$\zeta(5), \zeta(7).....\zeta(197), \zeta(199)$ *etc.*

are irrational number

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Abstract

I wondered why I would do this and I traced the course. However, I did not understand much.

Numerically, they match perfectly.

Also, this is considered to indicate that $\zeta(5)$, $\zeta(7)$, $\zeta(9)$, $\zeta(11)$, $\zeta(13).....\zeta(197)$, $\zeta(199)$ are irrational numbers. Because, $\zeta(3)$ is irrational number.

It can also be said that it is expressed by an expression using π^2.

key words
irrational number, $\zeta(5)$, $\zeta(7)$, $\zeta(197)$, $\zeta(199)$

1 Introduction

I use my previous paper[5],

$$\zeta(2m-1) = \frac{2^{2m-1}}{2^{2m-1}-1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{2m-1}} \tag{1}$$

m is a positive integer greater than or equal to 2

and

from Eq.(2) etc.

If $\zeta(3)$ is rational number. $\zeta(3) = \frac{n}{m}$, n and m are integer.
$\zeta(5) = \frac{no}{mp}$, o and p are integer.

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if $\zeta(5)$ is rational number. $\zeta(5) = \frac{s}{t}$, s and t are integer.
It equal $\zeta(3) = \frac{p}{o}\zeta(5) = \frac{ps}{ot}$

but, $\zeta(3) \neq \frac{ps}{ot}$
This is because $\zeta(3)$ is known to be an irrational number.
This contradicts.

$\zeta(5)$ is irrational number.

The same applies to $\zeta(7)$ etc.

2 Discussion

$$\zeta(5) = \zeta(3)\frac{28\sum_{n=1}^{\infty}\frac{1}{(2n-1)^7}}{31\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}} \tag{2}$$

$$\zeta(7) = \zeta(3)\frac{112\sum_{n=1}^{\infty}\frac{1}{(2n-1)^7}}{127\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}} \tag{3}$$

$$\zeta(9) = \zeta(3)\frac{64\sum_{n=1}^{\infty}\frac{1}{(2n-1)^9}}{73\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}} \tag{4}$$

$$\zeta(11) = \zeta(3)\frac{1792\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{11}}}{2047\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}} \tag{5}$$

$$\zeta(13) = \zeta(3)\frac{7168\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{13}}}{8191\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}} \tag{6}$$

$$\zeta(15) = \zeta(3)\frac{4096\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{15}}}{4681\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}} \tag{7}$$

$$\zeta(17) = \zeta(3)\frac{114688\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{17}}}{131071\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}} \tag{8}$$

$$\zeta(19) = \zeta(3)\frac{458752\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{19}}}{524287\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}} \tag{9}$$

$$\zeta(21) = \zeta(3)\frac{262144\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{21}}}{299593\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}} \tag{10}$$

$$\zeta(23) = \zeta(3)\frac{7340032\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{23}}}{8388607\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}} \tag{11}$$

$$\zeta(25) = \zeta(3)\frac{29360128\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{25}}}{33554431\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{12}$$

$$\zeta(27) = \zeta(3)\frac{16777216\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{27}}}{19173961\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{13}$$

$$\zeta(29) = \zeta(3)\frac{469762048\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{29}}}{536870911\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{14}$$

$$\zeta(31) = \zeta(3)\frac{1879048192\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{31}}}{2147483647\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{15}$$

$$\zeta(33) = \zeta(3)\frac{1073741824\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{33}}}{1227133513\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{16}$$

$$\zeta(35) = \zeta(3)\frac{30064771072\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{35}}}{34359738367\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{17}$$

$$\zeta(37) = \zeta(3)\frac{120259084288\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{37}}}{137438953471\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{18}$$

$$\zeta(39) = \zeta(3)\frac{68719476736\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{39}}}{78536544841\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{19}$$

$$\zeta(41) = \zeta(3)\frac{1924145348608\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{41}}}{2199023255551\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{20}$$

$$\zeta(43) = \zeta(3)\frac{7696581394432\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{43}}}{8796093022207\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{21}$$

$$\zeta(45) = \zeta(3)\frac{4398046511104\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{45}}}{5026338869833\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{22}$$

$$\zeta(45) = \zeta(3)\frac{4398046511104\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{45}}}{5026338869833\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{23}$$

$$\zeta(47) = \zeta(3)\frac{123145302310912\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{47}}}{140737488355327\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{24}$$

$$\zeta(49) = \zeta(3)\frac{492581209243648\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{49}}}{562949953421311\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{25}$$

