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Dark Energy and the Time Dependence of Fundamental Particle Constants

Bodo Lampe

II. Institut für theoretische Physik der Universität Hamburg

Luruper Chaussee 149, 22761 Hamburg, Germany

Abstract

The cosmic time dependencies of G, α, h and of SM parameters like the Higgs vev and elementary particle masses are studied in the framework of a new dark energy interpretation. Due to the associated time variation of rulers, many effects turn out to be invisible. However, a rather large time dependence is claimed to arise in association with dark energy measurements, and smaller ones in connection with the SM.

I. Introduction

Dirac was one of the first to suggest that fundamental physical constants may vary in time due to the expansion of the universe[1]. Dirac concentrated on Newton's constant G, but since then a time dependence of c, α and so on has been considered possible as well[2]. From the 21st century viewpoint it is clear that if fundamental constants are time dependent in this way, the observed dark energy effect must have to do with it, because dark energy dominates the present expansion of the universe.

We shall be working in the framework of the FLRW cosmology with a scale factor a(t) and a spatial curvature k, the latter assumed to be tiny (in accordance with observations). Furthermore, I will use the so-called 'cosmic coordinate system', i.e. cosmic time t and proper distances r as parameters. This will prove to be optimal for the presentation.

We start with the fundamental spacetime constants c, h and G, or equivalently the Planck length, time and mass L, T and M which describe the basic properties of space[m], time[s] and matter[kg]

$$L(t) = \sqrt{\frac{\hbar(t)G(t)}{c^3}} \qquad T(t) = \sqrt{\frac{\hbar(t)G(t)}{c^5}} \qquad M(t) = \sqrt{\frac{\hbar(t)c}{G(t)}} \tag{1}$$

One may invert these relations to obtain

$$c = \frac{L(t)}{T(t)} \qquad \hbar(t) = E(t)T(t) \qquad \kappa(t) = \frac{L(t)}{E(t)} \tag{2}$$

where $E = Mc^2$ is the Planck energy and $\kappa = G/c^4$ the Einstein constant.

I have anticipated a time dependence of these quantities here. $t = 0$ is taken to be the present, so we have the present day values $L_0 = L(0) =$ Planck length, $T_0 = T(0) =$ Planck time and $E_0 = E(0) =$ Planck energy. Numerical values are

$$L_0 = 1.6 \times 10^{-35}m \qquad M_0 = 2.2 \times 10^{-8}kg \qquad T_0 = 5.4 \times 10^{-44}s \tag{3}$$

I have not indicated a time dependence of c, because in my model there is none - at least if one uses the above mentioned cosmic coordinates t and r, in which case the FLRW solution of the Einstein equations has the line element

$$ds^2 = -c^2dt^2 + dr^2/(1 + ...) \tag{4}$$

with a constant i.e. time-independent speed of light.

c being constant, one only needs to consider time dependencies of G and h.

Equivalently, since one has T(t)=L(t)/c one only needs to consider time dependencies of the Planck length L(t) and Planck energy E(t).

I rewrite eq. (2) as

$$\hbar(t)c = E(t)L(t) \tag{5}$$

$$G(t) = c^4 L(t)/E(t) \tag{6}$$

So we want to derive:

-L(t)=the time dependence of the fundamental measure of space

-E(t)=the time dependence of the 'physically active' quantities - the 'quantities of motion', as Newton called them.

Remark: The time dependence of elementary particle couplings α, G_F and so on is a different story. It will be treated in section V and will boil down to determine the time dependence of one other quantity

-J(t)=the time dependence of the 'internal exchange energy' to be defined in section V.

To determine L(t) and E(t) I introduce 2 equations:

II. Measure-of-Space Equation

$$\ddot{L} = -\frac{4\pi}{3}G\rho L - \omega^2(L - L_s) + \frac{\Lambda}{3}c^2 L \tag{7}$$

The idea behind this is that the universe is an elastic medium which consists of elementary constituents called tetrons[3, 4], and the bond length of these constituents is given by the Planck length L(t), while the Planck energy E(t) measures the binding energy of every 2 bound constituents. It is to be noted that the tetrons are invisible to us. All (ordinary and dark) matter particles and radiation we know are quasi-particles/wave-excitations of them and can propagate on the elastic medium.[1]

[1] In the tetron-model[3] our universe is embedded in a higher-dimensional space, and as an elastic medium it can thus acquire the full 3+1 GR curvature within this space, including the timely curvature related to expansion.

