

**New possible mathematical developments concerning  $\zeta(2)$ ,  $\phi$ , the Rogers-Ramanujan identity: Mathematical connections with some sectors of Particles Physics and the Black Hole physical parameters.**

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**Abstract**

*In the present research thesis, we have obtained various and interesting new possible mathematical results concerning  $\zeta(2)$ ,  $\phi$  and the Rogers-Ramanujan identity. We obtain various mathematical connections with some sectors of Particles Physics and the Black Hole physical parameters.*

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## Rogers-Ramanujan type modular units




<http://www.maths.dur.ac.uk/lms/103/talks/0710ono0.pdf>

$$R(q) := \frac{q^{1/5}}{1 + \frac{q}{1 + \frac{q^2}{1 + \dots}}}$$

$$R(q) = \frac{\sqrt{5}-1}{2} \exp\left(-\frac{1}{5} \int_q^1 \frac{(1-t)^5(1-t^2)^5(1-t^3)^5 \dots dt}{(1-t^5)(1-t^{10})(1-t^{15}) \dots t}\right)$$

$$\ln\left(\sqrt{4\phi+3} \phi^2\right) = \frac{1}{5} \int_{e^{-2\pi}}^1 \frac{(1-t)^5(1-t^2)^5(1-t^3)^5 \dots dt}{(1-t^5)(1-t^{10})(1-t^{15}) \dots t}$$

# QUANTUM GRAVITY AND THE HOLOGRAPHIC MASS

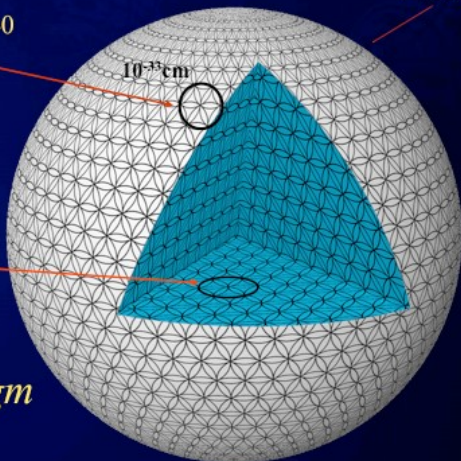


$$\eta = \frac{A_p}{A_{\ell c}} = 10^{40}$$


$$R = \frac{V_p}{V_{\ell s}} = 10^{60}$$

$$2 \frac{\eta}{R} m_{\ell} = 1.6714213 \times 10^{-24} \text{ gm}$$

**Proton**



Within  $0.0012 \times 10^{-24} \text{ gm}$  of CODATA

THE RESONANCE PROJECT 

We now proceed to calculate the rest mass of the proton as above, utilizing the new muonic hydrogen measured proton charge radius  $r_p = 0.84184 \times 10^{-13} \text{ cm}$  and find  $\eta = 4.340996 \times 10^{40}$ ,  $\eta_p = 9.448222 \times 10^{35} \text{ gm}$ , and  $R = 1.130561 \times 10^{60}$ . Again utilizing equation (24) we obtain

$$m_{p'} = 2 \frac{\eta_p}{R} = 1.6714213 \times 10^{-24} \text{ gm} . \tag{25}$$

$$\phi = \frac{\eta}{R} = \frac{\eta_p}{R_p} = 3.839682 \times 10^{-20}$$

$$m_{p'}^2 = 4\phi^2 \frac{\hbar c}{G} .$$

From these formulas, we obtain various expressions that we will analyze. We have:

$$c = 2.99 * 10^{10} \text{ cm/s} \quad \hbar = 1.054571817 \text{ e-32}$$

$$m_p = 2\phi m_e = 2\phi \sqrt{\frac{\hbar c}{G}}$$

$$m_p = 2.17645e-5$$

$$2((((2.17645e-5)*(4.340996e+40)))) / (1.130561e+60)$$

**Input interpretation:**

$$2 \times \frac{2.17645 \times 10^{-5} \times 4.340996 \times 10^{40}}{1.130561 \times 10^{60}}$$

**Result:**

$$1.6713756699903853042869867260590096421157283861728823... \times 10^{-24}$$

$$1.671375669... * 10^{-24}$$

$$G = ((((((4*(3.839682e-20)^2 * (1.054571817e-32) \text{ Newton cm } *(2.99e+10) \text{ cm})))))) / ((((((2*3.839682e-20*2.17645e-5 \text{ grams})^2)))))) \text{ (GRAVITATIONAL COUPLING CONSTANT)}$$

(Note that:  $2 \times 3.839682 \times 10^{-20} \times 2.17645 \times 10^{-5}$  grams =  $1.6714 \times 10^{-27}$  kg (kilograms))

**Input interpretation:**

$$\frac{(4(3.839682 \times 10^{-20})^2 \times 1.054571817 \times 10^{-32} \text{ Ncm (newton centimeters)} \times 2.99 \times 10^{10} \text{ cm (centimeters)})}{(2 \times 3.839682 \times 10^{-20} \times 2.17645 \times 10^{-5} \text{ grams})^2}$$

**Result:**

$$6.657 \times 10^{-11} \text{ m}^3/(\text{kg s}^2) \text{ (meters cubed per kilogram second squared)}$$

$$6.657 * 10^{-11}$$

**Unit conversions:**

$$6.657 \times 10^{-11} \text{ Nm}^2/\text{kg}^2 \text{ (newton square meters per kilogram squared)}$$

$$6.657 \times 10^{-8} \text{ dyne cm}^2/\text{g}^2 \text{ (dyne square centimeters per gram squared)}$$

$$3.431 \times 10^{-8} \text{ ft}^3/(\text{slug s}^2) \text{ (feet cubed per slug per second squared)}$$



**Interpretation:**

Newtonian gravitational coupling

Or:

$$\frac{\left(\left(\left(\left(4 \times (3.839682 \times 10^{-20})^2 \times (1.054571817 \times 10^{-32}) \text{ Newton cm} \times (2.99 \times 10^{10}) \text{ cm}\right)\right)\right)\right) / \left(\left(\left(\left(1.6714213 \times 10^{-24} \text{ grams}\right)^2\right)\right)\right)}$$

**Input interpretation:**

$$\frac{\left(4 \times (3.839682 \times 10^{-20})^2 \times 1.054571817 \times 10^{-32} \text{ N cm (newton centimeters)} \times 2.99 \times 10^{10} \text{ cm (centimeters)}\right) / \left(1.6714213 \times 10^{-24} \text{ grams}\right)^2}$$

**Result:**

$$6.656 \times 10^{-11} \text{ m}^3 / (\text{kg s}^2) \text{ (meters cubed per kilogram second squared)}$$

$$6.656 * 10^{-11}$$

**Unit conversions:**

$$6.656 \times 10^{-11} \text{ Nm}^2 / \text{kg}^2 \text{ (newton square meters per kilogram squared)}$$

$$6.656 \times 10^{-8} \text{ dyne cm}^2 / \text{g}^2 \text{ (dyne square centimeters per gram squared)}$$

$$3.43 \times 10^{-8} \text{ ft}^3 / (\text{slug s}^2) \text{ (feet cubed per slug per second squared)}$$

**Interpretation:**

Newtonian gravitational coupling

$$m_p = \sqrt{\left(\left(\left(\left(\left(\left(\left(\left(4 \times (3.839682 \times 10^{-20})^2 \times (1.054571817 \times 10^{-32}) \text{ Newton cm} \times (2.99 \times 10^{10}) \text{ cm}\right)\right)\right)\right)\right) / \left(\left(\left(6.657 \times 10^{-11} \text{ newton square meters per kilogram squared}\right)\right)\right)\right)\right)\right)\right)\right)\right)\right) \text{ (rest mass of the proton)}$$

**Input interpretation:**

$$\sqrt{\left(\left(4 \times (3.839682 \times 10^{-20})^2 \times 1.054571817 \times 10^{-32} \text{ N cm (newton centimeters)} \times 2.99 \times 10^{10} \text{ cm (centimeters)}\right) / \left(6.657 \times 10^{-11} \text{ Nm}^2 / \text{kg}^2 \text{ (newton square meters per kilogram squared)}\right)\right)}$$

**Result:**

$$1.671 \times 10^{-27} \text{ kg (kilograms)}$$

**Unit conversion:**

$$1.671 \times 10^{-24} \text{ grams}$$

$$1.671 * 10^{-24} \text{ gm}$$

### Comparisons as mass:

$$\approx 0.53 \times \text{tau particle mass } (\approx 3.2 \times 10^{-27} \text{ kg})$$

$$\approx 0.9992 \times \text{proton mass } (1.67262192 \times 10^{-27} \text{ kg})$$

### Comparison as mass of atom:

$$\approx 1.006 \times \text{unified atomic mass unit } (1 m_u)$$

### Corresponding quantities:

Relativistic energy  $E$  from  $E = mc^2$ :

$$938 \text{ MeV (megaelectronvolts)}$$

$$938 \text{ MeV}$$

Characteristic length  $L$  from  $L = h/(mc)$ :

$$1.3 \text{ fm (femtometers)}$$

Thermal de Broglie wavelength at 100 K from  $\lambda = h/(2\pi mkT)^{1/2}$ :

$$174 \text{ pm (picometers)}$$

Characteristic time  $T$  from  $T = h/(mc^2)$ :

$$4.4 \times 10^{-24} \text{ seconds}$$

Thermodynamic temperature  $T$  from  $kT = mc^2$ :

$$1.088 \times 10^{13} \text{ K (kelvins)}$$

Compton frequency  $\nu$  from  $\nu = mc^2/h$ :

$$2.267 \times 10^{23} \text{ Hz (hertz)}$$

Note that  $938 = 9^3 + 10^3 - (1010^3 - 1 - 812^3)^{1/3}$ , indeed:

$$9^3 + 10^3 - (((1010^3 - 1 - 812^3)^{1/3}))$$

### Input:

$$9^3 + 10^3 - \sqrt[3]{1010^3 - 1 - 812^3}$$

### Exact result:

$$938$$

$$938$$

$$((((((9^3 + 10^3 - (((1010^3 - 1 - 812^3)^{1/3}))))))))^{1/14}$$

### Input:

$$\sqrt[14]{9^3 + 10^3 - \sqrt[3]{1010^3 - 1 - 812^3}}$$

**Result:**

$$\sqrt[14]{938}$$

**Decimal approximation:**

1.630422660411353985834042575484597339951468120622461216215...

1.63042266... result practically equal to the value 1.629 (see Fig.)

**FORMULAS:**

For G, we have:

$$\frac{4\phi^2 \hbar c \text{ (reduced Planck constant speed of light)}}{(2\phi m_p \text{ (Planck masses)})^2} = 6.657 \times 10^{-11} = G \text{ (Newtonian gravitational constant)}$$

$$1 \hbar c / m_p^2 \text{ (reduced Planck constant speed of light per Planck mass squared)} = 6.657 \times 10^{-11} = G \text{ (Newtonian gravitational constant)}$$

For  $m_p'$ , we have:

$$\sqrt{\frac{4\phi^2 \hbar c \text{ (reduced Planck constant speed of light)}}{G \text{ (Newtonian gravitational constant)}}} = 1.6714213 \times 10^{-24} \text{ grams}$$

where:

$$\phi \approx 3.8398 \times 10^{-20}$$

Now:

$m_p' =$

$$\begin{aligned} &\sqrt{\left(\left(4(3.839682 \times 10^{-20})^2 \times 1.054571817 \times 10^{-32} \text{ N cm (newton centimeters)} \times \right. \right. \\ &\quad \left. \left. 2.99 \times 10^{10} \text{ cm (centimeters)}\right) / \right. \\ &\quad \left. \left(6.657 \times 10^{-11} \text{ Nm}^2/\text{kg}^2 \text{ (newton square meters per kilogram squared)}\right)\right) \\ &= 1.671 \times 10^{-24} \text{ grams} \\ &= 1.671 * 10^{-24} \text{ grams} \end{aligned}$$

We have, without units, for  $m_p$  :

$$\sqrt{\left(\left(\left(\left(\left(\left(4 \cdot (3.839682 \times 10^{-20})^2 \cdot (1.054571817 \times 10^{-32}) \cdot (2.99 \times 10^{10}) \cdot 10^4\right)\right)\right)\right)\right)\right) / \left(\left(6.657 \times 10^{-11}\right) \cdot 10^2\right)\right)}$$

where  $10^4$  are  $\text{cm}^2 = \text{convert } 1 \text{ m}^2 \text{ (square meter) to centimeters}$

$10000 \text{ cm}^2 \text{ (square centimeters)}$  and  $10^2 = 100 \text{ m} \cdot \text{kg}$  (100 kilogram-force centimeters)

$$\begin{aligned} m_p &= \sqrt{\frac{4(3.839682 \times 10^{-20})^2 \times 1.054571817 \times 10^{-32} \times 2.99 \times 10^{10} \times 10^4}{6.657 \times 10^{-11} \times 10^2}} \\ &= 1.67132... \times 10^{-24} \\ &= 1.67132... \cdot 10^{-24} \text{ gm} \end{aligned}$$

that is the **holographic derivation of the mass of the proton**, connected with the Gravitational constant.

With regard G, we have that:

Newton \* centimeter<sup>2</sup> / grams<sup>2</sup>

$100 \text{ m}^3 / (\text{kg s}^2)$  (meters cubed per kilogram second squared)

$100 \text{ Nm}^2 / \text{kg}^2$  (newton square meters per kilogram squared)

thence:  $10^2$

in dimensionless form, we have for G:

$$\left(\left(\left(\left(4 \cdot (3.839682 \times 10^{-20})^2 \cdot (1.054571817 \times 10^{-32}) \cdot (2.99 \times 10^{10}) \cdot 10^2\right)\right)\right)\right) / \left(\left(\left(\left(1.6714213 \times 10^{-24}\right)^2\right)\right)\right)$$

$$G = \frac{4(3.839682 \times 10^{-20})^2 \times 1.054571817 \times 10^{-32} \times 2.99 \times 10^{10} \times 10^2}{(1.6714213 \times 10^{-24})^2} =$$

$$= 6.6561943088359539989386568039782503097413911161978359... \times 10^{-11}$$

$$= 6.6561943... \times 10^{-11}$$

From:

## Quantum Gravity and the Holographic Mass

*Nassim Hamein*

We now proceed to calculate the rest mass of the proton as above, utilizing the new muonic hydrogen measured proton charge radius  $r_p = 0.84184 \times 10^{-13} \text{ cm}$  and find  $\eta = 4.340996 \times 10^{40}$ ,  $\eta_p = 9.448222 \times 10^{35} \text{ gm}$ , and  $R = 1.130561 \times 10^{60}$ . Again utilizing equation (24) we obtain

$$m_{p'} = 2 \frac{\eta_p}{R} = 1.6714213 \times 10^{-24} \text{ gm} . \quad (25)$$

$$m_{p'} = 2 \times \frac{\eta}{R} m_p = 1.6714213 \times 10^{-24} \text{ gm}$$

$$m_p = 2.17645 \times 10^{-5} \text{ grams} = \text{Planck Mass}$$

For to obtain G, we utilize this other formula

$$m_{p'}^2 = 4\phi^2 \frac{\hbar c}{G} .$$

We obtain:

$$m_{p'} = 2 \times \frac{\eta}{R} m_p = 1.6714213 \times 10^{-24} \text{ grams}$$

$$G = \frac{4\phi^2 \times \hbar \times c}{m_{p'}^2} = 6.6561943 \dots \times 10^{-11}$$

that is the **Gravitational coupling Constant** ( $G = 6.67430(15) \times 10^{-11} \text{ m}^3 \cdot \text{kg}^{-1} \cdot \text{s}^{-2}$ )

$$m_{p'} = \sqrt{\frac{4\phi^2 \times h \times c}{G}} = 1.6714213 \times 10^{-24} \text{ grams}$$

that is the **holographic derivation of the mass of the proton.**

There is a strong connection between the proton mass and the Gravitational coupling Constant. Indeed, from G we can to obtain  $m_{p'}$  and from  $m_{p'}$  we can to obtain G

We note that:

$$3.359885 * (((1.6714213 \times 10^{-24}) / 6.6561943 \times 10^{-11})))$$

where 3.359885... is the Prévo's Constant:

3.359885666243177553172011302918927179688905... that is equal to the sum of the reciprocals of the Fibonacci numbers:  $1/1 + 1/1 + 1/2 + 1/3 + 1/5 + 1/8 + 1/13 + 1/21 + 1/34 + 1/55 + 1/89 + ..$

We have:

$$\begin{aligned} & 3.359885 \times \frac{1.6714213 \times 10^{-24}}{6.6561943 \times 10^{-11}} \\ &= 8.4369282227090335989741164857522263134656390664557373... \times 10^{-14} \\ &= 8.4369282227... * 10^{-14} \end{aligned}$$

But:

$$0.8436928222709 \times 10^{-13} = 8.436928222709 \times 10^{-14}$$

Thence:

$0.8436928222709... * 10^{-13}$  that is a very good approximation to the following formula result:

$$r_{p'} = 4\ell \frac{m_\ell}{m_p} = 0.841236 \times 10^{-13} \text{ cm}$$

From:

N. Hamein, *The Schwarzschild Proton*, AIP CP 1303, ISBN 978-0-7354-0858-6, pp. 95-100, December 2010

## The Schwarzschild Proton

Nassim Hamein

	Mass	Log Mass	Radius	Log Radius
Schwarzschild Proton	8.89E+14	1.49E+01	1.32E-13	-12.88
Standard Proton	1.67E-24	-2.38E+01	2.97E+01	-12.88

We calculate the “anomalous” magnetic moment<sup>19</sup> of the proton using a simple model where the proton is a sphere with a Compton radius of 1.321 Fermi spinning at the speed of light,  $c$ , with a point proton charge at its equator. The magnetic moment is given as:

$$\mu = \frac{qv}{2} \tag{17}$$

where  $q$  is an elementary charge of  $1.60217653 \times 10^{-19}$  Coulombs, the proton radius is  $r_p = 1.321 \times 10^{-15}$  meters and the velocity  $v = 2.998 \times 10^8$  m/s giving a value of the magnetic moment of such a proton of  $3.17259 \times 10^{-26}$  Joules/Tesla.

The value of radius of Schwarzschild Proton, that is  $1.321 \times 10^{-15}$  m, is very closed to the result of the following Ramanujan mock theta function  $f(q) = 1.333425959\dots$

We have that:

$$1/2 * (1.60217653 \times 10^{-19} * 1.321 \times 10^{-15} * 2.998 \times 10^8)$$

### Input interpretation:

$$\frac{1}{2} (1.60217653 \times 10^{-19} \times 1.321 \times 10^{-15} \times 2.998 \times 10^8)$$

### Result:

$$3.17259631899887 \times 10^{-26}$$

$3.17259631899887 * 10^{-26} = \text{proton magnetic moment}$

From this result, we obtain also:

$$1/(2\pi) (3.17259631899887 * 10^{-26})^2 * 10^{24} * 10^4$$

Note that:

$$(3.17259631899887 * 10^{-26})^2$$

### Input interpretation:

$$\left( \frac{3.17259631899887}{10^{26}} \right)^2$$



**Result:**

$$1.00653674033251796933190612769 \times 10^{-51}$$

This result is a sub-multiple practically equal to the Ramanujan mock theta function 1.0061571663

**Input interpretation:**

$$\frac{1}{2\pi} \left( \frac{3.17259631899887}{10^{26}} \right)^2 (10^{24} \times 10^4)$$

**Result:**

$$1.60195297627524... \times 10^{-24}$$

$$1.6019529... * 10^{-24}$$

**Alternative representations:**

$$\frac{\left( \frac{3.172596318998870000}{10^{26}} \right)^2 (10^{24} \times 10^4)}{2\pi} = \frac{10^4 \times 10^{24} \left( \frac{3.172596318998870000}{10^{26}} \right)^2}{360^\circ}$$

•

$$\frac{\left( \frac{3.172596318998870000}{10^{26}} \right)^2 (10^{24} \times 10^4)}{2\pi} = - \frac{10^4 \times 10^{24} \left( \frac{3.172596318998870000}{10^{26}} \right)^2}{2i \log(-1)}$$

•

$$\frac{\left( \frac{3.172596318998870000}{10^{26}} \right)^2 (10^{24} \times 10^4)}{2\pi} = \frac{10^4 \times 10^{24} \left( \frac{3.172596318998870000}{10^{26}} \right)^2}{2 \cos^{-1}(-1)}$$

log(x) is the natural logarithm

i is the imaginary unit

cos<sup>-1</sup>(x) is the inverse cosine function

**Series representations:**

$$\frac{\left( \frac{3.172596318998870000}{10^{26}} \right)^2 (10^{24} \times 10^4)}{2\pi} = \frac{1.258170925415647462 \times 10^{-24}}{\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}}$$

•

$$\frac{\left(\frac{3.172596318998870000}{10^{26}}\right)^2 (10^{24} \times 10^4)}{2 \pi} = \frac{2.516341850831294923 \times 10^{-24}}{-1.000000000000000000 + \sum_{k=1}^{\infty} \frac{2^k}{\binom{2k}{k}}}$$

$$\frac{\left(\frac{3.172596318998870000}{10^{26}}\right)^2 (10^{24} \times 10^4)}{2 \pi} = \frac{5.03268370166258985 \times 10^{-24}}{\sum_{k=0}^{\infty} \frac{2^{-k} (-6+50k)}{\binom{3k}{k}}}$$

$\binom{n}{m}$  is the binomial coefficient

### Integral representations:

$$\frac{\left(\frac{3.172596318998870000}{10^{26}}\right)^2 (10^{24} \times 10^4)}{2 \pi} = \frac{2.516341850831294923 \times 10^{-24}}{\int_0^{\infty} \frac{1}{1+t^2} dt}$$

$$\frac{\left(\frac{3.172596318998870000}{10^{26}}\right)^2 (10^{24} \times 10^4)}{2 \pi} = \frac{1.258170925415647462 \times 10^{-24}}{\int_0^1 \sqrt{1-t^2} dt}$$

$$\frac{\left(\frac{3.172596318998870000}{10^{26}}\right)^2 (10^{24} \times 10^4)}{2 \pi} = \frac{2.516341850831294923 \times 10^{-24}}{\int_0^{\infty} \frac{\sin(t)}{t} dt}$$

If we insert the value of Ramanujan mock theta function 1.0061571663 with exponent  $10^{-51}$  in the above formula, we obtain about the same result. Indeed:

$$1/(2\pi) (1.0061571663 * 10^{-51}) 10^{24} * 10^4$$

### Input interpretation:

$$\frac{1}{2 \pi} \times \frac{1.0061571663}{10^{51}} (10^{24} \times 10^4)$$

### Result:

$$1.6013488654... \times 10^{-24}$$

$$1.6013488654... * 10^{-24}$$

**Alternative representations:**

$$\frac{1.00615716630000 (10^{24} \times 10^4)}{10^{51} (2 \pi)} = \frac{1.00615716630000 \times 10^4 \times 10^{24}}{10^{51} (360^\circ)}$$

•

$$\frac{1.00615716630000 (10^{24} \times 10^4)}{10^{51} (2 \pi)} = \frac{1.00615716630000 \times 10^4 \times 10^{24}}{10^{51} (-2 i \log(-1))}$$

•

$$\frac{1.00615716630000 (10^{24} \times 10^4)}{10^{51} (2 \pi)} = \frac{1.00615716630000 \times 10^4 \times 10^{24}}{10^{51} (2 \cos^{-1}(-1))}$$

$\log(x)$  is the natural logarithm

$i$  is the imaginary unit

$\cos^{-1}(x)$  is the inverse cosine function

**Series representations:**

$$\frac{1.00615716630000 (10^{24} \times 10^4)}{10^{51} (2 \pi)} = \frac{1.25769645787500 \times 10^{-24}}{\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}}$$

•

$$\frac{1.00615716630000 (10^{24} \times 10^4)}{10^{51} (2 \pi)} = \frac{2.51539291575000 \times 10^{-24}}{-1.0000000000000000 + \sum_{k=1}^{\infty} \frac{2^k}{\binom{2k}{k}}}$$

•

$$\frac{1.00615716630000 (10^{24} \times 10^4)}{10^{51} (2 \pi)} = \frac{5.03078583150000 \times 10^{-24}}{\sum_{k=0}^{\infty} \frac{2^{-k} (-6+50k)}{\binom{3k}{k}}}$$

$\binom{n}{m}$  is the binomial coefficient

**Integral representations:**

$$\frac{1.00615716630000 (10^{24} \times 10^4)}{10^{51} (2\pi)} = \frac{2.51539291575000 \times 10^{-24}}{\int_0^\infty \frac{1}{1+t^2} dt}$$

•

$$\frac{1.00615716630000 (10^{24} \times 10^4)}{10^{51} (2\pi)} = \frac{1.25769645787500 \times 10^{-24}}{\int_0^1 \sqrt{1-t^2} dt}$$

•

$$\frac{1.00615716630000 (10^{24} \times 10^4)}{10^{51} (2\pi)} = \frac{2.51539291575000 \times 10^{-24}}{\int_0^\infty \frac{\sin(t)}{t} dt}$$

From the radius of this Schwarzschild Proton, 1.321e-15 m, we obtain:

Mass = 8.896512e+11

Radius = 1.321000e-15

Temperature = 1.379421e+11

From the Ramanujan-Nardelli mock formula, we obtain:

sqrt[[[1/((((((4\*1.962364415e+19)/(5\*0.0864055^2))) \* 1/(8.896512e+11) \* sqrt[[-(((1.379421e+11 \* 4\*Pi\*(1.321000e-15)^3 - (1.321000e-15)^2)))] / ((6.67\*10^-11))))]]]]]

**Input interpretation:**

$$\sqrt{\left(1 / \left( \frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{8.896512 \times 10^{11}} \right) \sqrt{-\frac{1.379421 \times 10^{11} \times 4 \pi (1.321000 \times 10^{-15})^3 - (1.321000 \times 10^{-15})^2}{6.67 \times 10^{-11}}} \right)}$$

**Result:**

1.618249204300565050355062348663809483670930902980707958077...

1.6182492...

And:

1/sqrt[[[1/(((((((4\*1.962364415e+19)/(5\*0.0864055^2))))\*1/(8.896512e+11)\* sqrt[[-(((1.379421e+11 \* 4\*Pi\*(1.321000e-15)^3-(1.321000e-15)^2)))) / ((6.67\*10^-11))]]]]]]]

**Input interpretation:**

$$1 / \left( \sqrt{ \left( 1 / \left( \frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{8.896512 \times 10^{11}} \right) \sqrt{ - \frac{1.379421 \times 10^{11} \times 4 \pi (1.321000 \times 10^{-15})^3 - (1.321000 \times 10^{-15})^2}{6.67 \times 10^{-11}} } \right) } \right)$$

**Result:**

0.617951794657125805526971131347159581467907749388337508719...  
0.61795179....

From:

Preprint: N. Hamein, M. Hyson, E. A. Rauscher, *Proceedings of The Unified Theories Conference (2008)*, Scale Unification: **A Universal Scaling Law for Organized Matter**, in Cs Varga, I. Dienes & R.L. Amoroso (eds.)

$$M = \frac{8.988 \times 10^{20} \text{ cm}^2 / \text{s}^2 \times 1.321 \times 10^{-13} \text{ cm}}{2 \times 6.674 \times 10^{-8} \text{ cm}^3 / (\text{gm s}^2)} = \sim 8.898 \times 10^{14} \text{ gm}$$

that is the mass provided from the vacuum density of the Schwarzschild Proton.

and from:

**Quantum Gravity and the Holographic Mass**  
**Nassim Hamein**

We have that:

$$m_h = 1.683354 \times 10^{34} \text{ gm}$$

That is the “*holographic gravitational mass*”

From the difference between the values without exponent  $8.898 - 1.683354$ , we obtain 7,214646

$$m_p = 2.17645 \times 10^{-5} \text{ grams} = \text{Planck Mass}$$

From the mass of Schwarzschild Proton =  $8.898e+14$  gm, we have in conclusion:

$$-(34/10^3+8/10^3+3/10^3)*(1/10^{27})+\exp(7.214646)(\text{((((((((((((((1/2.17645e-5 * \text{[[[[[sqrt(\text{(((((((1/(8.898e+14))))^2(((2.17645e-5))))))]]))]]))]]))]]))]]))]]))]]))]]^2$$

**Input interpretation:**

$$-\left(\frac{34}{10^3} + \frac{8}{10^3} + \frac{3}{10^3}\right) \times \frac{1}{10^{27}} + \exp(7.214646) \left( \frac{1}{2.17645 \times 10^{-5}} \sqrt{\left(\frac{1}{8.898 \times 10^{14}}\right)^2 \times 2.17645 \times 10^{-5}} \right)$$

**Result:**

$$1.6717068650000460814749871746199805126462269918339186... \times 10^{-27}$$

$$1.671706865000046081474987174619980512646226992 \times 10^{-24} \text{ grams}$$

$$1.671706865... * 10^{-24} \text{ gm}$$

**Comparisons as mass:**

$$\approx 0.53 \times \text{tau particle mass } (\approx 3.2 \times 10^{-27} \text{ kg})$$

**Corresponding quantities:**

Relativistic energy  $E$  from  $E = mc^2$ :

$$938 \text{ MeV (megaelectronvolts)}$$

Note that  $938 = 9^3 + 10^3 - (1010^3 - 1 - 812^3)^{1/3}$ , indeed:

$$9^3 + 10^3 - \text{((((1010^3 - 1 - 812^3)^{1/3}}))$$

$$9^3 + 10^3 - \sqrt[3]{1010^3 - 1 - 812^3}$$

$$938$$

$$938$$

We observe that these numbers are sums of two cubes (see below Ramanujan’s manuscript)

ff

$$(i) \frac{1+53x+9x^2}{1-82x-82x^2+x^3} = a_0 + a_1x + a_2x^2 + a_3x^3 + \dots$$
$$\text{or } \frac{\alpha_0}{x} + \frac{\alpha_1}{x^2} + \frac{\alpha_2}{x^3} + \dots$$

$$(ii) \frac{2-26x-12x^2}{1-82x-82x^2+x^3} = b_0 + b_1x + b_2x^2 + b_3x^3 + \dots$$
$$\text{or } \frac{\beta_0}{x} + \frac{\beta_1}{x^2} + \frac{\beta_2}{x^3} + \dots$$

$$(iii) \frac{2+8x-10x^2}{1-82x-82x^2+x^3} = c_0 + c_1x + c_2x^2 + c_3x^3 + \dots$$
$$\text{or } \frac{\gamma_0}{x} + \frac{\gamma_1}{x^2} + \frac{\gamma_2}{x^3} + \dots$$

then

$$\left. \begin{aligned} a_n^3 + b_n^3 &= c_n^3 + (-1)^n \\ \text{and } \alpha_n^3 + \beta_n^3 &= \gamma_n^3 + (-1)^n \end{aligned} \right\}$$

Examples

$$135^3 + 138^3 = 172^3 - 1$$

$$11161^3 + 11468^3 = 14258^3 + 1$$

$$791^3 + 812^3 = 1010^3 - 1$$

$$9^3 + 10^3 = 12^3 + 1$$

$$6^3 + 8^3 = 9^3 - 1$$

Characteristic length  $L$  from  $L = h/(mc)$ :

1.3 fm (femtometers)

Thermal de Broglie wavelength at 100 K from  $\lambda = h/(2\pi mkT)^{1/2}$ :

174 pm (picometers)

Characteristic time  $T$  from  $T = h/(mc^2)$ :

$4.4 \times 10^{-24}$  seconds

Thermodynamic temperature  $T$  from  $kT = mc^2$ :

$1.088 \times 10^{13}$  K (kelvins)

Compton frequency  $\nu$  from  $\nu = mc^2/h$ :

$2.267 \times 10^{23}$  Hz (hertz)





$$G = \frac{4(3.839682 \times 10^{-20})^2 \times 1.054571817 \times 10^{-32} \times 2.99 \times 10^{10} \times 10^2}{(1.6714213 \times 10^{-24})^2} =$$

$$= 6.6561943088359539989386568039782503097413911161978359... \times 10^{-11}$$

$$= 6.6561943... * 10^{-11}$$

Further, we have:

For 2.26e-30 (see eq.15), 8.53e-30 (see eq.16) and 9.24e+55 (see eq.19), we obtain the following formula:

$$1.514376e+121 * ((( (9.24e+55 * (8.53e-30 + 2.26e-30)))) / 1.4018957360e+26$$

**Input interpretation:**

$$(1.514376 \times 10^{121}) \times \frac{9.24 \times 10^{55} (8.53 \times 10^{-30} + 2.26 \times 10^{-30})}{1.4018957360 \times 10^{26}}$$

**Result:**

$$1.0769893764025258437621783307856498109785248679863307... \times 10^{122}$$

$$1.0769893764... * 10^{122}$$

(We note that this result is very near to the following partial Ramanujan mock theta function:  $\varphi(q) = 1.075226 + 0.00572374 = 1.08094974$ , precisely to the value 1.075226)

*Thus, when the vacuum energy density of the Universe is considered in terms of the proton density and the protons PSU packing (i.e. its volume entropy, R) we find the density scales by a factor of  $10^{122}$*

Inserting this value  $1.0769893764 * 10^{122}$  in the formula to obtain G, together the other values already calculated, we obtain the following interesting equation:

$$(55/10^2 + 8/10^2 + (5+3)/10^3) * 1/10^{11} + ((((((1/(1.0769893764 * 10^{122}) * 1/((((4*(3.839682e-20)^2 * (1.054571817e-32) * (2.99e+10)*10^2)))))) / (((((9.10938e-28)^2))))))))))$$

**Input interpretation:**

$$\left( \frac{55}{10^2} + \frac{8}{10^2} + \frac{5+3}{10^3} \right) \times \frac{1}{10^{11}} +$$

$$\frac{1}{1.0769893764 \times 10^{122}} \times \frac{1}{\frac{4(3.839682 \times 10^{-20})^2 \times 1.054571817 \times 10^{-32} \times 2.99 \times 10^{10} \times 10^2}{(9.10938 \times 10^{-28})^2}}$$

**Result:**

6.6554582551865718690323325841431126250499266592470529... × 10<sup>-11</sup>

6.65545825... \* 10<sup>-11</sup> result practically equal to the

With regard the computation of G, we have also:

$$\alpha = \frac{\phi_e h}{8\pi r_l m_e c} = \frac{\phi_e \lambda_e}{8\pi r_l} = 7.29735(34) \times 10^{-3}$$

Where  $4\phi^2 = 5.897264 \times 10^{-39}$  is the exact value for the coupling constant between gravitation and confinement at the proton scale or the strong interaction. The typical computation given for the gravitational coupling constant is

$$\frac{F_g}{F_s} = \frac{F_g}{F_e} \frac{F_e}{F_s} = \frac{Gm_p m_p / r^2}{e^2 / r^2} \alpha = \frac{Gm_p^2}{e^2} \alpha = 5.905742 \times 10^{-39} \tag{53}$$

where  $e$  is the elementary charge and  $\alpha$  is the fine structure constant. Note that the slightly different value of equation (53) from  $4\phi^2$  of equation (52) is due to our utilization of the 2010 muonic measurement of the radius of the proton, and that utilizing our predicted radius  $r'_p$  from equation (30) yields the exact value.

$$(((5.905742e-39))) / (((((((((((((((0.0072973534 * (1.672622e-24)^2)))) / (1.602176e-19)^2)))))))))) * (8.988e+18)$$

**Input interpretation:**

$$\frac{5.905742 \times 10^{-39}}{\frac{0.0072973534 (1.672622 \times 10^{-24})^2}{(1.602176 \times 10^{-19})^2}} \times 8.988 \times 10^{18}$$

**Result:**

6.6741657706017542804754928814935962215932678504339047... × 10<sup>-8</sup>

6.67416577... \* 10<sup>-8</sup>

6.6741657706017542804754928814935962215932678504339047 × 10<sup>-8</sup> cm<sup>3</sup> \* gm<sup>-1</sup> \* s<sup>-2</sup>

**Input interpretation:**

6.674165770601754280475492881493596221593267850433905 × 10<sup>-8</sup> cm<sup>3</sup>/(g s<sup>2</sup>)  
(centimeters cubed per gram second squared)

**Result:**

$6.674165770601754280475492881493596221593267850433905 \times 10^{-11} \text{ m}^3/(\text{kg s}^2)$   
 (meters cubed per kilogram second squared)

$6.67416577... \times 10^{-11}$

**Interpretation:**

Newtonian gravitational coupling

Or, for  $1.6714213 \times 10^{-24}$ , we obtain:

**Input interpretation:**

$$\frac{5.905742 \times 10^{-39}}{\frac{0.0072973534 (1.6714213 \times 10^{-24})^2}{(1.602176 \times 10^{-19})^2}} \times 8.988 \times 10^{18}$$

**Result:**

$6.6837582644077303468404509459589164713210393908523003... \times 10^{-8}$

$6.6837582... \times 10^{-8}$

$6.6837582644077303468404509459589164713210393908523003 \times 10^{-8} \text{ cm}^3 * \text{ gm}^{-1} * \text{ s}^{-2}$

**Input interpretation:**

$6.6837582644077303468404509459589164713210393908523003 \times 10^{-8} \text{ cm}^3/(\text{g s}^2)$   
 (centimeters cubed per gram second squared)

**Result:**

$6.6837582644077303468404509459589164713210393908523 \times 10^{-11} \text{ m}^3/(\text{kg s}^2)$   
 (meters cubed per kilogram second squared)

$6.68375826... \times 10^{-11}$

**Interpretation:**

Newtonian gravitational coupling

$$-(\frac{55}{10^3} - \frac{3}{10^3}) * \frac{1}{10^8} + \frac{1}{4} * (((5.905742e-39))) / (((((((((((((0.0072973534 * (1.6714213e-24)^2)))) / (1.602176e-19)^2)))))))) * (8.988e+18)$$

**Input interpretation:**

$$-\left(\frac{55}{10^3} - \frac{3}{10^3}\right) \times \frac{1}{10^8} + \frac{1}{4} \times \frac{5.905742 \times 10^{-39}}{\frac{0.0072973534 (1.6714213 \times 10^{-24})^2}{(1.602176 \times 10^{-19})^2}} \times 8.988 \times 10^{18}$$

**Result:**

1.6189395661019325867101127364897291178302598477130750... × 10<sup>-8</sup>  
 1.6189395661... \* 10<sup>-8</sup>

This result is a sub-multiple that is a very good approximation to the value of the golden ratio 1,618033988749...

From the electron mass 9.10938e-28 gm (9.10938×10<sup>-31</sup> kg), we obtain:

$$\text{Mass} = 9.109380\text{e-}31$$

$$\text{Radius} = 1.352608\text{e-}57$$

$$\text{Temperature} = 1.347186\text{e+}53$$

From the Ramanujan-Nardelli mock formula, we obtain:

$$\sqrt{\left[ \left[ \left[ \left[ \left[ \left[ \frac{1}{\left( \frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \right) \times \frac{1}{9.109380 \times 10^{-31}} \right) \right] \right] \right] \right] \sqrt{\left[ \left[ \left[ \left[ \left[ \frac{1.347186 \times 10^{53} \times 4 \pi (1.352608 \times 10^{-57})^3 - (1.352608 \times 10^{-57})^2}{6.67 \times 10^{-11}} \right] \right] \right] \right] \right] \right]$$

**Input interpretation:**

$$\sqrt{\left( \left( \left( \left( \left( \frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \right) \times \frac{1}{9.109380 \times 10^{-31}} \right) \right) \right) \sqrt{\left( \frac{1.347186 \times 10^{53} \times 4 \pi (1.352608 \times 10^{-57})^3 - (1.352608 \times 10^{-57})^2}{6.67 \times 10^{-11}} \right) \right) \right) \right)$$

**Result:**

1.618249047788083177656530807889395871876048366303506550226...  
 1.61824904...

And:

$$1/\sqrt{\left[ \left[ \left[ \left[ \left[ \left[ \frac{1}{\left( \frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \right) \times \frac{1}{9.109380 \times 10^{-31}} \right) \right] \right] \right] \right] \sqrt{\left[ \left[ \left[ \left[ \left[ \frac{1.347186 \times 10^{53} \times 4 \pi (1.352608 \times 10^{-57})^3 - (1.352608 \times 10^{-57})^2}{6.67 \times 10^{-11}} \right] \right] \right] \right] \right] \right]$$

**Input interpretation:**

$$1/\left(\sqrt{\left(1/\left(\frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{9.109380 \times 10^{-31}}\right)\right.\right. \\ \left.\left.\sqrt{-\frac{1.347186 \times 10^{53} \times 4 \pi (1.352608 \times 10^{-57})^3 - (1.352608 \times 10^{-57})^2}{6.67 \times 10^{-11}}}\right)\right)}$$

**Result:**

0.617951854423679780438048053044947539632031140339181540028...

0.61795185...

$$27/10^3 + \text{sqrt}[\left[\left[\left[\left[\left[\left[\left[\left[1/\left(\frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2}\right) \times \frac{1}{9.109380 \times 10^{-31}}\right]\right]\right]\right]\right]\right]\right]\right] \\ * \text{sqrt}[\left[-\left(\left(\left(1.347186 \times 10^{53} * 4 * \pi * (1.352608 \times 10^{-57})^3 - (1.352608 \times 10^{-57})^2\right)\right)\right) / \right. \\ \left. \left(\left(6.67 * 10^{-11}\right)\right)\right]\right]\right]\right]$$

**Input interpretation:**

$$\frac{27}{10^3} + \sqrt{\left(1/\left(\frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{9.109380 \times 10^{-31}}\right)\right. \\ \left.\sqrt{-\frac{1.347186 \times 10^{53} \times 4 \pi (1.352608 \times 10^{-57})^3 - (1.352608 \times 10^{-57})^2}{6.67 \times 10^{-11}}}\right)}$$

**Result:**

1.645249047788083177656530807889395871876048366303506550226...

1.64524904...  $\approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$

And:

$$2\text{sqrt}(\left(\left(\left(6 * \left(\left(\left(27/10^3 + \text{sqrt}[\left[\left[\left[\left[\left[\left[\left[1/\left(\frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2}\right) \times \frac{1}{9.109380 \times 10^{-31}}\right]\right]\right]\right]\right]\right]\right]\right]\right] * \text{sqrt}[\left[-\left(\left(\left(1.347186 \times 10^{53} * 4 * \pi * (1.352608 \times 10^{-57})^3 - (1.352608 \times 10^{-57})^2\right)\right)\right) / \left(\left(6.67 * 10^{-11}\right)\right)\right]\right]\right]\right]\right)\right)$$

**Input interpretation:**

$$2 \sqrt{\left(6 \left(\frac{27}{10^3} + \sqrt{\left(1/\left(\frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{9.109380 \times 10^{-31}}\right)\right.}\right.\right. \\ \left.\left.\sqrt{\left(-\frac{1}{6.67 \times 10^{-11}} (1.347186 \times 10^{53} \times 4 (\pi (1.352608 \times 10^{-57})^3 - (1.352608 \times 10^{-57})^2)\right)\right)\right)\right)}$$

**Result:**

6.283786847667097329877946490266684818210441032142038821155...

$6.28378684\dots \approx 2\pi$

The difference with  $2\pi$  is:

$6.283786847667 / (2\pi)$

**Input interpretation:**

$$\frac{6.283786847667}{2\pi}$$

**Result:**

1.000095738142...

1.000095738142...

**Alternative representations:**

$$\frac{6.2837868476670000}{2\pi} = -\frac{6.2837868476670000}{2i \log(-1)}$$

$$\frac{6.2837868476670000}{2\pi} = \frac{6.2837868476670000}{2 \cos^{-1}(-1)}$$

**Series representations:**

$$\frac{6.2837868476670000}{2\pi} = \frac{0.78547335595837500}{\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}}$$

$$\frac{6.2837868476670000}{2\pi} = \frac{1.5709467119167500}{-1.000000000000000000 + \sum_{k=1}^{\infty} \frac{2^k}{\binom{2k}{k}}}$$

$$\frac{6.2837868476670000}{2\pi} = \frac{3.1418934238335000}{\sum_{k=0}^{\infty} \frac{2^{-k}(-6+50k)}{\binom{3k}{k}}}$$

$\binom{n}{m}$  is the binomial coefficient



**Integral representations:**

$$\frac{6.2837868476670000}{2\pi} = \frac{1.5709467119167500}{\int_0^\infty \frac{1}{1+t^2} dt}$$

$$\frac{6.2837868476670000}{2\pi} = \frac{0.78547335595837500}{\int_0^1 \sqrt{1-t^2} dt}$$

$$\frac{6.2837868476670000}{2\pi} = \frac{1.5709467119167500}{\int_0^\infty \frac{\sin(t)}{t} dt}$$

Thence we haven't perfect circumferences of unitary radii, because the circles in the context of M-Theory, are not stable, as they are subject to vibrations that are equivalent to various frequencies. This is the reason why the values of the golden ratio,  $\pi$  and  $\zeta(2)$ , vary, also if very little:  
1.000095738142

We have that:

Moreover, we find that the only radius at which the holographic ratio equals one (i.e.  $R = \eta$ ), where all the volume information is encoded on the surface, is the condition

$$r_{S_\ell} = \frac{2Gm_\ell}{c^2} = 2\ell \tag{Eqn. 8}$$

where  $r_{S_\ell}$  is the Schwarzschild radius of a black hole with mass  $m = m_\ell$ .

