Formula of ζ odd-numbers

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Abstract

I tried to find a new expression for zeta odd-numbers.
It may be a new expression and will be published here.
The correctness of this formula was confirmed by WolframAlpha to be numerically completely correct.

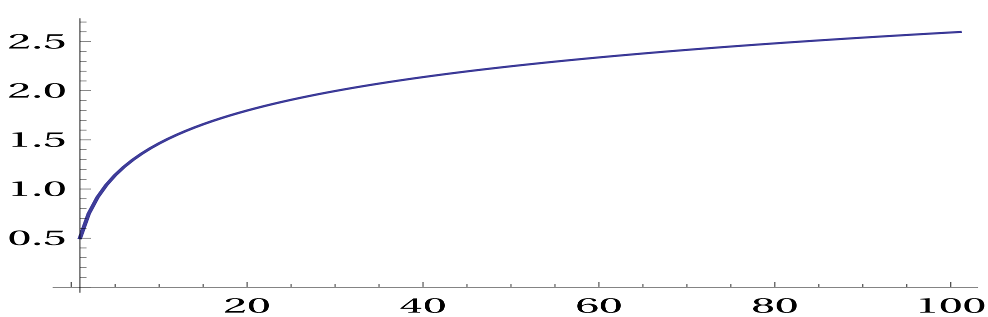
key words
zeta odd-numbers, new expression, formula

1 Introduction

$$\zeta(1)=\sum_{n=1}^{\infty}\frac{1}{(2n-1)^1}+\sum_{n=1}^{\infty}\frac{1}{(2n)^1}=\frac{1}{2^1}\sum_{n=1}^{\infty}\frac{1}{n^1}+\sum_{n=1}^{\infty}\frac{1}{(2n-1)^1}=\frac{1}{2^1}\zeta(1)+\sum_{n=1}^{\infty}\frac{1}{(2n-1)^1} \tag{1}$$

$$\zeta(1)=\frac{2^1}{2^1-1}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^1}=\frac{2}{1}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^1} \tag{2}$$

If n=$\infty = 10^{2199}$
$\zeta(1)$=2 $\times$ 762.403932858110634134549973588306337814533477397070613 7050...



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10^{2199} seemed to be the limit of the calculator.

do the same

$$\zeta(3)=\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}+\sum_{n=1}^{\infty}\frac{1}{(2n)^3}=\frac{1}{2^3}\sum_{n=1}^{\infty}\frac{1}{n^3}+\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}=\frac{1}{2^3}\zeta(3)+\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3} \tag{3}$$

$$\zeta(3)=\frac{2^3}{2^3-1}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}=\frac{8}{7}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3} \tag{4}$$

If ∞=10
$\zeta(3)\approx 1.2013...$
If ∞=20
$\zeta(3)\approx 1.2018...$
∞=40 seem to be the limit of the calculator.
If ∞=40
$\zeta(3)\approx 1.202012...$
$\zeta(3)=1.202056...$

2 Discussion

do the same

$$\zeta(5)=\sum_{n=1}^{\infty}\frac{1}{(2n-1)^5}+\sum_{n=1}^{\infty}\frac{1}{(2n)^5}=\frac{1}{2^5}\sum_{n=1}^{\infty}\frac{1}{n^5}+\sum_{n=1}^{\infty}\frac{1}{(2n-1)^5}=\frac{1}{2^5}\zeta(5)+\sum_{n=1}^{\infty}\frac{1}{(2n-1)^5} \tag{5}$$

$$\zeta(5)=\frac{2^5}{2^5-1}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^5}=\frac{32}{31}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^5} \tag{6}$$

If ∞=10
$\zeta(5)\approx 1.036926955...$
∞=20 seem to be the limit of the calculator.
If ∞=20
$\zeta(5)\approx 1.036927704...$
$\zeta(5)=1.036927755...$

do the same

$$\zeta(7)=\sum_{n=1}^{\infty}\frac{1}{(2n-1)^7}+\sum_{n=1}^{\infty}\frac{1}{(2n)^7}=\frac{1}{2^7}\sum_{n=1}^{\infty}\frac{1}{n^7}+\sum_{n=1}^{\infty}\frac{1}{(2n-1)^7}=\frac{1}{2^7}\zeta(7)+\sum_{n=1}^{\infty}\frac{1}{(2n-1)^7} \tag{7}$$

$$\zeta(7)=\frac{2^7}{2^7-1}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^7}=\frac{128}{127}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^7} \tag{8}$$

