\log(\log(18)))}}} = -\frac{35}{10^3} - \frac{3}{\frac{\log_e(\log(14))}{\frac{\log_e(\log(\log(14))) \log_e(\log(18))}{\log_e(\log(\log(18)))}}}$$

$$-\frac{34+1}{10^3} - \frac{3}{\frac{\log(\log(14))}{\frac{\log(\log(\log(14))) \log(\log(18))}{\log(\log(\log(18)))}}} = -\frac{35}{10^3} - \frac{3}{\frac{\log(a) \log_a(\log(14))}{(\log(a) \log_a(\log(\log(14)))) (\log(a) \log_a(\log(18)))} \log(a) \log_a(\log(\log(18)))}}$$

$$-\frac{34+1}{10^3} - \frac{3}{\frac{\log(\log(14))}{\frac{\log(\log(\log(14))) \log(\log(18))}{\log(\log(\log(18)))}}} = -\frac{35}{10^3} - \frac{3}{-\frac{\text{Li}_1(1-\log(14))}{\frac{\text{Li}_1(1-\log(\log(14))) (-\text{Li}_1(1-\log(18)))}{\text{Li}_1(1-\log(\log(18)))}}}$$

$\log_b(x)$ is the base- b logarithm

$\text{Li}_n(x)$ is the polylogarithm function

Series representations:

$$-\frac{34+1}{10^3} - \frac{3}{\frac{\log(\log(14))}{\frac{\log(\log(\log(14))) \log(\log(18))}{\log(\log(\log(18)))}}} = -\frac{7}{200} - \left(3 \left(2 i \pi \left[\frac{\arg(-x + \log(18))}{2 \pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k x^{-k} (-x + \log(18))^k}{k} \right) \right. \\ \left. \left(2 i \pi \left[\frac{\arg(-x + \log(\log(14)))}{2 \pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k x^{-k} (-x + \log(\log(14)))^k}{k} \right) \right) / \\ \left(\left(2 i \pi \left[\frac{\arg(-x + \log(14))}{2 \pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k x^{-k} (-x + \log(14))^k}{k} \right) \right. \\ \left. \left(2 i \pi \left[\frac{\arg(-x + \log(\log(18)))}{2 \pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k x^{-k} (-x + \log(\log(18)))^k}{k} \right) \right)$$

for $x < 0$

$$-\frac{34+1}{10^3} - \frac{3}{\frac{\log(\log(14))}{\frac{\log(\log(\log(14)))\log(\log(18))}{\log(\log(\log(18)))}}} = -\frac{7}{200} -$$

$$\left(3 \left(\log(z_0) + \left\lfloor \frac{\arg(\log(18) - z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (\log(18) - z_0)^k z_0^{-k}}{k} \right) \right. \\ \left. \left(\log(z_0) + \left\lfloor \frac{\arg(\log(\log(14)) - z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (\log(\log(14)) - z_0)^k z_0^{-k}}{k} \right) \right) \Bigg) / \\ \left(\left(\log(z_0) + \left\lfloor \frac{\arg(\log(14) - z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (\log(14) - z_0)^k z_0^{-k}}{k} \right) \right. \right. \\ \left. \left. \left(\log(z_0) + \left\lfloor \frac{\arg(\log(\log(18)) - z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (\log(\log(18)) - z_0)^k z_0^{-k}}{k} \right) \right) \right)$$

$$\begin{aligned}
& -\frac{34+1}{10^3} - \frac{3}{\frac{\log(\log(14))}{\frac{\log(\log(\log(14)))\log(\log(18))}{\log(\log(\log(18)))}}} = \\
& -\left(\left(2428 \pi^2 \left[\frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right]^2 - 2428 i \pi \left[\frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] \log(z_0) - \right. \right. \\
& \quad 607 \log^2(z_0) + 14 i \pi \left[\frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] \sum_{k=1}^{\infty} \frac{(-1)^k (\log(14) - z_0)^k z_0^{-k}}{k} + \\
& \quad 7 \log(z_0) \sum_{k=1}^{\infty} \frac{(-1)^k (\log(14) - z_0)^k z_0^{-k}}{k} + 1200 i \pi \left[\frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] \\
& \quad \sum_{k=1}^{\infty} \frac{(-1)^k (\log(18) - z_0)^k z_0^{-k}}{k} + 600 \log(z_0) \sum_{k=1}^{\infty} \frac{(-1)^k (\log(18) - z_0)^k z_0^{-k}}{k} + \\
& \quad 1200 i \pi \left[\frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] \sum_{k=1}^{\infty} \frac{(-1)^k (\log(\log(14)) - z_0)^k z_0^{-k}}{k} + \\
& \quad 600 \log(z_0) \sum_{k=1}^{\infty} \frac{(-1)^k (\log(\log(14)) - z_0)^k z_0^{-k}}{k} + \\
& \quad 14 i \pi \left[\frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] \sum_{k=1}^{\infty} \frac{(-1)^k (\log(\log(18)) - z_0)^k z_0^{-k}}{k} + \\
& \quad 7 \log(z_0) \sum_{k=1}^{\infty} \frac{(-1)^k (\log(\log(18)) - z_0)^k z_0^{-k}}{k} - \\
& \quad 600 \sum_{k_1=1}^{\infty} \sum_{k_2=1}^{\infty} \frac{(-1)^{k_1+k_2} (\log(18) - z_0)^{k_1} (\log(\log(14)) - z_0)^{k_2} z_0^{-k_1-k_2}}{k_1 k_2} - \\
& \quad \left. \left. 7 \sum_{k_1=1}^{\infty} \sum_{k_2=1}^{\infty} \frac{(-1)^{k_1+k_2} (\log(14) - z_0)^{k_1} (\log(\log(18)) - z_0)^{k_2} z_0^{-k_1-k_2}}{k_1 k_2} \right) / \right. \\
& \quad \left(200 \left(2\pi \left[\frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] - i \log(z_0) + i \sum_{k=1}^{\infty} \frac{(-1)^k (\log(14) - z_0)^k z_0^{-k}}{k} \right) \right. \\
& \quad \left. \left(2\pi \left[\frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] - i \log(z_0) + i \sum_{k=1}^{\infty} \frac{(-1)^k (\log(\log(18)) - z_0)^k z_0^{-k}}{k} \right) \right) \right)
\end{aligned}$$

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

Integral representation:

$$-\frac{34+1}{10^3} - \frac{3}{\frac{\log(\log(14))}{\log(\log(\log(14)))\log(\log(18))} \frac{1}{\log(\log(\log(18)))}}} =$$

$$-\left(\int_0^1 \int_0^1 \frac{1}{(1+(-1+\log(18))t_1)(1+(-1+\log(\log(14)))t_2)} dt_2 dt_1 + \right.$$

$$\left. \int_0^1 \int_0^1 \frac{1}{(1+(-1+\log(14))t_1)(1+(-1+\log(\log(18)))t_2)} dt_2 dt_1 \right) /$$

$$\left(200 \left(\int_1^{\log(14)} \frac{1}{t} dt \right) \int_1^{\log(\log(18))} \frac{1}{t} dt \right)$$

We have also that:

$$2\sqrt{\left(\left(\left(\left(\left(6 \cdot \left(\left(\left(-\frac{8}{10^3} + \frac{3}{\left(\frac{\ln(\ln 14)}{\ln(\ln(\ln 14))}) \right)} \right) \right) \right) \right) \right) \cdot \frac{1}{\left(\frac{\ln(\ln 18)}{\ln(\ln(\ln 18))}) \right)} \right) \right) \right) \right) \right) \right)$$

Input:

$$2 \sqrt{6 \left(-\frac{8}{10^3} - \frac{3}{\frac{\log(\log(14))}{\log(\log(\log(14)))} \times \frac{1}{\frac{\log(\log(18))}{\log(\log(\log(18)))}}} \right)}$$

$\log(x)$ is the natural logarithm

Exact result:

$$2 \sqrt{6 \left(-\frac{1}{125} - \frac{3 \log(\log(18)) \log(\log(\log(14)))}{\log(\log(14)) \log(\log(\log(18)))} \right)}$$

Decimal approximation:

6.284575038218643342085847877207543950572439720737770043251...

$$6.284575038... = C = 2\pi r$$

Alternate forms:

$$2 \sqrt{-\frac{6}{125} - \frac{18 \log(\log(18)) \log(\log(\log(14)))}{\log(\log(14)) \log(\log(\log(18)))}}$$

$$\frac{2}{5} \sqrt{\frac{6(-375 \log(\log(18)) \log(\log(\log(14))) - \log(\log(14)) \log(\log(\log(18))))}{5 \log(\log(14)) \log(\log(\log(18)))}}$$

$$\frac{2}{5} \sqrt{((6(-\log(\log(14)) \log(\log(\log(2) + 2 \log(3))) - 375 \log(\log(2) + 2 \log(3)) \log(\log(\log(14)))))) / (5 \log(\log(14)) \log(\log(\log(2) + 2 \log(3))))}$$

Alternative representations:

$$2 \sqrt{6 \left(-\frac{8}{10^3} - \frac{3}{\frac{\log(\log(14))}{\frac{\log(\log(18)) \log(\log(\log(14)))}{\log(\log(\log(18)))}}} \right)} = 2 \sqrt{6 \left(-\frac{8}{10^3} - \frac{3}{\frac{\log_e(\log(14))}{\frac{\log_e(\log(\log(14))) \log_e(\log(18))}{\log_e(\log(\log(18)))}}} \right)}$$

$$2 \sqrt{6 \left(-\frac{8}{10^3} - \frac{3}{\frac{\log(\log(14))}{\frac{\log(\log(18)) \log(\log(\log(14)))}{\log(\log(\log(18)))}}} \right)} = 2 \sqrt{6 \left(-\frac{8}{10^3} - \frac{3}{\frac{\log(a) \log_a(\log(14))}{(\log(a) \log_a(\log(\log(14)))) (\log(a) \log_a(\log(18)))}} \right)}$$

$$2 \sqrt{6 \left(-\frac{8}{10^3} - \frac{3}{\frac{\log(\log(14))}{\frac{\log(\log(18)) \log(\log(\log(14)))}{\log(\log(\log(18)))}}} \right)} = 2 \sqrt{6 \left(-\frac{8}{10^3} - \frac{3}{-\frac{\text{Li}_1(1-\log(14))}{-\frac{\text{Li}_1(1-\log(\log(14))) (-\text{Li}_1(1-\log(18)))}{\text{Li}_1(1-\log(\log(18)))}}} \right)}$$

$\log_b(x)$ is the base- b logarithm

$\text{Li}_n(x)$ is the polylogarithm function

Series representations:

$$\begin{aligned}
 & 2 \sqrt{6 \left(-\frac{8}{10^3} - \frac{3}{\frac{\log(\log(14))}{\frac{\log(\log(18)) \log(\log(\log(14)))}{\log(\log(\log(18)))}}} \right)} = 2 \sqrt{6} \\
 & \sqrt{\left(-\frac{1}{125} - \left(3 \left(2 i \pi \left[\frac{\arg(-x + \log(18))}{2 \pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k x^{-k} (-x + \log(18))^k}{k} \right) \right. \right. \\
 & \quad \left. \left(2 i \pi \left[\frac{\arg(-x + \log(\log(14)))}{2 \pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k x^{-k} (-x + \log(\log(14)))^k}{k} \right) \right) \Bigg/} \\
 & \left(\left(2 i \pi \left[\frac{\arg(-x + \log(14))}{2 \pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k x^{-k} (-x + \log(14))^k}{k} \right) \right. \\
 & \quad \left. \left(2 i \pi \left[\frac{\arg(-x + \log(\log(18)))}{2 \pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k x^{-k} (-x + \log(\log(18)))^k}{k} \right) \right) \Bigg) \Bigg) \text{ for } x < 0
 \end{aligned}$$

$$\begin{aligned}
 & 2 \sqrt{6 \left(-\frac{8}{10^3} - \frac{3}{\frac{\log(\log(14))}{\frac{\log(\log(18)) \log(\log(\log(14)))}{\log(\log(\log(18)))}}} \right)} = 2 \sqrt{6} \\
 & \sqrt{\left(-\frac{1}{125} - \left(3 \left(2 i \pi \left[\frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2 \pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (\log(18) - z_0)^k z_0^{-k}}{k} \right) \right. \right. \\
 & \quad \left. \left(2 i \pi \left[\frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2 \pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (\log(\log(14)) - z_0)^k z_0^{-k}}{k} \right) \right) \Bigg/} \\
 & \left(\left(2 i \pi \left[\frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2 \pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (\log(14) - z_0)^k z_0^{-k}}{k} \right) \right. \\
 & \quad \left. \left(2 i \pi \left[\frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2 \pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (\log(\log(18)) - z_0)^k z_0^{-k}}{k} \right) \right) \Bigg) \Bigg)
 \end{aligned}$$

$$2 \sqrt{6 \left(-\frac{8}{10^3} - \frac{3}{\frac{\log(\log(14))}{\frac{\log(\log(18)) \log(\log(\log(14)))}{\log(\log(\log(18)))}}} \right)} = \\
2 \sqrt{6} \sqrt{\left(-\frac{1}{125} - \left(3 \left(\log(z_0) + \left\lfloor \frac{\arg(\log(18) - z_0)}{2\pi} \right\rfloor \right) \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \right. \right. \\
\left. \left. \sum_{k=1}^{\infty} \frac{(-1)^k (\log(18) - z_0)^k z_0^{-k}}{k} \right) \left(\log(z_0) + \left\lfloor \frac{\arg(\log(\log(14)) - z_0)}{2\pi} \right\rfloor \right) \right. \\
\left. \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (\log(\log(14)) - z_0)^k z_0^{-k}}{k} \right) \right) / \\
\left(\left(\log(z_0) + \left\lfloor \frac{\arg(\log(14) - z_0)}{2\pi} \right\rfloor \right) \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \right. \\
\left. \sum_{k=1}^{\infty} \frac{(-1)^k (\log(14) - z_0)^k z_0^{-k}}{k} \right) \left(\log(z_0) + \left\lfloor \frac{\arg(\log(\log(18)) - z_0)}{2\pi} \right\rfloor \right) \right. \\
\left. \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (\log(\log(18)) - z_0)^k z_0^{-k}}{k} \right) \right) \right)$$

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

Integral representation:

$$2 \sqrt{6 \left(-\frac{8}{10^3} - \frac{3}{\frac{\log(\log(14))}{\frac{\log(\log(18)) \log(\log(\log(14)))}{\log(\log(\log(18)))}}} \right)} = \\
\frac{2}{5} \sqrt{\frac{6}{5}} \sqrt{\left(\left(\int_0^1 \int_0^1 \frac{1}{(1 + (-1 + \log(18)) t_1) (1 + (-1 + \log(\log(14))) t_2)} dt_2 dt_1 + \right. \right. \\
\left. \int_0^1 \int_0^1 \frac{1}{(1 + (-1 + \log(14)) t_1) (1 + (-1 + \log(\log(18))) t_2)} dt_2 dt_1 \right) / \\
\left(\left(\int_1^{\log(14)} \frac{1}{t} dt \right) \int_1^{\log(\log(18))} \frac{1}{t} dt \right) \right)$$

The radius of the circumference is:

$$1/(2\text{Pi}) * 2\text{sqrt}(((((((6*((((-8/10^3 + 3/ [((ln(ln14) / ln(ln(ln14)))))) * 1/((ln(ln18) / ln(ln(ln18))))))])))))))))$$

Input:

$$\frac{1}{2\pi} \times 2 \sqrt{6 \left(-\frac{8}{10^3} - \frac{3}{\frac{\log(\log(14))}{\log(\log(\log(14)))} \times \frac{1}{\frac{\log(\log(18))}{\log(\log(\log(18)))}}}} \right)}$$

$\log(x)$ is the natural logarithm

Exact result:

$$\frac{\sqrt{6 \left(-\frac{1}{125} - \frac{3 \log(\log(18)) \log(\log(\log(14)))}{\log(\log(14)) \log(\log(\log(18)))} \right)}}{\pi}$$

Decimal approximation:

1.000221182564434135948850359752650016695948332391919191633...