$$\zeta(51) = \zeta(3)\frac{281474976710656\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{51}}}{321685687669321\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{26}$$

$$\zeta(55) = \zeta(3)\frac{31525197391593472\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{55}}}{36028797018963967\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{27}$$

$$\zeta(57) = \zeta(3)\frac{18014398509481984\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{57}}}{20587884010836553\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}} \tag{28}$$

$$\zeta(59) = \zeta(3)\frac{504403158265495552\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{59}}}{576460752303423487\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}} \tag{29}$$

$$\zeta(61) = \zeta(3)\frac{2017612633061982208\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{61}}}{2305843009213693951\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}} \tag{30}$$

$$\zeta(63) = \zeta(3)\frac{11529215046068469760\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{63}}}{13176245766935394011\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}} \tag{31}$$

$$\zeta(65) = \zeta(3)\frac{32281802128991715328\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{65}}}{36893488147419103231\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}} \tag{32}$$

$$\zeta(67) = \zeta(3)\frac{129127208515966861312\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{67}}}{147573952589676412927\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}} \tag{33}$$

$$\zeta(69) = \zeta(3)\frac{73786976294838206464\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{69}}}{84327972908386521673\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}} \tag{34}$$

$$\zeta(71) = \zeta(3)\frac{2066035336255469780992\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{71}}}{2361183241434822606847\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}} \tag{35}$$

$$\zeta(73) = \zeta(3)\frac{826414134502187912396 8\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{73}}}{9444732965739290427391\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}} \tag{36}$$

$$\zeta(75) = \zeta(3)\frac{4722366482869645213696\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{75}}}{5396990266136737387081\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}} \tag{37}$$

$$\zeta(77) = \zeta(3)\frac{1322262615203500659834 88\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{77}}}{1511157274518286468382 71\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}} \tag{38}$$

$$\zeta(79) = \zeta(3)\frac{5289050460814002639339 52\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{79}}}{6044629098073145873530 87\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}} \tag{39}$$

$$\zeta(79) = \zeta(3)\frac{5289050460814002639339 52\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{79}}}{6044629098073145873530 87\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}} \tag{40}$$

$$\zeta(81) = \zeta(3)\frac{3022314549036572936765 44\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{81}}}{3454073770327511927731 93\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}} \tag{41}$$

$$\zeta(83) = \zeta(3)\frac{8462480737302404222943232\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{83}}}{9671406556917033397649407\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}} \tag{42}$$

$$\zeta(85) = \zeta(3)\frac{3384992294920961689177292 8\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{85}}}{3868562622766813359059763 1\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}} \tag{43}$$

$$\zeta(87)=\zeta(3)\frac{193428131133834066795298816\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{87}}}{221060721300960763374843 61\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{44}$$

$$\zeta(89)=\zeta(3)\frac{541598767187353870268366848\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{89}}}{618970019642690137449562111\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{45}$$

$$\zeta(91)=\zeta(3)\frac{216639506874941548107346739 2\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{91}}}{247588007857076054979824844 7\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{46}$$

$$\zeta(93)=\zeta(3)\frac{123794003928538027489912422 4\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{93}}}{141478861632614888559899911 3\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{47}$$

$$\zeta(95)=\zeta(3)\frac{346623210999906476971754782 72\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{95}}}{396140812571321687967719751 67\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{48}$$

$$\zeta(97)=\zeta(3)\frac{138649284399962590788701913088\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{97}}}{158456325028528675187087900671\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{49}$$

$$\zeta(99)=\zeta(3)\frac{792281625142643375935439503 36\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{99}}}{905464714448735286783359432 41\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{50}$$

$$\zeta(101)=\zeta(3)\frac{221838855039940145261923060940 8\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{101}}}{253530120045645880299340641075 1\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{51}$$

$$\zeta(103)=\zeta(3)\frac{887355420159760581047692243763 2\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{103}}}{101412048018258352119736256430 07\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{52}$$

$$\zeta(105)=\zeta(3)\frac{507060240091291760598681282150 4\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{105}}}{579497417247190583541350036743 3\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{53}$$

$$\zeta(107)=\zeta(3)\frac{141976867225561692967630759002 112\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{107}}}{162259276829213363391578010288 127\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{54}$$

$$\zeta(109)=\zeta(3)\frac{567907468902246771870523036008 448\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{109}}}{649037107316853453566312041152 511\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{55}$$

$$\zeta(111)=\zeta(3)\frac{324518553658426726783156020576 256\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{111}}}{370878347038201973466464023515 721\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{56}$$

