

$\zeta(5), \zeta(7), \dots, \zeta(331), \zeta(333)$

are irrational number

Toshiro Takami*
mmm82889@yahoo.co.jp

Abstract

Using the fact that $\zeta(3)$ is an irrational number, I prove that $\zeta(5), \zeta(7), \dots, \zeta(331)$ and $\zeta(333)$ are irrational numbers.

$\zeta(5), \zeta(7), \dots, \zeta(331)$ and $\zeta(333)$ are confirmed that they were in perfect numerical agreement.

This is because I created an odd-number formula for ζ , and the formula was created by dividing the odd-number for ζ itself into odd and even numbers.

key words

irrational number, $\zeta(3)$, odd-number formula for ζ , $\zeta(5)$, $\zeta(7)$, $\zeta(331)$, $\zeta(333)$

1 Introduction

Write the formula I finally got in advance.

$$\zeta(2m+1) = \frac{(2^{2m+1} - 4)}{(2^{2m+1} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{2m-1}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{2m-1}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{2m+1}} \quad (1)$$

m is a positive integer.

and

$$\zeta(2m+1) = \zeta(2m-1) \frac{(2^{2m+1} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{2m+1}}}{(2^{2m+1} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{2m-1}}} \quad (2)$$

m is a positive integer.

and

*47-8 Kuyamadai, Isahaya-shi, Nagasaki-prefecture, 854-0067 Japan

Eq.(1) and Eq.(2) are equation these are modification of Eq.(3).

$$\zeta(2m-1) = \frac{2^{2m-1}}{2^{2m-1}-1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{2m-1}} \quad (3)$$

m is a positive integer.

In detail

$$\zeta(3) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3} + \sum_{n=1}^{\infty} \frac{1}{(2n)^3} = \frac{1}{2^3} \sum_{n=1}^{\infty} \frac{1}{n^3} + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3} = \frac{1}{2^3} \zeta(3) + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3} \quad (4)$$

$$\zeta(3) = \frac{2^3}{2^3-1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3} = \frac{8}{7} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3} \quad (5)$$

do the same

$$\zeta(5) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^5} + \sum_{n=1}^{\infty} \frac{1}{(2n)^5} = \frac{1}{2^5} \sum_{n=1}^{\infty} \frac{1}{n^5} + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^5} = \frac{1}{2^5} \zeta(5) + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^5} \quad (6)$$

$$\zeta(5) = \frac{2^5}{2^5-1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^5} = \frac{32}{31} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^5} \quad (7)$$

do the same

$$\zeta(7) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^7} + \sum_{n=1}^{\infty} \frac{1}{(2n)^7} = \frac{1}{2^7} \sum_{n=1}^{\infty} \frac{1}{n^7} + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^7} = \frac{1}{2^7} \zeta(7) + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^7} \quad (8)$$

$$\zeta(7) = \frac{2^7}{2^7-1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^7} = \frac{128}{127} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^7} \quad (9)$$

Use these for $\zeta(9), \zeta(11), \zeta(13)$ etc.

and

Detailed description

$$\zeta(3) = \frac{2^3}{2^3-1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3} = \frac{8}{7} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3} \quad (10)$$

$$\zeta(5) = \frac{2^5}{2^5-1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^5} = \frac{32}{31} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^5} \quad (11)$$

Multiply $\zeta(3)$ and $\zeta(5)$

$$\zeta(5) \frac{8}{7} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3} = \zeta(3) \frac{32}{31} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^5} \quad (12)$$

$$\zeta(5) \frac{8}{7} = \zeta(3) \frac{32}{31} \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^5}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (13)$$

$$\zeta(5) = \zeta(3) \frac{28}{31} \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^5}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \frac{28}{31} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^3}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^5} \quad (14)$$

2 Discussion

Example 1

from Eq.(3)
if m=3

$$\zeta(5) = \frac{32}{31} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^5} \quad (15)$$

if m=4

$$\zeta(7) = \frac{128}{127} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^7} \quad (16)$$

Multiply $\zeta(5)$ and $\zeta(7)$.

$$\zeta(7) \frac{32}{31} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^5} = \zeta(5) \frac{128}{127} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^7} = \zeta(5) \frac{128}{127} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^7} \quad (17)$$

$$\zeta(7) \frac{32}{31} = \zeta(5) \frac{128}{127} \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^7}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^5}} \quad (18)$$

$$\zeta(7) = \zeta(5) \frac{31}{32} \frac{128}{127} \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^7}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^5}} \quad (19)$$

$$\zeta(7) = \zeta(5) \frac{124}{127} \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^7}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^5}} = \frac{124}{127} \zeta(5) \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^7}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^5}} \quad (20)$$

$$= \frac{124}{127} \left[\frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^5} + \sum_{n=1}^{\infty} \frac{1}{(2n)^5}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^5}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^7} = \frac{124}{127} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^5}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^5}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^7} \quad (21)$$

The new formula Eq.(1) has been followed.

Do the same for $\zeta(9), \zeta(11), \zeta(13)$ etc.

(Proof 1)

If $\zeta(5)$ is assumed to be rational number. $\zeta(5) = \frac{s}{t}$, s and t are integer.

from formula Eq.(1).

$$\zeta(5) = \zeta(3) \frac{28 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^5}}{31 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \qquad \frac{28 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^5}}{31 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \frac{o}{p} \qquad o, p \text{ are assumed to be integer.}$$

$$\zeta(5) = \zeta(3) \frac{o}{p} \qquad \text{it equal} \qquad \zeta(3) = \zeta(5) \frac{p}{o} = \frac{sp}{to} \qquad \text{But, } \zeta(3) \neq \frac{sp}{to}$$

This is because $\zeta(3)$ is known to be an irrational number.

This contradicts.

$\zeta(5)$ is irrational number.

(Proof end)

Do the same, sequentially prove that $\zeta(7), \zeta(9), \zeta(11)$ etc. are irrational numbers.

Assuming that this formula Eq.(1) holds even when $\zeta(3)$

$$\zeta(3) = \zeta(1) \frac{4 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}}{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^1}} = \frac{4}{7} \zeta(1) \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^1}} \qquad (22)$$

$$= \frac{4}{7} \left[\frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^1} + \sum_{n=1}^{\infty} \frac{1}{(2n)^1}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^1}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3} \qquad (23)$$

$$= \frac{4}{7} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^1}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^1}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3} \qquad (24)$$

If $\infty = 10000$

The calculator gives the following results:

$$\zeta(3) \approx \zeta(1) \frac{4 \sum_{n=1}^{10000} \frac{1}{(2n-1)^3}}{7 \sum_{n=1}^{10000} \frac{1}{(2n-1)^1}} = \infty \qquad (25)$$

$$\zeta(3) \approx \frac{4}{7} \left[1 + \frac{\sum_{n=1}^{10000} \frac{1}{(2n)^1}}{\sum_{n=1}^{10000} \frac{1}{(2n-1)^1}} \right] \sum_{n=1}^{10000} \frac{1}{(2n-1)^3} = \text{can not calculate.} \qquad (26)$$

$$\zeta(1) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^1} + \sum_{n=1}^{\infty} \frac{1}{(2n)^1} = \infty \quad \frac{\sum_{n=1}^{10000} \frac{1}{(2n)^1}}{\sum_{n=1}^{10000} \frac{1}{(2n-1)^1}} = \text{can not calculate.} \quad (27)$$

But

when assuming $\infty = 10^{12}$

$$\sum_{n=1}^{10^{12}} \frac{1}{(2n)^1} = 14.104118390415290534411204731480719794461021933075937155602... \quad (28)$$

$$\sum_{n=1}^{10^{12}} \frac{1}{(2n-1)^1} = 14.79726557097498584382843691543889636253652206743618459722... \quad (29)$$

$$\sum_{n=1}^{10^{12}} \frac{1}{(2n-1)^3} = 1.051799790264644999724770828822518741919363005797952146568... \quad (30)$$

$$\zeta(3) \approx \frac{4}{7} \left[1 + \frac{14.10411839041529}{14.79726557097498} \right] \times 1.0517997902646449997$$

$$= 1.173902973323675078443392566878600895507141148997041472187...$$

And

when assuming $\infty = 10^{16}$

$$\sum_{n=1}^{10^{16}} \frac{1}{(2n)^1} = 18.7092885764031319274471876825161144596632249... \quad (31)$$

$$\sum_{n=1}^{10^{16}} \frac{1}{(2n-1)^1} = 19.402435756963077211864419803974291652738725... \quad (32)$$

$$\sum_{n=1}^{10^{16}} \frac{1}{(2n-1)^3} = 1.051799790264644999724770891322518116919363... \quad (33)$$

$$\zeta(3) \approx \frac{4}{7} \left[1 + \frac{18.7092885764031319}{19.402435756963} \right] \times 1.0517997902646449997$$

$$= 1.180585311558045345860831390748451768142635776092183165622...$$

And

when assuming $\infty = 10^{22}$

$$\sum_{n=1}^{10^{22}} \frac{1}{(2n)^1} = 25.61704385538526895450118704656920749913319604246979786910... \quad (34)$$

$$\sum_{n=1}^{10^{22}} \frac{1}{(2n-1)^1} = 26.310191035945214263918394168027384067208696177455... \quad (35)$$

$$\sum_{n=1}^{10^{22}} \frac{1}{(2n-1)^3} = 1.051799790264644999724770891322518741919363... \quad (36)$$

$$\zeta(3) \approx \frac{4}{7} \left[1 + \frac{25.6170438553852689545}{26.3101910359452142639} \right] \times 1.0517997902646449997$$

=1.18622268986636194786...

