# Cubic Power Algorithm II Using polynomials

**Author and researcher**

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The discovery of a new algorithm, which went unnoticed for centuries, now comes to light to show its characteristics and its contribution to the use of polynomials.

Registrado en La ciudad de La Plata, Buenos Aires Argentina.

**Title**: Cubic Power Algorithm, using polynomials.
**Sub title:** Cube of a binomial, trinomial, tetranomial and pentanomial.
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**Comments:** 29 pages
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**Abstract:** This document develops and demonstrates the discovery of a new cube potentiation algorithm that works absolutely with all the numbers using the formula of the square of a binomial, trinomial, tetranomial and pentanomial. This presents the expansion of terms to the cube, the ideal order of the coefficients to obtain a sum that generates the results of the power.

## Chapter 1: Square of a binomial, trinomial, tetranomial and pentanomial.

### Example nº1 Binomial

$$(a+b)^3 = (a+b)*(a+b)*(a+b)$$

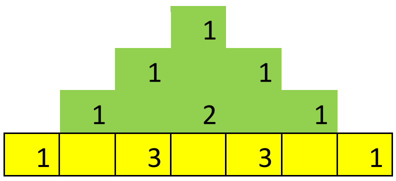
Right distribution of terms

$$\boxed{a^3 + 3a^2b + 3ab^2 + b^3}$$

Coefficient of terms

1331

We can get the coefficients of Pascal's triangle



# Demonstration of cubic power

**Example $(25)^3 = 15.625$**

a=2
b=5

Right distribution of terms

$$a^3 + 3a^2b + 3ab^2 + b^3$$

$$25^3 = 2^3 + 3 * 2^2 * 5 + 3 * 2 * 5^2 * 5^3$$
$$25^3 = 8 + 60 + 150 + 125$$

| | | | | | |
|---|---|---|---|---|---|
| | 8 | | | | *1.000 |
| | 6 | 0 | | | *100 |
| | 1 | 5 | 0 | | *10 |
| + | | 1 | 2 | 5 | *1 |
| 1 | 5 | 6 | 2 | 5 | Result |

The figure is a pattern that will be present in all the numbers of two digits squared.
We multiply the first term by 1000, the second term by 100, the third term by 10, and the four term by 1. In all cases when we use the cube of a whole number.

## Example n°2 Trinomial

$$(a+b+c)^3 = (a+b+c)*(a+b+c)*(a+b+c)$$

Right distribution of terms

$$a^3 + 3a^2b + 3a^2c + 3ab^2 + 6abc + 3ac^2 + b^3 + 3b^2c + 3bc^2 + c^3$$

Coefficient of terms

**1333631331**

We can obtain the coefficients by multiplying

Pascal's triangle

| | | |
|---|---|---|
| $(a+b)^0$ | 1 | x1 |
| $(a+b)^1$ | 1 1 | x3 |
| $(a+b)^2$ | 1 2 1 | x3 |
| $(a+b)^3$ | 1 3 3 1 | x1 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $(a+b+c)^3$ | 1 | 3 | 3 | 3 | 6 | 3 | 1 | 3 | 3 | 1 |

Demonstration of cubic power

Example $(132)^3 = 2.299.968$

**a=1**
**b=3**
**c=2**

Right distribution of terms

$$a^3 + 3a^2b + 3a^2c + 3ab^2 + 6abc + 3ac^2 + b^3 + 3b^2c + 3bc^2 + c^3$$

$$1^3 + 3*1^2*3 + 3*1^2*2 + 3*1*3^2 + 6*1*3*2 + 3*1*2^2 + 3^3 + 3*3^2*2 + 3*3^2*2 + 2^3$$

$$1 + 6 + 9 + 27 + 36 + 12 + 27 + 108 + 54 + 8$$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1 | | | | | | | *1.000.000 |
| 0 | 9 | | | | | | *100.000 |
| | 0 | 6 | | | | | <u>*10.000</u> |
| | 2 | 7 | | | | | *10.000 |
| | | 3 | 6 | | | | *1.000 |
| | | | 1 | 2 | | | <u>*100</u> |
| | | 2 | 7 | | | | *1.000 |
| | | | 5 | 4 | | | *100 |
| | | | | 3 | 6 | | *10 |
| | | | | | 0 | 8 | *1 |
| 2 | 2 | 9 | 9 | 9 | 6 | 8 | Result |

The shape that is formed here is a pattern that will always be formed when we have three cube digits.
We can see that the geometric figure contains the figure of example 1 (cube of a binomial).
We add following this model, ordering the numbers from left to right.

## Example n°3 Tetranomial

$$(a+b+c+d)^3 = (a+b+c+d)*(a+b+c+d)*(a+b+c+d)$$

Right distribution of terms

$a^3+3a^2b+3a^2c+3a^2d+3ab^2+6abc+6abd+3ac^2+6acd+3ad^2+b^3+3b^2c+$
$3b^2d+3bc^2+6bcd+3bd^2+c^3+3c^2d+3cd^2+d^3$

Coefficient of terms

13333663631333631331

We can obtain the coefficients by multiplying

Pyramid of three terms

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $(a+b+c)^0$ | 1 | | | | | | | | | | x1 |
| $(a+b+c)^1$ | 1 | 1 | 1 | | | | | | | | x3 |
| $(a+b+c)^2$ | 1 | 2 | 2 | 1 | 2 | 1 | | | | | x3 |
| $(a+b+c)^3$ | 1 | 3 | 3 | 3 | 6 | 3 | 1 | 3 | 3 | 1 | x1 |

| | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $(a+b+c+d)^3$ | 1 | 3 | 3 | 3 | 3 | 6 | 6 | 3 | 6 | 3 | 1 | 3 | 3 | 3 | 6 | 3 | 1 | 3 | 3 | 1 |

Demonstration of cubic power

Example $(1.234)^3$ = 1.879.080.904

**a =1**
**b =2**
**c =3**
**d =4**

$a^3+3a^2b+3a^2c+3a^2d+3ab^2+6abc+6abd+3ac^2+6acd+3ad^2+b^3+3b^2c+$
$3b^2d+3bc^2+6bcd+3bd^2+c^3+3c^2d+3cd^2+d^3$

$1^3+3*1^2*2+3*1^2*3+3*1^2*4+3*1*2^2+6*1*2*3+6*1*2*4+3*1*3^2+6*1*3*4+3*1*4^2+2^3+3*2^2*3$
$+3*2^2*4+3*2*3^2+6*2*3*4+3*2*4^2+3^3+3*3^2*4+3*3*4^2+4^3$

1+6+9+12+12+36+48+27+72+48+8+36+48+54+144+96+27+108+144+64

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | | | *1.000.000.000 |
| 0 | 6 | | | | | | | | | *100.000.000 |
| | 0 | 9 | | | | | | | | *10.000.000 |
| | | 1 | 2 | | | | | | | *1.000.000 |
| | 1 | 2 | | | | | | | | *10.000.000 |
| | | 3 | 6 | | | | | | | *1.000.000 |
| | | | 4 | 8 | | | | | | *100.000 |
| | | | 2 | 7 | | | | | | *100.000 |
| | | | | 7 | 2 | | | | | *10.000 |
| | | | | | 4 | 8 | | | | *1.000 |
| | | 0 | 8 | | | | | | | *1.000.000 |
| | | | 3 | 6 | | | | | | *100.000 |
| | | | | 4 | 8 | | | | | *10.000 |
| | | | | 5 | 4 | | | | | *10.000 |
| | | | | 1 | 4 | 4 | | | | *1.000 |
| | | | | | | 9 | 6 | | | *100 |
| | | | | | 2 | 7 | | | | *1.000 |
| | | | | | 1 | 0 | 8 | | | *100 |
| | | | | | | 1 | 4 | 4 | | *10 |
| | | | | | | | | 6 | 4 | *1 |
| **1** | **8** | **7** | **9** | **0** | **8** | **0** | **9** | **0** | **4** | Result |

The figure is a pattern that will be formed with all the numbers with a maximum of 4 digits.
To add we use this model, ordering the numbers from left to right.
This pattern contains the patterns of examples 1 and 2 within itself.

