I. Introduction

1. Main reference. This article is actually an extension of an older paper of the same author called “On the intrinsic paradox of the geometric point definition (solved using the Included Middle Logic) as the main cause of Euclid’s postulate “inaccuracy”, allowing the existence not only of non-Euclidean geometries but also of a new “t-meta-mathematics” used to redefine the basics of General relativity, Quantum field theory, Superstring theories and M-theory” [1]

2. The point-like masses and electromagnetic (EM) charges used in the Quantum Field Theory. In classical electromagnetism (cEM) but also in quantum field theory (QFT), electrons (and all the other QFT-theorized elementary particles [EPs] respectively) are idealized as zero-dimensional (0D) geometrical points (GPs) with possible non-zero rest energies/masses/EM-charges: the physicist Paul Dirac was the first to introduce his (Dirac) delta function (DF) (aka “δ function”) which is essentially a generalized function on the real (number) line that takes the value 0 everywhere except for the 0 argument, with an integral equal to 1 over the entire real line. DF is usually regarded as an infinitely high and thin spike at the 0-origin (of the real line), with total area (the integral of DF) equal to 1 under the spike, which spike physically represents an idealized point-like EP with possible rest mass/energy, EM charge, weak charge or color charge. [URL] [1]

II. On two proposed distinct types of imaginary (im) infinities (“imfinities”) in mathematics and meta-mathematics (including meta-geometry)

1. Preliminary discussion on generic infinitesimals. Regarding the generic infinitesimal \((1/\infty)\), this paper emphasizes that, although \(1/\infty\) may occasionally be approximated with zero \((1/\infty \approx 0)\) for some practical reasons, in theory \(1/\infty\) and 0 are distinct notions such as:

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[1] Andrei-Lucian Drăgoi (ALD) research pages on: ResearchGate (RG), Academia.edu, Vixra, G5Journal; See also ALD’s RG CV (long version), RG CV (short version)
and, more specifically, \( \frac{1}{\infty} > 0 \), with the important mention that only the generic infinitesimal \( 0/\infty \) reaches zero, such as \( 0/\infty = 0 \cdot (1/\infty) = 0 \), which is also equivalent to \( \infty \cdot 0 = 0 \) (standard infinity \([\text{StdInf}]\) multiplied with zero is always zero, by definition of this StdInf). [1]

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2. The zero-infinitesimal duality/(polarity) (ZID). As one may easily notice the fact that, on the real number line, there are actually an infinite number of distinct reals between the real number 0 (zero) and the positive real infinitesimal \( 1/\infty \), so that: \( \frac{1}{\infty} > 0 \Rightarrow \frac{1}{\infty} \neq 0 \), even if \( \frac{1}{\infty} \equiv 0 \) and asymptotic limit \( \frac{1}{\infty} \times \ln = \). This polarity between zero (0) and the real infinitesimal \( 1/\infty(>0) \) shall be called the zero-infinitesimal duality/(polarity) (ZID). Clearly enough, ZID is also deeply connected to Zeno’s Dichotomy paradox (but also to other paradoxes proposed by the pre-Socratic Greek philosopher Zeno of Elea, c. 490–430 BC) which states that: “That which is in locomotion must arrive at the half-way stage before it arrives at the goal. Suppose Homer wishes to walk to the end of a path. Before he can get there, he must get halfway there. Before he can get halfway there, he must get a quarter of the way there. Before traveling a quarter, he must travel one-eighth; before an eighth, one-sixteenth; and so on: this description requires one to complete an infinite number of tasks, which is an impossibility.” (as recounted by Aristotle in his Physics VI:9, 239b10). ZID additionally means that “even the (real) infinity \( \infty \) (in its current definition) isn’t sufficient to turn an infinite number of 0D GPs into an continuous segment or into a segment, line, plane etc”. [1]