And

when assuming $\infty = 10^{30}$

$$\sum_{n=1}^{10^{30}} \frac{1}{(2n)^1} = 34.8273842273614516905731278653069143295376... \quad (37)$$

$$\sum_{n=1}^{10^{30}} \frac{1}{(2n-1)^1} = 35.5205314079213969999903599867648408976131... \quad (38)$$

$$\sum_{n=1}^{10^{30}} \frac{1}{(2n-1)^3} = 1.051799790264644999724770891322518741919363... \quad (39)$$

$$\zeta(3) \approx \frac{4}{7} \left[1 + \frac{34.82738422736145169}{35.52053140792139699999} \right] \times 1.0517997902646449997$$

=1.190328441896879612155803403161594612984488432917342871853...

And

when assuming $\infty = 10^{38}$

$$\sum_{n=1}^{10^{38}} \frac{1}{(2n)^1} = 44.0377245993376344266450936840441211599445... \quad (40)$$

$$\sum_{n=1}^{10^{38}} \frac{1}{(2n-1)^1} = 44.7308717798975797360623258055022977280175... \quad (41)$$

$$\sum_{n=1}^{10^{38}} \frac{1}{(2n-1)^3} = 1.051799790264644999724770891322518741919363... \quad (42)$$

$$\zeta(3) \approx \frac{4}{7} \left[1 + \frac{44.037724599337634426645}{44.730871779897579736} \right] \times 1.05179979026464499972477$$

=1.19274339863639225876851128522673136626108...

And

when assuming $\infty = 10^{58}$

$$\sum_{n=1}^{10^{58}} \frac{1}{(2n)^1} = 67.06357552927809126682500823... \quad (43)$$

$$\sum_{n=1}^{10^{58}} \frac{1}{(2n-1)^1} = 67.75672270983803657624224... \quad (44)$$

$$\sum_{n=1}^{10^{58}} \frac{1}{(2n-1)^3} = 1.05179979026464499972477... \quad (45)$$

$$\zeta(3) \approx \frac{4}{7} \left[1 + \frac{67.063575529278}{67.756722709838} \right] \times 1.05179979026464499972477$$

$$= 1.19590841840431862809146600402932735842549\dots$$

And
when assuming $\infty = 10^{116}$

$$\sum_{n=1}^{10^{116}} \frac{1}{(2n)^1} \tag{46}$$

$$= 133.8385432261054161033467604167343252563849660084387944093\dots \tag{47}$$

$$\sum_{n=1}^{10^{116}} \frac{1}{(2n-1)^1} \tag{48}$$

$$= 134.5316904066653614127639925381925018244604661427990496634\dots \tag{49}$$

$$\sum_{n=1}^{10^{116}} \frac{1}{(2n-1)^3} \tag{50}$$

$$= 1.051799790264644999724770891322518741919363005797936521568\dots \tag{51}$$

$$\zeta(3) \approx \frac{4}{7} \left[1 + \frac{133.8385432261054161}{134.5316904066653614} \right] \times 1.05179979026464499972477$$

$$= 1.198960226269639697269085949373100951783586227399769978492\dots$$

And
when assuming $\infty = 10^{399}$

$$\sum_{n=1}^{10^{399}} \frac{1}{(2n)^1} \tag{52}$$

$$= 459.6543338847628803918925512545718606319408266494101705180\dots \tag{53}$$

$$\sum_{n=1}^{10^{399}} \frac{1}{(2n-1)^1} \tag{54}$$

$$= 460.3474810653228257013097833760300372000163267837704257721\dots \tag{55}$$

$$\sum_{n=1}^{10^{399}} \frac{1}{(2n-1)^3} \tag{56}$$

$$= 1.051799790264644999724770891322518741919363005797936521568\dots \tag{57}$$

$$\zeta(3) \approx \frac{4}{7} \left[1 + \frac{459.65433388476288}{460.3474810653228257} \right] \times 1.05179979026464499972477$$

$$= 1.201151932037161379199952769513411110608627462766818854800\dots$$

And
when assuming $\infty = 10^{1999}$

$$\sum_{n=1}^{10^{1999}} \frac{1}{(2n)^1} \tag{58}$$

$$= 2301.722408279999427606285715002063226712822017552428551344... \tag{59}$$

$$\sum_{n=1}^{10^{1999}} \frac{1}{(2n-1)^1} \tag{60}$$

$$= 2302.415555460559372915702947123521403280897517686788806598... \tag{61}$$

$$\sum_{n=1}^{10^{1999}} \frac{1}{(2n-1)^3} \tag{62}$$

$$= 1.051799790264644999724770891322518741919363005797936521568... \tag{63}$$

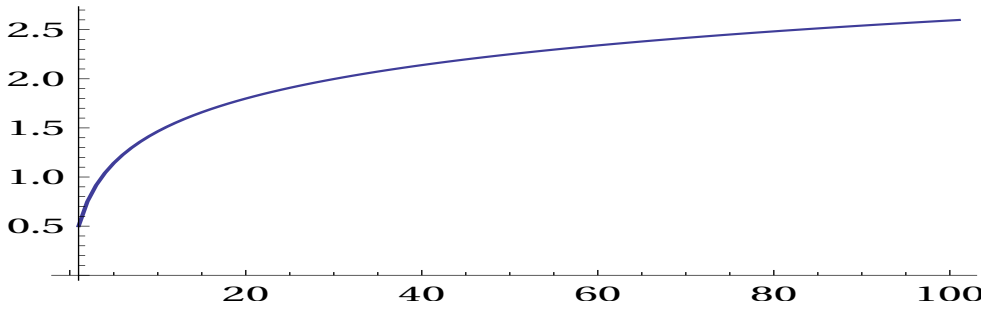
$$\zeta(3) \approx \frac{4}{7} \left[1 + \frac{2301.7224082799994276}{2302.4155554605593729157} \right] \times 1.05179979026464499972477$$

$$= 1.201875962244883729612910417552589813371555291298009320594...$$

And
when assuming $\infty = 10^{2199}$

The calculator could not calculate any more. It seems to be the upper limit.
The value increases as shown below.

Below is a graph of $\sum_{n=1}^{10^{2199}} \frac{1}{(2n)^1}$.



$$\sum_{n=1}^{10^{2199}} \frac{1}{(2n)^1} \tag{64}$$

$$= 2531.980917579403996008084860470499647472932166415305848948... \tag{65}$$

$$\sum_{n=1}^{10^{2199}} \frac{1}{(2n-1)^1} \tag{66}$$

$$= 2532.674064759963941317502092591957824041007666549666104202... \tag{67}$$

$$\sum_{n=1}^{10^{2199}} \frac{1}{(2n-1)^3} \quad (68)$$

$$= 1.051799790264644999724770891322518741919363005797936521568... \quad (69)$$

$$\zeta(3) \approx \frac{4}{7} \left[1 + \frac{2531.980917579403996}{2532.6740647599639413175} \right] \times 1.05179979026464499972477$$

$$= 1.201892412520058191091295104042207915620600363092370348104...$$

$$\zeta(3) = 1.20205690315959428539973816151144999...$$

The formula Eq.(1) seems to hold.

I thought $\zeta(1) = \infty$, but I was surprised to realize that it wasn't.

This may imply $\frac{\infty}{\infty} = 1$

My first formula Eq.(3)

when $m=1$ and from Eq.(66)Eq.(67)

$$\zeta(1) = \frac{2^1}{2^1 - 1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^1} \approx 2 \sum_{n=1}^{10^{2199}} \frac{1}{(2n-1)^1} \quad (70)$$

$$= 2 \times 2532.674064759963941317502092591957824041007666549666104202... \quad (71)$$

This is an error because $n=10^{2199}$.

If I transform Eq.(2)

Use $\infty = 10^{2199}$.

$$\zeta(1) = \zeta(3) \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^1}}{4 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \approx \zeta(3) \frac{7}{4} \times \frac{2532.674064759963941317502092591957824041007666549666104202...}{1.051799790264644999724770891322518741919363005797936521568...} \quad (72)$$

$$\approx \zeta(3) \frac{7}{4} \times \frac{2532.6740647599639413175}{1.05179979026464499972477} = 5603.7075056254558602670... \quad (73)$$

Even at this time, this is an error because $n=2199$.

