Refutation of Cook-Reckhow definition

Abstract: We evaluate the Cook-Reckhow definition as represented for conjectures of a polynomial equation and integer linear inequalities with both as *not* tautologous. Hence we do not evaluate the conjecture of a relational form on which the paper subsequently relies. These results form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ , · , ⊗ ; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊕;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Krajíček, J. (2019). The Cook-Reckhow definition. arxiv.org/pdf/1909.03691.pdf

The Cook-Reckhow paper .. introduced the notions of *propositional proof* systems and *polynomial simulations* among them, described several classes of logical propositional calculi and compared them with regards to their efficiency, and introduced the *pigeonhole principle tautology* PHP_n that is the prime example of a tautology hard to prove in weaker systems ever since … It was the Cook-Reckhow 1979 paper .. which defined the area of research we now call proof complexity.

Remark 0: We prove the stronger and weaker pigeon hole principles as theorems at: vixra.org/pdf/1902.0414v1.pdf .

[Remark 1.2:] As rendered in Eq. 1.2, the pigeon hole principle is tautologous and a trivial theorem. It is the stronger form of the theorem. The weaker form, to which the paper directs, in this context substitutes the antecedent clause of "some object implies the necessity of space" with "some object implies the *possibility* of space", for result of the same table.

1 Definition of proof systems

For example, a clause

p ∨ ¬q ∨ r (1.1.1)

(p+~q)+r ; TT**F**T TTTT TT**F**T TTTT (1.1.2)

together with the requirement that we look for 0−1 solution can be represented by polynomial equations

$$(1 - p)q(1 - r) = 0, \tag{1.2.1}$$

((((%s>#s)-p)&q)&((%s>#s)-r))=(s@s) ;

TTNT TTTT TTNT TTTT (1.2.2)

$$p^2 - p = 0,\ q^2 - q = 0,\ r^2 - r = 0 \tag{1.3.0}$$

Remark 1.3.0: Eq. 1.3.0 factors into the form of p(p - 1) = 0 to mean the solutions are 0 and 1, hence rendering p, q, r as always tautologous (and trivial for the instant example).

the first equation states that the clause contains a true literal while the last three equations force 0−1 solutions over any integral domain. In this case we can use a calculus deriving elements of the ideal generated by the equations representing similarly all clauses of the formula, trying to derive 1 as a member of the ideal and thus demonstrating the unsolvability of the equations and hence the unsatisfiability of the formula.

Another approach is to represent the clause as integer linear inequalities

$$p + (1 - q) + r \geq 1\ , \tag{1.4.1}$$

~((%s>#s)>((p+((%s>#s)-q))+r))=(s=s) ;

N**F**N**F** **FFFF** N**F**N**F** **FFFF** (1.4.2)

$$1 \geq p, q, r\ \geq 0 \tag{1.5.1}$$

~(~((p+(q+r))>(s@s))>(%s>#s))=(s=s) ;

FCCC CCCC **F**CCC CCCC (1.5.2)

and use some integer linear program[m]ing algorithm to derive the unsolvability of the system of inequalities representing the whole CNF [conjunctive normal form] formula.

Eqs. 1.2, 2.2, 4.2, and 5.2 as rendered are *no*t tautologous. This refutes the Cook-Reckhow definition on its face, without resorting to evaluation of "[p]roof systems… also defined equivalently in a relational form", on which the paper subsequently relies.