

## Refutation of Skolem witness and counter-example functions

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**Abstract:** We evaluate the Skolem witness function and counter-example function. The functions are *not* tautologous and *not* counter-exemplary. This refutes the conjecture of a witness function as opposite a counter-example function and forms a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, **N** as truthity (non-contingency), and **C** as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET  $\sim$  Not,  $\neg$ ; + Or,  $\vee$ ,  $\cup$ ,  $\sqcup$ ; - Not Or; & And,  $\wedge$ ,  $\cap$ ,  $\square$ ,  $\cdot$ ,  $\otimes$ ; \ Not And;  
 $>$  Imply, greater than,  $\rightarrow$ ,  $\Rightarrow$ ,  $\mapsto$ ,  $>$ ,  $\supset$ ,  $\rightarrow$ ;  $<$  Not Imply, less than,  $\in$ ,  $<$ ,  $\subset$ ,  $\neq$ ,  $\neq$ ,  $\ll$ ,  $\lesssim$ ;  
 $=$  Equivalent,  $\equiv$ ,  $:=$ ,  $\Leftrightarrow$ ,  $\leftrightarrow$ ,  $\triangleq$ ,  $\approx$ ,  $\simeq$ ; @ Not Equivalent,  $\neq$ ,  $\oplus$ ;  
 $\%$  possibility, for one or some,  $\exists$ ,  $\diamond$ , **M**; # necessity, for every or all,  $\forall$ ,  $\square$ , **L**;  
 $(z=z)$  **T** as tautology, **T**, ordinal 3;  $(z@z)$  **F** as contradiction,  $\emptyset$ , Null,  $\perp$ , zero;  
 $(\%z\>\#z)$  **N** as non-contingency,  $\Delta$ , ordinal 1;  $(\%z\<\#z)$  **C** as contingency,  $\nabla$ , ordinal 2;  
 $\sim(y < x)$  ( $x \leq y$ ), ( $x \subseteq y$ ), ( $x \sqsubseteq y$ );  $(A=B)$  ( $A\sim B$ ).  
 Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Komara, J. (2019). Efficient elimination of Skolem functions in first-order logic without equality. [arxiv.org/pdf/1909.01697.pdf](https://arxiv.org/pdf/1909.01697.pdf) komara@fmph.uniba.sk

### 1 Introduction

The Skolem functions  $f$  and  $g$  are called a witnessing function for

$$\forall x \exists y A \tag{1.1.1}$$

LET  $p, q, r: x, y, A.$

$$(\#p\&\%q)\&r ; \quad \text{FFFF FFFN FFFF FFFN} \tag{1.1.2}$$

and [respectively] a counterexample function to

$$\exists x \forall y A \tag{1.2.1}$$

$$(\%p\&\#q)\&r ; \quad \text{FFFF FFFN FFFF FFFN} \tag{1.2.2}$$

Eqs. 1.1.2 and 1.2.2 are *not* tautologous and *not* counter-exemplary. This refutes the conjecture of a witness function as opposite a counter-example function.