$$\zeta(113)=\zeta(3)\frac{908651950243594834992836857613 5168\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{113}}}{103845937170696552570609926584 40191\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{57}$$

$$\zeta(115)=\zeta(3)\frac{363460780097437933997134743045 40672\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{115}}}{415383748682786210282439706337 60767\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{58}$$

$$\zeta(117)=\zeta(3)\frac{207691874341393105141219853168803 84\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{117}}}{237362142104449263018536975050061 53\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{59}$$

$$\zeta(119)=\zeta(3)\frac{581537248155900694395415588872650752\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{119}}}{664613997892457936451903530140172287\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{60}$$

$$\zeta(121)=\zeta(3)\frac{232614899262360277758166235549060300 8\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{121}}}{265845599156983174580761412056068915 1\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{61}$$

$$\zeta(123)=\zeta(3)\frac{132922799578491587290380706028034457 6\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{123}}}{151911770946847528331863664032039380 1\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{62}$$

$$\zeta(125)=\zeta(3)\frac{372183838819776444413065976878496481 28\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{125}}}{425352958651173079329218259289710264 31\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{63}$$

$$\zeta(127)=\zeta(3)\frac{148873535527910577765226390751398592512\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{127}}}{170141183460469231731687303715884105727\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{64}$$

$$\zeta(129)=\zeta(3)\frac{850705917302346158658436518579420528 64\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{129}}}{972235334059824181323927449805052032 73\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{65}$$

$$\zeta(131)=\zeta(3)\frac{238197656844656924424362225202237748019 2\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{131}}}{272225893536750770770699685945414569164 7\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{66}$$

$$\zeta(133)=\zeta(3)\frac{952790627378627697697448900808950992076 8\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{133}}}{108890357414700308308279874378165827665 91\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{67}$$

$$\zeta(135)=\zeta(3)\frac{544451787073501541541399371890829138329 6\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{135}}}{622230613798287476047313567875233300948 1\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{68}$$

$$\zeta(137)=\zeta(3)\frac{152446500380580431631591824129432158732288\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{137}}}{174224571863520493293247799005065324265471\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{69}$$

$$\zeta(139)=\zeta(3)\frac{609786001522321726526367296517728634929152\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{139}}}{696898287454081973172991196020261297061887\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{70}$$

$$\zeta(141)=\zeta(3)\frac{348449143727040986586495598010130648530944\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{141}}}{398227592830903984670280683440149312606793\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{71}$$

$$\zeta(143)=\zeta(3)\frac{975657602435714762442187674428365815886643 2\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{143}}}{111503725992653115707678591363241807529902 07\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{72}$$

$$\zeta(145)=\zeta(3)\frac{390263040974285904976875069771346326354657 28\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{145}}}{446014903970612462830714365452967230119608 31\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{73}$$

$$\zeta(147)=\zeta(3)\frac{223007451985306231415357182726483615059804 16\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{147}}}{254865659411778550188979637401695560068347 61\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{74}$$

$$\zeta(149)=\zeta(3)\frac{624420865558857447963000111634154122167451648\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{149}}}{713623846352979940529142984724747568191373311\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{75}$$

$$\zeta(151)=\zeta(3)\frac{24976834622354297918520004465366164886698065 92\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{151}}}{28544953854119197621165719388989902727654932 47\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{76}$$

$$\zeta(153)=\zeta(3)\frac{14272476927059598810582859694494951363827466 24\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{153}}}{16311402202353827212094696793708515844374247 13\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{77}$$

$$\zeta(155)=\zeta(3)\frac{39962935395766876669632007144585863818716905472\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{155}}}{45671926166590716193865151022383844364247891967\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{78}$$

$$\zeta(157)=\zeta(3)\frac{15985174158306750667852802857834345527486762188 8\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{157}}}{18268770466636286477546060408953537745699156787 1\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{79}$$

$$\zeta(159)=\zeta(3)\frac{91343852333181432387730302044767688728495783936\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{159}}}{10439297409506449415740605947973450140399518164 1\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{80}$$

$$\zeta(161)=\zeta(3)\frac{25576278653290801068564484572534952843978819502 08\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{161}}}{29230032746618058364073696654325660393118650859 51\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{81}$$

$$\zeta(163)=\zeta(3)\frac{10230511461316320427425793829013981137591527800832\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{163}}}{11692013098647223345629478661730264157247460343807\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{82}$$

