A NEW PRIMALITY TEST USING FIBONNACI NUMBERS?

ABSTRACT. In this paper, we ask whether a heuristic test for prime numbers can be derived from the Fibonacci numbers. The results below test for values up to F_{75} show that we might have a heuristic test for prime numbers akin to Fermat's Little Theorem.

Introduction

In early mathematics, it was thought that Mersenne Primes of the form $2^n - 1$ were prime when n is also prime. This certainly holds true for the first few values of n:

$$2^{2} - 1 = 3$$

 $2^{3} - 1 = 7$
 $2^{5} - 1 = 31$
 $2^{7} - 1 = 127$

However, it does not hold for all primes n. For example,

$$2^{11} - 1 = 2047 = 23.89.$$

In 1640, Fermat showed that it was not true of n = 23 and n = 37:

$$2^{23} - 1 = 8388607 = 47.178481$$
$$2^{37} - 1 = 137438953471 = 223.616318177.$$

In the same year Fermat proved that if p is a prime number, then for any integer a, the number $a^p - a$ is an integer multiple of p. This is known as Fermat's Little Theorem. So for the first few, where a = 2, we get:

$$2^{2} - 2 = 2.1$$

$$2^{3} - 2 = 3.2$$

$$2^{5} - 2 = 5.6$$

$$2^{7} - 2 = 7.18$$

$$2^{11} - 2 = 11.186$$

$$2^{13} - 2 = 13.630$$

$$2^{17} - 2 = 17.7710$$

All the results for prime exponents to infinity are divisible by the prime exponent that produced them. But if p is composite then the result is not divisible by p. It is therefore the basis for the so-called 'Fermat primality test' and is one of the fundamental results of elementary number theory.

Date: Sept 2019.

²⁰¹⁰ Mathematics Subject Classification. Primary 11B39, 11A41.

Key words and phrases. Fibonnaci, Primes.

However, the converse is not true. In early China it was wrongly thought that a number, n, is prime if the expression $2^n - 2$ is a multiple of n. So $2^3 - 2$ is divisible by $3, 2^5 - 2$ is divisible by 5 and so on. But $2^{341} - 2$ is divisible by $341 (=11 \times 31)$.

Fibonacci Numbers. Here we examine a similar text for primes using the Fibonacci sequence. The Fibonacci numbers are:

They are the sequence of numbers $F_{n(n=1)}^{\infty}$ defined by the linear recurrence equation

$$F_n = F_{(n-1)} + F_{(n-2)}$$

where $F_0 = 0$ and $F_1 = F_2 = 1$.

In the following conjecture, we are not concerning ourselves directly with Fibonacci primes. A Fibonacci prime is a Fibonacci number F_n that is also a prime number, e.g. 2,3,5,13,89.... It is also known that every F_n that is prime must have a prime index n, with the exception of $F_4 = 3$. However, the converse is not true (i.e., not every prime index p gives a prime F_p), e.g. $F_{19} = 4181 = 37.113$.

Existing Fibonacci primality tests. A Fibonacci primality test already exists, but as I am aware, not for all primes. Lucas, and later Lehmer also explored using the Fibonacci sequence and more general Lucas sequences to test n for primality.¹ For example, if $p \equiv \pm 2 \pmod{5}$, then $u_{p+1} \equiv 0 \pmod{p}$, where u_k denotes the kth Fibonacci number. This can be turned into a primality criterion for numbers $n \equiv \pm 2 \pmod{5}$ provided you have the prime factorization of n + 1, or a large factored portion. For $n \not\equiv \pm 2 \pmod{5}$ we can use other Lucas sequences.

This has led John Selfridge to conjecture that if p is an odd number, and $p \equiv \pm \pmod{5}$, then p will be prime if both of the following hold:²

$$2^p - 1 \equiv 1 \pmod{p},$$

 $f_{p+1} \equiv 0 \pmod{p},$

where f_k is the k^{th} Fibonacci number. The first condition is the Fermat primality test using base 2.