In this case, the surface entropy  $\eta$  and the volume entropy  $R$  are thus calculated to be,

$$\eta_\ell = \frac{4\pi r_{S_\ell}^2}{\pi r_\ell^2} = \frac{4\pi(2\ell)^2}{\pi(\ell/2)^2} = 64 \tag{Eqn. 9}$$

$$R_\ell = \frac{r_{S_\ell}^3}{r_\ell^3} = \frac{(2\ell)^3}{(\ell/2)^3} = 64$$

Eqn. 10

Now, we multiplied eqs.(9) and (10):

$$\left(\frac{4\pi(2 \times 1.616252 \times 10^{-35})^2}{\pi\left(\frac{1}{2} \times 1.616252 \times 10^{-35}\right)^2}\right) \times \left(\frac{(2 \times 1.616252 \times 10^{-35})^3}{\left(\frac{1}{2} \times 1.616252 \times 10^{-35}\right)^3}\right)$$

**Input interpretation:**

$$\frac{4\pi(2 \times 1.616252 \times 10^{-35})^2}{\pi\left(\frac{1}{2} \times 1.616252 \times 10^{-35}\right)^2} \times \frac{(2 \times 1.616252 \times 10^{-35})^3}{\left(\frac{1}{2} \times 1.616252 \times 10^{-35}\right)^3}$$

**Result:**

4096

4096

$$\frac{1}{4} \left(\frac{4\pi(2 \times 1.616252 \times 10^{-35})^2}{\pi\left(\frac{1}{2} \times 1.616252 \times 10^{-35}\right)^2}\right) \times \left(\frac{(2 \times 1.616252 \times 10^{-35})^3}{\left(\frac{1}{2} \times 1.616252 \times 10^{-35}\right)^3}\right)$$

**Input interpretation:**

$$\frac{1}{4} \times \frac{4\pi(2 \times 1.616252 \times 10^{-35})^2}{\pi\left(\frac{1}{2} \times 1.616252 \times 10^{-35}\right)^2} \times \frac{(2 \times 1.616252 \times 10^{-35})^3}{\left(\frac{1}{2} \times 1.616252 \times 10^{-35}\right)^3}$$

**Result:**

1024

1024

and minus 5, we obtain:

**Input interpretation:**

$$-5 + \frac{1}{4} \times \frac{4\pi(2 \times 1.616252 \times 10^{-35})^2}{\pi\left(\frac{1}{2} \times 1.616252 \times 10^{-35}\right)^2} \times \frac{(2 \times 1.616252 \times 10^{-35})^3}{\left(\frac{1}{2} \times 1.616252 \times 10^{-35}\right)^3}$$

**Result:**

1019

1019 result practically equal to the rest mass of Phi meson 1019.445

$$27 \times \sqrt{\left(\frac{4\pi(2 \times 1.616252 \times 10^{-35})^2}{\pi\left(\frac{1}{2} \times 1.616252 \times 10^{-35}\right)^2}\right) \times \left(\frac{(2 \times 1.616252 \times 10^{-35})^3}{\left(\frac{1}{2} \times 1.616252 \times 10^{-35}\right)^3}\right)}$$

**Input interpretation:**

$$27 \sqrt{\frac{4\pi(2 \times 1.616252 \times 10^{-35})^2}{\pi\left(\frac{1}{2} \times 1.616252 \times 10^{-35}\right)^2} \times \frac{(2 \times 1.616252 \times 10^{-35})^3}{\left(\frac{1}{2} \times 1.616252 \times 10^{-35}\right)^3}}$$

**Result:**

1728

1728

$$55+27* \text{sqrt}(\frac{(((((4\text{Pi}*(2*1.616252\text{e-}35)^2) / ((\text{Pi}*(1/2*1.616252\text{e-}35)^2)))) * (((2*1.616252\text{e-}35)^3 / (1/2*1.616252\text{e-}35)^3))))))$$

**Input interpretation:**

$$55 + 27 \sqrt{\frac{4\pi(2 \times 1.616252 \times 10^{-35})^2}{\pi(\frac{1}{2} \times 1.616252 \times 10^{-35})^2} \times \frac{(2 \times 1.616252 \times 10^{-35})^3}{(\frac{1}{2} \times 1.616252 \times 10^{-35})^3}}$$

**Result:**

1783

1783

We have also that:

$$27*1/2*(((((4\text{Pi}*(2*1.616252\text{e-}35)^2) / ((\text{Pi}*(1/2*1.616252\text{e-}35)^2)))) + (((2*1.616252\text{e-}35)^3 / (1/2*1.616252\text{e-}35)^3))))$$

**Input interpretation:**

$$27 \times \frac{1}{2} \left( \frac{4\pi(2 \times 1.616252 \times 10^{-35})^2}{\pi(\frac{1}{2} \times 1.616252 \times 10^{-35})^2} + \frac{(2 \times 1.616252 \times 10^{-35})^3}{(\frac{1}{2} \times 1.616252 \times 10^{-35})^3} \right)$$

**Result:**

1728

1728

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson. Furthermore, 1728 occurs in the algebraic formula for the  $j$ -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

And:

$$55+27*1/2*(((((4\text{Pi}*(2*1.616252\text{e-}35)^2) / ((\text{Pi}*(1/2*1.616252\text{e-}35)^2)))) + (((2*1.616252\text{e-}35)^3 / (1/2*1.616252\text{e-}35)^3))))$$

**Input interpretation:**

$$55 + 27 \times \frac{1}{2} \left( \frac{4 \pi (2 \times 1.616252 \times 10^{-35})^2}{\pi (\frac{1}{2} \times 1.616252 \times 10^{-35})^2} + \frac{(2 \times 1.616252 \times 10^{-35})^3}{(\frac{1}{2} \times 1.616252 \times 10^{-35})^3} \right)$$

**Result:**

1783

1783 result in the range of the hypothetical mass of Gluino (gluino = 1785.16 GeV).

$$\left( \left( \left( \left( \left( \left( \left( \left( \left( \left( \left( \left( \left( 55 + 27 \times \frac{1}{2} \times \left( \frac{4 \pi (2 \times 1.616252 \times 10^{-35})^2}{\pi (\frac{1}{2} \times 1.616252 \times 10^{-35})^2} + \frac{(2 \times 1.616252 \times 10^{-35})^3}{(\frac{1}{2} \times 1.616252 \times 10^{-35})^3} \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right)^{1/15}$$

**Input interpretation:**

$$\sqrt[15]{55 + 27 \times \frac{1}{2} \left( \frac{4 \pi (2 \times 1.616252 \times 10^{-35})^2}{\pi (\frac{1}{2} \times 1.616252 \times 10^{-35})^2} + \frac{(2 \times 1.616252 \times 10^{-35})^3}{(\frac{1}{2} \times 1.616252 \times 10^{-35})^3} \right)}$$

**Result:**

1.647189...

$$1.647189... \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 ...$$

From the sum of  $\eta_l$  and  $R_l$ , that are the surface entropy and the volume entropy, we obtain 128. Inserting this value in the Hawking radiation calculator, we have:

Mass = 1.054165e-7

Radius = 1.565279e-34

Temperature = 1.164147e+30

From the Ramanujan-Nardelli mock formula, we obtain:

$$\sqrt{\left[ \left[ \left[ \left[ \frac{1}{\left( \frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \right)} \times \frac{1}{1.054165 \times 10^{-7}} \right] \times \sqrt{\left[ - \left( \frac{1.164147 \times 10^{30} \times 4 \times \pi \times (1.565279 \times 10^{-34})^3 - (1.565279 \times 10^{-34})^2 \right)}{\left( 6.67 \times 10^{-11} \right)} \right]} \right]} \right]} \right]$$

**Input interpretation:**

$$\sqrt{\left(1/\left(\frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{1.054165 \times 10^{-7}}\right) \sqrt{-\frac{1.164147 \times 10^{30} \times 4\pi (1.565279 \times 10^{-34})^3 - (1.565279 \times 10^{-34})^2}{6.67 \times 10^{-11}}}\right)}$$

**Result:**

1.618248985866481236768366985260938671526618405922319964377...

1.61824898...

And:

1/sqrt[[[1/((((((4\*1.962364415e+19)/(5\*0.0864055^2))) \* 1/(1.054165e-7) \* sqrt[[-(((1.164147e+30 \* 4\*Pi\*(1.565279e-34)^3 - (1.565279e-34)^2)))] / ((6.67\*10^-11)))]]]]]]

**Input interpretation:**

$$1/\left(\sqrt{\left(1/\left(\frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{1.054165 \times 10^{-7}}\right) \sqrt{-\frac{1.164147 \times 10^{30} \times 4\pi (1.565279 \times 10^{-34})^3 - (1.565279 \times 10^{-34})^2}{6.67 \times 10^{-11}}}\right)}$$

**Result:**

0.617951878069341901936766247077092767732184040091205524185...

0.61795187...

Now, from:

## THE ORIGIN OF SPIN: A CONSIDERATION OF TORQUE AND CORIOLIS FORCES IN EINSTEIN'S FIELD EQUATIONS AND GRAND UNIFICATION THEORY

N. Hameiri and E.A. Rauscher

We have that:

$$\hbar = 1.054571817 \cdot 10^{-34} \text{ J} \cdot \text{s}$$

$$\underline{L}^2 = \frac{3\hbar^2}{4}$$

$$\underline{\tau} = \dot{\underline{L}} = \frac{d\underline{L}}{dt} \neq 0$$

$$L^2 = 3*(1.054571817e-34)^2 / 4$$

**Input interpretation:**

$$3\left(\frac{1}{4}(1.054571817 \times 10^{-34})^2\right)$$

**Result:**

$$8.3409128790801111675 \times 10^{-69}$$

$$8.34091287908... * 10^{-69}$$

From which:

$$L = \text{sqrt}(\text{sqrt}(\text{sqrt}(\text{sqrt}(3*(1.054571817e-34)^2 / 4))))$$

**Input interpretation:**

$$\sqrt{3\left(\frac{1}{4}(1.054571817 \times 10^{-34})^2\right)}$$

**Result:**

$$9.132859836... \times 10^{-35}$$

$$9.132859836... * 10^{-35}$$

$$\underline{\tau} = \frac{d\underline{L}}{dt} = \dot{\underline{L}}$$

(39)

$$F = c^4 / G$$

in units of dynes. The units of the left side of the field equations are in  $\text{cm}^2$ , or length squared. The quantity  $\ell$  is in cm and

$$(40) \quad \ell = \left(\frac{G\hbar}{c^3}\right)^{1/2}$$

which is the Planck length and can be written as

$$(41) \quad \ell = \left(\frac{c\hbar}{F}\right)^{1/2}$$

for the fundamental force in equation (39). Now we can write the torque term as

$$(42) \quad \frac{8\pi}{F} \left(\frac{c\hbar}{F}\right)^{1/2} \tau^{\mu\nu} = \frac{8\pi(c\hbar)^{1/2}}{F^{3/2}} \tau^{\mu\nu}.$$

$$F = (((2.99 \times 10^8)^4)) / (((6.6561943 \times 10^{-11})))$$

**Input interpretation:**

$$\frac{(2.99 \times 10^8)^4}{6.6561943 \times 10^{-11}}$$

**Result:**

$$1.2007670510760180182841116882660711992737351432184003... \times 10^{44}$$

$$1.200767051076... \times 10^{44}$$

From:

$$\frac{8\pi}{F} \left( \frac{c\hbar}{F} \right)^{1/2} \tau^{\mu\nu} = \frac{8\pi(c\hbar)^{1/2}}{F^{3/2}} \tau^{\mu\nu} .$$

We obtain:

$$(9.132859836e-35) * 8 * \text{Pi} * \text{sqrt}(((2.99 * 10^8 * 1.054571817e-34))) / ((1.200767051076 * 10^{44})^{1.5})$$

$$9.132859836... * 10^{-35}$$

$$(9.132859836e-35) * (((8 * \text{Pi} * \text{sqrt}(((2.99 * 10^8 * 1.054571817e-34)))) / (((1.200767051076 * 10^{44})^{1.5}))))$$

**Input interpretation:**

$$9.132859836 \times 10^{-35} \left( 8\pi \times \frac{\sqrt{2.99 \times 10^8 \times 1.054571817 \times 10^{-34}}}{(1.200767051076 \times 10^{44})^{1.5}} \right)$$

**Result:**

$$3.097648840138602096455016437910830021154362930030031... \times 10^{-112}$$

$$3.0976488401386... * 10^{-112}$$

Now, we have that:

$$1 / (((((((((9.132859836e-35) * (((8 * \text{Pi} * \text{sqrt}(((2.99 * 10^8 * 1.054571817e-34)))) / (((1.200767051076 * 10^{44})^{1.5}))))))))))))))^{1/500}$$

**Input interpretation:**





This result is a very good approximation to the value of the golden ratio  
1,618033988749...

$$2 \times 55 + 10^3 / \left( \left( \left( \left( \left( \left( \left( 9.132859836 \times 10^{-35} \right) * \left( \left( 8 * \pi * \sqrt{\left( \left( 2.99 * 10^8 * 1.054571817 \times 10^{-34} \right) \right) / \left( \left( \left( 1.200767051076 * 10^{44} \right)^{1.5} \right) \right) \right) \right) \right) \right) \right) \right) \right)^{1/500}$$

**Input interpretation:**

$$2 \times 55 + \frac{10^3}{\sqrt[500]{9.132859836 \times 10^{-35} \left( 8 \pi \times \frac{\sqrt{2.99 \times 10^8 \times 1.054571817 \times 10^{-34}}}{(1.200767051076 \times 10^{44})^{1.5}} \right)}}$$

**Result:**

1781.160...

1781.16... result in the range of the hypothetical mass of Gluino (gluino = 1785.16 GeV).

$$-13 - 55 + 7 * \operatorname{colog} \left( \left( \left( \left( \left( \left( \left( 9.132859836 \times 10^{-35} \right) * \left( \left( 8 * \pi * \sqrt{\left( \left( 2.99 * 10^8 * 1.054571817 \times 10^{-34} \right) \right) / \left( \left( \left( 1.200767051076 * 10^{44} \right)^{1.5} \right) \right) \right) \right) \right) \right) \right) \right) \right)$$

**Input interpretation:**

$$-13 - 55 + 7 \left( -\log \left( 9.132859836 \times 10^{-35} \left( 8 \pi \times \frac{\sqrt{2.99 \times 10^8 \times 1.054571817 \times 10^{-34}}}{(1.200767051076 \times 10^{44})^{1.5}} \right) \right) \right)$$

log(x) is the natural logarithm

**Result:**

1729.31...

1729.31...

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

$$-21/(10^2)+\text{sqrt} [\text{colog} (\text{((((((((9.132859836e-35)* ((8*\text{Pi}*\text{sqrt}(((2.99*10^8 *1.054571817e-34)))) / (((1.200767051076*10^44)^1.5)))))))))])]$$

**Input interpretation:**

$$-\frac{21}{10^2} + \sqrt{-\log \left( 9.132859836 \times 10^{-35} \left( 8 \pi \times \frac{\sqrt{2.99 \times 10^8 \times 1.054571817 \times 10^{-34}}}{(1.200767051076 \times 10^{44})^{1.5}} \right) \right)}$$

log(x) is the natural logarithm

**Result:**

15.81370...

15.81370... result practically equal to the black hole entropy 15.8174

We have the following two equations:

From:

**Advanced Geometric Physics Solutions**

by Mark Rohrbaugh for <http://fractalU.com>

BSEE -minor in solid-state physics -University of Cincinnati

MSEE -Southern Methodist University - September 11, 2016

$$\mu = \frac{m_p}{m_e} = \frac{\alpha^2}{\pi r_p R_H} = 1836.15267 \dots$$

From:

**Quantum Gravity and the Holographic Mass**

*Nassim Haramein*

$$m_{p'} = 2 \frac{\eta_p}{R} = 1.6714213 \times 10^{-24} \text{ gm} .$$

From Wikipedia:

## Rydberg constant

The CODATA value is<sup>[2]</sup>

$$R_{\infty} = \frac{m_e e^4}{8\epsilon_0^2 \hbar^3 c} = 10\,973\,731.568\,160(21) \text{ m}^{-1},$$

where

$m_e$  is the rest mass of the electron,  
 $e$  is the elementary charge,  
 $\epsilon_0$  is the permittivity of free space,  
 $\hbar$  is the Planck constant, and  
 $c$  is the speed of light in vacuum.

The Rydberg constant for hydrogen may be calculated from the reduced mass of the electron:

$$R_H = R_{\infty} \frac{m_p}{m_e + m_p} \approx 1.09678 \times 10^7 \text{ m}^{-1},$$

where

$m_e$  is the mass of the electron,  
 $m_p$  is the mass of the nucleus (a proton).

$$R_H = \frac{R_{\infty}}{1 + m_e/m_p} = \frac{R_{\infty}}{1 + 1/1836}$$

The Rydberg constant “to infinity” is:

$$R_{\infty} = \frac{m_e e^4}{(4\pi\epsilon_0)^2 \hbar^3 4\pi c} = \frac{m_e c^2 e^4}{(4\pi\epsilon_0)^2 (\hbar c)^3 4\pi} = \frac{m_e c^2 \alpha^2}{2\hbar c} = \frac{m_e e^4}{8\epsilon_0^2 \hbar^3 c} = 1.097\,373\,156\,850\,8(65) \times 10^7 \text{ m}^{-1}$$

From the Ramanujan’s cube sum

$$791^3 + 812^3 = 1010^3 - 1;$$

$$791^3 = 1010^3 - 1 - 812^3; 791 = (1010^3 - 1 - 812^3)^{1/3}$$

$$812^3 = 1010^3 - 1 - 791^3; 812 = (1010^3 - 1 - 791^3)^{1/3}$$

$$(791 + 812) / 1836.15267 = 0,87302108707550990299733627269676... (1)$$

$$1/ (0,87302108707550990299733627269676)^4 = 1,7214763657539833898...$$

$$1,7214763657539833898 - 5/10^2 = 1,67147636575398338...$$

With the following Ramanujan mock theta function value 0.8730077..., that is very closed to the result of division (1) we obtain:

$$1/ (0,8730077)^4 - 0.05 = 1,671581959618447480....$$

We have, in conclusion:

$$((1/(((791 + 812)* 1/1836.15267))))^4 - 5/10^2$$

**Input interpretation:**

$$\left( \frac{1}{(791 + 812) \times \frac{1}{1836.15267}} \right)^4 - \frac{5}{10^2}$$

**Result:**

$$1.671476365753983389870264368782546302103094287709219356422...$$

$$1.67147636...$$

And, with the Ramanujan mock theta function:

$$1/ (0.8730077)^4 - 5/10^2$$

**Input interpretation:**

$$\frac{1}{0.8730077^4} - \frac{5}{10^2}$$

**Result:**

$$1.671581959618447480750458294352470315640326571296545806115...$$

$$1.671581956...$$

We note that the two above results are practically equals to the value of the formula:

$$m_{p'} = 2 \times \frac{\eta}{R} m_P = 1.6714213 \times 10^{-24} \text{ gm}$$

that is the holographic proton mass

Furthermore:



$$\pi \sqrt[15]{20x^5 - 223x^4 + 613x^3 - 450x^2 + 523x - 728} \text{ near } x = 7.33674 \approx 23.0490615431599233355654$$

$$\pi \sqrt[15]{10113x^3 - 70470x^2 - 28051x + 5214} \text{ near } x = 7.33674 \approx 23.04906154315992333325323$$

We have also:

$$24 \times 3 * -(-24.24857397644107 + 22.55083157783097 - 22.34777907520773) - 2$$

**Input interpretation:**

$$24 \times 3 \times (-1)(-24.24857397644107 + 22.55083157783097 - 22.34777907520773) - 2$$

**Result:**

$$1729.27754611488376$$

$$1729.27...$$

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson. Furthermore, 1728 occurs in the algebraic formula for the  $j$ -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

$$(((24 \times 3 * -(-24.24857397 + 22.55083157 - 22.34777907) - 2)))^{1/15}$$

**Input interpretation:**

$$\sqrt[15]{24 \times 3 \times (-1)(-24.24857397 + 22.55083157 - 22.34777907) - 2}$$

**Result:**

$$1.6438328189...$$

$$1.643832... \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 ...$$

And:

$$-(5^2)/(10^3) + (((24 \times 3 * -(-24.24857397 + 22.55083157 - 22.34777907) - 2)))^{1/15}$$

**Input interpretation:**

$$-\frac{5^2}{10^3} + \sqrt[15]{24 \times 3 \times (-1)(-24.24857397 + 22.55083157 - 22.34777907) - 2}$$

**Result:**

1.6188328189...

1.618832...

This result is a very good approximation to the value of the golden ratio

1,618033988749...

From 1836.15267, we obtain:

$$21/10^3 + (((1836.15267)))^{1/15}$$

**Input interpretation:**

$$\frac{21}{10^3} + \sqrt[15]{1836.15267}$$

**Result:**

1.671417877...

1.671417877... We note that 1.671417877... is a result practically equal to the value of the formula:

$$m_{p'} = 2 \times \frac{\eta}{R} m_P = 1.6714213 \times 10^{-24} \text{ gm}$$

that is the holographic proton mass

And:

$$(5*2)/10^3 - (21*2)/10^3 + (((1836.15267)))^{1/15}$$

**Input interpretation:**

$$\frac{5 \times 2}{10^3} - \frac{21 \times 2}{10^3} + \sqrt[15]{1836.15267}$$

**Result:**

1.618417877...

1.618417877

This result is a very good approximation to the value of the golden ratio

1,618033988749...

From the Rydberg constant “to infinity”, we obtain:



$$1.0973731568508/(1+1/1836.15267)$$

**Input interpretation:**

$$\frac{1.0973731568508}{1 + \frac{1}{1836.15267}}$$

**Result:**

$$1.096775834061697883625534507156664339714347202293209524062... \\ 1.09677583...$$

$$13-2+64 * \exp^3(1.09677583406)$$

**Input interpretation:**

$$13 - 2 + 64 \exp^3(1.09677583406)$$

**Result:**

$$1729.5059963... \\ 1729.5059...$$

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson. Furthermore, 1728 occurs in the algebraic formula for the  $j$ -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

$$((((13-2+64 * \exp^3(1.09677583406))))^1/15$$

**Input interpretation:**

$$\sqrt[15]{13 - 2 + 64 \exp^3(1.09677583406)}$$

**Result:**

$$1.643847295486... \\ 1.6438472... \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 ...$$

$$-(5^2/10^3)+((((13-2+64 * \exp^3(1.09677583406))))^1/15$$

**Input interpretation:**

$$-\frac{5^2}{10^3} + \sqrt[15]{13 - 2 + 64 \exp^3(1.09677583406)}$$

**Result:**

1.618847295486...

1.618847...

This result is a very good approximation to the value of the golden ratio  
1,618033988749...

Ratio between surface area of SMBH87 calculated by M and surface area of  
Schwarzschild proton

$$A = 4\pi R^2 = \frac{16\pi G^2}{c^4} M^2$$

Surface area SMBH87 calculated by M

$$\frac{(((((16 * \pi (6.6561943e-11)^2 * (13.12806e+39)^2))))))}{(2.99e+8)^4}$$

**Input interpretation:**

$$\frac{16 \pi ((6.6561943 \times 10^{-11})^2 (13.12806 \times 10^{39})^2)}{(2.99 \times 10^8)^4}$$

**Result:**4.80218... × 10<sup>27</sup>4.80218... \* 10<sup>27</sup> m<sup>2</sup>

Surface area of BH by Schwarzschild proton radius

$$4 * \pi (1.321e-15)^2$$

**Input interpretation:**

$$4 \pi (1.321 \times 10^{-15})^2$$

**Result:**2.19288... × 10<sup>-29</sup>2.19288... \* 10<sup>-29</sup> m<sup>2</sup>

From the inverse of previous formula, we obtain:

$$1/(((((((16*\text{Pi} (6.6561943\text{e-}11)^2 * (13.12806\text{e+}39)^2)))))/(2.99\text{e+}8)^4)))$$

**Input interpretation:**

$$\frac{1}{16\pi \left( (6.6561943 \times 10^{-11})^2 (13.12806 \times 10^{39})^2 \right)} \\ (2.99 \times 10^8)^4$$

**Result:**

$$2.08239... \times 10^{-28}$$

$$2.08239... * 10^{-28} \text{ m}^{-2}$$

We have from:

$$(((0,639 + 0,613873542)/2+0.637)))/2 = 0.6317183855$$

And:

$$(1/0.6317183855) * ((\text{Pi}^2)/6)*1/8 * \text{sqrt}((((4.80218 * 10^{27})/(2.19288 * 10^{-29}))))$$

**Input interpretation:**

$$\frac{1}{0.6317183855} \times \frac{\pi^2}{6} \times \frac{1}{8} \sqrt{\frac{4.80218 \times 10^{27}}{\frac{2.19288}{10^{29}}}}$$

**Result:**

$$4.8166686952058581026800142328526883479856907981606713... \times 10^{27}$$

$$4.816668... * 10^{27}$$

**Series representations:**

$$\sqrt{\frac{\frac{4.80218 \times 10^{27}}{2.19288}}{10^{29}}} \pi^2 \\ (0.631718 \times 8) 6 = 0.0329788 \pi^2 \sqrt{2.1899 \times 10^{56}} \sum_{k=0}^{\infty} e^{-129.729k} \left( \frac{1}{2} \right) \binom{1}{k}$$

•

$$\frac{\sqrt{\frac{4.80218 \times 10^{27}}{2.19288} \pi^2}}{(0.631718 \times 8)6} = 0.0329788 \pi^2 \sqrt{2.1899 \times 10^{56}} \sum_{k=0}^{\infty} \frac{(-4.56643 \times 10^{-57})^k \left(-\frac{1}{2}\right)_k}{k!}$$

$$\frac{\sqrt{\frac{4.80218 \times 10^{27}}{2.19288} \pi^2}}{(0.631718 \times 8)6} = \frac{0.0164894 \pi^2 \sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} e^{-129.729s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{\sqrt{\pi}}$$

$\binom{n}{m}$  is the binomial coefficient

$n!$  is the factorial function

$(a)_n$  is the Pochhammer symbol (rising factorial)

$\Gamma(x)$  is the gamma function

$\text{Res}_{z=z_0} f$  is a complex residue

Note that from the result  $2.08239... \times 10^{-28} \text{ m}^{-2}$ , we obtain:

$$(1.0061571663^8)/10 * 1/(((((((16*\text{Pi} (6.6561943\text{e-}11)^2 * (13.12806\text{e+}39)^2)))))/(2.99\text{e+}8)^4)))$$

where 1.0061571663 is a Ramanujan mock theta function

### Input interpretation:

$$\frac{1.0061571663^8}{10} \times \frac{1}{\frac{16\pi ((6.6561943 \times 10^{-11})^2 (13.12806 \times 10^{39})^2)}{(2.99 \times 10^8)^4}}$$

### Result:

$$2.18720... \times 10^{-29}$$

$$2.18720... \times 10^{-29}$$

Or:

$$(1.1424432422)/(13.9766-\pi) * 1/(((((((16*\pi (6.6561943e-11)^2 * (13.12806e+39)^2)))))))/(2.99e+8)^4))$$

Where 1.1424432422 and 13.9766 are two Ramanujan mock theta functions

**Input interpretation:**

$$\frac{1.1424432422}{13.9766 - \pi} \times \frac{1}{\frac{16\pi ((6.6561943 \times 10^{-11})^2 (13.12806 \times 10^{39})^2)}{(2.99 \times 10^8)^4}}$$

**Result:**

$$2.1956708351833389199950174820337008548596664004280414... \times 10^{-29}$$

$$2.19567... * 10^{-29}$$

From:

### Advanced Geometric Physics Solutions

by Mark Rohrbaugh for <http://fractalU.com>

BSEE -minor in solid-state physics -University of Cincinnati

MSEE -Southern Methodist University - September 11, 2016

We have:

$$\mu = \frac{m_p}{m_e} = \frac{\alpha^2}{\pi r_p R_H} = 1836.15267 ...$$

and:

$$\pi = \frac{m_e \alpha^2}{m_p r_p R_H}$$

For:

$$\begin{aligned}
r_p &= 0.841236 fm \\
&= 8.41236 \times 10^{-14} \text{ centimeters} \\
\alpha &= 0.0072973534 \\
m_p &= 1.6714213e-24 \text{ gm} \\
m_e &= 9.10938356e-28 \text{ gm} \\
R_H &= 1.09678e+7 \text{ m}^{-1} = 109678 \text{ cm}^{-1}
\end{aligned}$$

Indeed:

$$\frac{(((9.10938356e-28*(0.0072973534)^2)))}{(((1.6714213e-24*8.41236e-14*109678)))}$$

**Input interpretation:**

$$\frac{9.10938356 \times 10^{-28} \times 0.0072973534^2}{1.6714213 \times 10^{-24} \times 8.41236 \times 10^{-14} \times 109678}$$

**Result:**

$$3.145548910163822844175988619574201190900908729535051664881...$$

$$3.145548910... \approx \pi$$

Thence:

$$1/6 * (((((((((((9.10938356e-28*(0.0072973534)^2)))))))))) / (((1.6714213e-24*8.41236e-14*109678)))))))))^2$$

**Input interpretation:**

$$\frac{1}{6} \left( \frac{9.10938356 \times 10^{-28} \times 0.0072973534^2}{1.6714213 \times 10^{-24} \times 8.41236 \times 10^{-14} \times 109678} \right)^2$$

**Result:**

$$1.649079657705468939648099937536996114781607025809952272266...$$

$$1.6490796577... \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 ...$$

And:

$$-(21/10^3+8/10^3+2/10^3)+1/6 * (((((((((((9.10938356e-28*(0.0072973534)^2)))))))))) / (((1.6714213e-24*8.41236e-14*109678)))))))))^2$$

**Input interpretation:**

$$-\left(\frac{21}{10^3} + \frac{8}{10^3} + \frac{2}{10^3}\right) + \frac{1}{6} \left( \frac{9.10938356 \times 10^{-28} \times 0.0072973534^2}{1.6714213 \times 10^{-24} \times 8.41236 \times 10^{-14} \times 109678} \right)^2$$

**Result:**

1.618079657705468939648099937536996114781607025809952272266...

1.6180796577...

This result is a very good approximation to the value of the golden ratio  
1,618033988749...

We note also that:

$$0.841236 \text{ fm} = 8.41236 * 10^{-14} \text{ cm}$$

**Input interpretation:**

$8.41236 \times 10^{-14}$  cm (centimeters)

**Unit conversions:**

841.236 am (attometers)

0.841236 fm (femtometers)

$8.41236 \times 10^{-16}$  meters

$3.31195 \times 10^{-14}$  inches

**Comparisons as radius:**

$\approx 0.3 \times$  classical electron radius ( $\approx 2.8 \times 10^{-15}$  m)

$\approx$  proton root-mean-square charge radius ( $\approx 8.4 \times 10^{-16}$  m)

**Comparison as distance:**

$\approx 0.65 \times$  range of the strong nuclear force ( $\approx 1.3 \times 10^{-15}$  m)

**Comparison as angular wavelength:**

$\approx (0.0022 \approx 1/459) \times$  natural unit ( $\approx 3.9 \times 10^{-13}$  m)

**Comparisons as wavelength:**

$\approx (0.072 \approx 1/14) \times$  muon Compton wavelength ( $\approx 1.2 \times 10^{-14}$  m)

$\approx 0.64 \times$  proton Compton wavelength ( $\approx 1.3 \times 10^{-15}$  m)

$\approx 1.2 \times$  tau Compton wavelength ( $\approx 7 \times 10^{-16}$  m)

**Comparison as electromagnetic radiation wavelength:**

$\approx$  wavelength of high-energy gamma rays ( $\approx 1$  fm)

### Corresponding quantities:

Energy  $E$  of a photon in a vacuum from  $E = hc/\tilde{\lambda}$ :

235 MeV (megaelectronvolts)

Wavelength  $\lambda$  from  $\lambda = 2\pi\tilde{\lambda}$ :

$5.3 \times 10^{-15}$  meters

Spectroscopic wavenumber  $\tilde{\nu}$  from  $\tilde{\nu} = 2\pi/\tilde{\lambda}$ :

$7.46899 \times 10^{15} \text{ m}^{-1}$  (reciprocal meters)

Wavenumber  $k$  from  $k = 1/\tilde{\lambda}$ :

$1.18873 \times 10^{15} \text{ m}^{-1}$  (reciprocal meters)

Energy  $E$  of a photon in a vacuum from  $E = hc/\lambda$ :

1.5 GeV (gigaelectronvolts)

Angular wavelength  $\tilde{\lambda}$  from  $\tilde{\lambda} = \lambda/(2\pi)$ :

$1.3 \times 10^{-16}$  meters

Frequency  $\nu$  of a photon in a vacuum from  $\nu = c/\lambda$ :

$3.564 \times 10^{23}$  Hz (hertz)

Spectroscopic wavenumber  $\tilde{\nu}$  from  $\tilde{\nu} = 1/\lambda$ :

$1.18873 \times 10^{15} \text{ m}^{-1}$  (reciprocal meters)

Wavenumber  $k$  from  $k = 2\pi/\lambda$ :

$7.46899 \times 10^{15} \text{ m}^{-1}$  (reciprocal meters)

And that:

$1 + (0.637 + 0.361)/2 * \sqrt{2 * (0.841236)}$  fm

### Input interpretation:

$(1 + (\frac{1}{2} (0.637 + 0.361)) \sqrt{2 * 0.841236})$  fm (femtometers)

### Result:

1.647253590543922745723316920254399766973385341682417006108...

$1.6472535905 \approx \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$

1.647 fm (femtometers)



**Unit conversions:**

$1.647 \times 10^{-15}$  meters

$6.485 \times 10^{-14}$  inches

**Comparisons as radius:**

$\approx 0.58 \times$  classical electron radius ( $\approx 2.8 \times 10^{-15}$  m)

$\approx 2 \times$  proton root-mean-square charge radius ( $\approx 8.4 \times 10^{-16}$  m)

**Comparison as distance:**

$\approx 1.3 \times$  range of the strong nuclear force ( $\approx 1.3 \times 10^{-15}$  m)

**Comparison as angular wavelength:**

$\approx (0.0043 \approx 1/234) \times$  natural unit ( $\approx 3.9 \times 10^{-13}$  m)

**Comparisons as wavelength:**

$\approx (0.14 \approx 1/7) \times$  muon Compton wavelength ( $\approx 1.2 \times 10^{-14}$  m)

$\approx 1.2 \times$  proton Compton wavelength ( $\approx 1.3 \times 10^{-15}$  m)

$\approx 2.4 \times$  tau Compton wavelength ( $\approx 7 \times 10^{-16}$  m)

**Comparisons as electromagnetic radiation wavelength:**

$\approx (1 \times 10^{-6}$  to  $0.001) \times$  wavelength range of an X-ray ( $9 \times 10^{-13}$  to  $1 \times 10^{-9}$  m)

$\approx (0.0016 \approx 1/607) \times$  wavelength range of a gamma ray ( $\approx 1 \times 10^{-12}$  m)

$\approx 1.6 \times$  wavelength of high-energy gamma rays ( $\approx 1$  fm)

**Corresponding quantities:**

Energy  $E$  of a photon in a vacuum from  $E = hc/\lambda$ :

120 MeV (megaelectronvolts)

Wavelength  $\lambda$  from  $\lambda = 2\pi\tilde{\lambda}$ :

$1 \times 10^{-14}$  meters

Spectroscopic wavenumber  $\tilde{\nu}$  from  $\tilde{\nu} = 2\pi/\tilde{\lambda}$ :

$3.814 \times 10^{15} \text{ m}^{-1}$  (reciprocal meters)

Wavenumber  $k$  from  $k = 1/\tilde{\lambda}$ :

$6.071 \times 10^{14} \text{ m}^{-1}$  (reciprocal meters)

Energy  $E$  of a photon in a vacuum from  $E = hc/\lambda$ :

753 MeV (megaelectronvolts)

753

**This number is the sum of  $9^3 - 1 + 5^2 = 728 + 25 = 753$**

Angular wavelength  $\tilde{\lambda}$  from  $\tilde{\lambda} = \lambda/(2\pi)$ :

$2.6 \times 10^{-16}$  meters

Frequency  $\nu$  of a photon in a vacuum from  $\nu = c/\lambda$ :

$1.82 \times 10^{23}$  Hz (hertz)

Spectroscopic wavenumber  $\tilde{\nu}$  from  $\tilde{\nu} = 1/\lambda$ :

$6.071 \times 10^{14} \text{ m}^{-1}$  (reciprocal meters)

Wavenumber  $k$  from  $k = 2\pi/\lambda$ :

$$3.814 \times 10^{15} \text{ m}^{-1} \text{ (reciprocal meters)}$$

Note that:

$$3814; 28\sqrt{3814} = 28*61.757590626578 = 1729.212537...$$

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson. Furthermore, 1728 occurs in the algebraic formula for the  $j$ -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

$$\text{sqrt}[6*(((1+(0.637+0.361)/2*\text{sqrt}(((2*(0.841236)))))))]$$

**Input interpretation:**

$$\sqrt{6\left(1 + \left(\frac{1}{2}(0.637 + 0.361)\right)\sqrt{2 \times 0.841236}\right)}$$

**Result:**

$$3.143806855273322364974803200232060546646668358465462403979...$$

$$3.143806855... \approx \pi$$

And:

$$-(21/10^3+8/10^3)+(((1+(0.637+0.361)/2*\text{sqrt}(((2*(0.841236)))))))]$$

**Input interpretation:**

$$-\left(\frac{21}{10^3} + \frac{8}{10^3}\right) + \left(1 + \left(\frac{1}{2}(0.637 + 0.361)\right)\sqrt{2 \times 0.841236}\right)$$

**Result:**

$$1.618253590543922745723316920254399766973385341682417006108...$$

$$1.6182535905....$$

This result is a very good approximation to the value of the golden ratio 1,618033988749...

Now, we have:

$$\nu_1 = \frac{m_\ell}{m_e} = \frac{r_e}{4\ell} = \frac{\alpha^2}{4\pi\ell R_H} = 2.3893048e + 22$$

& Planck Mass to Proton Mass Ratio:

$$\varphi = \frac{m_\ell}{m_p} = \frac{r_p}{4\ell} = 1.30125608e + 19$$

Note that 2.3893048 is very near to the following sum of Ramanujan mock theta functions:

$$1.897512108 + 0.5097073445 = 2.4072194525$$

And

1.30125608 is very neat to the following sum of Ramanujan mock theta functions:

$$0.07612513678 + 1.22734321 = 1.30346834678$$

From the above formula of Planck mass to proton mass ratio, we obtain:

$$(2.17645e-5) / (1.6714213e-24)$$

**Input interpretation:**

$$\frac{2.17645 \times 10^{-5}}{1.6714213 \times 10^{-24}}$$

**Result:**

$$1.3021552375813327256269858473144981459791137040074815... \times 10^{19}$$

$$1.30215523758... * 10^{19} \text{ gm} = 1.3021552376 \times 10^{16} \text{ kg}$$

Note that 1.30215523758 is very near to the value of the following Ramanujan mock theta function **f(q) = 1.333425959...**

Inserting this value in the Hawking radiation calculator, we have:

$$\text{Mass} = 1.302155e+16$$

$$\text{Radius} = 1.933507e-11$$

$$\text{Temperature} = 9424400$$

From the Ramanujan-Nardelli mock formula, we obtain:

sqrt[[[1/((((((4\*1.962364415e+19)/(5\*0.0864055^2))) \* 1/(1.302155e+16) \* sqrt[-(((9424400 \* 4\*Pi\*(1.933507e-11)^3-(1.933507e-11)^2)))))) / ((6.67\*10^-11))]]]]]

**Input interpretation:**

$$\sqrt{\frac{1}{\frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{1.302155 \times 10^{16}} \sqrt{\frac{9424400 \times 4 \pi (1.933507 \times 10^{-11})^3 - (1.933507 \times 10^{-11})^2}{6.67 \times 10^{-11}}}}}$$

**Result:**

1.618249172018650859208083382685266487587078060003548739381...

1.618249172...

And:

1/sqrt[[[1/((((((4\*1.962364415e+19)/(5\*0.0864055^2))) \* 1/(1.302155e+16) \* sqrt[-(((9424400 \* 4\*Pi\*(1.933507e-11)^3-(1.933507e-11)^2)))))) / ((6.67\*10^-11))]]]]]

**Input interpretation:**

$$\sqrt{\frac{1}{\frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{1.302155 \times 10^{16}} \sqrt{\frac{9424400 \times 4 \pi (1.933507 \times 10^{-11})^3 - (1.933507 \times 10^{-11})^2}{6.67 \times 10^{-11}}}}}$$

**Result:**

0.617951806984440507317962330816090755886266461260118147070...

0.6179518...

Now, we have that:

From:

**Advanced Geometric Physics Solutions**

by *Mark Rohrbaugh* for <http://fractalU.com>

BSEE -minor in solid-state physics -University of Cincinnati

MSEE -Southern Methodist University

September 11, 2016

## Ionized Hydrogen Radii – Using Dan Winter’s Phi( $\phi$ ) Equation

- $r = \ell_p \times \phi^N$
- $r_i = \ell_p \times \phi^{116} = 0.282527789\text{\AA}$
- $r_{ii} = \ell_p \times \phi^{117} = 0.457139566\text{\AA}$
- $r_{iii} = \ell_p \times \phi^{118} = 0.739667355\text{\AA}$

• <http://precedings.nature.com/documents/2929/version/1/files/npre20092929-1.pdf>

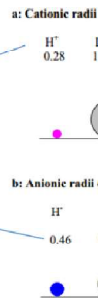


Figure 3. The Golden ratio based radii of hydrogen.  $d(H^+) = B1P = d(HH)/\phi^2$  and  $d(H^-) = B2P = d(HH)/\phi$ . P is the Golden point on  $d(HH) = B1B2 (= 0.74 \text{\AA}, 1, 0.75 \text{\AA}, 13)$ .

We obtain, for the Planck length in meters, and the golden ratio, the above values. Indeed:

$$(((1.61803398^{116}) * (1.616252e-35))) / 0.0000000001$$

(we have divided by 0.0000000001 because the value is expressed in Angstrom)

### Input interpretation:

$$\frac{1.61803398^{116} \times 1.616252 \times 10^{-35}}{1 \times 10^{-10}}$$

### Result:

0.282536832405310390307794081765210459226487640311027618414...

0.282536832 Angstrom result that is very closed to the above value

And:

$$(((1.61803398^{117}) * (1.616252e-35))) / 0.0000000001$$

### Input interpretation:

$$\frac{1.61803398^{117} \times 1.616252 \times 10^{-35}}{1 \times 10^{-10}}$$

### Result:

0.457154195433357343965073483139008904879861518073260455313...

0.457154195 Angstrom

$$(((1.61803398^{118}) * (1.616252e-35))) / 0.0000000001$$

**Input interpretation:**

$$\frac{1.61803398^{118} \times 1.616252 \times 10^{-35}}{1 \times 10^{-10}}$$

**Result:**

0.739691022310733008018036828915873471618203753936919546087...

0.739691022 Angstrom

From the sum of the three results, we obtain:

$$0.282536832405310390307794081765210459226487640311027618414 + \\ 0.739691022310733008018036828915873471618203753936919546087 + \\ 0.457154195433357343965073483139008904879861518073260455313$$

**Result:**

1.479382050149400742290904393820092835724552912321207619814...

1.479382050149....

Note that the result is very near to the following sum of Ramanujan mock theta functions:

$$(5.608437361 / 4) + 0.07612513678... = 1.47823447703$$

We note that the result 0.2825368324... is very near to the following formula that regard the Rogers-Ramanujan identity:

**Input:**

$$\frac{1}{\phi} \exp\left(\log\left(\sqrt{4\phi + 3} - \phi^2\right)\right)$$

log(x) is the natural logarithm

φ is the golden ratio

## Exact result:

$$\frac{\sqrt{4\phi + 3} - \phi^2}{\phi}$$

## Decimal approximation:

0.284079043840412296028291832393126169091088088445737582759...  
 0.2840790438...

From:

### Loop Quantum Dynamics of the Schwarzschild Interior

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<sup>2</sup>Institute of Cosmology & Gravitation, University of Portsmouth, Portsmouth PO1 2EG, UK

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(Dated: October 24, 2018)

From the previous equations it is clear that the asymptotic value of  $\bar{p}_c$  depends only on the constants  $\Delta$  and  $\gamma$  as

$$\bar{p}_c = \gamma^2 g(\Delta) \quad (68)$$

where  $g(\Delta)$  is some function determined from the solutions of equations (67) and (65). For the natural choice  $\Delta = 2\sqrt{3}\pi\gamma l_{\text{Pl}}^2$  the following asymptotic values are obtained

$$\begin{aligned} \bar{b} &\approx 0.156, & \bar{p}_c &\approx 0.182 l_{\text{Pl}}^2, & \alpha &\approx 0.670 \\ \bar{c}/\bar{p}_b &\approx -2.290 m_{\text{Pl}}^2, & \bar{N} &\approx 0.689, \end{aligned} \quad (69)$$

where  $\bar{N}$  is the asymptotic value of the lapse which also behaves as a constant. These values agree with the asymptotic region of the numerical solution that we studied, see in particular figure 3.

We have also considered the behavior of the Kretschmann invariant, the square of the Riemann tensor  $K = R_{abcd}R^{abcd}$ . For the metric (14), written out explicitly, this yields a rather complicated expression. However, plotting this quantity for our numerical solution also shows that the geometry of the spacetime changes significantly, from the classical Schwarzschild like behavior where  $K = 48m^2 \exp(-6T)$  (which diverges at  $T = -\infty$ ) towards a space of

constant curvature, as expected for the Nariai type metrics whose form is given by

$$K = 4 \left( \frac{\alpha^4}{N^2} + \frac{1}{\bar{p}_c^2} \right). \quad (70)$$

Plugging in the theoretical values in (69) gives an expected value

$$K \approx 124.36 m_{\text{Pl}}^4 \quad (71)$$

In physics, the **Planck length**, denoted  $\ell_P$ , is a unit of **length** that is the distance light travels in one unit of **Planck** time. It is equal to  $1.616255(18) \times 10^{-35}$  m. The Planck mass is equal to  $2.17645 \times 10^{-8}$  kg = Planck mass. From (71), we obtain:

$$124.36 * (2.17645e-8)^4$$

**Input interpretation:**

$$124.36 (2.17645 \times 10^{-8})^4$$

**Result:**

$$2.79045800691107755265725 \times 10^{-29}$$

$$2.79045800691107... * 10^{-29}$$

$$2(((124.36 * (2.17645e-8)^4)))^{1/340}$$

**Input interpretation:**

$$2^{340} \sqrt[340]{124.36 (2.17645 \times 10^{-8})^4}$$

**Result:**

$$1.648339584687485839984468579978382575883655376331929693703...$$

$$1.6483395846... \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 ...$$

And:

$$-(21/10^3 + 3^2/10^3) + 2(((124.36 * (2.17645e-8)^4)))^{1/340}$$

**Input interpretation:**

$$-\left(\frac{21}{10^3} + \frac{3^2}{10^3}\right) + 2^{340} \sqrt[340]{124.36 (2.17645 \times 10^{-8})^4}$$

**Result:**

$$1.618339584687485839984468579978382575883655376331929693703...$$

$$1.618339584...$$

This result is a very good approximation to the value of the golden ratio  
1,618033988749...



2.79045800691107... \* 10<sup>-29</sup> Planck mass = 1.522485e-36 kg

Inserting the mass 1.522485... \* 10<sup>-36</sup> kg in the Hawking radiation calculator, we obtain:

Mass = 1.522485e-36

Radius = 2.260664e-63

Temperature = 8.060527e+58

From the Ramanujan-Nardelli mock formula, we obtain:

sqrt[[[1/((((((4\*1.962364415e+19)/(5\*0.0864055^2)))\*1/( 1.522485e-36)\* sqrt[[-(((8.060527e+58 \* 4\*Pi\*(2.260664e-63)^3-(2.260664e-63)^2)))] / ((6.67\*10^-11))]]]]]]

**Input interpretation:**

$$\sqrt{\left(1 / \left( \frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{1.522485 \times 10^{-36}} \right) \sqrt{-\frac{8.060527 \times 10^{58} \times 4 \pi (2.260664 \times 10^{-63})^3 - (2.260664 \times 10^{-63})^2}{6.67 \times 10^{-11}}} \right)}$$

**Result:**

1.618249302288762981015352487157922149220521222912060806477...