$$\zeta(3) = \zeta(1) \frac{(2^3 - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}}{(2^3 - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^1}} = \frac{(2^3 - 4)}{(2^3 - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^1}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^1}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3} \quad (74)$$

$$\zeta(5) = \zeta(3) \frac{(2^5 - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^5}}{(2^5 - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \frac{(2^5 - 4)}{(2^5 - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^3}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^5} \quad (75)$$

$$\zeta(7) = \zeta(5) \frac{(2^7 - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^7}}{(2^7 - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^5}} = \frac{(2^7 - 4)}{(2^7 - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^5}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^5}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^7} \quad (76)$$

$$\zeta(9) = \zeta(7) \frac{(2^9 - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^9}}{(2^9 - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^7}} = \frac{(2^9 - 4)}{(2^9 - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^7}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^7}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^9} \quad (77)$$

$$\zeta(11) = \zeta(9) \frac{(2^{11} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{11}}}{(2^{11} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^9}} = \frac{(2^{11} - 4)}{(2^{11} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^9}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^9}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{11}} \quad (78)$$

$$\zeta(13) = \zeta(11) \frac{(2^{13} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{13}}}{(2^{13} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{11}}} = \frac{(2^{13} - 4)}{(2^{13} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{11}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{11}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{13}} \quad (79)$$

$$\zeta(15) = \zeta(13) \frac{(2^{15} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{15}}}{(2^{15} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{13}}} = \frac{(2^{15} - 4)}{(2^{15} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{13}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{13}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{15}} \quad (80)$$

$$\zeta(17) = \zeta(15) \frac{(2^{17} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{17}}}{(2^{17} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{15}}} = \frac{(2^{17} - 4)}{(2^{17} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{15}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{15}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{17}} \quad (81)$$

$$\zeta(19) = \zeta(17) \frac{(2^{19} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{19}}}{(2^{19} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{17}}} = \frac{(2^{19} - 4)}{(2^{19} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{17}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{17}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{19}} \quad (82)$$

$$\zeta(21) = \zeta(19) \frac{(2^{21} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{21}}}{(2^{21} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{19}}} = \frac{(2^{21} - 4)}{(2^{21} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{19}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{19}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{21}} \quad (83)$$

$$\zeta(23) = \zeta(21) \frac{(2^{23} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{23}}}{(2^{23} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{21}}} = \frac{(2^{23} - 4)}{(2^{23} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{21}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{21}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{23}} \quad (84)$$

$$\zeta(25) = \zeta(23) \frac{(2^{25} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{25}}}{(2^{25} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{23}}} = \frac{(2^{25} - 4)}{(2^{25} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{23}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{23}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{25}} \quad (85)$$

$$\zeta(27) = \zeta(25) \frac{(2^{27} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{27}}}{(2^{27} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{25}}} = \frac{(2^{27} - 4)}{(2^{27} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{25}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{25}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{27}} \quad (86)$$

$$\zeta(29) = \zeta(27) \frac{(2^{29} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{29}}}{(2^{29} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{27}}} = \frac{(2^{29} - 4)}{(2^{29} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{27}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{27}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{29}} \quad (87)$$

$$\zeta(31) = \zeta(29) \frac{(2^{31} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{31}}}{(2^{31} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{29}}} = \frac{(2^{31} - 4)}{(2^{31} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{29}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{29}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{31}} \quad (88)$$

$$\zeta(33) = \zeta(31) \frac{(2^{33} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{33}}}{(2^{33} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{31}}} = \frac{(2^{33} - 4)}{(2^{33} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{31}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{31}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{33}} \quad (89)$$

$$\zeta(35) = \zeta(33) \frac{(2^{35} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{35}}}{(2^{35} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{33}}} = \frac{(2^{35} - 4)}{(2^{35} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{33}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{33}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{35}} \quad (90)$$

$$\zeta(37) = \zeta(35) \frac{(2^{37} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{37}}}{(2^{37} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{35}}} = \frac{(2^{37} - 4)}{(2^{37} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{35}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{35}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{37}} \quad (91)$$

$$\zeta(39) = \zeta(37) \frac{(2^{39} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{39}}}{(2^{39} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{37}}} = \frac{(2^{39} - 4)}{(2^{39} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{37}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{37}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{39}} \quad (92)$$

$$\zeta(41) = \zeta(39) \frac{(2^{41} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{41}}}{(2^{41} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{39}}} = \frac{(2^{41} - 4)}{(2^{41} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{39}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{39}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{41}} \quad (93)$$

$$\zeta(43) = \zeta(41) \frac{(2^{43} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{43}}}{(2^{43} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{41}}} = \frac{(2^{43} - 4)}{(2^{43} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{41}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{41}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{43}} \quad (94)$$

$$\zeta(45) = \zeta(43) \frac{(2^{45} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{45}}}{(2^{45} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{43}}} = \frac{(2^{45} - 4)}{(2^{45} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{43}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{43}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{45}} \quad (95)$$

$$\zeta(47) = \zeta(45) \frac{(2^{47} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{47}}}{(2^{47} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{45}}} = \frac{(2^{47} - 4)}{(2^{47} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{45}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{45}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{47}} \quad (96)$$

$$\zeta(49) = \zeta(47) \frac{(2^{49} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{49}}}{(2^{49} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{47}}} = \frac{(2^{49} - 4)}{(2^{49} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{47}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{47}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{49}} \quad (97)$$

$$\zeta(51) = \zeta(49) \frac{(2^{51} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{51}}}{(2^{51} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{49}}} = \frac{(2^{51} - 4)}{(2^{51} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{49}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{49}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{51}} \quad (98)$$

$$\zeta(53) = \zeta(51) \frac{(2^{53} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{53}}}{(2^{53} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{51}}} = \frac{(2^{53} - 4)}{(2^{53} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{51}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{51}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{53}} \quad (99)$$

$$\zeta(55) = \zeta(53) \frac{(2^{55} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{55}}}{(2^{55} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{53}}} = \frac{(2^{55} - 4)}{(2^{55} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{53}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{53}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{55}} \quad (100)$$

$$\zeta(57) = \zeta(55) \frac{(2^{57} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{57}}}{(2^{57} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{55}}} = \frac{(2^{57} - 4)}{(2^{57} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{55}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{55}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{57}} \quad (101)$$

$$\zeta(59) = \zeta(57) \frac{(2^{59} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{59}}}{(2^{59} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{57}}} = \frac{(2^{59} - 4)}{(2^{59} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{57}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{57}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{59}} \quad (102)$$

$$\zeta(61) = \zeta(59) \frac{(2^{61} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{61}}}{(2^{61} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{59}}} = \frac{(2^{61} - 4)}{(2^{61} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{59}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{59}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{61}} \quad (103)$$

$$\zeta(63) = \zeta(61) \frac{(2^{63} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{63}}}{(2^{63} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{61}}} = \frac{(2^{63} - 4)}{(2^{63} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{61}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{61}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{63}} \quad (104)$$

$$\zeta(65) = \zeta(63) \frac{(2^{65} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{65}}}{(2^{65} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{63}}} = \frac{(2^{65} - 4)}{(2^{65} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{63}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{63}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{65}} \quad (105)$$

$$\zeta(67) = \zeta(65) \frac{(2^{67} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{67}}}{(2^{67} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{65}}} = \frac{(2^{67} - 4)}{(2^{67} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{65}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{65}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{67}} \quad (106)$$

$$\zeta(69) = \zeta(67) \frac{(2^{69} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{69}}}{(2^{69} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{67}}} = \frac{(2^{69} - 4)}{(2^{69} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{67}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{67}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{69}} \quad (107)$$

$$\zeta(71) = \zeta(69) \frac{(2^{71} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{71}}}{(2^{71} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{69}}} = \frac{(2^{71} - 4)}{(2^{71} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{69}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{69}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{71}} \quad (108)$$

$$\zeta(73) = \zeta(71) \frac{(2^{73} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{73}}}{(2^{73} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{71}}} = \frac{(2^{73} - 4)}{(2^{73} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{71}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{71}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{73}} \quad (109)$$

$$\zeta(75) = \zeta(73) \frac{(2^{75} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{75}}}{(2^{75} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{73}}} = \frac{(2^{75} - 4)}{(2^{75} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{73}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{73}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{75}} \quad (110)$$

$$\zeta(77) = \zeta(75) \frac{(2^{77} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{77}}}{(2^{77} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{75}}} = \frac{(2^{77} - 4)}{(2^{77} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{75}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{75}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{77}} \quad (111)$$

$$\zeta(79) = \zeta(77) \frac{(2^{79} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{79}}}{(2^{79} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{77}}} = \frac{(2^{79} - 4)}{(2^{79} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{77}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{77}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{79}} \quad (112)$$

$$\zeta(81) = \zeta(79) \frac{(2^{81} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{81}}}{(2^{81} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{79}}} = \frac{(2^{81} - 4)}{(2^{81} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{79}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{79}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{81}} \quad (113)$$

$$\zeta(83) = \zeta(81) \frac{(2^{83} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{83}}}{(2^{83} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{81}}} = \frac{(2^{83} - 4)}{(2^{83} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{81}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{81}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{83}} \quad (114)$$

$$\zeta(85) = \zeta(83) \frac{(2^{85} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{85}}}{(2^{85} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{83}}} = \frac{(2^{85} - 4)}{(2^{85} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{83}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{83}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{85}} \quad (115)$$

$$\zeta(87) = \zeta(85) \frac{(2^{87} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{87}}}{(2^{87} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{85}}} = \frac{(2^{87} - 4)}{(2^{87} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{85}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{85}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{87}} \quad (116)$$

$$\zeta(89) = \zeta(87) \frac{(2^{89} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{89}}}{(2^{89} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{87}}} = \frac{(2^{89} - 4)}{(2^{89} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{87}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{87}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{89}} \quad (117)$$

$$\zeta(91) = \zeta(89) \frac{(2^{91} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{91}}}{(2^{91} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{89}}} = \frac{(2^{91} - 4)}{(2^{91} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{89}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{89}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{91}} \quad (118)$$

$$\zeta(93) = \zeta(91) \frac{(2^{93} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{93}}}{(2^{93} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{91}}} = \frac{(2^{93} - 4)}{(2^{93} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{91}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{91}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{93}} \quad (119)$$

$$\zeta(95) = \zeta(93) \frac{(2^{95} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{95}}}{(2^{95} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{93}}} = \frac{(2^{95} - 4)}{(2^{95} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{93}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{93}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{95}} \quad (120)$$