In general, if $p \equiv a \pmod{x^2+4}$, where a is a quadratic non-residue (mod x^2+4) then p should be prime if the following conditions hold:

$$2^{p} - 1 \equiv 1 \pmod{p},$$

$$f(x)_{p+1} \equiv 0 \pmod{p},$$

then f(x)k is the k-th Fibonacci polynomial at x.

¹https://people.csail.mit.edu/vinodv/COURSES/MAT302-S13/pomerance.pdf

 $^{^{2}} https://en.wikipedia.org/wiki/Primality-testHeuristic-tests$

Conjecture. Here we conjecture that (except for p = 5) if p is prime, then p will always divide $F_p + 1$ (if F_p terminates in the digits 3 or 7) or will divide $F_p - 1$ (if F_p terminates in the digits 1 or 9). The same will happen only for 2p, where 2p will divide $F_{2p} \pm 1$ (under equivalent conditions). Stated alternatively, for all $p, F_p \equiv \pm 1 \pmod{p}$, $F_{2p} \equiv \pm 1 \pmod{2p}$. For all other composites, $F_n \not\equiv \pm 1 \pmod{n}$.

The table below gives the results up to n = 75. In the first column, n, the prime values of n are highlighted in bold; the second column is the Fibonacci sequence; the third and fourth rows are, respectively, the necessary calculations to 2 decimal places (integer results for p are marked in bold, and for 2p are marked []*); the last column shows whether 1 was added or subtracted. Note that the only case for which this does not work is n = 5, n = 10.

n	F_n	$\frac{F_n+1}{n}$	$\frac{F_n-1}{n}$	$\equiv \pm 1$
		16	16	\pmod{p}
1	1	2.00 (trivial)	0.00	-
2	1	1.00	0.00	+1
3	2	1.00	0.33	+1
4	3	[1.00]*	0.50	+1
5	5	1.20	0.80	-
6	8	1.50	1.17	-
7	13	2.00	1.71	+1
8	21	2.75	2.50	-
9	34	3.89	3.67	-
10	55	5.60	5.40	-
11	89	8.18	8.00	-1
12	144	12.08	11.92	-
13	233	18.00	17.85	+1
14	377	[27.00]*	26.86	-
15	610	40.73	40.60	-
16	987	61.75	61.63	-
17	1597	94.00	93.88	+1
18	2584	143.61	143.50	_
19	4181	220.11	220.00	-1
20	6765	338.30	338.20	-
21	10946	521.29	521.19	-
22	17711	805.09	[805.00]*	-
23	28657	1246.00	1245.91	+1
24	46368	1932.04	1931.96	-
25	75025	3001.04	3000.96	-
26	121393	[4669.00]*	4668.92	-
27	196418	7274.78	7274.70	_
28	317811	11350.43	11350.36	-
29	514229	17732.07	17732.00	-1
30	832040	27734.70	27734.63	-
31	1346269	43428.06	43428.00	-1