1.6182493...

And:

1/sqrt[[[1/((((((4\*1.962364415e+19)/(5\*0.0864055^2)))\*1/( 1.522485e-36)\* sqrt[[-(((8.060527e+58 \* 4\*Pi\*(2.260664e-63)^3-(2.260664e-63)^2)))] / ((6.67\*10^-11))]]]]]]

**Input interpretation:**

$$1 / \left( \sqrt{\left(1 / \left( \frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{1.522485 \times 10^{-36}} \right) \sqrt{-\frac{8.060527 \times 10^{58} \times 4 \pi (2.260664 \times 10^{-63})^3 - (2.260664 \times 10^{-63})^2}{6.67 \times 10^{-11}}} \right) \right)}$$

**Result:**

0.617951757238921650686436420601105844176229725072071815083...  
 0.6179517572...

From (69), we have that:

$$-2.290m_{Pl}^2$$

thence:

$$((-2.290 * (2.17645e-8)^2))$$

**Input interpretation:**

$$-2.29 (2.17645 \times 10^{-8})^2$$

**Result:**

$$-1.0847580239725 \times 10^{-15}$$

$-1.0847580239725 * 10^{-15} = 5.918483e-23$  kg, result very near to the Ramanujan mock theta function 1.08640555

Inserting the mass  $5.918483e-23$  kg in the Hawking radiation calculator, we obtain:

$$\text{Mass} = 5.918483e-23$$

$$\text{Radius} = 8.788069e-50$$

$$\text{Temperature} = 2.073510e+45$$

From the Ramanujan-Nardelli mock formula, we obtain:

$$\text{sqrt}[\text{[[[1/((((((4*1.962364415e+19)/(5*0.0864055^2)))*1/(-5.918483e-23)* \text{sqrt}[\text{[-} \\ \text{(((((-2.073510e+45 * 4*Pi*(-8.788069e-50)^3-(-8.788069e-50)^2)))] / ((6.67*10^- \\ 11))]]]]]]]$$

**Input interpretation:**

$$\sqrt{\left(1/\left(\frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \left(-\frac{1}{5.918483 \times 10^{-23}}\right)\right.\right. \\ \left.\left.\sqrt{-\frac{-2.073510 \times 10^{45} \times 4 \pi (-8.788069 \times 10^{-50})^3 - (-8.788069 \times 10^{-50})^2}{6.67 \times 10^{-11}}}\right)\right)}$$

**Result:**

1.618249195012986656517158445263416729318098265247336648301... i  
 1.6182491...i

Note that the value is imaginary. Perhaps there is any link with the imaginary time of “no-boundary proposal” theory (see paper “black hole and soft hair”) ?

For  $0.182 \times (1.616255e-35)^2$ , we obtain the value  $4.754350e-71$  (=  $1.925968e-105$  meters) that we consider a radius.

Mass =  $1.297078e-78$

Radius =  $1.925968e-105$

Temperature =  $9.461291e+100$

From the Ramanujan-Nardelli mock formula, we obtain:

$$\sqrt{\left[\left[\left[\left[\left[\left[\frac{1}{\left(\left(\left(\left(\left(\left(\frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2}\right)\right) \times \frac{1}{1.297078 \times 10^{-78}}\right) \times \sqrt{\left[\left[\left[\left[\left[\frac{1}{\left(\left(\left(\left(\left(\left(\frac{9.461291 \times 10^{100} \times 4 \pi (1.925968 \times 10^{-105})^3 - (1.925968 \times 10^{-105})^2\right)}{6.67 \times 10^{-11}}\right)\right]\right]\right]\right]\right]\right]\right]\right]\right]\right]\right]\right]\right]\right]$$

**Input interpretation:**

$$\sqrt{\frac{1}{\frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{1.297078 \times 10^{-78}} \sqrt{\frac{(9.461291 \times 10^{100}) \times 4 \pi \left(\frac{1.925968}{10^{105}}\right)^3 - \left(\frac{1.925968}{10^{105}}\right)^2}{6.67 \times 10^{-11}}}}$$

**Result:**

1.618249343515001554168836902710244455242839807781294859873...  
 1.6182493...

From:

## Quantum Black Holes, Localization & Mock Modular Forms

ATISH DABHOLKAR

CNRS - University of Paris VI - VII Regional Meeting in String Theory - 19 June 2013

$$F(\tau) = \frac{1}{\eta^{24}(\tau)} = \sum_{n=-1}^{\infty} c(n)q^n$$
$$d(n) := c(n) = p_{24}(n+1)$$

where  $\eta(\tau)$  is the familiar Dedekind eta function

$$\eta(\tau) = q^{\frac{1}{24}} \prod_{n=1}^{\infty} (1 - q^n) \quad \text{with} \quad q := e^{2\pi i\tau}$$

$$\vartheta_1(\tau, z) = q^{\frac{1}{8}} (y^{\frac{1}{2}} - y^{-\frac{1}{2}}) \prod_{n=1}^{\infty} (1 - q^n)(1 - yq^n)(1 - y^{-1}q^n)$$

with  $y := e^{2\pi iz}$ .

$$q = e^{2\pi i\tau} = e^{2\pi} \quad \text{for } i\tau > 0$$

$$y = e^{2\pi iz} = e^{-2\pi} \quad \text{for } z = -1$$

$$q = 535.49165: \quad y = 0.00186744273$$

$$(535.49165)^{0.125} * (0.00186744273^{0.5} - 0.00186744273^{-0.5}) *$$

product  $(1-535.49165^n)(1-0.00186744*535.49165^n)((1-(1/0.00186744)*535.49165^n))$ ,  $n=1..1.603498$

**Product:**

$$\prod_{n=1}^{1.6035} (1 - 535.492 \times 535.492^n)(1 - 535.492^n)(1 - 0.00186744 \times 535.492^n) = 225.78$$

where 1.603498 without exponent is given by:

$$m_{p'} = 2 \frac{\eta_p}{R} = 1.603498 \times 10^{-24} gm$$

where  $m_{p'}$  is the holographic derivation of the mass of the proton. The result is a close approximation to the measured CODATA value for the proton mass  $m_p = 1.672622 \times 10^{-24} gm$  with a  $0.069 \times 10^{-24} gm$  or ~4% deviation from the CODATA value.

225.78 partial result

Now:

$$225.78 * ((((((535.49165)^{0.125} * (0.00186744273^{0.5} - 0.00186744273^{-0.5}))))))$$

**Input interpretation:**

$$225.78 \left( 535.49165^{0.125} \left( \sqrt{0.00186744273} - \frac{1}{0.00186744273^{0.5}} \right) \right)$$

**Result:**

-11437.8...  
-11437.8...

$$\ln(-(((225.78 * ((((((535.49165)^{0.125} * (0.00186744273^{0.5} - 0.00186744273^{-0.5}))))))$$

**Input interpretation:**

$$\log\left(-\left(225.78 \left( 535.49165^{0.125} \left( \sqrt{0.00186744273} - \frac{1}{0.00186744273^{0.5}} \right) \right) \right)\right)$$

log(x) is the natural logarithm

**Result:**

9.34468...

9.34468... result practically equal to the black hole entropy 9.3664

**Alternative representations:**

$$\log\left(-225.78\left(535.492^{0.125}\left(\sqrt{0.00186744}-\frac{1}{0.00186744^{0.5}}\right)\right)\right)=$$

$$\log_e\left(-225.78\left(-\frac{1}{0.00186744^{0.5}}+\sqrt{0.00186744}\right)535.492^{0.125}\right)$$

•

$$\log\left(-225.78\left(535.492^{0.125}\left(\sqrt{0.00186744}-\frac{1}{0.00186744^{0.5}}\right)\right)\right)=$$

$$\log(a)\log_a\left(-225.78\left(-\frac{1}{0.00186744^{0.5}}+\sqrt{0.00186744}\right)535.492^{0.125}\right)$$

•

$$\log\left(-225.78\left(535.492^{0.125}\left(\sqrt{0.00186744}-\frac{1}{0.00186744^{0.5}}\right)\right)\right)=$$

$$-\text{Li}_1\left(1+225.78\left(-\frac{1}{0.00186744^{0.5}}+\sqrt{0.00186744}\right)535.492^{0.125}\right)$$

$\log_b(x)$  is the base- $b$  logarithm  
 $\text{Li}_n(x)$  is the polylogarithm function

**Series representations:**

$$\log\left(-225.78\left(535.492^{0.125}\left(\sqrt{0.00186744}-\frac{1}{0.00186744^{0.5}}\right)\right)\right)=$$

$$\log(11436.8)-\sum_{k=1}^{\infty}\frac{(-1)^k e^{-9.3446k}}{k}$$

•

$$\log\left(-225.78\left(535.492^{0.125}\left(\sqrt{0.00186744}-\frac{1}{0.00186744^{0.5}}\right)\right)\right)=$$

$$2i\pi\left[\frac{\arg(11437.8-x)}{2\pi}\right]+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k(11437.8-x)^k x^{-k}}{k} \text{ for } x < 0$$

•

$$\log\left(-225.78 \left(535.492^{0.125} \left(\sqrt{0.00186744} - \frac{1}{0.00186744^{0.5}}\right)\right)\right) = \left\lfloor \frac{\arg(11437.8 - z_0)}{2\pi} \right\rfloor \log\left(\frac{1}{z_0}\right) + \log(z_0) + \left\lfloor \frac{\arg(11437.8 - z_0)}{2\pi} \right\rfloor \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (11437.8 - z_0)^k z_0^{-k}}{k}$$

$\arg(z)$  is the complex argument

$\lfloor x \rfloor$  is the floor function

$i$  is the imaginary unit

### Integral representations:

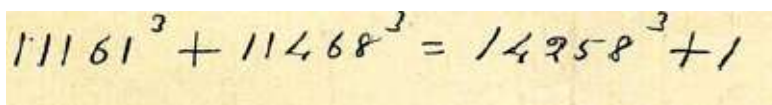
$$\log\left(-225.78 \left(535.492^{0.125} \left(\sqrt{0.00186744} - \frac{1}{0.00186744^{0.5}}\right)\right)\right) = \int_1^{11437.8} \frac{1}{t} dt$$

$$\log\left(-225.78 \left(535.492^{0.125} \left(\sqrt{0.00186744} - \frac{1}{0.00186744^{0.5}}\right)\right)\right) = \frac{1}{2i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-9.3446s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \text{ for } -1 < \gamma < 0$$

$\Gamma(x)$  is the gamma function

The result 11437.8 (in absolute value) is very near to a number that is in a Ramanujan sum of two cubes, precisely 11468.

From Ramanujan's manuscript, where are described the representations of the sum of two cubes:



We obtain the number 11468, simply :

$$11468^3 = 14258^3 + 1 - 11161^3$$

### Input:

$$\sqrt[3]{11468^3} = \sqrt[3]{14258^3 + 1 - 11161^3}$$

### Left hand side:

$$\sqrt[3]{11468^3} = 11468$$

### Right hand side:

$$\sqrt[3]{14258^3 + 1 - 11161^3} = 11468$$

We obtain a result practically equal to this number, with the previous expression, as follows:

$$-\left(\left(-\left(21+8+2\right)+225.78 * \left(\left(\left(\left(535.49165\right)^{0.125} * \left(0.00186744273^{0.5} - 0.00186744273^{-0.5}\right)\right)\right)\right)\right)\right)$$

**Input interpretation:**

$$-\left(-\left(21+8+2\right)+225.78\left(535.49165^{0.125}\left(\sqrt{0.00186744273}-\frac{1}{0.00186744273^{0.5}}\right)\right)\right)$$

**Result:**

11468.8...

11468.8...

$$\left(-\left(\left(-\left(21+8+2\right)+225.78 * \left(\left(\left(\left(535.49165\right)^{0.125} * \left(0.00186744273^{0.5} - 0.00186744273^{-0.5}\right)\right)\right)\right)\right)\right)\right)^{1/3}$$

**Input interpretation:**

$$\left(-\left(-\left(21+8+2\right)+225.78\left(535.49165^{0.125}\left(\sqrt{0.00186744273}-\frac{1}{0.00186744273^{0.5}}\right)\right)\right)\right)^{(1/3)}$$

**Result:**

22.5514...

22.5514... result very near to the black hole entropy 22.6589

Or:

$$\left(\left(\left(\left(21+8+2\right)-1 * \left(\left(-11437.8\right)\right)\right)\right)\right)^{1/3}$$

**Input interpretation:**

$$\sqrt[3]{\left(21+8+2\right)-1 * \left(-11437.8\right)}$$

**Result:**

22.5514...

22.5514... as above



Now, we have:

It turns out one can even evaluate the finite-dimensional integral to obtain

$$W_1(\Delta) = (-1)^{\Delta+1} 2\pi \left(\frac{\pi}{\Delta}\right)^{7/2} I_{\frac{7}{2}}(\pi\sqrt{\Delta}) .$$

where  $\Delta = q^2 p^2 - (p \cdot q)^2$  is the U-duality invariant and

$$I_\rho(z) = \frac{1}{2\pi i} \int_{\epsilon-i\infty}^{\epsilon+i\infty} \frac{d\sigma}{\sigma^{\rho+1}} \exp\left[\sigma + \frac{z^2}{4\sigma}\right]$$

is the Bessel function of first kind of index  $\rho$ .

$\Delta$	3	4	7	8	11	12
$C(\Delta)$	8	-12	39	-56	152	-208
$W_1(\Delta)$	7.972	12.201	38.986	55.721	152.041	208.455
$\exp(\pi\sqrt{\Delta})$	230.765	535.492	4071.93	7228.35	33506	53252

We obtain:

$\exp(\text{Pi}*\text{sqrt}(3))$

**Input:**

$\exp(\pi\sqrt{3})$

**Exact result:**

$e^{\sqrt{3}\pi}$

**Decimal approximation:**

230.7645883191458792400751539310090016878540780654754395554...

230.764588...

**Property:**

$e^{\sqrt{3}\pi}$  is a transcendental number

**Series representations:**

$$e^{\pi\sqrt{3}} = \sum_{k=0}^{\infty} \frac{3^{k/2} \pi^k}{k!}$$

•

$$e^{\pi\sqrt{3}} = \sum_{k=-\infty}^{\infty} I_k(\sqrt{3}\pi)$$

•

$$e^{\pi\sqrt{3}} = \sum_{k=0}^{\infty} \frac{3^k \pi^{2k} (1 + 2k + \sqrt{3}\pi)}{(1 + 2k)!}$$

$n!$  is the factorial function

$I_n(z)$  is the modified Bessel function of the first kind

$\exp(\pi\sqrt{4})$

**Input:**

$$\exp(\pi\sqrt{4})$$

**Exact result:**

$$e^{2\pi}$$

**Decimal approximation:**

535.4916555247647365030493295890471814778057976032949155072...

535.491655...

**Property:**

$e^{2\pi}$  is a transcendental number

**Series representations:**

$$e^{\pi\sqrt{4}} = e^{8 \sum_{k=0}^{\infty} (-1)^k / (1+2k)}$$

•

$$e^{\pi\sqrt{4}} = \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^{2\pi}$$

•

$$e^{\pi \sqrt{4}} = \left( \frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}} \right)^{2\pi}$$

$\exp(\text{Pi}*\text{sqrt}(7))$

**Input:**

$$\exp(\pi \sqrt{7})$$

**Exact result:**

$$e^{\sqrt{7} \pi}$$

**Decimal approximation:**

4071.932095225261098524568332557597916651844857963299169137...

4071.932095

**Property:**

$e^{\sqrt{7} \pi}$  is a transcendental number

**Series representations:**

$$e^{\pi \sqrt{7}} = \sum_{k=0}^{\infty} \frac{7^{k/2} \pi^k}{k!}$$

•

$$e^{\pi \sqrt{7}} = \sum_{k=-\infty}^{\infty} I_k(\sqrt{7} \pi)$$

•

$$e^{\pi \sqrt{7}} = \sum_{k=0}^{\infty} \frac{7^k \pi^{2k} (1 + 2k + \sqrt{7} \pi)}{(1 + 2k)!}$$

$n!$  is the factorial function

$I_n(z)$  is the modified Bessel function of the first kind

$\exp(\text{Pi}*\text{sqrt}(8))$

**Input:**

$$\exp(\pi \sqrt{8})$$

**Exact result:**

$$e^{2\sqrt{2}\pi}$$

**Decimal approximation:**

7228.348575847044171148885470209524956543650204303266888619...

7228.348575...

**Property:**

$e^{2\sqrt{2}\pi}$  is a transcendental number

**Series representations:**

$$e^{\pi\sqrt{8}} = \sum_{k=0}^{\infty} \frac{(\pi\sqrt{8})^k}{k!}$$

•

$$e^{\pi\sqrt{8}} = \sum_{k=-\infty}^{\infty} I_k(\pi\sqrt{8})$$

•

$$e^{\pi\sqrt{8}} = \sum_{k=-\infty}^{\infty} (-1)^k I_k(-\pi\sqrt{8})$$

$n!$  is the factorial function

$I_n(z)$  is the modified Bessel function of the first kind

$\exp(\text{Pi}*\text{sqrt}(11))$

**Input:**

$$\exp(\pi\sqrt{11})$$

•

**Exact result:**

$$e^{\sqrt{11}\pi}$$

**Decimal approximation:**

33506.14306559243876668155096281917791197332352226196825663...

33506.143065...

**Property:**

$e^{\sqrt{11} \pi}$  is a transcendental number

**Series representations:**

$$e^{\pi \sqrt{11}} = \sum_{k=0}^{\infty} \frac{11^{k/2} \pi^k}{k!}$$

•

$$e^{\pi \sqrt{11}} = \sum_{k=-\infty}^{\infty} I_k(\sqrt{11} \pi)$$

•

$$e^{\pi \sqrt{11}} = \sum_{k=0}^{\infty} \frac{11^k \pi^{2k} (1 + 2k + \sqrt{11} \pi)}{(1 + 2k)!}$$

$\exp(\text{Pi}*\text{sqrt}(12))$

**Input:**

$\exp(\pi \sqrt{12})$

**Exact result:**

$e^{2\sqrt{3} \pi}$

**Decimal approximation:**

53252.29522210487877132148420777237850995832779396156731812...

53252.295222...

**Property:**

$e^{2\sqrt{3} \pi}$  is a transcendental number

**Series representations:**

$$e^{\pi \sqrt{12}} = \sum_{k=0}^{\infty} \frac{(\pi \sqrt{12})^k}{k!}$$

•

$$e^{\pi \sqrt{12}} = \sum_{k=-\infty}^{\infty} I_k(\pi \sqrt{12})$$

•

$$e^{\pi\sqrt{12}} = \sum_{k=-\infty}^{\infty} (-1)^k I_k(-\pi\sqrt{12})$$

Note that, all these results are approximations to  $\pi$ . Indeed, for example, from this last formula, we obtain:

$$\left(\left(\left(\ln\left(\left(\left(53252.29522210487877132\right)\right)\right)\right)\right)\right) * 1 / \left(\left(\left(2 * \sqrt{3}\right)\right)\right)$$

**Input interpretation:**

$$\log(53\ 252.29522210487877132) \times \frac{1}{2\sqrt{3}}$$

$\log(x)$  is the natural logarithm

**Result:**

3.1415926535897932384626...

$\pi \approx 3.141592653589793238462643383$

3.14159265...

**Alternative representations:**

$$\frac{\log(53\ 252.295222104878771320000)}{2\sqrt{3}} = \frac{\log_e(53\ 252.295222104878771320000)}{2\sqrt{3}}$$

$$\frac{\log(53\ 252.295222104878771320000)}{2\sqrt{3}} = \frac{\log(a) \log_a(53\ 252.295222104878771320000)}{2\sqrt{3}}$$

$$\frac{\log(53\ 252.295222104878771320000)}{2\sqrt{3}} = - \frac{\text{Li}_1(-53\ 251.295222104878771320000)}{2\sqrt{3}}$$

$\log_b(x)$  is the base-  $b$  logarithm

$\text{Li}_n(x)$  is the polylogarithm function

**Series representations:**

$$\frac{\log(53\,252.295222104878771320000)}{2\sqrt{3}} = \frac{\log(53\,252.295222104878771320000)}{2\sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k}}$$

$$\frac{\log(53\,252.295222104878771320000)}{2\sqrt{3}} = \frac{\log(53\,251.295222104878771320000) - \sum_{k=1}^{\infty} \frac{(-1)^k e^{-10.8827774066956718970209949k}}{k}}{2\sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k}}$$

$$\frac{\log(53\,252.295222104878771320000)}{2\sqrt{3}} = \frac{\log(53\,251.295222104878771320000) - \sum_{k=1}^{\infty} \frac{(-1)^k e^{-10.8827774066956718970209949k}}{k}}{2\sqrt{2} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!}}$$

$\binom{n}{m}$  is the binomial coefficient

$n!$  is the factorial function

$(a)_n$  is the Pochhammer symbol (rising factorial)

### Integral representations:

$$\frac{\log(53\,252.295222104878771320000)}{2\sqrt{3}} = \frac{1}{2\sqrt{3}} \int_1^{53\,252.295222104878771320000} \frac{1}{t} dt$$

$$\frac{\log(53\,252.295222104878771320000)}{2\sqrt{3}} = \frac{1}{4i\pi\sqrt{3}} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-10.8827774066956718970209949s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \quad \text{for } -1 < \gamma < 0$$

$\Gamma(x)$  is the gamma function

$i$  is the imaginary unit

Now, we take the sum of all the results.

$$\exp(\text{Pi}*\text{sqrt}(3)) + \exp(\text{Pi}*\text{sqrt}(4)) + \exp(\text{Pi}*\text{sqrt}(7)) + \exp(\text{Pi}*\text{sqrt}(8)) + \exp(\text{Pi}*\text{sqrt}(11)) + \exp(\text{Pi}*\text{sqrt}(12))$$

**Input:**

$$\exp(\pi\sqrt{3}) + \exp(\pi\sqrt{4}) + \exp(\pi\sqrt{7}) + \exp(\pi\sqrt{8}) + \exp(\pi\sqrt{11}) + \exp(\pi\sqrt{12})$$

**Exact result:**

$$e^{2\pi} + e^{2\sqrt{2}\pi} + e^{\sqrt{3}\pi} + e^{2\sqrt{3}\pi} + e^{\sqrt{7}\pi} + e^{\sqrt{11}\pi}$$

**Decimal approximation:**

98824.97520261353342341961345687873547829280625415887198757...

98824.9752026.....

**Series representations:**

$$\begin{aligned} &\exp(\pi\sqrt{3}) + \exp(\pi\sqrt{4}) + \exp(\pi\sqrt{7}) + \exp(\pi\sqrt{8}) + \exp(\pi\sqrt{11}) + \exp(\pi\sqrt{12}) = \\ &\exp\left(\pi\sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k}\right) + \exp\left(\pi\sqrt{3} \sum_{k=0}^{\infty} 3^{-k} \binom{\frac{1}{2}}{k}\right) + \exp\left(\pi\sqrt{6} \sum_{k=0}^{\infty} 6^{-k} \binom{\frac{1}{2}}{k}\right) + \\ &\exp\left(\pi\sqrt{7} \sum_{k=0}^{\infty} 7^{-k} \binom{\frac{1}{2}}{k}\right) + \exp\left(\pi\sqrt{10} \sum_{k=0}^{\infty} 10^{-k} \binom{\frac{1}{2}}{k}\right) + \exp\left(\pi\sqrt{11} \sum_{k=0}^{\infty} 11^{-k} \binom{\frac{1}{2}}{k}\right) \end{aligned}$$

$$\begin{aligned} &\exp(\pi\sqrt{3}) + \exp(\pi\sqrt{4}) + \exp(\pi\sqrt{7}) + \exp(\pi\sqrt{8}) + \exp(\pi\sqrt{11}) + \exp(\pi\sqrt{12}) = \\ &\exp\left(\pi\sqrt{2} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!}\right) + \exp\left(\pi\sqrt{3} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{3}\right)^k \left(-\frac{1}{2}\right)_k}{k!}\right) + \\ &\exp\left(\pi\sqrt{6} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{6}\right)^k \left(-\frac{1}{2}\right)_k}{k!}\right) + \exp\left(\pi\sqrt{7} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{7}\right)^k \left(-\frac{1}{2}\right)_k}{k!}\right) + \\ &\exp\left(\pi\sqrt{10} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{10}\right)^k \left(-\frac{1}{2}\right)_k}{k!}\right) + \exp\left(\pi\sqrt{11} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{11}\right)^k \left(-\frac{1}{2}\right)_k}{k!}\right) \end{aligned}$$



$$\begin{aligned} & \exp(\pi \sqrt{3}) + \exp(\pi \sqrt{4}) + \exp(\pi \sqrt{7}) + \exp(\pi \sqrt{8}) + \exp(\pi \sqrt{11}) + \exp(\pi \sqrt{12}) = \\ & \exp\left(\pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (3 - z_0)^k z_0^{-k}}{k!}\right) + \\ & \exp\left(\pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (4 - z_0)^k z_0^{-k}}{k!}\right) + \\ & \exp\left(\pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (7 - z_0)^k z_0^{-k}}{k!}\right) + \\ & \exp\left(\pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (8 - z_0)^k z_0^{-k}}{k!}\right) + \\ & \exp\left(\pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (11 - z_0)^k z_0^{-k}}{k!}\right) + \\ & \exp\left(\pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (12 - z_0)^k z_0^{-k}}{k!}\right) \text{ for not } ((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0)) \end{aligned}$$

$\binom{n}{m}$  is the binomial coefficient

$n!$  is the factorial function

$(a)_n$  is the Pochhammer symbol (rising factorial)

$\mathbb{R}$  is the set of real numbers

$$\ln(\exp(\pi \sqrt{3}) + \exp(\pi \sqrt{4}) + \exp(\pi \sqrt{7}) + \exp(\pi \sqrt{8}) + \exp(\pi \sqrt{11}) + \exp(\pi \sqrt{12}))$$

**Input:**

$$\log(\exp(\pi \sqrt{3}) + \exp(\pi \sqrt{4}) + \exp(\pi \sqrt{7}) + \exp(\pi \sqrt{8}) + \exp(\pi \sqrt{11}) + \exp(\pi \sqrt{12}))$$

$\log(x)$  is the natural logarithm

**Exact result:**

$$\log(e^{2\pi} + e^{2\sqrt{2}\pi} + e^{\sqrt{3}\pi} + e^{2\sqrt{3}\pi} + e^{\sqrt{7}\pi} + e^{\sqrt{11}\pi})$$

**Decimal approximation:**

11.50110563724265425660891085778768278374289779905246694251...

11.501105637...

**Alternative representations:**

$$\log\left(\exp(\pi\sqrt{3}) + \exp(\pi\sqrt{4}) + \exp(\pi\sqrt{7}) + \exp(\pi\sqrt{8}) + \exp(\pi\sqrt{11}) + \exp(\pi\sqrt{12})\right) = \log_e\left(\exp(\pi\sqrt{3}) + \exp(\pi\sqrt{4}) + \exp(\pi\sqrt{7}) + \exp(\pi\sqrt{8}) + \exp(\pi\sqrt{11}) + \exp(\pi\sqrt{12})\right)$$

$$\log\left(\exp(\pi\sqrt{3}) + \exp(\pi\sqrt{4}) + \exp(\pi\sqrt{7}) + \exp(\pi\sqrt{8}) + \exp(\pi\sqrt{11}) + \exp(\pi\sqrt{12})\right) = \log(a) \log_a\left(\exp(\pi\sqrt{3}) + \exp(\pi\sqrt{4}) + \exp(\pi\sqrt{7}) + \exp(\pi\sqrt{8}) + \exp(\pi\sqrt{11}) + \exp(\pi\sqrt{12})\right)$$

$\log_b(x)$  is the base- $b$  logarithm

### Series representations:

- More

$$\log\left(\exp(\pi\sqrt{3}) + \exp(\pi\sqrt{4}) + \exp(\pi\sqrt{7}) + \exp(\pi\sqrt{8}) + \exp(\pi\sqrt{11}) + \exp(\pi\sqrt{12})\right) = 2i\pi \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k \left( e^{2\pi} + e^{2\sqrt{2}\pi} + e^{\sqrt{3}\pi} + e^{2\sqrt{3}\pi} + e^{\sqrt{7}\pi} + e^{\sqrt{11}\pi} - z_0 \right)^k z_0^{-k}}{k}$$

$$\log\left(\exp(\pi\sqrt{3}) + \exp(\pi\sqrt{4}) + \exp(\pi\sqrt{7}) + \exp(\pi\sqrt{8}) + \exp(\pi\sqrt{11}) + \exp(\pi\sqrt{12})\right) = \log\left(-1 + e^{2\pi} + e^{2\sqrt{2}\pi} + e^{\sqrt{3}\pi} + e^{2\sqrt{3}\pi} + e^{\sqrt{7}\pi} + e^{\sqrt{11}\pi}\right) - \sum_{k=1}^{\infty} \frac{\left( \frac{1}{-1 + e^{2\pi} + e^{2\sqrt{2}\pi} + e^{\sqrt{3}\pi} + e^{2\sqrt{3}\pi} + e^{\sqrt{7}\pi} + e^{\sqrt{11}\pi}} \right)^k}{k}$$

$$\log\left(\exp(\pi\sqrt{3}) + \exp(\pi\sqrt{4}) + \exp(\pi\sqrt{7}) + \exp(\pi\sqrt{8}) + \exp(\pi\sqrt{11}) + \exp(\pi\sqrt{12})\right) = 2i\pi \left[ \frac{\arg\left(e^{2\pi} + e^{2\sqrt{2}\pi} + e^{\sqrt{3}\pi} + e^{2\sqrt{3}\pi} + e^{\sqrt{7}\pi} + e^{\sqrt{11}\pi} - x\right)}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k \left( e^{2\pi} + e^{2\sqrt{2}\pi} + e^{\sqrt{3}\pi} + e^{2\sqrt{3}\pi} + e^{\sqrt{7}\pi} + e^{\sqrt{11}\pi} - x \right)^k x^{-k}}{k} \quad \text{for } x < 0$$

$\arg(z)$  is the complex argument

$\lfloor x \rfloor$  is the floor function

**Integral representations:**

$$\log\left(\exp\left(\pi\sqrt{3}\right) + \exp\left(\pi\sqrt{4}\right) + \exp\left(\pi\sqrt{7}\right) + \exp\left(\pi\sqrt{8}\right) + \exp\left(\pi\sqrt{11}\right) + \exp\left(\pi\sqrt{12}\right)\right) = \int_1^{e^{2\pi} + e^{2\sqrt{2}\pi} + e^{\sqrt{3}\pi} + e^{2\sqrt{3}\pi} + e^{\sqrt{7}\pi} + e^{\sqrt{11}\pi}} \frac{1}{t} dt$$

$$\log\left(\exp\left(\pi\sqrt{3}\right) + \exp\left(\pi\sqrt{4}\right) + \exp\left(\pi\sqrt{7}\right) + \exp\left(\pi\sqrt{8}\right) + \exp\left(\pi\sqrt{11}\right) + \exp\left(\pi\sqrt{12}\right)\right) = -\frac{i}{2\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\left(-1 + e^{2\pi} + e^{2\sqrt{2}\pi} + e^{\sqrt{3}\pi} + e^{2\sqrt{3}\pi} + e^{\sqrt{7}\pi} + e^{\sqrt{11}\pi}\right)^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \text{ for } -1 < \gamma < 0$$

$\Gamma(x)$  is the gamma function

Or:

$$34/10^2 + \ln\left(\left(\left(\left(\left(\left(\exp(\text{Pi}*\text{sqrt}(3)) + \exp(\text{Pi}*\text{sqrt}(4)) + \exp(\text{Pi}*\text{sqrt}(7)) + \exp(\text{Pi}*\text{sqrt}(8)) + \exp(\text{Pi}*\text{sqrt}(11)) + \exp(\text{Pi}*\text{sqrt}(12))\right)\right)\right)\right)\right)\right)$$

**Input:**

$$\frac{34}{10^2} + \log\left(\exp\left(\pi\sqrt{3}\right) + \exp\left(\pi\sqrt{4}\right) + \exp\left(\pi\sqrt{7}\right) + \exp\left(\pi\sqrt{8}\right) + \exp\left(\pi\sqrt{11}\right) + \exp\left(\pi\sqrt{12}\right)\right)$$

$\log(x)$  is the natural logarithm

**Exact result:**

$$\frac{17}{50} + \log\left(e^{2\pi} + e^{2\sqrt{2}\pi} + e^{\sqrt{3}\pi} + e^{2\sqrt{3}\pi} + e^{\sqrt{7}\pi} + e^{\sqrt{11}\pi}\right)$$

**Decimal approximation:**

11.84110563724265425660891085778768278374289779905246694251...

11.841105637... result practically equal to the black hole entropy 11.8458

**Alternate form:**

$$\frac{1}{50} \left(17 + 50 \log\left(e^{2\pi} + e^{2\sqrt{2}\pi} + e^{\sqrt{3}\pi} + e^{2\sqrt{3}\pi} + e^{\sqrt{7}\pi} + e^{\sqrt{11}\pi}\right)\right)$$

**Alternative representations:**

$$\frac{34}{10^2} + \log\left(\exp(\pi\sqrt{3}) + \exp(\pi\sqrt{4}) + \exp(\pi\sqrt{7}) + \exp(\pi\sqrt{8}) + \exp(\pi\sqrt{11}) + \exp(\pi\sqrt{12})\right) = \log_e\left(\exp(\pi\sqrt{3}) + \exp(\pi\sqrt{4}) + \exp(\pi\sqrt{7}) + \exp(\pi\sqrt{8}) + \exp(\pi\sqrt{11}) + \exp(\pi\sqrt{12})\right) + \frac{34}{10^2}$$

$$\frac{34}{10^2} + \log\left(\exp(\pi\sqrt{3}) + \exp(\pi\sqrt{4}) + \exp(\pi\sqrt{7}) + \exp(\pi\sqrt{8}) + \exp(\pi\sqrt{11}) + \exp(\pi\sqrt{12})\right) = \log(a) \log_a\left(\exp(\pi\sqrt{3}) + \exp(\pi\sqrt{4}) + \exp(\pi\sqrt{7}) + \exp(\pi\sqrt{8}) + \exp(\pi\sqrt{11}) + \exp(\pi\sqrt{12})\right) + \frac{34}{10^2}$$

$\log_b(x)$  is the base- $b$  logarithm

[More information »](#)

### Series representations:

$$\frac{34}{10^2} + \log\left(\exp(\pi\sqrt{3}) + \exp(\pi\sqrt{4}) + \exp(\pi\sqrt{7}) + \exp(\pi\sqrt{8}) + \exp(\pi\sqrt{11}) + \exp(\pi\sqrt{12})\right) = \frac{17}{50} + 2i\pi \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k \left( e^{2\pi} + e^{2\sqrt{2}\pi} + e^{\sqrt{3}\pi} + e^{2\sqrt{3}\pi} + e^{\sqrt{7}\pi} + e^{\sqrt{11}\pi} - z_0 \right)^k z_0^{-k}}{k}$$

$$\frac{34}{10^2} + \log\left(\exp(\pi\sqrt{3}) + \exp(\pi\sqrt{4}) + \exp(\pi\sqrt{7}) + \exp(\pi\sqrt{8}) + \exp(\pi\sqrt{11}) + \exp(\pi\sqrt{12})\right) = \frac{17}{50} + \log\left(-1 + e^{2\pi} + e^{2\sqrt{2}\pi} + e^{\sqrt{3}\pi} + e^{2\sqrt{3}\pi} + e^{\sqrt{7}\pi} + e^{\sqrt{11}\pi}\right) - \sum_{k=1}^{\infty} \frac{\left( -\frac{1}{-1 + e^{2\pi} + e^{2\sqrt{2}\pi} + e^{\sqrt{3}\pi} + e^{2\sqrt{3}\pi} + e^{\sqrt{7}\pi} + e^{\sqrt{11}\pi}} \right)^k}{k}$$

$$\frac{34}{10^2} + \log\left(\exp(\pi\sqrt{3}) + \exp(\pi\sqrt{4}) + \exp(\pi\sqrt{7}) + \exp(\pi\sqrt{8}) + \exp(\pi\sqrt{11}) + \exp(\pi\sqrt{12})\right) =$$

$$\frac{17}{50} + 2i\pi \left[ \frac{\arg\left(e^{2\pi} + e^{2\sqrt{2}\pi} + e^{\sqrt{3}\pi} + e^{2\sqrt{3}\pi} + e^{\sqrt{7}\pi} + e^{\sqrt{11}\pi} - x\right)}{2\pi} \right] + \log(x) -$$

$$\sum_{k=1}^{\infty} \frac{(-1)^k \left(e^{2\pi} + e^{2\sqrt{2}\pi} + e^{\sqrt{3}\pi} + e^{2\sqrt{3}\pi} + e^{\sqrt{7}\pi} + e^{\sqrt{11}\pi} - x\right)^k x^{-k}}{k} \quad \text{for } x < 0$$

$\arg(z)$  is the complex argument  
 $\lfloor x \rfloor$  is the floor function  
[More information »](#)

### Integral representations:

$$\frac{34}{10^2} + \log\left(\exp(\pi\sqrt{3}) + \exp(\pi\sqrt{4}) + \exp(\pi\sqrt{7}) + \exp(\pi\sqrt{8}) + \exp(\pi\sqrt{11}) + \exp(\pi\sqrt{12})\right) =$$

$$\frac{17}{50} + \int_1^{e^{2\pi} + e^{2\sqrt{2}\pi} + e^{\sqrt{3}\pi} + e^{2\sqrt{3}\pi} + e^{\sqrt{7}\pi} + e^{\sqrt{11}\pi}} \frac{1}{t} dt$$

$$\frac{34}{10^2} + \log\left(\exp(\pi\sqrt{3}) + \exp(\pi\sqrt{4}) + \exp(\pi\sqrt{7}) + \exp(\pi\sqrt{8}) + \exp(\pi\sqrt{11}) + \exp(\pi\sqrt{12})\right) = \frac{17}{50} - \frac{i}{2\pi}$$

$$\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\left(-1 + e^{2\pi} + e^{2\sqrt{2}\pi} + e^{\sqrt{3}\pi} + e^{2\sqrt{3}\pi} + e^{\sqrt{7}\pi} + e^{\sqrt{11}\pi}\right)^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \quad \text{for } -1 < \gamma < 0$$

$\Gamma(x)$  is the gamma function

$$-5^2/10^3 + 1/7 \ln\left(\left(\left(\left(\left(\left(\exp(\text{Pi}*\text{sqrt}(3)) + \exp(\text{Pi}*\text{sqrt}(4)) + \exp(\text{Pi}*\text{sqrt}(7)) + \exp(\text{Pi}*\text{sqrt}(8)) + \exp(\text{Pi}*\text{sqrt}(11)) + \exp(\text{Pi}*\text{sqrt}(12))\right)\right)\right)\right)\right)\right)$$

### Input:

$$-\frac{5^2}{10^3} + \frac{1}{7} \log\left(\exp(\pi\sqrt{3}) + \exp(\pi\sqrt{4}) + \exp(\pi\sqrt{7}) + \exp(\pi\sqrt{8}) + \exp(\pi\sqrt{11}) + \exp(\pi\sqrt{12})\right)$$

$\log(x)$  is the natural logarithm

### Exact result:

$$\frac{1}{7} \log\left(e^{2\pi} + e^{2\sqrt{2}\pi} + e^{\sqrt{3}\pi} + e^{2\sqrt{3}\pi} + e^{\sqrt{7}\pi} + e^{\sqrt{11}\pi}\right) - \frac{1}{40}$$

### Decimal approximation:

1.618015091034664893801272979683954683391842542721780991787...

1.61801509...

This result is a very good approximation to the value of the golden ratio  
1,618033988749...

**Alternate form:**

$$\frac{1}{280} \left( 40 \log \left( e^{2\pi} + e^{2\sqrt{2}\pi} + e^{\sqrt{3}\pi} + e^{2\sqrt{3}\pi} + e^{\sqrt{7}\pi} + e^{\sqrt{11}\pi} \right) - 7 \right)$$

**Alternative representations:**

$$-\frac{5^2}{10^3} + \frac{1}{7} \log \left( \exp(\pi \sqrt{3}) + \exp(\pi \sqrt{4}) + \exp(\pi \sqrt{7}) + \exp(\pi \sqrt{8}) + \exp(\pi \sqrt{11}) + \exp(\pi \sqrt{12}) \right) = \frac{1}{7} \log_e \left( \exp(\pi \sqrt{3}) + \exp(\pi \sqrt{4}) + \exp(\pi \sqrt{7}) + \exp(\pi \sqrt{8}) + \exp(\pi \sqrt{11}) + \exp(\pi \sqrt{12}) \right) - \frac{5^2}{10^3}$$

$$-\frac{5^2}{10^3} + \frac{1}{7} \log \left( \exp(\pi \sqrt{3}) + \exp(\pi \sqrt{4}) + \exp(\pi \sqrt{7}) + \exp(\pi \sqrt{8}) + \exp(\pi \sqrt{11}) + \exp(\pi \sqrt{12}) \right) = \frac{1}{7} \log(a) \log_a \left( \exp(\pi \sqrt{3}) + \exp(\pi \sqrt{4}) + \exp(\pi \sqrt{7}) + \exp(\pi \sqrt{8}) + \exp(\pi \sqrt{11}) + \exp(\pi \sqrt{12}) \right) - \frac{5^2}{10^3}$$

$\log_b(x)$  is the base- $b$  logarithm

**Series representations:**

$$-\frac{5^2}{10^3} + \frac{1}{7} \log \left( \exp(\pi \sqrt{3}) + \exp(\pi \sqrt{4}) + \exp(\pi \sqrt{7}) + \exp(\pi \sqrt{8}) + \exp(\pi \sqrt{11}) + \exp(\pi \sqrt{12}) \right) = -\frac{1}{40} + \frac{2}{7} i \pi \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] + \frac{\log(z_0)}{7} - \frac{1}{7} \sum_{k=1}^{\infty} \frac{(-1)^k \left( e^{2\pi} + e^{2\sqrt{2}\pi} + e^{\sqrt{3}\pi} + e^{2\sqrt{3}\pi} + e^{\sqrt{7}\pi} + e^{\sqrt{11}\pi} - z_0 \right)^k z_0^{-k}}{k}$$

$$\begin{aligned}
& -\frac{5^2}{10^3} + \frac{1}{7} \log(\exp(\pi \sqrt{3}) + \exp(\pi \sqrt{4}) + \\
& \quad \exp(\pi \sqrt{7}) + \exp(\pi \sqrt{8}) + \exp(\pi \sqrt{11}) + \exp(\pi \sqrt{12})) = \\
& -\frac{1}{40} + \frac{1}{7} \log(-1 + e^{2\pi} + e^{2\sqrt{2}\pi} + e^{\sqrt{3}\pi} + e^{2\sqrt{3}\pi} + e^{\sqrt{7}\pi} + e^{\sqrt{11}\pi}) - \\
& \frac{1}{7} \sum_{k=1}^{\infty} \frac{\left( -\frac{1}{-1 + e^{2\pi} + e^{2\sqrt{2}\pi} + e^{\sqrt{3}\pi} + e^{2\sqrt{3}\pi} + e^{\sqrt{7}\pi} + e^{\sqrt{11}\pi}} \right)^k}{k}
\end{aligned}$$

$$\begin{aligned}
& -\frac{5^2}{10^3} + \frac{1}{7} \log(\exp(\pi \sqrt{3}) + \exp(\pi \sqrt{4}) + \\
& \quad \exp(\pi \sqrt{7}) + \exp(\pi \sqrt{8}) + \exp(\pi \sqrt{11}) + \exp(\pi \sqrt{12})) = \\
& -\frac{1}{40} + \frac{2}{7} i \pi \left[ \frac{\arg(e^{2\pi} + e^{2\sqrt{2}\pi} + e^{\sqrt{3}\pi} + e^{2\sqrt{3}\pi} + e^{\sqrt{7}\pi} + e^{\sqrt{11}\pi} - x)}{2\pi} \right] + \frac{\log(x)}{7} - \\
& \frac{1}{7} \sum_{k=1}^{\infty} \frac{(-1)^k (e^{2\pi} + e^{2\sqrt{2}\pi} + e^{\sqrt{3}\pi} + e^{2\sqrt{3}\pi} + e^{\sqrt{7}\pi} + e^{\sqrt{11}\pi} - x)^k x^{-k}}{k} \quad \text{for } x < 0
\end{aligned}$$

$\arg(z)$  is the complex argument

$\lfloor x \rfloor$  is the floor function

### Integral representations:

$$\begin{aligned}
& -\frac{5^2}{10^3} + \frac{1}{7} \log(\exp(\pi \sqrt{3}) + \exp(\pi \sqrt{4}) + \\
& \quad \exp(\pi \sqrt{7}) + \exp(\pi \sqrt{8}) + \exp(\pi \sqrt{11}) + \exp(\pi \sqrt{12})) = \\
& -\frac{1}{40} + \frac{1}{7} \int_1^{e^{2\pi} + e^{2\sqrt{2}\pi} + e^{\sqrt{3}\pi} + e^{2\sqrt{3}\pi} + e^{\sqrt{7}\pi} + e^{\sqrt{11}\pi}} \frac{1}{t} dt
\end{aligned}$$

$$\begin{aligned}
& -\frac{5^2}{10^3} + \frac{1}{7} \log(\exp(\pi \sqrt{3}) + \exp(\pi \sqrt{4}) + \exp(\pi \sqrt{7}) + \\
& \quad \exp(\pi \sqrt{8}) + \exp(\pi \sqrt{11}) + \exp(\pi \sqrt{12})) = -\frac{1}{40} - \frac{i}{14\pi} \\
& \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{(-1 + e^{2\pi} + e^{2\sqrt{2}\pi} + e^{\sqrt{3}\pi} + e^{2\sqrt{3}\pi} + e^{\sqrt{7}\pi} + e^{\sqrt{11}\pi})^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \quad \text{for } -1 < \gamma < 0
\end{aligned}$$

$\Gamma(x)$  is the gamma function

$$(21/10^3+5/10^3+2/10^3)+1/7 \ln((((((\exp(\text{Pi}*\text{sqrt}(3)) + \exp(\text{Pi}*\text{sqrt}(4)) + \exp(\text{Pi}*\text{sqrt}(7)) + \exp(\text{Pi}*\text{sqrt}(8)) + \exp(\text{Pi}*\text{sqrt}(11)) + \exp(\text{Pi}*\text{sqrt}(12))))))))))$$

**Input:**

$$\left(\frac{21}{10^3} + \frac{5}{10^3} + \frac{2}{10^3}\right) + \frac{1}{7} \log\left(\exp(\pi \sqrt{3}) + \exp(\pi \sqrt{4}) + \exp(\pi \sqrt{7}) + \exp(\pi \sqrt{8}) + \exp(\pi \sqrt{11}) + \exp(\pi \sqrt{12})\right)$$

log(x) is the natural logarithm

**Exact result:**

$$\frac{7}{250} + \frac{1}{7} \log\left(e^{2\pi} + e^{2\sqrt{2}\pi} + e^{\sqrt{3}\pi} + e^{2\sqrt{3}\pi} + e^{\sqrt{7}\pi} + e^{\sqrt{11}\pi}\right)$$

**Decimal approximation:**

1.671015091034664893801272979683954683391842542721780991787...