∞=10 seem to be the limit of the calculator.
If ∞=10
$\zeta(7)\approx 1.0083492760...$
$\zeta(7)=1.0083492773...$

do the same

$$\zeta(9)=\sum_{n=1}^{\infty}\frac{1}{(2n-1)^9}+\sum_{n=1}^{\infty}\frac{1}{(2n)^9}=\frac{1}{2^9}\sum_{n=1}^{\infty}\frac{1}{n^9}+\sum_{n=1}^{\infty}\frac{1}{(2n-1)^9}=\frac{1}{2^9}\zeta(9)+\sum_{n=1}^{\infty}\frac{1}{(2n-1)^9} \tag{9}$$

$$\zeta(9)=\frac{2^9}{2^9-1}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^9}=\frac{512}{511}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^9} \tag{10}$$

∞=10 seem to be the limit of the calculator.
If ∞=10
$\zeta(9)\approx 1.0020083928237...$
$\zeta(9)=1.0020083928260...$

do the same

$$\zeta(11)=\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{11}}+\sum_{n=1}^{\infty}\frac{1}{(2n)^{11}}=\frac{1}{2^{11}}\sum_{n=1}^{\infty}\frac{1}{n^{11}}+\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{11}}=\frac{1}{2^{11}}\zeta(11)+\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{11}} \tag{11}$$

$$\zeta(11)=\frac{2^{11}}{2^{11}-1}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{11}}=\frac{2048}{2047}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{11}} \tag{12}$$

∞=10 seem to be the limit of the calculator.
If ∞=10
$\zeta(11)\approx 1.00049418860411147...$
$\zeta(11)=1.00049418860411194...$

do the same

$$\zeta(13)=\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{13}}+\sum_{n=1}^{\infty}\frac{1}{(2n)^{13}}=\frac{1}{2^{13}}\sum_{n=1}^{\infty}\frac{1}{n^{13}}+\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{13}}=\frac{1}{2^{13}}\zeta(13)+\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{13}} \tag{13}$$

$$\zeta(13)=\frac{2^{13}}{2^{13}-1}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{13}}=\frac{8192}{8191}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{13}} \tag{14}$$

∞=10 seem to be the limit of the calculator.
If ∞=10
$\zeta(13) \approx 1.000122713347578479...$
$\zeta(13) = 1.000122713347578489...$

do the same

$$\zeta(15) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{15}} + \sum_{n=1}^{\infty} \frac{1}{(2n)^{15}} = \frac{1}{2^{15}} \sum_{n=1}^{\infty} \frac{1}{n^{15}} + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{15}} = \frac{1}{2^{15}}\zeta(15) + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{15}} \tag{15}$$

$$\zeta(15) = \frac{2^{15}}{2^{15}-1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{15}} = \frac{32768}{32767} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{15}} \tag{16}$$

∞=10 seem to be the limit of the calculator.
If ∞=10
$\zeta(15) \approx$ 1.000030588236307020473....
$\zeta(15) =$ 1.000030588236307020493...

do the same

$$\zeta(17) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{17}} + \sum_{n=1}^{\infty} \frac{1}{(2n)^{17}} = \frac{1}{2^{17}} \sum_{n=1}^{\infty} \frac{1}{n^{17}} + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{17}} = \frac{1}{2^{17}}\zeta(17) + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{17}} \tag{17}$$

$$\zeta(17) = \frac{2^{17}}{2^{17}-1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{17}} = \frac{131072}{131071} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{17}} \tag{18}$$

∞=6 seem to be the limit of the calculator.
If ∞=6
$\zeta(17) \approx 1.0000076371976378996 3...$
$\zeta(17) = 1.0000076371976378997 6...$

do the same

$$\zeta(19) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{19}} + \sum_{n=1}^{\infty} \frac{1}{(2n)^{19}} = \frac{1}{2^{19}} \sum_{n=1}^{\infty} \frac{1}{n^{19}} + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{19}} = \frac{1}{2^{19}}\zeta(19) + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{19}} \tag{19}$$

$$\zeta(19) = \frac{2^{19}}{2^{19}-1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{19}} = \frac{524288}{524287} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{19}} \tag{20}$$

∞=6 seem to be the limit of the calculator.

If ∞=6
$\zeta(19) \approx 1.000001908212716553938 19...$
$\zeta(19) = 1.00000190821271655393892...$

do the same

$$\zeta(21) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{21}} + \sum_{n=1}^{\infty} \frac{1}{(2n)^{21}} = \frac{1}{2^{21}} \sum_{n=1}^{\infty} \frac{1}{n^{21}} + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{21}} = \frac{1}{2^{21}} \zeta(21) + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{21}} \tag{21}$$

$$\zeta(21) = \frac{2^{21}}{2^{21}-1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{21}} = \frac{2097152}{2097151} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{21}} \tag{22}$$