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3. The management of both zero-infinitesimal duality/(polarity) (ZID). To manage ZID, this paper proposes two major distinct types of mathematics (each distinct type being actually a metamathematics (MM) -- the study of mathematics itself using mathematical methods): [1]

a. the “dualistic/dichotomic mathematics” abbreviated as “d-mathematics” (all part of a “d-metamathematics” [dMM]) in which \( \frac{1}{\infty} > 0 \Rightarrow \frac{1}{\infty} \leq 0 \): dMM is based on the standard definition of the real infinite (\( \text{Inf} \) \( \infty \)) and of the real infinitesimal \( 1/\infty(>0) \). [1]

b. the “monadic mathematics” abbreviated as “m-mathematics” (mM) (all part of a “m-metamathematics” [mMM]) based on StdInf (\( \infty \)) BUT ALSO on an “imaginary” (im.) (real) infinite called here “im-infinite”/“imfinite”/“iminfinity” (or briefly “imfinite”/“imfinity” and abbreviated as “imf”), noted as \( [\infty_{im}] \) (so that “im” subscript won’t be confused with the imaginary part \( i = \sqrt{-1} \) of complex numbers, although the “imfinity” notion is somehow analogous with the “imaginary” concept \( [\infty_{im}] \) on which the “complex numbers” are based by definition) and definable in at least two major distinct ways, in trying to solve ZID): [1]

i. \( \text{imf1} \) is actually a set noted as \( [\infty_{im1}(r)] \) and defined as infinite 1D matrix bijective with the real set \( \Re \) (so that each unique real number \( r(e,\Re) \) corresponds to a unique \( r_{\infty}(=\infty) \) corresponding to
\[ \infty_{im1}(\infty) \text{ and vice versa} \] such as
\[ r / \infty_{im1}(r), \forall r \in \Re \quad \Leftrightarrow \quad r = 0 \cdot \infty_{im1}(r), \forall r \in \Re \] (with \( r/0 = \infty_{im1}(r) \) division-by-zero NOT being a non-sense, BUT having an imaginary mM “true” sense and \( \infty_{im1}(r) \neq \infty_{im1}(r) \) so that:
\[ \infty_{im1}(r_1) = \sum_{r_2} \infty_{im1}(r_2) / r_1 = \infty_{im1}(r_2) \forall r_1, r_2 \in \Re \).

this is obviously in contrast with the standard infinity (Inf) (\( \infty \)) (defined by the property \( \infty \cdot 0 = 0 \)), which is thus identified in mM with the special case \( \infty_{im1}(0) \) which is also defined by the same property that \( \infty_{im1}(0) \cdot 0 = 0 \), so that
\[ \infty_{im1}(0) \equiv \infty. \] Important note (redefinition of \( \infty \)). mM also states that standard infinity (\( \infty \)) may be also imaginarily defined as an infinite 1D matrix/set \( \infty_{r}(r) \) (also bijective with the real set \( \Re \)) having the property that \( \infty_{r}(r) \cdot 0 = 0, \forall r \in \Re \) (contrasting to \( \infty_{im1}(r) \cdot 0 = r, \forall r \in \Re \)). [1]

ii. \( \text{imf2} \) is a more “pretentious” imf alternative noted as \( \infty_{im2} \) which is not a set itself but is defined using a (necessary) new imaginary concept of a multiple-valued real zero (mvRZ) (or an imaginary [im.] zero)
\[ 0_{im}(r), with r \in \Re \] (with \( 0_{im}(r) \) being also an infinite 1D matrix bijective with the set of reals \( \Re \)) so that:
\[ 0_{im}(r) = r / \infty_{im2}, \forall r \in \Re \],
which is equivalent to
\[ r = 0_{im}(r) \cdot \infty_{im2}, \forall r \in \Re \],
with \( 0_{im}(0) \equiv 0 \) also by mvRZ definition) and the “imaginary integer part” of \( 0_{im}(r) \)
\[ \text{imfint}[0_{im}(r)] \equiv 0, \forall r \in \Re \] (defined as a new type of imaginary integer-valued function applicable on mvRZ only). [1]

* Important note (1). Both imf1 (\( \infty_{im1}(r) \)) and imf2 (\( \infty_{im2} \)) are notions similar to the infinity used in complex analysis (\( \infty_{C} \)), which has the property that \( \infty_{C} = z / 0, with z \in \Re \) (with \( \Re \) being the set of complex numbers). However, Imf1 is defined as a set and Imf2 has a clear distinction to \( \infty_{C} \) by its “attached” concept of mvRZ. In a checkpoint conclusion (1), both zero and infinite support multiple alternative (including imaginary) definitions as numbers OR sets. Imf1, Imf2 and mvRZ all support an extensive redefinition based on the more general \( \Re \) such as:
\[ c = 0 \cdot \infty_{im1}(c), \forall c \in \Re \], \[ c = 0_{im}(c) \cdot \infty_{im2}, \forall c \in \Re \]
and \[ \text{imfint}[0_{im}(c)] \equiv 0, \forall c \in \Re \]. In a checkpoint conclusion (2), imf1 and imf2 concepts emphasize and “exploit” the potential unlimited diversity “inside”/“within” concepts such as “infinite” and “zero”, in which \( \infty_{r} \neq \infty_{r} \) \( \Leftrightarrow \)
\[ \infty_{r} / 0 = \infty_{im1}(r_1) \neq \infty_{r} / 0 = \infty_{im1}(r_2) \]
\[ \Leftrightarrow \infty_{r} / \infty_{im2} \equiv [0_{im}(r)] \neq \infty_{r} / \infty_{im2} \equiv [0_{im}(r)] \]
\[ \forall r_1, r_2 \neq \infty \in \Re \]. [1]

