The characteristic of primes

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Abstract

In this paper, we propose the axiomatic regularity of prime numbers.

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1 Introduction

In 1859, Riemann [Rie59] showed a deep connection between non-trivial zeros of the Riemann zeta-function and the prime numbers. Our motivation is to axiomatize the structure of primes.

2 Results

These below are some patterns of number.

Let \( t_n \) denote the \( n \)th triangular number. Then

\[
t_n = \binom{n + 1}{2} \quad n \geq 1,
\]

where \( \binom{n}{k} \) is the binomial coefficients.

Let \( F_n \) be the \( n \)th Fibonacci number. Then

\[
F_n = \frac{(1 + \sqrt{5})^n - (1 - \sqrt{5})^n}{2^n \sqrt{5}},
\]

where \( n \) is a positive integer.
Let $B_n$ be the $n$th Bernoulli number. Then

$$B_n = (-1)^{n+1}n\zeta(1 - n),$$

where $\zeta(1 - n)$ is the Riemann zeta-function.

If $p(n)$ denotes the total number of partitions of $n$, then

$$p(n) \sim \frac{e^{\pi\sqrt{2n/3}}}{4n\sqrt{3}},$$

where $n$ is a positive integer.

**Postulate 2.1 (Peano Postulates).** Given the number 0, the set $\mathbb{N}$, and the function $\sigma$. Then:

1. $0 \in \mathbb{N}$.
2. $\sigma : \mathbb{N} \to \mathbb{N}$ is a function from $\mathbb{N}$ to $\mathbb{N}$.
3. $0 \notin \text{range}(\sigma)$.
4. The function $\sigma$ is one-to-one.
5. If $I \subset \mathbb{N}$ such that $0 \in I$ and $\sigma(n) \in I$ whenever $n \in I$, then $I = \mathbb{N}$.

We define $1 = \sigma(0)$, $2 = \sigma(1)$, $3 = \sigma(2)$, etc. Next, we propose the fundamental properties of prime numbers.

**Definition 2.2.** Given a positive integer $n$, let $\chi(n)$ denote the number of third positive divisor of $n$ and $\Delta(n)$ denote the number of positive divisors of $n$ besides 1 and $n$.

Indeed, $\chi(1) = 0$ and $\Delta(1) = 0.$

**Postulate 2.3.** Given a prime number $p$, $\sigma(n)$ denotes the sum of positive divisors of $n$. Then:

1. $2 \leq p$.
2. $4 \nmid p$.
3. $(-1)^{\chi(p)} = 1$.
4. $3 \leq \sigma(p)$.
5. $\Delta(p) = 0$.

By our observation, we get the estimation. Let $p_n$ be the $n$th prime, where $n$ is a positive integer. Then

$$\frac{p_{n+1}}{p_n} \leq 1.7.$$
References