Within such a picture it is rather clear, that in an expanding universe L and E will vary with time (and so will h and G as well as all particle physics constants), and we shall now be making the most straightforward ansätze for these variations.

First of all, when the universe (=the elastic medium) expands, the variation of the Planck or bond length L(t) must reflect the general expansion as described by the FLRW expansion parameter a(t). Eq. (7) relies on the simple assumption that on the average the bond length between 2 tetrons is always proportional to the scale parameter, i.e. $a \sim L$ or equivalently

$$\frac{a(t)}{a_0} = \frac{L(t)}{L_0} \tag{8}$$

Thus, the first term in (7) arises from the general relativistic deceleration of the universe through its matter content ρ, while the second term accounts for the dark energy phenomenon, however, not quite in the usual form of a cosmological constant (indicated in green), but of a harmonic force $-\omega^2(L - L_s)$, that expands the elastic medium towards an equilibrium value L_s of the bond length L.

Eq. (7) tells us that linear forces are acting, one induced by (ordinary and dark) matter and driving the system towards $L = 0$, the other induced by the ('dark energy') tetron binding and driving it towards the equilibrium binding distance L_s. Presently we are in the region $L_0 < L_s$, so that $-\omega^2(L - L_s)$ really is an expanding force. The value of ω can and will be determined from the fit to dark energy measurements.

In the course of time, i.e. with increasing L, the matter force becomes smaller because the matter density dilutes according to $\rho = \rho_0 L_0^3/L^3$. This is a well known effect and makes the first term on the RHS of (7) behave like $\sim 1/L$ instead of $\sim L$.

The differential equation (7) can easily be solved using initial values

$$L(0) = L_0 \qquad \dot{L}(0) = H_0 L_0 \tag{9}$$

where L_0 is the (present day) value of the Planck length and H_0 the Hubble constant (=present day value of the Hubble parameter $H(t) = \dot{a}/a = \dot{L}/L$). The solution will be given later in (20).

From the initial conditions it is immediately clear that ω is naturally of the order of H_0, In section IV this will be confirmed by fitting with observations. ω and H(t) are extremely small frequencies corresponding to an approximately harmonic movement of the universe as a whole and a priori have little to do with the Planck frequency $1/T_P$ which is the local response frequency of a single tetron in the elastic medium.

$$H_0 \, T_P \approx 1.18 \;\; 10^{-61} \tag{10}$$

So seemingly, there are 2 very different fundamental scales in the universe: one is the single tetron binding energy/Planck energy E and the other is the collective dark energy of the universe as a whole, which drives it to its equilibrium value. However, **this drive is just a reflection of the tetron binding energy having a minimum at bond length** L_s. Therefore, although E(t) and H(t) are vastly different, their time behavior is related. See eq. (18) later.

In other words, I will argue that, due to the homogeneity of the elastic medium, the time behavior of the microsopic tetron energy E(t) and that of the cosmological frequencies ω and H(t) are related.

III. Quantities-of-Motion Equation

If one thinks it over, a time dependent L(t) has long been observed, namely in the form of the cosmolgical redshift. Usually this time dependence is not put into L, G or h, as in eqs. (5) and (6), but into the redshifted photon frequency f and the expansion paramter a. This is possible, because these quantities always appear in products h*f and G*a, respectively. So one can choose whether to absorb the time dependence of L in h and G or in f and a. The conventional choice is to keep G and h constant. We shall follow this choice - as far as the variation of the Measure-of-Space equation is concerned.

From this point of view, a varying L(t) is not so much new [apart from the modified cosmological constant approach to dark energy with $-\omega^2(L - L_s)$ instead of a Λ-term].

As for the time dependence of the Planck energy E, the situation is different, i.e. there is something new.

E can be interpreted as the binding energy among the constituents of the elastic medium which is our universe. Not too far away from the equilibrium $L = L_s$ it has a quadratic dependence on L

$$E(L) = C + D(L - Ls)^2 + O(L - Ls)^4 \tag{11}$$

The constants C and D can be determined from the conditions that $E(L_0) = E_0$ and $E(L_s) = E_s$. One obtains

$$E(L) = E_s - (E_s - E_0)(\frac{L - L_s}{L_0 - L_s})^2 = E_s[1 - (1 - \frac{E_0}{E_s})(\frac{1 - L/L_s}{1 - L_0/L_s})^2] \tag{12}$$

As will turn out, the energy difference $E_0 - E_s$ triggers the harmonic dark energy term $\sim \omega^2$ in eq.(7).