* Important note (2). Both Imf1 and Imf2 “split” mM in two major subtypes. Regarded as a whole, MM is thus “split” in two main branches (dMM and mM), with mM being also splitted in two secondary (sub)branches: an Imf1-based mM and an Imf2-based mM.

* Important note on mM. In a GP-and-imf1-based mM, the GP and imf1 would be the only primitives (making mM a “convergent”/“coherent” unifying subtype of MM) because ALL the other geometrical primitives (the 1D line, the 2D plane, the 3D space etc) can be obtained from the same GP multiplied with imf1 on various directions/senses such as: GP multiplied by imf1 would generate the 1D line (more exactly/specifically \( L_{GP} = [0] \)) multiplied with
would generate a segment with length \( r \), and multiplied with \( \infty_{iml}(\infty) \) in one direction/sense would generate a semi-line with infinite “semi-length” and multiplied with \( \infty_{iml}(r) \) in both opposite directions/senses would generate a 1D line with infinite length \( L_{line} = \infty \); in the same way the imaginarily juxtaposition of 1D lines (each with zero width) can always generate a 2D plane (with non-zero width), the imaginarily juxtaposition of 2D lines (each with zero thickness/depth) can always generate a 3D space (with non-zero depth) and so on.

*In a checkpoint conclusion*, the GP&imf-based unifying”/”convergent” mM has obvious advantages over the “dez-unifying”/”divergent” dMM, in which each n-space is a distinct geometrical primitive by itself.

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**III. Final remarks**

**Final remark** (1). As GP is 0D, it is clearly an imaginary/virtual object that exists and non-exists in the same time, because “something” that has no-length/area/volume/n-volume has all the conditions of non-existence in both imaginary or real space (the conditions of a “non-object”): however, its non-existence doesn’t prevent any human being to work with such a virtual non-object as if it was a virtual object. The same with the 1D line and the 2D plane which are objects and non-objects as the same time, as they both exist and non-exist, because something with zero width or zero thickness/depth cannot exist (even if it has non-zero length OR non-zero length plus non-zero width; this happens because a zero-length dimension nullifies all the others by multiplication: \( L \cdot 0 = 0 \) [for a zero-width line], \( L \cdot L \cdot 0 = 0 \) [for a zero-thick plane]). The simultaneous existence and non-existence of GP contradicts the “standard”/classical excluded middle principle (EMP) and invokes the modern included middle principle (IMP) [URL2] (aka “the included third principle [ITP]”) firstly formulated by the Romanian philosopher Stefan Lupașcu (also known as “Stéphane Lupasco”), as he lived for many years in France and published there) in his book “The Principle of Antagonism and the Logic of Energy” (first published in 1951). In his book called “Logic and contradiction” (“Logique et contradiction, P. U. F., Paris, 1947), Lupașcu initially proposed the essential Asymptoticity Principle (conjecture) (AP) which states that: “Given any \( A \) defined as the representation of any real physical entities (objects, processes, events) and non-physical entities, NO real process implying \( A \) goes to the idealized, abstract limits of (binary) classical logic”. Using AP, Lupașcu essentially changed the classical affirmation(A)-negation (absolute) duality (the pair of conjugates \( A \) and \(-A \) ) to the actualization-potentialization (relative) duality (the pair of conjugates \( A^\exists = A \) and \(- (A^\exists) \neq (-A) \) (so that the “true” concept noted as \( A \) is replaced by the “actual” concept \( A^\exists \) [alternative sign proposed by the author] and the “false” concept noted as \(- A \) is replaced by the “potential” concept \(- A^\exists \) [alternative sign proposed by the author]). AP can also be formulated as: “No process of actualization or potentialization of any generic element \( A \) goes to 100% completeness [in none of the possible/imaginable spaces/times/spacetimes of that process associated with that element \( A \)].”