1.0002211825644.... = radius

•

Alternate forms:

$$\frac{\sqrt{-\frac{6}{125} - \frac{18 \log(\log(18)) \log(\log(\log(14)))}{\log(\log(14)) \log(\log(\log(18)))}}}{\pi}$$

•

$$\frac{\sqrt{\frac{6(-375 \log(\log(18)) \log(\log(\log(14))) - \log(\log(14)) \log(\log(\log(18))))}{5 \log(\log(14)) \log(\log(\log(18)))}}}{5\pi}$$

•

$$\frac{\sqrt{\frac{6(-\log(\log(14)) \log(\log(\log(2)+2 \log(3))) - 375 \log(\log(2)+2 \log(3)) \log(\log(\log(14))))}{5 \log(\log(14)) \log(\log(\log(2)+2 \log(3)))}}}{5\pi}$$

•

Alternative representations:

$$\frac{2 \sqrt{6 \left(-\frac{8}{10^3} - \frac{3}{\frac{\log(\log(14))}{\log(\log(\log(14))) \log(\log(18))}} \right)}}{2 \pi} = \frac{2 \sqrt{6 \left(-\frac{8}{10^3} - \frac{3}{\frac{\log_e(\log(14))}{\log_e(\log(\log(14))) \log_e(\log(18))}} \right)}}{2 \pi}$$

•

$$\frac{2 \sqrt{6 \left(-\frac{8}{10^3} - \frac{3}{\frac{\log(\log(14))}{\log(\log(\log(14))) \log(\log(18))}} \right)}}{2 \pi} = \frac{2 \sqrt{6 \left(-\frac{8}{10^3} - \frac{3}{\frac{\log(a) \log_a(\log(14))}{(\log(a) \log_a(\log(\log(14)))) (\log(a) \log_a(\log(18)))}} \right)}}{2 \pi}$$

•

$$\frac{2 \sqrt{6 \left(-\frac{8}{10^3} - \frac{3}{\frac{\log(\log(14))}{\log(\log(\log(14))) \log(\log(18))}} \right)}}{2 \pi} = \frac{2 \sqrt{6 \left(-\frac{8}{10^3} - \frac{3}{\frac{\text{Li}_1(1-\log(14))}{-\text{Li}_1(1-\log(\log(14))) (-\text{Li}_1(1-\log(18)))}} \right)}}{2 \pi}$$

$\log_b(x)$ is the base- b logarithm

$\text{Li}_n(x)$ is the polylogarithm function

•

Series representations:

$$\begin{aligned}
& \frac{2 \sqrt{6 \left(-\frac{8}{10^3} - \frac{3}{\frac{\log(\log(14))}{\log(\log(\log(14))) \log(\log(18))}} \right)}}{2 \pi} = \frac{1}{\pi} \sqrt{6} \\
& \sqrt{\left(-\frac{1}{125} - \left(3 \left[2 i \pi \left[\frac{\arg(-x + \log(18))}{2 \pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k x^{-k} (-x + \log(18))^k}{k} \right) \right. \right. \\
& \quad \left. \left(2 i \pi \left[\frac{\arg(-x + \log(\log(14)))}{2 \pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k x^{-k} (-x + \log(\log(14)))^k}{k} \right) \right) / \\
& \quad \left(\left(2 i \pi \left[\frac{\arg(-x + \log(14))}{2 \pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k x^{-k} (-x + \log(14))^k}{k} \right) \right. \\
& \quad \left. \left(2 i \pi \left[\frac{\arg(-x + \log(\log(18)))}{2 \pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k x^{-k} (-x + \log(\log(18)))^k}{k} \right) \right) \right) \text{ for } x < 0
\end{aligned}$$

$$\begin{aligned}
& \frac{2 \sqrt{6 \left(-\frac{8}{10^3} - \frac{3}{\frac{\log(\log(14))}{\log(\log(\log(14))) \log(\log(18))}} \right)}}{2 \pi} = \frac{1}{\pi} \sqrt{6} \\
& \sqrt{\left(-\frac{1}{125} - \left(3 \left[2 i \pi \left[\frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2 \pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (\log(18) - z_0)^k z_0^{-k}}{k} \right) \right. \right. \\
& \quad \left(2 i \pi \left[\frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2 \pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (\log(\log(14)) - z_0)^k z_0^{-k}}{k} \right) \right) / \\
& \quad \left(\left(2 i \pi \left[\frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2 \pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (\log(14) - z_0)^k z_0^{-k}}{k} \right) \right. \\
& \quad \left. \left(2 i \pi \left[\frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2 \pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (\log(\log(18)) - z_0)^k z_0^{-k}}{k} \right) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \frac{2 \sqrt{6 \left(-\frac{8}{10^3} - \frac{3}{\frac{\log(\log(14))}{\log(\log(\log(14))) \log(\log(18))}} \right)}}{2 \pi} = \frac{1}{\pi} \\
& \sqrt{6} \sqrt{\left(-\frac{1}{125} - \left(3 \left(\log(z_0) + \left\lfloor \frac{\arg(\log(18)) - z_0}{2 \pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \right. \right. \right. \\
& \quad \left. \left. \sum_{k=1}^{\infty} \frac{(-1)^k (\log(18) - z_0)^k z_0^{-k}}{k} \right) \left(\log(z_0) + \left\lfloor \frac{\arg(\log(\log(14)) - z_0)}{2 \pi} \right\rfloor \right. \right. \\
& \quad \left. \left. \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (\log(\log(14)) - z_0)^k z_0^{-k}}{k} \right) \right) \right) / \\
& \quad \left(\left(\log(z_0) + \left\lfloor \frac{\arg(\log(14)) - z_0}{2 \pi} \right\rfloor \right) \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \right. \\
& \quad \left. \sum_{k=1}^{\infty} \frac{(-1)^k (\log(14) - z_0)^k z_0^{-k}}{k} \right) \left(\log(z_0) + \left\lfloor \frac{\arg(\log(\log(18)) - z_0)}{2 \pi} \right\rfloor \right. \\
& \quad \left. \left. \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (\log(\log(18)) - z_0)^k z_0^{-k}}{k} \right) \right) \right)
\end{aligned}$$

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

Integral representation:

$$\begin{aligned}
& \frac{2 \sqrt{6 \left(-\frac{8}{10^3} - \frac{3}{\frac{\log(\log(14))}{\log(\log(\log(14))) \log(\log(18))}} \right)}}{2 \pi} = \frac{1}{5 \pi} \\
& \sqrt{\frac{6}{5}} \sqrt{\left(\left(\int_0^1 \int_0^1 \frac{1}{(1 + (-1 + \log(18)) t_1) (1 + (-1 + \log(\log(14))) t_2)} dt_2 dt_1 + \right. \right. \\
& \quad \left. \left. \int_0^1 \int_0^1 \frac{1}{(1 + (-1 + \log(14)) t_1) (1 + (-1 + \log(\log(18))) t_2)} dt_2 dt_1 \right) \right) / \\
& \quad \left(\int_1^{\log(14)} \frac{1}{t} dt \int_1^{\log(\log(18))} \frac{1}{t} dt \right)
\end{aligned}$$

We note that **1.0002211825644...** is very near to the value of $\zeta(12)$. Indeed:

zeta (12)

Input: $\zeta(12)$ $\zeta(s)$ is the Riemann zeta function**Exact result:**

$$\frac{691 \pi^{12}}{638512875}$$

Decimal approximation:

1.000246086553308048298637998047739670960416088458003404533...

1.0002460865533...

We observe that, from the Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} - \varphi + 1 \approx 0.9991104684$$

From the reciprocal, we obtain the following result:

1 / 0.9991104684

Input interpretation:

$$\frac{1}{0.9991104684}$$

Result:

1.000890323570950585387740893777627442983561075857505248015...

1.00089032357...

and by

-(47+18)/10^5+(1/0.9991104684)

where 47 and 18 are Lucas numbers, we obtain:

Input interpretation:

$$-\frac{47+18}{10^5} + \frac{1}{0.9991104684}$$

Result:

1.000240323570950585387740893777627442983561075857505248015...

1.00024032357...

This result is very near to the value of $\zeta(12)$ 1.0002460865533...and to the radius of the above circumference that is: 1.0002211825644...

Explicitly:

$$\exp(\frac{(-\pi) \times 1/\sqrt{5}}{\sqrt{5} \times \frac{1}{1+\sqrt[5]{5^{0.75} \phi^{2.5}-1}} - \phi + 1}})/(\frac{(\sqrt{5}) \times 1/(\frac{1+((5^{0.75} \times (\text{golden ratio})^{2.5}-1))^{1/5}}{1+\sqrt[5]{5^{0.75} \phi^{2.5}-1}} - \phi + 1)}}{1+\sqrt[5]{5^{0.75} \phi^{2.5}-1}} - \phi + 1))$$

Input:

$$\frac{\exp\left(-\pi \times \frac{1}{\sqrt{5}}\right)}{\sqrt{5} \times \frac{1}{1+\sqrt[5]{5^{0.75} \phi^{2.5}-1}} - \phi + 1}$$

ϕ is the golden ratio

Result:

0.999110468396793152282141577538458189641715455753675872795...

0.999110468396...

Series representations:

$$\frac{\exp\left(-\frac{\pi}{\sqrt{5}}\right)}{\frac{\sqrt{5}}{1+\sqrt[5]{5^{0.75} \phi^{2.5}-1}} - \phi + 1} = \frac{\left(1 + \sqrt[5]{-1 + 3.3437 \phi^{2.5}}\right) \exp\left(-\frac{\pi}{\sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k}}\right)}{-1 + \phi - \sqrt[5]{-1 + 3.3437 \phi^{2.5}} + \phi \sqrt[5]{-1 + 3.3437 \phi^{2.5}} - \sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k}}$$

$$\frac{\exp\left(-\frac{\pi}{\sqrt{5}}\right)}{\frac{\sqrt{5}}{1+\sqrt[5]{5^{0.75}\phi^{2.5}-1}} - \phi + 1} = \frac{\left(1 + \sqrt[5]{-1 + 3.3437\phi^{2.5}}\right) \exp\left(-\frac{\pi}{\sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)_k \left(-\frac{1}{2}\right)_k}{k!}}\right)}{-1 + \phi - \sqrt[5]{-1 + 3.3437\phi^{2.5}} + \phi \sqrt[5]{-1 + 3.3437\phi^{2.5}} - \sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)_k \left(-\frac{1}{2}\right)_k}{k!}}$$

$$\frac{\exp\left(-\frac{\pi}{\sqrt{5}}\right)}{\frac{\sqrt{5}}{1+\sqrt[5]{5^{0.75}\phi^{2.5}-1}} - \phi + 1} = -\left(\left(\left(1 + \sqrt[5]{-1 + 3.3437\phi^{2.5}}\right) \exp\left(-\frac{\pi}{\sqrt{z_0} \sum_{k=0}^{\infty} \frac{\left(-1\right)^k \left(-\frac{1}{2}\right)_k (5-z_0)^k z_0^{-k}}{k!}}\right)\right) / \left(-1 + \phi - \sqrt[5]{-1 + 3.3437\phi^{2.5}} + \phi \sqrt[5]{-1 + 3.3437\phi^{2.5}} - \sqrt{z_0} \sum_{k=0}^{\infty} \frac{\left(-1\right)^k \left(-\frac{1}{2}\right)_k (5-z_0)^k z_0^{-k}}{k!}\right)\right) \text{ for not } ((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

\mathbb{R} is the set of real numbers

$$-(47+18)/10^5 + 1/[\exp(\frac{-\pi}{\sqrt{5}}) * 1 / (((\sqrt{5} * 1 / (((1 + ((5^{0.75} * (\text{golden ratio})^{2.5} - 1)))^{1/5})))) - \text{golden ratio} + 1)]]]$$

Where 47 and 18 are Lucas numbers, we obtain:

Input:

$$-\frac{47+18}{10^5} + \frac{1}{\exp\left(-\frac{\pi}{\sqrt{5}}\right) \times \frac{1}{\sqrt{5} \times \frac{1}{1+\sqrt[5]{5^{0.75} \phi^{2.5}-1}}^{-\phi+1}}}$$

ϕ is the golden ratio

Result:

1.000240323574163145911824361683315922732170929436105556831...

1.000240323574....

