Einstein vs Bell? Bell's inequality refuted, Bell's error corrected.

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Abstract Bell's inequality is widely regarded as a profound proof that nature is nonlocal, not Einstein-local. Against this, and supporting Einstein, we refute Bell's inequality and correct his error. We thus advance the principle of true-local-realism (TLR); the union of *true-locality* (no beables move superluminally, after Einstein) and *true-realism* (some beables change interactively, after Bohr). Importantly, for STEM teachers: we believe our commonsense results require no knowledge of quantum mechanics. Let's see.

Keywords Bell's first error (BE1), the same-instance-rule (SIR), true-local-realism (TLR)

1. Introduction

1.1. (i) Bell (1964a) points us to Bell (1964b)—and the EPRB experiment therein—for *proof* that *any* latent-beable account of quantum mechanics (QM), and thus of nature, *must* have a grossly nonlocal character. (ii) Alas, Bell's proof relies on Bell's inequality (BI), Bell 1964b:(15), which is false. (iii) For BI is based on BE1 (Bell's first error): which, as we (and experiments) show, limits BI to experiments less-correlated than EPRB. (iv) We thus advance the principle of true-local-realism (TLR): the union of *true-locality* (no beables move superluminally, after Einstein) and *true-realism* (some beables change interactively, after Bohr); *true* here separating our terms from false or naive variants, *beables* being Bell's handy term for existents. (v) Importantly: our results require no knowledge of QM.

2. Bell's inequality (BI) refuted

2.1. Studying Bell (1964b), let us use: (i) E for expectations (reserving P for probabilities). (ii) a, b, c for Bell's unit-vectors $\vec{a}, \vec{b}, \vec{c}$ in spacetime. (iii) Bell-(1), short for Bell 1964b:(1); etc. (iv) Bell-(14a)-(14c) to identify the relations between Bell-(14) and Bell-(15). (v) nb: all integrals are over $\mathbf{\Lambda}$, the space of $\boldsymbol{\lambda}$.

2.2. Then, from Bell-(1), Bell-(2), and the first line on p.198, we have the expectations:

$$-1 \leq E(a,b) \leq 1,\ -1 \leq E(a,c) \leq 1,\ -1 \leq E(b,c) \leq 1. \tag{1}$$

$$\therefore E(a,b)[1+E(a,c)] \leq 1+E(a,c);\ \text{for, if } V \leq 1, \text{ and } 0 \leq W, \text{ then } VW \leq W. \tag{2}$$

So, with all of our analysis bound by TLR: from Bell's start-point LHS-Bell-(14a) and our (2), (3)

$$E(a,b) - E(a,c) \leq 1 - E(a,b)E(a,c).\ \text{Similarly, } E(a,c) - E(a,b) \leq 1 - E(a,b)E(a,c). \tag{4}$$

So, irrefutably from (4), we have our inequality: $|E(a,b) - E(a,c)| \leq 1 - E(a,b)E(a,c)$: ■ (5)

To be compared with Bell-(15); aka Bell's inequality (BI): $|E(a,b) - E(a,c)| \leq 1 + E(b,c)$. (6)

Thus, given (5)'s certainty: BI falls short, and is refuted, when $E(b,c) < -E(a,b)E(a,c)$. ■ (7)

Example: let $E(b,c) = -1/2 = E(a,b) = -E(a,c)$; then RHS-(6) $= 1/2$ vs RHS-(5) $= 5/4$. ■ (8)

2.3. (i) Via TLR and QM, the expectations in (8) are validly constrained by LHS Bell-(2) = RHS Bell-(3). (ii) Bell's second error (BE2), the claim below Bell-(3): relies on, and thus falls with, BE1.

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3. Bell's first error (BE1) defined and corrected

3.1. (i) From (8) we see the extent to which BI falls short. (ii) Further, by observation, we see that Bell-(14b) leads to BI. (iii) Yet, via (3)-(7), Bell-(14a) does not. (iv) So BE1 occurs in Bell's move from Bell-(14a) to Bell-(14b): *using* [sic] Bell-(1); as he remarks in the line below Bell-(14b). (v) nb: our *sic* indicates that Bell's *usage* is wrong: not Bell-(1). (vi) For Bell-(1) is fully consistent with TLR: as we now show in (11)-(12); en route to correct Bell's *misuse* of Bell-(1) and derive (5) anew.