## Example n°4 Pentanomial

$(a+b+c+d+e)^3 = (a+b+c+d+e) * (a+b+c+d+e) * (a+b+c+d+e)$

Right distribution of terms

$a^3 + 3a^2b + 3a^2c + 3a^2d + 3a^2e + 3ab^2 + 6abc + 6abd + 6abe + 3ac^2 + 6acd + 6ace + 3ad^2 + 6ade + 3ae^2 + b^3 + 3cb^2 + 3b^2d + 3b^2e + 3c^2b + 6bcd + 6bce + 3bd^2 + 6bde + 3be^2 + c^3 + 3dc^2 + 3c^2e + 3d^2c + 6cde + 3ce^2 + d^3 + 3d^2e + 3de^2 + e^3$

Coefficient of terms

**13333366636636313333663631333631331**

We can obtain the coefficients by multiplying

Pyramid of four terms

| | | |
|---|---|---|
| $(a+b+c+d)^0$ | 1 | x1 |
| $(a+b+c+d)^1$ | 1 1 1 1 | x3 |
| $(a+b+c+d)^2$ | 1 2 2 2 1 2 2 1 2 1 | x3 |
| $(a+b+c+d)^3$ | 1 3 3 3 3 6 6 3 6 3 1 3 3 3 6 3 1 3 3 1 | x1 |

$(a+b+c+d+e)^3$

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 3 | 3 | 3 | 3 | 3 | 6 | 6 | 6 | 3 | 6 | 6 | 3 | 6 | 3 | 1 | 3 | 3 | 3 | 3 | 6 | 6 | 3 | 6 | 3 | 1 | 3 | 3 | 3 | 6 | 3 | 1 | 3 | 3 | 1 |

Demonstration of cubic power

Example $(12.345)^3 = 1.881.365.963.625$

**a=1**
**b=2**
**c=3**
**d=4**
**e=5**

$1^3 + 3*1^2*2 + 3*1^2*3 + 3*1^2*4 + 3*1^2*5 + 3*1*2^2 + 6*1*2*3 + 6*1*2*4 + 6*1*2*5 + 3*1*3^2 + 6*1*3*4 + 6*1*3*5 + 3*1*4^2 + 6*1*4*5 + 3*1*5^2 + 2^3 + 3*3*2^2 + 3*2^2*4 + 3*2^2*5 + 3*3^2*2 + 6*2*3*4 + 6*2*3*5 + 3*2*4^2 + 6*2*4*5 + 3*2*5^2 + 3^3 + 3*4*3^2 + 3*3^2*5 + 3*4^2*3 + 6*3*4*5 + 3*3*5^2 + 4^3 + 3*4^2*5 + 3*4*5^2 + 5^3$

1 + 6 + 9 + 12 + 15+ 12 + 36 + 48 + 60 + 27 + 72 +90 + 48+ 120 + 75 + 8 + 36 + 48 + 60 + 54 + 144 + 180 + 96 + 240+ 150+ 27+ 108 + 135+ 144 + 360 +225 + 64+ 240 + 300+ 125

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | | | | | | *1.000.000.000.000 |
| 0 | 6 | | | | | | | | | | | | *100.000.000.000 |
| | 0 | 9 | | | | | | | | | | | *10.000.000.000 |
| | | 1 | 2 | | | | | | | | | | *1.000.000.000 |
| | | | 1 | 5 | | | | | | | | | *100.000.000 |
| | 1 | 2 | | | | | | | | | | | *10.000.000.000 |
| | | 3 | 6 | | | | | | | | | | *1.000.000.000 |
| | | | 4 | 8 | | | | | | | | | *100.000.000 |
| | | | | 6 | 0 | | | | | | | | *10.000.000 |
| | | | 2 | 7 | | | | | | | | | *100.000.000 |
| | | | | 7 | 2 | | | | | | | | *10.000.000 |
| | | | | | 9 | 0 | | | | | | | *1.000.000 |
| | | | | | 4 | 8 | | | | | | | *1.000.000 |
| | | | | | 1 | 2 | 0 | | | | | | 100.000 |
| | | | | | | | 7 | 5 | | | | | 10.000 |
| | | 0 | 8 | | | | | | | | | | *1.000.000.000 |
| | | | 3 | 6 | | | | | | | | | *100.000.000 |
| | | | | 4 | 8 | | | | | | | | *10.000.000 |
| | | | | | 6 | 0 | | | | | | | *1.000.000 |
| | | | | 5 | 4 | | | | | | | | 10.000.000 |
| | | | | 1 | 4 | 4 | | | | | | | 1.000.000 |
| | | | | | 1 | 8 | 0 | | | | | | 100.000 |
| | | | | | | 9 | 6 | | | | | | 100.000 |
| | | | | | | 2 | 4 | 0 | | | | | 10.000 |
| | | | | | | | 1 | 5 | 0 | | | | 1.000 |
| | | | | | 2 | 7 | | | | | | | *1.000.000 |
| | | | | | 1 | 0 | 8 | | | | | | *100.000 |
| | | | | | | 1 | 3 | 5 | | | | | *10.000 |
| | | | | | | 1 | 4 | 4 | | | | | *10.000 |
| | | | | | | | 3 | 6 | 0 | | | | *1.000 |
| | | | | | | | | 2 | 2 | 5 | | | *100 |
| | | | | | | | | 6 | 4 | | | | *1.000 |
| | | | | | | | | 2 | 4 | 0 | | | *100 |
| + | | | | | | | | | 3 | 0 | 0 | | *10 |
| | | | | | | | | | | 1 | 2 | 5 | *1 |
| 1 | 8 | 8 | 1 | 3 | 6 | 5 | 9 | 6 | 3 | 6 | 2 | 5 | Result |

The figure is a pattern that will be formed with all the numbers with a maximum of 5 digits.
To add we use this model, ordering the numbers from left to right.
This pattern contains the patterns of examples 1, 2 and 3 within itself.

The red numbers are the values that were cube in the formula. These are ordered multiplying each other by 1000.
Example: 1; 1000; 1.000.000; 1.000.000.000; 1.000.000.000.000

## The multinomial theorem

The multinomial theorem, which gives us a simple formula for any coefficient we might want. It is possible to "read off" the multinomial coefficients from the terms by using the multinomial coefficient formula.

$$(x_1 + x_2 + x_3 + \cdots x_m)^n$$

$$\boldsymbol{a^{k1} b^{k2} c^{k3}} \ldots . \&^{km} = \frac{\boldsymbol{n}!}{\boldsymbol{k1}!\,\boldsymbol{k2}!\,\boldsymbol{k3}! \ldots \ldots \boldsymbol{Km}!}$$

***Example***:

$$\boxed{(\boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c})^3 = \mathbf{a^3 + 3a^2b + 3a^2c + 3ab^2 + 6abc + 3ac^2 + b^3 + 3b^2c + 3bc^2 + c^3}}$$

$$a^2 b^1 c^0 = \frac{n!}{k1!\,k2!\,k3!} = \frac{3!}{2!\,1!\,0!} = \frac{6}{2} = \mathbf{3}$$

The coefficient for this term will be number 3.