$$\zeta(97) = \zeta(95) \frac{(2^{97} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{97}}}{(2^{97} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{95}}} = \frac{(2^{97} - 4)}{(2^{97} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{95}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{95}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{97}} \quad (121)$$

$$\zeta(99) = \zeta(97) \frac{(2^{99} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{99}}}{(2^{99} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{97}}} = \frac{(2^{99} - 4)}{(2^{99} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{97}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{97}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{99}} \quad (122)$$

$$\zeta(101) = \zeta(99) \frac{(2^{101} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{101}}}{(2^{101} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{99}}} = \frac{(2^{101} - 4)}{(2^{101} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{99}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{99}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{101}} \quad (123)$$

$$\zeta(127) = \zeta(125) \frac{(2^{127} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{127}}}{(2^{127} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{125}}} = \frac{(2^{127} - 4)}{(2^{127} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{125}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{125}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{127}} \quad (136)$$

$$\zeta(129) = \zeta(127) \frac{(2^{129} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{129}}}{(2^{129} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{127}}} = \frac{(2^{129} - 4)}{(2^{129} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{127}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{127}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{129}} \quad (137)$$

$$\zeta(131) = \zeta(129) \frac{(2^{131} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{131}}}{(2^{131} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{129}}} = \frac{(2^{131} - 4)}{(2^{131} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{129}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{129}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{131}} \quad (138)$$

$$\zeta(133) = \zeta(131) \frac{(2^{133} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{133}}}{(2^{133} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{131}}} = \frac{(2^{133} - 4)}{(2^{133} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{131}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{131}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{133}} \quad (139)$$

$$\zeta(135) = \zeta(133) \frac{(2^{135} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{135}}}{(2^{135} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{133}}} = \frac{(2^{135} - 4)}{(2^{135} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{133}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{133}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{135}} \quad (140)$$

$$\zeta(137) = \zeta(135) \frac{(2^{137} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{137}}}{(2^{137} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{135}}} = \frac{(2^{137} - 4)}{(2^{137} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{135}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{135}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{137}} \quad (141)$$

$$\zeta(139) = \zeta(137) \frac{(2^{139} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{139}}}{(2^{139} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{137}}} = \frac{(2^{139} - 4)}{(2^{139} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{137}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{137}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{139}} \quad (142)$$

$$\zeta(141) = \zeta(139) \frac{(2^{141} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{141}}}{(2^{141} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{139}}} = \frac{(2^{141} - 4)}{(2^{141} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{139}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{139}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{141}} \quad (143)$$

$$\zeta(143) = \zeta(141) \frac{(2^{143} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{143}}}{(2^{143} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{141}}} = \frac{(2^{143} - 4)}{(2^{143} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{141}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{141}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{143}} \quad (144)$$

$$\zeta(145) = \zeta(143) \frac{(2^{145} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{145}}}{(2^{145} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{143}}} = \frac{(2^{145} - 4)}{(2^{145} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{143}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{143}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{145}} \quad (145)$$

$$\zeta(147) = \zeta(145) \frac{(2^{147} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{147}}}{(2^{147} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{145}}} = \frac{(2^{147} - 4)}{(2^{147} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{145}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{145}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{147}} \quad (146)$$

$$\zeta(149) = \zeta(147) \frac{(2^{149} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{149}}}{(2^{149} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{147}}} = \frac{(2^{149} - 4)}{(2^{149} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{147}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{147}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{149}} \quad (147)$$

$$\zeta(199) = \zeta(197) \frac{(2^{199} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{199}}}{(2^{199} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{197}}} = \frac{(2^{199} - 4)}{(2^{199} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{197}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{197}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{199}} \quad (172)$$

$$\zeta(201) = \zeta(199) \frac{(2^{201} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{201}}}{(2^{201} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{199}}} = \frac{(2^{201} - 4)}{(2^{201} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{199}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{199}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{201}} \quad (173)$$

$$\zeta(203) = \zeta(201) \frac{(2^{203} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{203}}}{(2^{203} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{201}}} = \frac{(2^{203} - 4)}{(2^{203} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{201}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{201}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{203}} \quad (174)$$

$$\zeta(205) = \zeta(203) \frac{(2^{205} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{205}}}{(2^{205} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{203}}} = \frac{(2^{205} - 4)}{(2^{205} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{203}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{203}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{205}} \quad (175)$$

$$\zeta(207) = \zeta(205) \frac{(2^{207} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{207}}}{(2^{207} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{205}}} = \frac{(2^{207} - 4)}{(2^{207} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{205}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{205}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{207}} \quad (176)$$

$$\zeta(209) = \zeta(207) \frac{(2^{209} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{209}}}{(2^{209} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{207}}} = \frac{(2^{209} - 4)}{(2^{209} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{207}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{207}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{209}} \quad (177)$$

$$\zeta(211) = \zeta(209) \frac{(2^{211} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{211}}}{(2^{211} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{209}}} = \frac{(2^{211} - 4)}{(2^{211} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{209}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{209}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{211}} \quad (178)$$

$$\zeta(213) = \zeta(211) \frac{(2^{213} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{213}}}{(2^{213} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{211}}} = \frac{(2^{213} - 4)}{(2^{213} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{211}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{211}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{213}} \quad (179)$$

$$\zeta(215) = \zeta(213) \frac{(2^{215} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{215}}}{(2^{215} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{213}}} = \frac{(2^{215} - 4)}{(2^{215} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{213}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{213}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{215}} \quad (180)$$

$$\zeta(217) = \zeta(215) \frac{(2^{217} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{217}}}{(2^{217} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{215}}} = \frac{(2^{217} - 4)}{(2^{217} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{215}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{215}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{217}} \quad (181)$$

$$\zeta(219) = \zeta(217) \frac{(2^{219} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{219}}}{(2^{219} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{217}}} = \frac{(2^{219} - 4)}{(2^{219} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{217}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{217}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{219}} \quad (182)$$

$$\zeta(221) = \zeta(219) \frac{(2^{221} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{221}}}{(2^{221} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{219}}} = \frac{(2^{221} - 4)}{(2^{221} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{219}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{219}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{221}} \quad (183)$$

$$\zeta(247) = \zeta(245) \frac{(2^{247} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{247}}}{(2^{247} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{245}}} = \frac{(2^{247} - 4)}{(2^{247} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{245}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{245}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{247}} \quad (196)$$

$$\zeta(249) = \zeta(247) \frac{(2^{249} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{249}}}{(2^{249} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{247}}} = \frac{(2^{249} - 4)}{(2^{249} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{247}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{247}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{249}} \quad (197)$$

$$\zeta(251) = \zeta(249) \frac{(2^{251} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{251}}}{(2^{251} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{249}}} = \frac{(2^{251} - 4)}{(2^{251} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{249}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{249}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{251}} \quad (198)$$

$$\zeta(253) = \zeta(251) \frac{(2^{253} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{253}}}{(2^{253} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{251}}} = \frac{(2^{253} - 4)}{(2^{253} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{251}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{251}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{253}} \quad (199)$$

$$\zeta(255) = \zeta(253) \frac{(2^{255} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{255}}}{(2^{255} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{253}}} = \frac{(2^{255} - 4)}{(2^{255} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{253}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{253}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{255}} \quad (200)$$

$$\zeta(257) = \zeta(255) \frac{(2^{257} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{257}}}{(2^{257} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{255}}} = \frac{(2^{257} - 4)}{(2^{257} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{255}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{255}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{257}} \quad (201)$$

$$\zeta(259) = \zeta(257) \frac{(2^{259} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{259}}}{(2^{259} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{257}}} = \frac{(2^{259} - 4)}{(2^{259} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{257}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{257}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{259}} \quad (202)$$

$$\zeta(261) = \zeta(259) \frac{(2^{261} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{261}}}{(2^{261} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{259}}} = \frac{(2^{261} - 4)}{(2^{261} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{259}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{259}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{261}} \quad (203)$$

$$\zeta(263) = \zeta(261) \frac{(2^{263} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{263}}}{(2^{263} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{261}}} = \frac{(2^{263} - 4)}{(2^{263} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{261}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{261}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{263}} \quad (204)$$

$$\zeta(265) = \zeta(263) \frac{(2^{265} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{265}}}{(2^{265} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{263}}} = \frac{(2^{265} - 4)}{(2^{265} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{263}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{263}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{265}} \quad (205)$$

$$\zeta(267) = \zeta(265) \frac{(2^{267} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{267}}}{(2^{267} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{265}}} = \frac{(2^{267} - 4)}{(2^{267} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{265}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{265}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{267}} \quad (206)$$

$$\zeta(269) = \zeta(267) \frac{(2^{269} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{269}}}{(2^{269} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{267}}} = \frac{(2^{269} - 4)}{(2^{269} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{267}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{267}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{269}} \quad (207)$$

$$\zeta(271) = \zeta(269) \frac{(2^{271} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{271}}}{(2^{271} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{269}}} = \frac{(2^{271} - 4)}{(2^{271} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{269}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{269}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{271}} \quad (208)$$

$$\zeta(273) = \zeta(271) \frac{(2^{273} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{273}}}{(2^{273} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{271}}} = \frac{(2^{273} - 4)}{(2^{273} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{271}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{271}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{273}} \quad (209)$$

$$\zeta(275) = \zeta(273) \frac{(2^{275} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{275}}}{(2^{275} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{273}}} = \frac{(2^{275} - 4)}{(2^{275} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{273}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{273}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{275}} \quad (210)$$