Continued on next page

n r_n r	comm	E	F_n+1	F_n-1	— ⊥1
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49 7778742049 158749837.76 158749837.71 - 50 12586269025 251725380.52 251725380.48 - 51 20365011074 399313942.65 399313942.61 - 52 32951280099 633678463.46 633678463.42 - 53 53316291173 1005967758.00 100596775.96 +1 54 86267571272 1597547616.17 1597547616.13 - 55 139583862445 2537888408.11 2537888408.07 - 56 225851433717 4033061316.39 4033061316.36 - 57 365435296162 6411145546.72 6411145546.68 - 58 591286729879 10194598791.03 [10194598791.00]* - 59 956722026041 16215627560.03 16215627560.00 -1 60 1548008755920 25800145932.02 25800145931.98 - 61 2504730781961 41061160360.03 41061160360.00 -1 62 4052739537881 65366766740.03 [65366766740.00]* - 63 6557470319842 10	48	4807526976	100156812.02	100156811.98	-
50 12586269025 251725380.52 251725380.48 - 51 20365011074 399313942.65 399313942.61 - 52 32951280099 633678463.46 633678463.42 - 53 53316291173 1005967758.00 1005967757.96 +1 54 86267571272 1597547616.17 1597547616.13 - 55 139583862445 2537888408.11 2537888408.07 - 56 225851433717 4033061316.39 4033061316.36 - 57 365435296162 6411145546.72 6411145546.68 - 58 591286729879 10194598791.03 [10194598791.00]* - 60 1548008755920 25800145932.02 25800145931.98 - 61 2504730781961 41061160360.03 41061160360.00 -1 62 4052739537881 65366766740.03 [6576470319842 104086830473.67 - 63 6557470319842 104086830473.70 104086830473.67 - 64 10610209	49	7778742049	158749837.76	158749837.71	
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52 32951280099 633678463.46 633678463.42 - 53 53316291173 1005967758.00 1005967757.96 +1 54 86267571272 1597547616.17 1597547616.13 - 55 139583862445 2537888408.11 2537888408.07 - 56 225851433717 4033061316.39 4033061316.36 - 57 365435296162 6411145546.72 6411145546.68 - 58 591286729879 10194598791.03 [10194598791.00]* - 60 1548008755920 25800145932.02 25800145931.98 - 61 2504730781961 41061160360.03 41061160360.00 -1 62 4052739537881 65366766740.03 [65366766740.00]* - 63 6557470319842 104086830473.70 104086830473.67 - 64 10610209857723 165784529026.94 165784529026.91 - 65 17167680177565 264118156577.94 264118156577.94 - 66 27777890035288 420877121746.80 420877121746.77 - 67	51	20365011074	399313942.65	399313942.61	
53 53316291173 1005967758.00 1005967757.96 +1 54 86267571272 1597547616.17 1597547616.13 - 55 139583862445 2537888408.11 2537888408.07 - 56 225851433717 4033061316.39 4033061316.36 - 57 365435296162 6411145546.72 6411145546.68 - 58 591286729879 10194598791.03 [10194598791.00]* - 60 1548008755920 25800145932.02 25800145931.98 - 61 2504730781961 41061160360.03 41061160360.00 -1 62 4052739537881 65366766740.03 [65366766740.00]* - 63 6557470319842 104086830473.70 104086830473.67 - 64 10610209857723 165784529026.94 165784529026.91 - 65 17167680177565 264118156577.94 264118156577.91 - 66 27777890035288 420877121746.80 420877121746.77 - 67 4494557021285	52	32951280099	633678463.46	633678463.42	-
54 86267571272 1597547616.17 1597547616.13 - 55 139583862445 2537888408.11 2537888408.07 - 56 225851433717 4033061316.39 4033061316.36 - 57 365435296162 6411145546.72 6411145546.68 - 58 591286729879 10194598791.03 [10194598791.00]* - 59 956722026041 16215627560.03 16215627560.00 -1 60 1548008755920 25800145932.02 25800145931.98 - 61 2504730781961 41061160360.03 41061160360.00 -1 62 4052739537881 65366766740.03 [65366766740.00]* - 63 6557470319842 104086830473.70 104086830473.67 - 64 10610209857723 165784529026.94 165784529026.91 - 65 17167680177565 264118156577.94 264118156577.91 - 66 27777890035288 420877121746.80 420877121746.77 - 67 44945570212853 670829406162.00 670829406161.97 +1 <td< td=""><td>53</td><td>53316291173</td><td>1005967758.00</td><td>1005967757.96</td><td>+1</td></td<>	53	53316291173	1005967758.00	1005967757.96	+1
