A Proof of Goldbach's strong conjecture

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Abstract : The proof includes a series, and its aspects are treated in a special way. The concept of Unique Path of primes is explained and its effects are shown. In the midway of the proof, it is postponed for a while and a deviation from the course is taken to introduce a probably new axiom. Afterwards the proof restarts
again and using the axiom and other results the conjecture is proved.

Please note : Every symbol 'p' with or without any suffix or (*) denotes some prime number. a|b means a divides b and a®b means a doesn't divide b. n is a natural number. The word 'prime' will hereafter mean prime number and 'even' mean even positive integer. The sign '∃' means 'there exist'.

First stage : There is at least one prime p ($3\leq p<n$) for every $2n>6$ such that p®2n.
Proof : For any even $2n>6$, at least one of the evens 2n-2 and 2n+2 is not an integral power of 2. Now n-1 or n+1 is divisible by at least one prime p* ($3\leq p^*<n$).
So p*|2(n-1)⇒p*®2(n-1)+2⇒p*®2n or, alternatively p*|2(n+1)⇒p*®2(n+1)-2⇒p*®2n
Suitably using any of the above two alternative results we can prove the claim.

Second stage : Concider a prime p_1 ($3\leq p_1<n$) such that p_1®2n. Now let $2n-p_1$ is divisible by a prime p_2, where $p_2<n$.
So there can be a series

∃p_2, such that $p_2|2n-p_1$, where $p_2<n$

∃p_3, such that $p_3|2n-p_2$, where $p_3<n$

… … …

… … … ∃p_k, such that $p_k|2n-p_{k-1}$, where $p_k<n$

The primes p_2, p_3,… are taken in such a manner as far as possible that each one is different from all the other primes previously appeared in the series.

It can easily be proved that no such prime divides 2n (since any $p_k\neq p_{k-1}$).

The operation of getting $p_2,p_3,\ldots$ must end at some p_k, otherwise there will be infinite number of different primes<n. We henceforth shall call p_1 as 'starting prime'. We further call $p_2,p_3,\ldots, p_k$ (all being different, where p_k is the last of them) as different outputs or simply as outputs.

Let the course of the proof be postponed for a while to discuss a topic. It is a common sense that we can omit anything from a written or mentioned expression. For that purpose we simply need to wipe out or erase the purported object from the expression. But when the question comes to the dealing with its logical aspect, we need to introduce an axiom. Namely •••

Axiom of omission : We can omit or erase anything from an expression or a system of expressions if the rest of it bears some logical meaning.

It is a different question what the effect of this axiom should be in the context of other mathematical topics. It is just a logical interpretation of certain human discretion taken in common sense perspectives.

Return to the proof •••••

Observation (1) : For a particular p_1 we can choose arbitrarily particular $p_2, p_3, \ldots, p_k$ (p_k being the last available different output for the series where p_1 is the starting prime) and in this way they constitute an Unique Path of successively particular selections from the prime factors of various $2n\text{-}p_t$'s, p_t 's starting from p_1. p_1 is also included in the Unique Path and put in its place. Such Unique Path is always strictly ordered.

Observation (2) : p_k being the last different output ($<n$) from $2n\text{-}p_{k-1}$ in the series, proceeding similarly beyond it we get p_{k+1} from $2n\text{-}p_k$ where, $p_{k+1}|2n\text{-}p_k$, with the exception that in this case p_{k+1} not necessarily $<n$.

Now $p_{k+1}<n$ implies p_{k+1} is a recycled prime i.e, p_{k+1} is any of $p_1,p_2,p_3,\ldots, p_k$ (since p_k is the last available different output for the series of k-1 unique steps, p_1 being the starting prime).

We define 'a list' as a successive mentioning of items (ignore the commas) and 'a choice' as a selection of mentioned item/s. Evidently a list is an expression. We claim that,

we choose only one item from a particular list⇒ we omit the rest of the items from the list.

Proof : If not so,

since a list is an expression and we have to choose from the list, if we retain at least another item other than the intended one, there will be at least two items for a choice, where none can be excluded. But we have to choose only one item as per requirement which is also supported by axiom of choice.

Hence our claim is true.

Let p_{k+1} is a recycled prime.

Since p_{k+1} is a singularly mentioned prime (as evident from its identity), a recycled prime taken as p_{k+1} is also a singularly mentioned identity which implies we have to choose only one output from the list of different outputs.

Now for an arbitrarily particular p_1 we obtain an unique path of successively particular selection of primes, which must contain an unique starting prime and an unique last output w.r.t itself.

Omission of these two doesn't make any logical meaning for the expression containing any of the rest, when viewed in accordance with the above unique path, that view we are bound to take in this case [application of Axiom of omission in this context implies the ordered list here is devoid of a starting prime and a last output and thus the whole residual concept drawn from Observation (1) & (2) and related other concepts, goes undefined].

This means after every possible omission there remains at least two primes to choose as p_{k+1}, none of which is omittable.

So we can say that we can't omit, all primes other than that intended one for recycling purpose, from the list, which implies we can't choose only one recycled prime as p_{k+1}.

[Comment : Here it may be counterargued that there is no harm if we select the recycled prime from the list $p_1,p_2,\ldots, p_k$, ignoring its affiliation with the series and related arguments upto this point, and it doesn't make any difference if we change their proposed order to whatever else we want. In that case we say the recycled one must be chosen from the list only, which couldn't be derived otherwise so as to serve for the 'recycled choice' purpose [because ignoring of the said affiliation means no relevance to the word 'recycled' and giving provision for any possibility at which the list might be created, other than the one described above, which is impossible because the conceptual idea of p_{k+1} pertains a precondition of proceeding of the series in the same way as it does before this step, p_{k+1} behaving the same way as $p_2, p_3,\ldots, p_k$, with the exception that p_{k+1} not necessarily$<n$ {see Observation (2)}, hence refuting the very premise of the selection]. Further we can argue that changing of order contradicts the uniqe path of successively particular selections of primes, which is one of the unavoidable foundations behind the construction of the particular list (though change of the order may only create another unique path with the same elements of the original list, if it can be so constituted at least, posing no threat to the basic reasoning as followed in the case of original list and unique path; however it is convinient not to change the original order for better understanding.)]

Summing up the above discussions we conclude that p_{k+1} can't be recycled, that implies p_{k+1} isn't$<n$, and since $p_k<n$,

we are bound to accept the conclusion that $p_{k+1}>n \Rightarrow p_{k+1}=2n-p_k$.

[$2n-p_k$ can't have a factor that is grater than n and smaller than itself, and $p_{k+1}\neq n$ for obvious reasons.]

Therefore, $2n=p_k+p_{k+1}$

Contrary to what we have assumed at the beginning of the Second stage, if p_2 isn't $<n$, then as $p_1<n$, p_2 becomes$>n$. This implies $2n=p_1+p_2$
[Reasons are similar as above]

Finally over the question whether the integers 6 & 4 comply to Goldbach's strong conjecture, we write 6=3+3 and 4=2+2

Therefore Goldbach's strong conjecture holds for every $2n\geq 4$.