The qualitative behavior of E(L) is that of a parabola and together with the solution L(t) to (7) one deduces the time dependence E(t) as needed in eqs. (5) and (6). Since we have absorbed the factors L(t) in eqs. (5) and (6) into the redshift description, we only have to consider time dependencies according to

$$h(t) \sim E(t) \qquad\qquad G(t) \sim 1/E(t) \tag{13}$$

or equivalently

$$h(t) = h_0 \frac{E(L)}{E(L_0)} \qquad\qquad G(t) = G_0 \frac{E(L_0)}{E(L)} \tag{14}$$

with E(L) to be taken from (12).

Considered as a binding energy, E(t) is actually negative, so one should write $h(t) \sim |E(t)|$ and $G(t) \sim 1/|E(t)|$. Since E(t) is negative and presently becomes more negative as it approaches its minimum value E_s, one concludes that **Plancks constant presently goes up with time, whereas the gravitational coupling is decreasing**.

At this point one may worry, whether a varying E has a problem with energy conservation. Actually, this question also arises in connection with the redshift, and is usually answered by saying that energy 'goes into the metric'. Interpreting the universe as an elastic medium one can reformulate this by stating that energy goes into the total binding energy of the universe.

Nevertheless, the ω-term in (7) can be attributed to an 'energy'

$$W(L) = \frac{\omega_s^2}{2} - \frac{\omega^2}{2}(\frac{L - L_s}{L_0})^2 = \frac{\omega_s^2}{2}[1 - (1 - \frac{\omega_0^2}{\omega_s^2})(\frac{1 - L/L_s}{1 - L_0/L_s})^2] \tag{15}$$

with

$$\omega^2 = \frac{\omega_s^2 - \omega_0^2}{(1 - L_s/L_0)^2} \tag{16}$$

Note the similarity between (12) and (15). Since the dark energy phenomenon is a smooth collective effect of all tetron binding energies E having a minimum at bond length L_s, E and W are proportional. $E(L) \sim W(L)$ holds in a similar way as $L(t) \sim a(t)$ for the cosmic scale factor a and the bond/Planck length L, cf. (8):

$$\frac{W(L)}{W(L_0)} = \frac{E(L)}{E(L_0)} \tag{17}$$

Therefore we can write

$$\frac{\omega_0^2}{\omega_s^2} = \frac{E_0}{E_s} \tag{18}$$

The physical difference between W and E is that
-E is the microscopic tetron binding energy and remains roughly of the order of the present day Planck energy to be measured in Joule.
-the ω's are frequencies of the universe as a whole and measured in Hertz, and they are of the order of the Hubble parameter.

However, since the appearance of W is only a reflection of the tetron bond length driving towards its equilibrium value L_s (where the tetron binding energy E is having a minimum), the time evolution of W and E is absolutely parallel, in a similar way as the time evolution of a(t) is parallel to that of L(t).

IV. Comparison with Astrophysical Data

In the laboratory it is more or less impossible to observe time variations of G and h, because via (1) these quantities define our rulers for mass and energy. While the universe expands, the rulers will expand, too.

In case of the redshift, astronomers were able to obtain relevant information on L(t) from observations of distant galaxies. In contrast, it seems difficult to measure the time variation (12) of energy from such observations, because any process, which took place in the past in some distant galaxy, will do so with the energy/rulers relations valid at that time, and when the produced particles arrive on earth they will interact with the detectors with the energy/rulers relations valid now; so that the observer will see no difference between processes now and then.

Furthermore note that for considering time variations of h and G in the *early* universe, an approximation of the form (12) will not be sufficient, because at small bond length L a typical binding energy is known to be governed by a power behavior of the form $E(L) \sim L^{-n}$.

Not testable in particle processes, it turns out, however, that E(t) from eq. (12) can be directly observed in dark energy measurements. Dark energy observations do not usually concern the very early universe, so that the parabolic approximation (12) should be good enough. They are in effect testing eq. (7), and E(t) in (12) not only governs the ω-term but according to (14) and (13) also enters the G-term on the RHS of (7).