$$\zeta(165)=\zeta(3)\frac{58460065493236116728147393308651320786237301719 04\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{165}}}{66811503420841276260739878067030080898556916250 33\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{83}$$

$$\zeta(167)=\zeta(3)\frac{16368818338106112683881270126422369820146444448133 12\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{167}}}{18707220957835557353007165858768422651595936550092 7\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{84}$$

$$\zeta(169)=\zeta(3)\frac{65475273352424450735525080505689479280585777925324 8\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{169}}}{74828883831342229412028663435073690606383746200371 1\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{85}$$

$$\zeta(171)=\zeta(3)\frac{37414441915671114706014331717536845303191873100185 6\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{171}}}{42759362189338416806873521962899251775076426400212 1\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{86}$$

$$\zeta(173)=\zeta(3)\frac{10476043736387912117684012880910316684893724468051968\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{173}}}{11972621413014756705924586149611790497021399392059391\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{87}$$

$$\zeta(175)=\zeta(3)\frac{41904174945551648470736051523641266739574897872207872\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{175}}}{47890485652059026823698344598447161988085597568237567\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{88}$$

$$\zeta(177)=\zeta(3)\frac{23945242826029513411849172299223580994042798784118784\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{177}}}{27365991801176586756399054056255521136048912896135753\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{89}$$

$$\zeta(179)=\zeta(3)\frac{67046679912882637553177682437826026783319836595532595 2\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{179}}}{76624777043294442917917351357515459180936956109180108 7\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{90}$$

$$\zeta(181)=\zeta(3)\frac{26818671965153055021271072975130410713327934638213038\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{181}}}{30649910817317777167166940543006183672374782443672043\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{91}$$

$$\zeta(183)=\zeta(3)\frac{15324955408658888583583470271503091836187391221836021\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{183}}}{17514234752753015524095394596003533527071304253526882\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{92}$$

$$\zeta(185)=\zeta(3)\frac{42909875144244880340337167602086571413246954211408609\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{185}}}{49039857307708443467467104868809893875799651909875269631\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{93}$$

$$\zeta(187)=\zeta(3)\frac{17163950057697955213613486704083462856529878168456344371\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{187}}}{19615942923083377386986841947523957550319860763950107852\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{94}$$

$$\zeta(189)=\zeta(3)\frac{98079714615416886934934209737619787751599303819750539264\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{189}}}{11209110241761929935421052541442261457325634722257204487\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{95}$$

$$\zeta(191)=\zeta(3)\frac{27462320092316728341781578726533540570447805069530150993\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{191}}}{31385508676933403819178947116038332080511777222320172564\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{96}$$

$$\zeta(193)=\zeta(3)\frac{10984928036926691336712631490613416228179122027812060397568\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{193}}}{12554203470773361527671578846415332832204710888928069025791\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{97}$$

$$\zeta(195)=\zeta(3)\frac{62771017353866807638357894232076664161023554444640345128\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{195}}}{71738305547276351586694736265230473326884062222446108711\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{98}$$

$$\zeta(197)=\zeta(3)\frac{17575884859082706138740210384981465965086595244499296636108\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{197}}}{20086725553237378444274526154264532531527537422284910441267\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{99}$$

$$\zeta(199)=\zeta(3)\frac{70303539436330824554960841539925863860346380977997186544352\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{199}}}{80346902212949513777098104617058130126110149689139641765068\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{100}$$

$\zeta(201),\zeta(203)$ etc. can also be expressed by these equations

Since $\zeta(3)$ is an irrational number, $\zeta(5)$ and below are also irrational numbers.

$$\frac{\zeta(5)}{\zeta(3)}=\frac{28\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{5}}}{31\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{101}$$

$$\frac{\zeta(7)}{\zeta(5)}=\frac{124\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{7}}}{127\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{5}}} \tag{102}$$

$$\frac{\zeta(9)}{\zeta(7)} = \frac{508\sum_{n=1}^{\infty}\frac{1}{(2n-1)^9}}{511\sum_{n=1}^{\infty}\frac{1}{(2n-1)^7}} \tag{103}$$

$$\frac{\zeta(11)}{\zeta(9)} = \frac{2044\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{11}}}{2047\sum_{n=1}^{\infty}\frac{1}{(2n-1)^9}} \tag{104}$$

$$\frac{\zeta(13)}{\zeta(11)} = \frac{8188\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{13}}}{8191\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{11}}} \tag{105}$$

$$\frac{\zeta(15)}{\zeta(13)} = \frac{32764\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{15}}}{32767\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{13}}} \tag{106}$$