55 139583862445 2537888408.11 2537888408.07 - 56 225851433717 4033061316.39 4033061316.36 - 57 365435296162 6411145546.72 6411145546.68 - 58 591286729879 10194598791.03 [10194598791.00]* - 59 956722026041 16215627560.03 16215627560.00 -1 60 1548008755920 25800145932.02 25800145931.98 - 61 2504730781961 41061160360.03 41061160360.00 -1 62 4052739537881 65366766740.03 [65366766740.00]* - 63 6557470319842 104086830473.70 104086830473.67 - 64 10610209857723 165784529026.94 165784529026.91 - 65 17167680177565 264118156577.94 264118156577.91 - 66 27777890035288 420877121746.80 420877121746.77 - 67 44945570212853 670829406162.00 670829406161.97 +1 68 72723460248141 1069462650707.97 1069462650707.94 -	54	86267571272	1597547616.17	1597547616.13	-
56 225851433717 4033061316.39 4033061316.36 - 57 365435296162 6411145546.72 6411145546.68 - 58 591286729879 10194598791.03 $[10194598791.00]^*$ - 59 956722026041 16215627560.03 16215627560.00 -1 60 1548008755920 25800145932.02 25800145931.98 - 61 2504730781961 41061160360.03 41061160360.00 -1 62 4052739537881 65366766740.03 $[65366766740.00]^*$ - 63 6557470319842 104086830473.70 104086830473.67 - 64 10610209857723 165784529026.94 165784529026.91 - 65 17167680177565 264118156577.94 264118156577.91 - 66 27777890035288 420877121746.80 420877121746.77 - 67 44945570212853 670829406162.00 670829406161.97 +1 68 72723460248141 1069462650707.97 1069462650707.94 - 69 117669030460994 1705348267550.65 1705348267550.62 - 70 190392490709135 2719892724416.23 2719892724416.20 - 71 308061521170129 4338894664368.03 4338894664368.00 -1 72 498454011879264 6922972387212.01 6922972387211.99 - 73 806515533049393 $11048157986978.0011048157986978.00$ +1 74 1304969544928660 $[17634723580117.00]^*17634723580117$	55	139583862445	2537888408.11	2537888408.07	-
57 365435296162 6411145546.72 6411145546.68 - 58 591286729879 10194598791.03 $[10194598791.00]^*$ - 59 956722026041 16215627560.03 16215627560.00 -1 60 1548008755920 25800145932.02 25800145931.98 - 61 2504730781961 41061160360.03 41061160360.00 -1 62 4052739537881 65366766740.03 $[65366766740.00]^*$ - 63 6557470319842 104086830473.70 104086830473.67 - 64 10610209857723 165784529026.94 165784529026.91 - 65 17167680177565 264118156577.94 264118156577.91 - 66 27777890035288 420877121746.80 420877121746.77 - 67 44945570212853 670829406162.00 670829406161.97 +1 68 72723460248141 1069462650707.97 1069462650707.94 - 69 117669030460994 1705348267550.65 1705348267550.62 - 70 190392490709135 2719892724416.23 2719892724416.20 - 71 308061521170129 4338894664368.03 4338894664368.00 -1 72 498454011879264 6922972387212.01 6922972387211.99 - 73 806515533049393 $11048157986978.0011048157986978.00$ +1 74 1304969544928660 $[17634723580117.00]^* 17634723580117.00$ -	56	225851433717	4033061316.39	4033061316.36	-
58 591286729879 10194598791.03 [10194598791.00]* - 59 956722026041 16215627560.03 16215627560.00 -1 60 1548008755920 25800145932.02 25800145931.98 - 61 2504730781961 41061160360.03 41061160360.00 -1 62 4052739537881 65366766740.03 [65366766740.00]* - 63 6557470319842 104086830473.70 104086830473.67 - 64 10610209857723 165784529026.94 165784529026.91 - 65 17167680177565 264118156577.94 264118156577.91 - 66 27777890035288 420877121746.80 420877121746.77 - 67 44945570212853 670829406162.00 670829406161.97 +1 68 72723460248141 1069462650707.97 1069462650707.94 - 69 117669030460994 1705348267550.65 1705348267550.62 - 70 190392490709135 2719892724416.23 2719892724416.20 - 71 308061521170129 4338894664368.03 4338894664368.00 -1 <td>57</td> <td>365435296162</td> <td>6411145546.72</td> <td>6411145546.68</td> <td>-</td>	57	365435296162	6411145546.72	6411145546.68	-