1.67101509...

We note that 1.67101509... is a result very near to the value of the formula:

$$m_{p'} = 2 \times \frac{\eta}{R} m_P = 1.6714213 \times 10^{-24} \text{ gm}$$

that is the holographic proton mass

**Alternate form:**

$$\frac{49 + 250 \log\left(e^{2\pi} + e^{2\sqrt{2}\pi} + e^{\sqrt{3}\pi} + e^{2\sqrt{3}\pi} + e^{\sqrt{7}\pi} + e^{\sqrt{11}\pi}\right)}{1750}$$

**Alternative representations:**

$$\left(\frac{21}{10^3} + \frac{5}{10^3} + \frac{2}{10^3}\right) + \frac{1}{7} \log\left(\exp(\pi \sqrt{3}) + \exp(\pi \sqrt{4}) + \exp(\pi \sqrt{7}) + \exp(\pi \sqrt{8}) + \exp(\pi \sqrt{11}) + \exp(\pi \sqrt{12})\right) = \frac{1}{7} \log_e\left(\exp(\pi \sqrt{3}) + \exp(\pi \sqrt{4}) + \exp(\pi \sqrt{7}) + \exp(\pi \sqrt{8}) + \exp(\pi \sqrt{11}) + \exp(\pi \sqrt{12})\right) + \frac{28}{10^3}$$

•



$$\left(\frac{21}{10^3} + \frac{5}{10^3} + \frac{2}{10^3}\right) + \frac{1}{7} \log\left(\exp(\pi\sqrt{3}) + \exp(\pi\sqrt{4}) + \exp(\pi\sqrt{7}) + \exp(\pi\sqrt{8}) + \exp(\pi\sqrt{11}) + \exp(\pi\sqrt{12})\right) = \frac{1}{7} \log(a) \log_a\left(\exp(\pi\sqrt{3}) + \exp(\pi\sqrt{4}) + \exp(\pi\sqrt{7}) + \exp(\pi\sqrt{8}) + \exp(\pi\sqrt{11}) + \exp(\pi\sqrt{12})\right) + \frac{28}{10^3}$$

$\log_b(x)$  is the base- $b$  logarithm

### Series representations:

$$\left(\frac{21}{10^3} + \frac{5}{10^3} + \frac{2}{10^3}\right) + \frac{1}{7} \log\left(\exp(\pi\sqrt{3}) + \exp(\pi\sqrt{4}) + \exp(\pi\sqrt{7}) + \exp(\pi\sqrt{8}) + \exp(\pi\sqrt{11}) + \exp(\pi\sqrt{12})\right) = \frac{7}{250} + \frac{2}{7} i\pi \left| \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right| + \frac{\log(z_0)}{7} - \frac{1}{7} \sum_{k=1}^{\infty} \frac{(-1)^k \left(e^{2\pi} + e^{2\sqrt{2}\pi} + e^{\sqrt{3}\pi} + e^{2\sqrt{3}\pi} + e^{\sqrt{7}\pi} + e^{\sqrt{11}\pi} - z_0\right)^k z_0^{-k}}{k}$$

$$\left(\frac{21}{10^3} + \frac{5}{10^3} + \frac{2}{10^3}\right) + \frac{1}{7} \log\left(\exp(\pi\sqrt{3}) + \exp(\pi\sqrt{4}) + \exp(\pi\sqrt{7}) + \exp(\pi\sqrt{8}) + \exp(\pi\sqrt{11}) + \exp(\pi\sqrt{12})\right) = \frac{7}{250} + \frac{1}{7} \log\left(-1 + e^{2\pi} + e^{2\sqrt{2}\pi} + e^{\sqrt{3}\pi} + e^{2\sqrt{3}\pi} + e^{\sqrt{7}\pi} + e^{\sqrt{11}\pi}\right) - \frac{1}{7} \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{-1 + e^{2\pi} + e^{2\sqrt{2}\pi} + e^{\sqrt{3}\pi} + e^{2\sqrt{3}\pi} + e^{\sqrt{7}\pi} + e^{\sqrt{11}\pi}}\right)^k}{k}$$

$$\left(\frac{21}{10^3} + \frac{5}{10^3} + \frac{2}{10^3}\right) + \frac{1}{7} \log\left(\exp(\pi\sqrt{3}) + \exp(\pi\sqrt{4}) + \exp(\pi\sqrt{7}) + \exp(\pi\sqrt{8}) + \exp(\pi\sqrt{11}) + \exp(\pi\sqrt{12})\right) = \frac{7}{250} + \frac{2}{7} i\pi \left| \frac{\arg\left(e^{2\pi} + e^{2\sqrt{2}\pi} + e^{\sqrt{3}\pi} + e^{2\sqrt{3}\pi} + e^{\sqrt{7}\pi} + e^{\sqrt{11}\pi} - x\right)}{2\pi} \right| + \frac{\log(x)}{7} - \frac{1}{7} \sum_{k=1}^{\infty} \frac{(-1)^k \left(e^{2\pi} + e^{2\sqrt{2}\pi} + e^{\sqrt{3}\pi} + e^{2\sqrt{3}\pi} + e^{\sqrt{7}\pi} + e^{\sqrt{11}\pi} - x\right)^k x^{-k}}{k} \quad \text{for } x < 0$$

$\arg(z)$  is the complex argument

$\lfloor x \rfloor$  is the floor function

**Integral representations:**

$$\left(\frac{21}{10^3} + \frac{5}{10^3} + \frac{2}{10^3}\right) + \frac{1}{7} \log\left(\exp(\pi\sqrt{3}) + \exp(\pi\sqrt{4}) + \exp(\pi\sqrt{7}) + \exp(\pi\sqrt{8}) + \exp(\pi\sqrt{11}) + \exp(\pi\sqrt{12})\right) = \frac{7}{250} + \frac{1}{7} \int_1^{\infty} \frac{e^{2\pi} + e^{2\sqrt{2}\pi} + e^{\sqrt{3}\pi} + e^{2\sqrt{3}\pi} + e^{\sqrt{7}\pi} + e^{\sqrt{11}\pi}}{t} dt$$

$$\left(\frac{21}{10^3} + \frac{5}{10^3} + \frac{2}{10^3}\right) + \frac{1}{7} \log\left(\exp(\pi\sqrt{3}) + \exp(\pi\sqrt{4}) + \exp(\pi\sqrt{7}) + \exp(\pi\sqrt{8}) + \exp(\pi\sqrt{11}) + \exp(\pi\sqrt{12})\right) = \frac{7}{250} - \frac{i}{14\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\left(-1 + e^{2\pi} + e^{2\sqrt{2}\pi} + e^{\sqrt{3}\pi} + e^{2\sqrt{3}\pi} + e^{\sqrt{7}\pi} + e^{\sqrt{11}\pi}\right)^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \text{ for } -1 < \gamma < 0$$

$\Gamma(x)$  is the gamma function

$$72 + 144 \cdot \ln\left(\left(\left(\left(\left(\exp(\pi\sqrt{3}) + \exp(\pi\sqrt{4}) + \exp(\pi\sqrt{7}) + \exp(\pi\sqrt{8}) + \exp(\pi\sqrt{11}) + \exp(\pi\sqrt{12})\right)\right)\right)\right)\right)$$

**Input:**

$$72 + 144 \log\left(\exp(\pi\sqrt{3}) + \exp(\pi\sqrt{4}) + \exp(\pi\sqrt{7}) + \exp(\pi\sqrt{8}) + \exp(\pi\sqrt{11}) + \exp(\pi\sqrt{12})\right)$$

$\log(x)$  is the natural logarithm

**Exact result:**

$$72 + 144 \log\left(e^{2\pi} + e^{2\sqrt{2}\pi} + e^{\sqrt{3}\pi} + e^{2\sqrt{3}\pi} + e^{\sqrt{7}\pi} + e^{\sqrt{11}\pi}\right)$$

**Decimal approximation:**

1728.159211762942212951683163521426320858977283063555239721...

1728.1592117...

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson. Furthermore, 1728 occurs in the algebraic formula for the  $j$ -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

**Alternate form:**

$$72 \left( 1 + 2 \log \left( e^{2\pi} + e^{2\sqrt{2}\pi} + e^{\sqrt{3}\pi} + e^{2\sqrt{3}\pi} + e^{\sqrt{7}\pi} + e^{\sqrt{11}\pi} \right) \right)$$

**Alternative representations:**

$$72 + 144 \log \left( \exp(\pi\sqrt{3}) + \exp(\pi\sqrt{4}) + \exp(\pi\sqrt{7}) + \exp(\pi\sqrt{8}) + \exp(\pi\sqrt{11}) + \exp(\pi\sqrt{12}) \right) =$$

$$72 + 144 \log_e \left( \exp(\pi\sqrt{3}) + \exp(\pi\sqrt{4}) + \exp(\pi\sqrt{7}) + \exp(\pi\sqrt{8}) + \exp(\pi\sqrt{11}) + \exp(\pi\sqrt{12}) \right)$$

$$72 + 144 \log \left( \exp(\pi\sqrt{3}) + \exp(\pi\sqrt{4}) + \exp(\pi\sqrt{7}) + \exp(\pi\sqrt{8}) + \exp(\pi\sqrt{11}) + \exp(\pi\sqrt{12}) \right) =$$

$$72 + 144 \log(a) \log_a \left( \exp(\pi\sqrt{3}) + \exp(\pi\sqrt{4}) + \exp(\pi\sqrt{7}) + \exp(\pi\sqrt{8}) + \exp(\pi\sqrt{11}) + \exp(\pi\sqrt{12}) \right)$$

$\log_b(x)$  is the base- $b$  logarithm

**Series representations:**

$$72 + 144 \log \left( \exp(\pi\sqrt{3}) + \exp(\pi\sqrt{4}) + \exp(\pi\sqrt{7}) + \exp(\pi\sqrt{8}) + \exp(\pi\sqrt{11}) + \exp(\pi\sqrt{12}) \right) =$$

$$72 + 288 i \pi \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] + 144 \log(z_0) -$$

$$144 \sum_{k=1}^{\infty} \frac{(-1)^k \left( e^{2\pi} + e^{2\sqrt{2}\pi} + e^{\sqrt{3}\pi} + e^{2\sqrt{3}\pi} + e^{\sqrt{7}\pi} + e^{\sqrt{11}\pi} - z_0 \right)^k z_0^{-k}}{k}$$

$$72 + 144 \log \left( \exp(\pi\sqrt{3}) + \exp(\pi\sqrt{4}) + \exp(\pi\sqrt{7}) + \exp(\pi\sqrt{8}) + \exp(\pi\sqrt{11}) + \exp(\pi\sqrt{12}) \right) =$$

$$72 + 144 \log \left( -1 + e^{2\pi} + e^{2\sqrt{2}\pi} + e^{\sqrt{3}\pi} + e^{2\sqrt{3}\pi} + e^{\sqrt{7}\pi} + e^{\sqrt{11}\pi} \right) -$$

$$144 \sum_{k=1}^{\infty} \frac{\left( -\frac{1}{-1 + e^{2\pi} + e^{2\sqrt{2}\pi} + e^{\sqrt{3}\pi} + e^{2\sqrt{3}\pi} + e^{\sqrt{7}\pi} + e^{\sqrt{11}\pi}} \right)^k}{k}$$

$$\begin{aligned}
& 72 + 144 \log\left(\exp(\pi \sqrt{3}) + \exp(\pi \sqrt{4}) + \right. \\
& \quad \left. \exp(\pi \sqrt{7}) + \exp(\pi \sqrt{8}) + \exp(\pi \sqrt{11}) + \exp(\pi \sqrt{12})\right) = \\
& 72 + 288 i \pi \left[ \frac{\arg\left(e^{2\pi} + e^{2\sqrt{2}\pi} + e^{\sqrt{3}\pi} + e^{2\sqrt{3}\pi} + e^{\sqrt{7}\pi} + e^{\sqrt{11}\pi} - x\right)}{2\pi} \right] + \\
& 144 \log(x) - \\
& 144 \sum_{k=1}^{\infty} \frac{(-1)^k \left(e^{2\pi} + e^{2\sqrt{2}\pi} + e^{\sqrt{3}\pi} + e^{2\sqrt{3}\pi} + e^{\sqrt{7}\pi} + e^{\sqrt{11}\pi} - x\right)^k x^{-k}}{k} \quad \text{for } x < 0
\end{aligned}$$

$\arg(z)$  is the complex argument

$\lfloor x \rfloor$  is the floor function

### Integral representations:

$$\begin{aligned}
& 72 + 144 \log\left(\exp(\pi \sqrt{3}) + \exp(\pi \sqrt{4}) + \right. \\
& \quad \left. \exp(\pi \sqrt{7}) + \exp(\pi \sqrt{8}) + \exp(\pi \sqrt{11}) + \exp(\pi \sqrt{12})\right) = \\
& 72 + 144 \int_1^{e^{2\pi} + e^{2\sqrt{2}\pi} + e^{\sqrt{3}\pi} + e^{2\sqrt{3}\pi} + e^{\sqrt{7}\pi} + e^{\sqrt{11}\pi}} \frac{1}{t} dt
\end{aligned}$$

$$\begin{aligned}
& 72 + 144 \log\left(\exp(\pi \sqrt{3}) + \exp(\pi \sqrt{4}) + \exp(\pi \sqrt{7}) + \right. \\
& \quad \left. \exp(\pi \sqrt{8}) + \exp(\pi \sqrt{11}) + \exp(\pi \sqrt{12})\right) = 72 - \frac{72i}{\pi} \\
& \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\left(-1 + e^{2\pi} + e^{2\sqrt{2}\pi} + e^{\sqrt{3}\pi} + e^{2\sqrt{3}\pi} + e^{\sqrt{7}\pi} + e^{\sqrt{11}\pi}\right)^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \quad \text{for } -1 < \gamma < 0
\end{aligned}$$

$\Gamma(x)$  is the gamma function

$$128 + 144 * \ln\left(\left(\left(\left(\left(\left(\exp(\text{Pi} * \text{sqrt}(3)) + \exp(\text{Pi} * \text{sqrt}(4)) + \exp(\text{Pi} * \text{sqrt}(7)) + \exp(\text{Pi} * \text{sqrt}(8)) + \exp(\text{Pi} * \text{sqrt}(11)) + \exp(\text{Pi} * \text{sqrt}(12))\right)\right)\right)\right)\right)\right)$$

### Input:

$$128 + 144 \log\left(\exp(\pi \sqrt{3}) + \exp(\pi \sqrt{4}) + \exp(\pi \sqrt{7}) + \exp(\pi \sqrt{8}) + \exp(\pi \sqrt{11}) + \exp(\pi \sqrt{12})\right)$$

$\log(x)$  is the natural logarithm

### Exact result:

$$128 + 144 \log\left(e^{2\pi} + e^{2\sqrt{2}\pi} + e^{\sqrt{3}\pi} + e^{2\sqrt{3}\pi} + e^{\sqrt{7}\pi} + e^{\sqrt{11}\pi}\right)$$

**Decimal approximation:**

1784.159211762942212951683163521426320858977283063555239721...

1784.159211... result in the range of the hypothetical mass of Gluino (gluino = 1785.16 GeV).

**Alternate form:**

$$16\left(8 + 9 \log\left(e^{2\pi} + e^{2\sqrt{2}\pi} + e^{\sqrt{3}\pi} + e^{2\sqrt{3}\pi} + e^{\sqrt{7}\pi} + e^{\sqrt{11}\pi}\right)\right)$$

**Alternative representations:**

$$\begin{aligned} &128 + 144 \log\left(\exp(\pi\sqrt{3}) + \exp(\pi\sqrt{4}) + \right. \\ &\quad \left. \exp(\pi\sqrt{7}) + \exp(\pi\sqrt{8}) + \exp(\pi\sqrt{11}) + \exp(\pi\sqrt{12})\right) = \\ &128 + 144 \log_e\left(\exp(\pi\sqrt{3}) + \exp(\pi\sqrt{4}) + \exp(\pi\sqrt{7}) + \right. \\ &\quad \left. \exp(\pi\sqrt{8}) + \exp(\pi\sqrt{11}) + \exp(\pi\sqrt{12})\right) \end{aligned}$$

$$\begin{aligned} &128 + 144 \log\left(\exp(\pi\sqrt{3}) + \exp(\pi\sqrt{4}) + \right. \\ &\quad \left. \exp(\pi\sqrt{7}) + \exp(\pi\sqrt{8}) + \exp(\pi\sqrt{11}) + \exp(\pi\sqrt{12})\right) = \\ &128 + 144 \log_a\left(\exp(\pi\sqrt{3}) + \exp(\pi\sqrt{4}) + \exp(\pi\sqrt{7}) + \right. \\ &\quad \left. \exp(\pi\sqrt{8}) + \exp(\pi\sqrt{11}) + \exp(\pi\sqrt{12})\right) \end{aligned}$$

$\log_b(x)$  is the base- $b$  logarithm

**Series representations:**

$$\begin{aligned} &128 + 144 \log\left(\exp(\pi\sqrt{3}) + \exp(\pi\sqrt{4}) + \right. \\ &\quad \left. \exp(\pi\sqrt{7}) + \exp(\pi\sqrt{8}) + \exp(\pi\sqrt{11}) + \exp(\pi\sqrt{12})\right) = \\ &128 + 288 i \pi \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] + 144 \log(z_0) - \\ &144 \sum_{k=1}^{\infty} \frac{(-1)^k \left(e^{2\pi} + e^{2\sqrt{2}\pi} + e^{\sqrt{3}\pi} + e^{2\sqrt{3}\pi} + e^{\sqrt{7}\pi} + e^{\sqrt{11}\pi} - z_0\right)^k z_0^{-k}}{k} \end{aligned}$$

$$\begin{aligned}
& 128 + 144 \log\left(\exp(\pi\sqrt{3}) + \exp(\pi\sqrt{4}) + \right. \\
& \quad \left. \exp(\pi\sqrt{7}) + \exp(\pi\sqrt{8}) + \exp(\pi\sqrt{11}) + \exp(\pi\sqrt{12})\right) = \\
& 128 + 144 \log\left(-1 + e^{2\pi} + e^{2\sqrt{2}\pi} + e^{\sqrt{3}\pi} + e^{2\sqrt{3}\pi} + e^{\sqrt{7}\pi} + e^{\sqrt{11}\pi}\right) - \\
& 144 \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{-1 + e^{2\pi} + e^{2\sqrt{2}\pi} + e^{\sqrt{3}\pi} + e^{2\sqrt{3}\pi} + e^{\sqrt{7}\pi} + e^{\sqrt{11}\pi}}\right)^k}{k}
\end{aligned}$$

$$\begin{aligned}
& 128 + 144 \log\left(\exp(\pi\sqrt{3}) + \exp(\pi\sqrt{4}) + \right. \\
& \quad \left. \exp(\pi\sqrt{7}) + \exp(\pi\sqrt{8}) + \exp(\pi\sqrt{11}) + \exp(\pi\sqrt{12})\right) = \\
& 128 + 288 i \pi \left[ \frac{\arg\left(e^{2\pi} + e^{2\sqrt{2}\pi} + e^{\sqrt{3}\pi} + e^{2\sqrt{3}\pi} + e^{\sqrt{7}\pi} + e^{\sqrt{11}\pi} - x\right)}{2\pi} \right] + \\
& 144 \log(x) - \\
& 144 \sum_{k=1}^{\infty} \frac{(-1)^k \left(e^{2\pi} + e^{2\sqrt{2}\pi} + e^{\sqrt{3}\pi} + e^{2\sqrt{3}\pi} + e^{\sqrt{7}\pi} + e^{\sqrt{11}\pi} - x\right)^k x^{-k}}{k} \quad \text{for } x < 0
\end{aligned}$$

$\arg(z)$  is the complex argument

$\lfloor x \rfloor$  is the floor function

### Integral representations:

$$\begin{aligned}
& 128 + 144 \log\left(\exp(\pi\sqrt{3}) + \exp(\pi\sqrt{4}) + \right. \\
& \quad \left. \exp(\pi\sqrt{7}) + \exp(\pi\sqrt{8}) + \exp(\pi\sqrt{11}) + \exp(\pi\sqrt{12})\right) = \\
& 128 + 144 \int_1^{e^{2\pi} + e^{2\sqrt{2}\pi} + e^{\sqrt{3}\pi} + e^{2\sqrt{3}\pi} + e^{\sqrt{7}\pi} + e^{\sqrt{11}\pi}} \frac{1}{t} dt
\end{aligned}$$

$$\begin{aligned}
& 128 + 144 \log\left(\exp(\pi\sqrt{3}) + \exp(\pi\sqrt{4}) + \exp(\pi\sqrt{7}) + \right. \\
& \quad \left. \exp(\pi\sqrt{8}) + \exp(\pi\sqrt{11}) + \exp(\pi\sqrt{12})\right) = 128 - \frac{72i}{\pi} \\
& \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\left(-1 + e^{2\pi} + e^{2\sqrt{2}\pi} + e^{\sqrt{3}\pi} + e^{2\sqrt{3}\pi} + e^{\sqrt{7}\pi} + e^{\sqrt{11}\pi}\right)^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \quad \text{for } -1 < \gamma < 0
\end{aligned}$$

$$-(8^2 - 3 - (144 + 21) \ln(\exp(\pi\sqrt{3}) + \exp(\pi\sqrt{4}) + \exp(\pi\sqrt{7}) + \exp(\pi\sqrt{8}) + \exp(\pi\sqrt{11}) + \exp(\pi\sqrt{12})))$$

**Input:**

$$-(8^2 - 3 - (144 + 21) \log(\exp(\pi \sqrt{3}) + \exp(\pi \sqrt{4}) + \exp(\pi \sqrt{7}) + \exp(\pi \sqrt{8}) + \exp(\pi \sqrt{11}) + \exp(\pi \sqrt{12})))$$

$\log(x)$  is the natural logarithm

**Exact result:**

$$165 \log(e^{2\pi} + e^{2\sqrt{2}\pi} + e^{\sqrt{3}\pi} + e^{2\sqrt{3}\pi} + e^{\sqrt{7}\pi} + e^{\sqrt{11}\pi}) - 61$$

**Decimal approximation:**

1836.682430145037952340470291534967659317578136843657045514...

1836.6824... result very near to the following formula:

$$\mu = \frac{m_p}{m_e} = \frac{\alpha^2}{\pi r_p R_H} = \mathbf{1836.15267 \dots}$$

that is the ratio between proton mass and electron mass

**Alternative representations:**

$$-(8^2 - 3 - (144 + 21) \log(\exp(\pi \sqrt{3}) + \exp(\pi \sqrt{4}) + \exp(\pi \sqrt{7}) + \exp(\pi \sqrt{8}) + \exp(\pi \sqrt{11}) + \exp(\pi \sqrt{12}))) = 3 + 165 \log_e(\exp(\pi \sqrt{3}) + \exp(\pi \sqrt{4}) + \exp(\pi \sqrt{7}) + \exp(\pi \sqrt{8}) + \exp(\pi \sqrt{11}) + \exp(\pi \sqrt{12})) - 8^2$$

$$-(8^2 - 3 - (144 + 21) \log(\exp(\pi \sqrt{3}) + \exp(\pi \sqrt{4}) + \exp(\pi \sqrt{7}) + \exp(\pi \sqrt{8}) + \exp(\pi \sqrt{11}) + \exp(\pi \sqrt{12}))) = 3 + 165 \log(\alpha) \log_\alpha(\exp(\pi \sqrt{3}) + \exp(\pi \sqrt{4}) + \exp(\pi \sqrt{7}) + \exp(\pi \sqrt{8}) + \exp(\pi \sqrt{11}) + \exp(\pi \sqrt{12})) - 8^2$$

$\log_b(x)$  is the base- $b$  logarithm

**Series representations:**

$$\begin{aligned}
& -\left(8^2 - 3 - (144 + 21) \log\left(\exp(\pi \sqrt{3}) + \exp(\pi \sqrt{4}) + \right. \right. \\
& \quad \left. \left. \exp(\pi \sqrt{7}) + \exp(\pi \sqrt{8}) + \exp(\pi \sqrt{11}) + \exp(\pi \sqrt{12})\right)\right) = \\
& -61 + 330 i \pi \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2 \pi} \right] + 165 \log(z_0) - \\
& 165 \sum_{k=1}^{\infty} \frac{(-1)^k \left( e^{2\pi} + e^{2\sqrt{2}\pi} + e^{\sqrt{3}\pi} + e^{2\sqrt{3}\pi} + e^{\sqrt{7}\pi} + e^{\sqrt{11}\pi} - z_0 \right)^k z_0^{-k}}{k}
\end{aligned}$$

$$\begin{aligned}
& -\left(8^2 - 3 - (144 + 21) \log\left(\exp(\pi \sqrt{3}) + \exp(\pi \sqrt{4}) + \right. \right. \\
& \quad \left. \left. \exp(\pi \sqrt{7}) + \exp(\pi \sqrt{8}) + \exp(\pi \sqrt{11}) + \exp(\pi \sqrt{12})\right)\right) = \\
& -61 + 165 \log\left(-1 + e^{2\pi} + e^{2\sqrt{2}\pi} + e^{\sqrt{3}\pi} + e^{2\sqrt{3}\pi} + e^{\sqrt{7}\pi} + e^{\sqrt{11}\pi}\right) - \\
& 165 \sum_{k=1}^{\infty} \frac{\left( -\frac{1}{-1 + e^{2\pi} + e^{2\sqrt{2}\pi} + e^{\sqrt{3}\pi} + e^{2\sqrt{3}\pi} + e^{\sqrt{7}\pi} + e^{\sqrt{11}\pi}} \right)^k}{k}
\end{aligned}$$

$$\begin{aligned}
& -\left(8^2 - 3 - (144 + 21) \log\left(\exp(\pi \sqrt{3}) + \exp(\pi \sqrt{4}) + \right. \right. \\
& \quad \left. \left. \exp(\pi \sqrt{7}) + \exp(\pi \sqrt{8}) + \exp(\pi \sqrt{11}) + \exp(\pi \sqrt{12})\right)\right) = \\
& -61 + 330 i \pi \left[ \frac{\arg\left(e^{2\pi} + e^{2\sqrt{2}\pi} + e^{\sqrt{3}\pi} + e^{2\sqrt{3}\pi} + e^{\sqrt{7}\pi} + e^{\sqrt{11}\pi} - x\right)}{2 \pi} \right] + \\
& 165 \log(x) - \\
& 165 \sum_{k=1}^{\infty} \frac{(-1)^k \left( e^{2\pi} + e^{2\sqrt{2}\pi} + e^{\sqrt{3}\pi} + e^{2\sqrt{3}\pi} + e^{\sqrt{7}\pi} + e^{\sqrt{11}\pi} - x \right)^k x^{-k}}{k} \quad \text{for } x < 0
\end{aligned}$$

$\arg(z)$  is the complex argument

$[x]$  is the floor function

### Integral representations:

$$\begin{aligned}
& -\left(8^2 - 3 - (144 + 21) \log\left(\exp(\pi \sqrt{3}) + \exp(\pi \sqrt{4}) + \right. \right. \\
& \quad \left. \left. \exp(\pi \sqrt{7}) + \exp(\pi \sqrt{8}) + \exp(\pi \sqrt{11}) + \exp(\pi \sqrt{12})\right)\right) = \\
& -61 + 165 \int_1^{e^{2\pi} + e^{2\sqrt{2}\pi} + e^{\sqrt{3}\pi} + e^{2\sqrt{3}\pi} + e^{\sqrt{7}\pi} + e^{\sqrt{11}\pi}} \frac{1}{t} dt
\end{aligned}$$



$$\begin{aligned}
& -\left(8^2 - 3 - (144 + 21) \log\left(\exp(\pi \sqrt{3}) + \exp(\pi \sqrt{4}) + \exp(\pi \sqrt{7}) + \right. \right. \\
& \quad \left. \left. \exp(\pi \sqrt{8}) + \exp(\pi \sqrt{11}) + \exp(\pi \sqrt{12})\right)\right) = -61 - \frac{165 i}{2 \pi} \\
& \int_{-i \infty + \gamma}^{i \infty + \gamma} \frac{\left(-1 + e^{2 \pi} + e^{2 \sqrt{2} \pi} + e^{\sqrt{3} \pi} + e^{2 \sqrt{3} \pi} + e^{\sqrt{7} \pi} + e^{\sqrt{11} \pi}\right)^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} \\
& ds \text{ for } -1 < \gamma < 0
\end{aligned}$$

$\Gamma(x)$  is the gamma function

$$-9^3 - 34 - 3 + 2 * ((((((\exp(\text{Pi} * \text{sqrt}(3)) + \exp(\text{Pi} * \text{sqrt}(4)) + \exp(\text{Pi} * \text{sqrt}(7)) + \exp(\text{Pi} * \text{sqrt}(8)) + \exp(\text{Pi} * \text{sqrt}(11)) + \exp(\text{Pi} * \text{sqrt}(12))))))))))$$

**Input:**

$$\begin{aligned}
& -9^3 - 34 - 3 + \\
& 2 \left( \exp(\pi \sqrt{3}) + \exp(\pi \sqrt{4}) + \exp(\pi \sqrt{7}) + \exp(\pi \sqrt{8}) + \exp(\pi \sqrt{11}) + \exp(\pi \sqrt{12}) \right)
\end{aligned}$$

**Exact result:**

$$2 \left( e^{2 \pi} + e^{2 \sqrt{2} \pi} + e^{\sqrt{3} \pi} + e^{2 \sqrt{3} \pi} + e^{\sqrt{7} \pi} + e^{\sqrt{11} \pi} \right) - 766$$

**Decimal approximation:**

196883.9504052270668468392269137574709565856125083177439751...

196883.95...  $\approx$  196884 that is a value of the following partition function:

$$\begin{aligned}
Z_{24}(\tau) &= j(\tau) - 744 \\
&= q^{-1} + 196884 q + 21493760 q^2 + 864299970 q^3 + 20245856256 q^4 + \dots
\end{aligned}$$

**Alternate forms:**

$$2 \left( -383 + e^{2 \pi} + e^{2 \sqrt{2} \pi} + e^{\sqrt{3} \pi} + e^{2 \sqrt{3} \pi} + e^{\sqrt{7} \pi} + e^{\sqrt{11} \pi} \right)$$

$$-766 + 2 e^{2 \pi} + 2 e^{2 \sqrt{2} \pi} + 2 e^{\sqrt{3} \pi} + 2 e^{2 \sqrt{3} \pi} + 2 e^{\sqrt{7} \pi} + 2 e^{\sqrt{11} \pi}$$

**Series representations:**

$$\begin{aligned}
& -9^3 - 34 - 3 + 2 \left( \exp(\pi \sqrt{3}) + \exp(\pi \sqrt{4}) + \right. \\
& \quad \left. \exp(\pi \sqrt{7}) + \exp(\pi \sqrt{8}) + \exp(\pi \sqrt{11}) + \exp(\pi \sqrt{12}) \right) = \\
& 2 \left( -383 + \exp \left( \pi \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k} \right) + \exp \left( \pi \sqrt{3} \sum_{k=0}^{\infty} 3^{-k} \binom{\frac{1}{2}}{k} \right) + \right. \\
& \quad \exp \left( \pi \sqrt{6} \sum_{k=0}^{\infty} 6^{-k} \binom{\frac{1}{2}}{k} \right) + \exp \left( \pi \sqrt{7} \sum_{k=0}^{\infty} 7^{-k} \binom{\frac{1}{2}}{k} \right) + \\
& \quad \left. \exp \left( \pi \sqrt{10} \sum_{k=0}^{\infty} 10^{-k} \binom{\frac{1}{2}}{k} \right) + \exp \left( \pi \sqrt{11} \sum_{k=0}^{\infty} 11^{-k} \binom{\frac{1}{2}}{k} \right) \right)
\end{aligned}$$

$$\begin{aligned}
& -9^3 - 34 - 3 + 2 \left( \exp(\pi \sqrt{3}) + \exp(\pi \sqrt{4}) + \right. \\
& \quad \left. \exp(\pi \sqrt{7}) + \exp(\pi \sqrt{8}) + \exp(\pi \sqrt{11}) + \exp(\pi \sqrt{12}) \right) = \\
& 2 \left( -383 + \exp \left( \pi \sqrt{2} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right) + \exp \left( \pi \sqrt{3} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{3}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right) + \right. \\
& \quad \exp \left( \pi \sqrt{6} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{6}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right) + \exp \left( \pi \sqrt{7} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{7}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right) + \\
& \quad \left. \exp \left( \pi \sqrt{10} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{10}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right) + \exp \left( \pi \sqrt{11} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{11}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right) \right)
\end{aligned}$$

$$\begin{aligned}
& -9^3 - 34 - 3 + 2 \left( \exp(\pi \sqrt{3}) + \exp(\pi \sqrt{4}) + \right. \\
& \quad \left. \exp(\pi \sqrt{7}) + \exp(\pi \sqrt{8}) + \exp(\pi \sqrt{11}) + \exp(\pi \sqrt{12}) \right) = \\
& 2 \left( -383 + \exp \left( \pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (3 - z_0)^k z_0^{-k}}{k!} \right) + \right. \\
& \quad \exp \left( \pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (4 - z_0)^k z_0^{-k}}{k!} \right) + \\
& \quad \exp \left( \pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (7 - z_0)^k z_0^{-k}}{k!} \right) + \\
& \quad \exp \left( \pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (8 - z_0)^k z_0^{-k}}{k!} \right) + \\
& \quad \exp \left( \pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (11 - z_0)^k z_0^{-k}}{k!} \right) + \\
& \quad \left. \exp \left( \pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (12 - z_0)^k z_0^{-k}}{k!} \right) \right)
\end{aligned}$$

for not  $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

$\binom{n}{m}$  is the binomial coefficient

$n!$  is the factorial function  
 $(\alpha)_n$  is the Pochhammer symbol (rising factorial)  
 $\mathbb{R}$  is the set of real numbers

And:

$$(27 \times 4 + 1.08185^2 - 21/10^4 + 8/10^4 + 55/10^5 + 34/10^6 + 1/10^5) * (((-9^3 - 34 - 3 + 2 * ((((((\exp(\pi \sqrt{3})) + \exp(\pi \sqrt{4})) + \exp(\pi \sqrt{7})) + \exp(\pi \sqrt{8})) + \exp(\pi \sqrt{11})) + \exp(\pi \sqrt{12}))))))))))$$

Where 1.0185 is a Ramanujan mock theta function

**Input interpretation:**

$$\left( 27 \times 4 + 1.08185^2 - \frac{21}{10^4} + \frac{8}{10^4} + \frac{55}{10^5} + \frac{34}{10^6} + \frac{1}{10^5} \right) \left( -9^3 - 34 - 3 + 2 \left( \exp(\pi \sqrt{3}) + \exp(\pi \sqrt{4}) + \exp(\pi \sqrt{7}) + \exp(\pi \sqrt{8}) + \exp(\pi \sqrt{11}) + \exp(\pi \sqrt{12}) \right) \right)$$

**Result:**

$$2.14937605055493335291683521653218139518493991249074293... \times 10^7$$

21493760.50554933...  $\approx$  21493760 that is a value of the following partition function:

$$Z_{24}(\tau) = j(\tau) - 744 = q^{-1} + 196884q + 21493760q^2 + 864299970q^3 + 20245856256q^4 + \dots$$

**Series representations:**

$$\left( 27 \times 4 + 1.08185^2 - \frac{21}{10^4} + \frac{8}{10^4} + \frac{55}{10^5} + \frac{34}{10^6} + \frac{1}{10^5} \right) \left( -9^3 - 34 - 3 + 2 \left( \exp(\pi \sqrt{3}) + \exp(\pi \sqrt{4}) + \exp(\pi \sqrt{7}) + \exp(\pi \sqrt{8}) + \exp(\pi \sqrt{11}) + \exp(\pi \sqrt{12}) \right) \right) = 218.339 \left( -383 + \exp\left(\pi \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k}\right) + \exp\left(\pi \sqrt{3} \sum_{k=0}^{\infty} 3^{-k} \binom{\frac{1}{2}}{k}\right) + \exp\left(\pi \sqrt{6} \sum_{k=0}^{\infty} 6^{-k} \binom{\frac{1}{2}}{k}\right) + \exp\left(\pi \sqrt{7} \sum_{k=0}^{\infty} 7^{-k} \binom{\frac{1}{2}}{k}\right) + \exp\left(\pi \sqrt{10} \sum_{k=0}^{\infty} 10^{-k} \binom{\frac{1}{2}}{k}\right) + \exp\left(\pi \sqrt{11} \sum_{k=0}^{\infty} 11^{-k} \binom{\frac{1}{2}}{k}\right) \right)$$

$$\begin{aligned}
& \left( 27 \times 4 + 1.08185^2 - \frac{21}{10^4} + \frac{8}{10^4} + \frac{55}{10^5} + \frac{34}{10^6} + \frac{1}{10^5} \right) \\
& \left( -9^3 - 34 - 3 + 2 \left( \exp(\pi \sqrt{3}) + \exp(\pi \sqrt{4}) + \exp(\pi \sqrt{7}) + \right. \right. \\
& \quad \left. \left. \exp(\pi \sqrt{8}) + \exp(\pi \sqrt{11}) + \exp(\pi \sqrt{12}) \right) \right) = \\
& 218.339 \left( -383 + \exp \left[ \pi \sqrt{2} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right] + \exp \left[ \pi \sqrt{3} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{3}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right] + \right. \\
& \quad \exp \left[ \pi \sqrt{6} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{6}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right] + \exp \left[ \pi \sqrt{7} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{7}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right] + \\
& \quad \left. \exp \left[ \pi \sqrt{10} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{10}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right] + \exp \left[ \pi \sqrt{11} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{11}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right] \right)
\end{aligned}$$

$$\begin{aligned}
& \left( 27 \times 4 + 1.08185^2 - \frac{21}{10^4} + \frac{8}{10^4} + \frac{55}{10^5} + \frac{34}{10^6} + \frac{1}{10^5} \right) \\
& \left( -9^3 - 34 - 3 + 2 \left( \exp(\pi \sqrt{3}) + \exp(\pi \sqrt{4}) + \exp(\pi \sqrt{7}) + \right. \right. \\
& \quad \left. \left. \exp(\pi \sqrt{8}) + \exp(\pi \sqrt{11}) + \exp(\pi \sqrt{12}) \right) \right) = \\
& 218.339 \left( -383 + \exp \left[ \pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (3 - z_0)^k z_0^{-k}}{k!} \right] + \right. \\
& \quad \exp \left[ \pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (4 - z_0)^k z_0^{-k}}{k!} \right] + \\
& \quad \exp \left[ \pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (7 - z_0)^k z_0^{-k}}{k!} \right] + \\
& \quad \exp \left[ \pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (8 - z_0)^k z_0^{-k}}{k!} \right] + \\
& \quad \exp \left[ \pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (11 - z_0)^k z_0^{-k}}{k!} \right] + \\
& \quad \left. \exp \left[ \pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (12 - z_0)^k z_0^{-k}}{k!} \right] \right)
\end{aligned}$$

for not  $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

$\binom{n}{m}$  is the binomial coefficient

$n!$  is the factorial function

$(\alpha)_n$  is the Pochhammer symbol (rising factorial)

$\mathbb{R}$  is the set of real numbers

And:

$$1.0061571663 + 1 / (233 + 21 + 8 + 3) * (-729 - 34 - 3 + 2 * ((\exp(\pi * \sqrt{3})) + \exp(\pi * \sqrt{4}) + \exp(\pi * \sqrt{7}) + \exp(\pi * \sqrt{8}) + \exp(\pi * \sqrt{11}) + \exp(\pi * \sqrt{12}))))$$

Where 1.0061571663 is a Ramanujan mock theta function

**Input interpretation:**

$$\frac{1.0061571663 + 1}{233 + 21 + 8 + 3} \left( -729 - 34 - 3 + 2 \left( \exp(\pi \sqrt{3}) + \exp(\pi \sqrt{4}) + \exp(\pi \sqrt{7}) + \exp(\pi \sqrt{8}) + \exp(\pi \sqrt{11}) + \exp(\pi \sqrt{12}) \right) \right)$$

**Result:**

743.96446058225...

743.96446...  $\approx$  744 that is a value of the following partition function:

$$\begin{aligned} Z_{24}(\tau) &= j(\tau) - 744 \\ &= q^{-1} + 196884q + 21493760q^2 + 864299970q^3 + 20245856256q^4 + \dots \end{aligned}$$

Note that:  $744 - 10 - 6 = 728 = 9^3 - 1$  (see Fig. below “Ramanujan manuscript”)

**Series representations:**

$$\begin{aligned}
& 1.00615716630000 + \frac{1}{233 + 21 + 8 + 3} \\
& \left( -729 - 34 - 3 + 2 \left( \exp(\pi \sqrt{3}) + \exp(\pi \sqrt{4}) + \exp(\pi \sqrt{7}) + \exp(\pi \sqrt{8}) + \right. \right. \\
& \quad \left. \left. \exp(\pi \sqrt{11}) + \exp(\pi \sqrt{12}) \right) \right) = 0.0075471698113208 \\
& \left( -249.684175465250 + 1.00000000000000 \exp \left( \pi \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k} \right) + \right. \\
& \quad 1.00000000000000 \exp \left( \pi \sqrt{3} \sum_{k=0}^{\infty} 3^{-k} \binom{\frac{1}{2}}{k} \right) + \\
& \quad 1.00000000000000 \exp \left( \pi \sqrt{6} \sum_{k=0}^{\infty} 6^{-k} \binom{\frac{1}{2}}{k} \right) + \\
& \quad 1.00000000000000 \exp \left( \pi \sqrt{7} \sum_{k=0}^{\infty} 7^{-k} \binom{\frac{1}{2}}{k} \right) + \\
& \quad 1.00000000000000 \exp \left( \pi \sqrt{10} \sum_{k=0}^{\infty} 10^{-k} \binom{\frac{1}{2}}{k} \right) + \\
& \quad \left. 1.00000000000000 \exp \left( \pi \sqrt{11} \sum_{k=0}^{\infty} 11^{-k} \binom{\frac{1}{2}}{k} \right) \right)
\end{aligned}$$

$$\begin{aligned}
& 1.00615716630000 + \frac{1}{233 + 21 + 8 + 3} \\
& \left( -729 - 34 - 3 + 2 \left( \exp(\pi \sqrt{3}) + \exp(\pi \sqrt{4}) + \exp(\pi \sqrt{7}) + \exp(\pi \sqrt{8}) + \right. \right. \\
& \quad \left. \left. \exp(\pi \sqrt{11}) + \exp(\pi \sqrt{12}) \right) \right) = 0.0075471698113208 \\
& \left( -249.684175465250 + 1.00000000000000 \exp \left( \pi \sqrt{2} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right) + \right. \\
& \quad 1.00000000000000 \exp \left( \pi \sqrt{3} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{3}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right) + \\
& \quad 1.00000000000000 \exp \left( \pi \sqrt{6} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{6}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right) + \\
& \quad 1.00000000000000 \exp \left( \pi \sqrt{7} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{7}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right) + \\
& \quad 1.00000000000000 \exp \left( \pi \sqrt{10} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{10}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right) + \\
& \quad \left. 1.00000000000000 \exp \left( \pi \sqrt{11} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{11}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right) \right)
\end{aligned}$$

$$\begin{aligned}
& 1.00615716630000 + \frac{1}{233 + 21 + 8 + 3} \\
& \left( -729 - 34 - 3 + 2 \left( \exp(\pi \sqrt{3}) + \exp(\pi \sqrt{4}) + \exp(\pi \sqrt{7}) + \right. \right. \\
& \quad \left. \left. \exp(\pi \sqrt{8}) + \exp(\pi \sqrt{11}) + \exp(\pi \sqrt{12}) \right) \right) = \\
& 0.0075471698113208 \left( -249.684175465250 + 1.0000000000000000 \right. \\
& \quad \left. \exp \left( \pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (3 - z_0)^k z_0^{-k}}{k!} \right) + \right. \\
& 1.0000000000000000 \exp \left( \pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (4 - z_0)^k z_0^{-k}}{k!} \right) + \\
& 1.0000000000000000 \exp \left( \pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (7 - z_0)^k z_0^{-k}}{k!} \right) + \\
& 1.0000000000000000 \exp \left( \pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (8 - z_0)^k z_0^{-k}}{k!} \right) + \\
& 1.0000000000000000 \exp \left( \pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (11 - z_0)^k z_0^{-k}}{k!} \right) + \\
& \left. 1.0000000000000000 \exp \left( \pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (12 - z_0)^k z_0^{-k}}{k!} \right) \right)
\end{aligned}$$

for not  $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

$\binom{n}{m}$  is the binomial coefficient

$n!$  is the factorial function

$(\alpha)_n$  is the Pochhammer symbol (rising factorial)

$\mathbb{R}$  is the set of real numbers

These three results, 196884, 21493760 and 744 are values that are placed in the following expression:

$$\begin{aligned}
Z_{24}(\tau) &= j(\tau) - 744 \\
&= q^{-1} + 196884q + 21493760q^2 + 864299970q^3 + 20245856256q^4 + \dots
\end{aligned}$$

concerning the partition function that defines a very special theory among the 71 holomorphic CFTs believed to exist at  $c = 24$  (see paper “*Three-dimensional AdS gravity and extremal CFTs at  $c = 8m$* ”)

from:

$$W_1(\Delta) = (-1)^{\Delta+1} 2\pi \left(\frac{\pi}{\Delta}\right)^{7/2} I_{7/2}(\pi\sqrt{\Delta})$$

For the values of  $W_1(\Delta)$ , we obtain  $I_{7/2}$  for each expression.

$$7.972 / ((((-2\pi * (\pi/3)^{3.5} * (\pi * \sqrt{3}))))))$$

**Input:**

$$\frac{7.972}{2\pi \left(\frac{\pi}{3}\right)^{3.5} (\pi\sqrt{3})}$$

**Result:**

-0.198416...

-0.198416...