Lupașcu was inspired by Heisenberg’s uncertainty principle (HUP) (which was first proposed by the German physicist Werner Heisenberg in 1927). AP alternative formulation Every real phenomenon, element or event \( \overline{e} \) is always associated with an antiphenomenon, anti-element or anti-event \( \overline{\overline{e}} \), such that the actualization of \( \overline{e} \) entails the potentialization of \( \overline{\overline{e}} \) and vice versa, alternatively, without either ever disappearing completely. Aspects of phenomena that are generally considered independent can thus be understood as being in the dynamic opposition: one is “actualized” AND the other is “potentialized”. The included middle logic (IML). IMP was based on AP and both AP and IMP were the starting points of a new included middle logic (IML) which implies 3 states of the same assertion A: (1) A (the assertion) or the “actual” concept \( A^\exists = A \), (2) non-A (the negation of assertion A) or the “potential” concept \(- (A^\exists) \neq (-A) \) AND (3) the “T” state defined as \( T = [A \wedge (-A)] = [A^\exists \wedge (-A^\exists)] = \{1, if A=1 \} \{0, if A=0 \} \) (the so-called “included middle/third” labeled “T” from “third”; “1” and “0” notations stand for the logical attributes “true” and “false” respectively) in
which A (or \(A^\oplus\)) and non-A (or \(\neg (A^\oplus)\)) can be true at the same time OR false at the same time, so that \(T \in \{0,1\}\), which puts IMP in contrast with the classical EMP which states that

\[
T = [A \land \neg A]^{EMP} = \begin{cases} 
0, & \text{if } A = 1 \\
0, & \text{if } A = 0
\end{cases}.
\]

**meaning and the levels of reality.** T is not a “middle/third” in the sense of being “between” A and \(\neg A\), BUT rather in the sense that there is a 3rd position, another superior level of reality (alias “reality level” (RL)) which contains both A and \(\neg A\): in other words, A and \(\neg A\) exist at one inferior RL and T exists at another superior RL so that at the level of A and \(\neg A\), there are only the two contradictory possibilities, BUT at a higher RL, there is a larger domain where both elements could be possible. [1]

**Final remark (2).** IMP was further developed by the French-born American chemist and philosopher Joseph E. Brenner [URL2] and by the Romanian theoretical physicist Basarab Nicolescu [URL2, URL3, French-wiki, URL4-Romanian-wiki, URL5-work]. The continued the work of Lupasco and defined the concept of TransDisciplinarity (TD)[URL2, URL3-French_wiki] as supported by three different (BUT closely related) major “pillar”-concepts: (A) Complexity; (B) Levels of reality (alias reality-levels (RLs)); (C) IML (which is quite a “model of thinking” for TD). Note (1). IML overcomes binary dualism and simple linear causality, revealing a complex and multi-dimensional reality based on non-linear causality: IML is actually a robust logic having properties of both determinacy/indeterminacy, universal/ particular, part/whole and actuality/possibility (potentiality). Note (2). IML is essentially an analytic approach of the “Unity of opposites” (UO), which is a central category of dialectics deeply related to the notion of non-duality (aka nondualism): UO defines a situation in which “the existence or identity of a thing (or situation) depends on the co-existence of at least two conditions which are opposite to each other, yet dependent on each other and presupposing each other, within a field of tension.” [URL]. Cite from the American philosopher and economic theorist (at the New School for Social Research in New York) Melanie Swan. “Included Middle is a position of greater complexity and possibility for addressing any situation. Conceiving of a third space that holds two apparent contradictions of a problem is what the Included Middle might bring to contemporary challenges in consciousness, artificial intelligence, disease pathologies, and unified theories in physics and cosmology.” [URL1] [1]