In order to check our ideas with astrophysical data, we go over from L(t) to the redshift z defined by

$$z(t) = \frac{L}{L_0} - 1 \tag{19}$$

The most precise measurement of the dark energy effect comes from the study of type-Ia supernovae in distant galaxies. I shall compare my redshift prediction to those data in a small-t approximation. This is justified because on cosmic scales the times involved are not too large.

Under this condition, up to $O(t^4)$, the solution to (7) can be written as

$$\begin{aligned} z = tH_0 \quad &+ \quad \frac{t^2 H_0^2}{2}\left[-\frac{\Omega_M^0}{2} + \frac{\omega_0^2}{H_0^2}\frac{\frac{E_s}{E_P}-1}{\frac{L_s}{L_0}-1} + \frac{\Lambda c^2}{3H_0^2}\right] \\ &+ \quad \frac{t^3 H_0^3}{6}\left[\Omega_M^0\left(1+\frac{\frac{E_s}{E_P}-1}{\frac{L_s}{L_0}-1}\right) - \frac{\omega_0^2}{H_0^2}\frac{\frac{E_s}{E_P}-1}{(\frac{L_s}{L_0}-1)^2} + \frac{\Lambda c^2}{3H_0^2}\right] \end{aligned} \tag{20}$$

The term indicated in red is the contribution from the time dependent Newton constant, the terms in blue come from the harmonic dark energy ω contribution, and the terms in green from a cosmological constant (the latter to be ignored in the present model).

$$\Omega_M^0 = \frac{4\pi}{3}\frac{G_0 \rho_0}{H_0^2} \tag{21}$$

is the present day density parameter of matter in the universe, frequently used in this type of analysis. In the dark energy interpretation with a cosmological constant it comes out as roughly 0.3, which is usually considered a reasonable value.

As for any parabola, hidden in the parabolic dark energy (12) and (15) are 3 parameters, which need to be determined from observations. One can choose them as
(i) $\frac{E_s}{E_0} = \frac{\omega_s^2}{\omega_0^2} > 1$ = the ratio of the Planck energies resp dark energies at cosmic equilibrium and at present
(ii) $\frac{L_s}{L_0} > 1$ = the ratio of the tetron binding lengths at cosmic equilibrium and at present
(iii) $\frac{\omega_0^2}{H_0^2}$ = the ratio of the present dark energy over the present value of the Hubble constant.
Since we have more parameters here than in the case of a cosmological constant, the observations will only give relations between i, ii and iii. Furthermore, we have to take an estimate for Ω_M^0 from other sources. Nevertheless, our next aim is to see what the observations allow to say.

A fit to the redshifts of supernovae yields[6]

$$z = tH_0 + \frac{t^2 H_0^2}{2}(1.00 \pm 0.05) + \frac{t^3 H_0^3}{6}(0.54 \pm 0.05) \tag{22}$$

Comparing with (21) one deduces that it is easy to fit i, ii and iii to the data. For example, choosing $\Omega_M^0 = 0.3$ and
-$L_s = 10L_0$ one obtains $E_s = 1.34E_0$ and $\omega_0^2 = 3.4H_0^2$
-$L_s = 2L_0$ one obtains $E_s = 5.6E_0$ and $\omega_0^2 = 0.25H_0^2$

At first sight, the fact that data can be fitted this way so easily, seems to be a big surprise. After all, we are fitting numbers which usually are explained with an expontential increase due to a cosmological constant. The essential feature here is the contribution from the time variation of Newton's constant (red) which in combination with the harmonic dark energy contribution (blue) leads to an agreement with observations. The point is that since G(t) is going down with time, the retarding effect of (ordinary and dark) matter becomes smaller, and no exponential increase of the dark energy term as in the cosmological constant approach is needed.

In other words, although the harmonic force ansatz corresponds to a more moderate re-acceleration of the universe than the cosmological constant term, this is compensated by the time variation of energy as a whole which affects Newton's constant.

V. Cosmic Time Dependence of Particle Physics Parameters

Now we want to extend our analysis to the 'constants', which describe the particle physics interactions. We shall discuss all parameters of the Standard Model (SM) of particle physics, i.e.

-the 3 dimensionless gauge couplings: here we shall consider the weak and electromagnetic fine structure constants α_{weak} and α together with the QCD scale parameter Λ_{QCD}.