**Series representations:**

$$\frac{7.972}{2\pi\sqrt{3}\pi\left(\frac{\pi}{3}\right)^{3.5}} = -\frac{186.407}{\pi^{5.5}\sqrt{2}\sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k}}$$

•

$$\frac{7.972}{2\pi\sqrt{3}\pi\left(\frac{\pi}{3}\right)^{3.5}} = -\frac{186.407}{\pi^{5.5}\sqrt{2}\sum_{k=0}^{\infty} \frac{(-\frac{1}{2})^k (-\frac{1}{2})_k}{k!}}$$

•

$$\frac{7.972}{2\pi\sqrt{3}\pi\left(\frac{\pi}{3}\right)^{3.5}} = -\frac{372.814\sqrt{\pi}}{\pi^{5.5}\sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 2^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}$$

$\binom{n}{m}$  is the binomial coefficient

$n!$  is the factorial function

$(\alpha)_n$  is the Pochhammer symbol (rising factorial)

$\Gamma(x)$  is the gamma function

$\text{Res}_{s=z_0} f$  is a complex residue



$$12.201 / ((((-2\pi * (\pi/4)^{3.5} * (\pi * \sqrt{4}))))))$$

**Input interpretation:**

$$\frac{12.201}{2 \pi \left(\frac{\pi}{4}\right)^{3.5} (\pi \sqrt{4})}$$

**Result:**

-0.719815...

-0.719815...

**Series representations:**

$$\frac{12.201}{2 \pi \sqrt{4} \pi \left(\frac{\pi}{4}\right)^{3.5}} = -\frac{780.864}{\pi^{5.5} \sqrt{3} \sum_{k=0}^{\infty} 3^{-k} \binom{\frac{1}{2}}{k}}$$

•

$$\frac{12.201}{2 \pi \sqrt{4} \pi \left(\frac{\pi}{4}\right)^{3.5}} = -\frac{780.864}{\pi^{5.5} \sqrt{3} \sum_{k=0}^{\infty} \frac{(-\frac{1}{3})^k (-\frac{1}{2})_k}{k!}}$$

•

$$\frac{12.201}{2 \pi \sqrt{4} \pi \left(\frac{\pi}{4}\right)^{3.5}} = -\frac{1561.73 \sqrt{\pi}}{\pi^{5.5} \sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 3^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}$$

$\binom{n}{m}$  is the binomial coefficient

$n!$  is the factorial function

$(a)_n$  is the Pochhammer symbol (rising factorial)

$\Gamma(x)$  is the gamma function

$\text{Res}_{z=z_0} f$  is a complex residue

$$38.976 / ((((-2\pi*(\pi/7)^{3.5}*((\pi*\sqrt{7}))))))$$

**Input interpretation:**

$$\frac{38.976}{2\pi\left(\frac{\pi}{7}\right)^{3.5}(\pi\sqrt{7})}$$

**Result:**

-12.3236...

-12.3236...

**Series representations:**

$$\frac{38.976}{2\pi\sqrt{7}\pi\left(\frac{\pi}{7}\right)^{3.5}} = -\frac{17685.2}{\pi^{5.5}\sqrt{6}\sum_{k=0}^{\infty}6^{-k}\binom{\frac{1}{2}}{k}}$$

•

$$\frac{38.976}{2\pi\sqrt{7}\pi\left(\frac{\pi}{7}\right)^{3.5}} = -\frac{17685.2}{\pi^{5.5}\sqrt{6}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{6}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}$$

•

$$\frac{38.976}{2\pi\sqrt{7}\pi\left(\frac{\pi}{7}\right)^{3.5}} = -\frac{35370.4\sqrt{\pi}}{\pi^{5.5}\sum_{j=0}^{\infty}\text{Res}_{s=-\frac{1}{2}+j}6^{-s}\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}$$

$\binom{n}{m}$  is the binomial coefficient

$n!$  is the factorial function

$(\alpha)_n$  is the Pochhammer symbol (rising factorial)

$\Gamma(x)$  is the gamma function

$\text{Res}_{z=z_0} f$  is a complex residue

$$55.721 / ((((-2\pi * (\pi/8)^{3.5} * (\pi * \sqrt{8}))))))$$

**Input interpretation:**

$$\frac{55.721}{2 \pi \left(\frac{\pi}{8}\right)^{3.5} (\pi \sqrt{8})}$$

**Result:**

-26.2987...

-26.2987...

**Series representations:**

$$\frac{55.721}{2 \pi \sqrt{8} \pi \left(\frac{\pi}{8}\right)^{3.5}} = - \frac{40\,346.3}{\pi^{5.5} \sqrt{7} \sum_{k=0}^{\infty} 7^{-k} \binom{\frac{1}{2}}{k}}$$

•

$$\frac{55.721}{2 \pi \sqrt{8} \pi \left(\frac{\pi}{8}\right)^{3.5}} = - \frac{40\,346.3}{\pi^{5.5} \sqrt{7} \sum_{k=0}^{\infty} \frac{(-\frac{1}{7})^k (-\frac{1}{2})_k}{k!}}$$

•

$$\frac{55.721}{2 \pi \sqrt{8} \pi \left(\frac{\pi}{8}\right)^{3.5}} = - \frac{80\,692.6 \sqrt{\pi}}{\pi^{5.5} \sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 7^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}$$

$\binom{n}{m}$  is the binomial coefficient

$n!$  is the factorial function

$(a)_n$  is the Pochhammer symbol (rising factorial)

$\Gamma(x)$  is the gamma function

$\text{Res}_{z=z_0} f$  is a complex residue

$$152.041 / ((((-2\pi*(\pi/11)^{3.5}*(\pi*\sqrt{11}))))))$$

**Input interpretation:**

$$\frac{152.041}{2\pi\left(\frac{\pi}{11}\right)^{3.5}(\pi\sqrt{11})}$$

**Result:**

-186.545...

-186.545...

**Series representations:**

$$\frac{152.041}{2\pi\sqrt{11}\pi\left(\frac{\pi}{11}\right)^{3.5}} = -\frac{335587.}{\pi^{5.5}\sqrt{10}\sum_{k=0}^{\infty}10^{-k}\binom{\frac{1}{2}}{k}}$$

•

$$\frac{152.041}{2\pi\sqrt{11}\pi\left(\frac{\pi}{11}\right)^{3.5}} = -\frac{335587.}{\pi^{5.5}\sqrt{10}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{10}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}$$

•

$$\frac{152.041}{2\pi\sqrt{11}\pi\left(\frac{\pi}{11}\right)^{3.5}} = -\frac{671174.\sqrt{\pi}}{\pi^{5.5}\sum_{j=0}^{\infty}\text{Res}_{s=-\frac{1}{2}+j}10^{-s}\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}$$

$\binom{n}{m}$  is the binomial coefficient

$n!$  is the factorial function

$(a)_n$  is the Pochhammer symbol (rising factorial)

$\Gamma(x)$  is the gamma function

$\text{Res}_{z=z_0} f$  is a complex residue

$$208.455 / ((((-2\pi * (\pi/12)^{3.5} * (\pi * \sqrt{12}))))))$$

**Input interpretation:**

$$\frac{208.455}{2 \pi \left(\frac{\pi}{12}\right)^{3.5} (\pi \sqrt{12})}$$

**Result:**

-332.049...

-332.049...

**Series representations:**

$$\frac{208.455}{2 \pi \sqrt{12} \pi \left(\frac{\pi}{12}\right)^{3.5}} = - \frac{623902.}{\pi^{5.5} \sqrt{11} \sum_{k=0}^{\infty} 11^{-k} \binom{\frac{1}{2}}{k}}$$

$$\frac{208.455}{2 \pi \sqrt{12} \pi \left(\frac{\pi}{12}\right)^{3.5}} = - \frac{623902.}{\pi^{5.5} \sqrt{11} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{11}\right)^k \left(-\frac{1}{2}\right)_k}{k!}}$$

$$\frac{208.455}{2 \pi \sqrt{12} \pi \left(\frac{\pi}{12}\right)^{3.5}} = - \frac{1.2478 \times 10^6 \sqrt{\pi}}{\pi^{5.5} \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 11^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}$$

$\binom{n}{m}$  is the binomial coefficient

$n!$  is the factorial function

$(a)_n$  is the Pochhammer symbol (rising factorial)

$\Gamma(x)$  is the gamma function

$\operatorname{Res}_{z=z_0} f$  is a complex residue

The sum of  $I_{7/2}$  is:

(-0.198416 -0.719815 -12.3236 -26.2987 -186.545 -332.049)

**Input interpretation:**

-0.198416 - 0.719815 - 12.3236 - 26.2987 - 186.545 - 332.049

**Result:**

-558.134531

-558.134531

Note that:

$$W_1(\Delta) = (-1)^{\Delta+1} 2\pi \left(\frac{\pi}{\Delta}\right)^{7/2} I_{\frac{7}{2}}(\pi\sqrt{\Delta})$$

and:

$$I_\rho(z) = \frac{1}{2\pi i} \int_{\epsilon-i\infty}^{\epsilon+i\infty} \frac{d\sigma}{\sigma^{\rho+1}} \exp\left[\sigma + \frac{z^2}{4\sigma}\right]$$

is the Bessel function of first kind of index  $\rho$ .

From the sum of the values of  $W_1(\Delta)$ , we obtain:

7.972 + 12.201 + 38.986 + 55.721 + 152.041 + 208.455

**Input interpretation:**

7.972 + 12.201 + 38.986 + 55.721 + 152.041 + 208.455

**Result:**

475.376

475.376

From the ratio of the two results,  $I_{7/2}$  and  $W_1(\Delta)$ , we obtain:

((((( (-0.198416 -0.719815 -12.3236 -26.2987 -186.545 -332.049) / (7.972 + 12.201 + 38.986 + 55.721 + 152.041 + 208.455) ))))))))<sup>3</sup>

**Input interpretation:**

$$\left( \frac{-0.198416 - 0.719815 - 12.3236 - 26.2987 - 186.545 - 332.049}{7.972 + 12.201 + 38.986 + 55.721 + 152.041 + 208.455} \right)^3$$

**Result:**

-1.61847099833235229449448018655678632119255128641935602592...  
-1.6184709...

This result is a very good approximation to the value of the golden ratio 1,618033988749... with minus sign

And:

$$-\left(-\frac{55}{10^3} + \frac{2}{10^3}\right) - \left( \frac{-0.198416 - 0.719815 - 12.3236 - 26.2987 - 186.545 - 332.049}{7.972 + 12.201 + 38.986 + 55.721 + 152.041 + 208.455} \right)^3$$

**Input interpretation:**

$$-\left(-\frac{55}{10^3} + \frac{2}{10^3}\right) - \left( \frac{-0.198416 - 0.719815 - 12.3236 - 26.2987 - 186.545 - 332.049}{7.972 + 12.201 + 38.986 + 55.721 + 152.041 + 208.455} \right)^3$$

**Result:**

1.671470998332352294494480186556786321192551286419356025921...

1.6714709... result very near to the value of holographic proton mass 1.6714213 \* 10<sup>-24</sup> gm. Indeed, multiplied the expression by 10<sup>-24</sup>, we obtain:

**Input interpretation:**

$$\left( -\left(-\frac{55}{10^3} + \frac{2}{10^3}\right) - \left( \frac{-0.198416 - 0.719815 - 12.3236 - 26.2987 - 186.545 - 332.049}{7.972 + 12.201 + 38.986 + 55.721 + 152.041 + 208.455} \right)^3 \right) \times \frac{1}{10^{24}}$$

**Result:**

1.6714709983323522944944801865567863211925512864193560... × 10<sup>-24</sup>

1.6714709... \* 10<sup>-24</sup> (Haramain formula)

And:

$$\left( \frac{55}{10^3} - \frac{55}{10^3} - \frac{2}{10^3} - \frac{13}{10^3} \right) - \left( \frac{-0.198416 - 0.719815 - 12.3236 - 26.2987 - 186.545 - 332.049}{7.972 + 12.201 + 38.986 + 55.721 + 152.041 + 208.455} \right)^3$$

**Input interpretation:**

$$\left( \frac{55}{10^3} - \frac{55}{10^3} - \frac{2}{10^3} - \frac{13}{10^3} \right) - \left( \frac{-0.198416 - 0.719815 - 12.3236 - 26.2987 - 186.545 - 332.049}{7.972 + 12.201 + 38.986 + 55.721 + 152.041 + 208.455} \right)^3$$

**Result:**

1.603470998332352294494480186556786321192551286419356025921...

1.6034709...result practically equal to the following Hamein's formula

$$m_p = 2 \frac{\eta_p}{R} = 1.603498 \times 10^{-24} gm$$

From the sum of the two results,  $I_{7/2}$  and  $W_1(\Delta)$ , we obtain:

$$- 558,134531 + 475,376 = -82,758531$$

$$1 + \left( \left( \left( -1 / \left( \left( -0.198416 - 0.719815 - 12.3236 - 26.2987 - 186.545 - 332.049 \right) + \left( 7.972 + 12.201 + 38.986 + 55.721 + 152.041 + 208.455 \right) \right) \right)^{1/9} \right) \right)$$

**Input interpretation:**

$$1 + \left( -1 / \left( \left( -0.198416 - 0.719815 - 12.3236 - 26.2987 - 186.545 - 332.049 \right) + \left( 7.972 + 12.201 + 38.986 + 55.721 + 152.041 + 208.455 \right) \right) \right)^{1/9}$$

**Result:**

1.61222...

1.61222... result practically equal to the value 1.612 (see Fig. below)

From:

**COLLECTIVE COHERENT OSCILLATION PLASMA MODES IN SURROUNDING MEDIA OF BLACK HOLES AND VACUUM STRUCTURE - QUANTUM PROCESSES WITH CONSIDERATIONS OF SPACETIME TORQUE AND CORIOLIS FORCES**

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Whether classical or quantum plasma treatment is considered, the collective properties, as well as the single-particle properties must be considered. The collective properties of the plasma become important when it interacts with an external or self-generated radiation field. This occurs in the case where the electron plasma frequency,  $\omega_p$ , is of the same order of magnitude, or exceeds, the operating radiation frequency  $\omega$ , i.e.  $\omega_p \geq \omega$ . The value of  $\omega_p$  is of the order of  $10^5$  Hz or greater. A plasmon is defined as a collective mode of oscillation of a plasma gas and a

Again, we proceed from the usual definitions of the plasma frequency:

$$9. \quad \omega_p = \left(4\pi\rho e^2 / m_e\right)^{1/2}$$

where  $\rho$  is the electron density and  $m_e$  is the electron mass. The Debye screening length is given as  $\lambda_D = \left(4\pi\beta e^2 / \rho\right)^{-1/2}$ , where  $\beta$  is the Boltzmann temperature defined as  $1/KT$ ,  $K$  is the Boltzmann constant

We considered a simple example of an oscillatory imposed field  $E = E_0 e^{-i(kx - \omega t)}$ . If the frequency of oscillation of the field is high, then we must include the quantum mechanical properties of the medium, and when the photon energies are of the same order of magnitude as the electron rest energies, then the quantum properties of the radiation field must be included (see section VI). For the case of a high density plasma under low and high temperature conditions, we define a dimensionless quantity,  $r_s$ , which we will take to be small or of the order of the Debye screening length, divided by the Bohr radius. We define  $r_s \equiv r_0 / a$  where  $r_0$  is the interaction spacing of the order of  $\lambda_D$  and  $a$  is the Bohr radius. The volume per electron is  $1/3 \pi r_0^3$ . Terms in  $1/r_s^2$  are proportional to the electron density and  $r_s^2$  is proportional to  $e^2$ , the electromagnetic coupling constant. If

$$11. \quad r_s = \frac{e^2 m_e}{h^2 \rho^{1/2}}$$

then the Fermi energy is given as

$$12. \quad \varepsilon_F = \frac{3}{5} (9\pi/4)^{2/3} 1/r_s^2$$

and the maximum electron momentum is given as

$$13. \quad k = (9\pi/4)^{1/3} \hbar / r_0$$

The Fermi energy levels are defined in terms of the vacuum state. The collective correlation energy is proportional to  $\varepsilon_F$ . The ground state  $|\phi_0\rangle$  is the state of no electrons or holes and has the eigenvalue  $\varepsilon_F = \sum_{k_i > 1} \omega(k_i)$  for the momentum,  $k_i$ , of the  $i^{\text{th}}$  particle.

From (9):

$$\omega_p = \left(4\pi\rho e^2 / m_e\right)^{1/2}$$

We obtain  $\rho$ :

$$\left(\left(9.1093837e-31*(10^5)^2\right)\right) / \left(\left(4\pi * (1.602176e-19)^2\right)\right)$$

**Input interpretation:**

$$\frac{9.1093837 \times 10^{-31} (10^5)^2}{4\pi (1.602176 \times 10^{-19})^2}$$

**Result:**

$$2.823961... \times 10^{16}$$

$$2.823961... * 10^{16} = \rho \text{ (electron density)}$$

From (11), we obtain:

$$r_s = \frac{e^2 m_e}{\hbar^2 \rho^{1/2}}$$

$$\frac{(((1.602176e-19)^2 * 9.1093837e-31)))}{(((1.054571817e-34)^2 * (2.823961e+16)^{1/2}))}$$

**Input interpretation:**

$$\frac{(1.602176 \times 10^{-19})^2 \times 9.1093837 \times 10^{-31}}{(1.054571817 \times 10^{-34})^2 \sqrt{2.823961 \times 10^{16}}}$$

**Result:**

$$1.25120... \times 10^{-8}$$

$$1.25120... * 10^{-8}$$

From  $r_s = r_0 / a$ , we obtain  $r_0$ :

$$1.25120e-8 * 5.29177e-11 = 6.621062624 \times 10^{-19}$$

From (12), we obtain:

$$\varepsilon_\rho = \frac{3}{5} (9\pi/4)^{2/3} 1/r_s^2$$

$$3/5 * (((9\pi/4))^{(2/3)}) * 1/(1.25120e-8)^2$$

**Input interpretation:**

$$\frac{3}{5} \left( \frac{9\pi}{4} \right)^{2/3} \times \frac{1}{(1.25120 \times 10^{-8})^2}$$

**Result:**

$$1.41163... \times 10^{16}$$

1.41163... \* 10<sup>16</sup> that is the Fermi energy

From (13), we obtain:

$$k = (9\pi/4)^{1/3} \hbar / r_0$$

$$((9\pi)/4)^{1/3} * (1.054571817e-34/6.621062624e-19)$$

**Input interpretation:**

$$\sqrt[3]{\frac{9\pi}{4}} \times \frac{1.054571817 \times 10^{-34}}{6.621062624 \times 10^{-19}}$$

**Result:**

$$3.056745364... \times 10^{-16}$$

$$3.056745364... * 10^{-16}$$

From the ratio between  $k$  and  $\varepsilon_\rho$ , we obtain:

$$(55/10^2 + 21/10^2) * (3.056745364e-16/1.41163e+16) * 10^{32}$$

**Input interpretation:**

$$\left( \frac{55}{10^2} + \frac{21}{10^2} \right) \times \frac{3.056745364 \times 10^{-16}}{1.41163 \times 10^{16}} \times 10^{32}$$

**Result:**

$$1.645704948633848813074247501115731459376748864787515142069...$$

$$1.64570498... \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 ...$$

Or, from the ratio between  $\varepsilon_\rho$  and  $k$ , we obtain:

$$((((1.41163e+16/3.056745364e-16)*1/10^31))))-3$$

**Input interpretation:**

$$\frac{1.41163 \times 10^{16}}{3.056745364 \times 10^{-16}} \times \frac{1}{10^{31}} - 3$$

**Result:**

1.618081756580362642205351848862737000948306664382005749563...

1.61808175...

This result is a very good approximation to the value of the golden ratio  
1,618033988749...

Furthermore, we have also:

$$(1.41163e+16 / 3.056745364e-16)$$

**Input interpretation:**

$$\frac{1.41163 \times 10^{16}}{3.056745364 \times 10^{-16}}$$

**Result:**

4.6180817565803626422053518488627370009483066643820057...  $\times 10^{31}$

4.61808175658...  $\times 10^{31}$

And

$$(2.283961e+16 / 3.056745364e-16)$$

**Input interpretation:**

$$\frac{2.283961 \times 10^{16}}{3.056745364 \times 10^{-16}}$$

**Result:**

7.4718719684627286474883486565745880035298877450100877...  $\times 10^{31}$

7.4718719684...  $\times 10^{31}$

From the ratio between  $7.4718719684... \times 10^{31}$  and  $4.61808175658... \times 10^{31}$ , we obtain:

$$(2.283961e+16 / 3.056745364e-16) * 1 / 4.61808175658e+31$$

**Input interpretation:**

$$\frac{2.283961 \times 10^{16}}{3.056745364 \times 10^{-16}} \times \frac{1}{4.61808175658 \times 10^{31}}$$

**Result:**

1.617960088691923061755131318371707891191846012896532424949...

1.61796008869...

This result is a very good approximation to the value of the golden ratio  
1,618033988749...

Now:

$$(55/10^2 + 21/10^2 + 13/10^3 - 8/10^4 - 1/10^4 - 21/10^5) * (3.056745364e-16 / 1.41163e+16) * 10^{32}$$

**Input interpretation:**

$$\left( \frac{55}{10^2} + \frac{21}{10^2} + \frac{13}{10^3} - \frac{8}{10^4} - \frac{1}{10^4} - \frac{21}{10^5} \right) \times \frac{3.056745364 \times 10^{-16}}{1.41163 \times 10^{16}} \times 10^{32}$$

**Result:**

1.671451569474975737268264346889765731813577212158993503963...

1.67145156... result very near to the following value of Hamein's proton mass:

$$m_p = 2 \frac{\eta_p}{R} = 1.6714213 \times 10^{-24} \text{ gm} .$$

Or, from the previous formula:

$$(55/10^3 - 2/10^3) + (((1.41163e+16 / 3.056745364e-16) * 1/10^{31})) - 3$$

**Input interpretation:**

$$\left( \frac{55}{10^3} - \frac{2}{10^3} \right) + \frac{1.41163 \times 10^{16}}{3.056745364 \times 10^{-16}} \times \frac{1}{10^{31}} - 3$$

**Result:**

1.671081756580362642205351848862737000948306664382005749563...

1.671081756... result very near to the following value of Hamein's proton mass:

$$m_{p'} = 2 \frac{\eta_p}{R} = 1.6714213 \times 10^{-24} \text{ gm} .$$

From the ratio between  $k$  and  $r_0$ , we obtain:

$$(3.056745364e-16/6.621062624e-19)^{1/12}$$

**Input interpretation:**

$$\sqrt[12]{\frac{3.056745364 \times 10^{-16}}{6.621062624 \times 10^{-19}}}$$

**Result:**

1.6673532890...

1.667353289...

And:

$$(3.056745364e-16/6.621062624e-19)^{1/13}$$

**Input interpretation:**

$$\sqrt[13]{\frac{3.056745364 \times 10^{-16}}{6.621062624 \times 10^{-19}}}$$

**Result:**

1.6030555870...

1.603055587... result very near to the following value:

$$m_{p'} = 2 \frac{\eta_p}{R} = 1.603498 \times 10^{-24} \text{ gm}$$

where  $m_{p'}$  is the holographic derivation of the mass of the proton.

And:

$$(55/10^3+13/10^3)+(3.056745364e-16/6.621062624e-19)^{1/13}$$

**Input interpretation:**

$$\left(\frac{55}{10^3} + \frac{13}{10^3}\right) + \sqrt[13]{\frac{3.056745364 \times 10^{-16}}{6.621062624 \times 10^{-19}}}$$

**Result:**

1.6710555870...

1.671055587... result very near to the following value of Hamein's proton mass:

$$m_{p'} = 2 \frac{\eta_p}{R} = 1.6714213 \times 10^{-24} \text{ gm} .$$

And, in conclusion, we obtain also an excellent approximation to the Golden Ratio

$$(2/10^3 + 13/10^3) + (3.056745364e-16/6.621062624e-19)^{1/13}$$

**Input interpretation:**

$$\left(\frac{2}{10^3} + \frac{13}{10^3}\right) + \sqrt[13]{\frac{3.056745364 \times 10^{-16}}{6.621062624 \times 10^{-19}}}$$

**Result:**

1.6180555870...

1.618055587...

From the Fermi energy, we can to obtain the mass:

$$\left(\left(\frac{3}{5} * \left(\left(\frac{9\pi}{4}\right)^{2/3}\right)\right) * \frac{1}{(1.25120e-8)^2}\right) / \left(\left(\left(2.99 * 10^8 \text{ meter per seconds}\right)^2\right)\right)$$

**Input interpretation:**

$$\frac{\left(\frac{3}{5} \left(\frac{9\pi}{4}\right)^{2/3}\right) \times \frac{1}{(1.25120 \times 10^{-8})^2}}{\left(2.99 \times 10^8 \text{ m/s (meters per second)}\right)^2}$$

**Result:**

0.1579 s<sup>2</sup>/m<sup>2</sup> (seconds squared per meter squared)

**Unit conversion:**

157.9 g/J (grams per joule)





**Result:**

1.618249289974513151328594506537698102195063501150597211145...  
1.61824928...

And:

$$1/\sqrt{\left[\left[\left[\left[1/\left(\left(\left(\left(4 \times 1.962364415 \times 10^{19}\right)/\left(5 \times 0.0864055^2\right)\right)\right)\right] \times \left[1/\left(1.757427 \times 10^{-18}\right)\right] \right] \times \sqrt{\left[\left[-\left(\left(6.982954 \times 10^{40} \times 4 \times \pi \times \left(2.609518 \times 10^{-45}\right)^3 - \left(2.609518 \times 10^{-45}\right)^2\right)\right)\right] / \left(6.67 \times 10^{-11}\right)\right]} \right] \right] \right] \right]}$$

**Input interpretation:**

$$1/\left(\sqrt{\left[1/\left(\frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{1.757427 \times 10^{-18}}\right) \times \sqrt{\frac{6.982954 \times 10^{40} \times 4 \times \pi \times \left(2.609518 \times 10^{-45}\right)^3 - \left(2.609518 \times 10^{-45}\right)^2}{6.67 \times 10^{-11}}}\right] \right)}$$

**Result:**

0.617951761941294992344984237907592006745219711583521722762...  
0.61795176...

The corresponding entropy is:

3.557518e-20 = 0.0000000000000000003557518 (supersymmetric condition → 0)

In addition to the structures of individual systems, there have been very interesting surveys conducted on the distribution of superclusters which displays a remarkable periodicity. Battaner [87] and Battaner and Florido [88] have considered the large scale structures of the order of 100Mpc which is the deepest survey with a resolution to 10Mpc. They observe a network array of galactic clusters. Systematic statistical analysis indicates a strong probability that these clusters form an octahedral array in which the octahedrons are in contact at the vertexes and thus creating cuboctahedrons. It is believed that the magnetic fields of the radiation dominated Universe comprises a network of filaments produced by early large scale magnetic fields and may have produced the octahedral array. The observation of these arrays appears to be more fundamental than pattern recognition, as they are such dominant features observed in the deep survey [92]. Then we have,

144. 
$$N = \frac{2\pi^2}{\frac{4\pi}{3}\chi^3} = \frac{3\pi}{(2)^{5/2}} \left(\frac{a^*}{T}\right)^{3/2}$$

radius about 46 billion light years. 1ly = 9 460 730 472 581 km  
13,798 ± 0,037 billion years = 13798000000

The Schwarzschild cell method predicts a cycloidal relationship between radius of the Universe and proper co-time,  $T$ , which was formulated by Friedmann [93]. Then  $r = a \sin \chi$ , see equation (143). A path on a 3-sphere is given

as  $\eta = \left(\frac{2T}{a^*}\right)^{1/2}$ . The relationship between the Lindquist and Wheeler Schwarzschild sphere and the vertices of the

Battaner and Florido regular geometric structure of superclusters can be compared. For  $N$  vertices, each vertex can be equidistant from its nearest neighbor only when  $N = 5, 8, 16, 24, 120$ , or  $600$  [94]. The case where  $N = 8$  yields the simplest arrangement. In this lattice,  $N = 5, 16$ , and  $600$  correspond to a tetrahedron,  $N = 8$  to a cube,  $N = 24$  to an octahedron, and  $N = 120$  to a dodecahedron. Correspondence is made in terms of the ratio of the distance from a face to a corner of a cell of some volume of a regular polyhedron to a sphere.

$$\frac{\left(\left(\left(3 \cdot \pi \cdot (9460730472581000)^{1.5} / \left(\left(24 \cdot (2)^{2.5}\right)\right)\right)\right)\right)}{\left(\left(\left(3 \cdot \pi \cdot (9460730472581000)^{1.5} / \left(\left(120 \cdot (2)^{2.5}\right)\right)\right)\right)\right)}$$

**Input:**

$$\frac{3 \pi \times \frac{9460730472581000^{1.5}}{24 \times 2^{2.5}}}{3 \pi \times \frac{9460730472581000^{1.5}}{120 \times 2^{2.5}}}$$

**Result:**

5  
5

**Alternative representations:**

$$\frac{3 (\pi 9460730472581000^{1.5})}{(3 \pi 9460730472581000^{1.5})(24 \times 2^{2.5})} = \frac{540^\circ 9460730472581000^{1.5}}{(24 \times 2^{2.5})(540^\circ 9460730472581000^{1.5})}$$

- $$\frac{3 (\pi 9460730472581000^{1.5})}{(3 \pi 9460730472581000^{1.5})(24 \times 2^{2.5})} = - \frac{3 i \log(-1) 9460730472581000^{1.5}}{(24 \times 2^{2.5})(-3 i \log(-1) 9460730472581000^{1.5})}$$

- $$\frac{3 (\pi 9460730472581000^{1.5})}{(3 \pi 9460730472581000^{1.5})(24 \times 2^{2.5})} = \frac{3 \cos^{-1}(-1) 9460730472581000^{1.5}}{(24 \times 2^{2.5})(3 \cos^{-1}(-1) 9460730472581000^{1.5})}$$

$\log(x)$  is the natural logarithm

$i$  is the imaginary unit

$\cos^{-1}(x)$  is the inverse cosine function

$$\frac{\left(\left(\left(\left(3 \cdot \pi \cdot (9460730472581000)^{1.5} / \left((15 \cdot (2)^{2.5}\right)\right)\right)\right)\right)}{\left(\left(\left(\left(3 \cdot \pi \cdot (9460730472581000)^{1.5} / \left((120 \cdot (2)^{2.5}\right)\right)\right)\right)\right)}$$

**Input:**

$$3 \pi \times \frac{9460730472581000^{1.5}}{15 \times 2^{2.5}}$$

$$3 \pi \times \frac{9460730472581000^{1.5}}{120 \times 2^{2.5}}$$

**Result:**

8

8

**Alternative representations:**

$$\frac{3 (\pi 9460730472581000^{1.5})}{(3 \pi 9460730472581000^{1.5})(15 \times 2^{2.5})} = \frac{540^\circ 9460730472581000^{1.5}}{(15 \times 2^{2.5})(540^\circ 9460730472581000^{1.5})}$$

$$\frac{3 (\pi 9460730472581000^{1.5})}{120 \times 2^{2.5}} = \frac{540^\circ 9460730472581000^{1.5}}{120 \times 2^{2.5}}$$

- $$\frac{3 (\pi 9460730472581000^{1.5})}{(3 \pi 9460730472581000^{1.5})(15 \times 2^{2.5})} = - \frac{3 i \log(-1) 9460730472581000^{1.5}}{(15 \times 2^{2.5})(-3 i \log(-1) 9460730472581000^{1.5})}$$

$$\frac{3 (\pi 9460730472581000^{1.5})}{120 \times 2^{2.5}} = \frac{3 i \log(-1) 9460730472581000^{1.5}}{120 \times 2^{2.5}}$$

- $$\frac{3 (\pi 9460730472581000^{1.5})}{(3 \pi 9460730472581000^{1.5})(15 \times 2^{2.5})} = \frac{3 \cos^{-1}(-1) 9460730472581000^{1.5}}{(15 \times 2^{2.5})(3 \cos^{-1}(-1) 9460730472581000^{1.5})}$$

$$\frac{3 (\pi 9460730472581000^{1.5})}{120 \times 2^{2.5}} = \frac{3 \cos^{-1}(-1) 9460730472581000^{1.5}}{120 \times 2^{2.5}}$$

$\log(x)$  is the natural logarithm

$i$  is the imaginary unit

$\cos^{-1}(x)$  is the inverse cosine function

$$\frac{\left(\frac{3\pi \times (9460730472581000)^{1.5}}{5 \times 2^{2.5}}\right)}{\left(\frac{3\pi \times (9460730472581000)^{1.5}}{120 \times 2^{2.5}}\right)}$$

**Input:**

$$\frac{3\pi \times \frac{9460730472581000^{1.5}}{5 \times 2^{2.5}}}{3\pi \times \frac{9460730472581000^{1.5}}{120 \times 2^{2.5}}}$$

**Result:**

24

24

**Alternative representations:**

$$\frac{3(\pi 9460730472581000^{1.5})}{(3\pi 9460730472581000^{1.5})(5 \times 2^{2.5})} = \frac{540^\circ 9460730472581000^{1.5}}{(5 \times 2^{2.5})(540^\circ 9460730472581000^{1.5})}$$

•

$$\frac{3(\pi 9460730472581000^{1.5})}{(3\pi 9460730472581000^{1.5})(5 \times 2^{2.5})} = -\frac{3i \log(-1) 9460730472581000^{1.5}}{(5 \times 2^{2.5})(-3i \log(-1) 9460730472581000^{1.5})}$$

•

$$\frac{3(\pi 9460730472581000^{1.5})}{(3\pi 9460730472581000^{1.5})(5 \times 2^{2.5})} = \frac{3 \cos^{-1}(-1) 9460730472581000^{1.5}}{(5 \times 2^{2.5})(3 \cos^{-1}(-1) 9460730472581000^{1.5})}$$

$\log(x)$  is the natural logarithm

$i$  is the imaginary unit

$\cos^{-1}(x)$  is the inverse cosine function

$$\frac{\left(\frac{3\pi \times (9460730472581000)^{1.5}}{15 \times 2^{2.5}}\right)^2}{\left(\frac{3\pi \times (9460730472581000)^{1.5}}{120 \times 2^{2.5}}\right)^2}$$

**Input:**

$$\left(\frac{3\pi \times \frac{9460730472581000^{1.5}}{15 \times 2^{2.5}}}{3\pi \times \frac{9460730472581000^{1.5}}{120 \times 2^{2.5}}}\right)^2$$

**Result:**

64

64

$$\frac{\left(\frac{3\pi \times (9460730472581000)^{1.5}}{15 \times 2^{2.5}}\right)}{\left(\frac{3\pi \times (9460730472581000)^{1.5}}{120 \times 2^{2.5}}\right)}^3$$

**Input:**

$$\left(\frac{3\pi \times \frac{9460730472581000^{1.5}}{15 \times 2^{2.5}}}{3\pi \times \frac{9460730472581000^{1.5}}{120 \times 2^{2.5}}}\right)^3$$

**Result:**

512

512

$$\sqrt{729} \frac{\left(\frac{3\pi \times (9460730472581000)^{1.5}}{15 \times 2^{2.5}}\right)}{\left(\frac{3\pi \times (9460730472581000)^{1.5}}{120 \times 2^{2.5}}\right)}^2$$

**Input:**

$$\sqrt{729} \left(\frac{3\pi \times \frac{9460730472581000^{1.5}}{15 \times 2^{2.5}}}{3\pi \times \frac{9460730472581000^{1.5}}{120 \times 2^{2.5}}}\right)^2$$

**Result:**

1728

1728

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson. Furthermore, 1728 occurs in the algebraic formula for the  $j$ -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

While the number 729 within the square root is  $9^3$  value that is the Ramanujan’s cubes manuscript

**Series representations:**

$$\sqrt{729} \left(\frac{3\pi \frac{9460730472581000^{1.5}}{15 \times 2^{2.5}}}{3\pi \frac{9460730472581000^{1.5}}{120 \times 2^{2.5}}}\right)^2 = 64 \sqrt{728} \sum_{k=0}^{\infty} 728^{-k} \binom{\frac{1}{2}}{k}$$

$$\sqrt{729} \left( \frac{3 (\pi 9460730472581000^{1.5})}{(3 \pi 9460730472581000^{1.5})(15 \times 2^{2.5})} \right)^2 = 64 \sqrt{728} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{728}\right)^k \left(-\frac{1}{2}\right)_k}{k!}$$

$$\sqrt{729} \left( \frac{3 (\pi 9460730472581000^{1.5})}{(3 \pi 9460730472581000^{1.5})(15 \times 2^{2.5})} \right)^2 = \frac{32 \sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 728^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{\sqrt{\pi}}$$

$$\left( \left( \left( \left( \left( \left( \left( \sqrt{729} \left( \frac{3 \pi (9460730472581000)^{1.5}}{(15 \times 2^{2.5})} \right) \right) \right) \right) \right) \right) \right) \right) \left( \frac{3 \pi (9460730472581000)^{1.5}}{(120 \times 2^{2.5})} \right) \right)^2 \right)^{1/15}$$

**Input:**

$$\sqrt[15]{\sqrt{729} \left( \frac{3 \pi \times \frac{9460730472581000^{1.5}}{15 \times 2^{2.5}}}{3 \pi \times \frac{9460730472581000^{1.5}}{120 \times 2^{2.5}}} \right)^2}$$

**Result:**

1.643751829517225762308497936230979517383492589945475200411...

$$1.6437518... \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 ...$$

$$\frac{21}{10^3} + \frac{5}{10^3} + \frac{2}{10^3} + \left( \left( \left( \left( \left( \left( \left( \sqrt{728} \left( \frac{3 \pi (9460730472581000)^{1.5}}{(15 \times 2^{2.5})} \right) \right) \right) \right) \right) \right) \right) \left( \frac{3 \pi (9460730472581000)^{1.5}}{(120 \times 2^{2.5})} \right) \right)^2 \right)^{1/15}$$

**Input:**

$$\frac{21}{10^3} + \frac{5}{10^3} + \frac{2}{10^3} + \sqrt[15]{\sqrt{728} \left( \frac{3 \pi \times \frac{9460730472581000^{1.5}}{15 \times 2^{2.5}}}{3 \pi \times \frac{9460730472581000^{1.5}}{120 \times 2^{2.5}}} \right)^2}$$

**Result:**

1.671676619520322039938254399834921339019892158880692634873...

1.671676619... result very near to the following value of Hamein's proton mass:

$$m_{p'} = 2 \frac{\eta_p}{R} = 1.6714213 \times 10^{-24} \text{ gm} .$$

**Series representations:**

$$\frac{21}{10^3} + \frac{5}{10^3} + \frac{2}{10^3} + \sqrt[15]{\sqrt{728} \left( \frac{3 (\pi 9460730472581000^{1.5})}{(3 \pi 9460730472581000^{1.5})(15 \times 2^{2.5})} \right)^2} =$$

$$0.028 + 1.31951 \sqrt[15]{\sqrt{727} \sum_{k=0}^{\infty} 727^{-k} \binom{\frac{1}{2}}{k}}$$

•

$$\frac{21}{10^3} + \frac{5}{10^3} + \frac{2}{10^3} + \sqrt[15]{\sqrt{728} \left( \frac{3 (\pi 9460730472581000^{1.5})}{(3 \pi 9460730472581000^{1.5})(15 \times 2^{2.5})} \right)^2} =$$

$$0.028 + 1.31951 \sqrt[15]{\sqrt{727} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{727}\right)^k \binom{-\frac{1}{2}}{k}}{k!}}$$

•

$$\frac{21}{10^3} + \frac{5}{10^3} + \frac{2}{10^3} + \sqrt[15]{\sqrt{728} \left( \frac{3 (\pi 9460730472581000^{1.5})}{(3 \pi 9460730472581000^{1.5})(15 \times 2^{2.5})} \right)^2} =$$

$$0.028 + 1.25992 \sqrt[15]{\frac{\sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 727^{-s} \Gamma\left(-\frac{1}{2} - s\right) \Gamma(s)}{\sqrt{\pi}}}$$

$\binom{n}{m}$  is the binomial coefficient

$n!$  is the factorial function

$(\alpha)_n$  is the Pochhammer symbol (rising factorial)

$\Gamma(x)$  is the gamma function

$\text{Res}_{z=z_0} f$  is a complex residue

$$-(5^2/10^3)+((((((((sqrt(728)((((((((3*Pi*(9460730472581000)^1.5 / (((15*(2)^2.5)))))))))) / (((((3*Pi*(9460730472581000)^1.5 / (((120*(2)^2.5)))))))))))))^2))))))^(1/15)$$

**Input:**

$$-\frac{5^2}{10^3} + \sqrt[15]{\sqrt{728} \left( \frac{3\pi \times \frac{9460730472581000^{1.5}}{15 \times 2^{2.5}}}{3\pi \times \frac{9460730472581000^{1.5}}{120 \times 2^{2.5}}} \right)^2}$$

**Result:**

1.618676619520322039938254399834921339019892158880692634873...

1.618676619...

This result is a very good approximation to the value of the golden ratio 1,618033988749...

**Series representations:**

$$-\frac{5^2}{10^3} + \sqrt[15]{\sqrt{728} \left( \frac{3(\pi 9460730472581000^{1.5})}{(3\pi 9460730472581000^{1.5})(15 \times 2^{2.5})} \right)^2} =$$

$$-0.025 + 1.31951 \sqrt[15]{\sqrt{727} \sum_{k=0}^{\infty} 727^{-k} \binom{\frac{1}{2}}{k}}$$

•

$$-\frac{5^2}{10^3} + \sqrt[15]{\sqrt{728} \left( \frac{3(\pi 9460730472581000^{1.5})}{(3\pi 9460730472581000^{1.5})(15 \times 2^{2.5})} \right)^2} =$$

$$-0.025 + 1.31951 \sqrt[15]{\sqrt{727} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{727}\right)^k \left(-\frac{1}{2}\right)_k}{k!}}$$

•



$$-\frac{5^2}{10^3} + \sqrt[15]{\sqrt{728} \left( \frac{3(\pi 9460730472581000^{1.5})}{(3\pi 9460730472581000^{1.5})(15 \times 2^{2.5})} \right)^2} =$$

$$-0.025 + 1.25992 \sqrt[15]{\frac{\sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 727^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{\sqrt{\pi}}}$$

$$-(5^2/10^3+13/10^3+2/10^3)+(((((((\sqrt{728}(((((((3*\text{Pi}*(9460730472581000)^{1.5} / ((15* (2)^{2.5})))))))))))/((((3*\text{Pi}*(9460730472581000)^{1.5} / ((120*(2)^{2.5}))))))))))^{2}))))))^{1/15}$$

**Input:**

$$-\left(\frac{5^2}{10^3} + \frac{13}{10^3} + \frac{2}{10^3}\right) + \sqrt[15]{\sqrt{728} \left( \frac{3\pi \times \frac{9460730472581000^{1.5}}{15 \times 2^{2.5}}}{3\pi \times \frac{9460730472581000^{1.5}}{120 \times 2^{2.5}}} \right)^2}$$

**Result:**

1.603676619520322039938254399834921339019892158880692634873...

1.603676619... result very near to the following value:

$$m_{p'} = 2 \frac{\eta_p}{R} = 1.603498 \times 10^{-24} \text{ gm}$$

where  $m_{p'}$  is the holographic derivation of the mass of the proton.

**Series representations:**

$$-\left(\frac{5^2}{10^3} + \frac{13}{10^3} + \frac{2}{10^3}\right) + \sqrt[15]{\sqrt{728} \left( \frac{3(\pi 9460730472581000^{1.5})}{(3\pi 9460730472581000^{1.5})(15 \times 2^{2.5})} \right)^2} =$$

$$-0.04 + 1.31951 \sqrt[15]{\sqrt{727} \sum_{k=0}^{\infty} 727^{-k} \binom{\frac{1}{2}}{k}}$$



**Result:**

1.618249240495369875407663405949666785845309107623025839691...  
 1.61824924...

And:

$1/\sqrt{[1/(((4 \times 1.962364415 \times 10^{19}) / (5 \times 0.0864055^2)) \times 1 / (6.371499 \times 10^{42}) \times \sqrt{[1 - ((1.926082 \times 10^{-20} \times 4 \times \pi \times (9.460730 \times 10^{15})^3 - (9.460730 \times 10^{15})^2)]}) / ((6.67 \times 10^{-11})))]}]}$

**Input interpretation:**

$$1 / \left( \sqrt{ \left( 1 / \left( \frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{6.371499 \times 10^{42}} \right) \sqrt{ \frac{1.926082 \times 10^{-20} \times 4 \pi (9.460730 \times 10^{15})^3 - (9.460730 \times 10^{15})^2}{6.67 \times 10^{-11}} } \right) } \right)$$

**Result:**

0.617951780835617944315868311909963458929384044358822072009...  
 0.61795178...

The value of lifetime  $6.890975 \times 10^{104}$  is very near, perhaps more precise, to the value  $10^{100}$  years, the time within which the supermassive black holes evaporate according to the Hawking process (which, however, has claimed that not all information disappears, in order not to violate the laws of thermodynamics)

From the Entropy =  $4.676014 \times 10^{101}$ , we obtain:

$$\ln(4.676014 \times 10^{101})$$

**Input interpretation:**

$$\log(4.676014 \times 10^{101})$$

$\log(x)$  is the natural logarithm

**Result:**

234.103540...  
 234.103540...

**Alternative representations:**

$$\log(4.67601 \times 10^{101}) = \log_e(4.67601 \times 10^{101})$$

•

$$\log(4.67601 \times 10^{101}) = \log(a) \log_a(4.67601 \times 10^{101})$$

•

$$\log(4.67601 \times 10^{101}) = -\text{Li}_1(1 - 4.67601 \times 10^{101})$$

$\log_b(x)$  is the base- $b$  logarithm

$\text{Li}_n(x)$  is the polylogarithm function

### Series representations:

$$\log(4.67601 \times 10^{101}) = \log(4.67601 \times 10^{101}) - \sum_{k=1}^{\infty} \frac{(-1)^k e^{-234.104 k}}{k}$$

•

$$\log(4.67601 \times 10^{101}) = 2 i \pi \left\lfloor \frac{\arg(4.67601 \times 10^{101} - x)}{2 \pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (4.67601 \times 10^{101} - x)^k x^{-k}}{k} \quad \text{for } x < 0$$

•

$$\log(4.67601 \times 10^{101}) = \left\lfloor \frac{\arg(4.67601 \times 10^{101} - z_0)}{2 \pi} \right\rfloor \log\left(\frac{1}{z_0}\right) + \log(z_0) + \left\lfloor \frac{\arg(4.67601 \times 10^{101} - z_0)}{2 \pi} \right\rfloor \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (4.67601 \times 10^{101} - z_0)^k z_0^{-k}}{k}$$

$\arg(z)$  is the complex argument

$\lfloor x \rfloor$  is the floor function

$i$  is the imaginary unit

### Integral representations:

$$\log(4.67601 \times 10^{101}) = \int_1^{4.67601 \times 10^{101}} \frac{1}{t} dt$$

•

$$\log(4.67601 \times 10^{101}) = \frac{1}{2i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-234.104s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \quad \text{for } -1 < \gamma < 0$$

Note that (from OEIS):

A053279      A '7th order' mock theta function.

