Multi-photon processes in double-inverted Y system

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Abstract
In this article, the effect of multi-photon processes, occurring via different probe beam channels, on the absorption of probe beams in double-inverted Y system is investigated and the results are reported.

1 Introduction
Generation of atomic coherence, in multi-photon processes, between atomic energy levels which are not directly coupled, modifies the absorption and dispersion of a weak probe beam which is determined by the first order atomic susceptibility $\chi^{(1)}$. For example, in three level atomic systems, the resonant absorption of a weak probe beam, coupling the ground level and an excited level, is reduced due to the presence of a strong control beam coupling the excited level and a third atomic level. This reduction in resonant absorption, termed electromagnetically induced transparency (EIT), is due to the two-photon coherence generated between the ground level and third atomic level. EIT has wide applications such as ultraslow light propagation, optical switching, quantum memory etc., Adding another strong control beam that couples the third atomic level to a fourth level, creates a three-photon coherence between the ground level and fourth atomic level which gives rise to the phenomenon EITA in which the reduction in absorption due to EIT is reduced.

EIT systems, such as multi-Λ and multi-Ξ EIT systems, were reported to be suitable for frequency conversion, generation of entangled photons and entanglement detection. In these systems, the ground level is coupled to $n$ intermediate levels by $n$ probe beams and another energy level is coupled to the same $n$ intermediate levels by $n$ control beams. A probe beam transition, in such a system, is not only affected by EIT and EITA processes but also by the gain due to non-linear $\chi^{(3)}$ process occurring via other probe beam channels. Thus there is a cross-talk between different probe transitions. In double-Ξ system, a large gain due to $\chi^{(3)}$ process was predicted and experimentally verified, the cross-talk between two probe transitions was shown to be controlled by the interplay between $\chi^{(1)}$ and $\chi^{(3)}$ processes and that system was shown to be suitable for producing slow-fast light pairs.

In this paper, the effect of EIT, EITA and gain due to $\chi^{(3)}$ process on the absorption of two weak probe beams in a double-inverted Y system is investigated
using density matrix formalism. This system can also be viewed as double-Λ + double-Ξ system and it shows rich variations in the probe beam absorptions for different choices of control beam Rabi frequencies.

2 Double-inverted Y system

A five level double-inverted Y system is shown in Figure 1. It consists of five atomic energy levels and six transitions between them. The five atomic levels, with energies in increasing order, are labelled as states |0⟩, |1⟩, |2⟩, |3⟩ and |4⟩ respectively. The transitions |0⟩ → |2⟩ and |0⟩ → |3⟩ are coupled by two weak probe beams of angular frequencies ω_{p_1} and ω_{p_2}, and of Rabi frequencies Ω_{p_1} and Ω_{p_2}, respectively. Similarly, the transitions |1⟩ → |2⟩, |1⟩ → |3⟩, |2⟩ → |4⟩ and |3⟩ → |4⟩ are coupled by four strong control beams of angular frequencies ω_{c_1}, ω_{c_2}, ω_{c_3} and ω_{c_4}, and of Rabi frequencies Ω_{c_1}, Ω_{c_2}, Ω_{c_3} and Ω_{c_4} respectively. The detunings of probe and control beams are given as follows.

\[ \Delta_{p_j} = \omega_{p_j} - \left( \frac{\epsilon_{j+1} - \epsilon_0}{\hbar} \right), \]

\[ \Delta_{c_j} = \omega_{c_j} - \left( \frac{\epsilon_{j+1} - \epsilon_1}{\hbar} \right). \]
\[ \Delta_{cj+2} = \omega_{cj+2} - \left( \frac{\epsilon_j - \epsilon_{j+1}}{\hbar} \right), \quad (j = 1, 2). \]  \hspace{1cm} (1)

where \( \epsilon_k \) is the energy of the state \( |k\rangle \).