∞=6 seem to be the limit of the calculator.
If ∞=6
$\zeta(21) \approx 1.00000047693298678780645 8...$
$\zeta(21) = 1.000000476932986787806463...$

do the same

$$\zeta(23) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{23}} + \sum_{n=1}^{\infty} \frac{1}{(2n)^{23}} = \frac{1}{2^{23}} \sum_{n=1}^{\infty} \frac{1}{n^{23}} + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{23}} = \frac{1}{2^{23}} \zeta(23) + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{23}} \tag{23}$$

$$\zeta(23) = \frac{2^{23}}{2^{23}-1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{23}} = \frac{8388608}{8388607} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{23}} \tag{24}$$

∞=6 seem to be the limit of the calculator.
If ∞=6
$\zeta(23) \approx 1.000000119209...$
$\zeta(23) = 1.000000119219...$

do the same

$$\zeta(25) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{25}} + \sum_{n=1}^{\infty} \frac{1}{(2n)^{25}} = \frac{1}{2^{25}} \sum_{n=1}^{\infty} \frac{1}{n^{25}} + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{25}} = \frac{1}{2^{25}} \zeta(25) + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{25}} \tag{25}$$

$$\zeta(25) = \frac{2^{25}}{2^{25}-1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{25}} = \frac{33554432}{33554431} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{25}} \tag{26}$$

∞=6 seem to be the limit of the calculator.
If ∞=6
$\zeta(25) \approx 1.0000000298035035146522801 8591...$

$\zeta(25) = 1.000000029803503514652280186\ldots$

do the same

$$\zeta(27) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{27}} + \sum_{n=1}^{\infty} \frac{1}{(2n)^{27}} = \frac{1}{2^{27}} \sum_{n=1}^{\infty} \frac{1}{n^{27}} + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{27}} = \frac{1}{2^{27}}\zeta(27) + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{27}} \tag{27}$$

$$\zeta(27) = \frac{2^{27}}{2^{27}-1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{27}} = \frac{134217728}{134217727} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{27}} \tag{28}$$

∞=6 seem to be the limit of the calculator.
If ∞=6
$\zeta(27) \approx 1.0000000074507117898354294919801\ldots$
$\zeta(27) = 1.0000000074507117898354294919810\ldots$

do the same

$$\zeta(29) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{29}} + \sum_{n=1}^{\infty} \frac{1}{(2n)^{29}} = \frac{1}{2^{29}} \sum_{n=1}^{\infty} \frac{1}{n^{29}} + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{29}} = \frac{1}{2^{29}}\zeta(29) + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{29}} \tag{29}$$

$$\zeta(29) = \frac{2^{29}}{2^{29}-1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{29}} = \frac{536870912}{536870911} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{29}} \tag{30}$$

∞=6 seem to be the limit of the calculator.
If ∞=6
$\zeta(29) \approx 1.0000000018626597235130490064039049\ldots$
$\zeta(29) = 1.0000000018626597235130490064039099\ldots$

do the same

$$\zeta(31) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{31}} + \sum_{n=1}^{\infty} \frac{1}{(2n)^{31}} = \frac{1}{2^{31}} \sum_{n=1}^{\infty} \frac{1}{n^{31}} + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{31}} = \frac{1}{2^{31}}\zeta(31) + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{31}} \tag{31}$$

$$\zeta(31) = \frac{2^{31}}{2^{31}-1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{31}} = \frac{2147483648}{2147483647} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{31}} \tag{32}$$

∞=4 seem to be the limit of the calculator.
If ∞=4
$\zeta(31) \approx 1.000000000465662906503378407296\ldots$
$\zeta(31) = 1.000000000465662906503378407298\ldots$

do the same

$$\zeta(33)=\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{33}}+\sum_{n=1}^{\infty}\frac{1}{(2n)^{33}}=\frac{1}{2^{33}}\sum_{n=1}^{\infty}\frac{1}{n^{33}}+\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{33}}=\frac{1}{2^{33}}\zeta(33)+\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{33}} \tag{33}$$

$$\zeta(33)=\frac{2^{33}}{2^{33}-1}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{33}}=\frac{8589934592}{8589934591}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{33}} \tag{34}$$

∞=4 seem to be the limit of the calculator.
If ∞=4
$\zeta(33)\approx 1.0000000001164155017270051977592 64...$
$\zeta(33)=1.000000000116415501727005197759297...$

do the same

$$\zeta(35)=\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{35}}+\sum_{n=1}^{\infty}\frac{1}{(2n)^{35}}=\frac{1}{2^{35}}\sum_{n=1}^{\infty}\frac{1}{n^{35}}+\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{35}}=\frac{1}{2^{35}}\zeta(35)+\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{35}} \tag{35}$$

$$\zeta(35)=\frac{2^{35}}{2^{35}-1}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{35}}=\frac{34359738368}{34359738367}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{35}} \tag{36}$$

