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Number and arithmetic operation

The meaning (logical content) of the concept of numbers is revealed. Arithmetic operations are defined.

Definition of numbers

If the description of the situation is difficult and requires many words because of its complexity, it is replaced by a specific term (name of the situation) to achieve brevity and clarity in all judgments about this situation in which it should appear as a part of the sentence.

This applies to situations involving the object of our interest (OOI).

For example, absence of OOI is designated by the term "zero", they say "there is zero of objects" or "number zero is set" instead of "OOI is not present", "OOI are absent".

Another situation of our interest (SOI) – "There is an OOI and, except for it, there are no other objects that fall under the definition of OOI" – is briefly denoted by the term "one", they say "there is one object" or "number one is set", without description of the situation.

SOI "There is an OOI, there is another object that falls under the definition of an OOI, and there are no other objects that fall under the definition of an OOI apart from those mentioned" is denoted by the term "two", they say "there are two objects" or "number two is given".

SOI "there are two OOIs, there is another object that falls under the definition of an OOI, and there are no other objects that fall under the definition of an OOI apart from those mentioned" is denoted by the term "three", they say "there are three OOIs" or "number three is set", etc.

Therefore, the numbers are ***the names of the various SOIs related to the presence of OOI***. So, ***we know what a number is.***

Definitions of mathematics

This is very simple.

Mathematics has no direct definition of numbers. Neither preliminary, requiring clarification, as in Euclid, nor final. *Nothing whatsoever*.

There are statements about the "centuries-old experience of abstraction and generalization" of mankind, i.e. *non mathematicians*. Statements that coexist with opposite claims about the inability to abstract the "savages", i.e. the mankind on most of its history.

This is rarely said directly. For example:

"The concept of natural number is one of the simplest concepts. It can only be explained by a subject presentation.

Note: Euclid (III B.C.) defined the number (natural) as "a multitude made up of units"; such definitions can be found in many current textbooks. But the word "set" (or "assembly", or "population", etc.) is no clearer than the word "number"." [1]

The term "elementary mathematics" is used here to mislead. To embarrass the student to ask any questions. That is, to *disable* him/her, because everything here is "elementary". This intentional disabling is the reason why this issue is still unresolved. Although those who have mastered "elementary" mathematics are considered to have not elementary, but "secondary" education. However, even when they receive "higher" education, they do not return to it anymore. This issue is considered to be well studied at the "elementary" level. Or a subject of excessive philosophizing.

This is the first universal way to hide ignorance: *what cannot be defined should be called obvious or elementary.*

In mathematics, "knowledge of numbers" is just knowing the rules of its handling, ensuring the performance of "arithmetic operations", the meaning of which may also be unknown.

Here is the statement of the same source:

"The concept of the addition arises from such simple facts that it does not need to be defined and cannot be formally defined.

Note: there are many "definitions" such as "addition is the action by which several numbers are joined together into one" or "the process of determining how many ones are contained in several numbers together." But if somebody doesn't know what it means to "add", they don't know what it means to "join the numbers together", so all such "definitions" are just replacing one word with another."

Instead of explaining the meaning of the addition, it is stated that all these are "simple facts". Although criticism of Kant begins with the question about such "simple fact" at the suggestion of Leibniz [2]. This criticism has resulted in a large philosophical discourse. It is exactly the words of Kant that the summands "join together in one number" (sum), as if merging or "synthesizing", like atoms in the molecule. Such superficial analogy does not give a real understanding of the meaning of this operation.

This quote in the absence of a definition is certainly correct.

But the statement that the addition *"cannot be formally defined"* does not follow from here and remains only the author's opinion. Something like "unknown, therefore, impossible". Simple logical error.

Numeral system

Numbers are just SOI names. Therefore, its study comes down to the development of a method of naming. There can be only two of them, *random* and *non-random* naming. Both are used at once, forming a combined method called the *numeral system*.

SOI can always include one or more additional OOIs, which in turn form some SOI. The differences in SOIs obtained by this combination of other SOI can be infinite.

The numeral system problem is precisely this infinite variety of SOIs, which requires the same variety of names. Theoretically, it is not difficult to imagine this infinite variety. However, its practical implementation is *impossible*, because such list cannot be completed, not that it is learned. Therefore, all *infinity* of different SOIs should be covered by a *finite* set of different names. Memory capabilities are also finite and may require a small number of different names. Therefore, only ten random names are used in a written record: 0, 1, 2, ..., 9, although there may be fewer, for example, 0, 1, or more than ten.

Other names are *descriptions* of the way SOI is obtained.

They are formed as follows.

Random names are used repeatedly to indicate *different* SOIs.

These SOIs differ not by the presence of OOI, which is the same when random names coincide as defined, but by the OOI itself.

The initial OOI is random, all others are *non-random* and are formed by SOI.

These SOIs are always formed by the largest of the previous SOIs, including one additional OOI.

SOI of the 1st generation (SOI_1) is formed by the OOI of the 1st type (OOI_1), which can be a randomly selected object. SOI_1 has random names: 0, 1, 2, 3, …, 9.

SOI of the 2nd generation (SOI_2) is formed by OOI_2, which is $SOI_1 = 9OOI_1 + 1OOI_1$ ("+" means inclusion, "=" means identity) or $OOI_2 = (9 + 1)\ OOI_1$.