$$\frac{\zeta(17)}{\zeta(15)} = \frac{131068\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{17}}}{131071\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{15}}} \tag{107}$$

$$\frac{\zeta(19)}{\zeta(17)} = \frac{524284\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{19}}}{524287\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{17}}} \tag{108}$$

$$\frac{\zeta(21)}{\zeta(19)} = \frac{2097148\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{21}}}{2097151\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{19}}} \tag{109}$$

$$\frac{\zeta(23)}{\zeta(21)} = \frac{8388604\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{23}}}{8388607\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{21}}} \tag{110}$$

$$\frac{\zeta(25)}{\zeta(23)} = \frac{33554428\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{25}}}{33554431\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{23}}} \tag{111}$$

$$\frac{\zeta(27)}{\zeta(25)} = \frac{134217724\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{27}}}{134217727\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{25}}} \tag{112}$$

$$\frac{\zeta(29)}{\zeta(27)} = \frac{536870908\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{29}}}{536870911\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{27}}} \tag{113}$$

$$\frac{\zeta(31)}{\zeta(29)} = \frac{2147483644\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{31}}}{2147483647\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{29}}} \tag{114}$$

$$\frac{\zeta(33)}{\zeta(31)} = \frac{8589934588\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{33}}}{8589934591\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{31}}} \tag{115}$$

The following expressions can also be made.

$$\zeta(3) = \frac{-1}{3}\int_0^1 \frac{\log^3(1-t^2)}{t^3}dt \tag{116}$$

$$\zeta(5) = \frac{-1}{30}\int_0^1 \frac{\log^5(1-t^4)}{t^5}dt \tag{117}$$

$$\zeta(7) = \frac{-1}{840}\int_0^1 \frac{\log^7(1-t^6)}{t^7}dt \tag{118}$$

$$\zeta(9) = \frac{-1}{45360}\int_0^1 \frac{\log^9(1-t^8)}{t^9}dt \tag{119}$$

$$\zeta(11) = \frac{-1}{3991680}\int_0^1 \frac{\log^{11}(1-t^{10})}{t^{11}}dt \tag{120}$$

$$\zeta(13) = \frac{-1}{518918400}\int_0^1 \frac{\log^{13}(1-t^{12})}{t^{13}}dt \tag{121}$$

$$\zeta(15) = \frac{-1}{93405312000}\int_0^1 \frac{\log^{15}(1-t^{14})}{t^{15}}dt \tag{122}$$

$$\zeta(17) = \frac{-1}{22230464256000}\int_0^1 \frac{\log^{17}(1-t^{16})}{t^{17}}dt \tag{123}$$

$$\zeta(19) = \frac{-1}{6758061133824000}\int_0^1 \frac{\log^{19}(1-t^{18})}{t^{19}}dt \tag{124}$$

$$\zeta(21) = \frac{-1}{2554547108585472000}\int_0^1 \frac{\log^{21}(1-t^{20})}{t^{21}}dt \tag{125}$$

$$\zeta(23) = \frac{-1}{1175091669949317120000}\int_0^1 \frac{\log^{23}(1-t^{22})}{t^{23}}dt \tag{126}$$

$$\zeta(25) = \frac{-1}{646300418472124416000000}\int_0^1 \frac{\log^{25}(1-t^{24})}{t^{25}}dt \tag{127}$$

$$\zeta(27) = \frac{-1}{418802671169936621568000000}\int_0^1 \frac{\log^{27}(1-t^{26})}{t^{27}}dt \tag{128}$$

$$\zeta(29) = \frac{-1}{3157772140621322126622720000000}\int_0^1 \frac{\log^{29}(1-t^{28})}{t^{29}}dt \tag{129}$$

$$\zeta(31) = \frac{-1}{27409462180593076059085209600000}\int_0^1 \frac{\log^{31}(1-t^{30})}{t^{31}}dt \tag{130}$$

$$\zeta(33) = \frac{-1}{271353675587871452984943575040000000}\int_0^1 \frac{\log^{33}(1-t^{32})}{t^{33}}dt \tag{131}$$

It can also be expressed by these equations.
Because I use π^2, these are irrational numbers.

If π is rational number. $\pi = \frac{n}{m}$, n and m are integer.
$\pi^2 = \frac{n^2}{m^2}$,
if π^2 is rational number.

It equal $\pi = \frac{n}{m}$
but, $\pi \neq \frac{n}{m}$.
This is because π is known to be an irrational number.
This contradicts.