Series representations:

$$-\frac{47+18}{10^5} + \frac{1}{\frac{\exp\left(-\frac{\pi}{\sqrt{5}}\right)}{\frac{\sqrt{5}}{1+\sqrt[5]{5^{0.75} \phi^{2.5}-1}}^{-\phi+1}}} =$$

$$-\left(-20\,000 + 20\,000 \phi - 20\,000 \sqrt[5]{-1 + 3.3437 \phi^{2.5}} + 20\,000 \phi \sqrt[5]{-1 + 3.3437 \phi^{2.5}} + \right.$$

$$13 \exp\left(-\frac{\pi}{\sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k}} \right) + 13 \sqrt[5]{-1 + 3.3437 \phi^{2.5}}$$

$$\left. \exp\left(-\frac{\pi}{\sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k}} \right) - 20\,000 \sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k} \right) /$$

$$\left(20\,000 \left(1 + \sqrt[5]{-1 + 3.3437 \phi^{2.5}} \right) \exp\left(-\frac{\pi}{\sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k}} \right) \right)$$

$$\begin{aligned}
& -\frac{47+18}{10^5} + \frac{1}{\frac{\exp\left(-\frac{\pi}{\sqrt{5}}\right)}{\frac{\sqrt{5}}{1+\sqrt[5]{5^{0.75}\phi^{2.5}-1}}^{-\phi+1}}} = \\
& -\left(-20000 + 20000\phi - 20000\sqrt[5]{-1+3.3437\phi^{2.5}} + 20000\phi\sqrt[5]{-1+3.3437\phi^{2.5}} + \right. \\
& \quad 13 \exp\left(-\frac{\pi}{\sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!}} \right) + 13\sqrt[5]{-1+3.3437\phi^{2.5}} \\
& \quad \left. \exp\left(-\frac{\pi}{\sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!}} \right) - 20000\sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right) / \\
& \quad \left(20000\left(1 + \sqrt[5]{-1+3.3437\phi^{2.5}}\right) \exp\left(-\frac{\pi}{\sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!}} \right) \right) \Bigg)
\end{aligned}$$

$$\begin{aligned}
& -\frac{47+18}{10^5} + \frac{1}{\frac{\exp\left(-\frac{\pi}{\sqrt{5}}\right)}{\frac{\sqrt{5}}{1+\sqrt[5]{5^{0.75}\phi^{2.5}-1}}^{-\phi+1}}} = \\
& -\left(-20000 + 20000\phi - 20000\sqrt[5]{-1+3.3437\phi^{2.5}} + 20000\phi\sqrt[5]{-1+3.3437\phi^{2.5}} + \right. \\
& \quad 13 \exp\left(-\frac{\pi}{\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5-z_0)^k z_0^{-k}}{k!}} \right) + \\
& \quad 13\sqrt[5]{-1+3.3437\phi^{2.5}} \exp\left(-\frac{\pi}{\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5-z_0)^k z_0^{-k}}{k!}} \right) - \\
& \quad \left. 20000\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5-z_0)^k z_0^{-k}}{k!} \right) / \\
& \quad \left(20000\left(1 + \sqrt[5]{-1+3.3437\phi^{2.5}}\right) \exp\left(-\frac{\pi}{\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5-z_0)^k z_0^{-k}}{k!}} \right) \right) \Bigg)
\end{aligned}$$

for not $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(\alpha)_n$ is the Pochhammer symbol (rising factorial)

\mathbb{R} is the set of real numbers

From the radius 1.0002211825644..., computing the mass and temperature, considering the brane a black hole, we obtain, by the Ramanujan-Nardelli mock formula, the golden ratio. Indeed:

$$\sqrt{\left[\frac{1}{\left(\frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \right) \times \frac{1}{6.736174 \times 10^{26}} \sqrt{\frac{-0.0001821810 \times 4 \pi \times 1.000222^3 - 1.000222^2}{6.67 \times 10^{-11}}}} \right]}$$

Input interpretation:

$$\sqrt{\frac{1}{\frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{6.736174 \times 10^{26}} \sqrt{\frac{-0.0001821810 \times 4 \pi \times 1.000222^3 - 1.000222^2}{6.67 \times 10^{-11}}}}}$$

Result:

1.61825...

1.61825...

And the conjugate:

$$\frac{1}{\sqrt{\left[\frac{1}{\left(\frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \right) \times \frac{1}{6.736174 \times 10^{26}} \sqrt{\frac{-0.0001821810 \times 4 \pi \times 1.000222^3 - 1.000222^2}{6.67 \times 10^{-11}}}} \right]}}$$

Input interpretation:

$$\frac{1}{\sqrt{\frac{1}{\frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{6.736174 \times 10^{26}} \sqrt{\frac{-0.0001821810 \times 4 \pi \times 1.000222^3 - 1.000222^2}{6.67 \times 10^{-11}}}}}}}$$

Result:

0.617952...

0.617952...

From:

FAST EVALUATION OF THE HURWITZ ZETA FUNCTION AND DIRICHLET L-SERIES¹

E. A. Karatsuba

UDC 621.391.1 : 681.327

We have that:

$$1 \leq j \leq s, n \geq 2s \log 2s, r \geq n,$$

$$r \geq 3n, n \geq 2s \log 2s, s \geq 2, 1 \leq j \leq s.$$

$$\begin{aligned}
B'_j &= -e^{-t} \log^j t \Big|_n^\infty + j \int_n^\infty e^{-t} t^{-1} \log^{j-1} t dt \leq \dots \\
\dots &\leq e^{-n} \log^j n \frac{1 - \left(\frac{j}{n \log n}\right)^j}{1 - \frac{j}{n \log n}} \leq \frac{5}{3} e^{-n} \log^s n.
\end{aligned}
\tag{17}$$

For $s = 2, n = 5.545177444479$, we obtain:

$$4 \ln(4)$$

Input:

$$4 \log(4)$$

$\log(x)$ is the natural logarithm

Decimal approximation:

5.545177444479562475337856971665412544604001074882042032965...

5.545177444479.....

Property:

$4 \log(4)$ is a transcendental number

$$5/3 * e^{(-5.545177444479)} * \ln^2(5.545177444479)$$

Input interpretation:

$$\frac{\frac{5}{3} \log^2(5.545177444479)}{e^{5.545177444479}}$$

log(x) is the natural logarithm

Result:

0.0191023727928...

0.0191023727928....

Alternative representations:

$$\frac{1}{3} (e^{-5.5451774444790000} \log^2(5.5451774444790000)) 5 = \frac{5 \log_e^2(5.5451774444790000)}{3 e^{5.5451774444790000}}$$

$$\frac{1}{3} (e^{-5.5451774444790000} \log^2(5.5451774444790000)) 5 = \frac{5 (\log(a) \log_a(5.5451774444790000))^2}{3 e^{5.5451774444790000}}$$

$$\frac{1}{3} (e^{-5.5451774444790000} \log^2(5.5451774444790000)) 5 = \frac{5 (-\text{Li}_1(-4.5451774444790000))^2}{3 e^{5.5451774444790000}}$$

log_b(x) is the base- b logarithm

Li_n(x) is the polylogarithm function

Series representations:

$$\frac{1}{3} (e^{-5.5451774444790000} \log^2(5.5451774444790000)) 5 = \frac{5 \left(\log(4.5451774444790000) - \sum_{k=1}^{\infty} \frac{(-1)^k e^{-1.5140667685568842 k}}{k} \right)^2}{3 e^{5.5451774444790000}}$$

$$\frac{1}{3} (e^{-5.5451774444790000} \log^2(5.5451774444790000)) 5 = \frac{5 \left(2 i \pi \left\lfloor \frac{\arg(5.5451774444790000-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (5.5451774444790000-x)^k x^{-k}}{k} \right)^2}{3 e^{5.5451774444790000}}$$

for $x < 0$

$$\frac{1}{3} (e^{-5.5451774444790000} \log^2(5.5451774444790000)) 5 = \frac{1}{3 e^{5.5451774444790000}} 5 \left(\log(z_0) + \left\lfloor \frac{\arg(5.5451774444790000 - z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (5.5451774444790000 - z_0)^k z_0^{-k}}{k} \right)^2$$

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

Integral representations:

$$\frac{1}{3} (e^{-5.5451774444790000} \log^2(5.5451774444790000)) 5 = \frac{5 \left(\int_1^{5.5451774444790000} \frac{1}{t} dt \right)^2}{3 e^{5.5451774444790000}}$$

$$\frac{1}{3} (e^{-5.5451774444790000} \log^2(5.5451774444790000)) 5 = \frac{5 \left(\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-1.5140667685568842 s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right)^2}{12 e^{5.5451774444790000} i^2 \pi^2} \text{ for } -1 < \gamma < 0$$

$\Gamma(x)$ is the gamma function

$$1/((((5/3 * e^{(-5.545177444479)} * \ln^2(5.545177444479))))))$$

Input interpretation:

$$\frac{1}{\frac{5}{3} \log^2(5.545177444479)} \frac{1}{e^{5.545177444479}}$$

$\log(x)$ is the natural logarithm

Result:

52.3495175623...

52.3495175623....

•

Alternative representations:

$$\frac{1}{\frac{1}{3} (e^{-5.5451774444790000} \log^2(5.5451774444790000)) 5} = \frac{1}{\frac{5 \log_e^2(5.5451774444790000)}{3 e^{5.5451774444790000}}}$$

•

$$\frac{1}{\frac{1}{3} (e^{-5.5451774444790000} \log^2(5.5451774444790000)) 5} = \frac{1}{\frac{5 (\log(a) \log_a(5.5451774444790000))^2}{3 e^{5.5451774444790000}}}$$

•

$$\frac{1}{\frac{1}{3} (e^{-5.5451774444790000} \log^2(5.5451774444790000)) 5} = \frac{1}{\frac{5 (-\text{Li}_1(-4.5451774444790000))^2}{3 e^{5.5451774444790000}}}$$

$\log_b(x)$ is the base- b logarithm

$\text{Li}_n(x)$ is the polylogarithm function

•

Series representations:

$$\frac{1}{\frac{1}{3} (e^{-5.5451774444790000} \log^2(5.5451774444790000)) 5} = \frac{1}{3 e^{5.5451774444790000} \left(\log(4.5451774444790000) - \sum_{k=1}^{\infty} \frac{(-1)^k e^{-1.5140667685568842 k}}{k} \right)^2}$$

•

$$\frac{1}{\frac{1}{3} (e^{-5.5451774444790000} \log^2(5.5451774444790000)) 5} = \frac{5 \left(2 i \pi \left\lfloor \frac{\arg(5.5451774444790000 - x)}{2 \pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (5.5451774444790000 - x)^k x^{-k}}{k} \right)^2}{3 e^{5.5451774444790000}}$$

for $x < 0$

$$\frac{1}{\frac{1}{3} (e^{-5.5451774444790000} \log^2(5.5451774444790000)) 5} = (3 e^{5.5451774444790000}) / \left(5 \left(\log(z_0) + \left\lfloor \frac{\arg(5.5451774444790000 - z_0)}{2 \pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (5.5451774444790000 - z_0)^k z_0^{-k}}{k} \right)^2 \right)$$

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

Integral representations:

$$\frac{1}{\frac{1}{3} (e^{-5.5451774444790000} \log^2(5.5451774444790000)) 5} = \frac{3 e^{5.5451774444790000}}{5 \left(\int_1^{5.5451774444790000} \frac{1}{t} dt \right)^2}$$

$$\frac{1}{\frac{1}{3} (e^{-5.5451774444790000} \log^2(5.5451774444790000)) 5} = \frac{12 e^{5.5451774444790000} i^2 \pi^2}{5 \left(\int_{-i \infty + \gamma}^{i \infty + \gamma} \frac{e^{-1.5140667685568842 s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right)^2} \text{ for } -1 < \gamma < 0$$

$\Gamma(x)$ is the gamma function

$$1/(29+3) * 1/((((5/3 * e^{(-5.545177444479)} * \ln^2(5.545177444479))))))$$

Where 3 and 29 are Lucas numbers

Input interpretation:

$$\frac{1}{29 + 3} \times \frac{1}{\frac{\frac{5}{3} \log^2(5.545177444479)}{e^{5.545177444479}}}$$

$\log(x)$ is the natural logarithm

Result:

1.63592242382...

1.6359224...

•

Alternative representations:

$$\frac{1}{\frac{1}{3} (5 e^{-5.5451774444790000} \log^2(5.5451774444790000)) (29 + 3)} = \frac{1}{\frac{32 (5 \log_e^2(5.5451774444790000))}{3 e^{5.5451774444790000}}}$$

•

$$\frac{1}{\frac{1}{3} (5 e^{-5.5451774444790000} \log^2(5.5451774444790000)) (29 + 3)} = \frac{1}{\frac{32 (5 (\log(\alpha) \log_{\alpha}(5.5451774444790000))^2)}{3 e^{5.5451774444790000}}}$$

•

$$\frac{1}{\frac{1}{3} (5 e^{-5.5451774444790000} \log^2(5.5451774444790000)) (29 + 3)} = \frac{1}{\frac{32 (5 (-\text{Li}_1(-4.5451774444790000))^2)}{3 e^{5.5451774444790000}}}$$

$\log_b(x)$ is the base- b logarithm

$\text{Li}_n(x)$ is the polylogarithm function

•

Series representations:

$$\frac{1}{\frac{1}{3} (5 e^{-5.5451774444790000} \log^2(5.5451774444790000)) (29 + 3) 3 e^{5.5451774444790000}} =$$

$$160 \left(\log(4.5451774444790000) - \sum_{k=1}^{\infty} \frac{(-1)^k e^{-1.5140667685568842 k}}{k} \right)^2$$

$$\frac{1}{\frac{1}{3} (5 e^{-5.5451774444790000} \log^2(5.5451774444790000)) (29 + 3) 3 e^{5.5451774444790000}} =$$

$$160 \left(2 i \pi \left\lfloor \frac{\arg(5.5451774444790000 - x)}{2 \pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (5.5451774444790000 - x)^k x^{-k}}{k} \right)^2$$

for $x < 0$

$$\frac{1}{\frac{1}{3} (5 e^{-5.5451774444790000} \log^2(5.5451774444790000)) (29 + 3)} = (3 e^{5.5451774444790000}) /$$

$$\left(160 \left(\log(z_0) + \left\lfloor \frac{\arg(5.5451774444790000 - z_0)}{2 \pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (5.5451774444790000 - z_0)^k z_0^{-k}}{k} \right)^2 \right)$$

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

Integral representations:

$$\frac{1}{\frac{1}{3} (5 e^{-5.5451774444790000} \log^2(5.5451774444790000)) (29 + 3) 3 e^{5.5451774444790000}} =$$

$$160 \left(\int_1^{5.5451774444790000} \frac{1}{t} dt \right)^2$$

$$\frac{1}{\frac{1}{3} (5 e^{-5.5451774444790000} \log^2(5.5451774444790000)) (29 + 3) 3 e^{5.5451774444790000} i^2 \pi^2} =$$

$$40 \left(\int_{-i \infty + \gamma}^{i \infty + \gamma} \frac{e^{-1.5140667685568842 s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right)^2 \text{ for } -1 < \gamma < 0$$

$\Gamma(x)$ is the gamma function

$$(199+11)+29/\left(\left(\left(\frac{5}{3} * e^{(-5.545177444479)} * \ln^2(5.545177444479)\right)\right)\right)$$

Where 11, 29 and 199 are Lucas numbers

Input interpretation:

$$(199 + 11) + \frac{29}{\frac{\frac{5}{3} \log^2(5.545177444479)}{e^{5.545177444479}}}$$

$\log(x)$ is the natural logarithm

Result:

1728.13600931...

1728.136...