$$\zeta(277) = \zeta(275) \frac{(2^{277} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{277}}}{(2^{277} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{275}}} = \frac{(2^{277} - 4)}{(2^{277} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{275}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{275}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{277}} \quad (211)$$

$$\zeta(279) = \zeta(277) \frac{(2^{279} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{279}}}{(2^{279} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{277}}} = \frac{(2^{279} - 4)}{(2^{279} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{277}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{277}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{279}} \quad (212)$$

$$\zeta(281) = \zeta(279) \frac{(2^{281} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{281}}}{(2^{281} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{279}}} = \frac{(2^{281} - 4)}{(2^{281} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{279}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{279}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{281}} \quad (213)$$

$$\zeta(283) = \zeta(281) \frac{(2^{283} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{283}}}{(2^{283} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{281}}} = \frac{(2^{283} - 4)}{(2^{283} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{281}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{281}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{283}} \quad (214)$$

$$\zeta(285) = \zeta(283) \frac{(2^{285} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{285}}}{(2^{285} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{283}}} = \frac{(2^{285} - 4)}{(2^{285} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{283}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{283}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{285}} \quad (215)$$

$$\zeta(287) = \zeta(285) \frac{(2^{287} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{287}}}{(2^{287} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{285}}} = \frac{(2^{287} - 4)}{(2^{287} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{285}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{285}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{287}} \quad (216)$$

$$\zeta(289) = \zeta(287) \frac{(2^{289} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{289}}}{(2^{289} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{287}}} = \frac{(2^{289} - 4)}{(2^{289} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{287}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{287}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{289}} \quad (217)$$

$$\zeta(291) = \zeta(289) \frac{(2^{291} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{291}}}{(2^{291} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{289}}} = \frac{(2^{291} - 4)}{(2^{291} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{289}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{289}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{291}} \quad (218)$$

$$\zeta(293) = \zeta(291) \frac{(2^{293} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{293}}}{(2^{293} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{291}}} = \frac{(2^{293} - 4)}{(2^{293} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{291}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{291}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{293}} \quad (219)$$

$$\zeta(295) = \zeta(293) \frac{(2^{295} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{295}}}{(2^{295} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{293}}} = \frac{(2^{295} - 4)}{(2^{295} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{293}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{293}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{295}} \quad (220)$$

$$\zeta(297) = \zeta(295) \frac{(2^{297} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{297}}}{(2^{297} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{295}}} = \frac{(2^{297} - 4)}{(2^{297} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{295}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{295}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{297}} \quad (221)$$

$$\zeta(299) = \zeta(297) \frac{(2^{299} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{299}}}{(2^{299} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{297}}} = \frac{(2^{299} - 4)}{(2^{299} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{297}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{297}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{299}} \quad (222)$$

$$\zeta(301) = \zeta(299) \frac{(2^{301} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{301}}}{(2^{301} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{299}}} = \frac{(2^{301} - 4)}{(2^{301} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{299}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{299}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{301}} \quad (223)$$

$$\zeta(303) = \zeta(301) \frac{(2^{303} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{303}}}{(2^{303} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{301}}} = \frac{(2^{303} - 4)}{(2^{303} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{301}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{301}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{303}} \quad (224)$$

$$\zeta(305) = \zeta(303) \frac{(2^{305} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{305}}}{(2^{305} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{303}}} = \frac{(2^{305} - 4)}{(2^{305} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{303}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{303}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{305}} \quad (225)$$

$$\zeta(307) = \zeta(305) \frac{(2^{307} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{307}}}{(2^{307} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{305}}} = \frac{(2^{307} - 4)}{(2^{307} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{305}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{305}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{307}} \quad (226)$$

$$\zeta(309) = \zeta(307) \frac{(2^{309} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{309}}}{(2^{309} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{307}}} = \frac{(2^{309} - 4)}{(2^{309} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{307}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{307}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{309}} \quad (227)$$

$$\zeta(311) = \zeta(309) \frac{(2^{311} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{311}}}{(2^{311} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{309}}} = \frac{(2^{311} - 4)}{(2^{311} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{309}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{309}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{311}} \quad (228)$$

$$\zeta(313) = \zeta(311) \frac{(2^{313} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{313}}}{(2^{313} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{311}}} = \frac{(2^{313} - 4)}{(2^{313} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{311}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{311}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{313}} \quad (229)$$

$$\zeta(315) = \zeta(313) \frac{(2^{315} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{315}}}{(2^{315} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{313}}} = \frac{(2^{315} - 4)}{(2^{315} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{313}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{313}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{315}} \quad (230)$$

$$\zeta(317) = \zeta(315) \frac{(2^{317} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{317}}}{(2^{317} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{315}}} = \frac{(2^{317} - 4)}{(2^{317} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{315}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{315}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{317}} \quad (231)$$

$$\zeta(319) = \zeta(317) \frac{(2^{319} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{319}}}{(2^{319} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{317}}} = \frac{(2^{319} - 4)}{(2^{319} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{317}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{317}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{319}} \quad (232)$$

$$\zeta(321) = \zeta(319) \frac{(2^{321} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{321}}}{(2^{321} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{319}}} = \frac{(2^{321} - 4)}{(2^{321} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{319}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{319}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{321}} \quad (233)$$

$$\zeta(323) = \zeta(321) \frac{(2^{323} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{323}}}{(2^{323} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{321}}} = \frac{(2^{323} - 4)}{(2^{323} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{321}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{321}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{323}} \quad (234)$$

$$\zeta(325) = \zeta(323) \frac{(2^{325} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{325}}}{(2^{325} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{323}}} = \frac{(2^{325} - 4)}{(2^{325} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{323}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{323}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{325}} \quad (235)$$

$$\zeta(327) = \zeta(325) \frac{(2^{327} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{327}}}{(2^{327} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{325}}} = \frac{(2^{327} - 4)}{(2^{327} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{325}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{325}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{327}} \quad (236)$$

$$\zeta(329) = \zeta(327) \frac{(2^{329} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{329}}}{(2^{329} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{327}}} = \frac{(2^{329} - 4)}{(2^{329} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{327}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{327}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{329}} \quad (237)$$

$$\zeta(331) = \zeta(329) \frac{(2^{331} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{331}}}{(2^{331} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{329}}} = \frac{(2^{331} - 4)}{(2^{331} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{329}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{329}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{331}} \quad (238)$$

$$\zeta(333) = \zeta(331) \frac{(2^{333} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{333}}}{(2^{333} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{331}}} = \frac{(2^{333} - 4)}{(2^{333} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{331}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{331}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{333}} \quad (239)$$

$\zeta(335), \zeta(337)$ etc. can also be expressed by these equations.

$\zeta(5), \zeta(7), \dots, \zeta(331), \zeta(333)$ are irrational numbers.

Example 2

That $\zeta(5)$ is an irrational number is already proven at **Example 1** (proof 1).

(Proof 2)

If $\zeta(7)$ is assumed to be rational number. $\zeta(7) = \frac{s}{t}$, s and t are integer.

$$\zeta(7) = \zeta(3) \frac{112 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^7}}{127 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad \frac{112 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^7}}{127 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \frac{o}{p} \quad o, p \text{ are assumed to be integer.}$$

$$\zeta(7) = \zeta(3) \frac{o}{p} \quad \text{it equal} \quad \zeta(3) = \zeta(7) \frac{p}{o} = \frac{sp}{to} \quad \text{But, } \zeta(3) \neq \frac{sp}{to}$$

This is because $\zeta(3)$ is known to be an irrational number.

This contradicts.

$\zeta(7)$ is irrational number.

(Proof end)

Do the same for $\zeta(9), \zeta(11), \zeta(13)$ etc. prove that $\zeta(9), \zeta(11), \zeta(13)$ etc. are an irrational numbers.

and

Detailed description

$$\zeta(3) = \frac{2^3}{2^3 - 1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3} = \frac{8}{7} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3} \quad (240)$$

$$\zeta(7) = \frac{2^7}{2^7 - 1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^7} = \frac{128}{127} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^7} \quad (241)$$

Multiply $\zeta(3)$ and $\zeta(7)$

$$\zeta(7) \frac{8}{7} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3} = \zeta(3) \frac{128}{127} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^7} \quad (242)$$

$$\zeta(7) \frac{8}{7} = \zeta(3) \frac{128 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^7}}{127 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (243)$$

$$\zeta(7) = \zeta(3) \frac{128 \cdot 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^7}}{127 \cdot 8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{112 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^7}}{127 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (244)$$

Do the same for $\zeta(9), \zeta(11), \zeta(13)$ etc.