π^2 is irrational number.

$$\zeta(3) = \frac{15\zeta(4)}{\pi^2\zeta(2)} \tag{132}$$

$$\zeta(5) = \frac{21\zeta(6)}{2\pi^2\zeta(4)} \tag{133}$$

$$\zeta(7) = \frac{10\zeta(8)}{\pi^2\zeta(6)} \tag{134}$$

$$\zeta(9) = \frac{99\zeta(10)}{10\pi^2\zeta(8)} \tag{135}$$

$$\zeta(11) = \frac{6825\zeta(12)}{691\pi^2\zeta(10)} \tag{136}$$

$$\zeta(13) = \frac{691\zeta(14)}{70\pi^2\zeta(12)} \tag{137}$$

$$\zeta(15) = \frac{35700\zeta(16)}{3617\pi^2\zeta(14)} \tag{138}$$

$$\zeta(17) = \frac{4329549\zeta(18)}{438670\pi^2\zeta(16)} \tag{139}$$

$$\zeta(19) = \frac{12063425\zeta(20)}{1222277\pi^2\zeta(18)} \tag{140}$$

$$\zeta(21) = \frac{84337113\zeta(22)}{8545130\pi^2\zeta(20)} \tag{141}$$

$$\zeta(23) = \frac{2332820490\zeta(24)}{236364091\pi^2\zeta(22)} \tag{142}$$

$$\zeta(25) = \frac{1181820455\zeta(26)}{119743442\pi^2\zeta(24)} \tag{143}$$

$$\zeta(27) = \frac{33485398245\zeta(28)}{3392780147\pi^2\zeta(26)} \tag{144}$$

$$\zeta(29) = \frac{1700698904286669\zeta(30)}{17231682552010\pi^2\zeta(28)} \tag{145}$$

$$\zeta(31) = \frac{5858772067683400\zeta(32)}{593617720173709\pi^2\zeta(30)} \tag{146}$$

$$\zeta(33) = \frac{2544075943601 61\zeta(34)}{25776878583670\pi^2\zeta(32)} \tag{147}$$

$$\zeta(35) = \frac{25972131994721654 3325\zeta(36)}{26315271553053477373\pi^2\zeta(34)} \tag{148}$$

$$\zeta(37) = \frac{2631527155305347 7373\zeta(38)}{2666294461595818690\pi^2\zeta(36)} \tag{149}$$

$$\zeta(39) = \frac{25767831475279590625 50\zeta(40)}{26108271849644912205 1\pi^2\zeta(38)} \tag{150}$$

$$\zeta(41) = \frac{16503038636160549004843 71\zeta(42)}{16721074083098778829601 0\pi^2\zeta(40)} \tag{151}$$

$$\zeta(43) = \frac{17481122905057814230946 5\zeta(44)}{17712080641373379058651\pi^2\zeta(42)} \tag{152}$$

$$\zeta(45) = \frac{11773473032044333251414729\zeta(46)}{11929022231878243265559 22\pi^2\zeta(44)} \tag{153}$$

$$\zeta(47) = \frac{55362592178146926995460340020\zeta(48)}{56094033689978176862491275 47\pi^2\zeta(46)} \tag{154}$$

$$\zeta(49) = \frac{43192405941283196184118282 1119\zeta(50)}{43763056943311440901983013210\pi^2\zeta(48)} \tag{155}$$

$$\zeta(51) = \frac{66906981288331914455916337503 75\zeta(52)}{67790945380695304083285960849 1\pi^2\zeta(50)} \tag{156}$$

$$\zeta(53) = \frac{28769860938245444368363986457441 23\zeta(54)}{29149963634884862421418123812691 0\pi^2\zeta(52)} \tag{157}$$

$\zeta(55), \zeta(57)$ etc. can also be expressed by these equations

3 Conclusion

$\zeta(5), \zeta(7)$.....$\zeta(197), \zeta(199)$ are irrational numbers.

References

[1] B.Riemann.: Uber die Anzahl der Primzahlen unter einer gegebenen Grosse, Mon. Not. Berlin Akad pp.671-680, 1859

[2] John Derbyshire.: Prime Obsession: Bernhard Riemann and The Greatest Unsolved Problem in Mathematics, Joseph Henry Press, 2003

[3] S.Kurokawa.: Riemann hypothesis, Japan Hyoron Press, 2009

[4] Marcus du Sautoy.: The Music of The Primes, Zahar Press, 2007

[5] T. takami.: zeta odd-numbers, vixra.org/pdf/1810.0423v6.pdf