SOI_2 has the same random names as SOI_1: 0, 1, 2, 3, …, 9.

SOI of the 3rd generation (SOI_3) is formed by $OOI_3 = (9 + 1)\ OOI_2$. SOI_3 has the same random names as SOI_1, SOI_2: 0, 1, 2, 3, …, 9.

SOI_4 is formed by $OOI_4 = (9 + 1)\ OOI_3$, etc.

As a result, you can get as many *composite names* as you like for randomly set SOIs, which differ among themselves, using only 10 initial random names belonging to different SOI_1, SOI_2, etc.

So, there are not only random names from 0 to 9, but also composite names.

Composite names are *descriptions* of the method of obtaining SOI.

Arithmetic operation

The same SOI can be obtained in different ways, but with different descriptions.

For example, 7 + 5 or 12. The first SOI is obtained by combining SOI_1=7 OOI_1 with SOI_1=5 OOI_1, and the second SOI is obtained by combining SOI_2=1 OOI_2 with SOI_1=2 OOI_1.

As a result, the same SOI has different descriptions, determined by the way it is obtained. Which is expressed by the following equality: SOI = 7 + 5 = 12.

Depending on the way it is obtained, any SOI can have many different descriptions, not one.

How to identify such SOI, which has different descriptions used as names?

The answer is the following: only *one* description of all possible descriptions is accepted as a *standard* description, by which only SOI is recognized. All other descriptions are *non-standard*. They can be freely used to describe the actual method of obtaining SOI. However, the SOI itself is considered *unidentified*. Its identification requires a transition from a random non-standard description to a standard description.

This ***transition from a non-standard description to a standard description is called an arithmetic operation.***

This applies to any operation – addition, subtraction, multiplication, division, exponentiation or root extraction. Although some may be formally defined through others, for example, subtraction is the opposite of addition. But the *first* one, which "cannot be formally defined" in the mathematician's opinion, is according to this definition.

Standard description

The standard description is made according to the following rules:

1. The random names SOI_1, SOI_2, SOI_3, etc. are arranged in a certain sequence from right to left.
2. If there is SOI formed by OOI_2, OOI_3, etc., all SOIs formed by the previous OOIs should be specified.
3. The extreme left SOI cannot be equal to zero.
4. The SOI formed by only one SOI_1 may be equal to zero.
5. Each SOI_1, SOI_2, etc. can be used once in the description.
6. Each SOI_1, SOI_2, etc. can be included in the description of SOI by only one operation – the inclusion expressed as a "+" sign.

Only in this case the designations of all OOI1, OOI2, ... forming the description of SOI can be omitted together with the signs of their inclusion in the set SOI without violating its understanding.

Non-standard descriptions

Other SOI descriptions that specify different ways of obtaining it are non-standard. They are expressed by the arithmetic operations of subtraction, multiplication, division, exponentiation or root extraction. Or addition, if any SOI_1, SOI_2, etc. is used more than once in the description.

To identify SOI, any non-standard description should be given to a standard description expressed through random names SOI_1, SOI_2, etc. This is the meaning of arithmetic operations.

Illustrative examples

1. The description of SOI = 7 OOI_1+ 5OOI_1 is not standard because it contains SOI_1 more than once. Here, the OOI_1 designations themselves can be omitted without prejudice to understanding, and the description can be shortened to SOI = 7 + 5. But the "+" sign cannot be omitted, as it is not a standard description.

2. The description of the same SOI = 1 OOI_2 + 2 OOI_1 is standard. Both the designations of OOI_1, OOI_2, and the sign "+" of the inclusion of SOI_1, SOI_2 in the described SOI can be omitted without prejudice to understanding.

Therefore, the description of SOI can be reduced to SOI = 12 without prejudice to understanding.

3. The description of the same SOI = 2 OOI_1 + 1 OOI_2, expressing the possible real way to obtain it, is non-standard, because the correct sequence of the location of SOI_1, SOI_2 is broken. Therefore, its order must be changed, after which it receives a standard description, expressed in just 12.

4. The description of SOI = 1OOI_2 is non-standard because it does not contain the SOI formed by OOI_1. The standard description should look like SOI = 1 OOI_2 + 0 OOI_1, after which it can be omitted without prejudice to the understanding of both the designation of OOI_1, OOI_2, and the sign of inclusion of SOI_1, SOI_2 formed by it in the described SOI. This gives a standard abbreviated description of SOI = 10.

5. The description of SOI = 7 OOI_1 – 5 OOI_1 is non-standard, as it is formed not by the inclusion of SOI_1, designated by the sign "+", and the exception, designated by the sign "-". Its standard random name is SOI = 2.

The same applies to multiplication, division, exponentiation and root extraction operations. SOI descriptions expressed through these operations are non-standard and requires the SOI to be brought to a standard description for identification. This is the meaning of arithmetic operations.

References:

1. M.Y. Vygodskiy Elementary Mathematics Handbook.
2. I. Kant. Critique of Pure Reason.

Note. This publication is an abridgement of the article http://sciteclibrary.ru/rus/catalog/pages/8664.html dated 12.06.2007.