**Final remark (3).** IMP was also supported by Werner K. Heisenberg: Heisenberg noticed that EMP (which seemed to hold at large/macro length scales of our universe) doesn’t hold at the lowest (known) length scales (of our universe) and he pointed out how EMP has to be modified in quantum mechanics (QM), to accommodate the quantum superposition principle (QSP) (which states that “any two or more quantum states (QSs) can be added together ("superposed") and the result will be another valid quantum QS; and conversely, that every QS can be represented as a sum of two or more other distinct QSs.”), the quantum probability (QP) and the wave–particle duality (WPD) (which states that: “every particle or quantum entity may be partly described in terms not only of particles, but also of waves”: WPD essentially states the inability of the classical concepts "particle" or "wave" to fully describe the behavior of quantum entities); an additional logic term was thus needed to describe this third possible situation, hence the Included Middle. [1]

**Final remark (4).** The author of this paper has also extended IML by generalization and proposed a “generalized IML (GIML)” based on a “generalized IMP (GIMP)” [5.6]. IMP had already been applied in a variety of scientific domains and has a substantially wider potential applications. [1]

**Final remark (5).** In a specific sense, IMP also appears to be applicable on the symmetry breaking phenomenon in physics, in which the physical energy magnitude levels (which are actually physical information density or entropy magnitude levels) can be regarded as RLs, so that the (quantum) elementary particles (EPs) of the Standard model (of particle physics) (SM) appear different at an inferior-rank RL, but are strongly related to one another by being the "faces"("fragments"/"crosks"/"puzzle-pieces") of the same unified EP at a superior-rank RL. For example, the photon (a super-light EP with theoretical/predicted zero rest mass and only having non-zero relativistic energy/mass) and the W/Z bosons (very heavy EPs with quite large non-zero rest masses) all appear with quite “opposite” properties at some specific energy level (corresponding to a specific RL), BUT unify in a “super”-EP (the electroweak field/force/interaction quanta) at some superior energy level (superior-rank
RL), as initially predicted by the electroweak theory (proposed by physicists Sheldon Glashow, Abdus Salam and Steven Weinberg) and then experimentally demonstrated (in 1973) by: (1) the existence of neutral currents of Z-bosons (in neutrino scattering experiments conducted in the heavy liquid bubble chamber detector of CERN called “Gargamelle”, in operation between 1970 and 1979); (2) the discovery of the W and Z gauge bosons (in 1983) in proton–antiproton collisions by UA1 and UA2 collaborations at the converted Super Proton Synchrotron. [1]

Final remark (6). The EMP-generated ZID and Zeno’s dichotomy paradox also pose problems to Quantum field theory (QFT) and Quantum mechanics (QM) which both model (quantum) elementary particles (EPs) from the Standard Model (SM) as 0D point-like/GP-like entities: this GP-based approach strongly contradicts the 4D spacetime continuum conjectured by Einstein’s General relativity theory (GRT) (and modeled as a curved generalization of Minkowski space): it is clear that a GP-like 0D-EP will have to accomplish an infinite number of steps (each step being defined as a GP in such a spacetime continuum) thus taking an infinite amount of time (if the maximum speed in the universe is limited by GRT and QFT to the speed of light in vacuum) to reach from a GP “A” to another distinct GP “B” from the same spacetime continuum: this contra-argument obviously resembles Zeno’s dichotomy paradox (ZDP) and cannot be solved other than by IML or GIML; a step-by-step movement of a 0D-EP in a continuum spacetime cannot be accomplished in a finite time other than by a “teleportation”-like movement only (quantized movement in micro-steps identified with segments composed from an “infinitesimal” number of GPs), in which the GP-like EP can surpass “infinitesimal chunks” GPs. In conclusion, the IML-based mMM can “absorb” both QFT and GRT by avoiding such paradoxes. The mMM can also “absorb” all types n-dimensional Hilbert spaces and Minkowski spaces without generating paradoxes.

Final remark (6). As superstring theories (SSTs) and M-Theory (MT) also use 1D strings (aka “1-strings”) and 2D branes (aka “2-branes”) respectively, they cannot avoid ZID, BUT can be easily “absorbed” by mMM.

IV. End references (in the order of their apparition in this paper)

[1] Andrei-Lucian Drăgoi (August 2018). (IP-GP – version 1.0 – 15 pages – 14.08.2018) On the intrinsic paradox of the geometric point definition (solved using the Included Middle Logic) as the main cause of Euclid’s postulate “inaccuracy”, allowing the existence not only of non-Euclidean geometries but also of a new “t-metamathematics” used to redefine the basics of General relativity, Quantum field theory, Superstring theories and M-theory. Research Gate preprint. DOI: 10.13140/RG.2.2.32439.42405. URL (Research Gate source).