59 95672202604116215627560.03 16215627560.00 -160154800875592025800145932.0225800145931.98- 61 250473078196141061160360.03 41061160360.00 -162405273953788165366766740.03[65366766740.00]*-636557470319842104086830473.70104086830473.67-6410610209857723165784529026.94165784529026.91-6517167680177565264118156577.94264118156577.91-6627777890035288420877121746.80420877121746.77-6744945570212853 670829406162.00 670829406161.97+168727234602481411069462650707.971069462650707.94-691176690304609941705348267550.651705348267550.62-701903924907091352719892724416.232719892724416.20-713080615211701294338894664368.03 4338894664368.00 -1724984540118792646922972387212.016922972387211.99-73806515533049393 11048157986978.00 11048157986978.00+1741304969544928660[17634723580117.00]* 17634723580117.00-	58	591286729879	10194598791.03	[10194598791.00]*	-
60154800875592025800145932.0225800145931.98-61250473078196141061160360.0341061160360.00-162405273953788165366766740.03[65366766740.00]*-636557470319842104086830473.70104086830473.67-6410610209857723165784529026.94165784529026.91-6517167680177565264118156577.94264118156577.91-6627777890035288420877121746.80420877121746.77-6744945570212853 670829406162.00 670829406161.97+168727234602481411069462650707.971069462650707.94-691176690304609941705348267550.651705348267550.62-701903924907091352719892724416.232719892724416.20-713080615211701294338894664368.03 4338894664368.00 -1724984540118792646922972387212.016922972387211.99-73806515533049393 11048157986978.00 11048157986978.00+1741304969544928660[17634723580117.00]*17634723580117.00-	59	956722026041	16215627560.03	16215627560.00	-1
61250473078196141061160360.0341061160360.00-162405273953788165366766740.03[65366766740.00]*-636557470319842104086830473.70104086830473.67-6410610209857723165784529026.94165784529026.91-6517167680177565264118156577.94264118156577.91-6627777890035288420877121746.80420877121746.77-6744945570212853 670829406162.00 670829406161.97+168727234602481411069462650707.971069462650707.94-691176690304609941705348267550.651705348267550.62-701903924907091352719892724416.232719892724416.20-713080615211701294338894664368.03 4338894664368.00 -1724984540118792646922972387212.016922972387211.99-73806515533049393 11048157986978.00 +1741304969544928660[17634723580117.00]* 17634723580117.00-	60	1548008755920	25800145932.02	25800145931.98	-
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	61	2504730781961	41061160360.03	41061160360.00	-1
$\begin{array}{rrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrr$	62	4052739537881	65366766740.03	[65366766740.00]*	-
$\begin{array}{rrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrr$	63	6557470319842	104086830473.70	104086830473.67	-
$\begin{array}{rrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrr$	64	10610209857723	165784529026.94	165784529026.91	-
66 27777890035288 420877121746.80 420877121746.77 - 67 44945570212853 670829406162.00 670829406161.97 +1 68 72723460248141 1069462650707.97 1069462650707.94 - 69 117669030460994 1705348267550.65 1705348267550.62 - 70 190392490709135 2719892724416.23 2719892724416.20 - 71 308061521170129 4338894664368.03 4338894664368.00 -1 72 498454011879264 6922972387212.01 6922972387211.99 - 73 806515533049393 11048157986978.00 +1 74 1304969544928660 [17634723580117.00]* 17634723580117.00 -	65	17167680177565	264118156577.94	264118156577.91	-