This system can also be viewed as consisting of double-\( \Lambda \) + double-\( \Xi \) configuration. This five level double-inverted \( Y \) configuration can be realized in \( ^{87}\text{Rb} \) by choosing the hyperfine levels \( |F = 1, 2\rangle \) of \( ^{7}\text{Li} \) (\( |F' = 1\rangle \) of \( ^{5}\text{P}_{1/2} \), \( |F'' = 1\rangle \) of \( ^{5}\text{P}_{3/2} \) and \( |F''' = 1\rangle \) of \( ^{6}\text{S}_{1/2} \) as states \( |0\rangle \), \( |1\rangle \), \( |2\rangle \), \( |3\rangle \) and \( |4\rangle \) respectively.

### 3 Steady state solutions

In the interaction picture, the Hamiltonian of the system is written as

\[ \hat{H}_I = -\frac{\hbar}{2} \sum_{j=1}^{2} \left[ \Omega_{p_j} e^{-i\Delta_{p_j} t} |j+1\rangle \langle 0| + \Omega_{c_j} e^{-i\Delta_{c_j} t} |j+1\rangle \langle 1| + \Omega_{p_{j+1}} e^{-i\Delta_{p_{j+1}} t} |4\rangle \langle j+1| + h.c. \right]. \]  \hspace{1cm} (2)

The equation of motion for the atomic system is given by

\[ \frac{d\hat{\rho}}{dt} = -\frac{i}{\hbar} [\hat{H}_I, \hat{\rho}] + \text{decay terms}, \]  \hspace{1cm} (3)

where \( \hat{\rho} = [\rho_{ij}] \) is the density matrix of the atomic system.

Time evolution of the coherence terms are given by

\[ \frac{d\rho'_{10}}{dt} = -\gamma_{10} \rho'_{10} + \frac{i}{2} \left[ \Omega_{c_1} \rho'_{20} + \Omega_{c_2} \rho'_{30} - \Omega_{p_1} \rho'_{12} - \Omega_{p_2} \rho'_{13} \right], \]  \hspace{1cm} (4)

\[ \frac{d\rho'_{20}}{dt} = -\gamma_{20} \rho'_{20} + \frac{i}{2} \left[ \Omega_{p_1} (\rho'_{00} - \rho'_{22}) + \Omega_{c_1} \rho'_{10} + \Omega_{c_2} \rho'_{40} - \Omega_{p_2} \rho'_{23} \right], \]  \hspace{1cm} (5)

\[ \frac{d\rho'_{30}}{dt} = -\gamma_{30} \rho'_{30} + \frac{i}{2} \left[ \Omega_{p_2} (\rho'_{00} - \rho'_{33}) + \Omega_{c_1} \rho'_{10} + \Omega_{c_2} \rho'_{40} - \Omega_{p_1} \rho'_{32} \right], \]  \hspace{1cm} (6)

\[ \frac{d\rho'_{40}}{dt} = -\gamma_{40} \rho'_{40} + \frac{i}{2} \left[ \Omega_{c_2} \rho'_{20} + \Omega_{c_1} \rho'_{30} - \Omega_{p_1} \rho'_{42} - \Omega_{p_2} \rho'_{43} \right], \]  \hspace{1cm} (7)

where,

\[ \rho'_{00} = \rho_{00}, \]

\[ \rho'_{22} = \rho_{22}, \]

\[ \rho'_{33} = \rho_{33}, \]

\[ \rho'_{20} = \rho_{20} e^{i\Delta_{p_1} t}, \]

\[ \rho'_{30} = \rho_{30} e^{i\Delta_{p_2} t}, \]

\[ \rho'_{20} = \rho_{20} e^{i(\Delta_{p_1} - \Delta_{c_2} + \Delta_{s_1}) t}, \]

\[ \rho'_{30} = \rho_{30} e^{i(\Delta_{p_2} + \Delta_{c_2} - \Delta_{s_1}) t}, \]