$$\zeta(3) = \zeta(1) \frac{4 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}}{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^1}} = \zeta(1) \frac{4 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}}{(2^3 - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^1}} \quad (245)$$

$$\zeta(5) = \zeta(3) \frac{28 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^5}}{31 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{28 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^5}}{(2^5 - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (246)$$

$$\zeta(7) = \zeta(3) \frac{112 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^7}}{127 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{112 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^7}}{(2^7 - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (247)$$

$$\zeta(9) = \zeta(3) \frac{2^6 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^9}}{73 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (248)$$

$$\zeta(11) = \zeta(3) \frac{1792 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{11}}}{2047 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{1792 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{11}}}{(2^{11} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (249)$$

$$\zeta(13) = \zeta(3) \frac{7168 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{13}}}{8191 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{7168 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{13}}}{(2^{13} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (250)$$

$$\zeta(15) = \zeta(3) \frac{2^{12} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{15}}}{4681 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (251)$$

$$\zeta(17) = \zeta(3) \frac{114688 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{17}}}{131072 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{114688 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{17}}}{(2^{17} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (252)$$

$$\zeta(19) = \zeta(3) \frac{458752 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{19}}}{524287 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{458752 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{19}}}{(2^{19} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (253)$$

$$\zeta(21) = \zeta(3) \frac{2^{18} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{21}}}{299593 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (254)$$

$$\zeta(23) = \zeta(3) \frac{7340032 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{23}}}{8388607 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{7340032 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{23}}}{(2^{23} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (255)$$

$$\zeta(25) = \zeta(3) \frac{29360128 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{25}}}{33554431 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{29360128 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{25}}}{(2^{25} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (256)$$

$$\zeta(27) = \zeta(3) \frac{2^{24} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{27}}}{19173961 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (257)$$

$$\zeta(29) = \zeta(3) \frac{469762048 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{29}}}{536870911 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{469762048 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{29}}}{(2^{29} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (258)$$

$$\zeta(31) = \zeta(3) \frac{1879048192 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{31}}}{2147483647 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{1879048192 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{31}}}{(2^{31} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (259)$$

$$\zeta(33) = \zeta(3) \frac{2^{30} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{33}}}{1227133513 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (260)$$

$$\zeta(35) = \zeta(3) \frac{30064771072 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{35}}}{34359738367 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{30064771072 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{35}}}{(2^{35} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (261)$$

$$\zeta(37) = \zeta(3) \frac{120259084288 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{37}}}{137438953471 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{120259084288 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{37}}}{(2^{37} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (262)$$

$$\zeta(39) = \zeta(3) \frac{2^{36} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{39}}}{78536544841 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (263)$$

$$\zeta(41) = \zeta(3) \frac{1924145348608 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{41}}}{2199023255551 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{1924145348608 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{41}}}{(2^{41} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (264)$$

$$\zeta(43) = \zeta(3) \frac{7696581394432 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{43}}}{8796093022207 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{7696581394432 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{43}}}{(2^{43} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (265)$$

$$\zeta(45) = \zeta(3) \frac{2^{42} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{45}}}{5026338869833 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (266)$$

$$\zeta(47) = \zeta(3) \frac{123145302310912 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{47}}}{140737488355327 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{123145302310912 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{47}}}{(2^{47} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (267)$$

$$\zeta(49) = \zeta(3) \frac{492581209243648 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{49}}}{562949953421311 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{492581209243648 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{49}}}{(2^{49} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (268)$$

$$\zeta(51) = \zeta(3) \frac{2^{48} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{51}}}{321685687669321 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (269)$$

$$\zeta(53) = \zeta(3) \frac{7881299347898368 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{53}}}{9007199254740991 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{7881299347898368 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{53}}}{(2^{53} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (270)$$

$$\zeta(55) = \zeta(3) \frac{31525197391593472 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{55}}}{36028797018963967 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{31525197391593472 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{55}}}{(2^{55} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (271)$$

$$\zeta(57) = \zeta(3) \frac{2^{54} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{57}}}{20587884010836553 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (272)$$

$$\zeta(59) = \zeta(3) \frac{504403158265495552 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{59}}}{576460752303423487 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{504403158265495552 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{59}}}{(2^{59} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (273)$$

$$\zeta(61) = \zeta(3) \frac{2017612633061982208 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{61}}}{2305843009213693951 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2017612633061982208 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{61}}}{(2^{61} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (274)$$

$$\zeta(63) = \zeta(3) \frac{2^{60} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{63}}}{1317624576693539401 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (275)$$

$$\zeta(65) = \zeta(3) \frac{32281802128991715328 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{65}}}{36893488147419103231 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{32281802128991715328 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{65}}}{(2^{65} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (276)$$

$$\zeta(67) = \zeta(3) \frac{129127208515966861312 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{67}}}{147573952589676412927 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{129127208515966861312 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{67}}}{(2^{67} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (277)$$

$$\zeta(69) = \zeta(3) \frac{2^{66} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{69}}}{84327972908386521673 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (278)$$

$$\zeta(71) = \zeta(3) \frac{2066035336255469780992 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{71}}}{2361183241434822606847 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2066035336255469780992 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{71}}}{(2^{71} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (279)$$

$$\zeta(73) = \zeta(3) \frac{8264141345021879123968 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{73}}}{9444732965739290427391 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{8264141345021879123968 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{73}}}{(2^{73} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (280)$$

$$\zeta(75) = \zeta(3) \frac{2^{72} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{75}}}{5396990266136737387081 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (281)$$

$$\zeta(77) = \zeta(3) \frac{132226261520350065983488 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{77}}}{151115727451828646838271 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{132226261520350065983488 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{77}}}{(2^{77} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (282)$$

$$\zeta(79) = \zeta(3) \frac{528905046081400263933952 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{79}}}{604462909807314587353087 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{528905046081400263933952 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{79}}}{(2^{79} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (283)$$

$$\zeta(81) = \zeta(3) \frac{2^{78} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{81}}}{345407377032751192773193 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (284)$$

$$\zeta(83) = \zeta(3) \frac{8462480737302404222943232 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{83}}}{9671406556917033397649407 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{8462480737302404222943232 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{83}}}{(2^{83} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (285)$$

$$\zeta(85) = \zeta(3) \frac{33849922949209616891772928 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{85}}}{38685626227668133590597631 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{33849922949209616891772928 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{85}}}{(2^{85} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (286)$$

$$\zeta(87) = \zeta(3) \frac{2^{84} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{87}}}{22106072130096076337484361 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (287)$$

$$\zeta(89) = \zeta(3) \frac{541598767187353870268366848 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{89}}}{(2^{89} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (288)$$

$$\zeta(91) = \zeta(3) \frac{2166395068749415481073467392 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{91}}}{(2^{91} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (289)$$

$$\zeta(93) = \zeta(3) \frac{2^{90} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{93}}}{1414788616326148885598999113 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (290)$$

$$\zeta(95) = \zeta(3) \frac{34662321099990647697175478272 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{95}}}{(2^{95} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (291)$$

$$\zeta(97) = \zeta(3) \frac{138649284399962590788701913088 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{97}}}{(2^{97} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (292)$$

$$\zeta(99) = \zeta(3) \frac{2^{96} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{99}}}{90546471444873528678335943241 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (293)$$

$$\zeta(101) = \zeta(3) \frac{2218388550399401452619230609408 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{101}}}{(2^{101} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (294)$$

$$\zeta(103) = \zeta(3) \frac{8873554201597605810476922437632 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{103}}}{(2^{103} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (295)$$

$$\zeta(105) = \zeta(3) \frac{2^{102} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{105}}}{5794974172471905835413500367433 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (296)$$

$$\zeta(107) = \zeta(3) \frac{141976867225561692967630759002112 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{107}}}{(2^{107} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (297)$$

$$\zeta(109) = \zeta(3) \frac{567907468902246771870523036008448 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{109}}}{(2^{109} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (298)$$

$$\zeta(111) = \zeta(3) \frac{2^{108} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{111}}}{370878347038201973466464023515721 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (299)$$

$$\zeta(113) = \zeta(3) \frac{9086519502435948349928368576135168 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{113}}}{(2^{113} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (300)$$

$$\zeta(115) = \zeta(3) \frac{36346078009743793399713474304540672 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{115}}}{(2^{115} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (301)$$

$$\zeta(117) = \zeta(3) \frac{2^{114} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{117}}}{23736214210444926301853697505006153 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (302)$$

$$\zeta(119) = \zeta(3) \frac{581537248155900694395415588872650752 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{119}}}{(2^{119} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (303)$$

$$\zeta(121) = \zeta(3) \frac{2326148992623602777581662355490603008 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{121}}}{(2^{121} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (304)$$

$$\zeta(123) = \zeta(3) \frac{2^{120} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{123}}}{1519117709468475283318636640320393801 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (305)$$

$$\zeta(125) = \zeta(3) \frac{37218383881977644441306597687849648128 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{125}}}{(2^{125} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (306)$$

$$\zeta(127) = \zeta(3) \frac{148873535527910577765226390751398592512 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{127}}}{(2^{127} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (307)$$

$$\zeta(129) = \zeta(3) \frac{2^{126} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{129}}}{97223533405982418132392744980505203273 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (308)$$

$$\zeta(131) = \zeta(3) \frac{2381976568446569244243622252022377480192 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{131}}}{(2^{131} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (309)$$

$$\zeta(133) = \zeta(3) \frac{9527906273786276976974489008089509920768 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{133}}}{(2^{133} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (310)$$

$$\zeta(135) = \zeta(3) \frac{2^{132} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{135}}}{6222306137982874760473135678752333009481 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (311)$$

$$\zeta(137) = \zeta(3) \frac{152446500380580431631591824129432158732288 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{137}}}{(2^{137} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (312)$$

$$\zeta(139) = \zeta(3) \frac{609786001522321726526367296517728634929152 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{139}}}{(2^{139} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (313)$$

$$\zeta(141) = \zeta(3) \frac{2^{138} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{141}}}{398227592830903984670280683440149312606793 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (314)$$

$$\zeta(143) = \zeta(3) \frac{9756576024357147624421876744283658158866432 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{143}}}{(2^{143} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (315)$$

$$\zeta(145) = \zeta(3) \frac{39026304097428590497687506977134632635465728 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{145}}}{(2^{145} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (316)$$