This result is very near to the mass of candidate glueball $f_0(1710)$ meson. Furthermore, 1728 occurs in the algebraic formula for the j -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

•

Alternative representations:

$$(199 + 11) + \frac{29}{\frac{1}{3} (e^{-5.5451774444790000} \log^2(5.5451774444790000)) 5} =$$

$$210 + \frac{29}{\frac{5 \log_e^2(5.5451774444790000)}{3 e^{5.5451774444790000}}}$$

•

$$(199 + 11) + \frac{29}{\frac{1}{3} (e^{-5.5451774444790000} \log^2(5.5451774444790000)) 5} =$$

$$210 + \frac{29}{\frac{5 (\log(\alpha) \log_{\alpha}(5.5451774444790000))^2}{3 e^{5.5451774444790000}}}$$

$$(199 + 11) + \frac{29}{\frac{1}{3} (e^{-5.5451774444790000} \log^2(5.5451774444790000)) 5} =$$

$$210 + \frac{29}{\frac{5 (-\text{Li}_1(-4.5451774444790000))^2}{3 e^{5.5451774444790000}}}$$

$\log_b(x)$ is the base- b logarithm

$\text{Li}_n(x)$ is the polylogarithm function

Series representations:

$$(199 + 11) + \frac{29}{\frac{1}{3} (e^{-5.5451774444790000} \log^2(5.5451774444790000)) 5} =$$

$$210 + \frac{29}{87 e^{5.5451774444790000} \left(\log(4.5451774444790000) - \sum_{k=1}^{\infty} \frac{(-1)^k e^{-1.5140667685568842 k}}{k} \right)^2}$$

$$(199 + 11) + \frac{29}{\frac{1}{3} (e^{-5.5451774444790000} \log^2(5.5451774444790000)) 5} = 210 +$$

$$\frac{29}{87 e^{5.5451774444790000} \left(5 \left(2 i \pi \left[\frac{\arg(5.5451774444790000 - x)}{2 \pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (5.5451774444790000 - x)^k x^{-k}}{k} \right)^2 \right)}$$

for $x < 0$

$$(199 + 11) + \frac{29}{\frac{1}{3} (e^{-5.5451774444790000} \log^2(5.5451774444790000)) 5} =$$

$$210 + (87 e^{5.5451774444790000}) /$$

$$\left(5 \left(\log(z_0) + \left[\frac{\arg(5.5451774444790000 - z_0)}{2 \pi} \right] \right) \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (5.5451774444790000 - z_0)^k z_0^{-k}}{k} \right)^2$$

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

Integral representations:

$$(199 + 11) + \frac{29}{\frac{1}{3} (e^{-5.5451774444790000} \log^2(5.5451774444790000)) \frac{5}{87} e^{5.5451774444790000}} = 210 + \frac{29}{5 \left(\int_1^5 \frac{1}{t} dt \right)^2}$$

$$(199 + 11) + \frac{29}{\frac{1}{3} (e^{-5.5451774444790000} \log^2(5.5451774444790000)) \frac{5}{348} e^{5.5451774444790000} i^2 \pi^2} = 210 + \frac{29}{5 \left(\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-1.5140667685568842s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right)^2} \text{ for } -1 < \gamma < 0$$

$\Gamma(x)$ is the gamma function

$$\left(\frac{(199+11)+29/\left(\frac{5}{3}e^{-5.545177444479} \ln^2(5.545177444479)\right)}{e^{5.545177444479}} \right)^{1/15}$$

Input interpretation:

$$\sqrt[15]{(199 + 11) + \frac{29}{\frac{5}{3} \frac{\log^2(5.545177444479)}{e^{5.545177444479}}}}$$

$\log(x)$ is the natural logarithm

Result:

1.643760454414...

$$1.643760454414.... \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 ...$$

$$29/10^3 + (((((199+11)+29/((((5/3 * e^{-5.545177444479}) * \ln^2(5.545177444479))))))))^{1/15}$$

Input interpretation:

$$\frac{29}{10^3} + \sqrt[15]{(199 + 11) + \frac{29}{\frac{5}{3} \frac{\log^2(5.545177444479)}{e^{5.545177444479}}}}$$

log(x) is the natural logarithm

Result:

1.672760454414...

1.672760454414.... result very near to the proton mass

Alternative representations:

$$\frac{29}{10^3} + \sqrt[15]{(199 + 11) + \frac{29}{\frac{1}{3} (e^{-5.5451774444790000} \log^2(5.5451774444790000)) 5}} =$$

$$\frac{29}{10^3} + \sqrt[15]{210 + \frac{29}{\frac{5 \log_e^2(5.5451774444790000)}{3 e^{5.5451774444790000}}}}$$

$$\frac{29}{10^3} + \sqrt[15]{(199 + 11) + \frac{29}{\frac{1}{3} (e^{-5.5451774444790000} \log^2(5.5451774444790000)) 5}} =$$

$$\frac{29}{10^3} + \sqrt[15]{210 + \frac{29}{\frac{5 (\log(\alpha) \log_{\alpha}(5.5451774444790000))^2}{3 e^{5.5451774444790000}}}}$$

$$\frac{29}{10^3} + \sqrt[15]{(199 + 11) + \frac{29}{\frac{1}{3} (e^{-5.5451774444790000} \log^2(5.5451774444790000)) 5}} =$$

$$\frac{29}{10^3} + \sqrt[15]{210 + \frac{29}{\frac{5 (-\text{Li}_1(-4.5451774444790000))^2}{3 e^{5.5451774444790000}}}}$$

log_b(x) is the base- b logarithm

Series representations:

$$\frac{29}{10^3} + \sqrt[15]{(199 + 11) + \frac{29}{\frac{1}{3} (e^{-5.5451774444790000} \log^2(5.5451774444790000)) 5}} =$$

$$\frac{29}{1000} + \sqrt[15]{210 + \frac{87 e^{5.5451774444790000}}{5 \left(\log(4.5451774444790000) - \sum_{k=1}^{\infty} \frac{(-1)^k e^{-1.5140667685568842 k}}{k} \right)^2}}$$

$$\frac{29}{10^3} + \sqrt[15]{(199 + 11) + \frac{29}{\frac{1}{3} (e^{-5.5451774444790000} \log^2(5.5451774444790000)) 5}} =$$

$$\frac{29}{1000} + \left(210 + (87 e^{5.5451774444790000}) / \left(5 \left(2 i \pi \left[\frac{\arg(5.5451774444790000 - x)}{2 \pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (5.5451774444790000 - x)^k x^{-k}}{k} \right)^2 \right) \right)^{(1/15)}$$

15) for $x < 0$

$$\frac{29}{10^3} + \sqrt[15]{(199 + 11) + \frac{29}{\frac{1}{3} (e^{-5.5451774444790000} \log^2(5.5451774444790000)) 5}} =$$

$$\frac{29}{1000} + \left(210 + (87 e^{5.5451774444790000}) / \left(5 \left(\log(z_0) + \left[\frac{\arg(5.5451774444790000 - z_0)}{2 \pi} \right] \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (5.5451774444790000 - z_0)^k z_0^{-k}}{k} \right)^2 \right) \right)^{(1/15)}$$

$\arg(z)$ is the complex argument

$[x]$ is the floor function

i is the imaginary unit

Integral representations:

$$\frac{29}{10^3} + \sqrt[15]{(199 + 11) + \frac{29}{\frac{1}{3} (e^{-5.5451774444790000} \log^2(5.5451774444790000)) 5}} =$$

$$\frac{29}{1000} + \sqrt[15]{210 + \frac{87 e^{5.5451774444790000}}{5 \left(\int_1^{5.5451774444790000} \frac{1}{t} dt \right)^2}}$$

$$\frac{29}{10^3} + \sqrt[15]{(199 + 11) + \frac{29}{\frac{1}{3} (e^{-5.5451774444790000} \log^2(5.5451774444790000)) 5}} =$$

$$\frac{29}{1000} + \sqrt[15]{210 + \frac{348 e^{5.5451774444790000} i^2 \pi^2}{5 \left(\int_{-i \infty + \gamma}^{i \infty + \gamma} \frac{e^{-1.5140667685568842 s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right)^2}} \text{ for } -1 < \gamma < 0$$

$\Gamma(x)$ is the gamma function

$$-(18+7)/10^3 + (((((199+11)+29/(((5/3 * e^{(-5.545177444479)} * \ln^2(5.545177444479))))))))^{1/15}$$

Where 7, 11, 18, 29 and 199 are Lucas numbers

Input interpretation:

$$-\frac{18+7}{10^3} + \sqrt[15]{(199+11) + \frac{29}{\frac{5}{3} \frac{\log^2(5.545177444479)}{e^{5.545177444479}}}}$$

$\log(x)$ is the natural logarithm

Result:

1.618760454414...

1.618760454414....

This result is a very good approximation to the value of the golden ratio
1,618033988749...

Alternative representations:

$$-\frac{18+7}{10^3} + \sqrt[15]{(199+11) + \frac{29}{\frac{1}{3}(e^{-5.5451774444790000} \log^2(5.5451774444790000))} 5} =$$

$$-\frac{25}{10^3} + \sqrt[15]{210 + \frac{29}{\frac{5 \log_e^2(5.5451774444790000)}{3 e^{5.5451774444790000}}}}$$

- $$-\frac{18+7}{10^3} + \sqrt[15]{(199+11) + \frac{29}{\frac{1}{3}(e^{-5.5451774444790000} \log^2(5.5451774444790000))} 5} =$$

$$-\frac{25}{10^3} + \sqrt[15]{210 + \frac{29}{\frac{5 (\log(a) \log_a(5.5451774444790000))^2}{3 e^{5.5451774444790000}}}}$$

- $$-\frac{18+7}{10^3} + \sqrt[15]{(199+11) + \frac{29}{\frac{1}{3}(e^{-5.5451774444790000} \log^2(5.5451774444790000))} 5} =$$

$$-\frac{25}{10^3} + \sqrt[15]{210 + \frac{29}{\frac{5 (-\text{Li}_1(-4.5451774444790000))^2}{3 e^{5.5451774444790000}}}}$$

$\log_b(x)$ is the base- b logarithm

$\text{Li}_n(x)$ is the polylogarithm function

- ### Series representations:

- $$-\frac{18+7}{10^3} + \sqrt[15]{(199+11) + \frac{29}{\frac{1}{3}(e^{-5.5451774444790000} \log^2(5.5451774444790000))} 5} =$$

$$-\frac{1}{40} + \sqrt[15]{210 + \frac{87 e^{5.5451774444790000}}{5 \left(\log(4.5451774444790000) - \sum_{k=1}^{\infty} \frac{(-1)^k e^{-1.5140667685568842 k}}{k} \right)^2}}$$

$$-\frac{18+7}{10^3} + \sqrt[15]{(199+11) + \frac{29}{\frac{1}{3}(e^{-5.5451774444790000} \log^2(5.5451774444790000))5}} =$$

$$-\frac{1}{40} + \left(210 + (87 e^{5.5451774444790000}) / \left(5 \left(2i\pi \left\lfloor \frac{\arg(5.5451774444790000 - x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (5.5451774444790000 - x)^k x^{-k}}{k} \right)^2 \right) \right)^{1/15}$$

15) for $x < 0$

$$-\frac{18+7}{10^3} + \sqrt[15]{(199+11) + \frac{29}{\frac{1}{3}(e^{-5.5451774444790000} \log^2(5.5451774444790000))5}} =$$

$$-\frac{1}{40} + \left(210 + (87 e^{5.5451774444790000}) / \left(5 \left(\log(z_0) + \left\lfloor \frac{\arg(5.5451774444790000 - z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (5.5451774444790000 - z_0)^k z_0^{-k}}{k} \right)^2 \right) \right)^{1/15}$$

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

Integral representations:

$$-\frac{18+7}{10^3} + \sqrt[15]{(199+11) + \frac{29}{\frac{1}{3}(e^{-5.5451774444790000} \log^2(5.5451774444790000))5}} =$$

$$-\frac{1}{40} + \sqrt[15]{210 + \frac{87 e^{5.5451774444790000}}{5 \left(\int_1^{5.5451774444790000} \frac{1}{t} dt \right)^2}}$$

$$-\frac{18+7}{10^3} + \sqrt[15]{(199+11) + \frac{29}{\frac{1}{3}(e^{-5.5451774444790000} \log^2(5.5451774444790000))} 5} =$$

$$-\frac{1}{40} + \sqrt[15]{210 + \frac{348 e^{5.5451774444790000} i^2 \pi^2}{5 \left(\int_{-i \infty + \gamma}^{i \infty + \gamma} \frac{e^{-1.5140667685568842 s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right)^2}} \text{ for } -1 < \gamma < 0$$

$\Gamma(x)$ is the gamma function

$$2\sqrt{\left(\left(\left(\left(\left(\left(\left(6\left(\left(\left(\left(199+11\right)+29/\left(\left(\left(5/3*e^{\left(-5.545177444479\right)}\right)*\ln^2\left(5.545177444479\right)\right)\right)\right)\right)\right)\right)\right)\right)\right)^{1/15}\right)\right)\right)\right)\right)$$

Input interpretation:

$$2 \sqrt[6]{\sqrt[15]{(199 + 11) + \frac{29}{\frac{5 \log^2(5.545177444479)}{3 e^{5.545177444479}}}}}$$

$\log(x)$ is the natural logarithm

Result:

6.2809434725960...

$$6.2809434725960\dots = 2\pi r = C$$

•

Alternative representations:

$$2 \sqrt[6]{\sqrt[15]{(199 + 11) + \frac{29}{\frac{1}{3}(e^{-5.5451774444790000} \log^2(5.5451774444790000))} 5} =$$

$$2 \sqrt[6]{\sqrt[15]{210 + \frac{29}{\frac{5 \log_e^2(5.5451774444790000)}{3 e^{5.5451774444790000}}}}}$$

•

$$2 \sqrt{6 \sqrt[15]{(199 + 11) + \frac{29}{\frac{1}{3} (e^{-5.5451774444790000} \log^2(5.5451774444790000)) 5}}} =$$

$$2 \sqrt{6 \sqrt[15]{210 + \frac{29}{\frac{5 (\log(a) \log_a(5.5451774444790000))^2}{3 e^{5.5451774444790000}}}}}$$

$$2 \sqrt{6 \sqrt[15]{(199 + 11) + \frac{29}{\frac{1}{3} (e^{-5.5451774444790000} \log^2(5.5451774444790000)) 5}}} =$$

$$2 \sqrt{6 \sqrt[15]{210 + \frac{29}{\frac{5 (-\text{Li}_1(-4.5451774444790000))^2}{3 e^{5.5451774444790000}}}}}$$

$\log_b(x)$ is the base- b logarithm

$\text{Li}_n(x)$ is the polylogarithm function

Series representations:

$$2 \sqrt{6 \sqrt[15]{(199 + 11) + \frac{29}{\frac{1}{3} (e^{-5.5451774444790000} \log^2(5.5451774444790000)) 5}}} =$$

$$2 \sqrt{-1 + 6 \sqrt[15]{210 + \frac{87 e^{5.5451774444790000}}{5 \log^2(5.5451774444790000)}}}$$

$$\sum_{k=0}^{\infty} \binom{\frac{1}{2}}{k} \left(-1 + 6 \sqrt[15]{210 + \frac{87 e^{5.5451774444790000}}{5 \log^2(5.5451774444790000)}} \right)^{-k}$$

$$2 \sqrt{6 \sqrt[15]{(199 + 11) + \frac{29}{\frac{1}{3} (e^{-5.5451774444790000} \log^2(5.5451774444790000)) 5}}} =$$

$$2 \sqrt{6 \sqrt[15]{210 + \frac{87 e^{5.5451774444790000}}{5 \left(\log(4.5451774444790000) - \sum_{k=1}^{\infty} \frac{(-1)^k e^{-1.5140667685568842 k}}{k} \right)^2}}}$$

$$2 \sqrt{6 \sqrt[15]{(199 + 11) + \frac{29}{\frac{1}{3} (e^{-5.5451774444790000} \log^2(5.5451774444790000)) 5}}} =$$

$$2 \sqrt{-1 + 6 \sqrt[15]{210 + \frac{87 e^{5.5451774444790000}}{5 \log^2(5.5451774444790000)}}}$$

$$\sum_{k=0}^{\infty} \frac{(-1)^k \left(-1 + 6 \sqrt[15]{210 + \frac{87 e^{5.5451774444790000}}{5 \log^2(5.5451774444790000)}} \right)^{-k}}{k!} \left(-\frac{1}{2} \right)_k$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

Integral representations:

$$2 \sqrt{6 \sqrt[15]{(199 + 11) + \frac{29}{\frac{1}{3} (e^{-5.5451774444790000} \log^2(5.5451774444790000)) 5}}} =$$

$$2 \sqrt{6 \sqrt[15]{210 + \frac{87 e^{5.5451774444790000}}{5 \left(\int_1^{5.5451774444790000} \frac{1}{t} dt \right)^2}}}$$

$$2 \sqrt{6 \sqrt[15]{(199 + 11) + \frac{29}{\frac{1}{3} (e^{-5.5451774444790000} \log^2(5.5451774444790000)) 5}}} =$$

$$2 \sqrt{6 \sqrt[15]{210 + \frac{348 e^{5.5451774444790000} i^2 \pi^2}{5 \left(\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-1.5140667685568842 s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right)^2}} \quad \text{for } -1 < \gamma < 0$$

$\Gamma(x)$ is the gamma function

i is the imaginary unit

$$(2\pi) \times \frac{1}{\sqrt[2]{\sqrt[6]{\sqrt[15]{(199+11) + \frac{29}{\frac{5}{3} e^{-5.545177444479} \log^2(5.545177444479)}}}}}}$$

Input interpretation:

$$(2\pi) \times \frac{1}{\sqrt[2]{\sqrt[6]{\sqrt[15]{(199+11) + \frac{29}{\frac{5}{3} e^{-5.545177444479} \log^2(5.545177444479)}}}}}}$$

log(x) is the natural logarithm

Result:

1.0003569264066...