# Chapter 2: Coefficient of terms

Table 1

| Cube | Number of terms | Quantity of coefficients | Tetrahedral (or triangular piramidal numbers) |
|---|---|---|---|
| $(a)^3$ | **1** | 1 | 1 |
| $(a+b)^3$ | **2** | 1331 | 4 |
| $(a+b+c)^3$ | **3** | 1333631331 | 10 |
| $(a+b+c+d)^3$ | **4** | 13333663631333631331 | 20 |
| $(a+b+c+d+e)^3$ | **5** | 13333366636636313333663631333631331 | 35 |

The total sum of digits forms the sequence of tetrahedral numbers.
A000292 https://oeis.org/

## Tetrahedral Numbers

A number is termed as a tetrahedral number if it can be represented as a pyramid with a triangular base and three sides, called a tetrahedron. The $n^{th}$ tetrahedral number is the sum of the first n triangular numbers.
The first ten tetrahedral numbers are:
1, 4, 10, 20, 35, 56, 84, 120, 165, 220, …

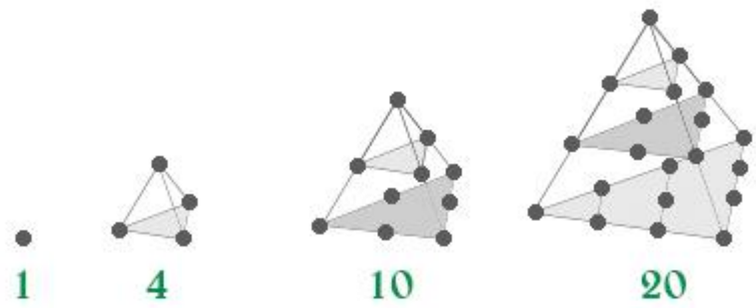


Table 2

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | sum |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 1 | 1 |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 1 | 3 | 3 | 1 | 8 |
| | | | | | | | | | | | | | | | | | | | | | | | | | 1 | 3 | 3 | 3 | 6 | 3 | 1 | 3 | 3 | 1 | 27 |
| | | | | | | | | | | | | | | | 1 | 3 | 3 | 3 | 3 | 6 | 6 | 3 | 6 | 3 | 1 | 3 | 3 | 3 | 6 | 3 | 1 | 3 | 3 | 1 | 64 |
| 1 | 3 | 3 | 3 | 3 | 3 | 6 | 6 | 6 | 3 | 6 | 6 | 3 | 6 | 3 | 1 | 3 | 3 | 3 | 3 | 6 | 6 | 3 | 6 | 3 | 1 | 3 | 3 | 3 | 6 | 3 | 1 | 3 | 3 | 1 | 125 |

The sum forms the sequence $x^3$
A000578 https://oeis.org/

In this table we can see how the upper sequences are integrated in the lower ones, repeating the same format and distribution.

Table 3

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | Total Nº 3 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 1 | 0 |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 1 | 3 | 3 | 1 | 2 |
| | | | | | | | | | | | | | | | | | | | | | | | | | 1 | 3 | 3 | 3 | 6 | 3 | 1 | 3 | 3 | 1 | 6 |
| | | | | | | | | | | | | | | | 1 | 3 | 3 | 3 | 3 | 6 | 6 | 3 | 6 | 3 | 1 | 3 | 3 | 3 | 6 | 3 | 1 | 3 | 3 | 1 | 12 |
| 1 | 3 | 3 | 3 | 3 | 3 | 6 | 6 | 6 | 3 | 6 | 6 | 3 | 6 | 3 | 1 | 3 | 3 | 3 | 3 | 6 | 6 | 3 | 6 | 3 | 1 | 3 | 3 | 3 | 6 | 3 | 1 | 3 | 3 | 1 | 20 |

The total number of numbers 3 forms the sequence of **oblong** numbers. Sequence (0, 2, 6, 12, 20, etc).

A002378 https://oeis.org/

Table 4

| | Total Nº 6 |
|---|---|
| 1 | 0 |
| 1 3 3 1 | 0 |
| 1 3 3 3 6 3 1 3 3 1 | 1 |
| 1 3 3 3 3 6 6 3 6 3 1 3 3 3 6 3 1 3 3 1 | 4 |
| 1 3 3 3 3 3 6 6 6 3 6 6 3 6 3 1 3 3 3 3 6 6 3 6 3 1 3 3 3 6 3 1 3 3 1 | 10 |

The total of numbers 6 forms the sequence of the **tetrahedral** numbers. Sequence (0, 1, 4, 10, etc).

A000292 https://oeis.org/
The total of numbers 1 matches the **natural** numbers.

## A) Hexagonal number centered

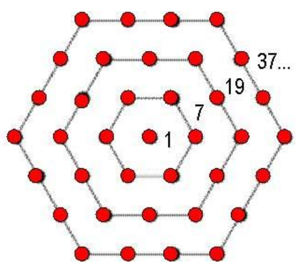
If we add the numbers according to their expansion we find a very interesting sequence.

| | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | | | | | | | 1 |
| | | | | | | | | | | | | | | | | | 1 | 3 | 3 | |
| | | | | | | | | | | | 1 | 3 | 3 | 3 | 6 | 3 | | | | |
| 1 | 3 | 3 | 3 | 3 | 6 | 6 | 3 | 6 | 3 | | | | | | | | | | | |

Reference A003215

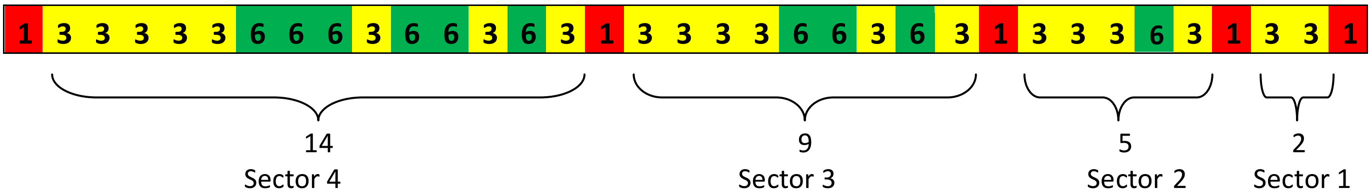
| | | | | |
|---|---|---|---|---|
| 1 3 3 3 3 3 6 6 6 3 6 6 3 6 3 | 1 3 3 3 3 6 6 3 6 3 | 1 3 3 3 6 3 | 1 3 3 | 1 |
| 61 | 37 | 19 | 7 | 1 |

Hexagonal number centered



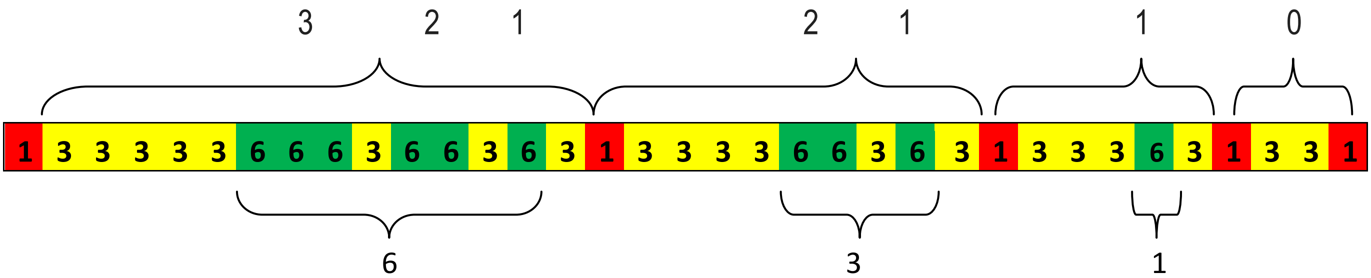
## B) Distribution of the sectors

Between the numbers 1 are formed combinations of digits of numbers 3 and 6. These form a sequence: 2,5,9,14,….etc. A000096 https://oeis.org/



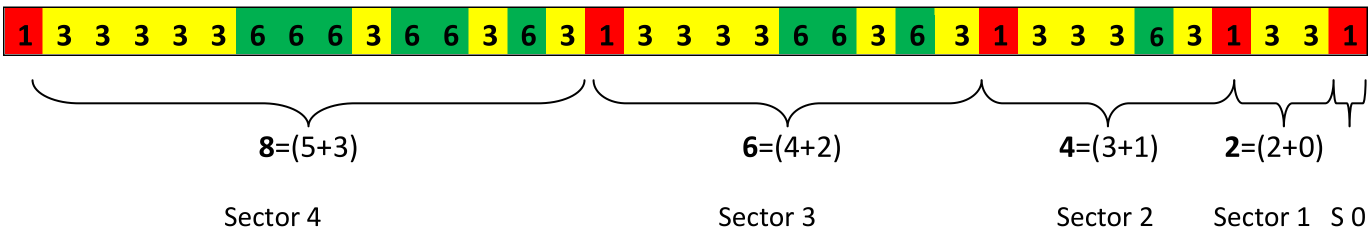

This sequence has the following characteristic.
$a\,(n) \;=\; n \,*\, (n \,+\, 3)\;/\;2.$