-the 2 parameters of the Higgs potential: here we shall consider the Higgs mass m_H and the vacuum expectation value v of the Higgs field. Note that using v is equivalent to using the Fermi coupling $G_F = 1/[\sqrt{2}v^2]$, and the quartic Higgs coupling is given by $\lambda = m_H^2/v^2$.

-the Yukawa couplings, which are all proportional to v.

Except for α_{weak} and α, all these parameters have dimension of energy. If one looks at the definition of the fine structure constant

$$\alpha = \frac{e^2}{4\pi\epsilon_0 \hbar c} \tag{23}$$

it is the only dimensionless combination which can be built from the quantities e^2/ϵ_0, h and c. As dimensionless, it is independent of the choice of rulers for time, length and energy. This is good news, because in looking for a cosmic time dependence of α one circumvents all the problems which one usually has in determining the time dependence of dimensionful quantities like E(t). The *bad news* in considering ratios like α is that most effects tend to drop out between numerator and denominator (see later).

An interesting point is that although α itself is not an energy, it can be written as a ratio of forces or energies. Namely we can rewrite (23) as

$$\alpha = \frac{e^2}{4\pi\epsilon_0 r^{(2)}} / \frac{G_0 M_0^2}{r^{(2)}} \tag{24}$$

i.e. as the ratio of the electrostatic Coulomb (force) energy and the gravitational (force) energy of 2 point particles with elementary charge e and Planck mass M_0 at an arbitrary distance r.

From this point of view the gravitational force is by no means small as compared to the electric force, but - for such tetron-like test particles - is 137 times stronger!

The key relation here is that from (2) one has

$$\hbar c = G_0 M_0^2 = L_0 E_0 \tag{25}$$

Defining $Q^2 = e^2/[4\pi\epsilon_0]$ and introducing possible time dependencies, we have

$$\alpha(t) = \frac{Q^2(t)/L(t)}{E(t)} \tag{26}$$

where Q^2 comprises the electromagnetic effect in a measurement-system independent way. Obviously, Q^2 has the dimension of length×energy. Since measurements and astrophysical observations show almost no time variation of α, the time dependence of Q^2/L must be the same as that of E(t) to a very good approximation.

Referring once again to the tetron model, this has to do with the fact, that the time dependence of Q^2 is determined by that of the binding energy E(t)[3], so that any time dependence of α drops out between numerator and denominator.

To understand this point in detail, one should note that the tetron model is more than a microscopic theory for the cosmic elastic medium. The tetrons actually appear in the form of tetrahedrons which extend into a 3-dimensional internal space and whose excitations can be shown to represent the complete 3-family elementary particle spectrum[3, 4].

The internal interactions among tetrons are typical quantum interactions in the sense that one always has 'exchange' energies in addition to 'direct' energies, simply because for 2 (or more) identical particles - tetrons in this case - with single wave functions f_1 and f_2 their total wave functions are either symmetric or antisymmetric of the form $f_1(x_1)f_2(x_2) \pm f_1(x_2)f_2(x_1)$. Correspondingly, the relevant 2-point function of the tetron Hamiltonian can be described as the sum of E(t) and a function J(t) usually called the exchange energy. In the present case it may be called 'internal exchange energy' because it arises as an integral including the internal space, in which the tetrahedrons are living.

$$E = \int d^6x_1 \int d^6x_2 f_1(x_1)f_2(x_2)V(1-2)f_1(x_1)f_2(x_2) \tag{27}$$

$$J = \int d^6x_1 \int d^6x_2 f_1(x_1)f_2(x_2)V(1-2)f_1(x_2)f_2(x_1) \tag{28}$$

where the integrals are actually 6-dimensional, because they extend over both internal and physical space. V(1-2) is the potential between 2 tetrons, and f_1 and f_2 are their wave functions.[2]

In a 6-dimensional environment the Green's function of the Laplace operator is r^{-4}, instead of r^{-1} in the 3-dimensional case. Therefore, the most promising choice seems to be

$$V(1-2) = \frac{N}{|x_1 - x_2|^4} \tag{29}$$

with some coupling constant N. A rough estimate of N can be obtained by equating V(1-2) at the Planck length to the Planck energy. This gives an estimate of the fundamental tetron coupling N:

$$\frac{N}{L_0^4} \approx E_0 \quad \Longrightarrow \quad N \approx 10^{-130} \frac{m^6 kg}{s^2} \tag{30}$$

When trying to calculate E and J according to (27) and (28), one naturally runs into the so-called hierarchy problem of physics. Namely the question, why the relevant energy scales of gravity ($E_0 \approx 10^{19}$GeV) and of particle physics ($J_0 = 1 - 100$GeV) are so much different. In the framework of the tetron model, the question can be reformulated: why is the exchange energy J so much smaller than the direct energy E?