1.0003569264066.... = radius

Alternative representations:

$$\frac{2\pi}{\sqrt[2]{\sqrt[6]{\sqrt[15]{(199+11) + \frac{29}{\frac{5}{3} e^{-5.5451774444790000} \log^2(5.5451774444790000)}}}}}} = \frac{2\pi}{\sqrt[2]{\sqrt[6]{\sqrt[15]{210 + \frac{29}{\frac{5}{3} e^{-5.5451774444790000} \log^2(5.5451774444790000)}}}}}}$$

$$\frac{2\pi}{\sqrt[2]{\sqrt[6]{\sqrt[15]{(199+11) + \frac{29}{\frac{5}{3} e^{-5.5451774444790000} \log^2(5.5451774444790000)}}}}}} = \frac{2\pi}{\sqrt[2]{\sqrt[6]{\sqrt[15]{210 + \frac{29}{\frac{5}{3} e^{-5.5451774444790000} (-\text{Li}_1(-4.5451774444790000))^2}}}}}}$$

$$\frac{2\pi}{2\sqrt{6\sqrt[15]{(199+11) + \frac{\frac{5}{3}e^{-5.5451774444790000}}{\log^2(5.5451774444790000)}}}} = \frac{2\sqrt{6\sqrt[15]{210 + \frac{29}{\frac{5(\log(a)\log_e(5.5451774444790000))^2}{3e^{5.5451774444790000}}}}}}{2\pi}$$

$\log_b(x)$ is the base- b logarithm

$\text{Li}_n(x)$ is the polylogarithm function

• **Series representations:**

$$\frac{2\pi}{2\sqrt{6\sqrt[15]{(199+11) + \frac{\frac{5}{3}e^{-5.5451774444790000}}{\log^2(5.5451774444790000)}}}} = \frac{\sqrt{6\sqrt[15]{210 + \frac{87e^{5.5451774444790000}}{5\left(\log(4.5451774444790000) - \sum_{k=1}^{\infty} \frac{(-1)^k e^{-1.5140667685568842k}}{k}\right)^2}}}}{\pi}$$

$$\frac{2\pi}{2\sqrt{6\sqrt[15]{(199+11) + \frac{\frac{5}{3}e^{-5.5451774444790000}}{\log^2(5.5451774444790000)}}}} = \frac{\pi}{\left(\sqrt{-1 + 6\sqrt[15]{210 + \frac{87e^{5.5451774444790000}}{5\log^2(5.5451774444790000)}}}} \sum_{k=0}^{\infty} \binom{\frac{1}{2}}{k} \left(\sqrt{-1 + 6\sqrt[15]{210 + \frac{87e^{5.5451774444790000}}{5\log^2(5.5451774444790000)}}}} \right)^{-k} \right)}$$

$$\begin{aligned}
& \frac{2\pi}{2\sqrt{6}\sqrt[15]{(199+11)+\frac{29}{\frac{5}{3}e^{-5.5451774444790000}\log^2(5.5451774444790000)}}}} = \\
& \pi / \left(\sqrt[15]{-1+6\sqrt[15]{210+\frac{87e^{5.5451774444790000}}{5\log^2(5.5451774444790000)}}}} \right. \\
& \left. \sum_{k=0}^{\infty} \frac{(-1)^k \left(-1+6\sqrt[15]{210+\frac{87e^{5.5451774444790000}}{5\log^2(5.5451774444790000)}}}\right)^{-k} \binom{-\frac{1}{2}}{k}}{k!} \right)
\end{aligned}$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(\alpha)_n$ is the Pochhammer symbol (rising factorial)

• **Integral representations:**

$$\begin{aligned}
& \frac{2\pi}{2\sqrt{6}\sqrt[15]{(199+11)+\frac{29}{\frac{5}{3}e^{-5.5451774444790000}\log^2(5.5451774444790000)}}}} = \\
& \frac{\pi}{\sqrt[15]{6\sqrt[15]{210+\frac{87e^{5.5451774444790000}}{5\left(\int_1^{\frac{1}{t}}e^{-5.5451774444790000}\frac{1}{t}dt\right)^2}}}
\end{aligned}$$

$$\begin{aligned}
& \frac{2\pi}{2\sqrt{6}\sqrt[15]{(199+11)+\frac{29}{\frac{5}{3}e^{-5.5451774444790000}\log^2(5.5451774444790000)}}}} = \\
& \frac{\pi}{\sqrt[15]{6\sqrt[15]{210+\frac{348e^{5.5451774444790000}i^2\pi^2}{5\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}e^{-1.5140667685568842s}\frac{\Gamma(-s)^2\Gamma(1+s)}{\Gamma(1-s)}ds\right)^2}}} \quad \text{for } -1 < \gamma < 0
\end{aligned}$$

$\Gamma(x)$ is the gamma function

i is the imaginary unit

We observe that 1.0003569264066.... = radius is a value very near to

$$\zeta(12) = 1 + \frac{1}{2^{12}} + \frac{1}{3^{12}} + \dots = \frac{691\pi^{12}}{638512875} = 1.000246\dots$$

And:

$$\frac{1}{2\pi} * [2\sqrt{((((6((((199+11)+29/((((5/3 * e^{(-5.545177444479)} * \ln^2(5.545177444479)))))))))^{1/15})})}]$$

Input interpretation:

$$\frac{1}{2\pi} \left(2 \sqrt{6 \sqrt[15]{(199 + 11) + \frac{29}{\frac{5}{3} \frac{\log^2(5.545177444479)}{e^{5.545177444479}}}}} \right)$$

log(x) is the natural logarithm

Result:

0.99964320094443...

0.99964320094....

- **Alternative representations:**
- More

$$\frac{2 \sqrt{6 \sqrt[15]{(199 + 11) + \frac{29}{\frac{5}{3} e^{-5.5451774444790000} \log^2(5.5451774444790000)}}}}{2\pi} = \frac{2 \sqrt{6 \sqrt[15]{210 + \frac{29}{\frac{5 \log_e^2(5.5451774444790000)}{3 e^{5.5451774444790000}}}}} }{2\pi}$$

$$\frac{2 \sqrt{6 \sqrt[15]{(199 + 11) + \frac{29}{\frac{5}{3} e^{-5.5451774444790000} \log^2(5.5451774444790000)}}}}{2\pi} = \frac{2 \sqrt{6 \sqrt[15]{210 + \frac{29}{\frac{5 (\log(\alpha) \log_{\alpha}(5.5451774444790000))^2}{3 e^{5.5451774444790000}}}}} }{2\pi}$$

$$\frac{2 \sqrt{6 \sqrt[15]{(199 + 11) + \frac{29}{3 e^{-5.5451774444790000} \log^2(5.5451774444790000)}}}}{2 \pi} = \frac{2 \sqrt{6 \sqrt[15]{210 + \frac{29}{3 e^{5.5451774444790000} 5 (-\text{Li}_1(-4.5451774444790000))^2}}}}{2 \pi}$$

$\log_b(x)$ is the base- b logarithm

$\text{Li}_n(x)$ is the polylogarithm function

Series representations:

$$\frac{2 \sqrt{6 \sqrt[15]{(199 + 11) + \frac{29}{3 e^{-5.5451774444790000} \log^2(5.5451774444790000)}}}}{2 \pi} = \frac{1}{\pi} \sqrt{-1 + 6 \sqrt[15]{210 + \frac{87 e^{5.5451774444790000}}{5 \log^2(5.5451774444790000)}}}} \sum_{k=0}^{\infty} \binom{\frac{1}{2}}{k} \left(-1 + 6 \sqrt[15]{210 + \frac{87 e^{5.5451774444790000}}{5 \log^2(5.5451774444790000)}} \right)^{-k}$$

$$\frac{2 \sqrt{6 \sqrt[15]{(199 + 11) + \frac{29}{3 e^{-5.5451774444790000} \log^2(5.5451774444790000)}}}}{2 \pi} = \frac{\sqrt{6 \sqrt[15]{210 + \frac{87 e^{5.5451774444790000}}{5 \left(\log(4.5451774444790000) - \sum_{k=1}^{\infty} \frac{(-1)^k e^{-1.5140667685568842 k}}{k} \right)^2}}}}{\pi}$$

$$2 \sqrt{6 \sqrt[15]{(199 + 11) + \frac{29}{3 e^{-5.5451774444790000} \log^2(5.5451774444790000)}}} = \frac{2 \pi}{\frac{1}{\pi} \sqrt{-1 + 6 \sqrt[15]{210 + \frac{87 e^{5.5451774444790000}}{5 \log^2(5.5451774444790000)}}}} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-1 + 6 \sqrt[15]{210 + \frac{87 e^{5.5451774444790000}}{5 \log^2(5.5451774444790000)}} \right)^{-k}}{k!} \left(-\frac{1}{2} \right)_k$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(\alpha)_n$ is the Pochhammer symbol (rising factorial)

[More information »](#)

Integral representations:

$$2 \sqrt{6 \sqrt[15]{(199 + 11) + \frac{29}{3 e^{-5.5451774444790000} \log^2(5.5451774444790000)}}} = \frac{2 \pi}{\sqrt{6 \sqrt[15]{210 + \frac{87 e^{5.5451774444790000}}{5 \left(\int_1^{\infty} \frac{1}{t} dt \right)^2}}}} \pi$$

$$2 \sqrt{6 \sqrt[15]{(199 + 11) + \frac{29}{3 e^{-5.5451774444790000} \log^2(5.5451774444790000)}}} = \frac{2 \pi}{\sqrt{6 \sqrt[15]{210 + \frac{348 e^{5.5451774444790000} i^2 \pi^2}{5 \left(\int_{-i \infty + \gamma}^{i \infty + \gamma} \frac{e^{-1.5140667685568842 s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right)^2}}} \pi \quad \text{for } -1 < \gamma < 0$$

$\Gamma(x)$ is the gamma function

i is the imaginary unit

This result 0.99964320094... is very near to the value of the following Rogers-Ramanujan continued fraction

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

Now, we have that:

$$A_j'' \leq \frac{1}{(r+1)!} \left(\left| \int_0^1 t^{r+a} \log^j t \, dt \right| + \left| \int_1^n t^{r+a} \log^j t \, dt \right| \right) \leq \frac{1}{(r+2)!} (1 + n^{r+2} \log^s n). \tag{19}$$

For $s = 2$, $n = 5.545177444479$ and $r = 6$, we obtain:

$$1/(6+2)! * ((1+5.545177444479^8 * \ln^2(5.545177444479))$$

Input interpretation:

$$\frac{1}{(6+2)!} (1 + 5.545177444479^8 \log^2(5.545177444479))$$

$n!$ is the factorial function

$\log(x)$ is the natural logarithm

Result:

65.0551613614...

65.0551613614...