FORMULA

G.f.:  $g(q^2, q^7)$ , where

$$g(x, q) = \sum_{n \geq 1} q^{n(n-1)} / ((1-x)(1-q/x)(1-qx)(1-q^2/x) \dots (1-q^{n-1}x)(1-q^n/x)).$$

$$a(n) \sim \exp(\text{Pi} \cdot \text{sqrt}(2 \cdot n/21)) / (2^{3/2} \cdot \sin(2 \cdot \text{Pi}/7) \cdot \text{sqrt}(7 \cdot n)). -$$

$$a(n) \sim \exp(\text{Pi} \cdot \text{sqrt}(2 \cdot n/21)) / (2^{3/2} \cdot \sin(2 \cdot \text{Pi}/7) \cdot \text{sqrt}(7 \cdot n))$$

for  $n = 94$ , we have  $a(n) \approx 234$ . Developing the formula, we obtain two results:

$$\exp(\text{Pi} \cdot \text{sqrt}(2 \cdot 94/21)) / (2^{3/2} \cdot \sin(2 \cdot \text{Pi}/7) \cdot \text{sqrt}(7 \cdot 94))$$

**Input:**

$$\frac{\exp\left(\pi \sqrt{2 \times \frac{94}{21}}\right)}{2^{3/2} \sin\left(2 \times \frac{\pi}{7}\right) \sqrt{7 \times 94}}$$

**Exact result:**

$$\frac{e^{2\sqrt{47/21}\pi} \sec\left(\frac{3\pi}{14}\right)}{4\sqrt{329}}$$

$\sec(x)$  is the secant function

**Decimal approximation:**

213.0665244695760356781932566158558179785755714425970617078...

213.0665244...

**Property:**

$$\frac{e^{2\sqrt{47/21}\pi} \sec\left(\frac{3\pi}{14}\right)}{4\sqrt{329}} \text{ is a transcendental number}$$

**Alternate forms:**

$$\frac{e^{2\sqrt{47/21}\pi} \cos\left(\frac{3\pi}{14}\right)}{2\sqrt{329} \left(1 + \sin\left(\frac{\pi}{14}\right)\right)}$$

•

$$\frac{e^{2\sqrt{47/21}\pi}}{2\sqrt{329} (e^{-3i\pi/14} + e^{3i\pi/14})}$$

•

$$e^{2\sqrt{47/21}\pi}$$

root of  $15\,953\,857\,472\,x^6 - 24\,245\,984\,x^4 + 9212\,x^2 - 1$  near  $x = 0.017629$

**Alternative representations:**

$$\frac{\exp\left(\pi\sqrt{\frac{2 \times 94}{21}}\right)}{2^{3/2} \sin\left(\frac{2\pi}{7}\right) \sqrt{7 \times 94}} = \frac{\exp\left(\pi\sqrt{\frac{188}{21}}\right)}{\cos\left(\frac{\pi}{2} - \frac{2\pi}{7}\right) 2^{3/2} \sqrt{658}}$$

•

$$\frac{\exp\left(\pi\sqrt{\frac{2 \times 94}{21}}\right)}{2^{3/2} \sin\left(\frac{2\pi}{7}\right) \sqrt{7 \times 94}} = -\frac{\exp\left(\pi\sqrt{\frac{188}{21}}\right)}{\cos\left(\frac{\pi}{2} + \frac{2\pi}{7}\right) 2^{3/2} \sqrt{658}}$$

•

$$\frac{\exp\left(\pi\sqrt{\frac{2 \times 94}{21}}\right)}{2^{3/2} \sin\left(\frac{2\pi}{7}\right) \sqrt{7 \times 94}} = \frac{\exp\left(\pi\sqrt{\frac{188}{21}}\right)}{\frac{2^{3/2} (-e^{-2i\pi/7} + e^{2i\pi/7}) \sqrt{658}}{2i}}$$

*i* is the imaginary unit

**Series representations:**

$$\frac{\exp\left(\pi \sqrt{\frac{2 \times 94}{21}}\right)}{2^{3/2} \sin\left(\frac{2\pi}{7}\right) \sqrt{7 \times 94}} = -\frac{e^{2\sqrt{47/21} \pi} \sum_{k=1}^{\infty} (-1)^k q^{-1+2k}}{2\sqrt{329}} \text{ for } q = (-1)^{3/14}$$

•

$$\frac{\exp\left(\pi \sqrt{\frac{2 \times 94}{21}}\right)}{2^{3/2} \sin\left(\frac{2\pi}{7}\right) \sqrt{7 \times 94}} = \frac{7 \sqrt{\frac{7}{47}} e^{2\sqrt{47/21} \pi} \sum_{k=0}^{\infty} \frac{(-1)^k (1+2k)}{10+49k+49k^2}}{4\pi}$$

•

$$\frac{\exp\left(\pi \sqrt{\frac{2 \times 94}{21}}\right)}{2^{3/2} \sin\left(\frac{2\pi}{7}\right) \sqrt{7 \times 94}} = \frac{e^{2\sqrt{47/21} \pi} \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{3\pi}{14}\right)^{2k} E_{2k}}{(2k)!}}{4\sqrt{329}}$$

$E_n$  is the  $n^{\text{th}}$  Euler number

$n!$  is the factorial function

### Integral representation:

$$\frac{\exp\left(\pi \sqrt{\frac{2 \times 94}{21}}\right)}{2^{3/2} \sin\left(\frac{2\pi}{7}\right) \sqrt{7 \times 94}} = \frac{e^{2\sqrt{47/21} \pi}}{2\sqrt{329} \pi} \int_0^{\infty} \frac{t^{3/7}}{1+t^2} dt$$

### Multiple-argument formulas:

$$\frac{\exp\left(\pi \sqrt{\frac{2 \times 94}{21}}\right)}{2^{3/2} \sin\left(\frac{2\pi}{7}\right) \sqrt{7 \times 94}} = \frac{e^{2\sqrt{47/21} \pi} \sec^2\left(\frac{3\pi}{28}\right)}{4\sqrt{329} \left(2 - \sec^2\left(\frac{3\pi}{28}\right)\right)}$$

•

$$\frac{\exp\left(\pi \sqrt{\frac{2 \times 94}{21}}\right)}{2^{3/2} \sin\left(\frac{2\pi}{7}\right) \sqrt{7 \times 94}} = \frac{e^{2\sqrt{47/21} \pi} \sec^3\left(\frac{\pi}{14}\right)}{\sqrt{329} \left(16 - 12 \sec^2\left(\frac{\pi}{14}\right)\right)}$$

If instead of 2 we insert  $\sqrt{((1+e^2)/2)} = 2.048054698$ , we get:

$$\frac{\exp(\pi \sqrt{\sqrt{\frac{1+e^2}{2}} \times \frac{94}{21}})}{2^{3/2} \sin(\sqrt{\frac{1+e^2}{2}} \times \frac{\pi}{7}) \sqrt{7 \times 94}}$$

**Input:**

$$\frac{\exp\left(\pi \sqrt{\sqrt{\frac{1+e^2}{2}} \times \frac{94}{21}}\right)}{2^{3/2} \sin\left(\sqrt{\frac{1+e^2}{2}} \times \frac{\pi}{7}\right) \sqrt{7 \times 94}}$$

**Exact result:**

$$\frac{e^{\sqrt{47/21}} \sqrt[4]{2(1+e^2)} \pi \operatorname{csc}\left(\frac{1}{7} \sqrt{\frac{1+e^2}{2}} \pi\right)}{4 \sqrt{329}}$$

$\operatorname{csc}(x)$  is the cosecant function

**Decimal approximation:**

234.4019975474714084902811248172160943355890000022534586221...

234.4019975...

**Alternate forms:**

$$\frac{e^{\sqrt{47/21}} \sqrt[4]{2(1+e^2)} \pi \sin\left(\frac{1}{7} \sqrt{\frac{1+e^2}{2}} \pi\right)}{2 \sqrt{329} \left(\cos\left(\frac{1}{7} \sqrt{2(1+e^2)} \pi\right) - 1\right)}$$

$$\frac{i e^{\sqrt{47/21}} \sqrt[4]{2(1+e^2)} \pi}{2 \sqrt{329} \left( e^{-1/7 i \sqrt{1/2(1+e^2)} \pi} - e^{1/7 i \sqrt{1/2(1+e^2)} \pi} \right)}$$

**Alternative representations:**



$$\frac{\exp\left(\pi \sqrt{\frac{1}{21} \sqrt{\frac{1}{2}(1+e^2)} 94}\right)}{2^{3/2} \sin\left(\frac{1}{7} \sqrt{\frac{1}{2}(1+e^2)} \pi\right) \sqrt{7 \times 94}} = \frac{\exp\left(\pi \sqrt{\frac{94}{21} \sqrt{\frac{1}{2}(1+e^2)}}\right)}{\cos\left(\frac{\pi}{2} - \frac{1}{7} \pi \sqrt{\frac{1}{2}(1+e^2)}\right) 2^{3/2} \sqrt{658}}$$

•

$$\frac{\exp\left(\pi \sqrt{\frac{1}{21} \sqrt{\frac{1}{2}(1+e^2)} 94}\right)}{2^{3/2} \sin\left(\frac{1}{7} \sqrt{\frac{1}{2}(1+e^2)} \pi\right) \sqrt{7 \times 94}} = - \frac{\exp\left(\pi \sqrt{\frac{94}{21} \sqrt{\frac{1}{2}(1+e^2)}}\right)}{\cos\left(\frac{\pi}{2} + \frac{1}{7} \pi \sqrt{\frac{1}{2}(1+e^2)}\right) 2^{3/2} \sqrt{658}}$$

•

$$\frac{\exp\left(\pi \sqrt{\frac{1}{21} \sqrt{\frac{1}{2}(1+e^2)} 94}\right)}{2^{3/2} \sin\left(\frac{1}{7} \sqrt{\frac{1}{2}(1+e^2)} \pi\right) \sqrt{7 \times 94}} = \frac{\exp\left(\pi \sqrt{\frac{1}{21} \sqrt{\frac{1}{2}(1+z^2)} 94}\right)}{(2^{3/2} \sqrt{7 \times 94}) (-1) \cos\left(\frac{\pi}{2} + \frac{1}{7} \pi \sqrt{\frac{1}{2}(1+e^2)} \pi\right)} \text{ for } z = e$$

### Series representations:

$$\frac{\exp\left(\pi \sqrt{\frac{1}{21} \sqrt{\frac{1}{2}(1+e^2)} 94}\right)}{2^{3/2} \sin\left(\frac{1}{7} \sqrt{\frac{1}{2}(1+e^2)} \pi\right) \sqrt{7 \times 94}} = \frac{e^{\sqrt{47/21} 4 \sqrt{2(1+e^2)} \pi} \sqrt{\frac{7}{94} (1+e^2)} \sum_{k=-\infty}^{\infty} \frac{(-1)^k}{1+e^2 - 98 k^2}}{2 \pi}$$

•

$$\frac{\exp\left(\pi \sqrt{\frac{1}{21} \sqrt{\frac{1}{2}(1+e^2)} 94}\right)}{2^{3/2} \sin\left(\frac{1}{7} \sqrt{\frac{1}{2}(1+e^2)} \pi\right) \sqrt{7 \times 94}} = -\frac{i e^{\sqrt{47/21} 4 \sqrt{2(1+e^2)} \pi} \sum_{k=1}^{\infty} q^{-1+2k}}{2 \sqrt{329}}$$

for  $q = e^{1/7 i \sqrt{1/2(1+e^2)} \pi}$

$$\frac{\exp\left(\pi \sqrt{\frac{1}{21} \sqrt{\frac{1}{2}(1+e^2)} 94}\right)}{2^{3/2} \sin\left(\frac{1}{7} \sqrt{\frac{1}{2}(1+e^2)} \pi\right) \sqrt{7 \times 94}} = \frac{e^{\sqrt{47/21} 4 \sqrt{2(1+e^2)} \pi} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{\pi}{2} + \frac{1}{7} \sqrt{\frac{1}{2}(1+e^2)} \pi\right)^{2k} E_{2k}}{(2k)!}}{4 \sqrt{329}}$$

$E_n$  is the  $n^{\text{th}}$  Euler number

$n!$  is the factorial function

### Integral representation:

$$\frac{\exp\left(\pi \sqrt{\frac{1}{21} \sqrt{\frac{1}{2}(1+e^2)} 94}\right)}{2^{3/2} \sin\left(\frac{1}{7} \sqrt{\frac{1}{2}(1+e^2)} \pi\right) \sqrt{7 \times 94}} = \frac{e^{\sqrt{47/21} 4 \sqrt{2(1+e^2)} \pi}}{4 \sqrt{329} \pi} \int_0^{\infty} \frac{t^{1/7} \sqrt{1/2(1+e^2)}}{t+t^2} dt$$

### Multiple-argument formulas:

$$\frac{\exp\left(\pi \sqrt{\frac{1}{21} \sqrt{\frac{1}{2}(1+e^2) 94}}\right)}{2^{3/2} \sin\left(\frac{1}{7} \sqrt{\frac{1}{2}(1+e^2) \pi}\right) \sqrt{7 \times 94}} = \frac{e^{\sqrt{47/21} 4 \sqrt{2(1+e^2) \pi}} \csc\left(\frac{1}{14} \sqrt{\frac{1}{2}(1+e^2) \pi}\right) \sec\left(\frac{1}{14} \sqrt{\frac{1}{2}(1+e^2) \pi}\right)}{8 \sqrt{329}}$$

$$\frac{\exp\left(\pi \sqrt{\frac{1}{21} \sqrt{\frac{1}{2}(1+e^2) 94}}\right)}{2^{3/2} \sin\left(\frac{1}{7} \sqrt{\frac{1}{2}(1+e^2) \pi}\right) \sqrt{7 \times 94}} = \frac{e^{\sqrt{47/21} 4 \sqrt{2(1+e^2) \pi}} \csc\left(\sqrt{\frac{1}{2}(1+e^2) \pi}\right)}{4 \sqrt{329} U_{-6/7}\left(\cos\left(\sqrt{\frac{1}{2}(1+e^2) \pi}\right)\right)}$$

$$\frac{\exp\left(\pi \sqrt{\frac{1}{21} \sqrt{\frac{1}{2}(1+e^2) 94}}\right)}{2^{3/2} \sin\left(\frac{1}{7} \sqrt{\frac{1}{2}(1+e^2) \pi}\right) \sqrt{7 \times 94}} = \frac{e^{\sqrt{47/21} 4 \sqrt{2(1+e^2) \pi}} \csc^3\left(\frac{1}{21} \sqrt{\frac{1}{2}(1+e^2) \pi}\right)}{4 \sqrt{329} \left(-4 + 3 \csc^2\left(\frac{1}{21} \sqrt{\frac{1}{2}(1+e^2) \pi}\right)\right)}$$

$\sec(x)$  is the secant function

$U_n(x)$  is the Chebyshev polynomial of the second kind

Or:

$$21 + \frac{\exp(\pi \sqrt{2 \times 94 / 21})}{2^{3/2} \sin(2 \times \pi / 7) \sqrt{7 \times 94}}$$

**Input:**

$$21 + \frac{\exp\left(\pi \sqrt{2 \times \frac{94}{21}}\right)}{2^{3/2} \sin\left(2 \times \frac{\pi}{7}\right) \sqrt{7 \times 94}}$$

**Exact result:**

$$21 + \frac{e^{2\sqrt{47/21}\pi} \sec\left(\frac{3\pi}{14}\right)}{4\sqrt{329}}$$

$\sec(x)$  is the secant function

### Decimal approximation:

234.0665244695760356781932566158558179785755714425970617078...

234.066524...

### Property:

$$21 + \frac{e^{2\sqrt{47/21}\pi} \sec\left(\frac{3\pi}{14}\right)}{4\sqrt{329}} \text{ is a transcendental number}$$

### Alternate forms:

$$\frac{27636 + \sqrt{329} e^{2\sqrt{47/21}\pi} \sec\left(\frac{3\pi}{14}\right)}{1316}$$

•

$$21 + \frac{e^{2\sqrt{47/21}\pi} \cos\left(\frac{3\pi}{14}\right)}{2\sqrt{329} \left(1 + \sin\left(\frac{\pi}{14}\right)\right)}$$

•

$$\frac{\left(e^{2\sqrt{47/21}\pi} + 84\sqrt{329} \cos\left(\frac{3\pi}{14}\right)\right) \sec\left(\frac{3\pi}{14}\right)}{4\sqrt{329}}$$

### Alternative representations:

$$21 + \frac{\exp\left(\pi \sqrt{\frac{2 \times 94}{21}}\right)}{2^{3/2} \sin\left(\frac{2\pi}{7}\right) \sqrt{7 \times 94}} = 21 + \frac{\exp\left(\pi \sqrt{\frac{188}{21}}\right)}{\cos\left(\frac{\pi}{2} - \frac{2\pi}{7}\right) 2^{3/2} \sqrt{658}}$$

•

$$21 + \frac{\exp\left(\pi \sqrt{\frac{2 \times 94}{21}}\right)}{2^{3/2} \sin\left(\frac{2\pi}{7}\right) \sqrt{7 \times 94}} = 21 + - \frac{\exp\left(\pi \sqrt{\frac{188}{21}}\right)}{\cos\left(\frac{\pi}{2} + \frac{2\pi}{7}\right) 2^{3/2} \sqrt{658}}$$

•

$$21 + \frac{\exp\left(\pi \sqrt{\frac{2 \times 94}{21}}\right)}{2^{3/2} \sin\left(\frac{2\pi}{7}\right) \sqrt{7 \times 94}} = 21 + \frac{\exp\left(\pi \sqrt{\frac{188}{21}}\right)}{\frac{2^{3/2} (-e^{-(2i\pi)/7} + e^{(2i\pi)/7}) \sqrt{658}}{2i}}$$

$i$  is the imaginary unit

### Series representations:

$$21 + \frac{\exp\left(\pi \sqrt{\frac{2 \times 94}{21}}\right)}{2^{3/2} \sin\left(\frac{2\pi}{7}\right) \sqrt{7 \times 94}} = 21 - \frac{e^{2\sqrt{47/21}\pi} \sum_{k=1}^{\infty} (-1)^k q^{-1+2k}}{2\sqrt{329}} \quad \text{for } q = (-1)^{3/14}$$

$$21 + \frac{\exp\left(\pi \sqrt{\frac{2 \times 94}{21}}\right)}{2^{3/2} \sin\left(\frac{2\pi}{7}\right) \sqrt{7 \times 94}} = 21 + \frac{7\sqrt{\frac{7}{47}} e^{2\sqrt{47/21}\pi} \sum_{k=0}^{\infty} \frac{(-1)^k (1+2k)}{10+49k+49k^2}}{4\pi}$$

$$21 + \frac{\exp\left(\pi \sqrt{\frac{2 \times 94}{21}}\right)}{2^{3/2} \sin\left(\frac{2\pi}{7}\right) \sqrt{7 \times 94}} = 21 + \frac{e^{2\sqrt{47/21}\pi} \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{3\pi}{14}\right)^{2k} E_{2k}}{(2k)!}}{4\sqrt{329}}$$

$E_n$  is the  $n^{\text{th}}$  Euler number

$n!$  is the factorial function

### Integral representation:

$$21 + \frac{\exp\left(\pi \sqrt{\frac{2 \times 94}{21}}\right)}{2^{3/2} \sin\left(\frac{2\pi}{7}\right) \sqrt{7 \times 94}} = 21 + \frac{e^{2\sqrt{47/21}\pi}}{2\sqrt{329}\pi} \int_0^{\infty} \frac{t^{3/7}}{1+t^2} dt$$

### Multiple-argument formulas:

$$21 + \frac{\exp\left(\pi \sqrt{\frac{2 \times 94}{21}}\right)}{2^{3/2} \sin\left(\frac{2\pi}{7}\right) \sqrt{7 \times 94}} = 21 + \frac{e^{2\sqrt{47/21} \pi} \sec^2\left(\frac{3\pi}{28}\right)}{4 \sqrt{329} \left(2 - \sec^2\left(\frac{3\pi}{28}\right)\right)}$$

$$21 + \frac{\exp\left(\pi \sqrt{\frac{2 \times 94}{21}}\right)}{2^{3/2} \sin\left(\frac{2\pi}{7}\right) \sqrt{7 \times 94}} = 21 + \frac{e^{2\sqrt{47/21} \pi} \sec^3\left(\frac{\pi}{14}\right)}{4 \sqrt{329} \left(4 - 3 \sec^2\left(\frac{\pi}{14}\right)\right)}$$

We observe that:

$$\left(\left(\left(\left(5 \cdot \left(\left(21 + \exp\left(\pi \sqrt{\frac{2 \times 94}{21}}\right)\right) / \left(2^{3/2} \cdot \sin\left(\frac{2\pi}{7}\right) \cdot \sqrt{7 \times 94}\right)\right)\right)\right)\right)\right)^{1/14}$$

**Input:**

$$\sqrt[14]{5 \left( 21 + \frac{\exp\left(\pi \sqrt{\frac{2 \times 94}{21}}\right)}{2^{3/2} \sin\left(\frac{2\pi}{7}\right) \sqrt{7 \times 94}} \right)}$$

**Exact result:**

$$\sqrt[14]{5 \left( 21 + \frac{e^{2\sqrt{47/21} \pi} \sec\left(\frac{3\pi}{14}\right)}{4 \sqrt{329}} \right)}$$

sec(x) is the secant function

**Decimal approximation:**

1.656398966210733860635454655653670853857280912366787815293...

1.65639896... is very near to the 14th root of the following Ramanujan's class invariant  $Q = (G_{505}/G_{101/5})^3 = 1164,2696$  i.e. 1,65578...

**Property:**

$$\sqrt[14]{5 \left( 21 + \frac{e^{2\sqrt{47/21} \pi} \sec\left(\frac{3\pi}{14}\right)}{4 \sqrt{329}} \right)}$$
 is a transcendental number

**Alternate forms:**

$$\sqrt[14]{105 + \frac{5 e^{2\sqrt{47/21} \pi} \sec\left(\frac{3\pi}{14}\right)}{4 \sqrt{329}}}$$

$$\frac{\sqrt[14]{\frac{5}{329} \left(27636 + \sqrt{329} e^{2\sqrt{47/21} \pi} \sec\left(\frac{3\pi}{14}\right)\right)}}{\sqrt[7]{2}}$$

$$\sqrt[14]{5 \left(21 + \frac{e^{2\sqrt{47/21} \pi} \cos\left(\frac{3\pi}{14}\right)}{2 \sqrt{329} \left(1 + \sin\left(\frac{\pi}{14}\right)\right)}\right)}$$

**All 14th roots of  $5 \left(21 + \frac{e^{2\sqrt{47/21} \pi} \sec\left(\frac{3\pi}{14}\right)}{4 \sqrt{329}}\right)$ :**

$$e^{0} \sqrt[14]{5 \left(21 + \frac{e^{2\sqrt{47/21} \pi} \sec\left(\frac{3\pi}{14}\right)}{4 \sqrt{329}}\right)} \approx 1.6564 \text{ (real, principal root)}$$

$$e^{(i\pi)/7} \sqrt[14]{5 \left(21 + \frac{e^{2\sqrt{47/21} \pi} \sec\left(\frac{3\pi}{14}\right)}{4 \sqrt{329}}\right)} \approx 1.4924 + 0.7187 i$$

$$e^{(2i\pi)/7} \sqrt[14]{5 \left(21 + \frac{e^{2\sqrt{47/21} \pi} \sec\left(\frac{3\pi}{14}\right)}{4 \sqrt{329}}\right)} \approx 1.0327 + 1.2950 i$$

$$e^{(3i\pi)/7} \sqrt[14]{5 \left(21 + \frac{e^{2\sqrt{47/21} \pi} \sec\left(\frac{3\pi}{14}\right)}{4 \sqrt{329}}\right)} \approx 0.3686 + 1.6149 i$$

$$e^{(4i\pi)/7} \sqrt[14]{5 \left( 21 + \frac{e^{2\sqrt{47/21} \pi} \sec\left(\frac{3\pi}{14}\right)}{4\sqrt{329}} \right)} \approx -0.3686 + 1.6149i$$

•

### Alternative representations:

$$\sqrt[14]{5 \left( 21 + \frac{\exp\left(\pi \sqrt{\frac{2 \times 94}{21}}\right)}{2^{3/2} \sin\left(\frac{2\pi}{7}\right) \sqrt{7 \times 94}} \right)} = \sqrt[14]{5 \left( 21 + \frac{\exp\left(\pi \sqrt{\frac{188}{21}}\right)}{\cos\left(\frac{\pi}{2} - \frac{2\pi}{7}\right) 2^{3/2} \sqrt{658}} \right)}$$

•

$$\sqrt[14]{5 \left( 21 + \frac{\exp\left(\pi \sqrt{\frac{2 \times 94}{21}}\right)}{2^{3/2} \sin\left(\frac{2\pi}{7}\right) \sqrt{7 \times 94}} \right)} = \sqrt[14]{5 \left( 21 + \frac{\exp\left(\pi \sqrt{\frac{188}{21}}\right)}{\cos\left(\frac{\pi}{2} + \frac{2\pi}{7}\right) 2^{3/2} \sqrt{658}} \right)}$$

•

$$\sqrt[14]{5 \left( 21 + \frac{\exp\left(\pi \sqrt{\frac{2 \times 94}{21}}\right)}{2^{3/2} \sin\left(\frac{2\pi}{7}\right) \sqrt{7 \times 94}} \right)} = \sqrt[14]{5 \left( 21 + \frac{\exp\left(\pi \sqrt{\frac{188}{21}}\right)}{\frac{2^{3/2} (-e^{-(2i\pi)/7} + e^{(2i\pi)/7}) \sqrt{658}}{2i}} \right)}$$

$i$  is the imaginary unit

•

### Series representations:

$$\sqrt[14]{5 \left( 21 + \frac{\exp\left(\pi \sqrt{\frac{2 \times 94}{21}}\right)}{2^{3/2} \sin\left(\frac{2\pi}{7}\right) \sqrt{7 \times 94}} \right)} = \sqrt[14]{105 - \frac{5 e^{2\sqrt{47/21} \pi} \sum_{k=1}^{\infty} (-1)^k q^{-1+2k}}{2\sqrt{329}}}$$

for  $q = (-1)^{3/14}$

•



$$\sqrt[14]{5 \left( 21 + \frac{\exp\left(\pi \sqrt{\frac{2 \times 94}{21}}\right)}{2^{3/2} \sin\left(\frac{2\pi}{7}\right) \sqrt{7 \times 94}} \right)} = \sqrt[14]{5} \sqrt[14]{21 - \frac{e^{2\sqrt{47/21} \pi} \sum_{k=1}^{\infty} (-1)^k q^{-1+2k}}{2 \sqrt{329}}}}$$

for  $q = (-1)^{3/14}$

$$\sqrt[14]{5 \left( 21 + \frac{\exp\left(\pi \sqrt{\frac{2 \times 94}{21}}\right)}{2^{3/2} \sin\left(\frac{2\pi}{7}\right) \sqrt{7 \times 94}} \right)} = \frac{\sqrt[14]{420 + \frac{35 \sqrt{\frac{7}{47}} e^{2\sqrt{47/21} \pi} \sum_{k=0}^{\infty} \frac{(-1)^k (1+2k)}{10+49k+49k^2}}{\pi}}}{\sqrt[7]{2}}$$

**Integral representation:**

$$\sqrt[14]{5 \left( 21 + \frac{\exp\left(\pi \sqrt{\frac{2 \times 94}{21}}\right)}{2^{3/2} \sin\left(\frac{2\pi}{7}\right) \sqrt{7 \times 94}} \right)} = \sqrt[14]{5} \sqrt[14]{21 + \frac{e^{2\sqrt{47/21} \pi}}{2 \sqrt{329} \pi} \int_0^{\infty} \frac{t^{3/7}}{1+t^2} dt}}$$

**Multiple-argument formulas:**

$$\sqrt[14]{5 \left( 21 + \frac{\exp\left(\pi \sqrt{\frac{2 \times 94}{21}}\right)}{2^{3/2} \sin\left(\frac{2\pi}{7}\right) \sqrt{7 \times 94}} \right)} = \sqrt[14]{5 \left( 21 + \frac{e^{2\sqrt{47/21} \pi} \sec^2\left(\frac{3\pi}{28}\right)}{4 \sqrt{329} \left(2 - \sec^2\left(\frac{3\pi}{28}\right)\right)} \right)}$$

$$\sqrt[14]{5 \left( 21 + \frac{\exp\left(\pi \sqrt{\frac{2 \times 94}{21}}\right)}{2^{3/2} \sin\left(\frac{2\pi}{7}\right) \sqrt{7 \times 94}} \right)} = \sqrt[14]{5 \left( 21 + \frac{e^{2\sqrt{47/21} \pi} \sec^3\left(\frac{\pi}{14}\right)}{4 \sqrt{329} \left(4 - 3 \sec^2\left(\frac{\pi}{14}\right)\right)} \right)}$$

$$\left(\frac{13}{10^3} + \frac{2}{10^3}\right) + \left(\left(\left(\left(\left(\left(5 \cdot \left(\frac{21 + \exp(\pi \cdot \sqrt{2 \cdot 94/21})}{2^{3/2} \cdot \sin(2 \cdot \pi/7)} \cdot \sqrt{7 \cdot 94}\right)\right)\right)\right)\right)\right)^{1/14}$$

**Input:**

$$\left(\frac{13}{10^3} + \frac{2}{10^3}\right) + \sqrt[14]{5 \left(21 + \frac{\exp\left(\pi \sqrt{2 \times \frac{94}{21}}\right)}{2^{3/2} \sin\left(2 \times \frac{\pi}{7}\right) \sqrt{7 \times 94}}\right)}$$

**Exact result:**

$$\frac{3}{200} + \sqrt[14]{5 \left(21 + \frac{e^{2 \sqrt{47/21} \pi} \sec\left(\frac{3\pi}{14}\right)}{4 \sqrt{329}}\right)}$$

$\sec(x)$  is the secant function

**Decimal approximation:**

1.671398966210733860635454655653670853857280912366787815293...

1.671398966...

We note that 1.671398966... is a result practically equal to the value of the formula:

$$m_{p'} = 2 \times \frac{\eta}{R} m_p = 1.6714213 \times 10^{-24} \text{ gm}$$

that is the holographic proton mass

**Property:**

$$\frac{3}{200} + \sqrt[14]{5 \left(21 + \frac{e^{2 \sqrt{47/21} \pi} \sec\left(\frac{3\pi}{14}\right)}{4 \sqrt{329}}\right)}$$
 is a transcendental number

•

**Alternate forms:**

•

$$\frac{3}{200} + \sqrt[14]{105 + \frac{5 e^{2 \sqrt{47/21} \pi} \sec\left(\frac{3\pi}{14}\right)}{4 \sqrt{329}}}$$

•

$$\frac{3}{200} + \sqrt[14]{5 \left( 21 + \frac{e^{2\sqrt{47/21}\pi} \cos\left(\frac{3\pi}{14}\right)}{2\sqrt{329} \left(1 + \sin\left(\frac{\pi}{14}\right)\right)} \right)}$$

$$\frac{987 + 100 \times 2^{6/7} \times 329^{13/14} \sqrt[14]{5 \left( 27636 + \sqrt{329} e^{2\sqrt{47/21}\pi} \sec\left(\frac{3\pi}{14}\right) \right)}}{65800}$$

**Alternative representations:**

$$\left( \frac{13}{10^3} + \frac{2}{10^3} \right) + \sqrt[14]{5 \left( 21 + \frac{\exp\left(\pi \sqrt{\frac{2 \times 94}{21}}\right)}{2^{3/2} \sin\left(\frac{2\pi}{7}\right) \sqrt{7 \times 94}} \right)} =$$

$$\frac{15}{10^3} + \sqrt[14]{5 \left( 21 + \frac{\exp\left(\pi \sqrt{\frac{188}{21}}\right)}{\cos\left(\frac{\pi}{2} - \frac{2\pi}{7}\right) 2^{3/2} \sqrt{658}} \right)}$$

$$\left( \frac{13}{10^3} + \frac{2}{10^3} \right) + \sqrt[14]{5 \left( 21 + \frac{\exp\left(\pi \sqrt{\frac{2 \times 94}{21}}\right)}{2^{3/2} \sin\left(\frac{2\pi}{7}\right) \sqrt{7 \times 94}} \right)} =$$

$$\frac{15}{10^3} + \sqrt[14]{5 \left( 21 + \frac{\exp\left(\pi \sqrt{\frac{188}{21}}\right)}{\cos\left(\frac{\pi}{2} + \frac{2\pi}{7}\right) 2^{3/2} \sqrt{658}} \right)}$$

$$\left( \frac{13}{10^3} + \frac{2}{10^3} \right) + \sqrt[14]{5 \left( 21 + \frac{\exp\left(\pi \sqrt{\frac{2 \times 94}{21}}\right)}{2^{3/2} \sin\left(\frac{2\pi}{7}\right) \sqrt{7 \times 94}} \right)} =$$

$$\frac{15}{10^3} + \sqrt[14]{5 \left( 21 + \frac{\exp\left(\pi \sqrt{\frac{188}{21}}\right)}{\frac{2^{3/2} \left( -e^{-(2i\pi)/7} + e^{(2i\pi)/7} \right) \sqrt{658}}{2i}} \right)}$$

$i$  is the imaginary unit

**Series representations:**

$$\left(\frac{13}{10^3} + \frac{2}{10^3}\right) + \sqrt[14]{5 \left( 21 + \frac{\exp\left(\pi \sqrt{\frac{2 \times 94}{21}}\right)}{2^{3/2} \sin\left(\frac{2\pi}{7}\right) \sqrt{7 \times 94}} \right)} =$$

$$\frac{3}{200} + \sqrt[14]{105 - \frac{5 e^{2\sqrt{47/21} \pi} \sum_{k=1}^{\infty} (-1)^k q^{-1+2k}}{2 \sqrt{329}}} \quad \text{for } q = (-1)^{3/14}$$

$$\left(\frac{13}{10^3} + \frac{2}{10^3}\right) + \sqrt[14]{5 \left( 21 + \frac{\exp\left(\pi \sqrt{\frac{2 \times 94}{21}}\right)}{2^{3/2} \sin\left(\frac{2\pi}{7}\right) \sqrt{7 \times 94}} \right)} =$$

$$\frac{3}{200} + \sqrt[14]{5} \sqrt[14]{21 - \frac{e^{2\sqrt{47/21} \pi} \sum_{k=1}^{\infty} (-1)^k q^{-1+2k}}{2 \sqrt{329}}} \quad \text{for } q = (-1)^{3/14}$$

$$\left(\frac{13}{10^3} + \frac{2}{10^3}\right) + \sqrt[14]{5 \left( 21 + \frac{\exp\left(\pi \sqrt{\frac{2 \times 94}{21}}\right)}{2^{3/2} \sin\left(\frac{2\pi}{7}\right) \sqrt{7 \times 94}} \right)} =$$

$$\frac{3}{200} + \frac{\sqrt[14]{420 + \frac{35 \sqrt{\frac{7}{47}} e^{2\sqrt{47/21} \pi} \sum_{k=0}^{\infty} \frac{(-1)^k (1+2k)}{10+49k+49k^2}}{\pi}}}{\sqrt[7]{2}}$$

**Integral representation:**

$$\left(\frac{13}{10^3} + \frac{2}{10^3}\right) + \sqrt[14]{5 \left( 21 + \frac{\exp\left(\pi \sqrt{\frac{2 \times 94}{21}}\right)}{2^{3/2} \sin\left(\frac{2\pi}{7}\right) \sqrt{7 \times 94}} \right)} =$$

$$\frac{3}{200} + \sqrt[14]{5} \sqrt[14]{21 + \frac{e^{2\sqrt{47/21} \pi}}{2 \sqrt{329} \pi} \int_0^{\infty} \frac{t^{3/7}}{1+t^2} dt}$$

**Multiple-argument formulas:**

$$\left(\frac{13}{10^3} + \frac{2}{10^3}\right) + \sqrt[14]{5 \left( 21 + \frac{\exp\left(\pi \sqrt{\frac{2 \times 94}{21}}\right)}{2^{3/2} \sin\left(\frac{2\pi}{7}\right) \sqrt{7 \times 94}} \right)} = \frac{3}{200} + \sqrt[14]{5 \left( 21 + \frac{e^{2\sqrt{47/21} \pi} \sec^2\left(\frac{3\pi}{28}\right)}{4\sqrt{329} \left(2 - \sec^2\left(\frac{3\pi}{28}\right)\right)} \right)}$$

$$\left(\frac{13}{10^3} + \frac{2}{10^3}\right) + \sqrt[14]{5 \left( 21 + \frac{\exp\left(\pi \sqrt{\frac{2 \times 94}{21}}\right)}{2^{3/2} \sin\left(\frac{2\pi}{7}\right) \sqrt{7 \times 94}} \right)} = \frac{3}{200} + \sqrt[14]{5 \left( 21 + \frac{e^{2\sqrt{47/21} \pi} \sec^3\left(\frac{\pi}{14}\right)}{4\sqrt{329} \left(4 - 3 \sec^2\left(\frac{\pi}{14}\right)\right)} \right)}$$

$$(8/10^3+13/10^3-34/10^3)+(((((((5*(((21+\exp(\text{Pi}*\text{sqrt}(2*94/21)))/(2^(3/2)*\sin(2*\text{Pi}/7)*\text{sqrt}(7*94))))))))))))))^(1/14)$$

**Input:**

$$\left(\frac{8}{10^3} + \frac{13}{10^3} - \frac{34}{10^3}\right) + \sqrt[14]{5 \left( 21 + \frac{\exp\left(\pi \sqrt{2 \times \frac{94}{21}}\right)}{2^{3/2} \sin\left(2 \times \frac{\pi}{7}\right) \sqrt{7 \times 94}} \right)}$$

**Exact result:**

$$\sqrt[14]{5 \left( 21 + \frac{e^{2\sqrt{47/21} \pi} \sec\left(\frac{3\pi}{14}\right)}{4\sqrt{329}} \right)} - \frac{13}{1000}$$

**Decimal approximation:**

1.643398966210733860635454655653670853857280912366787815293...

$$1.6433989662... \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 ...$$

**Property:**

$$-\frac{13}{1000} + \sqrt[14]{5 \left( 21 + \frac{e^{2\sqrt{47/21}\pi} \sec\left(\frac{3\pi}{14}\right)}{4\sqrt{329}} \right)}$$
 is a transcendental number

•

**Alternate forms:**

$$\sqrt[14]{105 + \frac{5 e^{2\sqrt{47/21}\pi} \sec\left(\frac{3\pi}{14}\right)}{4\sqrt{329}}} - \frac{13}{1000}$$

•

$$\sqrt[14]{5 \left( 21 + \frac{e^{2\sqrt{47/21}\pi} \cos\left(\frac{3\pi}{14}\right)}{2\sqrt{329} \left(1 + \sin\left(\frac{\pi}{14}\right)\right)} \right)} - \frac{13}{1000}$$

•

$$\frac{500 \times 2^{6/7} \times 329^{13/14} \sqrt[14]{5 \left( 27636 + \sqrt{329} e^{2\sqrt{47/21}\pi} \sec\left(\frac{3\pi}{14}\right) \right)} - 4277}{329000}$$

•

**Alternative representations:**

$$\left(\frac{8}{10^3} + \frac{13}{10^3} - \frac{34}{10^3}\right) + \sqrt[14]{5 \left( 21 + \frac{\exp\left(\pi \sqrt{\frac{2 \times 94}{21}}\right)}{2^{3/2} \sin\left(\frac{2\pi}{7}\right) \sqrt{7 \times 94}} \right)} =$$

$$-\frac{13}{10^3} + \sqrt[14]{5 \left( 21 + \frac{\exp\left(\pi \sqrt{\frac{188}{21}}\right)}{\cos\left(\frac{\pi}{2} - \frac{2\pi}{7}\right) 2^{3/2} \sqrt{658}} \right)}$$

•

$$\left(\frac{8}{10^3} + \frac{13}{10^3} - \frac{34}{10^3}\right) + \sqrt[14]{5 \left( 21 + \frac{\exp\left(\pi \sqrt{\frac{2 \times 94}{21}}\right)}{2^{3/2} \sin\left(\frac{2\pi}{7}\right) \sqrt{7 \times 94}} \right)} =$$

$$-\frac{13}{10^3} + \sqrt[14]{5 \left( 21 + \frac{\exp\left(\pi \sqrt{\frac{188}{21}}\right)}{\cos\left(\frac{\pi}{2} + \frac{2\pi}{7}\right) 2^{3/2} \sqrt{658}} \right)}$$

•

$$\left(\frac{8}{10^3} + \frac{13}{10^3} - \frac{34}{10^3}\right) + \sqrt[14]{5 \left( 21 + \frac{\exp\left(\pi \sqrt{\frac{2 \times 94}{21}}\right)}{2^{3/2} \sin\left(\frac{2\pi}{7}\right) \sqrt{7 \times 94}} \right)} =$$

$$-\frac{13}{10^3} + \sqrt[14]{5 \left( 21 + \frac{\exp\left(\pi \sqrt{\frac{188}{21}}\right)}{\frac{2^{3/2} (-e^{-(2i\pi)/7} + e^{(2i\pi)/7}) \sqrt{658}}{2i}} \right)}$$

$i$  is the imaginary unit

•

**Series representations:**

$$\left(\frac{8}{10^3} + \frac{13}{10^3} - \frac{34}{10^3}\right) + \sqrt[14]{5 \left( 21 + \frac{\exp\left(\pi \sqrt{\frac{2 \times 94}{21}}\right)}{2^{3/2} \sin\left(\frac{2\pi}{7}\right) \sqrt{7 \times 94}} \right)} =$$

$$-\frac{13}{1000} + \sqrt[14]{105 - \frac{5 e^{2\sqrt{47/21} \pi} \sum_{k=1}^{\infty} (-1)^k q^{-1+2k}}{2 \sqrt{329}}} \quad \text{for } q = (-1)^{3/14}$$

•

$$\left(\frac{8}{10^3} + \frac{13}{10^3} - \frac{34}{10^3}\right) + \sqrt[14]{5 \left( 21 + \frac{\exp\left(\pi \sqrt{\frac{2 \times 94}{21}}\right)}{2^{3/2} \sin\left(\frac{2\pi}{7}\right) \sqrt{7 \times 94}} \right)} =$$

$$-\frac{13}{1000} + \sqrt[14]{5} \sqrt[14]{21 - \frac{e^{2\sqrt{47/21} \pi} \sum_{k=1}^{\infty} (-1)^k q^{-1+2k}}{2 \sqrt{329}}} \quad \text{for } q = (-1)^{3/14}$$

•

$$\left(\frac{8}{10^3} + \frac{13}{10^3} - \frac{34}{10^3}\right) + \sqrt[14]{5 \left( 21 + \frac{\exp\left(\pi \sqrt{\frac{2 \times 94}{21}}\right)}{2^{3/2} \sin\left(\frac{2\pi}{7}\right) \sqrt{7 \times 94}} \right)} =$$

$$-\frac{13}{1000} + \sqrt[14]{420 + \frac{35 \sqrt{\frac{7}{47}} e^{2\sqrt{47/21} \pi} \sum_{k=0}^{\infty} \frac{(-1)^k (1+2k)}{10+49k+49k^2}}{\pi}} \sqrt[14]{2}$$

•

**Integral representation:**

$$\left(\frac{8}{10^3} + \frac{13}{10^3} - \frac{34}{10^3}\right) + \sqrt[14]{5 \left( 21 + \frac{\exp\left(\pi \sqrt{\frac{2 \times 94}{21}}\right)}{2^{3/2} \sin\left(\frac{2\pi}{7}\right) \sqrt{7 \times 94}} \right)} =$$

$$-\frac{13}{1000} + \sqrt[14]{5} \sqrt[14]{21 + \frac{e^{2\sqrt{47/21} \pi}}{2 \sqrt{329} \pi} \int_0^{\infty} \frac{t^{3/7}}{1+t^2} dt}$$



**Multiple-argument formulas:**

$$\left(\frac{8}{10^3} + \frac{13}{10^3} - \frac{34}{10^3}\right) + \sqrt[14]{5 \left(21 + \frac{\exp\left(\pi \sqrt{\frac{2 \times 94}{21}}\right)}{2^{3/2} \sin\left(\frac{2\pi}{7}\right) \sqrt{7 \times 94}}\right)} =$$

$$-\frac{13}{1000} + \sqrt[14]{5 \left(21 + \frac{e^{2\sqrt{47/21} \pi} \sec^2\left(\frac{3\pi}{28}\right)}{4\sqrt{329} \left(2 - \sec^2\left(\frac{3\pi}{28}\right)\right)}\right)}$$

$$\left(\frac{8}{10^3} + \frac{13}{10^3} - \frac{34}{10^3}\right) + \sqrt[14]{5 \left(21 + \frac{\exp\left(\pi \sqrt{\frac{2 \times 94}{21}}\right)}{2^{3/2} \sin\left(\frac{2\pi}{7}\right) \sqrt{7 \times 94}}\right)} =$$

$$-\frac{13}{1000} + \sqrt[14]{5 \left(21 + \frac{e^{2\sqrt{47/21} \pi} \sec^3\left(\frac{\pi}{14}\right)}{4\sqrt{329} \left(4 - 3 \sec^2\left(\frac{\pi}{14}\right)\right)}\right)}$$

$(1/10^3-5/10^3-34/10^3)+(((((((5*(((21+\exp(\text{Pi}*\text{sqrt}(2*94/21)) / (2^(3/2) * \sin(2*\text{Pi}/7) * \text{sqrt}(7*94))))))))))))))^{1/14}$

**Input:**

$$\left(\frac{1}{10^3} - \frac{5}{10^3} - \frac{34}{10^3}\right) + \sqrt[14]{5 \left(21 + \frac{\exp\left(\pi \sqrt{2 \times \frac{94}{21}}\right)}{2^{3/2} \sin\left(2 \times \frac{\pi}{7}\right) \sqrt{7 \times 94}}\right)}$$

**Exact result:**

$$\sqrt[14]{5 \left(21 + \frac{e^{2\sqrt{47/21} \pi} \sec\left(\frac{3\pi}{14}\right)}{4\sqrt{329}}\right)} - \frac{19}{500}$$

sec(x) is the secant function

**Decimal approximation:**

1.618398966210733860635454655653670853857280912366787815293...

1.61839896621...

This result is a very good approximation to the value of the golden ratio  
1,618033988749...

