On the value of the function 
\[ \exp(ax)/f(a) \text{ at } a = 0 \text{ for } f(a) = 0 \]

Saburou Saitoh
Institute of Reproducing Kernels
Kawauchi-cho, 5-1648-16, Kiryu 376-0041, JAPAN
saburou.saitoh@gmail.com

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Abstract: In this short note, we will consider the value of the function \( \exp(ax)/f(a) \) at \( a = 0 \) for \( f(a) = 0 \). This case appears for the construction of the special solution of some differential operator \( f(D) \) for the polynomial case of \( D \) with constant coefficients. We would like to show the power of the new method of the division by zero calculus, simply and typically.

Key Words: Division by zero calculus, construction of special solutions, ordinary differential equation.

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1 Introduction

In this short note, we will consider the value of the function \( \exp(ax)/f(a) \) at \( a = 0 \) for \( f(a) = 0 \). This case appears for the construction of the special solution of some differential operator \( f(D) \) for the polynomial case of \( D \) with constant coefficients. We would like to show the power of the new method of the division by zero calculus, simply and typically.
2 Division by zero calculus

For the statement of the conclusion, we will recall the division by zero calculus.

For any Laurent expansion around \( z = a \),

\[
f(z) = \sum_{n=-\infty}^{-1} C_n(z - a)^n + C_0 + \sum_{n=1}^{\infty} C_n(z - a)^n,
\]

we define the division by zero calculus by the identity

\[
f(a) = C_0.
\]

For many basic properties and applications of the division by zero calculus, see [7] and the references.

3 Conclusion

From the definition of the division by zero calculus, directly, we obtain the theorem, simply

**Theorem:** For the function

\[
\frac{\exp(ax)}{f(a)}, \quad f(a) = 0
\]

if \( f(z) \) is analytic around \( z = 0 \) and \( f'(a) = f''(a) = ... = f^{(m)}(a) = 0 \) and \( f^{(m+1)}(a) \neq 0 \), by the division by zero calculus, we obtain the identity

\[
x^{m+1} \frac{\exp(ax)}{f^{(m+1)}(a)}.
\]

When \( f(D) \) is an (polynomial) ordinary differential operator with \( D = d/dx \) and with constant coefficients, in the ordinary differential equation

\[
f(D)y = \exp(ax),
\]

if \( f'(a) = f''(a) = ... = f^{(m)}(a) = 0 \) and \( f^{(m+1)}(a) \neq 0 \), then it gives a special solution.
References


