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Abstract: We evaluate bitstring semantics and its follow-on by partition. Its ordered set of exhaustive predicates is *not* bivalent but a probabilistic vector space. Its calculus of relations is *not* tautologous. Hence its broader framework of question-answer semantics (QAS) is *not* tautologous. The conjecture of "generalizing the Aristotelian square within one common gathering" is denied. What is affirmed is the Meth8 corrected, modern, revised square of opposition is a square, to mean the following conjectures are probabilistic vector spaces: collapsed number line of opposition; non-standard quadrilateral of oppositions; and colored square of oppositions. Bitstring semantics and the extended QAS form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, \neg ; + Or, \lor , \cup , \sqcup ; - Not Or; & And, \land , \cap , \neg , \cdot , \otimes ; \land Not And; > Imply, greater than, \rightarrow , \Rightarrow , \mapsto , \succ , \neg , \Rightarrow ; < Not Imply, less than, \in , \prec , \subset , \nvDash , \nvDash , \notin , \leq ; = Equivalent, \equiv , :=, \Leftrightarrow , \leftrightarrow , \triangleq , \approx , \simeq ; @ Not Equivalent, \neq , \oplus ; % possibility, for one or some, \exists , \diamond , M; # necessity, for every or all, \forall , \Box , L; (z=z) T as tautology, \top , ordinal 3; (z@z) **F** as contradiction, Ø, Null, \bot , zero; (%z>#z) N as non-contingency, Δ , ordinal 1; (%z<#z) C as contingency, ∇ , ordinal 2; ~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B). Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Schang, F. (2019). End of the square? sa-logic.org/sajl-v4-i2/11-Schang-SAJL.pdf

1 Introduction: oppositions

In the third section, we will introduce a special semantics provided to account for the meaning of oppositional relations through opposite-forming operators: a *bitstring* semantics, where the basicality of opposition stems from a common analysis of logical space in terms of *partition*. By doing so, we will complete the preceding proposal by generalizing the Aristotelian square within one common gathering.

1 Oppositions with a square

[T]he kinds of logical opposition are depicted by functional expressions *ct* (for contrariety), *cd* (for contradictoriness), *sct* (for subcontrariety), *sb* (for subalternation), and *sp* (for superalternation).

3 Oppositions with another square

3.1 Bi[t]string semantics

The following semantics is a special application of a broader semantic framework: *Question-Answer Semantics* [QAS], where ... results from an ordered set of exhaustive predicates.

Num.	QAS	QAS	M8	M8	QA bitstring vs	Note	
	bitstring	value	script	result	M8 truth table		
3.1.0	$\beta(\perp)$	0000	p@p	FFFF	ok		
3.1.1	$\beta(p \land q)$	1000	p&q	FFFT	Read script right to left		
3.1.2	$\beta(\neg(p \rightarrow q))$	0100	~(p>q)=(p=p)	FTFF	Read script left to right	2=14	
3.1.3	$\beta(\neg(p \leftarrow q))$	0010	~(p <q)=(p=p)< td=""><td>TFTT</td><td>Negate script right to left</td><td>3= 8</td></q)=(p=p)<>	TFTT	Negate script right to left	3= 8	
3.1.4	$\beta(\neg(p \lor q))$	0001	~(p+q)=(p=p)	TFFF	Read script right to left		
3.1.5	β(p)	1100	p=(p=p)	FTFT	undecipherable		
3.1.6	$\beta(\neg(p \leftrightarrow q))$	0110	~(p=q)=(p=p)	FTTF	ok		
3.1.7	β(¬p)	0011	~p=(p=p)	TFTF	undecipherable		
3.1.8	$\beta(p \rightarrow q)$	1001	p>q	TFTT	undecipherable	8= 3	
3.1.9	β(¬q)	0101	~q=(p=p)	TTFF	undecipherable		
3.1.10	β(q)	1010	q=(p=p)	FFTT	undecipherable		
3.1.11	$\beta(p \lor q)$	1110	p+q	FTTT	Read script fight to left		
3.1.12	$\beta(\neg(p \land q))$	0111	~(p&q)=(p=p)	TTTF	Read right to left		
3.1.13	$\beta(p \leftrightarrow q))$	1011	p=q	TFFT	unreadable		
3.1.14	$\beta(p \leftarrow q))$	1101	p <q< td=""><td>FTFF</td><td>Reverse script right to left</td><td>14=2</td></q<>	FTFF	Reverse script right to left	14=2	
3.1.15	$\beta(\top)$	1111	p=p	TTTT	ok		

Remark 3.1: Of the 16 claimed bitstring values, three are bivalent mappings as represented in Meth8(M8) script and result: Eqs. 3.1.0, 3.1.6, and 3.1.15. These are for respectively contradiction (none), not equivalent, and tautology (all). Four bitstrings using the imply or not imply connectives are equivalents and hence are *not* unique values of the 16 as claimed.