**Property:**

$$-\frac{19}{500} + \sqrt[14]{5 \left( 21 + \frac{e^{2\sqrt{47/21}\pi} \sec\left(\frac{3\pi}{14}\right)}{4\sqrt{329}} \right)} \text{ is a transcendental number}$$

•

**Alternate forms:**

$$\sqrt[14]{105 + \frac{5 e^{2\sqrt{47/21}\pi} \sec\left(\frac{3\pi}{14}\right)}{4\sqrt{329}}} - \frac{19}{500}$$

•

$$\sqrt[14]{5 \left( 21 + \frac{e^{2\sqrt{47/21}\pi} \cos\left(\frac{3\pi}{14}\right)}{2\sqrt{329} \left(1 + \sin\left(\frac{\pi}{14}\right)\right)} \right)} - \frac{19}{500}$$

•

$$\frac{250 \times 2^{6/7} \times 329^{13/14} \sqrt[14]{5 \left( 27636 + \sqrt{329} e^{2\sqrt{47/21}\pi} \sec\left(\frac{3\pi}{14}\right) \right)} - 6251}{164500}$$

**Alternative representations:**

$$\left(\frac{1}{10^3} - \frac{5}{10^3} - \frac{34}{10^3}\right) + \sqrt[14]{5 \left( 21 + \frac{\exp\left(\pi \sqrt{\frac{2 \times 94}{21}}\right)}{2^{3/2} \sin\left(\frac{2\pi}{7}\right) \sqrt{7 \times 94}} \right)} =$$

$$-\frac{38}{10^3} + \sqrt[14]{5 \left( 21 + \frac{\exp\left(\pi \sqrt{\frac{188}{21}}\right)}{\cos\left(\frac{\pi}{2} - \frac{2\pi}{7}\right) 2^{3/2} \sqrt{658}} \right)}$$

$$\left(\frac{1}{10^3} - \frac{5}{10^3} - \frac{34}{10^3}\right) + \sqrt[14]{5 \left( 21 + \frac{\exp\left(\pi \sqrt{\frac{2 \times 94}{21}}\right)}{2^{3/2} \sin\left(\frac{2\pi}{7}\right) \sqrt{7 \times 94}} \right)} =$$

$$-\frac{38}{10^3} + \sqrt[14]{5 \left( 21 + \frac{\exp\left(\pi \sqrt{\frac{188}{21}}\right)}{\cos\left(\frac{\pi}{2} + \frac{2\pi}{7}\right) 2^{3/2} \sqrt{658}} \right)}$$

$$\left(\frac{1}{10^3} - \frac{5}{10^3} - \frac{34}{10^3}\right) + \sqrt[14]{5 \left( 21 + \frac{\exp\left(\pi \sqrt{\frac{2 \times 94}{21}}\right)}{2^{3/2} \sin\left(\frac{2\pi}{7}\right) \sqrt{7 \times 94}} \right)} =$$

$$-\frac{38}{10^3} + \sqrt[14]{5 \left( 21 + \frac{\exp\left(\pi \sqrt{\frac{188}{21}}\right)}{\frac{2^{3/2} (-e^{-(2i\pi)/7} + e^{(2i\pi)/7}) \sqrt{658}}{2i}} \right)}$$

$i$  is the imaginary unit

**Series representations:**

$$\left(\frac{1}{10^3} - \frac{5}{10^3} - \frac{34}{10^3}\right) + \sqrt[14]{5 \left( 21 + \frac{\exp\left(\pi \sqrt{\frac{2 \times 94}{21}}\right)}{2^{3/2} \sin\left(\frac{2\pi}{7}\right) \sqrt{7 \times 94}} \right)} =$$

$$-\frac{19}{500} + \sqrt[14]{105 - \frac{5 e^{2\sqrt{47/21} \pi} \sum_{k=1}^{\infty} (-1)^k q^{-1+2k}}{2 \sqrt{329}}} \quad \text{for } q = (-1)^{3/14}$$

•

$$\left(\frac{1}{10^3} - \frac{5}{10^3} - \frac{34}{10^3}\right) + \sqrt[14]{5 \left( 21 + \frac{\exp\left(\pi \sqrt{\frac{2 \times 94}{21}}\right)}{2^{3/2} \sin\left(\frac{2\pi}{7}\right) \sqrt{7 \times 94}} \right)} =$$

$$-\frac{19}{500} + \sqrt[14]{5} \sqrt[14]{21 - \frac{e^{2\sqrt{47/21} \pi} \sum_{k=1}^{\infty} (-1)^k q^{-1+2k}}{2 \sqrt{329}}} \quad \text{for } q = (-1)^{3/14}$$

•

$$\left(\frac{1}{10^3} - \frac{5}{10^3} - \frac{34}{10^3}\right) + \sqrt[14]{5 \left( 21 + \frac{\exp\left(\pi \sqrt{\frac{2 \times 94}{21}}\right)}{2^{3/2} \sin\left(\frac{2\pi}{7}\right) \sqrt{7 \times 94}} \right)} =$$

$$-\frac{19}{500} + \frac{\sqrt[14]{420 + \frac{35 \sqrt{\frac{7}{47}} e^{2\sqrt{47/21} \pi} \sum_{k=0}^{\infty} \frac{(-1)^k (1+2k)}{10+49k+49k^2}}{\pi}}}{\sqrt[7]{2}}$$

**Integral representation:**

$$\left(\frac{1}{10^3} - \frac{5}{10^3} - \frac{34}{10^3}\right) + \sqrt[14]{5 \left( 21 + \frac{\exp\left(\pi \sqrt{\frac{2 \times 94}{21}}\right)}{2^{3/2} \sin\left(\frac{2\pi}{7}\right) \sqrt{7 \times 94}} \right)} =$$

$$-\frac{19}{500} + \sqrt[14]{5} \sqrt[14]{21 + \frac{e^{2\sqrt{47/21} \pi}}{2 \sqrt{329} \pi} \int_0^{\infty} \frac{t^{3/7}}{1+t^2} dt}$$

**Multiple-argument formulas:**

$$\left(\frac{1}{10^3} - \frac{5}{10^3} - \frac{34}{10^3}\right) + \sqrt[14]{5 \left( 21 + \frac{\exp\left(\pi \sqrt{\frac{2 \times 94}{21}}\right)}{2^{3/2} \sin\left(\frac{2\pi}{7}\right) \sqrt{7 \times 94}} \right)} =$$

$$-\frac{19}{500} + \sqrt[14]{5 \left( 21 + \frac{e^{2\sqrt{47/21} \pi} \sec^2\left(\frac{3\pi}{28}\right)}{4\sqrt{329} \left(2 - \sec^2\left(\frac{3\pi}{28}\right)\right)} \right)}$$

$$\left(\frac{1}{10^3} - \frac{5}{10^3} - \frac{34}{10^3}\right) + \sqrt[14]{5 \left( 21 + \frac{\exp\left(\pi \sqrt{\frac{2 \times 94}{21}}\right)}{2^{3/2} \sin\left(\frac{2\pi}{7}\right) \sqrt{7 \times 94}} \right)} =$$

$$-\frac{19}{500} + \sqrt[14]{5 \left( 21 + \frac{e^{2\sqrt{47/21} \pi} \sec^3\left(\frac{\pi}{14}\right)}{4\sqrt{329} \left(4 - 3 \sec^2\left(\frac{\pi}{14}\right)\right)} \right)}$$

$(1/10^3 - 5/10^3 - 34/10^3 - 13/10^3 - 2/10^3) + ((((((5 * (((21 + \exp(\pi * \sqrt{2 * 94 / 21})) / (2^{3/2} * \sin(2 * \pi / 7) * \sqrt{7 * 94}))))))))))^{1/14}$

**Input:**

$$\left(\frac{1}{10^3} - \frac{5}{10^3} - \frac{34}{10^3} - \frac{13}{10^3} - \frac{2}{10^3}\right) + \sqrt[14]{5 \left( 21 + \frac{\exp\left(\pi \sqrt{2 \times \frac{94}{21}}\right)}{2^{3/2} \sin\left(2 \times \frac{\pi}{7}\right) \sqrt{7 \times 94}} \right)}$$

**Exact result:**

$$\sqrt[14]{5 \left( 21 + \frac{e^{2\sqrt{47/21} \pi} \sec\left(\frac{3\pi}{14}\right)}{4\sqrt{329}} \right)} - \frac{53}{1000}$$

sec(x) is the secant function

**Decimal approximation:**

1.603398966210733860635454655653670853857280912366787815293...

1.60339896621... result practically equal to the following Haremeïn's formula

$$m_{p'} = 2 \frac{\eta_{\rho}}{R} = 1.603498 \times 10^{-24} \text{ gm}$$

**Property:**

$$-\frac{53}{1000} + \sqrt[14]{5 \left( 21 + \frac{e^{2\sqrt{47/21} \pi} \sec\left(\frac{3\pi}{14}\right)}{4\sqrt{329}} \right)} \text{ is a transcendental number}$$

•

**Alternate forms:**

$$\sqrt[14]{105 + \frac{5 e^{2\sqrt{47/21} \pi} \sec\left(\frac{3\pi}{14}\right)}{4\sqrt{329}}} - \frac{53}{1000}$$

•

$$\sqrt[14]{5 \left( 21 + \frac{e^{2\sqrt{47/21} \pi} \cos\left(\frac{3\pi}{14}\right)}{2\sqrt{329} \left(1 + \sin\left(\frac{\pi}{14}\right)\right)} \right)} - \frac{53}{1000}$$

•

$$\frac{500 \times 2^{6/7} \times 329^{13/14} \sqrt[14]{5 \left( 27636 + \sqrt{329} e^{2\sqrt{47/21} \pi} \sec\left(\frac{3\pi}{14}\right) \right)} - 17437}{329000}$$

**Alternative representations:**

$$\left( \frac{1}{10^3} - \frac{5}{10^3} - \frac{34}{10^3} - \frac{13}{10^3} - \frac{2}{10^3} \right) + \sqrt[14]{5 \left( 21 + \frac{\exp\left(\pi \sqrt{\frac{2 \times 94}{21}}\right)}{2^{3/2} \sin\left(\frac{2\pi}{7}\right) \sqrt{7 \times 94}} \right)} =$$

$$-\frac{53}{10^3} + \sqrt[14]{5 \left( 21 + \frac{\exp\left(\pi \sqrt{\frac{188}{21}}\right)}{\cos\left(\frac{\pi}{2} - \frac{2\pi}{7}\right) 2^{3/2} \sqrt{658}} \right)}$$

•

$$\left(\frac{1}{10^3} - \frac{5}{10^3} - \frac{34}{10^3} - \frac{13}{10^3} - \frac{2}{10^3}\right) + \sqrt[14]{5 \left( 21 + \frac{\exp\left(\pi \sqrt{\frac{2 \cdot 94}{21}}\right)}{2^{3/2} \sin\left(\frac{2\pi}{7}\right) \sqrt{7 \times 94}} \right)} =$$

$$-\frac{53}{10^3} + \sqrt[14]{5 \left( 21 + \frac{\exp\left(\pi \sqrt{\frac{188}{21}}\right)}{\cos\left(\frac{\pi}{2} + \frac{2\pi}{7}\right) 2^{3/2} \sqrt{658}} \right)}$$

$$\left(\frac{1}{10^3} - \frac{5}{10^3} - \frac{34}{10^3} - \frac{13}{10^3} - \frac{2}{10^3}\right) + \sqrt[14]{5 \left( 21 + \frac{\exp\left(\pi \sqrt{\frac{2 \cdot 94}{21}}\right)}{2^{3/2} \sin\left(\frac{2\pi}{7}\right) \sqrt{7 \times 94}} \right)} =$$

$$-\frac{53}{10^3} + \sqrt[14]{5 \left( 21 + \frac{\exp\left(\pi \sqrt{\frac{188}{21}}\right)}{\frac{2^{3/2} (-e^{-2i\pi/7} + e^{2i\pi/7}) \sqrt{658}}{2i}} \right)}$$

$i$  is the imaginary unit

### Series representations:

$$\left(\frac{1}{10^3} - \frac{5}{10^3} - \frac{34}{10^3} - \frac{13}{10^3} - \frac{2}{10^3}\right) + \sqrt[14]{5 \left( 21 + \frac{\exp\left(\pi \sqrt{\frac{2 \cdot 94}{21}}\right)}{2^{3/2} \sin\left(\frac{2\pi}{7}\right) \sqrt{7 \times 94}} \right)} =$$

$$-\frac{53}{1000} + \sqrt[14]{105 - \frac{5 e^{2\sqrt{47/21}\pi} \sum_{k=1}^{\infty} (-1)^k q^{-1+2k}}{2\sqrt{329}}} \quad \text{for } q = (-1)^{3/14}$$

$$\left(\frac{1}{10^3} - \frac{5}{10^3} - \frac{34}{10^3} - \frac{13}{10^3} - \frac{2}{10^3}\right) + \sqrt[14]{5 \left( 21 + \frac{\exp\left(\pi \sqrt{\frac{2 \times 94}{21}}\right)}{2^{3/2} \sin\left(\frac{2\pi}{7}\right) \sqrt{7 \times 94}} \right)} =$$

$$-\frac{53}{1000} + \sqrt[14]{5} \sqrt[14]{21 - \frac{e^{2\sqrt{47/21}} \pi \sum_{k=1}^{\infty} (-1)^k q^{-1+2k}}{2\sqrt{329}}} \quad \text{for } q = (-1)^{3/14}$$

$$\left(\frac{1}{10^3} - \frac{5}{10^3} - \frac{34}{10^3} - \frac{13}{10^3} - \frac{2}{10^3}\right) + \sqrt[14]{5 \left( 21 + \frac{\exp\left(\pi \sqrt{\frac{2 \times 94}{21}}\right)}{2^{3/2} \sin\left(\frac{2\pi}{7}\right) \sqrt{7 \times 94}} \right)} =$$

$$-\frac{53}{1000} + \frac{\sqrt[14]{420 + \frac{35\sqrt{7}}{47} e^{2\sqrt{47/21}} \pi \sum_{k=0}^{\infty} \frac{(-1)^k (1+2k)}{10+49k+49k^2}}}{\sqrt[7]{2}}$$

**Integral representation:**

$$\left(\frac{1}{10^3} - \frac{5}{10^3} - \frac{34}{10^3} - \frac{13}{10^3} - \frac{2}{10^3}\right) + \sqrt[14]{5 \left( 21 + \frac{\exp\left(\pi \sqrt{\frac{2 \times 94}{21}}\right)}{2^{3/2} \sin\left(\frac{2\pi}{7}\right) \sqrt{7 \times 94}} \right)} =$$

$$-\frac{53}{1000} + \sqrt[14]{5} \sqrt[14]{21 + \frac{e^{2\sqrt{47/21}} \pi}{2\sqrt{329}} \int_0^{\infty} \frac{t^{3/7}}{1+t^2} dt}$$

**Multiple-argument formulas:**



$$\left(\frac{1}{10^3} - \frac{5}{10^3} - \frac{34}{10^3} - \frac{13}{10^3} - \frac{2}{10^3}\right) + \sqrt[14]{5 \left(21 + \frac{\exp\left(\pi \sqrt{\frac{2 \times 94}{21}}\right)}{2^{3/2} \sin\left(\frac{2\pi}{7}\right) \sqrt{7 \times 94}}\right)} =$$

$$-\frac{53}{1000} + \sqrt[14]{5 \left(21 + \frac{e^{2\sqrt{47/21} \pi} \sec^2\left(\frac{3\pi}{28}\right)}{4\sqrt{329} \left(2 - \sec^2\left(\frac{3\pi}{28}\right)\right)}\right)}$$

$$\left(\frac{1}{10^3} - \frac{5}{10^3} - \frac{34}{10^3} - \frac{13}{10^3} - \frac{2}{10^3}\right) + \sqrt[14]{5 \left(21 + \frac{\exp\left(\pi \sqrt{\frac{2 \times 94}{21}}\right)}{2^{3/2} \sin\left(\frac{2\pi}{7}\right) \sqrt{7 \times 94}}\right)} =$$

$$-\frac{53}{1000} + \sqrt[14]{5 \left(21 + \frac{e^{2\sqrt{47/21} \pi} \sec^3\left(\frac{\pi}{14}\right)}{4\sqrt{329} \left(4 - 3 \sec^2\left(\frac{\pi}{14}\right)\right)}\right)}$$

Now, we have that:

Superstring theory represents both bosonic and fermionic particle states. The usual string theories occupy a 26-dimensional spacetime, representing bosonic particle states. A quantum state of identical bosonic particles is symmetric under the exchange of any two particles. A quantum state of identical fermionic particles is antisymmetric under the exchange of any two particles to include the photon and gravitation. Then we have  $8 \times 8 = 64$  dimensional states in some superstring theories. The closed string theory is called a type II string theory, which has the doubly fermionic states included, for a total of  $2 \times 8 \times 8 = 128$  fermionic states [96].

In addition to the type II, there are two heterotic superstring theories which involve closed strings. Out of the 26-L bosonic coordinates of the bosonic factor, only ten are matched to R-bosonic coordinates of the superstring factor, hence this theory effectively exists in ten-dimensional spacetime. Heterotic strings comes in two versions, that is  $E_8 \times E_8$  and the  $SO(32)$  type. The Ramond vacuum is included and  $E_8$  is the highest dimensional exceptional group. The  $E_8 \times E_8$  superstring theory is derived from the compilation of  $M$  – theory. One of the most promising superstring theories that unifies the four forces is the  $E_8 \times E_8$  reflection space. This is possible only because reflection embedding provides for an embedding of  $A_4$  in  $E_8$  [97]. In our paper reference [3] we present the symmetry group relationship between  $A_4$  and the 24 element octahedral group. This procedure operates along the lines of the relationship between the  $SO(32)$  heterotic string theory which also utilizes the  $E_8 \times E_8$  formalism. However, we believe our approach to gravitation and strong interactions, which considers the inclusion of torque and Coriolis effects will result in a simplification and a more fundamental formalism with fewer free parameters.

Thence:

$$2 * \left( \frac{3 \pi (9460730472581000)^{1.5}}{15 \times 2^{2.5}} \right) / \left( \frac{3 \pi (9460730472581000)^{1.5}}{120 \times 2^{2.5}} \right) ^2$$

**Input:**

$$2 \left( \frac{3 \pi \times \frac{9460730472581000^{1.5}}{15 \times 2^{2.5}}}{3 \pi \times \frac{9460730472581000^{1.5}}{120 \times 2^{2.5}}} \right)^2$$

**Result:**

128

128 that is 16 \* 8, where 8 is a the number of vibration modes in Superstring theory/M-theory

**Alternative representations:**

$$2 \left( \frac{3 (\pi 9460730472581000^{1.5})}{(3 \pi 9460730472581000^{1.5})(15 \times 2^{2.5})} \right)^2 = 2 \left( \frac{540^\circ 9460730472581000^{1.5}}{(15 \times 2^{2.5})(540^\circ 9460730472581000^{1.5})} \right)^2$$

•

$$2 \left( \frac{3 (\pi 9460730472581000^{1.5})}{(3 \pi 9460730472581000^{1.5})(15 \times 2^{2.5})} \right)^2 = 2 \left( \frac{3 i \log(-1) 9460730472581000^{1.5}}{(15 \times 2^{2.5})(-3 i \log(-1) 9460730472581000^{1.5})} \right)^2$$

•

$$2 \left( \frac{3 (\pi 9460730472581000^{1.5})}{(3 \pi 9460730472581000^{1.5})(15 \times 2^{2.5})} \right)^2 = 2 \left( \frac{3 \cos^{-1}(-1) 9460730472581000^{1.5}}{(15 \times 2^{2.5})(3 \cos^{-1}(-1) 9460730472581000^{1.5})} \right)^2$$

$\log(x)$  is the natural logarithm

$i$  is the imaginary unit

$\cos^{-1}(x)$  is the inverse cosine function

**And:**

$$-8 + [4 \left( \frac{3 \pi (9460730472581000)^{1.5}}{15 \times 2^{2.5}} \right) / \left( \frac{3 \pi (9460730472581000)^{1.5}}{120 \times 2^{2.5}} \right) ^2 ]$$

**Input:**

$$-8 + 4 \left( \frac{3 \pi \times \frac{9460730472581000^{1.5}}{15 \times 2^{2.5}}}{3 \pi \times \frac{9460730472581000^{1.5}}{120 \times 2^{2.5}}} \right)^2$$

**Result:**

248

248 that is the dimensions number of  $E_8$

**Alternative representations:**

$$-8 + 4 \left( \frac{3 (\pi 9460730472581000^{1.5})}{\frac{(3 \pi 9460730472581000^{1.5})(15 \times 2^{2.5})}{120 \times 2^{2.5}}} \right)^2 =$$

$$-8 + 4 \left( \frac{540^\circ 9460730472581000^{1.5}}{\frac{(15 \times 2^{2.5})(540^\circ 9460730472581000^{1.5})}{120 \times 2^{2.5}}} \right)^2$$

•

$$-8 + 4 \left( \frac{3 (\pi 9460730472581000^{1.5})}{\frac{(3 \pi 9460730472581000^{1.5})(15 \times 2^{2.5})}{120 \times 2^{2.5}}} \right)^2 =$$

$$-8 + 4 \left( -\frac{3 i \log(-1) 9460730472581000^{1.5}}{\frac{(15 \times 2^{2.5})(-3 i \log(-1) 9460730472581000^{1.5})}{120 \times 2^{2.5}}} \right)^2$$

•

$$-8 + 4 \left( \frac{3 (\pi 9460730472581000^{1.5})}{\frac{(3 \pi 9460730472581000^{1.5})(15 \times 2^{2.5})}{120 \times 2^{2.5}}} \right)^2 =$$

$$-8 + 4 \left( \frac{3 \cos^{-1}(-1) 9460730472581000^{1.5}}{\frac{(15 \times 2^{2.5})(3 \cos^{-1}(-1) 9460730472581000^{1.5})}{120 \times 2^{2.5}}} \right)^2$$

$\log(x)$  is the natural logarithm

$i$  is the imaginary unit

$\cos^{-1}(x)$  is the inverse cosine function

$$-16+[8((((((((3*\pi*(9460730472581000)^{1.5} / (((15* (2)^{2.5})))))))))) / (((((3*\pi*(9460730472581000)^{1.5} / (((120* (2)^{2.5}))))))))))^{2} ]$$

**Input:**

$$-16 + 8 \left( \frac{3 \pi \times \frac{9460730472581000^{1.5}}{15 \times 2^{2.5}}}{3 \pi \times \frac{9460730472581000^{1.5}}{120 \times 2^{2.5}}} \right)^2$$

**Result:**

496

496 that is the dimensions number of  $E_8 \times E_8$

**Alternative representations:**

$$-16 + 8 \left( \frac{3 (\pi 9460730472581000^{1.5})}{\frac{(3 \pi 9460730472581000^{1.5})(15 \times 2^{2.5})}{120 \times 2^{2.5}}} \right)^2 =$$

$$-16 + 8 \left( \frac{540^\circ 9460730472581000^{1.5}}{\frac{(15 \times 2^{2.5})(540^\circ 9460730472581000^{1.5})}{120 \times 2^{2.5}}} \right)^2$$

•

$$-16 + 8 \left( \frac{3 (\pi 9460730472581000^{1.5})}{\frac{(3 \pi 9460730472581000^{1.5})(15 \times 2^{2.5})}{120 \times 2^{2.5}}} \right)^2 =$$

$$-16 + 8 \left( \frac{3 i \log(-1) 9460730472581000^{1.5}}{\frac{(15 \times 2^{2.5})(-3 i \log(-1) 9460730472581000^{1.5})}{120 \times 2^{2.5}}} \right)^2$$

•

$$-16 + 8 \left( \frac{3 (\pi 9460730472581000^{1.5})}{\frac{(3 \pi 9460730472581000^{1.5})(15 \times 2^{2.5})}{120 \times 2^{2.5}}} \right)^2 =$$

$$-16 + 8 \left( \frac{3 \cos^{-1}(-1) 9460730472581000^{1.5}}{\frac{(15 \times 2^{2.5})(3 \cos^{-1}(-1) 9460730472581000^{1.5})}{120 \times 2^{2.5}}} \right)^2$$

$\log(x)$  is the natural logarithm

$i$  is the imaginary unit

$\cos^{-1}(x)$  is the inverse cosine function

## NOTE

This is the Ramanujan fundamental formula for obtain a beautiful and highly precise golden ratio:

$$\sqrt[5]{\left(\frac{1}{\frac{1}{32}(-1+\sqrt{5})^5+5e^{(-\sqrt{5}\pi)^5}} - \frac{11 \times 5e^{(-\sqrt{5}\pi)^5}}{2\left(\frac{1}{32}(-1+\sqrt{5})^5+5e^{(-\sqrt{5}\pi)^5}\right)} - \frac{5\sqrt{5} \times 5e^{(-\sqrt{5}\pi)^5}}{2\left(\frac{1}{32}(-1+\sqrt{5})^5+5e^{(-\sqrt{5}\pi)^5}\right)}\right)}$$

$$1/\left(\left(\frac{1}{32}(-1+\sqrt{5})^5+5e^{(-\sqrt{5}\pi)^5}\right)\right)$$

**Input:**

$$\frac{1}{\frac{1}{32}(-1+\sqrt{5})^5+5e^{(-\sqrt{5}\pi)^5}}$$

**Exact result:**

$$\frac{1}{\frac{1}{32}(\sqrt{5}-1)^5+5e^{-25\sqrt{5}\pi^5}}$$

**Decimal approximation:**

11.09016994374947424102293417182819058860154589902881431067...

11.09016994374947424102293417182819058860154589902881431067

$$(11 \times 5 \times (e^{(-\sqrt{5}\pi)^5})) / \left(2 \times \left(\frac{1}{32}(-1+\sqrt{5})^5+5e^{(-\sqrt{5}\pi)^5}\right)\right)$$

**Input:**

$$\frac{11 \times 5 e^{(-\sqrt{5}\pi)^5}}{2\left(\frac{1}{32}(-1+\sqrt{5})^5+5e^{(-\sqrt{5}\pi)^5}\right)}$$

**Exact result:**

$$\frac{55 e^{-25 \sqrt{5} \pi^5}}{2 \left( \frac{1}{32} (\sqrt{5} - 1)^5 + 5 e^{-25 \sqrt{5} \pi^5} \right)}$$

**Decimal approximation:**

$$9.99290225070718723070536304129457122742436976265255... \times 10^{-7428}$$

$$9.99290225070718723070536304129457122742436976265255 \times 10^{-7428}$$

$$(5\sqrt{5}) * 5 * (e^{((-sqrt{5}) * Pi)^5}) / (((2 * (((1/32 * (-1 + sqrt{5})^5 + 5 * (e^{((-sqrt{5}) * Pi)^5))))$$

**Input:**

$$\frac{5 \sqrt{5} \times 5 e^{(-\sqrt{5} \pi)^5}}{2 \left( \frac{1}{32} (-1 + \sqrt{5})^5 + 5 e^{(-\sqrt{5} \pi)^5} \right)}$$

**Exact result:**

$$\frac{25 \sqrt{5} e^{-25 \sqrt{5} \pi^5}}{2 \left( \frac{1}{32} (\sqrt{5} - 1)^5 + 5 e^{-25 \sqrt{5} \pi^5} \right)}$$

**Decimal approximation:**

$$1.01567312386781438874777576295646917898823529098784... \times 10^{-7427}$$

$$1.01567312386781438874777576295646917898823529098784 \times 10^{-7427}$$

**Input interpretation:**

$$\left( 1 / \left( \left( \frac{1}{32} (-1 + \sqrt{5})^5 + 5 e^{(-\sqrt{5} \pi)^5} \right) - \frac{9.99290225070718723070536304129457122742436976265255}{10^{7428}} - \frac{1.01567312386781438874777576295646917898823529098784}{10^{7427}} \right) \right)^{(1/5)}$$

**Result:**

$$1.618033988749894848204586834365638117720309179805762862135...$$

$$1.61803398.....$$

From the following ratio, we obtain:

$$9.9929022507071e-7428 / 1.0156731238678e-7427$$

**Input interpretation:**

$$\frac{9.9929022507071}{10^{7428}}$$

$$\frac{1.0156731238678}{10^{7427}}$$

**Result:**

0.983869910099912816158369150955437117342004992260298363449...

0.98386991...

0.983869910099912816158369150955437117342004992260298363449 \* 10<sup>3</sup>  
MeV = kg

**Input interpretation:**

convert

983.86991009991281615836915095543711734200499226029836345 MeV/c<sup>2</sup>  
to kilograms

983.86991... ≈ mass of f<sub>0</sub>(980) scalar meson

**Result:**

1.75390710272792181665793288101899419720767159485681648539 × 10<sup>-27</sup> kg  
(kilograms)

1.7539071027... \* 10<sup>-27</sup> kg

**Additional conversion:**

1.75390710272792181665793288101899419720767159485681648539 × 10<sup>-24</sup>  
grams

**Comparisons as mass:**

≈ 0.52 × deuteron mass (≈ 3.3 × 10<sup>-27</sup> kg)

≈ 0.55 × tau particle mass (≈ 3.2 × 10<sup>-27</sup> kg)

≈ proton mass (≈ 1.7 × 10<sup>-27</sup> kg)

**Comparison as mass of atom:**

≈ 1.1 × unified atomic mass unit (1 m<sub>u</sub>)

**Comparisons as mass of molecule:**

≈ ( 0.018 ≈ 1/55 ) × molecular mass of sodium chloride (≈ 58 u)

≈ ( 0.059 ≈ 1/17 ) × molecular mass of water (≈ 18 u)

≈ 0.52 × molecular mass of hydrogen gas (≈ 2 u)

**Corresponding quantities:**

Relativistic energy  $E$  from  $E = mc^2$ :

0.98 GeV (gigaelectronvolts)





**Result:**

1.618249079808970946303228449831390475115879495889544474480...

1.61824907...

And:

$$1/\sqrt{[1/(((((((4*1.962364415e+19)/(5*0.0864055^2))) * 1/(1.753907e-27) * \sqrt{[-(((6.996968e+49 * 4*Pi*(2.604292e-54)^3 - (2.604292e-54)^2))]) / ((6.67*10^-11)))])]]}$$

**Input interpretation:**

$$1 / \left( \sqrt{ \left( \frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{1.753907 \times 10^{-27}} \sqrt{ \frac{6.996968 \times 10^{49} \times 4\pi (2.604292 \times 10^{-54})^3 - (2.604292 \times 10^{-54})^2}{6.67 \times 10^{-11}} } \right) } \right)$$

**Result:**

0.617951842196039904836458307387143050439949386485928899149...

0.61795184...

From:

**Golden Ratio and a Ramanujan-Type Integral**

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Ramanujan discovered the following remarkable integral formula regarding  $R(q)$ , which is recorded in his "lost" notebook:

$$R(q) = \frac{\sqrt{5} - 1}{2} \exp \left( -\frac{1}{5} \int_q^1 \frac{(1-t)^5(1-t^2)^5(1-t^3)^5 \dots dt}{(1-t^5)(1-t^{10})(1-t^{15}) \dots t} \right) \tag{2}$$

This was first proved by Andrews [4]. See also Section 14.4 of [5] and Chapter 15 of [3]. This article is a pedagogical introduction to this remarkable identity. As a corollary, we will derive the following integral identity:

$$\ln(\sqrt{4\phi + 3} - \phi^2) = -\frac{1}{5} \int_{e^{-2\pi}}^1 \frac{(1-t)^5(1-t^2)^5(1-t^3)^5 \dots dt}{(1-t^5)(1-t^{10})(1-t^{15}) \dots t} \quad (3)$$

Here,  $\phi = (1 + \sqrt{5})/2$  is the Golden Ratio. The integrand in this equation is the same integrand that appears in Equation (2).

First we recall the differential version of Equation (2):

$$5q \frac{d}{dq} \ln R(q) = \prod_{n=1}^{\infty} \frac{(1-q^n)^5}{(1-q^{5n})} \quad (4)$$

This will allow us to find a formula analogous to Equation (2) in the next step.

In order to make our discussion self-contained and motivated, we will give some details on one of the known proofs, following the work of Dobbie [6] (as it illustrates many nice tricks in dealing with  $q$ -series).

Equation (4) is not easy to prove. One may ask, "How do we take the derivative of the *continued fraction*  $R(q)$  on the left-hand side of it?" What comes to rescue us is the following remarkable identity due to Rogers and Ramanujan (who discovered it independently):

$$R(q) = q^{1/5} \prod_{n=1}^{\infty} \frac{(1-q^{5n-1})(1-q^{5n-4})}{(1-q^{5n-2})(1-q^{5n-3})} \quad (5)$$

From (5):

$$R(q) = q^{1/5} \prod_{n=1}^{\infty} \frac{(1-q^{5n-1})(1-q^{5n-4})}{(1-q^{5n-2})(1-q^{5n-3})}$$

for  $q = e^{2\pi i}$  and  $q = 0.5$ , we obtain:

$$535.49165^{0.2} \prod_{n=1}^{1152} ((1-535.49165^{5n-1})*(1-535.49165^{5n-4})) / (((1-535.49165^{5n-2})*(1-535.49165^{5n-3}))), n=1..1152$$

**Input interpretation:**

$$535.49165^{0.2} \prod_{n=1}^{1152} (1 - 535.49165^{5n-1}) \times \frac{1 - 535.49165^{5n-4}}{(1 - 535.49165^{5n-2})(1 - 535.49165^{5n-3})}$$

**Result:**

3.50704

3.50704

$$(0.233 \times 2 - 0.0021 \times 2 - 0.00021 \times 2) * 535.49165^{0.2} \text{ product } ((1 - 535.49165^{(5n-1)}) * ((1 - 535.49165^{(5n-4)})) / (((1 - 535.49165^{(5n-2)}) * (1 - 535.49165^{(5n-3)}))))), n=1..1152$$

**Input interpretation:**

$$(0.233 \times 2 - 0.0021 \times 2 - 0.00021 \times 2) \times 535.49165^{0.2} \prod_{n=1}^{1152} (1 - 535.49165^{5n-1}) \times \frac{1 - 535.49165^{5n-4}}{(1 - 535.49165^{5n-2})(1 - 535.49165^{5n-3})}$$

**Result:**

1.61808

1.61808

And:

$$[ 0.5^{0.2} \text{ product } ((1 - 0.5^{(5n-1)}) * ((1 - 0.5^{(5n-4)})) / (((1 - 0.5^{(5n-2)}) * (1 - 0.5^{(5n-3)}))))), n=1..4096]$$

**Input interpretation:**

$$0.5^{0.2} \prod_{n=1}^{4096} (1 - 0.5^{5n-1}) \times \frac{1 - 0.5^{5n-4}}{(1 - 0.5^{5n-2})(1 - 0.5^{5n-3})}$$

**Result:**

0.618018

0.618018

$$1/[ 0.5^{0.2} \text{ product } ((1 - 0.5^{(5n-1)}) * ((1 - 0.5^{(5n-4)})) / (((1 - 0.5^{(5n-2)}) * (1 - 0.5^{(5n-3)}))))), n=1..4096]$$

**Input interpretation:**

$$\frac{1}{0.5^{0.2} \prod_{n=1}^{4096} (1 - 0.5^{5n-1}) \times \frac{1 - 0.5^{5n-4}}{(1 - 0.5^{5n-2})(1 - 0.5^{5n-3})}}$$

**Result:**

1.61807

1.61807

From:

$$\ln\left(\sqrt{4\phi+3}-\phi^2\right)=-\frac{1}{5} \int_{e^{-2\pi}}^1 \frac{(1-t)^5(1-t^2)^5(1-t^3)^5 \dots dt}{(1-t^5)(1-t^{10})(1-t^{15}) \dots t} \quad (3)$$

we obtain:

$$\ln(\sqrt{4\phi+3}-\phi^2)$$

**Input:**

$$\log\left(\sqrt{4\phi+3}-\phi^2\right)$$

$\log(x)$  is the natural logarithm

$\phi$  is the golden ratio

**Decimal approximation:**

-0.77729093110436136528743433002266396133373091418023686588...

-0.777290931...

**Property:**

$\log\left(-\phi^2+\sqrt{3+4\phi}\right)$  is a transcendental number

**Alternate forms:**

$$\log\left(\frac{1}{2}\left(2\sqrt{4\phi+3}-3-\sqrt{5}\right)\right)$$

$$\log\left(2\sqrt{4\phi+3}-3-\sqrt{5}\right)-\log(2)$$

$$\log\left(\frac{1}{2}\left(-3-\sqrt{5}+2\sqrt{5+2\sqrt{5}}\right)\right)$$

**Alternative representations:**

$$\log\left(\sqrt{4\phi+3}-\phi^2\right)=\log_e\left(-\phi^2+\sqrt{3+4\phi}\right)$$

•

$$\log\left(\sqrt{4\phi+3}-\phi^2\right)=\log(a)\log_a\left(-\phi^2+\sqrt{3+4\phi}\right)$$

•

$$\log(\sqrt{4\phi+3} - \phi^2) = -\text{Li}_1(1 + \phi^2 - \sqrt{3+4\phi})$$

$\log_b(x)$  is the base- $b$  logarithm

$\text{Li}_n(x)$  is the polylogarithm function

### Series representations:

$$\log(\sqrt{4\phi+3} - \phi^2) = -\sum_{k=1}^{\infty} \frac{(-1)^k (-1 - \phi^2 + \sqrt{3+4\phi})^k}{k}$$

•

$$\log(\sqrt{4\phi+3} - \phi^2) = 2i\pi \left\lfloor \frac{\arg(-\phi^2 + \sqrt{3+4\phi} - x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (-\phi^2 + \sqrt{3+4\phi} - x)^k x^{-k}}{k} \quad \text{for } x < 0$$

•

$$\log(\sqrt{4\phi+3} - \phi^2) = 2i\pi \left\lfloor \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right\rfloor + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (-\phi^2 + \sqrt{3+4\phi} - z_0)^k z_0^{-k}}{k}$$

$\arg(z)$  is the complex argument

$\lfloor x \rfloor$  is the floor function

### Integral representation:

$$\log(\sqrt{4\phi+3} - \phi^2) = \int_1^{-\phi^2 + \sqrt{3+4\phi}} \frac{1}{t} dt$$

From:

$$R(q) = \frac{\sqrt{5}-1}{2} \exp\left(-\frac{1}{5} \int_q^1 \frac{(1-t)^5(1-t^2)^5(1-t^3)^5 \dots dt}{(1-t^5)(1-t^{10})(1-t^{15}) \dots t}\right) \quad (2)$$

and

$$\ln\left(\sqrt{4\phi+3}-\phi^2\right) = -\frac{1}{5} \int_{e^{-2\pi}}^1 \frac{(1-t)^5(1-t^2)^5(1-t^3)^5 \dots dt}{(1-t^5)(1-t^{10})(1-t^{15}) \dots t} \quad (3)$$

We obtain:

**Input:**

$$\frac{1}{\phi} \exp\left(\log\left(\sqrt{4\phi+3}-\phi^2\right)\right)$$

log(x) is the natural logarithm

$\phi$  is the golden ratio

**Exact result:**

$$\frac{\sqrt{4\phi+3}-\phi^2}{\phi}$$

**Decimal approximation:**

0.284079043840412296028291832393126169091088088445737582759...

0.284079043...

**Alternate forms:**

$$\frac{1}{2} \left( -1 - \sqrt{5} + \sqrt{2(5 + \sqrt{5})} \right)$$

•

$$\frac{2 \left( \sqrt{3+2(1+\sqrt{5})} - \frac{1}{4}(1+\sqrt{5})^2 \right)}{1+\sqrt{5}}$$

•

$$-\frac{3}{1+\sqrt{5}} - \frac{\sqrt{5}}{1+\sqrt{5}} + \frac{2\sqrt{3+2(1+\sqrt{5})}}{1+\sqrt{5}}$$

**Alternative representations:**

$$\frac{\exp\left(\log\left(\sqrt{4\phi+3}-\phi^2\right)\right)}{\phi} = \frac{\exp\left(\log_e\left(-\phi^2+\sqrt{3+4\phi}\right)\right)}{\phi}$$

•

$$\frac{\exp\left(\log\left(\sqrt{4\phi+3}-\phi^2\right)\right)}{\phi} = \frac{\exp\left(\log(a)\log_a\left(-\phi^2+\sqrt{3+4\phi}\right)\right)}{\phi}$$

$\log_b(x)$  is the base- $b$  logarithm

**Series representation:**

$$\frac{\exp\left(\log\left(\sqrt{4\phi+3}-\phi^2\right)\right)}{\phi} = \frac{\exp\left(-\sum_{k=1}^{\infty} \frac{(-1)^k \left(-1-\phi^2+\sqrt{3+4\phi}\right)^k}{k}\right)}{\phi}$$

**Integral representation:**

$$\frac{\exp\left(\log\left(\sqrt{4\phi+3}-\phi^2\right)\right)}{\phi} = \frac{\exp\left(\int_1^{-\phi^2+\sqrt{3+4\phi}} \frac{1}{t} dt\right)}{\phi}$$

$\ln\left(\left(\left(\left(\left(\left(\left(\left(\left(\left(\left(\left(\left(\left(\left(\left(\frac{1}{\text{golden ratio}}\right) * \exp\left(\left(\ln\left(\left(\sqrt{4*\text{golden ratio}+3}\right) - (\text{golden ratio})^2\right)\right)\right)\right)\right)\right)\right)\right)\right)\right)\right)\right)\right)\right)\right)\right)\right)\right)$

**Input:**

$$\log\left(\frac{1}{\phi} \exp\left(\log\left(\sqrt{4\phi+3}-\phi^2\right)\right)\right)$$

$\log(x)$  is the natural logarithm

$\phi$  is the golden ratio

**Exact result:**

$$\log\left(\frac{\sqrt{4\phi+3}-\phi^2}{\phi}\right)$$

### Decimal approximation:

-1.25850275616396481278519324344703238446891524856589738554...

-1.2585027...

### Property:

$\log\left(\frac{-\phi^2 + \sqrt{3+4\phi}}{\phi}\right)$  is a transcendental number

### Alternate forms:

$$\log\left(\frac{1}{2}\left(-1 - \sqrt{5} + \sqrt{2(5 + \sqrt{5})}\right)\right)$$

- $$\log\left(\frac{2\sqrt{4\phi+3} - 3 - \sqrt{5}}{1 + \sqrt{5}}\right)$$

$$\log(2\sqrt{4\phi+3} - 3 - \sqrt{5}) - \log(1 + \sqrt{5})$$

### Alternative representations:

$$\log\left(\frac{\exp(\log(\sqrt{4\phi+3} - \phi^2))}{\phi}\right) = \log_e\left(\frac{\exp(\log(-\phi^2 + \sqrt{3+4\phi}))}{\phi}\right)$$

- $$\log\left(\frac{\exp(\log(\sqrt{4\phi+3} - \phi^2))}{\phi}\right) = \log(a) \log_a\left(\frac{\exp(\log(-\phi^2 + \sqrt{3+4\phi}))}{\phi}\right)$$

$\log_b(x)$  is the base- $b$  logarithm

### Series representations:

- $$\log\left(\frac{\exp(\log(\sqrt{4\phi+3} - \phi^2))}{\phi}\right) = -\sum_{k=1}^{\infty} \frac{(-1)^k \left(-1 + \frac{-\phi^2 + \sqrt{3+4\phi}}{\phi}\right)^k}{k}$$



$$\log\left(\frac{\exp(\log(\sqrt{4\phi+3-\phi^2}))}{\phi}\right) = 2i\pi \left\lfloor \frac{\arg\left(\frac{-\phi^2+\sqrt{3+4\phi}}{\phi} - x\right)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{-\phi^2+\sqrt{3+4\phi}}{\phi} - x\right)^k x^{-k}}{k} \quad \text{for } x < 0$$

$$\log\left(\frac{\exp(\log(\sqrt{4\phi+3-\phi^2}))}{\phi}\right) = 2i\pi \left\lfloor \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right\rfloor + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{-\phi^2+\sqrt{3+4\phi}}{\phi} - z_0\right)^k z_0^{-k}}{k}$$

$\arg(z)$  is the complex argument

$\lfloor x \rfloor$  is the floor function

### Integral representation:

$$\log\left(\frac{\exp(\log(\sqrt{4\phi+3-\phi^2}))}{\phi}\right) = \int_1^{\frac{-\phi^2+\sqrt{3+4\phi}}{\phi}} \frac{1}{t} dt$$

$-5/2 * \ln(((((((1/(\text{golden ratio}) * \exp((((\ln((((\text{sqrt}(4*\text{golden ratio}+3) - (\text{golden ratio})^2))))))))))))))))))$

### Input:

$$-\frac{5}{2} \log\left(\frac{1}{\phi} \exp(\log(\sqrt{4\phi+3-\phi^2}))\right)$$

$\log(x)$  is the natural logarithm

$\phi$  is the golden ratio

**Exact result:**

$$-\frac{5}{2} \log \left( \frac{\sqrt{4\phi + 3} - \phi^2}{\phi} \right)$$

**Decimal approximation:**

3.146256890409912031962983108617580961172288121414743463855...

$$3.14625689\dots \approx \pi$$

**Property:**

$$-\frac{5}{2} \log \left( \frac{-\phi^2 + \sqrt{3 + 4\phi}}{\phi} \right) \text{ is a transcendental number}$$

**Alternate forms:**

$$\frac{5}{2} \log \left( \frac{\phi}{\sqrt{4\phi + 3} - \phi^2} \right)$$

•

$$-\frac{5}{2} \log \left( \frac{1}{2} \left( -1 - \sqrt{5} + \sqrt{2(5 + \sqrt{5})} \right) \right)$$

•

$$-\frac{5}{2} \left( \log(2\sqrt{4\phi + 3} - 3 - \sqrt{5}) - \log(1 + \sqrt{5}) \right)$$

**Alternative representations:**

$$\frac{1}{2} \log \left( \frac{\exp(\log(\sqrt{4\phi + 3} - \phi^2))}{\phi} \right)_{(-5)} = -\frac{5}{2} \log_e \left( \frac{\exp(\log(-\phi^2 + \sqrt{3 + 4\phi}))}{\phi} \right)$$

•

$$\frac{1}{2} \log \left( \frac{\exp(\log(\sqrt{4\phi + 3} - \phi^2))}{\phi} \right)_{(-5)} = -\frac{5}{2} \log(a) \log_a \left( \frac{\exp(\log(-\phi^2 + \sqrt{3 + 4\phi}))}{\phi} \right)$$

$\log_b(x)$  is the base- $b$  logarithm

**Series representations:**

$$\frac{1}{2} \log \left( \frac{\exp(\log(\sqrt{4\phi+3} - \phi^2))}{\phi} \right) (-5) = \frac{5}{2} \sum_{k=1}^{\infty} \frac{(-1)^k \left( -1 + \frac{-\phi^2 + \sqrt{3+4\phi}}{\phi} \right)^k}{k}$$

$$\frac{1}{2} \log \left( \frac{\exp(\log(\sqrt{4\phi+3} - \phi^2))}{\phi} \right) (-5) = -5i\pi \left\lfloor \frac{\arg \left( \frac{-\phi^2 + \sqrt{3+4\phi}}{\phi} - x \right)}{2\pi} \right\rfloor - \frac{5 \log(x)}{2} + \frac{5}{2} \sum_{k=1}^{\infty} \frac{(-1)^k \left( \frac{-\phi^2 + \sqrt{3+4\phi}}{\phi} - x \right)^k}{k} x^{-k} \quad \text{for } x < 0$$

$$\frac{1}{2} \log \left( \frac{\exp(\log(\sqrt{4\phi+3} - \phi^2))}{\phi} \right) (-5) = -5i\pi \left\lfloor \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right\rfloor - \frac{5 \log(z_0)}{2} + \frac{5}{2} \sum_{k=1}^{\infty} \frac{(-1)^k \left( \frac{-\phi^2 + \sqrt{3+4\phi}}{\phi} - z_0 \right)^k}{k} z_0^{-k}$$

$\arg(z)$  is the complex argument

$\lfloor x \rfloor$  is the floor function

### Integral representation:

$$\frac{1}{2} \log \left( \frac{\exp(\log(\sqrt{4\phi+3} - \phi^2))}{\phi} \right) (-5) = -\frac{5}{2} \int_1^{\frac{-\phi^2 + \sqrt{3+4\phi}}{\phi}} \frac{1}{t} dt$$

### Input:

$$21 \times \frac{2}{10^3} + 1 - \frac{1}{2} \log \left( \frac{1}{\phi} \exp(\log(\sqrt{4\phi+3} - \phi^2)) \right)$$

$\log(x)$  is the natural logarithm

$\phi$  is the golden ratio

**Exact result:**

$$\frac{521}{500} - \frac{1}{2} \log \left( \frac{\sqrt{4\phi + 3} - \phi^2}{\phi} \right)$$

**Decimal approximation:**

1.671251378081982406392596621723516192234457624282948692771...