3.2. (i) Under TLR, and consistent with Bell (1964b), we allow pristine particle-pairs (ie, twinned-particles, immediately ex-source; pre-interaction) to be pairwise anticorrelated via the pairwise conservation of total angular momentum. (ii) In other words: if one particle carries the latent-beable $\boldsymbol{\lambda}$, then its twin carries $\boldsymbol{\lambda}^- \equiv -\boldsymbol{\lambda}$ (for clarity). (iii) So paired results are spacelike separated.

3.3. (i) Now, under TLR and respecting what we call SIR (the *same-instance-rule*) in the line before Bell-(1)—and incorporating the *vital assumption re locality* in the line below Bell-(1)—let $\{.\}$ denote such an instance. (ii) Then $\{.\}$ reminds us of this crucial fact under EPRB: correlations and expectations arise only from the union of paired-results obtained in *the same irreducible instance*. (iii) And such instances $\{.\}$ are defined by the commuting results $(.)$ that they bind:

$$\text{ie, commuting results, } \{A(a,\boldsymbol{\lambda})A(b,\boldsymbol{\lambda})\} = \{A(b,\boldsymbol{\lambda})A(a,\boldsymbol{\lambda})\}\text{: but bound, and thus} \tag{9}$$

$$\text{not travelling results; } \{A(a,\boldsymbol{\lambda})A(b,\boldsymbol{\lambda})\}\{A(a,\boldsymbol{\lambda})A(c,\boldsymbol{\lambda})\} \neq \{A(a,\boldsymbol{\lambda})A(a,\boldsymbol{\lambda})\}\{A(b,\boldsymbol{\lambda})A(c,\boldsymbol{\lambda})\}. \tag{10}$$

3.4. Thus, from Bell-(1)-(2) and Bell-(13)-(14); all now combined under SIR:

$$E(a,b) \equiv \textstyle\int d\lambda\, \rho(\boldsymbol{\lambda})\{A(a,\boldsymbol{\lambda}) = \pm 1.\, B(b,\boldsymbol{\lambda}) = \mp 1 = A(b,\boldsymbol{\lambda}^-) = -A(b,\boldsymbol{\lambda})\} \tag{11}$$

$$= -\textstyle\int d\lambda\, \rho(\boldsymbol{\lambda})\{A(a,\boldsymbol{\lambda})A(b,\boldsymbol{\lambda})\}. \ \text{[nb: } = -\cos(a,b) \text{ under QM and TLR.]} \tag{12}$$

3.5. (i) That is—*in the same instance*—the correlations between spacelike-separated events are TLR-explicable via a *common function* A of *local variables*. (ii) From (11): if one result is a function A of local beables $a, \boldsymbol{\lambda}$ (thus truly-local), then the paired result is likewise truly-local via the same function A of $b, \boldsymbol{\lambda}^-$. (iii) So, from Bell's start-point—ie, Bell-(14a) as in (3)—but now using (12) under SIR:

$$E(a,b) - E(a,c) = -\textstyle\int d\lambda\, \rho(\boldsymbol{\lambda})\{A(a,\boldsymbol{\lambda})A(b,\boldsymbol{\lambda})\} + \int d\lambda\, \rho(\boldsymbol{\lambda})\{A(a,\boldsymbol{\lambda})A(c,\boldsymbol{\lambda})\} \tag{13}$$

$$= \textstyle\int d\lambda\, \rho(\boldsymbol{\lambda})\{A(a,\boldsymbol{\lambda})A(b,\boldsymbol{\lambda})\}[\{A(a,\boldsymbol{\lambda})A(b,\boldsymbol{\lambda})\}\{A(a,\boldsymbol{\lambda})A(c,\boldsymbol{\lambda})\} - 1]. \tag{14}$$

$$\text{Then} : \text{taking absolute values; and since } \{A(a,\boldsymbol{\lambda})A(b,\boldsymbol{\lambda})\} \leq 1; \tag{15}$$

$$|E(a,b) - E(a,c)| \leq \textstyle\int d\lambda \rho(\boldsymbol{\lambda})[1 - \{A(a,\boldsymbol{\lambda})A(b,\boldsymbol{\lambda})\}\{A(a,\boldsymbol{\lambda})A(c,\boldsymbol{\lambda})\}] \tag{16}$$