Looking at (27) and (28), one sees that $J \ll E$ can happen, if the tetron wave functions are strongly localized. In the extreme case of delta functions one even finds, that the exchange integral vanishes, while the direct integral attains the value (30). Such an extreme localization is of course unnatural. In order to get $J \approx 10^{-17} E$, it is enough to demand that f(x) drops from its maximum value at $x = 0$ by about

[2] If one looks into the details of the tetron model[3], the situation is a bit more complicated than described here. First of all, f_1 and f_2 are the wave functions of tetron-antitetron *pairs*, and V(1-2) is the potential between these 2 pairs. Secondly, to really calculate E and J from the 6-dimensional integrals one has to take the configuration of 2 adjacent tetrahedrons with at least 8 tetrons into account. Furthermore, there are actually 2 types of exchange integrals, one corresponding to the *inter*-tetrahedral interactions, which gives rise to the Fermi scale and is responsible for the large masses m_t, m_W and m_H of order 100 GeV, and another one corresponding to the *inner*-tetrahedral interactions, which gives rise to the lighter fermion masses and the QCD scale.

a factor of 10 at $x = L_0$. This is because J is a multidimensional integral and to integrate the product $dx_2 f_2(x_2)V(1-2)f_1(x_2)$ will give a suppression factor of roughly ~ 0.1 for each of the 6 dimensions. Similarly for the x_1-integration.

Except for α, which is constant, I will argue that **J(t) gives a universal time dependence for all internal/particle interactions in a similar way as the Planck energy E(t) for the spacetime quantities of motion**. In other words, while the time-dependence of all dimensionful spacetime quantities is dictated by E(t), the time dependencies of dimensionful SM particle properties like v, m_H, m_W and all quark and lepton masses can be described in terms J(t).

To see how this works in detail, one should relate J to the electroweak symmetry breaking scale. This was already done in [3], where it was shown that the critical energy of the electroweak phase transition is given by an exchange integral J of the form (28). This is because in the tetron model the electroweak phase transition corresponds to an alignment of the tetrahedrons in the internal spaces, and the Curie energy of this phase transition is given by J. Since the critical energy of the electroweak phase transition is approximately given by the Higgs vev v, one has $v = J$ or, equivalently

$$G_F(t) = \frac{1}{\sqrt{2}J^2(t)} \tag{31}$$

It is well known that all particle masses in the SM are proportional to v. Therefore, J enters all dimensionful parameters of the electroweak SM - the fermion masses, the Higgs vev and the masses of the weak gauge bosons - in a linear way. All these quantities are $\sim J(t)$.

Just as for E, the time-dependence of J arises through the time variation of the bond length L(t). If one would calculate the integrals J and E (27) and (28) as a function of L, then knowing L(t) according to (7), one could deduce from that the time-dependence of the Fermi constant (31) and compare it to present upper limits[7].

Unfortunately, the situation is not that simple. First of all, as mentioned in footnote 2, the integrals are difficult to calculate. Secondly, in everything we do, in every experiment we undertake, we encounter the Planck energy E(t) as a ruler, whose time dependence influences our perception of dimensionful quantities like G_F, m_W

and so on. To say it plainly, the time dependence we can perceive is not that of J(t) but that of the ratio J(t)/E(t).

If, for example, J and E would have an identical time dependence, the time dependence of dimensionful SM parameters like (31) could never be measured.

By analyzing the structure of the direct and the exchange integrals E and J in some detail, one can indeed show, that their dependence on the bond length L is quite similar, both with an extremum at nearly the same value L_s. Making a similar ansatz for J(L) as for E(L) in (12)

$$J(L) = J_s - (J_s - J_0)(\frac{L - L_s}{L_0 - L_s})^2 = J_s[1 - (1 - \frac{J_0}{J_s})(\frac{1 - L/L_s}{1 - L_0/L_s})^2] \tag{32}$$

one sees that the crucial part is the ratio J_0/J_s. To the extent that the equality

$$\frac{J_s}{J_0} = \frac{E_s}{E_0} \tag{33}$$

holds, a time dependence of SM parameters cannot be measured.[3] Conversely, any observed time dependence in a SM parameter can be traced back to a deviation from (33).