$$\zeta(147) = \zeta(3) \frac{2^{144} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{147}}}{25486565941177855018897963740169556006834761 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (317)$$

$$\zeta(149) = \zeta(3) \frac{624420865558857447963000111634154122167451648 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{149}}}{(2^{149} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (318)$$

$$\zeta(151) = \zeta(3) \frac{2497683462235429791852000446536616488669806592 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{151}}}{(2^{151} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (319)$$

$$\zeta(153) = \zeta(3) \frac{2^{150} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{153}}}{1631140220235382721209469679370851584437424713 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (320)$$

$$\zeta(155) = \zeta(3) \frac{39962935395766876669632007144585863818716905472 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{155}}}{(2^{155} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (321)$$

$$\zeta(157) = \zeta(3) \frac{159851741583067506678528028578343455274867621888 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{157}}}{(2^{157} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (322)$$

$$\zeta(159) = \zeta(3) \frac{2^{156} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{159}}}{104392974095064494157406059479734501403995181641 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (323)$$

$$\zeta(161) = \zeta(3) \frac{2557627865329080106856448457253495284397881950208 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{161}}}{(2^{161} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (324)$$

$$\zeta(163) = \zeta(3) \frac{10230511461316320427425793829013981137591527800832 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{163}}}{(2^{163} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (325)$$

$$\zeta(165) = \zeta(3) \frac{2^{162} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{165}}}{6681150342084127626073987806703008089855691625033 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (326)$$

$$\zeta(167) = \zeta(3) \frac{163688183381061126838812701264223698201464444813312 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{167}}}{(2^{167} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (327)$$

$$\zeta(169) = \zeta(3) \frac{654752733524244507355250805056894792805857779253248 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{169}}}{(2^{169} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (328)$$

$$\zeta(171) = \zeta(3) \frac{2^{168} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{171}}}{427593621893384168068735219628992517750764264002121 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (329)$$

$$\zeta(173) = \zeta(3) \frac{10476043736387912117684012880910316684893724468051968 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{173}}}{(2^{173} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (330)$$

$$\zeta(175) = \zeta(3) \frac{41904174945551648470736051523641266739574897872207872 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{175}}}{(2^{175} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (331)$$

$$\zeta(177) = \zeta(3) \frac{2^{174} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{177}}}{27365991801176586756399054056255521136048912896135753 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (332)$$

$$\zeta(179) = \zeta(3) \frac{670466799128826375531776824378260267833198365955325952 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{179}}}{(2^{179} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (333)$$

$$\zeta(181) = \zeta(3) \frac{2681867196515305502127107297513041071332793463821303808 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{181}}}{(2^{181} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (334)$$

$$\zeta(183) = \zeta(3) \frac{2^{180} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{183}}}{1751423475275301552409539459600353352707130425352688201 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (335)$$

$$\zeta(185) = \zeta(3) \frac{42909875144244888034033716760208657141324695421140860928 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{185}}}{(2^{185} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (336)$$

$$\zeta(187) = \zeta(3) \frac{171639500576979552136134867040834628565298781684563443712 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{187}}}{(2^{187} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (337)$$

$$\zeta(189) = \zeta(3) \frac{2^{186} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{189}}}{112091102417619299354210525414422614573256347222572044873 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (338)$$

$$\zeta(191) = \zeta(3) \frac{2746232009231672834178157872653354057044780506953015099392 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{191}}}{(2^{191} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (339)$$

$$\zeta(193) = \zeta(3) \frac{10984928036926691336712631490613416228179122027812060397568 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{193}}}{(2^{193} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (340)$$

$$\zeta(195) = \zeta(3) \frac{2^{192} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{195}}}{7173830554727635158669473626523047332688406222244610871881 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (341)$$

$$\zeta(197) = \zeta(3) \frac{175758848590827061387402103849814659650865952444992966361088 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{197}}}{(2^{197} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (342)$$

$$\zeta(199) = \zeta(3) \frac{703035394363308245549608415399258638603463809779971865444352 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{199}}}{(2^{199} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (343)$$

$$\zeta(201) = \zeta(3) \frac{2^{198} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{201}}}{459125155502568650154846312097475029292057998223655095800393 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (344)$$

$$\zeta(203) = \zeta(3) \frac{11248566309812931928793734646388138217655420956479549847109632 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{203}}}{(2^{203} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (345)$$

$$\zeta(205) = \zeta(3) \frac{4499426523925172771517493858552552870621683825918199388438528 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{205}}}{(2^{205} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (346)$$

$$\zeta(207) = \zeta(3) \frac{2^{204} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{207}}}{29384009952164393609910163974238401874691711886313926131225161 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (347)$$

$$\zeta(209) = \zeta(3) \frac{719908243828027643442799017368840845929946941214691190215016448 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{209}}}{(2^{209} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (348)$$

$$\zeta(211) = \zeta(3) \frac{2879632975312110573771196069475363383719787764858764760860065792 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{211}}}{(2^{211} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (349)$$

$$\zeta(213) = \zeta(3) \frac{2^{210} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{213}}}{1880576636938521191034250494351257719980269560724091272398410313 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (350)$$

$$\zeta(215) = \zeta(3) \frac{46074127604993769180339137111605814139516604237740236173761052672 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{215}}}{(2^{215} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (351)$$

$$\zeta(217) = \zeta(3) \frac{46074127604993769180339137111605814139516604237740236173761052672 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{217}}}{(2^{217} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (352)$$

$$\zeta(219) = \zeta(3) \frac{2^{216} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{219}}}{120356904764065356226192031638480494078737251886341841433498260041 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (353)$$

$$\zeta(221) = \zeta(3) \frac{2948744166719601227541704775142772104929062671215375115120707371008}{(2^{221} - 1)} \quad (354)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{221}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (355)$$

$$\zeta(223) = \zeta(3) \frac{11794976666878404910166819100571088419716250684861500460482829484032}{(2^{223} - 1)} \quad (356)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{223}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (357)$$

$$\zeta(225) = \zeta(3) \frac{2^{222}}{7702841904900182798476290024862751621039184120725877851743888642633} \quad (358)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{225}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (359)$$

$$\zeta(227) = \zeta(3) \frac{188719626670054478562669105609137414715460010957784007367725271744512}{(2^{227} - 1)} \quad (360)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{227}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (361)$$

$$\zeta(229) = \zeta(3) \frac{754878506680217914250676422436549658861840043831136029470901086978048}{(2^{229} - 1)} \quad (362)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{229}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (363)$$

$$\zeta(231) = \zeta(3) \frac{2^{228}}{492981881913611699102482561591216103746507783726456182511608873128521} \quad (364)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{231}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (365)$$

$$\zeta(233) = \zeta(3) \frac{12078056106883486628010822758984794541789440701298176471534417391648768}{(2^{233} - 1)} \quad (366)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{233}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (367)$$

$$\zeta(235) = \zeta(3) \frac{48312224427533946512043291035939178167157762805192705886137669566595072}{(2^{235} - 1)} \quad (368)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{235}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (369)$$

$$\zeta(237) = \zeta(3) \frac{2^{234}}{31550840442471148742558883941837830639776498158493195680742967880225353} \quad (370)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{237}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (371)$$

$$\zeta(239) = \zeta(3) \frac{772995590840543144192692656575026850674524204883083294178202713065521152}{(2^{239} - 1)} \quad (372)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{239}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (373)$$

$$\zeta(241) = \zeta(3) \frac{3091982363362172576770770626300107402698096819532333176712810852262084608}{(2^{241} - 1)} \quad (374)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{241}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (375)$$

$$\zeta(243) = \zeta(3) \frac{2^{240}}{2019253788318153519523768572277621160945695882143564523567549944334422601} \quad (376)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{243}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (377)$$

$$\zeta(245) = \zeta(3) \frac{49471717813794761228332330020801718443169549112517330827404973636193353728}{(2^{245} - 1)} \quad (378)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{245}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (379)$$

$$\zeta(247) = \zeta(3) \frac{197886871255179044913329320083206873772678196450069323309619894544773414912}{(2^{247} - 1)} \quad (380)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{247}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (381)$$

$$\zeta(249) = \zeta(3) \frac{2^{246}}{129232242452361825249521188625767754300524536457188129508323196437403046473} \quad (382)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{249}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (383)$$

$$\zeta(251) = \zeta(3) \frac{3166189940082864718613269121331309980362851143201109172953918312716374638592}{(2^{251} - 1)} \quad (384)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{251}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (385)$$

$$\zeta(253) = \zeta(3) \frac{12664759760331458874453076485325239921451404572804436691815673250865498554368}{(2^{253} - 1)} \quad (386)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{253}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (387)$$

$$\zeta(255) = \zeta(3) \frac{2^{252}}{8270863516951156815969356072049136275233570333260040288532684571993794974281} \quad (388)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{255}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (389)$$

$$\zeta(257) = \zeta(3) \frac{202636156165303341991249223765203838743222473164870987069050772013847976869888}{(2^{257} - 1)} \quad (390)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{257}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (391)$$

$$\zeta(259) = \zeta(3) \frac{810544624661213367964996895060815354972889892659483948276203088055391907479552}{(2^{259} - 1)} \quad (392)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{259}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (393)$$

$$\zeta(261) = \zeta(3) \frac{2^{258}}{529335265084874036222038788611144721614948501328642578466091812607602878353993} \quad (394)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{261}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (395)$$