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\[ \rho'_{10} = \rho_{10} e^{i(\Delta_{p_1} - \Delta_{c_1})t}, \]
\[ \rho'_{40} = \rho_{40} e^{i(\Delta_{p_1} + \Delta_{c_3})t}, \]
\[ \rho'_{12} = \rho_{12} e^{-i\Delta_{c_1}t}, \]
\[ \rho'_{13} = \rho_{13} e^{-i\Delta_{c_2}t}, \]
\[ \rho'_{42} = \rho_{42} e^{i\Delta_{c_3}t}, \]
\[ \rho'_{43} = \rho_{43} e^{i\Delta_{c_4}t}, \]
\[ \rho'_{32} = \rho_{32} e^{i(\Delta_{p_2} - \Delta_{p_1})t}, \] (8)

and
\[ \tilde{\gamma}_{10} = \left( \frac{\Gamma_0 + \Gamma_1}{2} \right) - i(\Delta_{p_1} - \Delta_{c_1}) = \gamma_{10} - i(\Delta_{p_1} - \Delta_{c_1}), \]
\[ \tilde{\gamma}_{20} = \left( \frac{\Gamma_0 + \Gamma_2}{2} \right) - i\Delta_{p_1} = \gamma_{20} - i\Delta_{p_1}, \]
\[ \tilde{\gamma}_{30} = \left( \frac{\Gamma_0 + \Gamma_3}{2} \right) - i(\Delta_{p_1} - \delta) = \gamma_{30} - i(\Delta_{p_1} - \delta), \]
\[ \tilde{\gamma}_{40} = \left( \frac{\Gamma_0 + \Gamma_4}{2} \right) - i(\Delta_{p_1} + \Delta_{c_3}) = \gamma_{40} - i(\Delta_{p_1} + \Delta_{c_3}), \] (9)

with \( \Gamma_k \) representing the decay rate of the state \( |k\rangle \). In writing above equations, the detunings are assumed to satisfy the conditions: \((\Delta_{p_1} - \Delta_{c_1}) = (\Delta_{p_2} - \Delta_{c_2})\), \((\Delta_{p_1} + \Delta_{c_3}) = (\Delta_{p_2} + \Delta_{c_4})\) and \((\Delta_{p_1} - \Delta_{p_2}) = \delta\).

Under weak probe beam approximation, assuming \( \rho'_{00} \approx 1, \rho'_{ij} \approx 0 \) for \( j = 1, 2, 3 \) & 4 keeping the terms which are first-order in probe beam Rabi frequencies, the steady state solutions for \( \rho'_{20} \) and \( \rho'_{30} \) are obtained as follows:
\[ \rho'_{20} = \Omega_{p_1} \Lambda^{(1)}_{20} + \Omega_{p_2} \Lambda^{(3)}_{20}, \] (10)
\[ \rho'_{30} = \Omega_{p_2} \Lambda^{(1)}_{30} + \Omega_{p_1} \Lambda^{(3)}_{30}, \] (11)

where,
\[ \Lambda^{(1)}_{20} = \frac{i}{2\tilde{\gamma}_{20} F_1} \left[ 1 + \frac{|\Omega_{c_1}|^2}{4\tilde{\gamma}_{30}\tilde{\gamma}_{10}} + \frac{|\Omega_{c_3}|^2}{4\tilde{\gamma}_{30}\tilde{\gamma}_{40}} \right], \] (12)
\[ \Lambda^{(3)}_{20} = -\frac{i}{2\tilde{\gamma}_{20} F_1} \left[ \Omega_{c_2}^{*}\Omega_{c_1} + \frac{\Omega_{c_3}^{*}\Omega_{c_4}}{4\tilde{\gamma}_{30}\tilde{\gamma}_{40}} \right], \] (13)
\[ \Lambda^{(1)}_{30} = \frac{i}{2\tilde{\gamma}_{30} F_1} \left[ 1 + \frac{|\Omega_{c_1}|^2}{4\tilde{\gamma}_{20}\tilde{\gamma}_{10}} + \frac{|\Omega_{c_3}|^2}{4\tilde{\gamma}_{20}\tilde{\gamma}_{40}} \right], \] (14)
\[ \Lambda_{30}^{(3)} = -\frac{i}{2\gamma_{30}} F_1 \left[ \frac{\Omega_{c1}^* \Omega_{c2} + \Omega_{c1} \Omega_{c2}^*}{\gamma_{20}^2 \gamma_{10}} \right], \]  