Actually, there is no reason, why (33) should be an exact relation. First of all, the integrals (27) and (28) are definitely distinct. Secondly, particle physics interactions have to do with inner symmetries not contained in the energetic analysis of the elastic universe [governed by E(t)], and their cosmic time dependence should therefore follow its own rule [given by J(t)].

VI. Discussion

In this study a theory concerning the time dependence of all known fundamental physical parameters has been developed. It rests on the idea that dark energy is a harmonic rather than an exponential effect, which is furthermore related to the binding energy of the underlying constituents of the universe. As has been shown, one is led to a time-dependence of Newton's and Planck's constant.

[3] Such a statement would be in accord with the present status of observations, which only give upper limits on time dependencies of SM parameters.

Furthermore
-microsopic (L) und cosmic (a) length scales are connected in a simple linear kind of way ('the universe expands in the same manner as the tetron bonds expand'), cf. (8).
-In an analogous fashion, Planck energies E(t) and dark energies ω are linearly related ('the total dark energy of the universe increases linearly with the tetron binding energy') via (17).

Since the universe is rather cool by now and apparently expands in a rather homogeneous way, these assumptions appear to be very good approximations. In particular, it was proven that they lead to agreement with present day dark energy observations. Thereby it has turned out that there is a significant contribution to the observed dark energy effect from the time variation of Newton's constant. Since G(t) is going down with time, the retarding effect of ordinary matter becomes smaller, and no exponential increase of the dark energy effect as in the cosmological constant approach is needed.

The Planck energy E_0 and its time-dependent generalization E(t) play a central role in the considerations presented here, see (5), (6) and (12). Actually, E_0 has been used in this paper with 2 meanings:
-it represents the gravitational energy of the interaction of 2 matter particles with Planck mass M_0 at Planck distance L_0, i.e. $E_0 = G_0 M_0^2 / L_0$.
-it describes the binding energy of 2 tetrons bound at distance L_0.

Concerning the fundamental parameters of particle physics, we have seen that they depend on cosmic time via the internal exchange function J, whose dependence on L is similar but not exactly the same as that of E.

A remaining problem is the calculation of E (Planck energy) and J (internal exchange energy) from first principles, i.e. from fundamental tetron interactions.

Another problem is the question of energy conservation in a theory with a varying G(t). Energy conservation is an uneasy business in GR anyhow[5], but assuming a time varying G makes the set of the ordinary FLRW equations

$$\frac{1}{2}\dot{a}^2 - \frac{4\pi}{3} G \rho a^2 = 0 \tag{34}$$

$$a\ddot{a} + \frac{1}{2}\dot{a}^2 = 0 \tag{35}$$

which comprises a 'force' equation for $\ddot{a}$ and an 'energy' equation for $\dot{a}^2/2$, inconsistent. [The FLRW equations have been written down here taking $\Lambda = 0$, $p = 0$ and $k = 0$]. (34) and (35) are 2 differential equations for one function a(t) and are only consistent, as long as the product $G\rho$ behaves like $\sim a^{-3}$ corresponding to a uniformly diluting mass density and no variation of the Newton constant at all.

The underlying reason is that Einstein's theory itself relies on a constant, time-independent G. This has to do with the fact that it is a theory for an elastic medium whose basic properties and couplings do not change when the medium expands. For the large expansion factors, however, which we encounter on the cosmic scale, such an assumption seems unrealistic.

In order to solve the conflict between (34) and (35) in case of a varying G I am therefore retreating to the point of view that a Hooke-type of force

$$\ddot{a} = -\frac{4\pi}{3} G(a)\rho(a)a \tag{36}$$

is induced by matter, with non-constant coefficients $\rho(a) \sim a^{-3}$ and $G(a) \sim 1/E(a)$, and use this as the basic starting point for (7).

In a similar way as it does not allow a time-dependent G, general relativity does not include an ω-term like in (7). In other words, the harmonic expansion describing the behavior of the elastic medium for $L \to L_s$ is not part of Einstein's theory. This is not a big surprise, because GR is a theory of local curvature induced by energy-momentum and does not know about the equilibrium of the unterlying elastic medium.

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