Alternative representations:

$$\frac{1 + 5.5451774444790000^8 \log^2(5.5451774444790000)}{(6 + 2)!} = \frac{1 + 5.5451774444790000^8 (\log_a(a) \log_a(5.5451774444790000))^2}{\Gamma(9)}$$

•

$$\frac{1 + 5.5451774444790000^8 \log^2(5.5451774444790000)}{(6 + 2)!} = \frac{1 + 5.5451774444790000^8 \log^2(5.5451774444790000)}{7!! \times 8!!}$$

•

$$\frac{1 + 5.5451774444790000^8 \log^2(5.5451774444790000)}{(6 + 2)!} = \frac{1 + 5.5451774444790000^8 \log_e^2(5.5451774444790000)}{(1)_8}$$

$\Gamma(x)$ is the gamma function

$\log_b(x)$ is the base- b logarithm

$n!!$ is the double factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

•

Series representations:

$$\frac{1 + 5.5451774444790000^8 \log^2(5.5451774444790000)}{(6 + 2)!} = \frac{1 + 893\,971.3161384143 \log^2(5.5451774444790000)}{\sum_{k=0}^{\infty} \frac{(8-n_0)^k \Gamma^{(k)}(1+n_0)}{k!}}$$

for $((n_0 \notin \mathbb{Z} \text{ or } n_0 \geq 0) \text{ and } n_0 \rightarrow 8)$

•

$$\frac{1 + 5.5451774444790000^8 \log^2(5.5451774444790000)}{(6 + 2)!} =$$

$$\left(893\,971.31613841 \left(1.11860412291480 \times 10^{-6} + \right. \right.$$

$$1.0000000000000000 \log^2(4.5451774444790000) - 2.0000000000000000$$

$$\log(4.5451774444790000) \sum_{k=1}^{\infty} \frac{(-1)^k e^{-1.5140667685568842k}}{k} +$$

$$1.0000000000000000 \left(\sum_{k=1}^{\infty} \frac{(-1)^k e^{-1.5140667685568842k}}{k} \right)^2 \Bigg) /$$

$$\left(\sum_{k=0}^{\infty} \frac{(8 - n_0)^k \Gamma^{(k)}(1 + n_0)}{k!} \right) \text{ for } ((n_0 \notin \mathbb{Z} \text{ or } n_0 \geq 0) \text{ and } n_0 \rightarrow 8)$$

$$\frac{1 + 5.5451774444790000^8 \log^2(5.5451774444790000)}{(6 + 2)!} =$$

$$\left(1 + 893\,971.3161384143 \left(\log(z_0) + \left[\frac{\arg(5.5451774444790000 - z_0)}{2\pi} \right] \left(\log\left(\frac{1}{z_0}\right) + \right. \right. \right.$$

$$\left. \left. \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (5.5451774444790000 - z_0)^k z_0^{-k}}{k} \right)^2 \Bigg) /$$

$$\left(\sum_{k=0}^{\infty} \frac{(8 - n_0)^k \Gamma^{(k)}(1 + n_0)}{k!} \right) \text{ for } ((n_0 \notin \mathbb{Z} \text{ or } n_0 \geq 0) \text{ and } n_0 \rightarrow 8)$$

\mathbb{Z} is the set of integers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

Integral representations:

$$\frac{1 + 5.5451774444790000^8 \log^2(5.5451774444790000)}{(6 + 2)!} =$$

$$\frac{1}{\int_0^{\infty} e^{-t} t^8 dt} 893\,971.31613841$$

$$(1.11860412291480 \times 10^{-6} + 1.0000000000000000 \log^2(5.5451774444790000))$$

•

$$\frac{1 + 5.5451774444790000^8 \log^2(5.5451774444790000)}{(6 + 2)!} =$$

$$\frac{1}{\int_0^1 \log^8\left(\frac{1}{t}\right) dt} 893\,971.31613841$$

$$\left(1.11860412291480 \times 10^{-6} + 1.0000000000000000 \left(\int_1^{5.5451774444790000} \frac{1}{t} dt\right)^2\right)$$

•

$$\frac{1 + 5.5451774444790000^8 \log^2(5.5451774444790000)}{(6 + 2)!} =$$

$$\frac{1}{\int_0^\infty e^{-t} t^8 dt} 893\,971.31613841$$

$$\left(1.11860412291480 \times 10^{-6} + 1.0000000000000000 \left(\int_1^{5.5451774444790000} \frac{1}{t} dt\right)^2\right)$$

$$(-123-47+11)+29 * 1/(6+2)! * ((1+5.545177444479^8 * \ln^2(5.545177444479))$$

Where 11, 29, 47 and 123 are Lucas numbers

Input interpretation:

$$(-123 - 47 + 11) + 29 \times \frac{1}{(6 + 2)!} (1 + 5.545177444479^8 \log^2(5.545177444479))$$

n! is the factorial function

log(x) is the natural logarithm

Result:

1727.59967948...

1727.59967...

This result is very near to the mass of candidate glueball $f_0(1710)$ meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

Alternative representations:

$$(-123 - 47 + 11) + \frac{29 (1 + 5.5451774444790000^8 \log^2(5.5451774444790000))}{(6 + 2)!} =$$

$$-159 + \frac{29 (1 + 5.5451774444790000^8 (\log_a(5.5451774444790000))^2)}{\Gamma(9)}$$

- $$(-123 - 47 + 11) + \frac{29 (1 + 5.5451774444790000^8 \log^2(5.5451774444790000))}{(6 + 2)!} =$$

$$-159 + \frac{29 (1 + 5.5451774444790000^8 \log^2(5.5451774444790000))}{7!! \times 8!!}$$

- $$(-123 - 47 + 11) + \frac{29 (1 + 5.5451774444790000^8 \log^2(5.5451774444790000))}{(6 + 2)!} =$$

$$-159 + \frac{29 (1 + 5.5451774444790000^8 \log_e^2(5.5451774444790000))}{(1)_8}$$

$\Gamma(x)$ is the gamma function

$\log_b(x)$ is the base- b logarithm

$n!!$ is the double factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

- ### Series representations:

$$(-123 - 47 + 11) + \frac{29 (1 + 5.5451774444790000^8 \log^2(5.5451774444790000))}{(6 + 2)!} = -159 +$$

$$\frac{29.000000000000000 + 2.59251681680140 \times 10^7 \log^2(5.5451774444790000)}{\sum_{k=0}^{\infty} \frac{(8-n_0)^k \Gamma^{(k)}(1+n_0)}{k!}}$$

for $((n_0 \notin \mathbb{Z} \text{ or } n_0 \geq 0) \text{ and } n_0 \rightarrow 8)$

$$\begin{aligned}
& (-123 - 47 + 11) + \frac{29 (1 + 5.5451774444790000^8 \log^2(5.5451774444790000))}{(6 + 2)!} = \\
& - \left(\left(159.00000000000000 \left(-0.1823899371069182 - 163\,051.3721258743 \right. \right. \right. \\
& \quad \log^2(4.5451774444790000) + 326\,102.744251749 \\
& \quad \log(4.5451774444790000) \sum_{k=1}^{\infty} \frac{(-1)^k e^{-1.5140667685568842 k}}{k} - \\
& \quad 163\,051.3721258743 \left(\sum_{k=1}^{\infty} \frac{(-1)^k e^{-1.5140667685568842 k}}{k} \right)^2 + \\
& \quad \left. \left. \left. 1.0000000000000000 \sum_{k=0}^{\infty} \frac{(8 - n_0)^k \Gamma^{(k)}(1 + n_0)}{k!} \right) \right) \right) / \\
& \left(\sum_{k=0}^{\infty} \frac{(8 - n_0)^k \Gamma^{(k)}(1 + n_0)}{k!} \right) \text{ for } ((n_0 \notin \mathbb{Z} \text{ or } n_0 \geq 0) \text{ and } n_0 \rightarrow 8)
\end{aligned}$$

$$\begin{aligned}
& (-123 - 47 + 11) + \frac{29 (1 + 5.5451774444790000^8 \log^2(5.5451774444790000))}{(6 + 2)!} = \\
& -159 + \left(29 \left(1 + 893\,971.3161384143 \right. \right. \\
& \quad \left(\log(z_0) + \left\lfloor \frac{\arg(5.5451774444790000 - z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \right. \\
& \quad \left. \left. \left. \sum_{k=1}^{\infty} \frac{(-1)^k (5.5451774444790000 - z_0)^k z_0^{-k}}{k} \right)^2 \right) \right) / \\
& \left(\sum_{k=0}^{\infty} \frac{(8 - n_0)^k \Gamma^{(k)}(1 + n_0)}{k!} \right) \text{ for } ((n_0 \notin \mathbb{Z} \text{ or } n_0 \geq 0) \text{ and } n_0 \rightarrow 8)
\end{aligned}$$

\mathbb{Z} is the set of integers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

Integral representations:

$$\begin{aligned}
& (-123 - 47 + 11) + \frac{29 (1 + 5.5451774444790000^8 \log^2(5.5451774444790000))}{(6 + 2)!} = \\
& - \frac{1}{\int_0^{\infty} e^{-t} t^8 dt} \\
& 159.00000000000000 \left(-0.182389937106918 + 1.000000000000000 \int_0^{\infty} e^{-t} t^8 dt - \right. \\
& \quad \left. 163\,051.372125874 \log^2(5.5451774444790000) \right)
\end{aligned}$$

$$\begin{aligned}
& \bullet \\
& (-123 - 47 + 11) + \frac{29 (1 + 5.5451774444790000^8 \log^2(5.5451774444790000))}{(6 + 2)!} = \\
& - \frac{1}{\int_0^1 \log^8\left(\frac{1}{t}\right) dt} 159.0000000000000 \\
& \quad \left(-0.182389937106918 - 163\,051.372125874 \left(\int_1^{5.5451774444790000} \frac{1}{t} dt \right)^2 + \right. \\
& \quad \left. 1.000000000000000 \int_0^1 \log^8\left(\frac{1}{t}\right) dt \right)
\end{aligned}$$

$$\begin{aligned}
& \bullet \\
& (-123 - 47 + 11) + \frac{29 (1 + 5.5451774444790000^8 \log^2(5.5451774444790000))}{(6 + 2)!} = \\
& - \frac{1}{\int_0^\infty e^{-t} t^8 dt} 159.0000000000000 \\
& \quad \left(-0.182389937106918 - 163\,051.372125874 \left(\int_1^{5.5451774444790000} \frac{1}{t} dt \right)^2 + \right. \\
& \quad \left. 1.000000000000000 \int_0^\infty e^{-t} t^8 dt \right)
\end{aligned}$$

$$\begin{aligned}
& (((((-123-47+11)+29 * 1/(6+2)! * ((1+5.545177444479^8 * \\
& \ln^2(5.545177444479))))))^{1/15}
\end{aligned}$$

Input interpretation:

$$\sqrt[15]{(-123 - 47 + 11) + 29 \times \frac{1}{(6 + 2)!} (1 + 5.545177444479^8 \log^2(5.545177444479))}$$

$n!$ is the factorial function
 $\log(x)$ is the natural logarithm

Result:

- More digits

1.643726439906...

$$1.643726439\dots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934\dots$$

$$\begin{aligned}
& 29/10^3 + (((((-123-47+11)+29 * 1/(6+2)! * ((1+5.545177444479^8 * \\
& \ln^2(5.545177444479))))))^{1/15}
\end{aligned}$$

Input interpretation:

$$\frac{29}{10^3} + \sqrt[15]{(-123 - 47 + 11) + 29 \times \frac{1}{(6 + 2)!} (1 + 5.545177444479^8 \log^2(5.545177444479))}$$

$n!$ is the factorial function

$\log(x)$ is the natural logarithm

Result:

1.672726439906...

1.672726439.... result very near to the proton mass

•

Alternative representations:

$$\begin{aligned} & \frac{29}{10^3} + \sqrt[15]{(-123 - 47 + 11) + \frac{29 (1 + 5.5451774444790000^8 \log^2(5.5451774444790000))}{(6 + 2)!}} \\ &= \frac{29}{10^3} + \sqrt[15]{-159 + \frac{29 (1 + 5.5451774444790000^8 (\log(a) \log_a(5.5451774444790000))^2)}{\Gamma(9)}} \end{aligned}$$

•

$$\begin{aligned} & \frac{29}{10^3} + \sqrt[15]{(-123 - 47 + 11) + \frac{29 (1 + 5.5451774444790000^8 \log^2(5.5451774444790000))}{(6 + 2)!}} \\ &= \frac{29}{10^3} + \sqrt[15]{-159 + \frac{29 (1 + 5.5451774444790000^8 \log^2(5.5451774444790000))}{7!! \times 8!!}} \end{aligned}$$

•

$$\frac{29}{10^3} + \sqrt[15]{(-123 - 47 + 11) + \frac{29(1 + 5.5451774444790000^8 \log^2(5.5451774444790000))}{(6+2)!}}$$

$$= \frac{29}{10^3} + \sqrt[15]{-159 + \frac{29(1 + 5.5451774444790000^8 \log^2(5.5451774444790000))}{(1)_8}}$$

$\Gamma(x)$ is the gamma function

$\log_b(x)$ is the base- b logarithm

$n!!$ is the double factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

• **Series representations:**

$$\frac{29}{10^3} + \left((-123 - 47 + 11) + \frac{29(1 + 5.5451774444790000^8 \log^2(5.5451774444790000))}{(6+2)!} \right)^{(1/15)} =$$

$$\frac{29}{1000} + \left(-159 + (29.00000000000000 + 2.59251681680140 \times 10^7 \log^2(5.5451774444790000)) / \left(\sum_{k=0}^{\infty} \frac{(8-n_0)^k \Gamma^{(k)}(1+n_0)}{k!} \right) \right)^{(1/15)}$$

(1/15) for $(n_0 \notin \mathbb{Z} \text{ or } n_0 \geq 0)$ and $n_0 \rightarrow 8$

•

$$\frac{29}{10^3} + \left((-123 - 47 + 11) + \frac{29 (1 + 5.5451774444790000^8 \log^2(5.5451774444790000))}{(6 + 2)!} \right)^{\wedge (1/15)} = \frac{1}{1000} \left(29 + 1000 \left(-159 + \left(29 \left(1 + 893\,971.3161384143 \left(\log(4.5451774444790000) - \sum_{k=1}^{\infty} \frac{(-1)^k e^{-1.5140667685568842k}}{k} \right)^2 \right) \right) \right) / \left(\sum_{k=0}^{\infty} \frac{(8 - n_0)^k \Gamma^{(k)}(1 + n_0)}{k!} \right) \right)^{\wedge (1/15)}$$

for $(n_0 \notin \mathbb{Z} \text{ or } n_0 \geq 0)$ and $n_0 \rightarrow 8$

$$\frac{29}{10^3} + \left((-123 - 47 + 11) + \frac{29 (1 + 5.5451774444790000^8 \log^2(5.5451774444790000))}{(6 + 2)!} \right)^{\wedge (1/15)} = \frac{1}{1000} \left(29 + 1000 \left(-159 + \left(29 \left(1 + 893\,971.3161384143 \left(2i\pi \left\lfloor \frac{\arg(5.5451774444790000 - x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (5.5451774444790000 - x)^k x^{-k}}{k} \right)^2 \right) \right) / \left(\sum_{k=0}^{\infty} \frac{(8 - n_0)^k \Gamma^{(k)}(1 + n_0)}{k!} \right) \right)^{\wedge (1/15)}$$

for $(n_0 \notin \mathbb{Z} \text{ or } n_0 \geq 0)$ and $x < 0$ and $n_0 \rightarrow 8$

\mathbb{Z} is the set of integers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

Integral representations:

$$\frac{29}{10^3} + \sqrt[15]{(-123 - 47 + 11) + \frac{29(1 + 5.5451774444790000^8 \log^2(5.5451774444790000))}{(6+2)!}}$$

$$= \frac{29}{1000} + \left(-159 + \frac{1}{\int_0^\infty e^{-t} t^8 dt} (29.00000000000000 + 2.59251681680140 \times 10^7 \log^2(5.5451774444790000)) \right)^{(1/15)}$$

$$\frac{29}{10^3} + \sqrt[15]{(-123 - 47 + 11) + \frac{29(1 + 5.5451774444790000^8 \log^2(5.5451774444790000))}{(6+2)!}}$$

$$= \frac{29 + 1000 \sqrt[15]{-159 + \frac{29.00000000000000 + 2.59251681680140 \times 10^7 \left(\int_1^{5.5451774444790000} \frac{1}{t} dt \right)^2}{\int_0^1 \log^8\left(\frac{1}{t}\right) dt}}}{1000}$$

$$\frac{29}{10^3} + \sqrt[15]{(-123 - 47 + 11) + \frac{29(1 + 5.5451774444790000^8 \log^2(5.5451774444790000))}{(6+2)!}}$$

$$= \frac{29 + 1000 \sqrt[15]{-159 + \frac{29.00000000000000 + 2.59251681680140 \times 10^7 \left(\int_1^{5.5451774444790000} \frac{1}{t} dt \right)^2}{\int_0^\infty e^{-t} t^8 dt}}}{1000}$$

$-(47+7-29)/10^3 + (((((-123-47+11)+29 * 1/(6+2)! * ((1+5.545177444479^8 * \ln^2(5.545177444479))))))^{1/15}$

Input interpretation:

$$-\frac{47 + 7 - 29}{10^3} + \sqrt[15]{(-123 - 47 + 11) + 29 \times \frac{1}{(6+2)!} (1 + 5.545177444479^8 \log^2(5.545177444479))}$$

$n!$ is the factorial function

$\log(x)$ is the natural logarithm

Result:

1.618726439906...