## C) Total of numbers 6 by sector and distribution of the numbers 6.



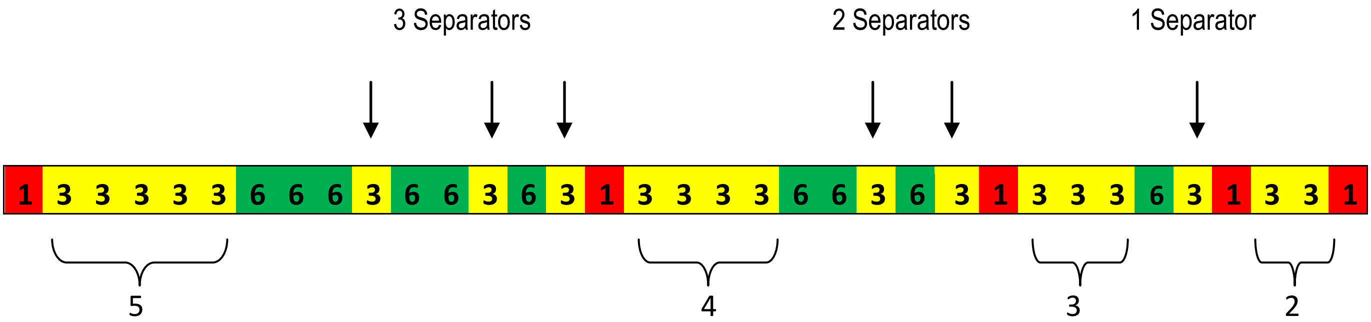

The total of numbers 6 by sector forms the sequence of triangular numbers.
0,1,3,6,....etc. A000217 https://oeis.org/
The numbers 6 are ordered in increasing order, respecting the format of the previous sectors, first 1, then 1 and 2 and finally, 1, 2 and 3.

## D) Total of numbers 3 by sector.




Pair numbers A005843 https://oeis.org/

## E) Analysis of the distribution of number 3



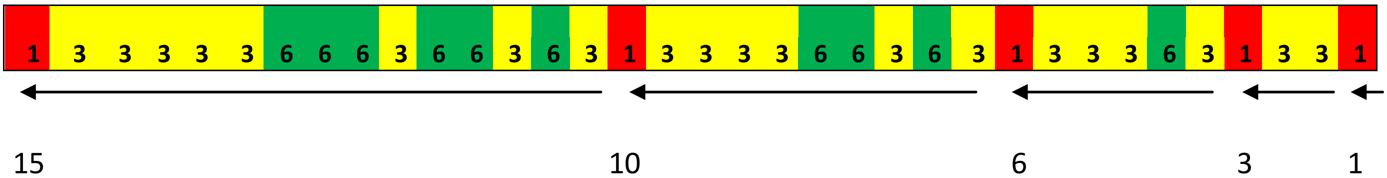

The consecutive 3 numbers on the right are in increasing order if we start on the left.
First two numbers 3, then three numbers 3, then 4 numbers 3 and finally 5 numbers 3.

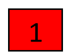
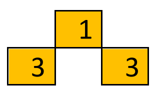
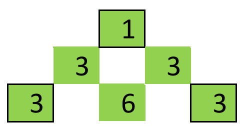
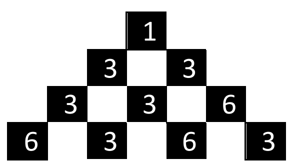
The numbers 3 that accompany the 6 in each sector are as separators. They also increase in a sustained way and their sequence belongs to the natural numbers. (0,1,2,3, etc).

## F) Analysis of the distribution of number 1

The numbers 1 are arranged under the sequences of trinagular numbers.

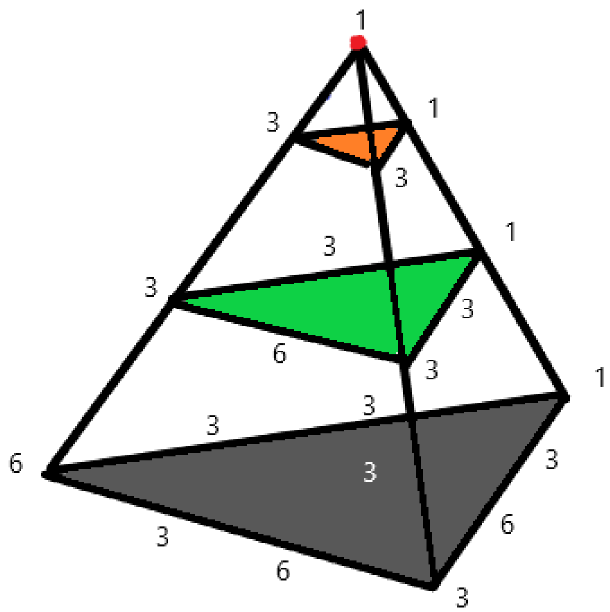
Coefficient of terms

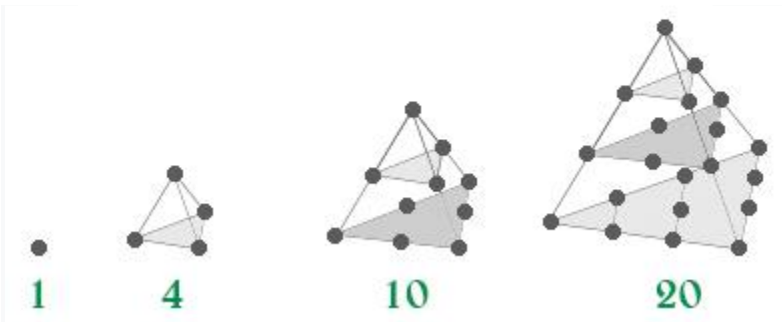



| Coeficients | Triangular numbers | Quantity | Sum |
|---|---|---|---|
| $a^3 = 1$ |  | 1 | 1 |
| $(a+b)^3 = 1331$ |  | 3 | 7 |
| $(a+b+c)^3 = 1333631331$ |  | 6 | 19 |
| $(a+b+c+d)^3 =$ 13333663631333631331 |  | 10 | 37 |

| | | | |
|---|---|---|---|
| $(a+b+c+d+e)^3 =$<br>133333666366363133333313333663631333631331 | 1<br>3 3<br>3 3 3<br>6 6 6 3<br>6 6 3 6 3 | 15 | 61 |