$$\zeta(263) = \zeta(3) \frac{12968713994579413887439950320973045679566238282551743172419249408886270519672832}{(2^{263} - 1)} \quad (396)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{263}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (397)$$

$$\zeta(265) = \zeta(3) \frac{51874855978317655549759801283892182718264953130206972689676997635545082078691328}{(2^{265} - 1)} \quad (398)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{265}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (399)$$

$$\zeta(267) = \zeta(3) \frac{2^{264}}{33877456965431938318210482471113262183356704085033125021829876006886584214655561} \quad (400)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{267}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (401)$$

$$\zeta(269) = \zeta(3) \frac{829997695653082488796156820542274923492239250083311563034831962168721313259061248}{(2^{269} - 1)} \quad (402)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{269}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (403)$$

$$\zeta(271) = \zeta(3) \frac{3319990782612329955184627282169099693968957000333246252139327848674885253036244992}{(2^{271} - 1)} \quad (404)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{271}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (405)$$

$$\zeta(273) = \zeta(3) \frac{2^{270}}{2168157245787644052365470878151248779734829061442120001397112064440741389737955913} \quad (406)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{273}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (407)$$

$$\zeta(275) = \zeta(3) \frac{53119852521797279282954036514705595103503312005331940034229245578798164048579919872}{(2^{275} - 1)} \quad (408)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{275}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (409)$$

$$\zeta(277) = \zeta(3) \frac{212479410087189117131816146058822380414013248021327760136916982315192656194319679488}{(2^{277} - 1)} \quad (410)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{277}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (411)$$

$$\zeta(279) = \zeta(3) \frac{2^{276}}{138762063730409219351390136201679921903029059932295680089415172124207448943229178441} \quad (412)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{279}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (413)$$

$$\zeta(281) = \zeta(3) \frac{3399670561395025874109058336941158086624211968341244162190671717043082499109114871808}{(2^{281} - 1)} \quad (414)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{281}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (415)$$

$$\zeta(283) = \zeta(3) \frac{13598682245580103496436233347764632346496847873364976648762686868172329996436459487232}{(2^{283} - 1)} \quad (416)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{283}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (417)$$

$\zeta(285) =$

$$\zeta(3) \frac{2^{282}}{8880772078746190038488968716907515001793859835666923525722571015949276732366667420233} \quad (418)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{285}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (419)$$

$$\zeta(287) = \zeta(3) \frac{217578915929281655942979733564234117543949565973839626380202989890757279942983351795712}{(2^{287} - 1)} \quad (420)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{287}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (421)$$

$$\zeta(289) = \zeta(3) \frac{870315663717126623771918934256936470175798263895358505520811959563029119771933407182848}{(2^{289} - 1)} \quad (422)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{289}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (423)$$

$$\zeta(291) = \zeta(3) \frac{2^{288}}{568369413039756162463293997882080960114807029482683105646244545020753710871466714894921} \quad (424)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{291}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (425)$$

$$\zeta(293) = \zeta(3) \frac{13925050619474025980350702948110983522812772222325736088332991353008465916350934514925568}{(2^{293} - 1)} \quad (426)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{293}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (427)$$

$$\zeta(295) = \zeta(3) \frac{55700202477896103921402811792443934091251088889302944353331965412033863665403738059702272}{(2^{295} - 1)} \quad (428)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{295}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (429)$$

$$\zeta(297) = \zeta(3) \frac{2^{294}}{36375642434544394397650815864453181447347649886891718761359650881328237495773869753274953} \quad (430)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{297}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (431)$$

$$\zeta(299) = \zeta(3) \frac{891203239646337662742444988679102945460017422228847109653311446592541818646459808955236352}{(2^{299} - 1)} \quad (432)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{299}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (433)$$

$$\zeta(301) = \zeta(3) \frac{3564812958585350650969779954716411781840069688915388438613245786370167274585839235820945408}{(2^{301} - 1)} \quad (434)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{301}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (435)$$

$$\zeta(303) = \zeta(3) \frac{2^{300}}{2328041115810841241449652215325003612630249592761070000727017656405007199729527664209597001} \quad (436)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{303}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (437)$$

$$\zeta(305) = \zeta(3) \frac{57037007337365610415516479275462588509441115022646215017811932581922676393373427773135126528}{(2^{305} - 1)} \quad (438)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{305}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (439)$$

$$\zeta(307) = \zeta(3) \frac{228148029349462441662065917101850354037764460090584860071247730327690705573493711092540506112}{(2^{307} - 1)} \quad (440)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{307}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (441)$$

$$\zeta(309)=$$

$$\zeta(3) \frac{2^{306}}{148994631411893839452777741780800231208335973936708480046529130009920460782689770509414208073} \quad (442)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{309}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (443)$$

$$\zeta(311)=$$

$$\zeta(3) \frac{3650368469591399066593054673629605664604231361449357761139963685243051289175899377480648097792}{(2^{311} - 1)} \quad (444)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{311}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (445)$$

$$\zeta(313)=$$

$$\zeta(3) \frac{14601473878365596266372218694518422658416925445797431044559854740972205156703597509922592391168}{(2^{313} - 1)} \quad (446)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{313}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (447)$$

$$\zeta(315)=$$

$$\zeta(3) \frac{2^{312}}{9535656410361205724977775473971214797333502331949342722977864320634909490092145312602509316681} \quad (448)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{315}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (449)$$

$$\zeta(317)=\zeta(3) \times$$

$$\frac{233623582053849540261955499112294762534670807132758896712957675855555282507257560158761478258688}{(2^{317} - 1)} \quad (450)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{317}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (451)$$

$$\zeta(319)=\zeta(3) \times$$

$$\frac{934494328215398161047821996449179050138683228531035586851830703422221130029030240635045913034752}{(2^{319} - 1)} \quad (452)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{319}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (453)$$

$$\zeta(321) = \zeta(3) \times$$

$$\frac{2^{318}}{610282010263117166398577630334157747029344149244757934270583316520634207365897300006560596267593} \quad (454)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{321}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (455)$$

$$\zeta(323) = \zeta(3) \times$$

$$\frac{14951909251446370576765151943186864802218931656496569389629291254755538080464483850160734608556032}{(2^{323} - 1)} \quad (456)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{323}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (457)$$

$$\zeta(325) = \zeta(3) \times$$

$$\frac{59807637005785482307060607772747459208875726625986277558517165019022152321857935400642938434224128}{(2^{325} - 1)} \quad (458)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{325}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (459)$$

$$\zeta(327) = \zeta(3) \times$$

$$\frac{2^{324}}{39058048656839498649508968341386095809878025551664507793317332257320589271417427200419878161125961} \quad (460)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{327}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (461)$$

$$\zeta(329) = \zeta(3) \times$$

$$\frac{956922192092567716912969724363959347342011626015780440936274640304354437149726966410287014947586048}{(2^{329} - 1)} \quad (462)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{329}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (463)$$

$$\zeta(331) = \zeta(3) \times \frac{3827688768370270867651878897455837389368046504063121763745098561217417748598907865641148059790344192}{(2^{331} - 1)} \tag{464}$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{331}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \tag{465}$$

$$\zeta(333) = \zeta(3) \times \frac{2499715114037727913568573973848710131832193635306528498772309264468517713370715340826872202312061513}{2^{330}} \tag{466}$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{333}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \tag{467}$$

$\zeta(335), \zeta(337)$ etc. can also be expressed by these equations

3 Conclusion

$\zeta(3), \zeta(5), \dots, \zeta(331), \zeta(333)$ are irrational numbers.

4 Postscript

$$\zeta(\pi) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^\pi} + \sum_{n=1}^{\infty} \frac{1}{(2n)^\pi} \tag{468}$$

$$\zeta(\pi) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^\pi} + \frac{1}{2^\pi} \sum_{n=1}^{\infty} \frac{1}{n^\pi} \tag{469}$$

$$\zeta(\pi) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{\pi}} + \frac{1}{2^{\pi}} \zeta(\pi) \quad (470)$$

$$\left(1 - \frac{1}{2^{\pi}}\right) \zeta(\pi) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{\pi}} \quad (471)$$

$$\left(\frac{2^{\pi}-1}{2^{\pi}}\right) \zeta(\pi) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{\pi}} \quad (472)$$

$$\zeta(\pi) = \frac{2^{\pi}}{2^{\pi}-1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{\pi}} \quad (473)$$

=1.17624173838258275887215045193805209116973899002165583496050834623040872376815861833572083732
557183113894566008145...

Do the same

$$\zeta(e) = \frac{2^e}{2^e-1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^e} \quad (474)$$

=1.26900960433571711576556986660086110885640446257719048833581592870816524294827307650849451745
076054575828347684218...

I believe that $\zeta(\pi), \zeta(e)$ are irrational numbers.

That is, I believe that the irrational number ζ are irrational numbers.

I also believe that all even value, as well as odd values of ζ , are irrational numbers.

The figures in this paper have been fully verified by WolframAlpha.

References

- [1] B.Riemann.: Uber die Anzahl der Primzahlen unter einer gegebenen Grosse, Mon. Not. Berlin Akad pp.671-680, 1859
- [2] John Derbyshire.: Prime Obsession: Bernhard Riemann and The Greatest Unsolved Problem in Mathematics, Joseph Henry Press, 2003
- [3] S.Kurokawa.: Riemann hypothesis, Japan Hyoron Press, 2009
- [4] Marcus du Sautoy.: The Music of The Primes, Zahar Press, 2007
- [5] T. takami.: Zeta odd-numbers, vixra:1909.0276