1.618726439...

This result is a very good approximation to the value of the golden ratio
1,618033988749...

Alternative representations:

$$\begin{aligned}
 & -\frac{47+7-29}{10^3} + \\
 & \sqrt[15]{(-123-47+11) + \frac{29(1+5.5451774444790000^8 \log^2(5.5451774444790000))}{(6+2)!}} \\
 & = -\frac{25}{10^3} + \\
 & \sqrt[15]{-159 + \frac{29(1+5.5451774444790000^8 (\log(a) \log_a(5.5451774444790000))^2)}{\Gamma(9)}}
 \end{aligned}$$

- $$\begin{aligned}
 & -\frac{47+7-29}{10^3} + \\
 & \sqrt[15]{(-123-47+11) + \frac{29(1+5.5451774444790000^8 \log^2(5.5451774444790000))}{(6+2)!}} \\
 & = -\frac{25}{10^3} + \\
 & \sqrt[15]{-159 + \frac{29(1+5.5451774444790000^8 \log^2(5.5451774444790000))}{7!! \times 8!!}}
 \end{aligned}$$
-

$$\begin{aligned}
& -\frac{47+7-29}{10^3} + \\
& \sqrt[15]{(-123-47+11) + \frac{29(1+5.5451774444790000^8 \log^2(5.5451774444790000))}{(6+2)!}} \\
& = -\frac{25}{10^3} + \\
& \sqrt[15]{-159 + \frac{29(1+5.5451774444790000^8 \log^2(5.5451774444790000))}{(1)_8}}
\end{aligned}$$

$\Gamma(x)$ is the gamma function

$\log_b(x)$ is the base- b logarithm

$n!!$ is the double factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

•

Series representations:

$$\begin{aligned}
& -\frac{47+7-29}{10^3} + \left((-123-47+11) + \frac{29(1+5.5451774444790000^8 \log^2(5.5451774444790000))}{(6+2)!} \right)^{(1/15)} = \\
& -\frac{1}{40} + \left(-159 + (29.00000000000000 + 2.59251681680140 \times 10^7 \right. \\
& \quad \left. \log^2(5.5451774444790000)) / \left(\sum_{k=0}^{\infty} \frac{(8-n_0)^k \Gamma^{(k)}(1+n_0)}{k!} \right) \right)^{(1/15)} \\
& \quad (1/15) \text{ for } ((n_0 \notin \mathbb{Z} \text{ or } n_0 \geq 0) \text{ and } n_0 \rightarrow 8)
\end{aligned}$$

•

$$-\frac{47+7-29}{10^3} + \left((-123-47+11) + \frac{29(1+5.5451774444790000^8 \log^2(5.5451774444790000))}{(6+2)!} \right)^{\wedge (1/15)} =$$

$$\frac{1}{40} \left(-1 + 40 \left(-159 + \left(29 \left(1 + 893971.3161384143 \left(\log(4.5451774444790000) - \sum_{k=1}^{\infty} \frac{(-1)^k e^{-1.5140667685568842k}}{k} \right)^2 \right) \right) \right) / \left(\sum_{k=0}^{\infty} \frac{(8-n_0)^k \Gamma^{(k)}(1+n_0)}{k!} \right)^{\wedge (1/15)} \right)$$

for $(n_0 \notin \mathbb{Z} \text{ or } n_0 \geq 0)$ and $n_0 \rightarrow 8$

$$-\frac{47+7-29}{10^3} + \left((-123-47+11) + \frac{29(1+5.5451774444790000^8 \log^2(5.5451774444790000))}{(6+2)!} \right)^{\wedge (1/15)} =$$

$$\frac{1}{40} \left(-1 + 40 \left(-159 + \left(29 \left(1 + 893971.3161384143 \left(2i\pi \left\lfloor \frac{\arg(5.5451774444790000-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (5.5451774444790000-x)^k x^{-k}}{k} \right)^2 \right) \right) / \left(\sum_{k=0}^{\infty} \frac{(8-n_0)^k \Gamma^{(k)}(1+n_0)}{k!} \right)^{\wedge (1/15)} \right) \right)$$

for $(n_0 \notin \mathbb{Z} \text{ or } n_0 \geq 0)$ and $x < 0$ and $n_0 \rightarrow 8$

\mathbb{Z} is the set of integers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

Integral representations:

$$\begin{aligned}
& -\frac{47+7-29}{10^3} + \\
& \sqrt[15]{(-123-47+11) + \frac{29(1+5.5451774444790000^8 \log^2(5.5451774444790000))}{(6+2)!}} \\
& = -\frac{1}{40} + \left(-159 + \frac{1}{\int_0^\infty e^{-t} t^8 dt} (29.00000000000000 + \right. \\
& \quad \left. 2.59251681680140 \times 10^7 \log^2(5.5451774444790000)) \right)^{(1/15)}
\end{aligned}$$

$$\begin{aligned}
& -\frac{47+7-29}{10^3} + \\
& \sqrt[15]{(-123-47+11) + \frac{29(1+5.5451774444790000^8 \log^2(5.5451774444790000))}{(6+2)!}} \\
& = \frac{1}{40} \left(-1 + 40 \left(-159 + \frac{1}{\int_0^1 \log^8\left(\frac{1}{t}\right) dt} \left(29.00000000000000 + \right. \right. \right. \\
& \quad \left. \left. \left. 2.59251681680140 \times 10^7 \left(\int_1^{5.5451774444790000} \frac{1}{t} dt \right)^2 \right) \right) \right)^{(1/15)}
\end{aligned}$$

$$\begin{aligned}
& -\frac{47+7-29}{10^3} + \\
& \sqrt[15]{(-123-47+11) + \frac{29(1+5.5451774444790000^8 \log^2(5.5451774444790000))}{(6+2)!}} \\
& = \frac{1}{40} \left(-1 + 40 \left(-159 + \frac{1}{\int_0^\infty e^{-t} t^8 dt} \left(29.00000000000000 + \right. \right. \right. \\
& \quad \left. \left. \left. 2.59251681680140 \times 10^7 \left(\int_1^{5.5451774444790000} \frac{1}{t} dt \right)^2 \right) \right) \right)^{(1/15)}
\end{aligned}$$

2sqrt((((6*((((-123-47+11)+29 * 1/(6+2)! * ((1+5.545177444479^8 * ln^2(5.545177444479))))))^1/15))))))

Input interpretation:

$$2 \sqrt[15]{\left(6 \left((-123 - 47 + 11) + \frac{29}{(6+2)!} (1 + 5.545177444479^8 \log^2(5.545177444479)) \right) \right)^{(1/15)}}$$

$n!$ is the factorial function

$\log(x)$ is the natural logarithm

Result:

6.280878486146...

$$6.280878486146\dots = 2\pi r = C$$

Alternative representations:

$$2 \sqrt{\left(6 \left((-123 - 47 + 11) + \frac{29 (1 + 5.5451774444790000^8 \log^2(5.5451774444790000))}{(6 + 2)!} \right)^{1/15} \right)} = 2 \sqrt{6 \sqrt[15]{-159 + \frac{29 (1 + 5.5451774444790000^8 \log^2(5.5451774444790000))}{(1)_8}}}$$

- $$2 \sqrt{\left(6 \left((-123 - 47 + 11) + \frac{29 (1 + 5.5451774444790000^8 \log^2(5.5451774444790000))}{(6 + 2)!} \right)^{1/15} \right)} = 2 \sqrt{6 \sqrt[15]{-159 + \frac{29 (1 + 5.5451774444790000^8 \log^2(5.5451774444790000))}{7!! \times 8!!}}}$$

-

$$2 \sqrt{\left(6 \left(-123 - 47 + 11\right) + \frac{29 \left(1 + 5.5451774444790000^8 \log^2(5.5451774444790000)\right)}{(6 + 2)!}\right)^{\wedge (1/15)}} = 2 \sqrt{6 \sqrt[15]{-159 + \frac{29 \left(1 + 5.5451774444790000^8 \log_e^2(5.5451774444790000)\right)}{(1)_8}}}$$

$(\alpha)_n$ is the Pochhammer symbol (rising factorial)

$n!!$ is the double factorial function

$\log_b(x)$ is the base- b logarithm

• **Series representations:**

$$2 \sqrt{\left(6 \left(-123 - 47 + 11\right) + \frac{29 \left(1 + 5.5451774444790000^8 \log^2(5.5451774444790000)\right)}{(6 + 2)!}\right)^{\wedge (1/15)}} = 2 \exp\left(i \pi \left[\frac{1}{2 \pi} \arg\left(-x + 8.4122284578998498 \left(-\frac{1}{8!}(-0.1823899371069182 + 8! - 163\,051.3721258743 \log^2(5.5451774444790000))\right)^{\wedge (1/15)}\right)\right]\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{1}{k!} (-1)^k x^{-k} \left(-x + 8.4122284578998498 \left(-\frac{1}{8!}(-0.1823899371069182 + 8! - 163\,051.3721258743 \log^2(5.5451774444790000))\right)^{\wedge (1/15)}\right)^k \left(-\frac{1}{2}\right)_k \text{ for } (x \in \mathbb{R} \text{ and } x < 0)$$

$$2 \sqrt{\left(6 \left(-123 - 47 + 11\right) + \frac{29 \left(1 + 5.5451774444790000^8 \log^2 \left(5.5451774444790000\right)\right)}{\left(6 + 2\right)!}\right)^{\wedge \left(1 / 15\right)} = 2 \left(\frac{1}{z_0}\right)^{1/2} \left[\arg \left(8.4122284578998498 \sqrt[15]{-\frac{-0.1823899371069182+8!-163051.3721258743 \log^2 \left(5.5451774444790000\right)}{8!}} - z_0\right) / (2 \pi)\right]$$

$$1/2 \left(1 + \arg \left(8.4122284578998498 \sqrt[15]{-\frac{-0.1823899371069182+8!-163051.3721258743 \log^2 \left(5.5451774444790000\right)}{8!}} - z_0\right) / (2 \pi)\right)$$

$$\sum_{k=0}^{z_0} \frac{1}{k!} (-1)^k \left(-\frac{1}{2}\right)_k \left(8.4122284578998498 \left(-\frac{1}{8!} (-0.1823899371069182 + 8! - 163051.3721258743 \log^2 (5.5451774444790000))\right)^{\wedge (1 / 15) - z_0}\right)^k z_0^{-k}$$

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

\mathbb{R} is the set of real numbers

Integral representations:

$$2 \sqrt{\left(6 \left(-123 - 47 + 11\right) + \frac{29 \left(1 + 5.5451774444790000^8 \log^2 \left(5.5451774444790000\right)\right)}{\left(6 + 2\right)!}\right)^{\wedge \left(1 / 15\right)} = 2 \sqrt{\left(6 \left(-159 + \frac{1}{\int_0^\infty e^{-t} t^8 dt} (29.00000000000000 + 2.59251681680140 \times 10^7 \log^2 \left(5.5451774444790000\right)\right)\right)^{\wedge \left(1 / 15\right)}\right)}$$

•

$$2 \sqrt{\left(6 \left(-123 - 47 + 11\right) + \frac{29 \left(1 + 5.5451774444790000^8 \log^2(5.5451774444790000)\right)}{(6+2)!}\right)}^{(1/15)} = 2 \sqrt{\left(6 \left(-159 + \frac{1}{\int_0^1 \log^8\left(\frac{1}{t}\right) dt}\left(29.00000000000000 + 2.59251681680140 \times 10^7 \left(\int_1^{5.5451774444790000} \frac{1}{t} dt\right)^2\right)\right)\right)}^{(1/15)}$$

$$2 \sqrt{\left(6 \left(-123 - 47 + 11\right) + \frac{29 \left(1 + 5.5451774444790000^8 \log^2(5.5451774444790000)\right)}{(6+2)!}\right)}^{(1/15)} = 2 \sqrt{\left(6 \left(-159 + \frac{1}{\int_0^\infty e^{-t} t^8 dt}\left(29.00000000000000 + 2.59251681680140 \times 10^7 \left(\int_1^{5.5451774444790000} \frac{1}{t} dt\right)^2\right)\right)\right)}^{(1/15)}$$

We have:

$$1/(2\pi) * 2\sqrt{\left(\left(\left(\left(\left(\left(-123-47+11\right)+29 * 1/\left(6+2\right)! * \left(\left(1+5.545177444479^8 * \ln^2\left(5.545177444479\right)\right)\right)\right)\right)\right)\right)\right)^{1/15}})$$

Input interpretation:

$$\frac{1}{2\pi} \times 2 \sqrt{\left(6 \left(-123 - 47 + 11\right) + 29 \times \frac{1}{(6+2)!} \left(1 + 5.545177444479^8 \log^2(5.545177444479)\right)\right)}^{(1/15)}$$

$n!$ is the factorial function

$\log(x)$ is the natural logarithm

Result:

0.9996328580297...

0.9996328580297...