<u>The triangles form the tetrahedron</u>



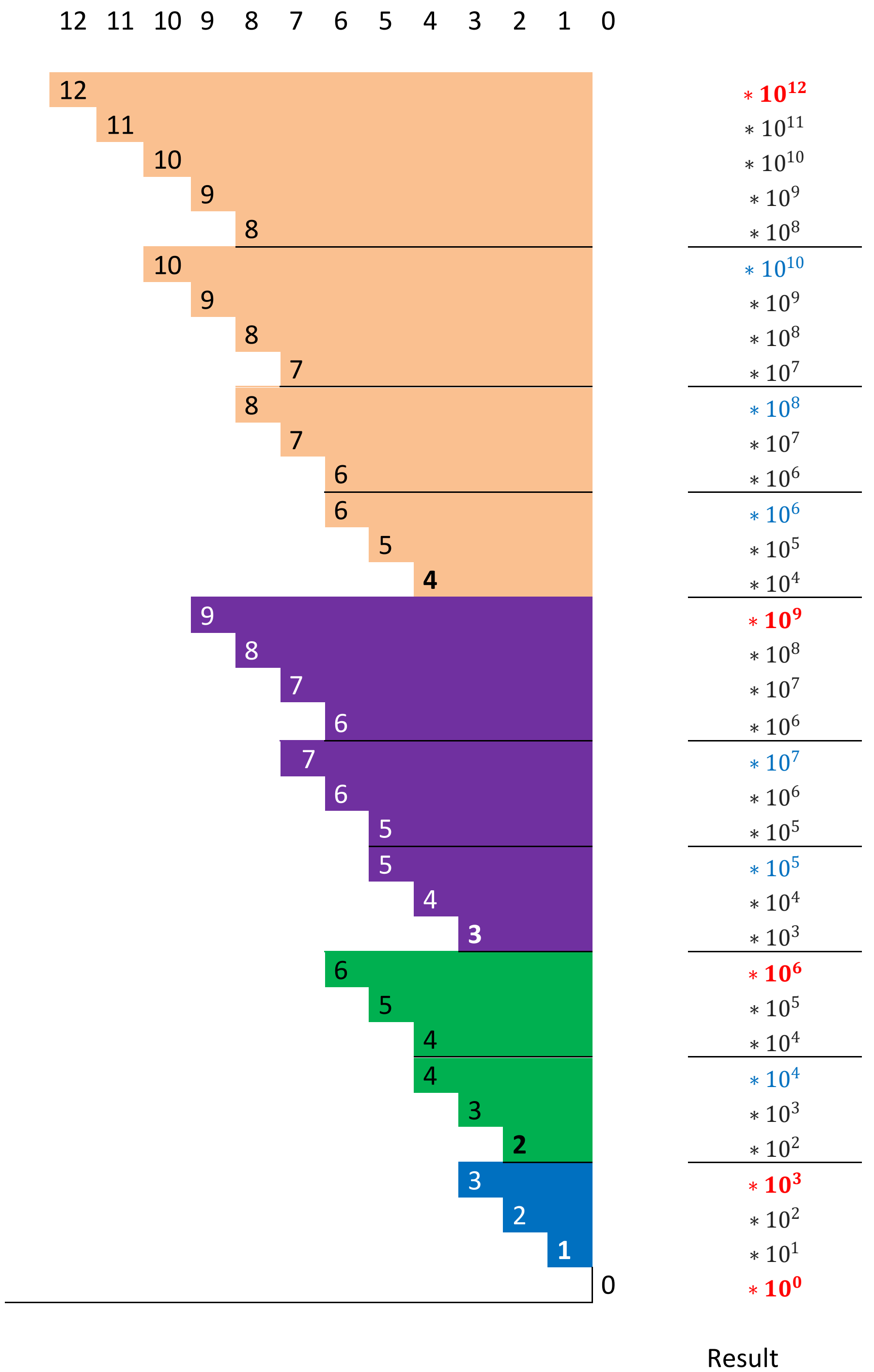
## G) Analysis and geometric distribution of sums.

The zone of color blue has a single section, the zone of green color has two sections, the zone of violet color has 3 sections, the zone of pink color has 4 sections.

Sequence of exponents: 0, 1, 2, 3, 2, 3, 4, 4, 5, 6, 3, 4, 5, 5 , 6, 7, 6, 7, 8, 9, 4, 5, 6, 6, 7, 8, 7, 8, 9, 10, 8, 9, 10, 11, 12,…..,etc.

A070770 https://oeis.org/

## Pattern of exponents



## H) Distribution of exponents

The 4 sectors have very well developed divisions and following the sequence of natural numbers.

Example

Zone 1 has no divisions
Zone 2 has two divisions
Zone 3 has 3 divisions
Zone 4 has 4 divisions

Each zone has well defined its exponents

Coefficient of terms

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 3 | 3 | 3 | 3 | 3 | 6 | 6 | 6 | 3 | 6 | 6 | 3 | 6 | 3 | 1 | 3 | 3 | 3 | 3 | 6 | 6 | 3 | 6 | 3 | 1 | 3 | 3 | 3 | 6 | 3 | 1 | 3 | 3 | 1 |

Exponents

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **12** | 11 | 10 | 9 | 8 | 10 | 9 | 8 | 7 | 8 | 7 | 6 | 6 | 5 | 4 | **9** | 8 | 7 | 6 | 7 | 6 | 5 | 5 | 4 | 3 | **6** | 5 | 4 | 4 | 3 | 2 | **3** | 2 | 1 | **0** |

Sector 4 Sector 3 Sector 2 Sector 1 S 0

[A070770](https://oeis.org/) https://oeis.org/

## Exponents

S= Sector

| | | |
|---|---|---|
| 0 | Sector 0 | $S0 = a^3$ |
| 1 2 3 | Sector 1 | $S\,0 + S1 = (a+b)^3$ |
| 2 3 4<br>4 5 6 | Sector 2 | $S\,0 + S1 + S2 = (a+b+c)^3$ |
| 3 4 5<br>5 6 7<br>6 7 8 9 | Sector3 | $S\,0 + S1 + S2 + S3 = (a+b+c+d)^3$ |
| 4 5 6<br>6 7 8<br>7 8 9 10<br>8 9 10 11 12 | Sector 4 | $S\,0 + S1 + S2 + S3 + S4 = (a+b+c+d+e)^3$ |
| 5 6 7<br>7 8 9<br>8 9 10 11<br>9 10 11 12 13<br>10 11 12 13 14 15 | Sector 5 | $S\,0 + S1 + S2 + S3 + S4 + S5 = (a+b+c+d+e+f)^3$ |
| 6 7 8<br>8 9 10<br>9 10 11 12<br>10 11 12 13 14<br>11 12 13 14 15 16<br>12 13 14 15 16 17 18 | Sector 6 | $S\,0 + S1 + S2 + S3 + S4 + S5 + S6 = (a+b+c+d+e+f+g)^3$ |

## Another way to organize the exponents

The steps that form this geometric figure belong to the natural numbers.

| Places | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Sector 0 | 0 | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Sector 1 | 1 | 2 | 3 | | | | | | | | | | | | | | | | | | | | | | | | | |
| Sector 2 | 2 | 3 | 4 | 4 | 5 | 6 | | | | | | | | | | | | | | | | | | | | | | |
| Sector 3 | 3 | 4 | 5 | 5 | 6 | 7 | 6 | 7 | 8 | 9 | | | | | | | | | | | | | | | | | | |
| Sector 4 | 4 | 5 | 6 | 6 | 7 | 8 | 7 | 8 | 9 | 10 | 8 | 9 | 10 | 11 | 12 | | | | | | | | | | | | | |
| Sector 5 | 5 | 6 | 7 | 7 | 8 | 9 | 8 | 9 | 10 | 11 | 9 | 10 | 11 | 12 | 13 | 10 | 11 | 12 | 13 | 14 | 15 | | | | | | | |
| Sector 6 | 6 | 7 | 8 | 8 | 9 | 10 | 9 | 10 | 11 | 12 | 10 | 11 | 12 | 13 | 14 | 11 | 12 | 13 | 14 | 15 | 16 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |

**Nº- 0** **Nº- 1** **Nº- 2** **Nº- 3** **Nº- 4**

The blue numbers present a very well organized sequence, in which the next number is generated under a subtraction. The order of the numbers in columns is under the sequence of the natural numbers.