with

\[
F_1 = 1 + \frac{\Omega_{c1}^2}{\gamma_{20}^2 \gamma_{10}} + \frac{\Omega_{c2}^2}{\gamma_{20}^2 \gamma_{40}} + \frac{\Omega_{c3}^2}{\gamma_{30}^2 \gamma_{10}} + \frac{\Omega_{c4}^2}{\gamma_{30}^2 \gamma_{40}} + \frac{\Omega_{c1}^* \Omega_{c3}^* - \Omega_{c2}^* \Omega_{c3}}{16\gamma_{20}^2 \gamma_{10}^2 \gamma_{40}^2}.
\]

Taking \(\rho_{20}^{(1)} = \Omega_{p1} \Lambda_{20}^{(1)}\), \(\rho_{20}^{(3)} = \Omega_{p2} \Lambda_{20}^{(3)}\), \(\rho_{30}^{(1)} = \Omega_{p2} \Lambda_{30}^{(1)}\) and \(\rho_{30}^{(3)} = \Omega_{p1} \Lambda_{30}^{(3)}\), eqs. (10) and (11) can be written as

\[
\rho_{20}^{'} = \rho_{20}^{(1)} + \rho_{20}^{(3)}
\]

\[
\rho_{30}^{'} = \rho_{30}^{(1)} + \rho_{30}^{(3)}
\]

The imaginary part of \(\rho_{20}^{(1)}\) describes the absorption of probe beam coupling the transition \(|0\rangle \rightarrow |2\rangle\) and its modification by EIT and EITA processes. The imaginary part of \(\rho_{20}^{(3)}\) describes the gain due to \(\chi^{(3)}\) process occurring via other probe beam channel. Both of them together describes the net absorption of the probe beam. When the second probe beam is not present (\(\Omega_{p2} = 0\)), the gain part becomes zero and imaginary part of \(\rho_{20}^{(1)}\) alone describes the absorption process. The imaginary parts of \(\rho_{30}^{(1)}\) and \(\rho_{30}^{(3)}\) describe the same for the probe beam coupling the transition \(|0\rangle \rightarrow |3\rangle\).

### 4 Results

The effect of EIT, EITA and gain on the absorption of probe beams in the five level double-inverted Y system is analyzed by plotting the imaginary parts of \(\rho_{20}^{(1)}\), \(\rho_{20}^{(3)}\), \(\rho_{20}^{'}\), \(\rho_{30}^{(1)}\), \(\rho_{30}^{(3)}\) and \(\rho_{30}^{'}\) as functions of probe beam detuning \(\Delta_{p1}\). They are shown in Figures 2 - 4. From eqs. (9), the variation of \(\Delta_{p1}\) indicates the simultaneous frequency scanning of both probe beams. However, the condition, \(\Delta_{p1} = \Delta_{p2} = \delta\), implies that the single-photon resonance conditions for both probe beams are satisfied at two different points with separation \(\delta\). For all analysis, the values of decay rates, detunings and probe beam Rabi frequencies are kept fixed as given in the following: the decay rates are assumed as \(\Gamma_0 = \Gamma_1 = 2\pi \times 0.001\, \text{MHz}\), \(\Gamma_2 = 2\pi \times 5.75\, \text{MHz}\), \(\Gamma_3 = 2\pi \times 6.07\, \text{MHz}\) and \(\Gamma_4 = 2\pi \times 3.5\, \text{MHz}\); the detunings of all control beams and the value of \(\delta\) are assumed to be zero; the Rabi frequencies of probe beams are taken as \(\Omega_{p1} = 0.001\gamma_{20}\) and \(\Omega_{p2} = 0.001\gamma_{30}\). In all the graphs, the blue (red) curves are for the probe beam coupling the transition \(|0\rangle \rightarrow |2\rangle\) (\(|0\rangle \rightarrow |3\rangle\))