67 44945570212853 670829406162.00 670829406161.97 +1 68 72723460248141 1069462650707.97 1069462650707.94 - 69 117669030460994 1705348267550.65 1705348267550.62 - 70 190392490709135 2719892724416.23 2719892724416.20 - 71 308061521170129 4338894664368.03 4338894664368.00 -1 72 498454011879264 6922972387212.01 6922972387211.99 - 73 806515533049393 11048157986978.00 +1 74 1304969544928660 [17634723580117.00]* 17634723580117.00 -	66	27777890035288	420877121746.80	420877121746.77	-
68 72723460248141 1069462650707.97 1069462650707.94 - 69 117669030460994 1705348267550.65 1705348267550.62 - 70 190392490709135 2719892724416.23 2719892724416.20 - 71 308061521170129 4338894664368.03 4338894664368.00 -1 72 498454011879264 6922972387212.01 6922972387211.99 - 73 806515533049393 11048157986978.00 11048157986978.00 +1 74 1304969544928660 [17634723580117.00]* 17634723580117.00 -	67	44945570212853	670829406162.00	670829406161.97	+1
69 117669030460994 1705348267550.65 1705348267550.62 - 70 190392490709135 2719892724416.23 2719892724416.20 - 71 308061521170129 4338894664368.03 4338894664368.00 -1 72 498454011879264 6922972387212.01 6922972387211.99 - 73 806515533049393 11048157986978.00 11048157986978.00 +1 74 1304969544928660 [17634723580117.00]* 17634723580117.00 -	68	72723460248141	1069462650707.97	1069462650707.94	-
70 190392490709135 2719892724416.23 2719892724416.20 - 71 308061521170129 4338894664368.03 4338894664368.00 -1 72 498454011879264 6922972387212.01 6922972387211.99 - 73 806515533049393 11048157986978.00 11048157986978.00 +1 74 1304969544928660 [17634723580117.00]* 17634723580117.00 -	69	117669030460994	1705348267550.65	1705348267550.62	-
71 308061521170129 4338894664368.03 4338894664368.00 -1 72 498454011879264 6922972387212.01 6922972387211.99 - 73 806515533049393 11048157986978.00 +1 74 1304969544928660 [17634723580117.00]* 17634723580117.00 -	70	190392490709135	2719892724416.23	2719892724416.20	-
72 498454011879264 6922972387212.01 6922972387211.99 - 73 806515533049393 11048157986978.0011048157986978.00 +1 74 1304969544928660 [17634723580117.00]* 17634723580117.00 -	71	308061521170129	4338894664368.03	4338894664368.00	-1
73 806515533049393 11048157986978.0011048157986978.00 +1 74 1304969544928660 [17634723580117.00]* 17634723580117.00 -	72	498454011879264	6922972387212.01	6922972387211.99	-
74 1304969544928660 [17634723580117.00]* 17634723580117.00 -	73	806515533049393	11048157986978.0	011048157986978.00	+1
	74	1304969544928660	[17634723580117.00]	* 17634723580117.00	-

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n	F_n	$\underline{F_n+1}$	F_{n-1}	$\equiv \pm 1$	
		n	n	\pmod{p}	
75	2111485077978050	28153134373040.70	28153134373040.70	-	

Table 1: It ends from the previous page.

Interestingly, the composite 341, a pseudoprime for Fermat's Little Theorem, properly follows the algorithm:

 $F_{341} = 82281144336295989585340713815384441479925901307982452831610787275979941$ And indeed,

$$\frac{F_{341} - 1}{341} =$$

645161290322580645161290322580645161290322580645161290323.

...a fraction, as expected.

But the critical question is: are there any pseudoprimes using this test?

References

1. http://mathworld.wolfram.com/FibonacciNumber.html. Last accessed 21.09.19.

 $2.\ http://www.maths.surrey.ac.uk/hosted-sites/R.Knott/Fibonacci/fibtable301.html$

3. http://www.maths.surrey.ac.uk/hosted-sites/R.Knott/contactron.htmlsection1.1

4. https://en.wikipedia.org/wiki/Primality-testHeuristic-tests

5. https://people.csail.mit.edu/vinodv/COURSES/MAT302-S13/pomerance.pdf

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