## Sum of the exponents

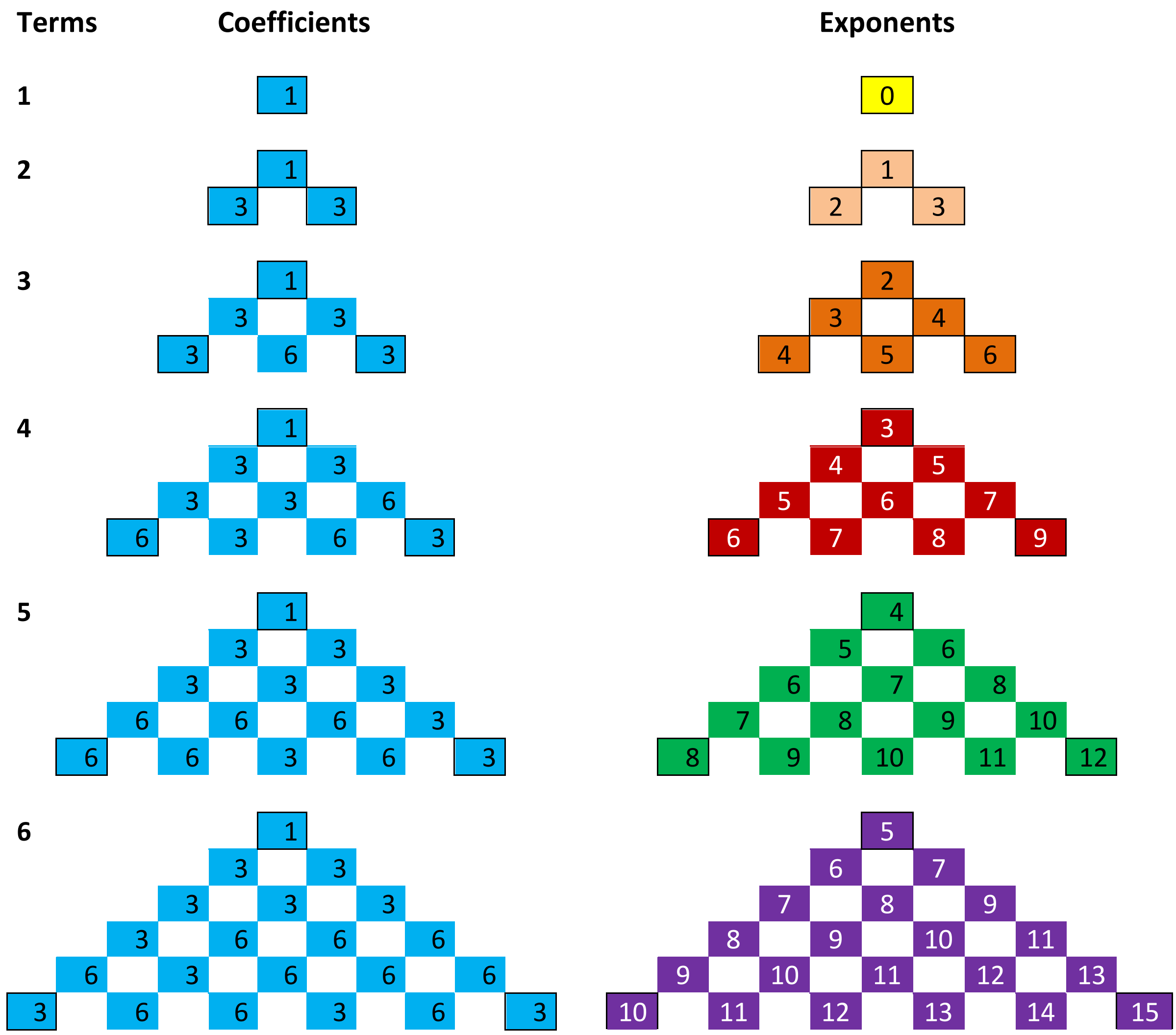
| Sum | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 6 | 1 | 2 | 3 | | | | | | | | | | | | | | | | | | | | | | | | | |
| 24 | 2 | 3 | 4 | 4 | 5 | 6 | | | | | | | | | | | | | | | | | | | | | | |
| 60 | 3 | 4 | 5 | 5 | 6 | 7 | 6 | 7 | 8 | 9 | | | | | | | | | | | | | | | | | | |
| 120 | 4 | 5 | 6 | 6 | 7 | 8 | 7 | 8 | 9 | 10 | 8 | 9 | 10 | 11 | 12 | | | | | | | | | | | | | |
| 210 | 5 | 6 | 7 | 7 | 8 | 9 | 8 | 9 | 10 | 11 | 9 | 10 | 11 | 12 | 13 | 10 | 11 | 12 | 13 | 14 | 15 | | | | | | | |
| 336 | 6 | 7 | 8 | 8 | 9 | 10 | 9 | 10 | 11 | 12 | 10 | 11 | 12 | 13 | 14 | 11 | 12 | 13 | 14 | 15 | 16 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |

$$\boldsymbol{Sum} = 6 * n$$

$$n = thetraedral\ number$$

| Sector | Sum | Formula |
|---|---|---|
| Sector 0 | 0 | 6*0 |
| Sector 1 | 6 | 6*1 |
| Sector 2 | 24 | 6*4 |
| Sector 3 | 60 | 6*10 |
| Sector 4 | 120 | 6*20 |
| Sector 5 | 210 | 6*35 |
| Sector 6 | 336 | 6*56 |

I) The relationship between the sectors of the pyramid, the coefficients and exponents.



# Formula of the exponents

## A) Inicial number

$T = \text{Number of terms} > 0$
$I = \text{Initial number}$

$$\boldsymbol{I = T - 1}$$

*Example: 6 terms*

$$\boldsymbol{I = 6 - 1}$$
$$\boldsymbol{I = 5}$$

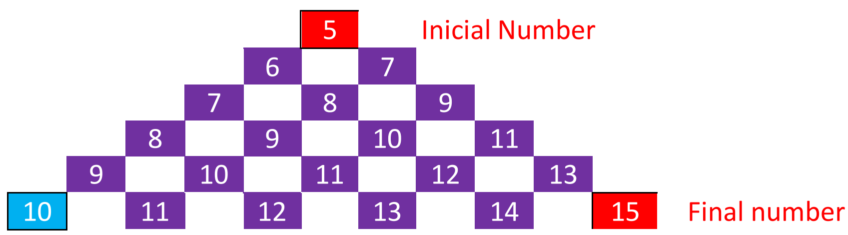
## B) Final number

$T = \text{Number of terms} > 0$
$F = \text{final number}$

$$\boldsymbol{F = 3(T - 1)}$$

*Example: 6 terms*

$$\boldsymbol{F = 3(6 - 1)}$$
$$\boldsymbol{F = 15}$$




## C) Middle number

$T = \text{Number of terms} > 0$
$M = \text{Middle number}$

$$\boldsymbol{M = 2(Inicial\ number)}$$

*Example: 6 terms*

$$\boldsymbol{M = 2 * 5}$$
$$\boldsymbol{M = 10}$$

## J) Coefficients arranged one on top of the other.

These form a rectangular triangle.

In the columns 3 very clear sequences are formed.

A= 3n+0, A1=3n+3, A2=3n+6, A3=3n+9,.... (digital root 3,6,9)
B=3n+1, B1=3n+4, B2=3n+7, B3=3n+10,.... (digital root 1,4,7)
C=3n+2, C1=3n+2, C2=3n+5, C3=3n+8,.... (digital root 2,5,8)

| C3 | B3 | A3 | C2 | B2 | A2 | C1 | B1 | A1 | C | B | A |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | 0 |
| | | | | | | | | | | 1 | |
| | | | | | | | | | 2 | | 3 |
| | | | | | | | | 3 | | 4 | |
| | | | | | | | 4 | | 5 | | 6 |
| | | | | | | 5 | | 6 | | 7 | |
| | | | | | 6 | | 7 | | 8 | | 9 |
| | | | | 7 | | 8 | | 9 | | 10 | |
| | | | 8 | | 9 | | 10 | | 11 | | 12 |
| | | 9 | | 10 | | 11 | | 12 | | 13 | |
| | 10 | | 11 | | 12 | | 13 | | 14 | | 15 |

In the diagonals from the top to the right, the numbers are ordered in pairs and odd numbers.
In the opposite diagonals they are arranged in the sequence of natural numbers.

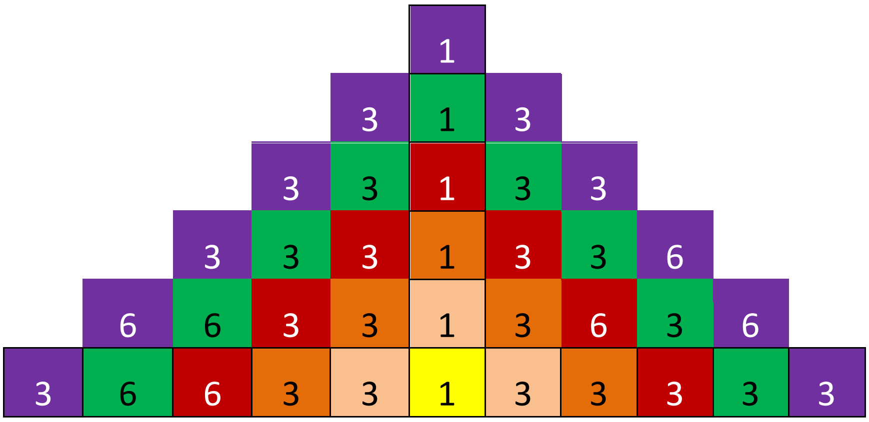
## View from above of the pyramid of Pascal (Exponents)

The exponents triangles are located one on top of the other.
The numbers belong to the exponents, which allowed us to order it in this way.
The number 0 would be the tip of the pyramid.
This model allows you to apply your expansion with ease.