### 5 Conclusion

In this paper, the effect of EIT, EITA and gain due to \(\chi^{(3)}\) process occurring via other probe beam channel, on the absorption of probe beams in double-inverted Y system is investigated using density matrix formalism. The results suggest that this system shows lot of variations in the probe beam absorptions for various choices of control beam Rabi frequencies.
References


Figure 2: Double-inverted Y system. For graphs a, e and i: $\Omega_{c_1} = 0.2\gamma_{20}$, $\Omega_{c_2} = 0.2\gamma_{30}$, $\Omega_{c_3} = 0.5\gamma_{20}$ and $\Omega_{c_4} = 0.5\gamma_{30}$; For graphs b, f and j: $\Omega_{c_1} = 0.2\gamma_{20}$, $\Omega_{c_2} = 0.8\gamma_{30}$, $\Omega_{c_3} = 0.5\gamma_{20}$ and $\Omega_{c_4} = 2.0\gamma_{30}$; For graphs c, g and k: $\Omega_{c_1} = 0.2\gamma_{20}$, $\Omega_{c_2} = 0.8\gamma_{30}$, $\Omega_{c_3} = 0.5\gamma_{20}$ and $\Omega_{c_4} = 0.5\gamma_{30}$; For graphs d, h and l: $\Omega_{c_1} = 0.2\gamma_{20}$, $\Omega_{c_2} = 0.2\gamma_{30}$, $\Omega_{c_3} = 0.5\gamma_{20}$ and $\Omega_{c_4} = 2.0\gamma_{30}$.
Figure 3: Inverted Y + Ξ system. \( \Omega_c = 0 \) for all graphs. For graphs a, e and i: \( \Omega_{c_1} = 0.2\gamma_{20}, \Omega_{c_3} = 0.5\gamma_{20} \) and \( \Omega_{c_4} = 0.5\gamma_{30} \); For graphs b, f and j: \( \Omega_{c_1} = 0.2\gamma_{20}, \Omega_{c_3} = 2.0\gamma_{20} \) and \( \Omega_{c_4} = 0.5\gamma_{30} \); For graphs c, g and k: \( \Omega_{c_1} = 0.2\gamma_{20}, \Omega_{c_3} = 0.5\gamma_{20} \) and \( \Omega_{c_4} = 2.0\gamma_{30} \); For graphs d, h and l: \( \Omega_{c_1} = 0.2\gamma_{20}, \Omega_{c_3} = 2.0\gamma_{20} \) and \( \Omega_{c_4} = 2.0\gamma_{30} \).
Figure 4: Inverted Y + Λ system. $\Omega_{c_4} = 0$ for all graphs. For graphs a, e and i: $\Omega_{c_1} = 0.2\gamma_{20}$, $\Omega_{c_2} = 0.2\gamma_{30}$ and $\Omega_{c_3} = 0.5\gamma_{20}$; For graphs b, f and j: $\Omega_{c_1} = 0.2\gamma_{20}$, $\Omega_{c_2} = 1.2\gamma_{30}$ and $\Omega_{c_3} = 0.5\gamma_{20}$; For graphs c, g and k: $\Omega_{c_1} = 1.2\gamma_{20}$, $\Omega_{c_2} = 0.2\gamma_{30}$ and $\Omega_{c_3} = 0.5\gamma_{20}$; For graphs d, h and l: $\Omega_{c_1} = 1.2\gamma_{20}$, $\Omega_{c_2} = 1.2\gamma_{30}$ and $\Omega_{c_3} = 0.5\gamma_{20}$. 