∞=4 seem to be the limit of the calculator.
If ∞=4
$\zeta(35)\approx 1.0000000000291038504449709968692938...$
$\zeta(35)=1.0000000000291038504449709968692942...$

do the same

$$\zeta(37)=\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{37}}+\sum_{n=1}^{\infty}\frac{1}{(2n)^{37}}=\frac{1}{2^{37}}\sum_{n=1}^{\infty}\frac{1}{n^{37}}+\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{37}}=\frac{1}{2^{37}}\zeta(37)+\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{37}} \tag{37}$$

$$\zeta(37)=\frac{2^{37}}{2^{37}-1}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{37}}=\frac{137438953472}{137438953471}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{37}} \tag{38}$$

∞=4 seem to be the limit of the calculator.
If ∞=4
$\zeta(37)\approx 1.000000000007275959835057481014520864 0...$
$\zeta(37)=1.0000000000072759598350574810145208690...$

do the same

$$\zeta(39)=\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{39}}+\sum_{n=1}^{\infty}\frac{1}{(2n)^{39}}=\frac{1}{2^{39}}\sum_{n=1}^{\infty}\frac{1}{n^{39}}+\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{39}}=\frac{1}{2^{39}}\zeta(39)+\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{39}} \tag{39}$$

$$\zeta(39)=\frac{2^{39}}{2^{39}-1}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{39}}=\frac{549755813888}{549755813888}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{39}} \tag{40}$$

∞=4 seem to be the limit of the calculator.
If ∞=4
$\zeta(39) \approx 1.0000000000018189896503070659475848320\,3...$
$\zeta(39) = 1.0000000000018189896503070659475848321\,0...$

do the same

$$\zeta(41)=\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{41}}+\sum_{n=1}^{\infty}\frac{1}{(2n)^{41}}=\frac{1}{2^{41}}\sum_{n=1}^{\infty}\frac{1}{n^{41}}+\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{41}}=\frac{1}{2^{41}}\zeta(41)+\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{41}} \tag{41}$$

$$\zeta(41)=\frac{2^{41}}{2^{41}-1}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{41}}=\frac{2199023255552}{2199023255551}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{41}} \tag{42}$$

∞=4 seem to be the limit of the calculator.
If ∞=4
$\zeta(41) \approx 1.000000000000454747378304215402679911202\,19...$
$\zeta(41) = 1.000000000000454747378304215402679911202\,94...$

do the same

$$\zeta(43)=\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{43}}+\sum_{n=1}^{\infty}\frac{1}{(2n)^{43}}=\frac{1}{2^{43}}\sum_{n=1}^{\infty}\frac{1}{n^{43}}+\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{43}}=\frac{1}{2^{43}}\zeta(43)+\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{43}} \tag{43}$$

$$\zeta(43)=\frac{2^{43}}{2^{43}-1}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{43}}=\frac{8796093022208}{8796093022207}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{43}} \tag{44}$$

∞=4 seem to be the limit of the calculator.
If ∞=4
$\zeta(43) \approx 1.000000000000113686840768022784934910483\,79...$
$\zeta(43) = 1.000000000000113686840768022784934910483\,80...$

do the same

$$\zeta(45)=\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{45}}+\sum_{n=1}^{\infty}\frac{1}{(2n)^{45}}=\frac{1}{2^{45}}\sum_{n=1}^{\infty}\frac{1}{n^{45}}+\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{45}}=\frac{1}{2^{45}}\zeta(45)+\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{45}} \tag{45}$$

$$\zeta(45)=\frac{2^{45}}{2^{45}-1}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{45}}=\frac{35184372088832}{35184372088831}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{45}} \tag{46}$$

∞=4 seem to be the limit of the calculator.
If ∞=4
$\zeta(45) \approx 1.000000000000028421709768893018554550737049\,31...$
$\zeta(45) = 1.000000000000028421709768893018554550737049\,42...$

3 Conclusion

make official

$$\zeta(2m-1)=\frac{2^{2m-1}}{2^{2m-1}-1}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{2m-1}} \tag{47}$$

m is a positive integer.

And this is just a variation of the above formula

$$\zeta(2m+1)=\frac{(2^{2m+1}-4)}{(2^{2m+1}-1)}\left[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{2m-1}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{2m-1}}}\right]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{2m+1}} \tag{48}$$

m is a positive integer.

And this is just a variation of the above formula

$$\zeta(2m+1)=\zeta(2m-1)\frac{(2^{2m+1}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{2m+1}}}{(2^{2m+1}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{2m-1}}} \tag{49}$$

m is a positive integer.

And I declare a new formula.

$$\zeta(2m+1)=\zeta(3)\frac{2^{2m-2}\times 7}{(2^{2m+1}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{2m+1}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{50}$$

m is a positive integer of 2 or more.

4 Postscript

Using WolframAlpha, it was confirmed numerically completely correct.

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