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | 5 | | | | | |
| | | | | 6 | 4 | 7 | | | | |
| | | | 7 | 5 | 3 | 6 | 9 | | | |
| | | 8 | 6 | 4 | 2 | 5 | 8 | 11 | | |
| | 9 | 7 | 5 | 3 | 1 | 4 | 7 | 10 | 13 | |
| 10 | 8 | 6 | 4 | 2 | 0 | 3 | 6 | 9 | 12 | 15 |

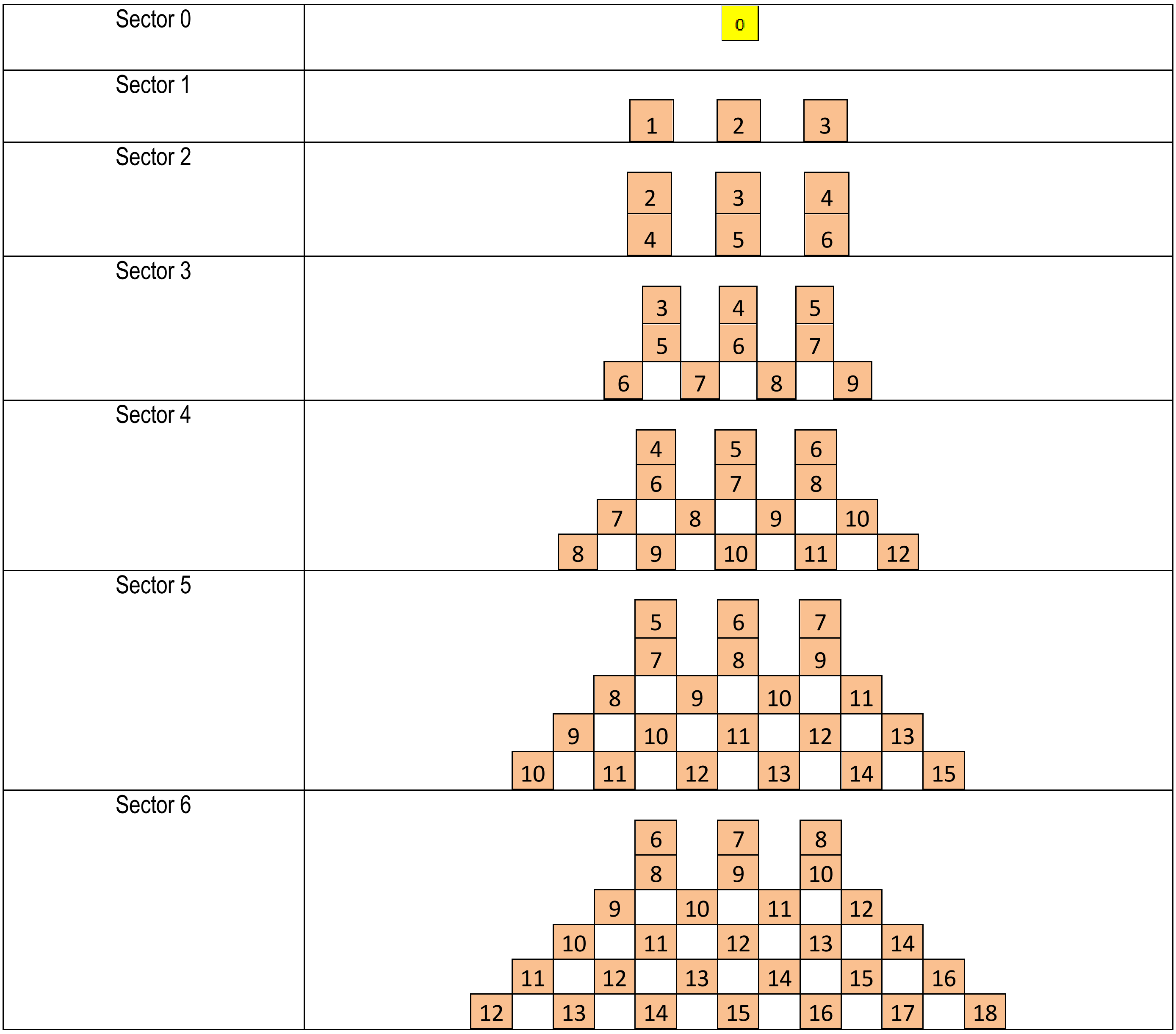
View from above of the pyramid of Pascal (Coeficients)

We use the previous model and replace it with the coefficient triangles. These are also located one on top of the other.

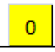
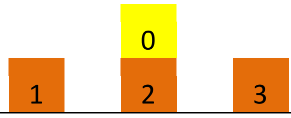
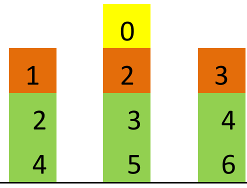
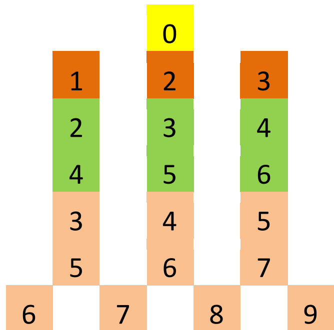
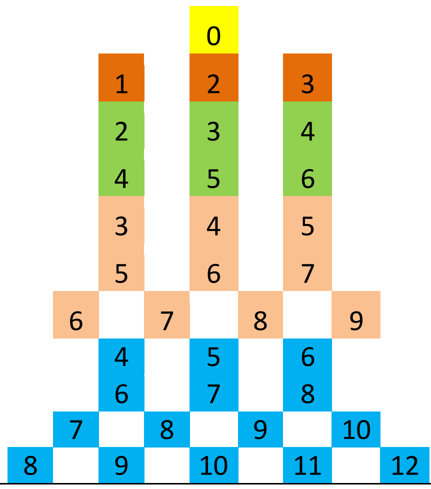


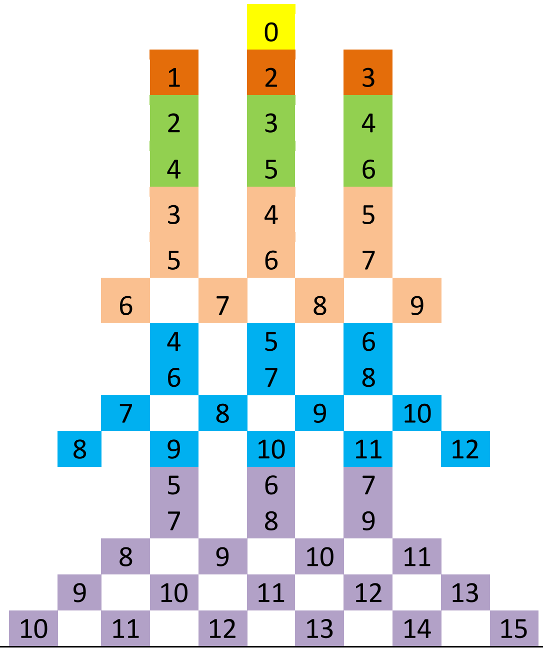
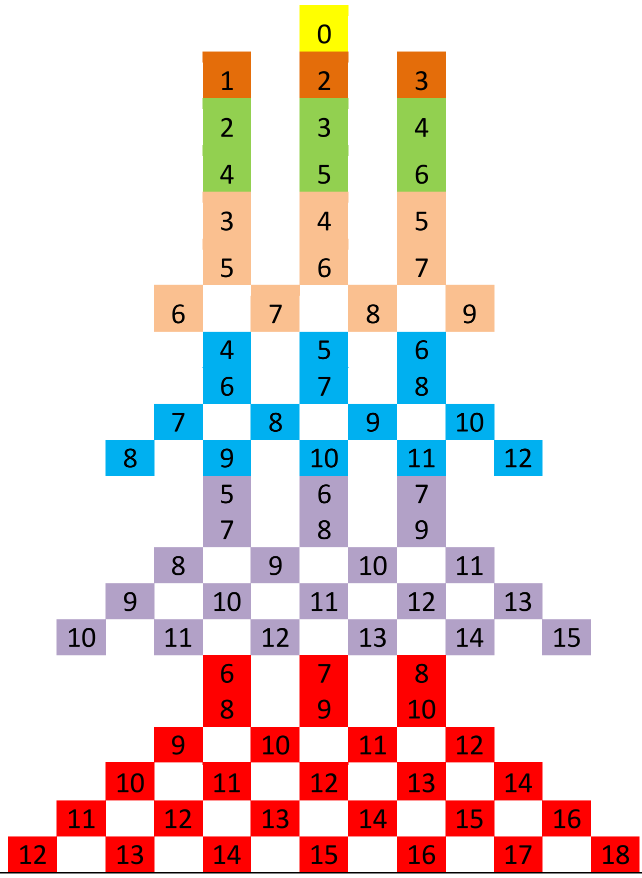
## Representation of the coefficients ordered in another way