1.671251378...

We note that 1.671251378... is a result practically equal to the value of the formula:

$$m_{p'} = 2 \times \frac{\eta}{R} m_P = 1.6714213 \times 10^{-24} \text{ gm}$$

that is the holographic proton mass

**Property:**

$$\frac{521}{500} - \frac{1}{2} \log \left( \frac{-\phi^2 + \sqrt{3 + 4\phi}}{\phi} \right) \text{ is a transcendental number}$$

**Alternate forms:**

$$\frac{1}{2} \log \left( \frac{\phi}{\sqrt{4\phi + 3} - \phi^2} \right) + \frac{521}{500}$$

•

$$\frac{1}{500} \left( 521 - 250 \log \left( \frac{\sqrt{4\phi + 3} - \phi^2}{\phi} \right) \right)$$

•

$$\frac{521}{500} - \frac{1}{2} \log \left( \frac{1}{2} \left( -1 - \sqrt{5} + \sqrt{2(5 + \sqrt{5})} \right) \right)$$

**Alternative representations:**

$$\frac{21 \times 2}{10^3} + 1 - \frac{1}{2} \log \left( \frac{\exp(\log(\sqrt{4\phi + 3} - \phi^2))}{\phi} \right) =$$

$$1 - \frac{1}{2} \log_e \left( \frac{\exp(\log(-\phi^2 + \sqrt{3 + 4\phi}))}{\phi} \right) + \frac{42}{10^3}$$

$$\frac{21 \times 2}{10^3} + 1 - \frac{1}{2} \log \left( \frac{\exp(\log(\sqrt{4\phi + 3} - \phi^2))}{\phi} \right) =$$

$$1 - \frac{1}{2} \log(a) \log_a \left( \frac{\exp(\log(-\phi^2 + \sqrt{3 + 4\phi}))}{\phi} \right) + \frac{42}{10^3}$$

$\log_b(x)$  is the base- $b$  logarithm

### Series representations:

$$\frac{21 \times 2}{10^3} + 1 - \frac{1}{2} \log \left( \frac{\exp(\log(\sqrt{4\phi + 3} - \phi^2))}{\phi} \right) = \frac{521}{500} + \frac{1}{2} \sum_{k=1}^{\infty} \frac{(-1)^k \left( -1 + \frac{-\phi^2 + \sqrt{3 + 4\phi}}{\phi} \right)^k}{k}$$

$$\frac{21 \times 2}{10^3} + 1 - \frac{1}{2} \log \left( \frac{\exp(\log(\sqrt{4\phi + 3} - \phi^2))}{\phi} \right) = \frac{521}{500} -$$

$$i\pi \left[ \frac{\arg \left( \frac{-\phi^2 + \sqrt{3 + 4\phi}}{\phi} - x \right)}{2\pi} \right] - \frac{\log(x)}{2} + \frac{1}{2} \sum_{k=1}^{\infty} \frac{(-1)^k \left( \frac{-\phi^2 + \sqrt{3 + 4\phi}}{\phi} - x \right)^k x^{-k}}{k} \quad \text{for } x < 0$$

$$\frac{21 \times 2}{10^3} + 1 - \frac{1}{2} \log \left( \frac{\exp(\log(\sqrt{4\phi + 3} - \phi^2))}{\phi} \right) =$$

$$\frac{521}{500} - i\pi \left[ \frac{\pi - \arg \left( \frac{1}{z_0} \right) - \arg(z_0)}{2\pi} \right] - \frac{\log(z_0)}{2} + \frac{1}{2} \sum_{k=1}^{\infty} \frac{(-1)^k \left( \frac{-\phi^2 + \sqrt{3 + 4\phi}}{\phi} - z_0 \right)^k z_0^{-k}}{k}$$

$\arg(z)$  is the complex argument

$\lfloor x \rfloor$  is the floor function

**Integral representation:**

$$\frac{21 \times 2}{10^3} + 1 - \frac{1}{2} \log \left( \frac{\exp(\log(\sqrt{4\phi + 3} - \phi^2))}{\phi} \right) = \frac{521}{500} - \frac{1}{2} \int_1^{\frac{-\phi^2 + \sqrt{3+4\phi}}{\phi}} \frac{1}{t} dt$$

From:

$$R(q) = \frac{\sqrt{5}-1}{2} \exp \left( -\frac{1}{5} \int_q^1 \frac{(1-t)^5(1-t^2)^5(1-t^3)^5 \dots dt}{(1-t^5)(1-t^{10})(1-t^{15}) \dots t} \right) \tag{2}$$

and

$$5q \frac{d}{dq} \ln R(q) = \prod_{n=1}^{\infty} \frac{(1-q^n)^5}{(1-q^{5n})} \tag{4}$$

We have:

**Input:**

$$5 \log \left( \frac{1}{\phi} \exp(\log(\sqrt{4\phi + 3} - \phi^2)) \right)$$

$\log(x)$  is the natural logarithm

$\phi$  is the golden ratio

**Exact result:**

$$5 \log \left( \frac{\sqrt{4\phi + 3} - \phi^2}{\phi} \right)$$

**Decimal approximation:**

-6.29251378081982406392596621723516192234457624282948692771...

-6.29251378...  $\approx -2\pi$

**Property:**

$5 \log\left(\frac{-\phi^2 + \sqrt{3 + 4\phi}}{\phi}\right)$  is a transcendental number

**Alternate forms:**

$$5 \log\left(\frac{1}{2}\left(-1 - \sqrt{5} + \sqrt{2(5 + \sqrt{5})}\right)\right)$$

•

$$5\left(\log\left(2\sqrt{4\phi + 3} - 3 - \sqrt{5}\right) - \log\left(1 + \sqrt{5}\right)\right)$$

$$5 \log\left(2\sqrt{4\phi + 3} - 3 - \sqrt{5}\right) - 5 \log\left(1 + \sqrt{5}\right)$$

**Alternative representations:**

$$5 \log\left(\frac{\exp\left(\log\left(\sqrt{4\phi + 3} - \phi^2\right)\right)}{\phi}\right) = 5 \log_e\left(\frac{\exp\left(\log\left(-\phi^2 + \sqrt{3 + 4\phi}\right)\right)}{\phi}\right)$$

•

$$5 \log\left(\frac{\exp\left(\log\left(\sqrt{4\phi + 3} - \phi^2\right)\right)}{\phi}\right) = 5 \log(a) \log_a\left(\frac{\exp\left(\log\left(-\phi^2 + \sqrt{3 + 4\phi}\right)\right)}{\phi}\right)$$

$\log_b(x)$  is the base- $b$  logarithm

**Series representations:**

$$5 \log\left(\frac{\exp\left(\log\left(\sqrt{4\phi + 3} - \phi^2\right)\right)}{\phi}\right) = -5 \sum_{k=1}^{\infty} \frac{(-1)^k \left(-1 + \frac{-\phi^2 + \sqrt{3 + 4\phi}}{\phi}\right)^k}{k}$$

•

$$5 \log \left( \frac{\exp(\log(\sqrt{4\phi + 3 - \phi^2}))}{\phi} \right) =$$

$$10 i \pi \left[ \frac{\arg\left(\frac{-\phi^2 + \sqrt{3+4\phi}}{\phi} - x\right)}{2\pi} \right] + 5 \log(x) - 5 \sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{-\phi^2 + \sqrt{3+4\phi}}{\phi} - x\right)^k x^{-k}}{k} \quad \text{for } x < 0$$

$$5 \log \left( \frac{\exp(\log(\sqrt{4\phi + 3 - \phi^2}))}{\phi} \right) =$$

$$10 i \pi \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] + 5 \log(z_0) - 5 \sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{-\phi^2 + \sqrt{3+4\phi}}{\phi} - z_0\right)^k z_0^{-k}}{k}$$

$\arg(z)$  is the complex argument

$[x]$  is the floor function

### Integral representation:

$$5 \log \left( \frac{\exp(\log(\sqrt{4\phi + 3 - \phi^2}))}{\phi} \right) = 5 \int_1^{\frac{-\phi^2 + \sqrt{3+4\phi}}{\phi}} \frac{1}{t} dt$$

From

$$5q \frac{d}{dq} \ln R(q) = \prod_{n=1}^{\infty} \frac{(1 - q^n)^5}{(1 - q^{5n})} \quad (4)$$

from the right hand side

$$\prod_{n=1}^{\infty} \frac{(1 - q^n)^5}{(1 - q^{5n})}$$

we obtain:



product  $((1-0.5^n)^5 / ((1-0.5^{5n})), n=1..infinity$

**Input interpretation:**

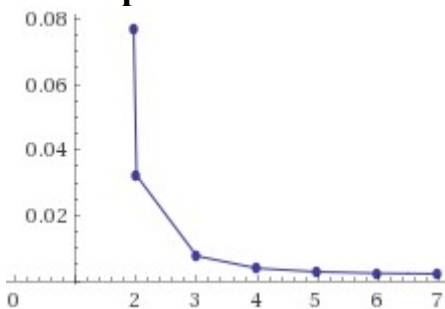
$$\prod_{n=1}^{\infty} \frac{(1 - 0.5^n)^5}{1 - 0.5^{5n}}$$

**Infinite product:**

$$\prod_{n=1}^{\infty} \frac{(1 - 0.5^n)^5}{1 - 0.5^{5n}} = 0.00207549991045000$$

0.0020754999...

**Partial products:**



**Partial product formula:**

$$\prod_{n=1}^m \frac{(1 - 0.5^n)^5}{1 - 0.5^{5n}} = \frac{(5((2; 2)_m)^4) / ((-0.809017 - 0.587785 i; 2)_{m+1} (-0.809017 + 0.587785 i; 2)_{m+1})}{(0.309017 - 0.951057 i; 2)_{m+1} (0.309017 + 0.951057 i; 2)_{m+1}}$$

$(a; q)_n$  gives the q-Pochhammer symbol

Integrating the differential equation (4) gives

$$R(q) = A \exp \left( -\frac{1}{5} \int_q^{q^*} \prod_{n=1}^{\infty} \frac{(1 - t^n)^5}{(1 - t^{5n})} \cdot \frac{dt}{t} \right) \tag{22}$$

we obtain:

$$\exp((( -1/5 * \text{integrate [product } ((1-0.5^n)^5 / ((1-0.5^{5n})), n=1..infinity ]]))$$

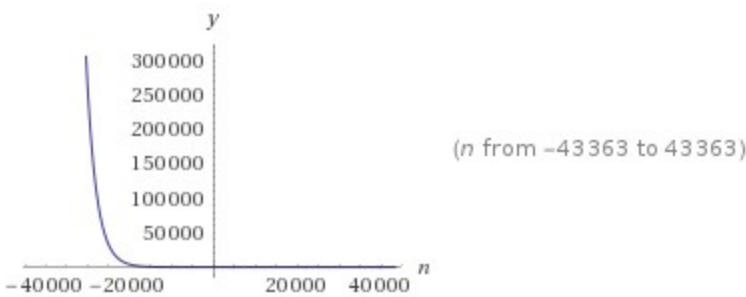
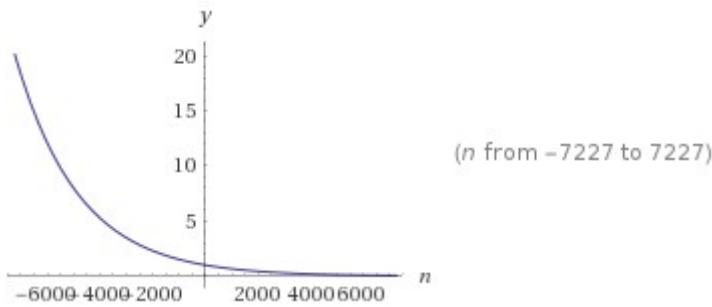
**Input interpretation:**

$$\exp\left(-\frac{1}{5} \int \left(\prod_{n=1}^{\infty} \frac{(1-0.5^n)^5}{1-0.5^{5n}}\right) dn\right)$$

**Result:**

$$e^{-0.000415099982089999n}$$

**Plots:**



**Values:**

n	1	2	3	4	5
$e^{-0.000415099982089999n}$	0.999584: 9861599: 87962	0.999170: 1445564: 63326	0.998755: 4751179: 45565	0.998340: 9777729: 8382	0.997926: 6524501: 5688

**Series expansion of the integral at n = 0:**

$$1 - 0.000415099982089999n + 8.6153997565559 \times 10^{-8} n^2 - 1.19208409488151 \times 10^{-11} n^3 + 1.23708521608772 \times 10^{-15} n^4 + O(n^5)$$

(Taylor series)

Big-O notation »

**Indefinite integral assuming all variables are real:**

$$-2409.05816224099 e^{-0.000415099982089999n} + \text{constant}$$

where  $A = R(q^*)$ .

Let us set  $q^* = 1$ . As  $q \rightarrow 1^-$ , we have

$$R(q) \rightarrow \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \dots}}} = \frac{1}{\phi}$$

Hence  $A = 1/\phi$ . This gives Equation (2).

From  $1/\phi = 0.61803398... \approx 0.618018$ , we obtain:

$$0.618018 * \exp\left(\left(-\frac{1}{5} * \int \left[ \prod_{n=1}^{\infty} \frac{(1-0.5^n)^5}{(1-0.5^{5n})} \right] dn\right)\right)$$

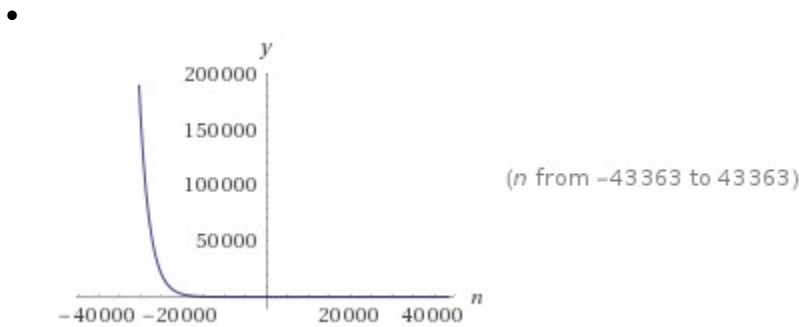
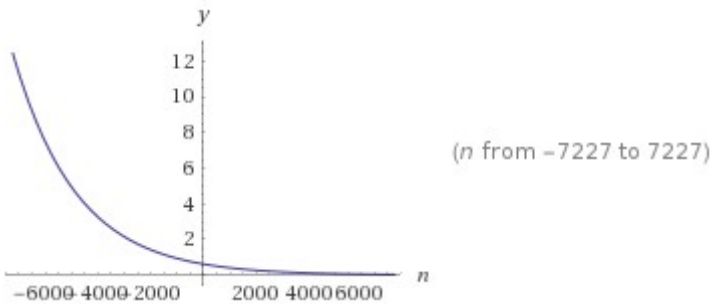
**Input interpretation:**

$$0.618018 \exp\left(-\frac{1}{5} \int \left(\prod_{n=1}^{\infty} \frac{(1-0.5^n)^5}{1-0.5^{5n}}\right) dn\right)$$

**Result:**

$$0.618018 e^{-0.000415099982089999n}$$

**Plots:**



**Values:**

$n$	1	2	3	4	5
$0.618018 e^{-0.000415099982089999n}$	0.617762	0.617505	0.617249	0.616993	0.616737

that are the various results

**Alternate form assuming  $n$  is real:**

$$0.618018 e^{-0.000415099982089999n} + 0$$

**Series expansion of the integral at  $n = 0$ :**

$$0.618018 - 0.000256539 n + 5.32447 \times 10^{-8} n^2 - 7.36729 \times 10^{-12} n^3 + 7.64541 \times 10^{-16} n^4 + O(n^5)$$

(Taylor series)

Big-O notation »

**Indefinite integral assuming all variables are real:**

$$-1488.84 e^{-0.000415099982089999n} + \text{constant}$$

From

$$\frac{1}{\phi} \exp\left(\log\left(\sqrt{4\phi + 3} - \phi^2\right)\right) =$$

$$= 0.284079043840412296028291832393126169091088088445737582759\dots$$

With this understood, we let  $q^* = e^{-2\pi}$  in Equation (22), and we have (with Equation (23)),

$$R(q) = (\sqrt{2 + \phi} - \phi) \exp\left(-\frac{1}{5} \int_q^{e^{-2\pi}} \prod_{n=1}^{\infty} \frac{(1 - t^n)^5}{(1 - t^{5n})} \cdot \frac{dt}{t}\right) \tag{24}$$

Where  $(\sqrt{2 + \phi} - \phi) = 0,2840790502902611$

we obtain:

$$0.284079050 \exp\left(\left(-\frac{1}{5} \int_q^{e^{-2\pi}} \prod_{n=1}^{\infty} \frac{(1 - t^n)^5}{(1 - t^{5n})} \cdot \frac{dt}{t}\right)\right)$$

**Input interpretation:**

$$0.284079050 \exp\left(-\frac{t}{5} \int \left(\prod_{n=1}^{\infty} \frac{(1-0.5^n)^5}{1-0.5^{5n}}\right) dn\right)$$

**Result:**

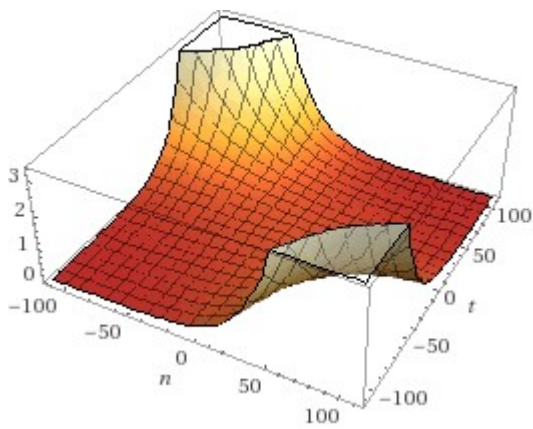
$$0.284079 e^{-0.000415099982089999 n t}$$

**Values:**

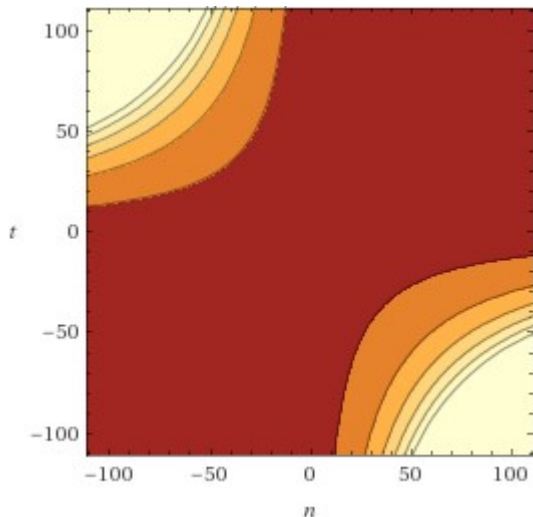
$n$	
0	0.284079
1	$0.284079 e^{-0.000415099982089999 t}$
2	$0.284079 e^{-0.000830199964179999 t}$
3	$0.284079 e^{-0.001245299946269998 t}$

that are the various results

**3D plot:**



• **Contour plot:**



- **Alternate form assuming n and t are real:**

$$0.284079 e^{-0.000415099982089999nt} + 0$$

- **Series expansion of the integral at t = 0:**

$$0.284079 - 0.000117921 nt + 2.44745 \times 10^{-8} n^2 t^2 - 3.38646 \times 10^{-12} n^3 t^3 + 3.5143 \times 10^{-16} n^4 t^4 + O(t^5)$$

(Taylor series)

Big-O notation »

- **Indefinite integral assuming all variables are real:**

$$-\frac{684.363 e^{-0.000415099982089999nt}}{n} + \text{constant}$$

$$0.284079050 \exp\left(\left(-\frac{1}{5} \int \left[ \prod_{n=1}^{\infty} \frac{(1-0.5^n)^5}{1-0.5^{5n}} \right] dn\right) \times 0.5\right)$$

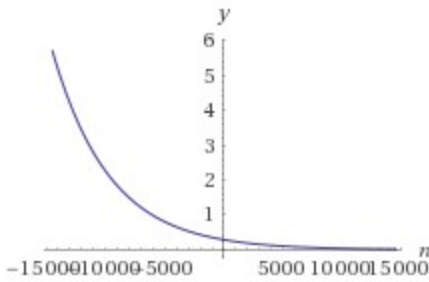
**Input interpretation:**

$$0.284079050 \exp\left(-\frac{1}{5} \left( \int \left( \prod_{n=1}^{\infty} \frac{(1-0.5^n)^5}{1-0.5^{5n}} \right) dn \right) \times 0.5 \right)$$

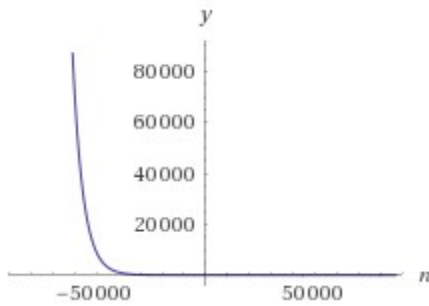
**Result:**

$$0.284079 e^{-0.00020755 n}$$

**Plots:**



(n from -14454 to 14454)



(n from -86726 to 86726)

**Values:**

n	1	2	3	4	5
$0.284079 e^{-0.00020755 n}$	0.28402	0.283961	0.283902	0.283843	0.283784

that are the various results

**Alternate form assuming n is real:**

$$0.284079 e^{-0.00020755 n} + 0$$

**Series expansion of the integral at n = 0:**

$$0.284079 - 0.0000589606 n + 6.11864 \times 10^{-9} n^2 - 4.23308 \times 10^{-13} n^3 + 2.19644 \times 10^{-17} n^4 + O(n^5)$$

(Taylor series)

Big-O notation »

**Indefinite integral assuming all variables are real:**

$$-1368.73 e^{-0.00020755 n} + \text{constant}$$

The sum of the various results is:

$$0,283784 + 0,283843 + 0,283902 + 0,283961 + 0,28402 = 1,41951;$$

The mean is: 0,283902

(Note that  $1.41951 - 0.777290931$ , that is the previous result, is equal to  $0.642219069$ , that  $+ 1 = 1.642219069$ )

And, we obtain again a good approximation to the golden ratio:

$$-\left(\frac{1}{\frac{1}{2} \log(0.283902)} - \frac{3}{10^2}\right)$$

**Input interpretation:**

$$-\left(\frac{1}{\frac{1}{2} \log(0.283902)} - \frac{3}{10^2}\right)$$

$\log(x)$  is the natural logarithm

**Result:**

1.618403169111213547862759173153163704864818705034231246255...

1.618403169...

This result is a very good approximation to the value of the golden ratio  
1,618033988749...

**Alternative representations:**

$$-\left(\frac{1}{\frac{\log(0.283902)}{2}} - \frac{3}{10^2}\right) = -\frac{1}{\frac{\log_e(0.283902)}{2}} + \frac{3}{10^2}$$

•

$$-\left(\frac{1}{\frac{\log(0.283902)}{2}} - \frac{3}{10^2}\right) = -\frac{1}{\coth^{-1}\left(-\frac{1.2839}{0.716098}\right)} + \frac{3}{10^2}$$

•

$$-\left(\frac{1}{\frac{\log(0.283902)}{2}} - \frac{3}{10^2}\right) = -\frac{1}{\tanh^{-1}\left(-\frac{0.716098}{1.2839}\right)} + \frac{3}{10^2}$$

$\log_b(x)$  is the base- $b$  logarithm



$\coth^{-1}(x)$  is the inverse hyperbolic cotangent function

$\tanh^{-1}(x)$  is the inverse hyperbolic tangent function

**Series representations:**

$$-\left(\frac{1}{\frac{\log(0.283902)}{2}} - \frac{3}{10^2}\right) = \frac{3}{100} + \frac{2}{\sum_{k=1}^{\infty} \frac{(-1)^k (-0.716098)^k}{k}}$$

- $$-\left(\frac{1}{\frac{\log(0.283902)}{2}} - \frac{3}{10^2}\right) = \frac{3}{100} - \frac{2}{2i\pi \left\lfloor \frac{\arg(0.283902-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (0.283902-x)^k x^{-k}}{k}}$$
 for  $x < 0$

- $$-\left(\frac{1}{\frac{\log(0.283902)}{2}} - \frac{3}{10^2}\right) = \frac{3}{100} - \frac{2}{\log(z_0) + \left\lfloor \frac{\arg(0.283902-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0)\right) - \sum_{k=1}^{\infty} \frac{(-1)^k (0.283902-z_0)^k z_0^{-k}}{k}}$$

$\arg(z)$  is the complex argument

$\lfloor x \rfloor$  is the floor function

$i$  is the imaginary unit

**Integral representation:**

$$-\left(\frac{1}{\frac{\log(0.283902)}{2}} - \frac{3}{10^2}\right) = \frac{3}{100} - \int_1^{0.283902} \frac{1}{t} dt$$

$$-((((2/((\ln(0.283902)))))-89/10^3+(2*3)/10^3))))$$

**Input interpretation:**

$$-\left(\frac{2}{\log(0.283902)} - \frac{89}{10^3} + \frac{2 \times 3}{10^3}\right)$$

$\log(x)$  is the natural logarithm

**Result:**

1.671403169111213547862759173153163704864818705034231246255...

1.671403169...

We note that 1.671403169... is a result practically equal to the value of the formula:

$$m_{p'} = 2 \times \frac{\eta}{R} m_p = 1.6714213 \times 10^{-2} \text{ gm}$$

that is the holographic proton mass

**Alternative representations:**

$$-\left(\frac{2}{\log(0.283902)} - \frac{89}{10^3} + \frac{2 \times 3}{10^3}\right) = -\frac{2}{\log_e(0.283902)} + \frac{83}{10^3}$$

•

$$-\left(\frac{2}{\log(0.283902)} - \frac{89}{10^3} + \frac{2 \times 3}{10^3}\right) = -\frac{2}{\log(a) \log_a(0.283902)} + \frac{83}{10^3}$$

•

$$-\left(\frac{2}{\log(0.283902)} - \frac{89}{10^3} + \frac{2 \times 3}{10^3}\right) = \frac{-2}{-\text{Li}_1(0.716098)} + \frac{83}{10^3}$$

$\log_b(x)$  is the base- $b$  logarithm

$\text{Li}_n(x)$  is the polylogarithm function

**Series representations:**

$$-\left(\frac{2}{\log(0.283902)} - \frac{89}{10^3} + \frac{2 \times 3}{10^3}\right) = \frac{83}{1000} + \frac{2}{\sum_{k=1}^{\infty} \frac{(-1)^k (-0.716098)^k}{k}}$$

•

$$-\left(\frac{2}{\log(0.283902)} - \frac{89}{10^3} + \frac{2 \times 3}{10^3}\right) = \frac{2}{1000 - \frac{2i\pi \left[ \frac{\arg(0.283902-x)}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (0.283902-x)^k x^{-k}}{k}}}$$
 for  $x < 0$

$$-\left(\frac{2}{\log(0.283902)} - \frac{89}{10^3} + \frac{2 \times 3}{10^3}\right) = \frac{2}{1000 - \frac{\log(z_0) + \left[ \frac{\arg(0.283902-z_0)}{2\pi} \right] \left( \log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (0.283902-z_0)^k z_0^{-k}}{k}}$$

$\arg(z)$  is the complex argument

$\lfloor x \rfloor$  is the floor function

$i$  is the imaginary unit

### Integral representation:

$$-\left(\frac{2}{\log(0.283902)} - \frac{89}{10^3} + \frac{2 \times 3}{10^3}\right) = \frac{83}{1000} - \frac{2}{\int_1^{0.283902} \frac{1}{t} dt}$$

In conclusion, we have that:

$$\phi(\sqrt{2+\phi} - \phi) = \exp\left(-\frac{1}{5} \int_{e^{-2\pi}}^1 \prod_{n=1}^{\infty} \frac{(1-t^n)^5}{(1-t^{5n})} \cdot \frac{dt}{t}\right)$$

which is equivalent to our key result Equation (3).

$$\ln\left(\sqrt{4\phi+3} - \phi^2\right) = -\frac{1}{5} \int_{e^{-2\pi}}^1 \frac{(1-t)^5(1-t^2)^5(1-t^3)^5 \dots dt}{(1-t^5)(1-t^{10})(1-t^{15}) \dots t}$$

$\ln(\text{((((golden ratio(((sqrt(2+golden ratio)-golden ratio))))))))$

### Input:

$$\log\left(\phi\left(\sqrt{2+\phi} - \phi\right)\right)$$

$\log(x)$  is the natural logarithm

$\phi$  is the golden ratio

### Decimal approximation:

-0.77729093110436136528743433002266396133373091418023686588...

-0.777290931...

### Property:

$\log(\phi(-\phi + \sqrt{2 + \phi}))$  is a transcendental number

### Alternate forms:

$$\log\left(\frac{1}{2}\left(-3 - \sqrt{5} + 2\sqrt{5 + 2\sqrt{5}}\right)\right)$$

•

$$\log\left(\frac{1}{2}\left(-1 - \sqrt{5}\right) + \sqrt{2 + \frac{1}{2}\left(1 + \sqrt{5}\right)}\right) + \operatorname{csch}^{-1}(2)$$

•

$$-2 \log(2) + \log(1 + \sqrt{5}) + \log\left(-1 - \sqrt{5} + \sqrt{2(5 + \sqrt{5})}\right)$$

$\operatorname{csch}^{-1}(x)$  is the inverse hyperbolic cosecant function

### Alternative representations:

$$\log(\phi(\sqrt{2 + \phi} - \phi)) = \log_e(\phi(-\phi + \sqrt{2 + \phi}))$$

•

$$\log(\phi(\sqrt{2 + \phi} - \phi)) = \log(a) \log_a(\phi(-\phi + \sqrt{2 + \phi}))$$

•

$$\log(\phi(\sqrt{2 + \phi} - \phi)) = -\operatorname{Li}_1(1 - \phi(-\phi + \sqrt{2 + \phi}))$$

$\log_b(x)$  is the base- $b$  logarithm

$\operatorname{Li}_n(x)$  is the polylogarithm function

**Series representations:**

$$\log(\phi(\sqrt{2+\phi}-\phi)) = -\sum_{k=1}^{\infty} \frac{(-1)^k (-1+\phi(-\phi+\sqrt{2+\phi}))^k}{k}$$

•

$$\log(\phi(\sqrt{2+\phi}-\phi)) = 2i\pi \left\lfloor \frac{\arg(\phi(-\phi+\sqrt{2+\phi})-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (\phi(-\phi+\sqrt{2+\phi})-x)^k x^{-k}}{k} \text{ for } x < 0$$

•

$$\log(\phi(\sqrt{2+\phi}-\phi)) = 2i\pi \left\lfloor \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right\rfloor + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (\phi(-\phi+\sqrt{2+\phi})-z_0)^k z_0^{-k}}{k}$$

$\arg(z)$  is the complex argument

$\lfloor x \rfloor$  is the floor function

**Integral representation:**

$$\log(\phi(\sqrt{2+\phi}-\phi)) = \int_1^{\phi(-\phi+\sqrt{2+\phi})} \frac{1}{t} dt$$

$\ln(((\sqrt{4*\text{golden ratio}+3})-(\text{golden ratio})^2)))$

**Input:**

$$\log(\sqrt{4\phi+3}-\phi^2)$$

$\log(x)$  is the natural logarithm

$\phi$  is the golden ratio

### Decimal approximation:

-0.77729093110436136528743433002266396133373091418023686588...

-0.777290931...

### Property:

$\log(-\phi^2 + \sqrt{3 + 4\phi})$  is a transcendental number

### Alternate forms:

$$\log\left(\frac{1}{2} \left( 2\sqrt{4\phi + 3} - 3 - \sqrt{5} \right)\right)$$

$$\log\left(2\sqrt{4\phi + 3} - 3 - \sqrt{5}\right) - \log(2)$$

$$\log\left(\frac{1}{2} \left( -3 - \sqrt{5} + 2\sqrt{5 + 2\sqrt{5}} \right)\right)$$

### Alternative representations:

$$\log\left(\sqrt{4\phi + 3} - \phi^2\right) = \log_e\left(-\phi^2 + \sqrt{3 + 4\phi}\right)$$

•

$$\log\left(\sqrt{4\phi + 3} - \phi^2\right) = \log(a) \log_a\left(-\phi^2 + \sqrt{3 + 4\phi}\right)$$

•

$$\log\left(\sqrt{4\phi + 3} - \phi^2\right) = -\text{Li}_1\left(1 + \phi^2 - \sqrt{3 + 4\phi}\right)$$

$\log_b(x)$  is the base- $b$  logarithm

$\text{Li}_n(x)$  is the polylogarithm function

### Series representations:

$$\log\left(\sqrt{4\phi + 3} - \phi^2\right) = -\sum_{k=1}^{\infty} \frac{(-1)^k \left(-1 - \phi^2 + \sqrt{3 + 4\phi}\right)^k}{k}$$

•

$$\log(\sqrt{4\phi+3} - \phi^2) = 2i\pi \left\lfloor \frac{\arg(-\phi^2 + \sqrt{3+4\phi-x})}{2\pi} \right\rfloor +$$

$$\log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (-\phi^2 + \sqrt{3+4\phi-x})^k x^{-k}}{k} \quad \text{for } x < 0$$

$$\log(\sqrt{4\phi+3} - \phi^2) =$$

$$2i\pi \left\lfloor \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right\rfloor + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (-\phi^2 + \sqrt{3+4\phi-z_0})^k z_0^{-k}}{k}$$

$\arg(z)$  is the complex argument

$\lfloor x \rfloor$  is the floor function

### Integral representation:

$$\log(\sqrt{4\phi+3} - \phi^2) = \int_1^{-\phi^2 + \sqrt{3+4\phi}} \frac{1}{t} dt$$

Thence:

$$\ln(\sqrt{4\phi+3} - \phi^2) = \phi(\sqrt{2+\phi} - \phi)$$

$$-0.777290931\dots = -0.777290931\dots$$

Now:

$$-\frac{1}{5} \int_{e^{-2\pi}}^1 \frac{(1-t)^5(1-t^2)^5(1-t^3)^5 \dots dt}{(1-t^5)(1-t^{10})(1-t^{15}) \dots t}$$

is equal to

$$\exp\left(-\frac{1}{5} \int_{e^{-2\pi}}^1 \prod_{n=1}^{\infty} \frac{(1-t^n)^5}{(1-t^{5n})} \cdot \frac{dt}{t}\right)$$

And:

$$\frac{1}{\phi} \exp \left( -\frac{1}{5} \int_q^1 \prod_{n=1}^{\infty} \frac{(1-t^n)^5}{(1-t^{5n})} \cdot \frac{dt}{t} \right) - (\sqrt{2+\phi} - \phi) \exp \left( -\frac{1}{5} \int_q^{e^{-2\tau}} \prod_{n=1}^{\infty} \frac{(1-t^n)^5}{(1-t^{5n})} \cdot \frac{dt}{t} \right)$$

the right hand side is equal to:

(((((sqrt(2+golden ratio)-golden ratio))))))

**Input:**

$$\sqrt{2+\phi} - \phi$$

$\phi$  is the golden ratio

**Decimal approximation:**

0.284079043840412296028291832393126169091088088445737582759...

0.284079043... partial result

**Alternate forms:**

$$\frac{1}{2} \left( -1 - \sqrt{5} + \sqrt{2(5 + \sqrt{5})} \right)$$

•

$$-\frac{1}{2} - \frac{\sqrt{5}}{2} + \sqrt{\frac{1}{2}(5 + \sqrt{5})}$$

•

$$-\frac{1}{2} - \frac{\sqrt{5}}{2} + \sqrt{2 + \frac{1}{2}(1 + \sqrt{5})}$$

•

**Minimal polynomial:**

$$x^4 + 2x^3 - 6x^2 - 2x + 1$$

**Series representations:**

$$\sqrt{2+\phi} - \phi = -\phi + \sqrt{1+\phi} \sum_{k=0}^{\infty} (1+\phi)^{-k} \binom{\frac{1}{2}}{k}$$

•

$$\sqrt{2+\phi} - \phi = -\phi + \sqrt{1+\phi} \sum_{k=0}^{\infty} \frac{(-1)^k (1+\phi)^{-k} \left(-\frac{1}{2}\right)_k}{k!}$$

•



$$\sqrt{2+\phi} - \phi = -\phi + \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2+\phi-z_0)^k z_0^{-k}}{k!}$$

for not  $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

$$(((\sqrt{5}-(0.055+0.00721))))*(0.284079 \exp(((((-1/5*\integrate [(((1-0.5)^5 (1-0.5^2)^5 (1-0.5^3)^5)) / (((1-0.5^5)(1-0.5^{10})(1-0.5^{15}))))] t,[1, e^{(-2\pi)}])]))))$$

**Input interpretation:**

$$\left( \sqrt{5} - (0.055 + 0.00721) \right) \left( 0.284079 \exp \left( -\frac{1}{5} \int_1^{e^{-2\pi}} \frac{(1-0.5)^5 (1-0.5^2)^5 (1-0.5^3)^5}{(1-0.5^5)(1-0.5^{10})(1-0.5^{15})} t dt \right) \right)$$

**Result:**

0.61779

0.61779

Or:

**Input interpretation:**

$$\left( \sqrt{5} - \left( 0.055 + \frac{1}{137} - 0.000068 \right) \right) \left( 0.284079 \exp \left( -\frac{1}{5} \int_1^{e^{-2\pi}} \frac{(1-0.5)^5 (1-0.5^2)^5 (1-0.5^3)^5}{(1-0.5^5)(1-0.5^{10})(1-0.5^{15})} t dt \right) \right)$$

where 1/137 is the reciprocal of the fine-structure constant

**Result:**

0.617784

0.617784

And:

$$0.61803398 \exp(((((-1/5*\integrate [(((1-0.5)^5 (1-0.5^2)^5 (1-0.5^3)^5)) / (((1-0.5^5)(1-0.5^{10})(1-0.5^{15}))))] t,[ e^{(-2\pi)}, 1])]))))$$

**Input interpretation:**

$$0.61803398 \exp \left( -\frac{1}{5} \int_{e^{-2\pi}}^1 \frac{(1-0.5)^5 (1-0.5^2)^5 (1-0.5^3)^5}{(1-0.5^5)(1-0.5^{10})(1-0.5^{15})} t dt \right)$$

**Result:**

0.617791

0.617791

Furthermore, the inverse of this equation is:

$$\frac{1}{\left(\frac{0.61803398 \exp\left(\int_{e^{-2\pi}}^1 \frac{(1-0.5)^5 (1-0.5^2)^5 (1-0.5^3)^5}{(1-0.5^5)(1-0.5^{10})(1-0.5^{15})} t dt\right)}{(1-0.5^5)(1-0.5^{10})(1-0.5^{15})}\right) t, [e^{-2\pi}, 1]}$$

**Input interpretation:**

$$\frac{1}{0.61803398 \exp\left(-\frac{1}{5} \int_{e^{-2\pi}}^1 \frac{(1-0.5)^5 (1-0.5^2)^5 (1-0.5^3)^5}{(1-0.5^5)(1-0.5^{10})(1-0.5^{15})} t dt\right)}$$

**Result:**

1.61867

1.61867

We observe that:

$$\frac{55}{10^3} - \frac{2}{10^3} + \frac{1}{\left(\frac{0.61803398 \exp\left(\int_{e^{-2\pi}}^1 \frac{(1-0.5)^5 (1-0.5^2)^5 (1-0.5^3)^5}{(1-0.5^5)(1-0.5^{10})(1-0.5^{15})} t dt\right)}{(1-0.5^5)(1-0.5^{10})(1-0.5^{15})}\right) t, [e^{-2\pi}, 1]}$$

**Input interpretation:**

$$\frac{55}{10^3} - \frac{2}{10^3} + \frac{1}{0.61803398 \exp\left(-\frac{1}{5} \int_{e^{-2\pi}}^1 \frac{(1-0.5)^5 (1-0.5^2)^5 (1-0.5^3)^5}{(1-0.5^5)(1-0.5^{10})(1-0.5^{15})} t dt\right)}$$

**Result:**

1.67167

1.67167... a result very near to the value of the formula:

$$m_{p'} = 2 \times \frac{\eta}{R} m_p = 1.6714213 \times 10^{-24} \text{ gm}$$

that is the holographic proton mass

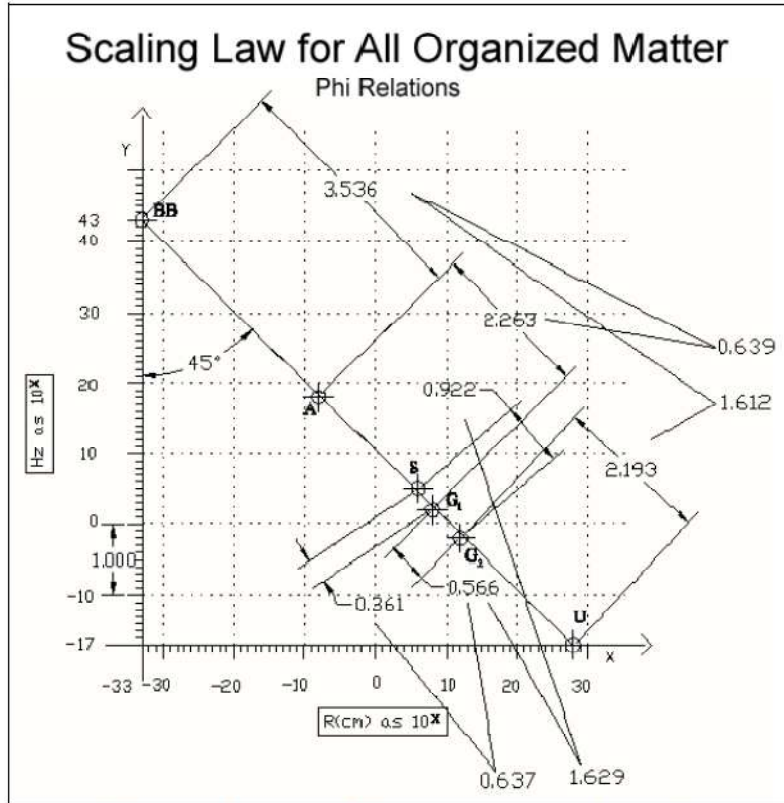


Figure 2b. We note that the distance between the data points on our graph, when divided with each other as in Figure 2b, yields a very close approximation to the familiar  $\Phi(\text{phi})$  ratio given by  $(1 + \sqrt{5})/2 \approx 1.618$  and its inverse  $(1 - \sqrt{5})/2 \approx 0.618$ . It is both appropriate and significant that the so called “golden ratio” is reflected in our scaling law (which maps energy dynamics at all scales), since it is prominently found everywhere in nature and has marked the evolution of cosmological mechanics and modern physics [18], from Kepler’s solar system modeling [20] to aperiodic Penrose tilings [21], including recent work on the thermodynamic phase transition of black holes showing a change of state from negative specific heat to positive specific heat at  $(1 - \sqrt{5})/2 \approx 0.618$  [22].

Ramanujan's manuscript. The representations of 1729 as the sum of two cubes appear in the bottom right corner. The equation expressing the near counter examples to Fermat's last theorem appears further up:  $\alpha^3 + \beta^3 = \gamma^3 + (-1)^n$ . Image courtesy Trinity College library.

ff

(i)  $\frac{1 + 53x + 9x^2}{1 - 82x - 82x^2 + x^3} = a_0 + a_1x + a_2x^2 + a_3x^3 + \dots$   
 or  $\frac{\alpha_0}{x} + \frac{\alpha_1}{x^2} + \frac{\alpha_2}{x^3} + \dots$

(ii)  $\frac{2 - 26x - 12x^2}{1 - 82x - 82x^2 + x^3} = b_0 + b_1x + b_2x^2 + b_3x^3 + \dots$   
 or  $\frac{\beta_0}{x} + \frac{\beta_1}{x^2} + \frac{\beta_2}{x^3} + \dots$

(iii)  $\frac{2 + 8x - 10x^2}{1 - 82x - 82x^2 + x^3} = c_0 + c_1x + c_2x^2 + c_3x^3 + \dots$   
 or  $\frac{\gamma_0}{x} + \frac{\gamma_1}{x^2} + \frac{\gamma_2}{x^3} + \dots$

then

$$\left. \begin{aligned} a_n^3 + b_n^3 &= c_n^3 + (-1)^n \\ \text{and } \alpha_n^3 + \beta_n^3 &= \gamma_n^3 + (-1)^n \end{aligned} \right\}$$

Examples

$$135^3 + 138^3 = 172^3 - 1$$

$$11161^3 + 11468^3 = 14258^3 + 1$$

$$791^3 + 812^3 = 1010^3 - 1$$

$$9^3 + 10^3 = 12^3 + 1$$

$$6^3 + 8^3 = 9^3 - 1$$

[https://plus.maths.org/content/sites/plus.maths.org/files/news/2015/ramanujan/page\\_large.jpg](https://plus.maths.org/content/sites/plus.maths.org/files/news/2015/ramanujan/page_large.jpg)

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### **Collective Coherent Oscillation Plasma Modes in Surrounding Media of Black Holes and Vacuum Structure – Quantum Processes with considerations of Spacetime Torque and Coriolis Forces**

*N. Hamein and E.A. Rauscher*

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