Alternative representations:

$$\frac{2 \sqrt{6^{15} \sqrt{(-123 - 47 + 11) + \frac{2^9 (1 + 5.5451774444790000)^8 \log^2(5.5451774444790000)}}{(6+2)!}}}{2 \pi} = \frac{2 \sqrt{6^{15} \sqrt{-159 + \frac{2^9 (1 + 5.5451774444790000)^8 \log^2(5.5451774444790000)}}{(1)_8}}}{2 \pi}$$

- $$\frac{2 \sqrt{6^{15} \sqrt{(-123 - 47 + 11) + \frac{2^9 (1 + 5.5451774444790000)^8 \log^2(5.5451774444790000)}}{(6+2)!}}}{2 \pi} = \frac{2 \sqrt{6^{15} \sqrt{-159 + \frac{2^9 (1 + 5.5451774444790000)^8 \log^2(5.5451774444790000)}}{7!! \times 8!!}}}{2 \pi}$$

- $$\frac{2 \sqrt{6^{15} \sqrt{(-123 - 47 + 11) + \frac{2^9 (1 + 5.5451774444790000)^8 \log^2(5.5451774444790000)}}{(6+2)!}}}{2 \pi} = \frac{2 \sqrt{6^{15} \sqrt{-159 + \frac{2^9 (1 + 5.5451774444790000)^8 \log_e^2(5.5451774444790000)}}{(1)_8}}}{2 \pi}$$

$(a)_n$ is the Pochhammer symbol (rising factorial)

$n!!$ is the double factorial function

$\log_b(x)$ is the base- b logarithm

- Series representations:**

$$\begin{aligned}
& \frac{2 \sqrt{6^{15} \sqrt{(-123 - 47 + 11) + \frac{29(1+5.5451774444790000)^8 \log^2(5.5451774444790000)}{(6+2)!}}}}{2 \pi} = \\
& \frac{1}{\pi} \exp\left(i \pi \left[\frac{1}{2 \pi} \arg\left(-x + 8.4122284578998498 \right. \right. \right. \\
& \quad \left. \left. \left. \left(-\frac{1}{8!}(-0.1823899371069182 + 8! - 163\,051.3721258743 \log^2(5.5451774444790000))\right) \wedge (1/15)\right)\right]\right) \sqrt{x} \\
& \sum_{k=0}^{\infty} \frac{1}{k!} (-1)^k x^{-k} \left(-x + 8.4122284578998498 \left(-\frac{1}{8!}(-0.1823899371069182 + 8! - \right. \right. \right. \\
& \quad \left. \left. \left. 163\,051.3721258743 \log^2(5.5451774444790000))\right) \wedge \right. \\
& \quad \left. (1/15)\right)^k \left(-\frac{1}{2}\right)_k \text{ for } (x \in \mathbb{R} \text{ and } x < 0)
\end{aligned}$$

$$\begin{aligned}
& \frac{2 \sqrt{6^{15} \sqrt{(-123 - 47 + 11) + \frac{29(1+5.5451774444790000)^8 \log^2(5.5451774444790000)}{(6+2)!}}}}{2 \pi} = \\
& \frac{1}{\pi} \left(\frac{1}{z_0}\right) \left[\frac{1}{2} \left| \arg \left(6^{15} \sqrt{-159 + \frac{29(1+893971.3161384143 \log^2(5.5451774444790000))}{8!}} - z_0 \right) / (2 \pi) \right| \right] \\
& \frac{1}{2} \left(1 + \left| \arg \left(6^{15} \sqrt{-159 + \frac{29(1+893971.3161384143 \log^2(5.5451774444790000))}{8!}} - z_0 \right) / (2 \pi) \right| \right) \\
& \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(6^{15} \sqrt{-159 + \frac{29(1+893971.3161384143 \log^2(5.5451774444790000))}{8!}} - z_0 \right)^k}{k!} z_0^{-k}
\end{aligned}$$

$$2 \sqrt{6^{15} \sqrt{(-123 - 47 + 11) + \frac{2^9(1+5.5451774444790000)^8 \log^2(5.5451774444790000)}{(6+2)!}}} = \frac{1}{\pi}$$

$$\left(\frac{1}{z_0}\right)^{1/2} \left[\arg \left(8.4122284578998498 \sqrt[15]{-\frac{-0.1823899371069182+8!-163051.3721258743 \log^2(5.5451774444790000)}{8!}} - z_0 \right) / (2\pi) \right]$$

$$1/2 \left(1 + \arg \left(8.4122284578998498 \sqrt[15]{-\frac{-0.1823899371069182+8!-163051.3721258743 \log^2(5.5451774444790000)}{8!}} - z_0 \right) / (2\pi) \right)$$

$$\sum_{k=0}^{\infty} \frac{1}{k!} (-1)^k \left(-\frac{1}{2}\right)_k \left(8.4122284578998498 \left(-\frac{1}{8!}(-0.1823899371069182 + 8! - 163051.3721258743 \log^2(5.5451774444790000))\right)^{(1/15) - z_0} z_0^{-k} \right)$$

$\arg(z)$ is the complex argument

$[x]$ is the floor function

i is the imaginary unit

\mathbb{R} is the set of real numbers

Integral representations:

$$2 \sqrt{6^{15} \sqrt{(-123 - 47 + 11) + \frac{2^9(1+5.5451774444790000)^8 \log^2(5.5451774444790000)}{(6+2)!}}} = \frac{2\pi}{\sqrt{6^{15} \sqrt{-159 + \frac{2^9(1+5.5451774444790000)^8 \log^2(5.5451774444790000)}{\int_0^{\infty} e^{-t} t^8 dt}}}}$$

$$\frac{2 \sqrt{6^{15} \sqrt{(-123 - 47 + 11) + \frac{29(1+5.5451774444790000)^8 \log^2(5.5451774444790000)}}{(6+2)!}}}{2 \pi} = \frac{\sqrt{6^{15} \sqrt{-159 + \frac{29.000000000000000+2.59251681680140 \times 10^7 \left(\int_1^{5.5451774444790000} \frac{1}{t} dt\right)^2}}{\int_0^1 \log^8\left(\frac{1}{t}\right) dt}}}{\pi}$$

$$\frac{2 \sqrt{6^{15} \sqrt{(-123 - 47 + 11) + \frac{29(1+5.5451774444790000)^8 \log^2(5.5451774444790000)}}{(6+2)!}}}{2 \pi} = \frac{\sqrt{6^{15} \sqrt{-159 + \frac{29.000000000000000+2.59251681680140 \times 10^7 \left(\int_1^{5.5451774444790000} \frac{1}{t} dt\right)^2}}{\int_0^\infty e^{-t} t^8 dt}}}{\pi}$$

This result 0.9996328580297... is very near to the value of the following Rogers-Ramanujan continued fraction

$$\frac{\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}}}{1 + \sqrt[5]{\sqrt{\varphi^5 \sqrt[4]{5^3} - 1}} - \varphi + 1} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}} \approx 0.9991104684$$

And:

$$(2\pi) / ((((((2\sqrt{((((6*((((-123-47+11))+29 * 1/(6+2)! * ((1+5.545177444479^8 * \ln^2(5.545177444479))))))^{1/15})^4))))))))$$

Input interpretation:

$$\frac{2 \sqrt{6^{15} \sqrt{(-123 - 47 + 11) + 29 \times \frac{1}{(6+2)!} (1 + 5.545177444479^8 \log^2(5.545177444479))}}}{2 \pi}$$

$n!$ is the factorial function

$\log(x)$ is the natural logarithm

Result:

1.000367276813...

1.000367276813 = radius

Alternative representations:

$$\frac{2\pi}{2\sqrt{6^{15}\sqrt{(-123-47+11)+\frac{29(1+5.5451774444790000)^8\log^2(5.5451774444790000)}{(6+2)!}}}} = \frac{2\pi}{2\sqrt{6^{15}\sqrt{-159+\frac{29(1+5.5451774444790000)^8\log^2(5.5451774444790000)}{(1)_8}}}}$$

- $$\frac{2\pi}{2\sqrt{6^{15}\sqrt{(-123-47+11)+\frac{29(1+5.5451774444790000)^8\log^2(5.5451774444790000)}{(6+2)!}}}} = \frac{2\pi}{2\sqrt{6^{15}\sqrt{-159+\frac{29(1+5.5451774444790000)^8\log^2(5.5451774444790000)}{7!!\times 8!!}}}}$$

- $$\frac{2\pi}{2\sqrt{6^{15}\sqrt{(-123-47+11)+\frac{29(1+5.5451774444790000)^8\log^2(5.5451774444790000)}{(6+2)!}}}} = \frac{2\pi}{2\sqrt{6^{15}\sqrt{-159+\frac{29(1+5.5451774444790000)^8\log_b^2(5.5451774444790000)}{(1)_8}}}}$$

$(a)_n$ is the Pochhammer symbol (rising factorial)

$n!!$ is the double factorial function

$\log_b(x)$ is the base- b logarithm

Series representations:

$$\begin{aligned}
 & \frac{2\pi}{2\sqrt{6^{15}\sqrt{(-123-47+11)+\frac{29(1+5.5451774444790000^8\log^2(5.5451774444790000))}{(6+2)!}}}} = \\
 & \pi / \left(\exp\left(i\pi\left[\frac{1}{2\pi}\arg\left(-x+8.4122284578998498\right.\right.\right.\right. \\
 & \quad \left.\left.\left.\left(-\frac{1}{8!}(-0.1823899371069182+8!-163051.3721258743\right.\right.\right.\right. \\
 & \quad \quad \left.\left.\left.\left.\log^2(5.5451774444790000)\right)\right)\right]^{\wedge}(1/15)\right)\right) \\
 & \sqrt{x} \sum_{k=0}^{\infty} \frac{1}{k!} (-1)^k x^{-k} \left(-x+8.4122284578998498\right. \\
 & \quad \left(-\frac{1}{8!}(-0.1823899371069182+8!-163051.3721258743\right. \\
 & \quad \quad \left.\log^2(5.5451774444790000)\right)\right)^{\wedge} \\
 & \quad \left.(1/15)\right)^k \left(-\frac{1}{2}\right)_k \text{ for } (x \in \mathbb{R} \text{ and } x < 0)
 \end{aligned}$$

$$\begin{aligned}
 & \frac{2\pi}{2\sqrt{6^{15}\sqrt{(-123-47+11)+\frac{29(1+5.5451774444790000^8\log^2(5.5451774444790000))}{(6+2)!}}}} = \\
 & \left(\pi \left(\frac{1}{z_0}\right)^{-1/2} \left[\arg\left(6^{15}\sqrt{-159+\frac{29(1+893971.3161384143\log^2(5.5451774444790000))}{8!}}\right) - z_0 \right] / (2\pi) \right) \\
 & \left. z_0 \left(\left[-1 - \arg\left(6^{15}\sqrt{-159+\frac{29(1+893971.3161384143\log^2(5.5451774444790000))}{8!}}\right) - z_0 \right] / (2\pi) \right) \right) \\
 & \left(\sum_{k=0}^{\infty} \frac{1}{k!} (-1)^k \left(-\frac{1}{2}\right)_k \right. \\
 & \quad \left. \left(6\left(-159+\frac{29(1+893971.3161384143\log^2(5.5451774444790000))}{8!}\right)\right)^{\wedge} \right. \\
 & \quad \left. (1/15)-z_0\right)^k z_0^{-k}
 \end{aligned}$$

$$2 \sqrt{6}^{15} \sqrt{(-123 - 47 + 11) + \frac{2^9(1+5.5451774444790000)^8 \log^2(5.5451774444790000)}{(6+2)!}} = \left[\begin{aligned} & \frac{2\pi}{\left(\frac{1}{z_0}\right)^{-1/2} \left[\arg \left(8.4122284578998498 \sqrt[15]{-\frac{-0.1823899371069182+8!-163051.3721258743 \log^2(5.5451774444790000)}{8!}} - z_0 \right) \right] / (2\pi)} \\ & \frac{1/2 \left[-1 - \arg \left(8.4122284578998498 \sqrt[15]{-\frac{-0.1823899371069182+8!-163051.3721258743 \log^2(5.5451774444790000)}{8!}} - z_0 \right) \right] / (2\pi)}{z_0} \\ & \left. \frac{1}{\left(\sum_{k=0}^{\infty} \frac{1}{k!} (-1)^k \left(-\frac{1}{2}\right)_k \left(8.4122284578998498 \right. \right. \right.} \right. \\ & \left. \left. \left. \left(-\frac{1}{8!} (-0.1823899371069182 + 8! - 163051.3721258743 \right. \right. \right. \right. \\ & \left. \left. \left. \left. \log^2(5.5451774444790000) \right) \right)^k (1/15 - z_0)^k z_0^{-k} \right) \right] \right] \end{aligned} \right] \pi$$

$\arg(z)$ is the complex argument

$[x]$ is the floor function

i is the imaginary unit

\mathbb{R} is the set of real numbers

Integral representations:

$$2 \sqrt{6}^{15} \sqrt{(-123 - 47 + 11) + \frac{2^9(1+5.5451774444790000)^8 \log^2(5.5451774444790000)}{(6+2)!}} = \frac{2\pi}{\pi}$$

$$\sqrt{6}^{15} \sqrt{-159 + \frac{2^9.000000000000000+2.59251681680140 \times 10^7 \log^2(5.5451774444790000)}{\int_0^{\infty} e^{-t} t^8 dt}}$$

•

$$\begin{aligned}
& \frac{2\pi}{\sqrt{6^{15} \sqrt{(-123 - 47 + 11) + \frac{29(1+5.5451774444790000)^8 \log^2(5.5451774444790000)}{(6+2)!}}}} = \\
& \sqrt{6^{15} \sqrt{-159 + \frac{29.000000000000000+2.59251681680140 \times 10^7 \left(\int_1^{5.5451774444790000} \frac{1}{t} dt\right)^2}{\int_0^1 \log^8\left(\frac{1}{t}\right) dt}}}
\end{aligned}$$

$$\begin{aligned}
& \frac{2\pi}{\sqrt{6^{15} \sqrt{(-123 - 47 + 11) + \frac{29(1+5.5451774444790000)^8 \log^2(5.5451774444790000)}{(6+2)!}}}} = \\
& \sqrt{6^{15} \sqrt{-159 + \frac{29.000000000000000+2.59251681680140 \times 10^7 \left(\int_1^{5.5451774444790000} \frac{1}{t} dt\right)^2}{\int_0^\infty e^{-t} t^8 dt}}}
\end{aligned}$$

We note that the result 1.000367276813, that is a radius, is very near to

$$\zeta(12) = 1 + \frac{1}{2^{12}} + \frac{1}{3^{12}} + \dots = \frac{691\pi^{12}}{638512875} = 1.000246\dots$$

Thence, we have the following mathematical connection with the Dirichlet conditions concerning the D-branes:

$$\zeta(12) = 1 + \frac{1}{2^{12}} + \frac{1}{3^{12}} + \dots = \frac{691\pi^{12}}{638512875} = 1.000246\dots$$

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

$$\frac{1 + \sqrt[5]{\sqrt{\varphi^5 4 \sqrt{5^3}} - 1}}{\sqrt{5}} - \varphi + 1$$

$$\int_0^\pi d\sigma \partial_\sigma X^{r25} = i \int_0^\pi d\sigma \partial_\tau X^{25} = 2\pi\alpha' p^{25} = \frac{2\pi\alpha' n}{R} = 2\pi n R'.$$

Example:

$$\frac{2\pi}{2\sqrt{6^{15}\sqrt{(-123-47+11)+29\times\frac{1}{(6+2)!}(1+5.545177444479^8\log^2(5.545177444479))}}}$$

$$= 1.000367276813... \Rightarrow$$

$$\Rightarrow \zeta(12) = 1 + \frac{1}{2^{12}} + \frac{1}{3^{12}} + \dots = \frac{691\pi^{12}}{638512875} = 1.000246...$$

$$1/\zeta(12) = 1 / 1.000246... = 0,999754060501 \Rightarrow$$

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}}$$

$$\frac{1 + \sqrt[5]{\sqrt{\varphi^5 4\sqrt{5^3} - 1}}}{\sqrt{5}} - \varphi + 1 \approx 0.9991104684$$

$$\Rightarrow$$

$$\Rightarrow \int_0^\pi d\sigma \partial_\sigma X^{r25} = i \int_0^\pi d\sigma \partial_\tau X^{25} = 2\pi\alpha' p^{25} = \frac{2\pi\alpha' n}{R} = 2\pi n R'.$$

References

Berndt, B. et al. "*The Rogers–Ramanujan Continued Fraction*",

<http://www.math.uiuc.edu/~berndt/articles/rrcf.pdf>

Berndt, B. et al. "*The Rogers–Ramanujan Continued Fraction*"