| | 1 | 2 | 3 | 4 | 5 | 6 | |
|---|---|---|---|---|---|---|---|
| Sector0 | 0 | | | | | | |
| Sector1 | 1 | 2 3 | | | | | |
| Sector2 | 2 | 3 4 | 4 5 6 | | | | |
| Sector3 | 3 | 4 5 | 5 6 7 | 6 7 8 9 | | | |
| Sector4 | 4 | 5 6 | 6 7 8 | 7 8 9 10 | 8 9 10 11 12 | | |
| Sector5 | 5 | 6 7 | 7 8 9 | 8 9 10 11 | 9 10 11 12 13 | 10 11 12 13 14 15 | |
| Sector6 | 6 | 7 8 | 8 9 10 | 9 10 11 12 | 10 11 12 13 14 | 11 12 13 14 15 16 | 12 13 14 15 16 17 18 |



Representation of the exponents together in expansion

| | |
|---|---|
| $S\,0 = a^3$ |   |
| $S\,0 + S1 =$<br>$(a+b)^3$ |   |
| $S\,0 + S1 + S2 =$<br>$(a+b+c)^3$ |   |
| $S\,0 + S1 + S2 + S3 =$<br>$(a+b+c+d)^3$ |   |
| $S\,0 + S1 + S2 + S3 + S4 =$<br>$(a+b+c+d+e)^3$ |   |

| | |
|---|---|
| $S\,0+S1+S2+S3+S4+S5=$ <br> $(a+b+c+d+e+f)^3$ |   |
| $S\,0+S1+S2+S3+S4+S5+S6=$ <br> $(a+b+c+d+e+f+g)^3$ |   |

<u>Representation of the previous model with the coefficients for:</u>

$$s0 + s1 + s2 + s3 + s4 + s5 + s6 + s7$$

$$(a + b + c + d + e + f + g)^3$$

This geometric representation of the polynomial cube expansion is really surprising since it is ordered under very well defined sequences. No one of the coefficient is messy, everything is in the right place.

| Sector | Floor | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| S 0 | 0 | | | | | | | 1 | | | | | | |
| S 1 | 1 | | | | | 3 | | 3 | | 1 | | | | |
| S 2 | 2 | | | | | 3 | | 6 | | 3 | | | | |
| | 3 | | | | | 3 | | 3 | | 1 | | | | |
| S 3 | 4 | | | | | 3 | | 6 | | 3 | | | | |
| | 5 | | | | | 6 | | 6 | | 3 | | | | |
| | 6 | | | | 3 | | 3 | | 3 | | 1 | | | |
| S 4 | 7 | | | | | 3 | | 6 | | 3 | | | | |
| | 8 | | | | | 6 | | 6 | | 3 | | | | |
| | 9 | | | | 6 | | 6 | | 6 | | 3 | | | |
| | 10 | | | 3 | | 3 | | 3 | | 3 | | 1 | | |
| S 5 | 11 | | | | | 3 | | 6 | | 3 | | | | |
| | 12 | | | | | 6 | | 6 | | 3 | | | | |
| | 13 | | | | 6 | | 6 | | 6 | | 3 | | | |
| | 14 | | | 6 | | 6 | | 6 | | 6 | | 3 | | |
| | 15 | | 3 | | 3 | | 3 | | 3 | | 3 | | 1 | |
| S 6 | 16 | | | | | 3 | | 6 | | 3 | | | | |
| | 17 | | | | | 6 | | 6 | | 3 | | | | |
| | 18 | | | | 6 | | 6 | | 6 | | 3 | | | |
| | 19 | | | 6 | | 6 | | 6 | | 6 | | 3 | | |
| | 20 | | 6 | | 6 | | 6 | | 6 | | 6 | | 3 | |
| | 21 | 3 | | 3 | | 3 | | 3 | | 3 | | 3 | | 1 |

Sector 0 has 0 floor, Sector 1 has 1 floor, sector 2 has 2 floors, sector 3 has 3 floors, sector 4 has 4 floors, ……..

Sector (n) has n floors

We can see how in the number 331 it expands in a row 6 (3331), in a row 10 (33331), in a row 15 (333331), and in a row 21 (3333331).

331*10+21=3331
3331*10+21=33331
33331*10+21=333331

The number 331 and its expansion appear in the rows 1, 3, 6, 10, 15, 21,... An expansion sequence is formed.
**Triangular numbers.**
$n \geq 0$

$$a(n) = \frac{n\,(n\; +\; 1)}{2}$$

The same happens with the number 663 (row 5), it expands in a row 9 (6663), in a row 14 (66663), in a row 20 (66663)

663*10+33=6663
6663*10+33=66663
66663*10+33=666663

Sequence 5, 9, 14, 20,.... An expansion sequence is formed.
$n > 1$

$$a\,(n)\; = \frac{\mathrm{n} \,*\, (\mathrm{n}\; +\; 3)}{2}$$

The number 363 appears in the rows 2, 4, 7, 11, 16, …. A repetition sequence is formed.
**Central polygonal numbers**

$n > 0$

$$a(n) = \frac{n\,(n\; +\; 1)}{2} + 1$$

The number 663 appears in the rows 5, 8, 12, 17,…. A repetition sequence is formed.
$n > 3$

$$a(n) = \frac{(n^{\,2} - \;3\,n\; +\; 6)}{2}$$

**<u>Sum of the coefficients by sectors</u>**

S0 =1; S1=7; S2=19; S3=37; S4=61….

**<u>Hexagonal number centered</u>**

$n \geq 0$
$a(n) = 3 * n * (n + 1) + 1$

Sequence 1,7,19, 37, 61….

## Conclusion

This new algorithm presents a surprising precision, which transforms it into a reliable system or method for performing cube number operations.
This is simply different, it is a novel and interesting alternative.
The correct setting of the coefficients of the terms is fundamental to reselect the final addition operations.
It is also very important to use the correct form coefficients to locate the numbers and make the addition in columns.
This potentiation algorithm opens the door for the development of polynomials elevated to the fourth, fifth, etc.
The geometric representations of the coefficients developed in this document are novel and show a predictable, calculable and amazing expansion.

.Teacher Zeolla Gabriel Martin

**Works of the author linked to paper**

Zeolla Gabriel Martin, New multiplication algorithm, http://vixra.org/abs/1811.0320
Zeolla Gabriel Martin, Algoritmo de multiplicación distributivo, http://vixra.org/abs/1903.0167
Zeolla Gabriel Martin, Simple Tesla algorithm, http://vixra.org/abs/1909.0215
Zeolla Gabriel Martin, New cubic potentiation algorithm, http://vixra.org/abs/1905.0098
Zeolla Gabriel Martin, Square Power Algorithm, Square of a Binomial, Trinomial, Tetranomial and Pentanomial. http://vixra.org/abs/1904.0446
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