First Integrals and Lagrangian Analysis of Nonlinear Differential Equations

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Abstract

We propose in this paper first integrals and Lagrangian analysis of nonlinear differential equations.

Theory

In a recent paper [1], the first differential equations

$$I(x,\dot{x}) = a\dot{x}^{l} g^{m}(x) e^{\alpha\int h(x)dx} + b\dot{x}^{q} f^{n}(x) e^{\beta\int \varphi(x)dx} \tag{1}$$

has been used in the context of Riccati transformation of equations. In the present theory, let

$$I(x,\dot{x}) = k \tag{2}$$

be a first integral of a differential equation where k is a time-independent constant. In this respect, using (2) the Lagrangian may be written as [2]

$$L(x,\dot{x},t) = \frac{a}{l-1} g^{m}(x) e^{\alpha\int h(x)dx}\dot{x}^{l} + \frac{b}{q-1} f^{n}(x) e^{\beta\int \varphi(x)dx}\dot{x}^{q} \tag{3}$$

Therefore one may secure the Euler-Lagrange equation

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = 0 \tag{4}$$

in the form

$$\frac{\ddot{x}}{\dot{x}^{2}}\left[al\, g^{m}(x) e^{\alpha\int h(x)dx}\dot{x}^{l} + bqf^{n}(x) e^{\beta\int \varphi(x)dx}\dot{x}^{q}\right] + ag^{m}(x) e^{\alpha\int h(x)dx}\left[m\frac{g'(x)}{g(x)} + \alpha h(x)\right]\dot{x}^{l} + bf^{n}(x) e^{\beta\int \varphi(x)dx}\left[n\frac{f'(x)}{f(x)} + \beta\varphi(x)\right]\dot{x}^{q} = 0 \tag{5}$$

Now, putting $l = q,$ one may ensure the interesting equation

$$\ddot{x}+\left\{ag^{m}(x)e^{\alpha\int h(x)dx}\left[m\frac{g'(x)}{g(x)}+\alpha h(x)\right]+bf^{n}(x)e^{\beta\int\varphi(x)dx}\left[n\frac{f'(x)}{f(x)}+\beta\varphi(x)\right]\right\}\times \tag{6}$$

$$\frac{k^{\frac{2}{l}}}{l\left[ag^{m}(x)e^{\alpha\int h(x)dx}+bf^{n}(x)e^{\beta\int\varphi(x)dx}\right]^{\frac{2}{l}+1}}=0$$

where the quadratic term $\dot{x}^2$ is eliminated using (2).

References

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