$$\leq 1 - E(a,b)E(a,c) = \text{irrefutable (5).} \quad \blacksquare \tag{17}$$

3.6. However, from the same start-point, Bell-(14a), and via the note below Bell-(14b), here is BE1:

$$\text{From (13): } E(a,b) - E(a,c) = -\textstyle\int d\lambda\, \rho(\boldsymbol{\lambda})[\{A(a,\boldsymbol{\lambda})A(b,\boldsymbol{\lambda})\} - \{A(a,\boldsymbol{\lambda})A(c,\boldsymbol{\lambda})\}] \tag{18}$$

$$= \textstyle\int d\lambda\, \rho(\boldsymbol{\lambda})\{A(a,\boldsymbol{\lambda})A(b,\boldsymbol{\lambda})\}[A(b,\boldsymbol{\lambda})A(c,\boldsymbol{\lambda}) - 1] = \text{invalid Bell-(14b): } \textit{using } [A(b,\boldsymbol{\lambda})]^2 = 1. \tag{19}$$

3.7. (i) Now true (14) and false (19) are from the same start-point. (ii) And $[A(b,\boldsymbol{\lambda})]^2 = 1$ only holds under $\{A(b,\boldsymbol{\lambda})A(b,\boldsymbol{\lambda})\}$: which is not an instance here. (iii) Further, BE1 reduces the product of $\{A(a,\boldsymbol{\lambda})A(b,\boldsymbol{\lambda})\}$ and $\{A(a,\boldsymbol{\lambda})A(c,\boldsymbol{\lambda})\}$ to $A(b,\boldsymbol{\lambda})A(c,\boldsymbol{\lambda})$: which is also not an instance here. (iv) Hence the consequences of BE1: it is impossible for BI to hold generally under EPRB.

3.8. And Bell proves our point: (i) For Bell uses $A(b,\boldsymbol{\lambda})A(c,\boldsymbol{\lambda})$ from (19) to derive $E(b,c)$, as formatted in (6). (ii) BI is consequently false; see (7). (iii) Here's BI's further difficulty under two simplifications: let a, b, c be coplanar, with $0 < (a,b) = (b,c) = (a,c)/2 < \pi/2$. (iv) Then, under EPRB and via the note in (12): BI is false under TLR and over the entire range of these reasonable conditions.

3.9. The completions of irrefutable (5), and (6)-corrected—which is BI-corrected—follow via exhaustion over relevant expectations:

$$0 \leq |E(a,b) - E(a,c)| + E(a,b)E(a,c) \leq 1. -1 \leq |E(a,b) - E(a,c)| - E(b,c) \leq \tfrac{3}{2}. \quad \blacksquare \tag{20}$$

4. Conclusions

4.1. In short: (i) BI, refuted at (7)-(8), is corrected at RHS (20). (ii) BE1 arises from the fact that instances cannot be broken to form new instances; see ¶3.3. (iii) Comparing (18)-(19) with (13)-(17), BE1 is the false step from Bell's start-point, Bell-(14a), to false Bell-(14b).

4.2. We close with an account of the Bellian difficulties that we've resolved.

> Pg.5: 'I cannot say that action at a distance (AAD) is required in physics. But I can say that you cannot get way with no AAD. You cannot separate off what happens in one place and what happens in another. Somehow they have to be described and explained jointly.' Pg.6: 'The Einstein program fails, that's too bad for Einstein, but should we worry about that? So what? ... it might be that we have to learn to accept not so much AAD, but the inadequacy of no AAD.' Pg.7: 'And that is the dilemma. We are led by analysing this situation to admit that in somehow distant things are connected, or at least not disconnected. ... So the connections have to be very subtle, and I have told you all that I know about them.' Pg.9: 'It's my feeling that all this AAD and no AAD business will go the same way [as the ether]. But someone will come up with the answer, with a reasonable way of looking at these things. If we are lucky it will be to some big new development like the theory of relativity. Maybe someone will just point out that we were being rather silly, and it won't lead to a big new development. But anyway, I believe the questions will be resolved.' Pg.10: 'I think somebody will find a way of saying that [relativity and QM] are compatible. But I haven't seen it yet. For me it's very hard to put them together, but I think somebody will put them together, and we'll just see that my imagination was too limited.' Pg.12: 'I don't know any conception of locality that works with QM. So I think we're stuck with nonlocality.' Pg.13: '... I step back from asserting that there is AAD, and I say only that you cannot get away with locality. You cannot explain things by events in their neighbourhood. But I am careful not to assert that there is AAD,' after Bell (1990).

5. References

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