Calculus of logical relations.

 $cd(\beta(x)) = 1 \Leftrightarrow \beta(x) = 0$, i.e. $cd(\beta(x)) = 1 \Rightarrow \beta(x) = 0$ and $cd(\beta(x)) = 0 \Rightarrow \beta(x) = 1$ (3.2.1)

LET p, q, r, s: cd (for contradictority), β , x, s.

Remark 3.2.2: Eq. 3.2.2 as rendered is *not* tautologous. Hence that logical relation is refuted, to color the entire claimed calculus.

3.3 Iterated oppositions

Remark 3.3.0.1: We take the edges of the corrected square of opposition from: James, C. (2019). Refutation of hexagons of opposition for statistical modalities. vixra.org/abs/1901.0192. We set the logical oppositions to those of the text in italics.

Source type Def. Meth8 corrected script Valid as

Corner	А	#(s= p)		NFNF	NFNF	F N FN	FNFN	
	Е	#(s=~p)		FNFN	FNFN	NFNF	NFNF	
	Ι	%(s= p)		TCTC	TCTC	CTCT	CTCT	
	0	%(s=~p)		CTCT	CTCT	TCTC	TCTC	
Contrarity	AE	#(s= p)\#(s=~p)	$A \setminus E$	TTTT	TTTT	TTTT	TTTT	ct
Superalternity	AI	#(s=p)>%(s=p)	A > I	TTTT	TTTT	TTTT	TTTT	sp
Contradictority	AO	#(s= p)\%(s=~p)	$A \backslash O$	TTTT	TTTT	TTTT	TTTT	cd
Contradictority	EI	$\#(s=\sim p) \otimes (s=p)$	$E \setminus I$	TTTT	TTTT	TTTT	TTTT	cd
Subalternity	EO	#(s=~p)>%(s=~p)	E > O	TTTT	TTTT	TTTT	TTTT	sb
Subcontrarity	ΙΟ	%(s=p)+%(s=~p)	I + O	TTTT	TTTT	TTTT	TTTT	sci

Remark 3.3.0.2: These formulas for the edges of the corrected, modern, revised square of opposition are tautologous, but not found anywhere in the instant text or its references. The point is that the square of opposition need not be a rectangle, reduced to one dimension, or abandoned as such.

Nevertheless, another parallel way to characterize subalternation is to define it by means of iterated functions. Here is the central point of the present paper: logical relations ... can be reduced after all to an iteration of basic oppositions. To begin with such a process, any subaltern of an arbitrary formula x is to be defined as the contradictory of a contrary of x:

$$sb(\beta(x)) = cd(ct(\beta(x)))$$
(3.3.1.1)

Conversely to (3.3.1.1), any superaltern of x is to be defined as the contrary of the contradictory of x:

$$sp(\beta(x)) = ct(cd(\beta(x)))$$
(3.3.2.1)

A subcontrary of any x is the contradictory of the superaltern of x or, by substituting the latter relation for its iterative definition, the contradictory of the contradictory of the contradictory of x:

$$\operatorname{sct}(\beta(x)) = \operatorname{cd}(\operatorname{sp}(\beta(x))) = \operatorname{cd}(\operatorname{ct}(\operatorname{cd}(\beta(x))))$$
(3.3.3.1)

Remark 3.3.0.3: As expected, Eqs. 3.3.1.1-..3.1 are tautologous. This means the iterated oppositions are obvious and not new per se or a recent advance.

From the sections above, we refute *bitstring* semantics, its follow-on by *partition*, its broader framework of question-answer semantics (QAS), and "generalizing the Aristotelian square within one common gathering". What is affirmed is the Meth8 corrected, modern, revised square of opposition, to mean the following conjectures are probabilistic vector spaces: collapsed number line of opposition; non-standard quadrilateral of oppositions; and colored square of oppositions.